



# education

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Department:  
Education  
**REPUBLIC OF SOUTH AFRICA**

**NASIONALE  
SENIOR SERTIFIKAAT**

**GRAAD 12**

**WISKUNDE V2**

**FEBRUARIE/MAART 2009**

**MEMORANDUM**

**Hierdie memorandum bestaan uit 13 bladsye.**

**VRAAG 1**

1.1	$m_{BC} = \frac{1 - 0}{6 - 3}$ $m_{BC} = \frac{1}{3}$	✓ substitusie in gradiënt-formule ✓ antwoord (2)
1.2	$m_{AD} = m_{BC}$ $m_{AD} = \frac{1}{3} \text{ ----- AB//BC}$ <p>∴ Vergelyking van AD is:</p> $y = \frac{1}{3}x + c$ $6 = \frac{1}{3}(1) + c$ $c = \frac{17}{3}$ $\therefore y = \frac{1}{3}x + \frac{17}{3}$ <p><b>OF</b></p> $y - 6 = \frac{1}{3}(x - 1)$ $y - 6 = \frac{1}{3}x - \frac{1}{3}$ $y = \frac{1}{3}x + \frac{17}{3}$	✓ $m_{AC} = \frac{1}{3}$  ✓ substitusie of (1 ; 6) in 'n regitlynvergelyking  ✓ vergelyking (3)  ✓ $m_{AC} = \frac{1}{3}$  ✓ substitusie of (1 ; 6) in 'n regitlynvergelyking  ✓ vergelyking (3)
1.3	$y = \frac{1}{3}x + \frac{17}{3}$ $t = \frac{1}{3}(7) + \frac{17}{3}$ $t = 8$ <p><b>OF</b></p> $\frac{t - 6}{7 - 1} = \frac{1}{3}$ $t - 6 = 2$ $\therefore t = 8$	✓ ✓ substitusie van $x$ -waarde in 'n regitlynvergelyking.  (2)

1.4	$AD = \sqrt{(8-6)^2 + (-1-3)^2}$ $AD = \sqrt{40}$ $AD = 2\sqrt{10}$ $BC = \sqrt{(6-3)^2 + (1-0)^2}$ $BC = \sqrt{10}$ $AB = \sqrt{(6-0)^2 + (1-3)^2}$ $AB = \sqrt{40}$ $AB = 2\sqrt{10}$	✓ gebruik afstandsformule  ✓ antwoord vir AD  ✓ Antwoord vir BC  ✓ antwoord vir AB  (4)
1.5	$m_{AB} = \frac{6-0}{1-3}$ $m_{AB} = -3$ $m_{BC} = \frac{1-0}{6-3} = \frac{1}{3}$ $m_{AB} \cdot m_{BC} = \frac{1}{3} \times -3$ $= -1$ $\therefore AB \perp BC$	✓ $m_{AB} = -3$  ✓ $m_{AB} \times m_{BC} = -1$  ✓ gevolgtrekking  (3)
1.6	Area van vierh ABCD = area van $\Delta ADC$ + area van ABC $= \frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ $= 20 + 10$ $= 30 \text{ vierkante eenheid}$ Of Area van ABCD = $\frac{1}{2}(\text{som van parallel sye}) \times h$ $= \frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$ $= \sqrt{10}(3\sqrt{10})$ $= 30 \text{ vierkante eenheid}$	✓ formule van area van $\Delta$ ✓✓ $\frac{1}{2}(2\sqrt{10})(2\sqrt{10}) + \frac{1}{2}(\sqrt{10})(2\sqrt{10})$ ✓ antwoord  (4)  ✓ formule van area van trapezium  ✓✓ $\frac{1}{2}(2\sqrt{10} + \sqrt{10})2\sqrt{10}$  ✓ antwoord  (4)
1.7	Uit 1.1 $m_{BC} = \frac{1}{3}$ $\tan \theta = \frac{1}{3}$ $\therefore \theta = 18,43^\circ$	✓ ✓ $\tan \theta = \frac{1}{3}$  ✓ $\theta = 18,43^\circ$  (3) <b>[21]</b>

## VRAAG 2

2.1	Middelpunt $AB$ $\left(\frac{-8+0}{2}; \frac{1+5}{2}\right)$ $= (-4; 3)$	✓ substitusie in middelpuntformule (1)
2.2	$M_{AD} = \frac{5+1}{0-3} = \frac{-2}{1}$ $y - y_1 = m(x - x_1)$ $y - 5 = -2(x - 0)$ $y = -2x + 5$	✓ substitusie in gradiënt formule ✓ $M_{AD} = \frac{-2}{1}$ ✓ substitusie van $(0; 5)$ in 'n reguitlynvergelyking ✓ antwoord (4)
2.3	$AM^2 = (5-3)^2 + (0+4)^2$ $AM^2 = 2^2 + 4^2$ $AM = \sqrt{20}$	✓ substitusie in afstands- formule ✓ vereenvoudiging ✓ antwoord (3)
2.4	$(x+4)^2 + (y-3)^2 = (\sqrt{20})^2$ $(x+4)^2 + (y-3)^2 = 20$ $x^2 + y^2 + 8x - 6y + 5 = 0$	✓ $(x+4)^2$ ✓ $(y-3)^2$ ✓ 20 ✓ antwoord (4)
2.5	$AT = TK = 6$ $CD \perp AK$  Dus ACKD is 'n vlieër want diagonaal CD halveer diagonaal AK reghoekig.  <b>OF</b> $\hat{CAD} = 90^\circ$  $M_{KC} \cdot M_{KD} = \frac{6}{-12} \cdot \frac{6}{3} = -1$  $\therefore \hat{CKD} = 90^\circ$ $\Delta CAD$ & $\Delta CKD$ is reghoekig & kongruent ACKD is 'n vlieër	✓ $AT = TK$ ✓ CD loodreg op AK  ✓ Vlieër ✓ rede (4)  ✓✓ $M_{KC} \cdot M_{KD} = \frac{6}{-12} \cdot \frac{6}{3} = -1$ ✓ $\Delta CAD$ & $\Delta CKD$ is reghoekig & kongruent  ✓ ACKD is 'n vlieër (4) <b>[16]</b>

**VRAAG 3**

3.1.1	$P'(-\sqrt{3}; -2)$	✓ ✓ koördinate $P'$ (2)
3.1.2	$P'(-\sqrt{3}; 2)$	✓ ✓ koördinate $P'$ (2)
3.2.1	$Q'(2; 2)$	✓ ✓ koördinate $Q'$ (2)
3.2.2		✓ koördinate $P'$ ✓ koördinate $Q'$ ✓ koördinate $R'$ ✓ koördinate $S'$ (4)
3.2.3	$P''(4; 6)$	✓ ✓ antwoord (2)
3.2.4	Nie star. Die vorm bly dieselfde, maar die grootte verander.	✓ nie star ✓ verduideliking (2)
3.2.5	$(x; y) \rightarrow (y; -x)$ $(y; -x) \rightarrow (2y; -2x)$ $\therefore (x; y) \rightarrow (2y; -2x)$	✓ $(y; -x)$ ✓ ✓ $(2y; -2x)$ (3)
3.2.6	Area PQRS : area $P''Q''R''S''$ $= 1^2 : 2^2$ $= 1 : 4$	✓ kwadrering ✓ antwoord (2) <b>[19]</b>

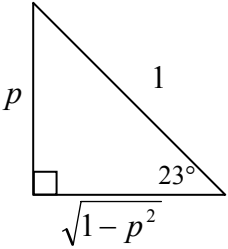
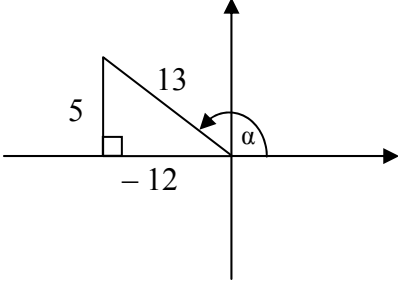
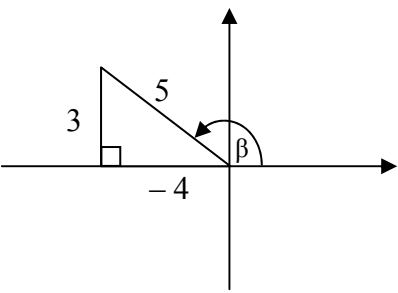
**VRAAG 4**

4.1	$x' = x \cos(135^\circ) - y \sin(135^\circ)$ $x' = -x \cos 45^\circ - y \sin 45^\circ$ $x' = x \left( \frac{-\sqrt{2}}{2} \right) - y \left( \frac{\sqrt{2}}{2} \right)$ $x' = -\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y$ <p>en</p> $y' = y \cos(135^\circ) + x \sin(135^\circ)$ $y' = -y \cos 45^\circ + x \sin 45^\circ$ $y' = y \left( -\frac{\sqrt{2}}{2} \right) + x \left( \frac{\sqrt{2}}{2} \right)$ $y' = -\frac{\sqrt{2}}{2}y + \frac{\sqrt{2}}{2}x$	✓ 135°  ✓ substitusie   ✓ antwoord vir $x$       ✓ antwoord vir $y$  (4)
4.2	$x' = -\frac{\sqrt{2}}{2}(2) - \frac{\sqrt{2}}{2}(4)$ $x' = -\sqrt{2} - 2\sqrt{2}$ $x' = -3\sqrt{2}$ $y' = -\frac{\sqrt{2}}{2}(4) + \frac{\sqrt{2}}{2}(2)$ $y' = -\sqrt{2}$ $\therefore M(-3\sqrt{2}; -\sqrt{2})$	✓ $x$ koördinate ✓ $y$ koördinate  (2) <b>[6]</b>

**VRAAG 5**

5.1	$\frac{\tan(180^\circ + x) \cos(360^\circ - x)}{\sin(180^\circ - x) \cos(90^\circ + x) + \cos(540^\circ + x) \cos(-x)}$ $= \frac{\tan x \cdot (\cos x)}{(\sin x) \cdot (-\sin x) - \cos x \cdot \cos x}$ $= \frac{\frac{\sin x}{\cos x} \cos x}{-\sin^2 x - \cos^2 x}$ $= \frac{\sin x}{-(\sin^2 x + \cos^2 x)}$ $= -\sin x$	<ul style="list-style-type: none"> <li>✓ <math>\tan x</math></li> <li>✓ <math>\cos x</math></li> <li>✓ <math>-\sin x</math></li> <li>✓ <math>-\sin x</math></li> <li>✓ <math>-\cos x</math></li> <li>✓ <math>\cos x</math></li> <li>✓ vereenvoudiging</li> <li>✓ antwoord</li> </ul> <p style="text-align: right;">(8)</p>
5.2	$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1 - (1 - 2 \sin^2 x) - \sin x}{2 \sin x \cdot \cos x - \cos x}$ $= \frac{2 \sin^2 x - \sin x}{2 \sin x \cdot \cos x - \cos x}$ $= \frac{\sin x(2 \sin x - 1)}{\cos x(2 \sin x - 1)}$ $= \frac{\sin x}{\cos x}$ $= \tan x$	<ul style="list-style-type: none"> <li>✓ <math>1 - 2 \sin^2 x</math></li> <li>✓ <math>2 \sin x \cdot \cos x</math></li>   <li>✓✓ faktorisering</li>   <li>✓ antwoord</li> </ul> <p style="text-align: right;">(5) [13]</p>

**VRAAG 6**

6.1.1	$\begin{aligned} \cos 113^\circ &= \cos (90^\circ + 23^\circ) \\ &= -\sin 23^\circ \\ &= -p \end{aligned}$	✓ reduksie ✓ antwoord (2)
6.1.2	$\begin{aligned} \cos 23^\circ &= \sqrt{1-p^2} \end{aligned}$ <p><b>OF</b></p> $\begin{aligned} \cos^2 23^\circ + \sin^2 23^\circ &= 1 \\ \cos^2 23^\circ &= 1-p^2 \\ \cos 23^\circ &= \sqrt{1-p^2} \end{aligned}$	 ✓ diagram ✓ antwoord <p><b>OF</b></p> ✓ identiteit ✓ antwoord (2)
6.1.3	$\begin{aligned} \sin 46^\circ &= 2\sin 23^\circ \cdot \cos 23^\circ \\ &= 2p\sqrt{1-p^2} \end{aligned}$	✓ uitbreiding ✓ antwoord (2)
6.2.1	$\begin{aligned} \sin \alpha &= \frac{5}{13} \\ y_\alpha &= 5 \quad r_\alpha = 13 \\ x_\alpha &= -12 \\ \cos \alpha &= -\frac{12}{13} \end{aligned}$	 ✓ vereenvoudiging ✓ diagram ✓ antwoord (3)
6.2.2	$\begin{aligned} \tan \beta &= -\frac{3}{4} \\ y_\beta &= 3 \quad x_\beta = -4 \\ r &= 5 \end{aligned}$ $\begin{aligned} \cos(\alpha + \beta) &= \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta \\ &= \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right) - \left(\frac{5}{13}\right)\left(\frac{3}{5}\right) \\ &= \frac{48-15}{65} \\ &= \frac{33}{65} \end{aligned}$	 ✓ diagram ✓ uitbreiding ✓ $-\frac{4}{5}$ ✓ $\frac{3}{5}$ ✓ antwoord (5)

6.3	$\frac{1}{2} \cos x = 0,435$ $\cos x = 0,87$ $x = 29,54^\circ \text{ or } x = 330,46^\circ$	✓ vereenvoudiging ✓✓ antwoorde (3) <b>[17]</b>
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**VRAAG 7**

7.1	$\frac{7}{PB} = \sin 18^\circ$ $PB = \frac{7}{\sin 18^\circ}$ $PB = 22,65 \text{ m } (22,65247584\dots)$	✓ verhouding  ✓ antwoord (2)
7.2	$\frac{18}{PA} = \cos 23^\circ$ $PA = \frac{18}{\cos 23^\circ}$ $PA = 19,55 \text{ m } (19,55448679\dots)$	✓ verhouding  ✓ antwoord (2)
7.3	$AB^2 = (22,65)^2 + (19,55)^2 - 2(22,65)(19,55) \cdot \cos 42^\circ$ $= 237,0847954\dots$ $AB = 15,40 \text{ m } (15,3975581\dots)$	✓ gebruik van cos-reel ✓ substitusie ✓ 237,0847... ✓ antwoord (4) <b>[8]</b>

**VRAAG 8**

<p>8.1</p>		<p>tan-grafiek                  ✓ vorm                  ✓ asimptote                  ✓ snypunte</p> <p>Sin-grafiek                  ✓ vorm                  ✓ snypunte                  ✓ periode</p> <p style="text-align: right;">(6)</p>
<p>8.2</p>	$\sin 2x = \frac{1}{2} \tan x$ $2 \sin x \cdot \cos x = \frac{\sin x}{2 \cos x}$ $4 \sin x \cdot \cos^2 x - \sin x = 0$ $\sin x(4 \cos^2 x - 1) = 0$ <p> <math>\sin x = 0</math>  <math>x = 0^\circ</math> or <math>180^\circ</math> or                 <math>\cos^2 x = \frac{1}{4}</math>  <math>\cos x = \pm \frac{1}{2}</math>  <math>x = 60^\circ ; -60^\circ</math> or <math>120^\circ</math> </p>	<p>✓ gelykstelling</p> <p>✓ <math>2 \cdot \sin x \cdot \cos x</math></p> <p>✓ <math>\frac{\sin x}{2 \cos x}</math></p> <p>✓ vereenvoudiging</p> <p>✓ faktoriserings</p> <p>✓ <math>\sin x = 0</math></p> <p>✓ <math>x = 0^\circ</math> of <math>180^\circ</math></p> <p>✓ <math>\cos^2 x = \frac{1}{4}</math></p> <p>✓ <math>\cos x = \pm \frac{1}{2}</math></p> <p>✓ antwoorde</p> <p style="text-align: right;">(10)</p>
<p>8.3</p>	<p><math>\{x \mid -60^\circ &lt; x &lt; 0^\circ\} \cup \{x \mid 60^\circ &lt; x &lt; 90^\circ\} \cup \{x \mid 120^\circ &lt; x &lt; 180^\circ\}</math></p> <p><b>OF</b>  <math>x \in (-60^\circ ; 0^\circ) \cup (60^\circ ; 90^\circ) \cup (120^\circ ; 180^\circ)</math></p> <p><b>OF</b>  <math>-60^\circ &lt; x &lt; 0^\circ</math> or <math>60^\circ &lt; x &lt; 90^\circ</math> or <math>120^\circ &lt; x &lt; 180^\circ</math></p>	<p>✓✓✓ antwoorde</p> <p style="text-align: right;">(3) [19]</p>



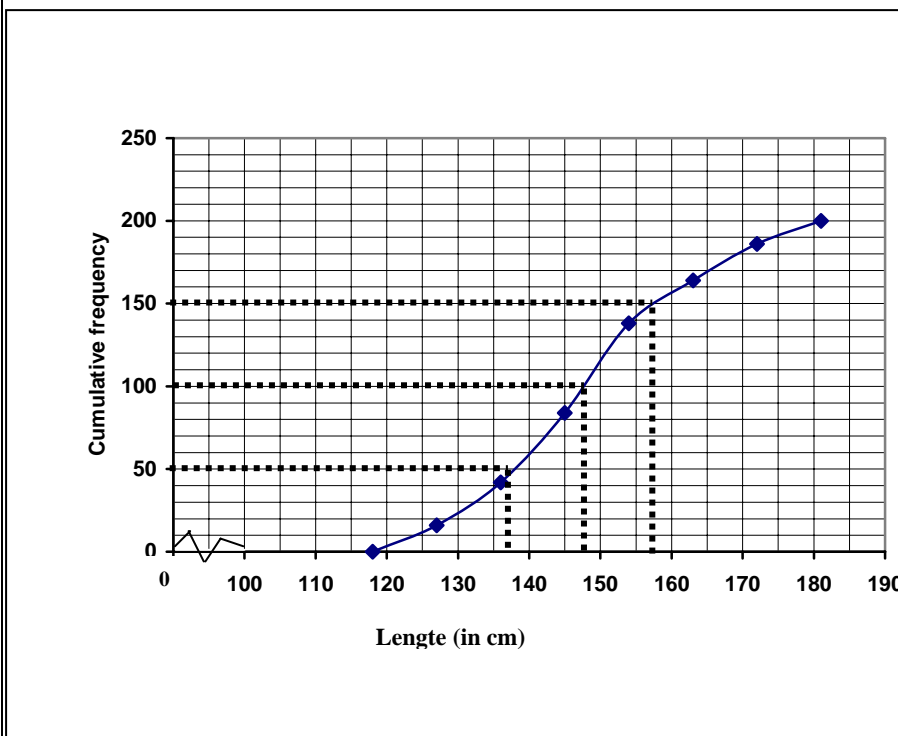
**VRAAG 11**

11.1

LENGTE (IN CM)	FREKWENSIE	KUMULATIEWE FREKWENSIE
$118 \leq h < 127$	16	16
$127 \leq h < 136$	26	42
$136 \leq h < 145$	42	84
$145 \leq h < 154$	54	138
$154 \leq h < 163$	26	164
$163 \leq h < 172$	22	186
$172 \leq h < 181$	14	200

✓✓ antwoorde in kumulatiewe frekwensiekolom (2)

11.2



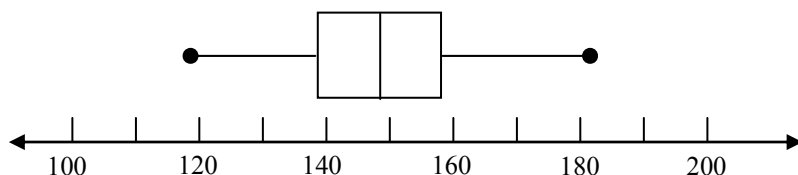
✓ kumulatiewe totale  
✓ punte by boonste limiete van intervalle  
✓ kurwes (3)

11.3

Onderste kwartiel  $\approx 138$  cm  
Mediaan  $\approx 148$  cm  
Boonste kwartiel  $\approx 158$  cm

✓✓✓ Lees korrek vanaf ogief (3)

11.4



✓ minimum en maksimum waardes  
✓ kwartiele en mediaan  
✓ snorbaarde (3)

11.5

Die lengtes van die leerders is redelik eweredig versprei.

✓ eweredig versprei (1)

11.6

100 leerders val in hierdie interval.

✓ 100 (1)

[13]

**VRAAG 12**

<p>12.1</p>	<p style="text-align: center;"><b>Spreidiagram van spoed versus brandstof verbruik</b></p> <table border="1"> <caption>Data points from the scatter plot</caption> <thead> <tr> <th>Spoed (km/h)</th> <th>Brandstofverbruik (l/100 km)</th> </tr> </thead> <tbody> <tr> <td>60</td> <td>11.5</td> </tr> <tr> <td>75</td> <td>10.0</td> </tr> <tr> <td>85</td> <td>9.2</td> </tr> <tr> <td>95</td> <td>8.8</td> </tr> <tr> <td>110</td> <td>7.8</td> </tr> <tr> <td>115</td> <td>8.5</td> </tr> <tr> <td>120</td> <td>8.8</td> </tr> </tbody> </table>	Spoed (km/h)	Brandstofverbruik (l/100 km)	60	11.5	75	10.0	85	9.2	95	8.8	110	7.8	115	8.5	120	8.8	<p>✓✓ plot van punte ✓ benaminge</p> <p style="text-align: right;">(3)</p>
Spoed (km/h)	Brandstofverbruik (l/100 km)																	
60	11.5																	
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95	8.8																	
110	7.8																	
115	8.5																	
120	8.8																	
<p>12.2</p>	<p>Kwadratiese</p>	<p>✓ kwadratiese</p> <p style="text-align: right;">(1)</p>																
<p>12.3</p>	<p>Die kwadratiese patroon wys dat die beste brandstofverbruik plaasvind wanneer die motor teen 110 km/h bestuur word. Om die brandstofrekening van die maatskappy minimaal te hou, moet die bestuurders aangemoedig word om sover moontlik teen die genoemde spoed te ry.</p>	<p>✓✓ antwoord</p> <p style="text-align: right;">(2) <b>[6]</b></p>																

**TOTAAL: 150**