basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2014

MEMORANDUM

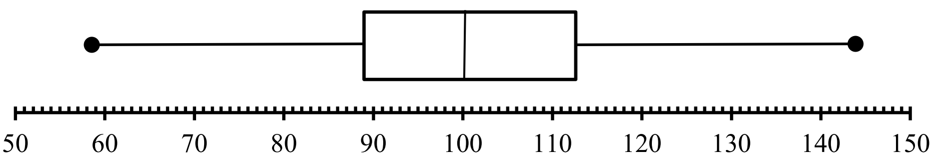
MARKS/*PUNTE*: 150

This memorandum consists of 14 pages.
Hierdie memorandum bestaan uit 14 bladsye.

NOTE:
- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out question.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die eerste poging.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid is DEURGAANS in ALLE aspekte van die memorandum van toepassing.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

QUESTION/*VRAAG* 1

1.1	Mean/*Gemiddelde* $= \frac{\sum x}{n} = \frac{1522}{15} = 101{,}47$	✓ 1522 ✓ 101,47 (2)
1.2	Standard deviation/*standaardafwyking* = 19,07	✓19,07 ✓2 decimal places (2)
1.3	lower quartile/*onderste (eerste) kwartiel* = 89 upper quartile/*boonste (derde) kwartiel* = 113	✓ 89 ✓ 113 (2)
1.4		✓ M at 100 ✓ min = 58 and max = 145 ✓ Q_1 = 89 and Q_2 = 113 (3)
1.5	$(\bar{x} - 1\sigma \ ; \ \bar{x} + 1\sigma) = (82{,}4 \ ; \ 120{,}54)$ $\therefore$ 2 days/*dae* Answer only: full marks	✓✓ interval ✓ answer (3) **[12]**

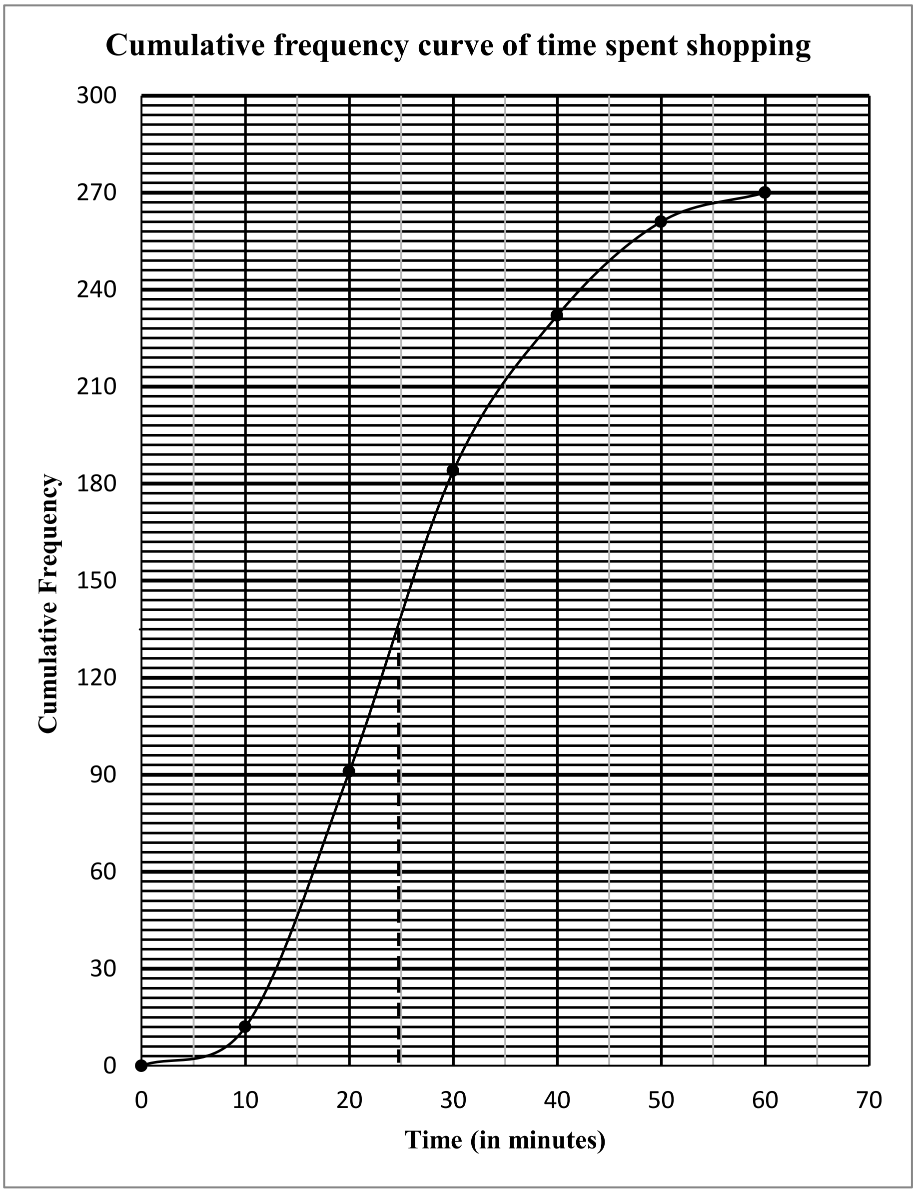
QUESTION/*VRAAG* 2

2.1

TIME IN MINUTES	NUMBER OF CUSTOMERS (frequency)	CUMULATIVE FREQUENCY
$0 < x \leq 10$	12	12
$10 < x \leq 20$	79	91
$20 < x \leq 30$	93	184
$30 < x \leq 40$	48	232
$40 < x \leq 50$	29	261
$50 < x \leq 60$	9	270

✓ completing frequency column
✓ ✓ cumulative frequency
(3)

2.2



✓ plot against upper limit
✓ cumulative frequency
✓ anchored
✓ smooth curve
(4)

2.3 Median time spent shopping is approximately 25 minutes.
(Allow 24–25 minutes)

✓ ✓ answer
(2)

2.4 The data is skewed to the right or positively skewed/
Die data is skeef na regs of positief skeef.

✓ correct skewness
(1)
[10]

QUESTION/*VRAAG* 3

3.1	(41 ; 26)	✓ correct outlier (1)
3.2	quadratic/*kwadraties*	✓ correct answer (1)
3.3	The younger or older the participants are, the longer they will take to complete the item. They do not have the required strength, fitness and stamina. *Hoe jonger of ouer die deelnemers is, hoe langer sal hulle neem om die item te voltooi. Hulle het nie die vereiste krag, fiksheid en stamina (energie) nie.* **OR** It would appear that swimmers close to 19 years completed the item in the shortest time. Swimmers of that age are normally in good physical condition and have lots of stamina. *Dit wil voorkom of swemmers rondom 19 jaar die item in die kortste tyd voltooi het. Swemmers van daardie ouderdom is normaalweg in goeie fisiese kondisie en het baie energie en stamina.*	✓ younger/older *jonger/ouer* ✓ lack of strength/ *tekort aan krag* (2) ✓ 19 years/*jaar* ✓ peak fitness/ *top fiks* (2)
3.4.1	The standard deviation will become smaller/decrease./ *Die standaardafwyking sal kleiner word/verminder.*	✓ decrease/ *verminder* (1)
3.4.2	The mean will become smaller/decrease./ *Die gemiddelde sal kleiner word/verminder.*	✓ decrease/ *verminder* (1) **[6]**

QUESTION/*VRAAG* 4

4.1	$y=-3x+k$ $-3=(-3)(-1)+k$ $k=-6$ **OR** By inspection, using the gradient: $k=-6$	✓ substitution of $(-1\,;-3)$ ✓ $k=-6$ (2)
4.2	$\frac{x_A+x_B}{2}=x_P$ $\frac{-1+x_B}{2}=\frac{5}{2}$ $x_B=6$ **and** $\frac{y_A+y_B}{2}=y_P$ $\frac{-3+y_B}{2}=1$ $y_B=5$ $\therefore$B (6 ; 5) **OR** By using translation: B(6 ; 5)	✓ 6 ✓ 5 (2)
4.3	$m_{AB}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{5-(-3)}{6-(-1)}$ $=\frac{8}{7}$ **OR** $m_{AB}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{1-(-3)}{2{,}5-(-1)}$ $=\frac{8}{7}$	✓ substitution ✓ gradient (2)

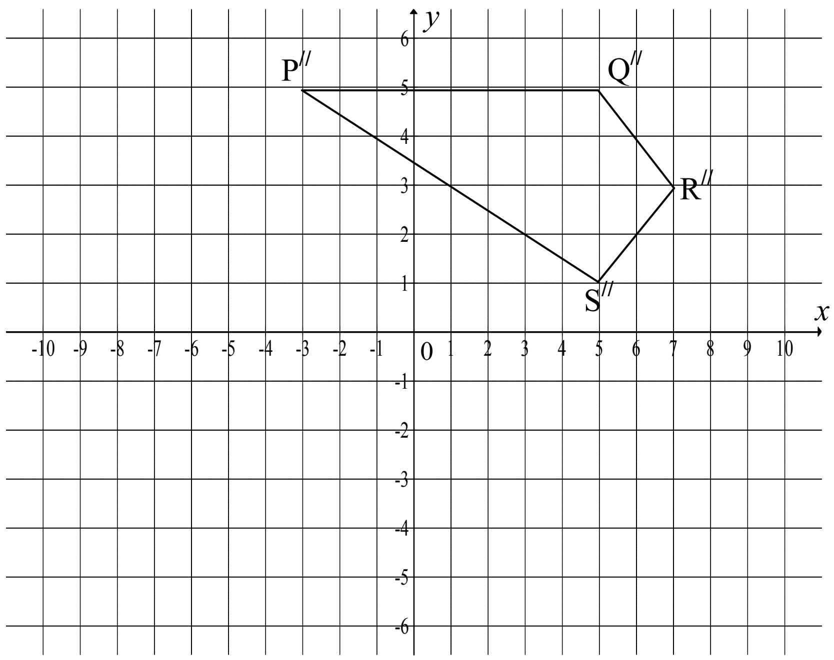
4.4	$\tan\beta = m_{AD} = -3$ $\beta = 108,43°$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48,81°$ $\theta = 108,43° - 48,81°$ $\theta = 59,62°$ **OR** $\tan\beta = m_{AD} = -3$ $\beta = 108,43°$ $C\hat{D}O = 18,43°$ $\tan\alpha = m_{AB} = \frac{8}{7}$ $\alpha = 48,81°$ $\theta = 18,43° + (90° - 48,81°)$ $\theta = 59,62°$	✓ $\tan\beta = -3$ ✓ $\beta = 108,43°$ ✓ $\tan\alpha = \frac{8}{7}$ ✓ $\alpha = 48,81°$ ✓ $\theta = 59,62°$ (5) ✓ $\tan\beta = -3$ ✓ $\beta = 108,43°$ ✓ $\tan\alpha = \frac{8}{7}$ ✓ $\alpha = 48,81°$ ✓ $\theta = 59,62°$ (5)
4.5	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0+1)^2 + (-6+3)^2}$ $= \sqrt{10}$	✓ substitution into distance formula ✓ $\sqrt{10}$ (2)
4.6	AC = 2 AD $= 2\sqrt{10}$ $CB^2 = AC^2 + AB^2 - 2AC.AB.\cos\theta$ $= (2\sqrt{10})^2 + (\sqrt{113})^2 - 2(2\sqrt{10})(\sqrt{113})\cos 59,62°$ $= 84,998...$ CB = 9,22 units. **OR** D(0 ; –6), A(–1 ; –3), AC = 2AD So $x_c - x_A = 2(x_A - x_D)$ $x_C + 1 = 2(-1 - 0)$, $x_C = -3$ $y_c - y_A = 2(y_A - y_D)$ $y_C + 3 = 2(-3 + 6)$, $y_C = 3$ The coordinates of C are (–3 ; 3). $CB = \sqrt{(6-(-3))^2 + (5-3)^2}$ $= 9,22$ units	✓ $AC = 2\sqrt{10}$ ✓ using cosine rule ✓ substitution ✓ 84,998... ✓ 9,22 (5) ✓✓✓ C(–3 ; 3) ✓ substitution into distance formula ✓ 9,22 (5) **[18]**

QUESTION/*VRAAG* 5

5.1	M(8 ; – 4)	✓ coordinates (1)
5.2	$OM = \sqrt{(8-0)^2 + (-4-0)^2}$ $= \sqrt{80}$ or $4\sqrt{5}$ units	✓ substitution into distance formula ✓ $\sqrt{80}$ *or* $4\sqrt{5}$ (2)
5.3	$ON = OM - NM$ $= \sqrt{80} - \sqrt{45}$ $= 4\sqrt{5} - 3\sqrt{5}$ $= \sqrt{5}$ *units*	✓ $ON = OM - NM$ ✓ length of NM ✓ answer (3)
5.4	$M\hat{T}P = 90°$ (tangent/*raaklyn* ⊥ radius) $\therefore$ $O\hat{M}T = 90°$ (alternate ∠'s /*verwissellende* ∠'*e* ; TP | | OM)	✓ Statement + reason ✓ answer (2)
5.5	$m_{MT} \,.\, m_{OM} = -1$ $m_{OM} = \frac{-4-0}{8-0} = -\frac{1}{2}$ $m_{MT} = 2$ $y + 4 = 2(x-8)$ $y = 2x - 20$ **OR** $y = 2x + c$ $-4 = 2(8) + c$ $c = -20$ $y = 2x - 20$	✓ ✓ m_{OM} ✓ m_{MT} ✓ substitution of *m* and (8 ; –4) ✓ equation MT (5)
5.6	$(x-8)^2 + (y+4)^2 = 45$ $(x-8)^2 + (2x-20+4)^2 = 45$ $(x-8)^2 + (2x-16)^2 = 45$ $x^2 - 16x + 64 + 4x^2 - 64x + 256 - 45 = 0$ $5x^2 - 80x + 275 = 0$ $x^2 - 16x + 55 = 0$ $(x-11)(x-5) = 0$ $x = 11$ $y = 2(11) - 20$ $y = 2$ $\therefore$ T(11 ; 2)	✓ substitution ✓ expansion ✓ standard form ✓ factors ✓ $x = 11$ ✓ substitution (6) **[19]**

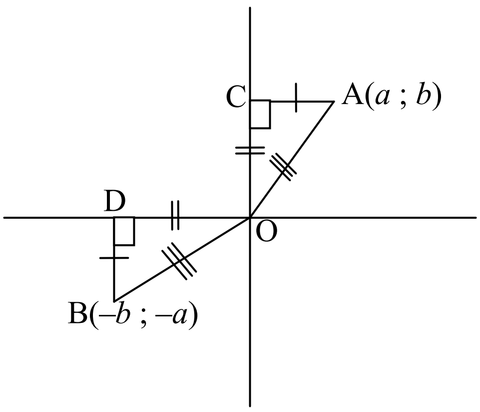
QUESTION/*VRAAG* 6

6.1.1	Rotation by 90° anti-clockwise about the origin/ *Rotasie van 90° antikloksgewys om die oorsprong.*	✓ rotation/*rotasie* ✓ 90° anti-clockwise/ *antikloksgewys* (2)
6.1.2	$(x;y)\rightarrow(y;x)$	✓ y ✓ x (2)
6.1.3	P$^{/}$(–3 ; –5)	✓ x value ✓ y value (2)
6.2.1	Q$^{/}$ (– 2 ; 4)	✓ x value and y value (1)
6.2.2(a)	$(x;y)\rightarrow(2x;2y)\rightarrow(-2x;2y)\rightarrow(-2x+3\ ;\ 2y+1)$	✓ $(2x;2y)$ ✓ $(-2x;2y)$ ✓ $(-2x+3\ ;\ 2y+1)$ (3)

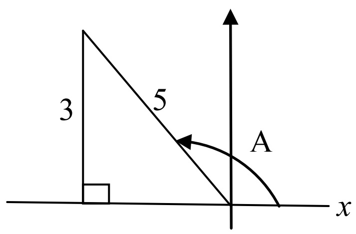
6.2.2(b)	
	P$^{/}$(6 ; 4) → P$^{//}$ (–3 ; 5)
	Q$^{/}$(–2 ; 4) → Q$^{//}$ (5 ; 5)
	R$^{/}$(–4 ; 2) → R$^{//}$ (7 ; 3)
	S$^{/}$(–2 ; 0) → S$^{//}$ (5 ; 1)
		✓ P$^{//}$ (–3 ; 5)
		✓ Q$^{//}$ (5 ; 5)
		✓ R$^{//}$ (7 ; 3)
		✓ S$^{//}$ (5 ; 1)
		✓ joining the points
		(5)

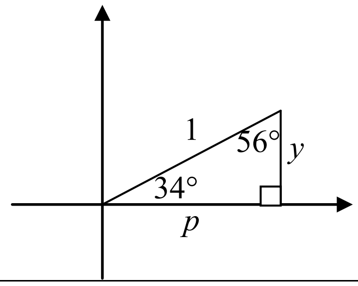
6.2.3	Perimeter/*Omtrek* PQRS = t × Perimeter/*Omtrek* $P^{//}Q^{//}R^{//}S^{//}$ Perimeter/*Omtrek* PQRS = t × 2 Perimeter/*Omtrek* PQRS $\therefore \quad t = \frac{1}{2}$	✓ 2 Perimeter PQRS/ 2 *omtrek PQRS* ✓ $\frac{1}{2}$ (2) **[17]**

QUESTION/*VRAAG* 7

	A regular octagon has 8 equal sides. Therefore each side subtends an angle of 45° at the centre of the octagon./*'n Reëlmatige oktagoon het 8 gelyke sye. Dus elke sy onderspan 'n hoek van 45° by die middelpunt van die oktagoon.* Angle of rotation is/*Hoek van rotasie is*: $A\hat{O}B = 135°$ $x^{/} = x\cos\theta - y\sin\theta$ $= 8{,}42\cos135° - 20{,}33\sin135°$ $= -20.33$ $y^{/} = y\cos\theta + x\sin\theta$ $= 20{,}33\cos135° + 8{,}42\sin135°$ $= -8{,}42$ B (– 20,33 ; – 8,42) **OR** Draw a sketch:  ΔAOC ≡ ΔBOD (S ; ∠ ; S *or* S ; S ; S) ∴ B($-b$; $-a$) ∴ B(–20,33 ; –8,42)	✓ $\frac{360°}{8} = 45°$ ✓ rotation of 135°/ *rotasie van 135°* ✓ substitution into correct formula ✓ – 20,33 ✓ substitution into correct formula ✓ – 8,42 **[6]** ✓ ✓✓ Sketch drawn showing the equal parts **OR** proving the Δ's ≡ ✓ B($-b$; $-a$) ✓ – 20,33 ✓ –8,42 **[6]**

QUESTION/*VRAAG* 8

8.1.1	$\sin A = \frac{3}{5}$ (given) $\sin(-A)$ $= -\sin A$ $= -\frac{3}{5}$	✓ sin A ✓ value (2)
8.1.2	$\sin^2 A + \cos^2 A = 1$ $\cos^2 A = 1 - \frac{9}{25} = \frac{16}{25}$ $\cos A = -\frac{4}{5}$ $\tan A = \frac{\sin A}{\cos A}$ $= \frac{3}{5} \times -\frac{5}{4}$ $= -\frac{3}{4}$	✓ $\cos^2 A = \frac{16}{25}$ ✓ $\cos A = -\frac{4}{5}$ ✓ ratio (3)
	OR $x = -4$ $\tan A = -\frac{3}{4}$ 	✓ sketch in correct quadrant ✓ $x = -4$ ✓ ratio (3)
8.2.1	cos 214° $=$ cos (180° + 34°) $= -$ cos 34° $= -p$	✓ – cos 34° ✓ $-p$ (2)
8.2.2	cos 68° $=$ cos[2(34°)] $= 2\cos^2 34° - 1$ $= 2p^2 - 1$	✓ cos [2(34°)] ✓ $2p^2 - 1$ (2)

8.2.3	$\tan 56° = \frac{\sin 56°}{\cos 56°}$ $= \frac{\cos 34°}{\sin 34°}$ $= \frac{\cos 34°}{\sqrt{1-\cos^2 34°}}$ $= \frac{p}{\sqrt{1-p^2}}$	✓ identity ✓ co-functions ✓ $\sqrt{1-\cos^2 34°}$ ✓ answer (4)
	OR	
	$y^2 = 1 - p^2$ $y = \sqrt{1-p^2}$ $\therefore \tan 56° = \frac{p}{\sqrt{1-p^2}}$  	✓ sketch with 34° ✓ $y = \sqrt{1-p^2}$ ✓ $90° - 34° = 56°$ ✓ answer (4)
8.3	$\cos 350° \sin 40° - \cos 440° \cos 40°$ $= \cos 10° \sin 40° - \cos 80° \cos 40°$ $= \cos 10° \sin 40° - \sin 10° \cos 40°$ $= \sin 40° \cos 10° - \cos 40° \sin 10°$ $= \sin(40° - 10°)$ $= \sin 30°$ $= \frac{1}{2}$	✓ cos 10° ✓ cos 80° ✓ sin10° ✓ sin 30° ✓ answer (5)
	OR	
	$\cos 350° \sin 40° - \cos 440° \cos 40°$ $= \cos 10° \sin 40° - \cos 80° \cos 40°$ $= \cos 10° \cos 50° - \sin 10° \sin 50°$ $= \cos(10° + 50°)$ $= \cos 60°$ $= \frac{1}{2}$ **NOTE:** There are many solutions.	✓ cos 10° ✓ cos 80° ✓ cos 50° and sin 50° ✓ cos 60° ✓ answer (5) **[18]**

QUESTION/*VRAAG* 9

9.1	$\cos(x-45°)=-2\sin x$ $\cos x\cos 45°+\sin x\sin 45°=-2\sin x$ $\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x=-2\sin x$ $\sqrt{2}\cos x=(-4-\sqrt{2})\sin x$ $\sqrt{2}=\frac{(-4-\sqrt{2})\sin x}{\cos x}$ $\tan x=\frac{\sqrt{2}}{-4-\sqrt{2}}=-0,2612$	✓ expansion ✓ $\frac{\sqrt{2}}{2}$ ✓ simplification ✓ dividing by cos x (4)
9.2	$\tan x=-0,2612...$ $x=165,36°+180°k;\ k\in Z$ $x=-14,64°$ *or* $165,36°$	✓ general solution ✓ ✓ values of x (3)
9.3	T (135° ; 0)	✓ x value ✓ y value (2)
9.4	$f(x)\geq g(x)$ $-14,64°\leq x\leq 165,36°$ **OR** $x\in[-14,64°\ ;\ 165,36°]$	✓ extreme values ✓ notation (2)
9.5	$-135°<x<-90°$ **OR** $x\in(-135°\ ;\ -90°)$	✓✓ extreme values ✓ notation (3)
9.6	$h(x)=\cos(x-45°-45°)$ $=\cos(x-90°)$ $=\sin x$ Answer only: full marks	✓ $\cos(x-90°)$ ✓ $\sin x$ (2) **[16]**

QUESTION/*VRAAG* 10

10.1	In ΔTRQ : $\dfrac{\text{TR}}{\sin\ \text{R}\hat{\text{Q}}\text{T}}=\dfrac{\text{TQ}}{\sin\ \text{Q}\hat{\text{R}}\text{T}}$ $\dfrac{\text{TR}}{\sin\ 60°}=\dfrac{k}{\sin[180°-(\theta+60°)]}$ $\text{TR}=\dfrac{k\sin\ 60°}{\sin[180°-(\theta+60°)]}$ *or* $\text{TR}=\dfrac{k\sin\ 60°}{\sin(120°-\theta)}$	✓ using sine rule ✓ correct substitution ✓ rewrite TR as subject (3)
10.2	In ΔTRS: $\dfrac{\text{RS}}{\text{TR}}=\sin\ \text{R}\hat{\text{T}}\text{S}$ $\text{RS}=\text{TR}.\sin 60°$ $=\dfrac{k\sin 60°}{\sin[180°-(\theta+60°)]}.\sin\ 60°$ $=\dfrac{k\sin 60°}{\sin(120°-\theta)}.\sin\ 60°$ $=\dfrac{k(\frac{\sqrt{3}}{2})^2}{\sin 120°\cos\theta-\cos 120°\sin\theta}$ $=\dfrac{3k}{4\left(\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta\right)}$ $=\dfrac{3k}{2(\sqrt{3}\cos\theta+\sin\theta)}$	✓ using sine ratio ✓ substitution of TR ✓ simplification ✓ $k(\frac{\sqrt{3}}{2})^2$ or $\frac{3k}{4}$ ✓ expansion of denominator ✓ value of sin 120° ✓ value of cos 120° (7) **[10]**

QUESTION/*VRAAG* 11

11.1.1	$f(x) = y = 3 - 2\sin^2 x$ $0 \le \sin^2 x \le 1$ $-2 \le -2\sin^2 x \le 0$ $1 \le 3 - 2\sin^2 x \le 3$ $1 \le y \le 3$ **OR** $y \in [1\ ;\ 3]$ **OR** $f(x) = y = 3 - 2\sin^2 x$ $= 2 + (1 - 2\sin^2 x)$ $= \cos 2x + 2$ $-1 \le \cos 2x \le 1$ $1 \le \cos 2x + 2 \le 3$ $1 \le y \le 3$ **OR** $y \in [1\ ;\ 3]$ **OR** $f(x) = y = 3 - 2\sin^2 x$ $= 3 - 2(1 - \cos^2 x)$ $= 1 + 2\cos^2 x$ $0 \le 2\cos^2 x \le 2$ $1 \le 1 + 2\cos^2 x \le 3$ $1 \le y \le 3$ **OR** $y \in [1\ ;\ 3]$	✓✓ $-2 \le -2\sin^2 x \le 0$ ✓✓ $1 \le 3 - 2\sin^2 x \le 3$ (4) ✓ rewriting ✓ $\cos 2x + 2$ ✓ $-1 \le \cos 2x \le 1$ ✓ $1 \le \cos 2x + 2 \le 3$ (4) ✓ rewriting ✓ $1 + 2\cos^2 x$ ✓ $0 \le 2\cos^2 x \le 2$ ✓ $1 \le 1 + 2\cos^2 x \le 3$ (4)
11.1.2	f has a minimum when $\sin^2 x = 1$ $\therefore$ $\sin x = \pm 1$ $\therefore$ $x = 90°$ *or* $-90°$ **OR** f has a minimum when $\cos 2x = -1$ $\therefore$ $2x = 180°$ *or* $-180°$ $\therefore$ $x = 90°$ *or* $-90°$	✓ $\sin^2 x = 1$ ✓ $90°$ ✓ $-90°$ (3) ✓ $\cos 2x = -1$ ✓ $90°$ ✓ $-90°$ (3)
11.2.1	$LHS = 1 - \cos 2Q$ $= 1 - (1 - 2\sin^2 Q)$ $= 2\sin^2 Q$ $= RHS$	✓ identity (1)
11.2.2(a)	$LHS = \sin 2R$ $= \sin 2[180° - (P + Q)]$ $= \sin[360° - 2(P + Q)]$ $= -\sin 2(P + Q)$ $= -\sin(2P + 2Q)$ $= RHS$	✓ $R = 180° - (P + Q)$ ✓ $360° - 2(P + Q)$ ✓ $-2(P + Q)$ (3)

11.2.2(b)	$LHS = \sin 2P + \sin 2Q + \sin 2R$ $= \sin 2P + \sin 2Q - \sin(2P + 2Q)$ $= \sin 2P + \sin 2Q - [\sin 2P \cos 2Q + \cos 2P \sin 2Q]$ $= \sin 2P + \sin 2Q - \sin 2P \cos 2Q - \cos 2P \sin 2Q$ $= \sin 2P(1 - \cos 2Q) + \sin 2Q(1 - \cos 2P)$ $= \sin 2P(2\sin^2 Q) + \sin 2Q(2\sin^2 P)$ $= 2\sin P \cos P.2\sin^2 Q + 2\sin Q \cos Q.2\sin^2 P$ $= 4\sin P \sin Q(\sin Q \cos P + \cos Q \sin P)$ $= 4\sin P \sin Q(\sin(Q + P))$ $= 4\sin P \sin Q(\sin[180° - (Q + P)])$ $= 4\sin P \sin Q \sin R$ $= RHS$	✓ substitution ✓ expansion ✓ factorising ✓ substitution ✓ identities ✓ factorising ✓ $[180° - (Q + P)]$ (7) **[18]**

TOTAL/*TOTAAL*: 150