education

Department:
Education
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MATHEMATICS

EXAMINATION GUIDELINES

GRADE 12
2009

This guideline consists of 21 pages.

MATHEMATICS

EXAMINATION GUIDELINE

- This guideline document must be read in conjunction with the National Curriculum Statement (NCS), the Subject Assessment Guidelines (SAGs): Mathematics – January 2008 and the Learning Programme Guidelines (LPGs) for Mathematics – January 2008.

- The Core Assessment Standards will be tested in Paper 1 and 2 and the Optional Assessment Standards will be tested in Paper 3 in the 2009 to 2012 final Grade 12 examination.

MATHEMATICS EXAMINATION PAPERS FOR GRADES 10 AND 11 AND 12

1. GRADE 10 AND 11

The end of year examination papers for Grades 10 and 11 will be internally set, internally marked and internally moderated, unless otherwise instructed by provincial departments of education.

Table 1: Suggested number of examination papers and times in Grades 10 and 11

GRADE 10		GRADE 11
TERM 2	Paper 1: 2 hours (100 marks)	Paper 1: 2 hours (100 marks) Paper 2: 2 hours (100 marks) Paper 3: discretion of school
TERM 4	Paper 1: 2 hours (100 marks) Paper 2: 2 hours (100 marks) Paper 3: minimum of 1 hour if offered (50 marks at least)	Paper 1: 3 hours (150 marks) Paper 2: 3 hours (150 marks) Paper 3: 2 hours if offered (100 marks)

2. GRADE 12

The Grade 12 final end-of-year examination is nationally set, marked and moderated.

External assessment

The external Mathematics examination covers the Assessment Standards of Grades 11 and 12. The examination will consist of two compulsory papers (Paper 1 and Paper 2) and one optional paper (Paper 3). The structure, time allocation and marks of the Grade 12 national Mathematics examinations are provided in Table 2.

> The nature of mathematics is such that knowledge learned in earlier grades is assumed in later grades and is incorporated in assessment tasks and examinations.

Table 2: Summary of the National Senior Certificate external Grade 12 assessment

EXAM PAPER	LEARNING OUTCOMES	TIME ALLOCATION	TOTAL MARKS
Paper 1	LO1 and LO2 (see Appendix 1)	3 hours	150 marks
Paper 2	LO3 and LO4 (see Appendix 1)	3 hours	150 marks
Paper 3	LO3 and LO4 (see Appendix 2)	2 hours	100 marks

The National Senior Certificate Mathematics examinations will be structured in line with the weightings indicated in Table 3. The level of complexity of the mathematical questions in the examinations will be in line with the taxonomical categories given in Table 4. Table 5 gives a detailed explanation of taxonomical categories of mathematical demand.

Table 3: Suggested guide for mark distribution for Mathematics NCS papers from Grade 10 - 12

Mark distribution for Mathematics NCS papers: Grades 10 - 12				
PAPER 1 : Bookwork max 6 marks				
LO	**Description**	**Gr 10**	**Gr 11**	**Gr. 12**
1	Patterns & Sequences	± 20	± 35	± 30
1	Annuities & Finance	± 20	± 25	± 15
2	Functions & Graphs	± 35	± 50	± 35
2	Algebra and Equations (and inequalities)	± 25	± 25	± 20
2	Calculus			± 35
2	Linear Programming		± 15	± 15
	TOTAL	100	150	150
PAPER 2 : Bookwork 0 marks				
LO	**Description**	**Gr 10**	**Gr 11**	**Gr 12**
3	Mensuration	± 10	± 10	
3	Coordinate Geometry	± 20	± 35	± 40
3	Transformations	± 15	± 20	± 25
3	Trigonometry	± 25	± 50	± 60
4	Data Handling	± 20	± 35	± 25
	TOTAL	100	150	150
PAPER 3 : Bookwork max 15 marks				
LO	**Description**	**Gr 10**	**Gr 11**	**Gr 12**
1	Sequences defined recursively			± 5
3	Geometry	± 15	± 50	± 40
4	Descriptive Stats & Interpretation	± 10	± 20	± 20
4	Probability	± 25	± 30	± 20
4	Bivariate Data			± 15
	TOTAL	50	100	100

Note:

- Modelling as a process should be included in all papers, thus contextual questions can be asked in any topic.
- Questions will not necessarily be compartmentalised in sections as this table indicates. Various topics can be integrated in the same question.
- Sequences defined recursively will occasionally be used to replace some of the marks of bivariate data questions.

3. TAXONOMICAL DIFFERENTATION OF QUESTIONS

Table 4: Taxonomical differentiation of questions on Grade 12 question papers

TAXONOMICAL CATEGORIES	APPROXIMATE PROPORTION OF THE PAPER		
	%	150 mark paper	150 mark paper
Knowledge	± 25	25 – 35	25 – 35
Performing routine procedures	± 30	30 – 40	30 - 40
Performing complex procedures	± 30	30 – 40	30 - 40
Problem Solving	± 15	15 – 25	15 – 25

The above taxonomical categories are based on the 1999 TIMSS Mathematics survey. The four categories of cognitive levels and their related skills are provided below.

Table 5: Table describing the four cognitive levels and their related skills

COGNITIVE LEVELS	EXPLANATION OF SKILLS TO BE DEMONSTRATED
KNOWLEDGE (25%)	• Algorithms • Estimation; appropriate rounding of numbers • Theorems • Straight recall • Identifying from data sheet • Simple mathematical facts • Know and use of appropriate vocabulary • Knowledge and use of formulae
ROUTINE PROCEDURES (30%)	• Problems are not necessarily unfamiliar and can involve the integration of different LOs • Perform well-known procedures • Simple applications and calculations which must have many steps and may require interpretation from given information • Identifying and manipulating of formulae
COMPLEX PROCEDURES (30%)	• Problems are mainly unfamiliar and learners are expected to solve by integrating different LOs • Problems do not have a direct route to the solution but involve: - using higher level calculation skills and reasoning to solve problems - mathematical reasoning processes • These problems are not necessarily based on real world contexts and may be abstract requiring fairly complex procedures in finding the solutions.

SOLVING PROBLEMS (15%)	• Solving non-routine, unseen problems by demonstrating higher level understanding and cognitive processes • Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts • Using higher level cognitive skills and reasoning to solve non-routine problems • Being able to break down a problem into its constituent parts – identifying what is required to be solved and then using appropriate methods in solving the problem • Non-routine problems based on real contexts

3. PURPOSE OF THE CLARIFICATION OF THE ASSESSMENT STANDARDS

- The clarification of the assessment standards is to give guidance to the teacher in terms of:
  - further clarification of assessment standards
  - the depth of the content necessary for examination purposes.

- Integration of topics is encouraged as learners understand Mathematics as a holistic discipline. Questions can thus be asked integrating various assessment standards. For example: transformations and functions can be integrated.

- No additional information has been provided where assessment standards are clear in terms of content and depth.

EXAMINATION GUIDELINES : GRADE 11 & 12 : CORE PAPER 1

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
NUMBER SYSTEM & EXPONENTS	11.1.1	Understand that not all numbers are real. (This requires the recognition but not the study of non-real numbers.)	• Distinguish between real and non-real numbers. Questions on this assessment standard may be integrated in other questions.
	11.1.2	(a) Simplify expressions using the laws of exponents for rational exponents. (b) Add, subtract, multiply and divide simple surds (e.g. see subject statement) (c) Demonstrate an understanding of error margins.	• Error margins are seen in context of rounding off answers correct to one, two or three decimal places where required. *Example:* *Rounding off the interest rate to one decimal place may not yield the desired result.*
NUM BER PATTERNS	11.1.3	Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence: (a) make conjectures and generalisations (b) provide explanations and justifications and attempt to prove conjectures.	• Investigate and identify number patterns including but not limited to those with - constant difference between consecutive terms (linear patterns) - constant second difference (quadratic patterns) - constant ratios (exponential patterns) • Extend the pattern and explain how the terms are generated. • Determine the general term • Calculate the term value and the number of terms in a sequence of any pattern.

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
	12.1.3	(a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series.	• Links must be clearly established between patterns done in earlier grades so that for example, learners understand that an arithmetic sequence is a linear pattern and a geometric sequence is an exponential pattern. • Calculate the term value and the number of terms in a sequence of any pattern.
		(b) Correctly interpret sigma notation.	• Convert fluently between Σ notation and expanded notation.
		(c) Prove and correctly select the formula for and calculate the sum of series, including: $\sum_{i=1}^{n} 1 = n$; $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$; $\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}[2a + (n-1)d]$ $\sum_{i=1}^{n} a.r^{i-1} = \frac{a(r^n - 1)}{r-1}; r \neq 1$ $\sum_{i=1}^{\infty} a.r^{i-1} = \frac{a}{1-r}$ $for -1 < r < 1$	• Proofs of the sum of arithmetic and geometric series are examinable. **NOTE:** Recursive formulae are part of number patterns but are tested ONLY in the optional paper 3.

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
FINANCIAL MATHEMATICS	11.1.4	Use simple and compound decay formulae to solve problems (including straight line depreciation and depreciation on a reducing balance) (*link to Learning Outcome 2*).	
	11.1.5	Demonstrate an understanding of different periods of compounding growth and decay (including effective compounding growth and decay and including effective and nominal interest rates).	• Understand the difference between nominal and effective interest rates and convert fluently between them for the following compounding periods: - Monthly - Quarterly - Half-Yearly or semi-annually **NOTE:** Daily (365 days in a year) and biennually (every two years) could be tested in assignments or tutorials. • If patterns and functions are linked, then simple interest can be seen as a straight line function and compound interest as an exponential function.
	12.1.4	(a) Calculate the value of n in the formula $A = P(1 \pm i)^n$ (b) Apply knowledge of geometric series to solving annuity, bond and sinking fund problems, with or without the use of the formulae: $F_v = \frac{x[(1+i)^n - 1]}{i}$; $P_v = \frac{x[1-(1+i)^{-n}]}{i}$	• Candidates are expected to calculate any of the following - A, - P, - i, (but not in the F_v and P_v formulae) - n, (by using logarithms) - x, - F_v - P_v • Timelines are a useful strategy to solve problems in Financial Mathematics.
	12.1.5	Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).	• Pyramid and micro-lenders' schemes will not be examined in the examination but can be assessed by means of Grade 12 school-based assessment tasks.

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
	11.1.6 12.1.6	Solve non-routine, unseen problems. Solve non-routine, unseen problems.	• Questions of this nature are not specifically taught in class and do not have a direct route to the solution. Learners are required to use their knowledge of Mathematics in a creative way to solve problems of this nature. • Learners should, however, be exposed to these types of problems in the teaching process so that they develop problem-solving strategies.
FUNCTIONS	11.2.1	(a) Demonstrate the ability to work with various types of functions including those listed in the following Assessment Standard. (b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).	• Use and interpret functional notation. In the teaching process learners must be able to understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$, $f(ax)$, $af(x)$ • The above necessitates an understanding of how transformations and functions are integrated.
	11.2.2	Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters *k*, *p*, *a* and *q* for functions including: $y = a(x+p)^2 + q$ $y = \frac{a}{x+p} + q$ $y = ab^{x+p} + q, b > 0$ $y = \sin kx$ $y = \cos kx$ $y = \tan kx$ $y = \sin(x+p)$ $y = \cos(x+p)$ $y = \tan(x+p)$	**NOTE:** • Trigonometric functions are tested in Paper 1 with respect to the characteristics of the functions as outlined in 11.2.3 • Not more than **TWO** variations will be examined on one function simultaneously. ***For example:*** Sketch the graphs of $y = \sin 2x$ and $y = 2\cos(x + 30°)$ on the same system of axes
	11.2.3	Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed above: (a) domain and range; (b) intercepts with the axes; (c) turning points, minima and maxima; (d) asymptotes; (e) shape and symmetry; (f) periodicity and amplitude; (g) average gradient (average rate of change); (h) intervals on which the function increases/decreases; (i) the discrete or continuous nature of the graph.	

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
FUNCTIONS, INVERSES & LOGARITHMS	12.1.2	Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems (e.g. growth and decay see 12.1.4(a)).	• Definition of a logarithm – understand that the logarithmic function is the inverse of the exponential function. • Convert fluently between logarithmic form and exponential form. • The NCS emphasises the use of logarithms to solve practical problems. Thus - Complicated logarithm law simplification is not in the spirit of the NCS. - Solving logarithm equations and inequalities must be seen in the context of functions.
	12.2.1	(a) Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard. (b) Demonstrate knowledge of the formal definition of a function.	• Given the relationship between x and y in - a set of graphs - tables - words - algebraic formulae determine whether the given information represents a function.
	12.2.2	(a) Investigate and generate graphs of the inverse relations of functions, in particular the inverses of: $y = ax + q$ $y = ax^2$ $y = a^x$; $a > 0$ (b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.	• Use and interpret functional notation. In the teaching process learners must understand how $f(x)$ has been transformed to generate $f(-x)$, $-f(x)$, $f(x+a)$, $f(x)+a$, $f(ax)$, $af(x)$ and $x = f(y)$
	12.2.3	Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above: (a) domain and range; (b) intercepts with the axes; (c) turning points, minima and maxima; (d) asymptotes; (e) shape and symmetry; (f) average gradient (average rate of change); intervals on which the function increases/decreases.	

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
ALGEBRA	11.2.4	Manipulate algebraic expressions: (a) by completing the square; (b) simplifying algebraic fractions with binomial denominators.	• To avoid unnecessary assumptions the binomial denominators should be linear expressions. • Completing the square is necessary in the teaching process to determine the turning point of a parabola.
	11.2.5	Solve: (a) quadratic equations (by factorisation, by completing the square, and by using the quadratic formula) and quadratic inequalities in one variable and interpret the solution graphically; (b) equations in two unknowns, one of which is linear and one of which is quadratic, algebraically or graphically.	• Solving quadratic equations by completing the square will not be examined. • Solution of non-quadratic inequalities should be seen in context of the graphs of functions. ***For example:*** Determine the value(s) if x for which $\frac{8}{x-3} \leq 2$ (The solution to this question should be read off the graph of the two functions.)
AVERAGE GRADIENT	11.2.7	Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point.	

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
CALCULUS	12.2.4	Factorise third degree polynomials (including examples which require the factor theorem)	• Besides using the Factor Theorem, methods of synthetic division or long division could also be used to factorise cubic polynomials and solve cubic equations. **NOTE:** This knowledge is necessary for the sketching of cubic functions.
	12.2.7	(a) Investigate and use instantaneous rate of change of a variable when interpreting models of situations: • demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient at a point; • establishing the derivatives of the following functions from first principles: $f(x) = b$; $f(x) = x$; $f(x) = x^2$; $f(x) = x^3$; $f(x) = \frac{1}{x}$ and then generalise to the derivative of $f(x) = x^n$;	• Calculate from first principles the derivatives of the following functions: Function $f(x) = ax$ $f(x) = ax + b$ $f(x) = ax^2$ $f(x) = ax^2 + b$ **NOTE:** $a \in \mathrm{Z}$
		(b) Use the following rules of differentiation: $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ $\frac{d}{dx}[k.f(x)] = k.\frac{d}{dx}[f(x)]$	• Differentiate by using the power rule (If $f(x) = ax^n$, then $f'(x) = an.x^{n-1}$) ***Examples:*** Differentiate (a) $f(x) = \sqrt[3]{x^2} + \frac{1}{2x^4} - 1$ (b) $y = \frac{t^2 - 3t + 2}{t - 2}$ (c) $f(x) = (x^2 - 2)(x^{\frac{1}{3}} + 1)$ (d) $y = \frac{2x^5 - 3x + 1}{x}$ **NOTE:** The following notations can be used: • $f'(x)$ • D_x • dy/dx • y'
		(c) Determine the equations of tangents to graphs. (d) Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the x-axis. (e) Solve practical problems involving optimisation and rates of change.	• Candidates are expected to be able to interpret cubic functions - By determining the equation of a cubic function from a given graph. - Using the second derivative or any other means to determine a point of inflection where applicable. - Discuss the nature of stationary points including local maximum, local minimum and points of inflection. - Integration with transformation. • Candidates are expected to interpret the graph of the derivative of a function.

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
LINEAR PROGRAMMING	11.2.8	(a) Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region. (b) Solve a system of linear equations to find the co-ordinates of the vertices of the feasible region.	• Candidates are expected to determine the optimal solution of a linear programming problem by substituting coordinates of the vertices of the feasible region into the objective function. • Problems of the following types can be examined: - Given the graphs and feasible region, determine the constraints and answer questions on the optimal function. - Given the constraints, sketch the feasible region and answer questions on the optimal function. - Given the problem situation and one constraint, determine the other constraints, sketch the feasible region and answer questions on the optimal function. - Given the problem situation, determine all the constraints, sketch the feasible region and answer questions on the optimal function. • Graph paper will be provided where necessary. • Constraints of the type $ax + by \leq c$ and $ax + by \geq c$ where $a, b \neq 0$ will be limited to a maximum of 3.
	12.2.8	Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.	

EXAMINATION GUIDELINES : GRADE 11 & 12 : CORE PAPER 2

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
SURFACE AREA & VOLUME	11.3.1	Use the formulae for surface area and volume of right pyramids, right cones, spheres and combinations of these geometric objects.	• Composite shapes could be formed by combining a maximum of **TWO** of the stated shapes. • These formulae will not be provided on the formula sheet. Relevant formulae, from which candidates have to select the correct one, will be provided in the specific questions.
ANALYTICAL GEOMETRY	11.3.3	Use a Cartesian co-ordinate system to derive and apply: (a) the equation of a line through two given points (b) the equation of a line through one point and parallel or perpendicular to a given line (c) the inclination of a line.	• Assessment standards from Grade 10 will be assumed knowledge. • The concept of collinearity must be understood. • Prove properties of polygons using analytical methods. ***For example:*** Prove that the quadrilateral A (–1 ; –2), B(–4 ; 2), C(1 ; 2) and D(4 ; –2) is a rhombus.
	12.3.3	Use a two-dimensional Cartesian co-ordinate system to derive and apply: (a) the equation of a circle (any centre); (b) the equation of a tangent to a circle given a point on the circle.	• All assessment standards from Grade 10 and 11 will be assumed knowledge. • Learners are expected to know and be able to use as an axiom: "The tangent to a circle is perpendicular to the radius drawn to the point of contact." • Concepts such as - Inscribed-centre - Circumscribed - Incentre - Orthocentre - Centroid will not be examined.

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
TRANSFORMATION GEOMETRY	11.3.4	Investigate, generalise and apply the effect on the co-ordinates of: (a) the point $(x\ ;\ y)$ after rotation around the origin through an angle of 90^o or 180^o; (b) the vertices $(x_1\ ;\ y_1)$, $(x_2\ ;\ y_2)$, . . . $(x_n\ ;\ y_n)$ of a polygon after enlargement through the origin, by a constant factor k.	• Transformations & Enlargements - Determine the rule of transformations that has occurred. - Determine the factor of a dilation (enlargement or reduction). - The factor of dilation is an element of the rational numbers. - Use a transformation rule to sketch images of transformations of shapes, determine points of the image of a transformation of a shape and determine the relationship of the area of the image in relation to its original shape. • Rotations that are generated in an anticlockwise direction are regarded as positive whilst clockwise rotations are regarded as negative.
	12.3.4	(a) Use the compound angle identities to generalise the effect on the co-ordinates of a point $(x\ ;\ y)$ after rotation about the origin through an angle θ. (b) Demonstrate the knowledge that rigid transformations (translations, reflections, rotations and glide reflections) preserve shape and size, and that enlargement preserves shape but not size.	

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
TRIGONOMETRY	11.3.5	(a) Derive and use the values of the trigonometric functions (in surd form where applicable) of 30º, 45ºand 60º. (b) Derive and use the following identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$ (c) Derive the reduction formulae for: $\sin(90^\circ \pm \theta)$, $\cos(90^\circ \pm \theta)$, $\sin(180^\circ \pm \theta)$, $\cos(180^\circ \pm \theta)$, $\tan(180^\circ \pm \theta)$, $\sin(360^\circ \pm \theta)$, $\cos(360^\circ \pm \theta)$, $\tan(360^\circ \pm \theta)$, $\sin(-\theta)$, $\cos(-\theta)$, $\tan(-\theta)$	• Simplify and solve Pythagorean trigonometric problems using the definitions of trigonometric functions. • Simplify expressions and prove trigonometric identities involving - Reduction formulae - Special angles - Negative angles - Complementary ratios (sin 25° = cos 65°) - and using the identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$
		(d) Determine the general solution of trigonometric equations	• Solve trigonometric equations with or without the use of a calculator and determining both general and specific solutions to the equation. Determining the solution to a trigonometric equation can be integrated with a graph question, specifically determining the point of intersection or in the form of an inequality.
	11.3.6	(e) Establish and apply the sine, cosine and area rules. Solve problems in two dimensions by using the sine, cosine and area rules; and by constructing and interpreting geometric and trigonometric models.	• Solution of non-right-angled triangles specifically including - Area formula - Sine rule - Cosine rule - Solve 2-D problems using the above rules. **NOTE:** Proofs are NOT required for examination purposes but should be part of the learning process to enhance understanding.
	11.2.2	Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters *k*, *p*, *a* and *q* for functions including: $y = \sin kx$, $y = \cos kx$, $y = \tan kx$, $y = \sin(x + p)$, $y = \cos(x + p)$, $y = \tan(x + p)$	• The focus of trigonometric graphs in paper 2 is on the relationships, simplification and determining points of intersection by solving equations, although the characteristics of the graphs should not be excluded.
	11.2.3	Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed above: (a) domain and range; (b) intercepts with the axes; (c) turning points, minima and maxima; (d) asymptotes; (e) shape and symmetry; (f) periodicity and amplitude; (g) average gradient (average rate of change); (h) intervals on which the function increases/decreases; (i) the discrete or continuous nature of the graph.	

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
	12.3.5	Derive and use the following compound angle identities: (a) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ (b) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$ (c) $\sin 2\alpha = 2\sin\alpha\cos\alpha$ (d) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$	• Use the compound angle formula for $\cos(\alpha - \beta)$ and derive the formulae for $\sin(\alpha \pm \beta)$ and $\cos(\alpha + \beta)$. **NOTE:** Proofs are NOT required for examination purposes but should be part of the learning process to enhance understanding. • Use the compound angle formulae in - Simplifying trigonometric expressions - Proving identities - Solving trigonometric equations (both specific and general solutions) - Solving trigonometric equations where the denominator of an identity is undefined. - Integration with transformation geometry.
	12.3.6	Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models	• Solution of non-right-angled triangles specifically including - Area formula - Sine rule - Cosine rule - Solve 2-D & 3-D problems using the above rules. .

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
STATISTICS AND DATA HANDLING	11.4.1	(a) Calculate and represent measures of central tendency and dispersion in univariate numerical data by: • five number summary (maximum, minimum and quartiles); • box and whisker diagrams; • ogives; • calculating the variance and standard deviation of sets of data manually (for small sets of data) and using available technology (for larger sets of data), and representing results graphically using histograms and frequency polygons. (b) Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health, social, economic, cultural, political and environmental issues).	• Use and apply concepts including - Measures of central tendency - Graphical representation of data - Measures of dispersion • Write down a five number summary and draw box and whisker diagrams. Interpret and make comparisons between different box and whisker diagrams. • Draw and interpret ogives. • Calculate variance and standard deviation (use of the calculator is advised). Interpret standard deviations for normal distributions. • Represent bivariate data including scatter plots and describe the line of best fit and interpret the graphs.
			There is no new content that needs to be covered in Grade 12 for the core papers. All data handling that is taught in Grade 10 and 11 will be examined in Grade 12.

EXAMINATION GUIDELINES : GRADE 11 & 12 OPTIONAL PAPER 3

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
NUMBER PATTERNS	12.1.3	(d) Correctly interpret recursive formulae: (e.g. $T_{n+1} = T_n + T_{n-1}$)	• Generate a sequence given a formula. • Derive a formula for a given sequence.
EUCLIDEAN GEOMETRY	11.3.2	a) Investigate necessary and sufficient conditions for polygons to be similar. b) Prove and use (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (the Mid-point Theorem as a special case of this theorem); • that equiangular triangles are similar; • that triangles with sides in proportion are similar; • the Pythagorean Theorem by similar triangles.	**NOTE:** The proofs of these theorems are required for examination purposes.
	12.3.2	(a) Accepting as axioms all results established in earlier grades and the fact that a tangent is perpendicular to the radius, drawn at the point of contact with the circle, and then investigate and **prove** the theorems of the geometry of circles: • the line drawn from the centre of a circle, perpendicular to a chord, bisects the chord and its converse; • the perpendicular bisector of a chord passes through the centre of the circle; • the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle; • angles subtended by a chord at the circle on the same side of the cord are equal and its converse; • the opposite angles of a cyclic quadrilateral are supplementary and its converse; • two tangents drawn to a circle from the same point outside the circle are equal in length; • the angles between a tangent and a chord, drawn to the point of contact of the chord, are equal to the angles which the chord subtends in the alternate chord segment and its converse. (b) Use the theorems listed above to: make and prove or disprove conjectures; prove riders.	• Corrolaries derived from the theorems and axioms in 12.3.2 are necessary - angles in a semi-circle - angles in the same segment - equal chords subtend equal angles at the circumference - equal chords subtend equal angles at the centre - Exterior angle of a cyclic quadrilateral and its converse

TOPIC	Assessment Standards	ASSESSMENT STANDARDS	CLARIFICATION OF ASSESSMENT STANDARDS
DESCRIPTIVE STATISTICS & PROBABILITY	11.4.2	(a) Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events: P(A and B) = P(A).P(B) (b) Use tree and Venn diagrams to solve probability problems (where events are not necessarily independent).	• Statistics studied as part of the core curriculum is assumed knowledge and therefore can be integrated into questions in this paper.
	11.4.3	(a) Identify potential sources of bias, error in measurement, potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here). (b) Effectively communicate conclusions and predictions that can be made from the analysis of data.	
	11.4.4	Differentiate between symmetric and skewed data and make relevant deductions.	
	12.4.1	(c) Demonstrate the ability to draw a suitable sample from a population and understand the importance of sample size in predicting the mean and standard deviation of a population. (d) Use available technology to calculate the regression function which best fits a given set of bivariate numerical data. (e) Use available technology to calculate the correlation co-efficient of a set of bivariate numerical data to make relevant deductions.	
	12.4.2	Generalise the fundamental counting principle (successive choices from m_1 then m_2 then m_3 … options create $m_1.m_2.m_3$ … different combined options) and solve problems using the fundamental counting principle.	
	12.4.3	(a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here). (b) Effectively communicate conclusions and predictions that can be made from the analysis of data. Identify data which is normally distributed about a mean by investigating appropriate histograms and frequency polygons.	
	12.4.4	Identify data which is normally distributed about a mean by investigating histograms and frequency polygons.	

