# Revised National Curriculum Statement Grades R-9 (Schools)

# Teacher's Guide for the Development of Learning Programmes

## Mathematics

DEPARTMENT OF EDUCATION

Department of Education

Sol Plaatje House
123 Schoeman Street
Private Bag X895
Pretoria 0001
South Africa
Tel: +27 12 312-5911
Fax: +27 12 321-6770

120 Plein Street
Private Bag X9023
Cape Town 8000
South Africa
Tel: +27 21 465-1701
Fax: +27 21 461-8110

http://education.pwv.gov.za

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This document must be read as part of the
Revised National Curriculum Statement Grades R-9 (Schools).

The Revised National Curriculum Statement Grades R-9 (Schools) includes:

1. An Overview
2. Eight Learning Area Statements:
    - Languages
    - Mathematics
    - Natural Sciences
    - Social Sciences
    - Arts and Culture
    - Life Orientation
    - Economic and Management Sciences
    - Technology

# FOREWORD

The majority of South African teachers have grappled with an education system that has been in the throes of rapid transformation sparked by the student cohort of 1976. Throughout the 1980's, education served as one of the focal areas that characterised resistance to the injustices of apartheid.

The 1990's, and the advent of change characterised by negotiations, saw the education system enter the current period where changes in education reflected systematic initiatives, research-based programmes and policy-driven, large-scale transformation. Teachers are now challenged to exert their professional judgment, curriculum expertise, teaching prowess and management skills in the interest of learners, schools, communities and the nation.

We are convinced that teachers implementing Curriculum 2005 have gained skills, experience, knowledge and techniques that have provided them with a base for engaging with the Revised National Curriculum Statement Grades R-9 (Schools). This Teacher's Guide for the Development of Learning Programmes builds on and enhances that base.

The Revised National Curriculum Statement Grades R-9 (Schools) will be implemented in schools by means of Learning Programmes. Learning Programmes are structured and systematic arrangements of activities that promote the attainment of Learning Outcomes and Assessment Standards for the phase. Learning Programmes ensure that all Learning Outcomes and Assessment Standards are effectively pursued and that each Learning Area is allocated its prescribed time and emphasis. Learning Programmes are based on relationships amongst outcomes and Assessment Standards without compromising the integrity of Learning Areas.

These Guidelines have been produced as a support mechanism to teachers. Over time, teachers will enhance their capacity to develop their own Learning Programmes. These Learning Programmes will take cognisance of the diverse learning contexts, availability of resources, different learning styles, multiple intelligences of learners and the barriers learners may experience.

These Guidelines are geared to assist teachers in accommodating Learning Outcomes and Assessment Standards that are prescribed, yet create space and possibilities for the use of judgments and insights based on particular contexts and a diverse learner population. As insights that are informed by practice, research and refinement, emerge from these Guidelines, it is anticipated that over a period of time teachers will develop as curriculum leaders. The majority of teachers within the apartheid education system were not encouraged to be creative, imaginative and lead curriculum development and design. They were controlled followers and were forced to practise through prescription. As a consequence, many teachers were not participants in the exciting process of curriculum development.

The development of these Guidelines was rooted within the framework of the Revised National Curriculum Statement Grades R-9 (Schools). Therefore, it is expected that these Guidelines should be read within a sound understanding of the Revised National Curriculum Statement Grades R-9 (Schools).

Teachers, schools management teams, departmental officials, teacher unions, non-governmental organisations, community-based organisations and service providers are invited to use these Guidelines not as a doctrine but as an enabling mechanism that will contribute to the delivery of quality, life-long learning.

**T.D. Mseleku**
Director-General: Education

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# SECTION 1
# GUIDELINES FOR THE DEVELOPMENT OF LEARNING PROGRAMMES

## 1.1 INTRODUCTION

Curriculum and teacher development theories and practices in recent times have focused on the role of teachers and specialists in the development and implementation of effective teaching, learning and assessment practices and materials. In the Revised National Curriculum Statement Grades R-9 (Schools) (RNCS), mention was made of policy guidelines for Learning Programmes (RNCS, Overview, p.16). It has been agreed that these will now be called Teacher's Guide for the Development of Learning Programmes. As stated in the RNCS, the curriculum is to be implemented in schools by means of Learning Programmes. Teachers are encouraged to develop and implement their own Learning Programmes, and this should happen within the policy framework provided in the RNCS. This Teacher's Guide for the Development of Learning Programmes (hereafter called 'these Guidelines') have been developed to support teachers to do so.

Although this document is primarily written for **teachers** who have to develop their own Learning Programmes, cognisance is taken of the fact that other Learning Programme developers will also use these Guidelines.

## 1.2 PURPOSE OF THE TEACHER'S GUIDE FOR THE DEVELOPMENT OF LEARNING PROGRAMMES

These policy Guidelines have been developed at national level (with provincial participation) to assure that teaching, learning and assessment practices are developed effectively so that learners can achieve the Learning Outcomes as set out in the RNCS.

The purposes of these Guidelines are to guide teachers in the development of a Learning programme by:

- providing Guidelines to teachers on *how to* develop a Learning Programme;
- providing the ***essential features and underlying principles*** of a Learning Programme;
- promoting and encouraging adherence to the RNCS and support for its implementation; and
- providing a framework for teacher development and training.

These Guidelines are intended to be implemented in conjunction with other policies that promote and support education transformation so that the Critical and Developmental Outcomes, which underpin teaching and learning across the South African school curriculum, are attained. For example, the *White Paper 6: Special Needs Education – Building an Inclusive Education and Training System* needs to be read to provide background information on issues related to barriers to learning, as these have crucial impact on what happens in the classroom. The *Assessment Guidelines for Inclusive Education* document stresses the need for alternative teaching and assessment strategies, and provides recommendations on how to overcome barriers to learning. Addressing barriers to learning is an important responsibility of teachers when developing Learning Programmes.

These Guidelines also need to be read together with the RNCS. The RNCS contains Learning Area Statements for each Learning Area that spell out the Learning Outcomes (LOs) and Assessment Standards (AS) per grade and the Overview.

## 1.3 DEFINITIONS

The RNCS indicates that LEARNING PROGRAMMES should be organised as follows:

- Planning for the whole phase. This is called a **Learning Programme.**
- Planning for a year and grade within a phase. This is called a **Work Schedule.**
- Planning for groups of linked activities or single activities. These are called **Lesson Plans.**

### 1.3.1 Learning Programme

A **Learning Programme** is a ***phase-long plan*** that provides a framework for planning, organising and managing classroom practice for each phase. It specifies the scope for teaching, learning and assessment for the phase and is a "structured and systematic arrangement of activites that promote the attainment of Learning Outcomes and Assessment Standards for the Phase" (RNCS Overview, 2002). A Learning Programme is a tool for ensuring that the Learning Outcomes for each Learning Area are effectively and comprehensively attended to in a sequential and balanced way across the phase.

The Learning Programme thus interprets and sequences the Learning Outcomes and Assessment Standards as spelt out in the RNCS into planned teaching, learning and assessment activities for a phase. It spells out what core knowledge and concepts will be used in attaining the Learning Outcomes for the phase. It plans for how different contexts and local realities, like the needs of the community, school and learners, will be considered.

The Learning Programme also considers how integration within and across Learning Areas will happen, as well as what resources are available and needed to deliver teaching and learning activities.

A Learning Programme will in turn, be translated into yearlong, grade specific ***Work Schedules*** and shorter activity-long ***Lesson Plans.***

### 1.3.2 Work Schedule

A **Work Schedule** is a ***yearlong programme*** that shows how teaching, learning and assessment will be sequenced and paced in a *particular grade.* It is a delivery tool, a means of working towards the achievement of the Learning Outcomes specified in the Learning Programme, and incorporates the Assessment Standards that will be achieved in that grade.

### 1.3.3 Lesson Plan

A **Lesson Plan** is the next level of planning and is drawn directly from the Work Schedule. It describes concretely and in detail teaching, learning and assessment activities that are "to be implemented in any given period[of time]" (RNCS Overview, 2002). A Lesson Plan could range

in duration from a single activity to a term's teaching, learning and assessment and, in terms of actual time, may last from a day to a week or a month. It includes *HOW* (i.e. teaching style, approach and methodology) teaching, learning and assessment activities are to be managed in the classroom.

## 1.4 THE PURPOSE OF A LEARNING PROGRAMME, WORK SCHEDULE AND LESSON PLAN

Learning Programmes, Work Schedules and Lesson Plans represent different stages of planning. While the team of teachers that teach in a phase develops a Learning Programme, the teachers of a particular grade within a phase develop a Work Schedule from the Learning Programme. The class/Learning Area teacher, in turn, develops the Lesson Plans for his/her class. At each level of planning more detail is added to that of the previous level as is described below. Quite apart from the detail shown at each stage, the whole process is informed (at each level) by the same and very important factors described in 1.5.

### 1.4.1 From the RNCS to the Learning Programme

A Learning Programme translates the RNCS into phase-long plans that detail (at a minimum):

- The sequencing of Learning Outcomes and Assessment Standards across the phase to ensure a coherent teaching, learning and assessment programme;
- The core knowledge and concepts or knowledge foci selected to be used to attain the Learning Outcomes;
- The context that ensures that teaching and learning is appropriate to the needs that exist in the community, school and classroom; and
- The time allocation and weighting given to the different Learning Outcomes and Assessment standards in the phase.

When developing the Learning Programme teachers also need to ***consider***:

- how integration within and across the Learning Areas will happen;
- the resources needed and those to be used when determining the teaching, learning and assessment activities; and
- any special or national events likely to be included in the school calendar.

These considerations are taken to more depth and given much more detail when planning the Work Schedule and Lesson Plans.

A team planning approach will promote coherence, integration and cohesion in the Learning Programme for the phase. Such an approach also provides for a framework for the development and effective use of Learning and Teaching Support Materials.

### 1.4.2 From the Learning Programme to the Work Schedule

A Work Schedule provides the teachers in a grade with a yearlong programme based on the Learning Programme. It develops on the sequencing, context, and core knowledge and concepts choices made at Learning Programme level. The teachers responsible for the Learning Programme for a particular grade within a phase will produce the Work Schedule for their grade

group by drawing on the Learning Programme for that phase.

In addition to the detail already provided in the Learning Programme, teachers will in developing the Work Schedule, plan:

- The assessment programme for the year. They will need to ensure a spread of different assessment forms across the year in keeping with the assessment guidelines for each Learning Area;
- The use of resources needed; and
- Integration within and across Learning Areas.

### 1.4.3 From the Work Schedule to the Lesson Plan

The Lesson Plan provides detailed structure for teaching, learning and assessment activities. It could range from a single lesson to a few months of activities. It provides the day-to-day details for teaching, learning and assessment. It also enables, for example, events of major importance internationally, nationally or locally, to be incorporated in the curriculum in a structured, yet flexible way. The World Summit on Sustainable Development held in Johannesburg in 2002, national commemoration and holidays, and other examples are opportunities around which a Lesson Plan could be built. The Lesson Plans are designed to ensure opportunities for learners to achieve the Learning Outcomes and Assessment Standards of that Learning Area.

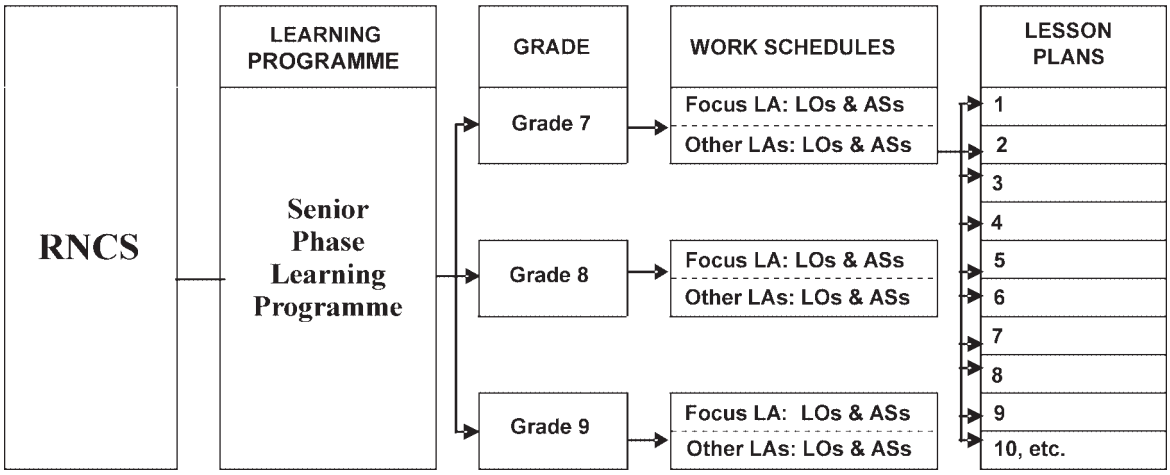
The following elements should be contained in the Lesson Plan:

- Those elements already determined in the Learning Programme and Work Schedule, namely:
  - The Learning Outcomes and Assessment Standards,
  - The context and/or core knowledge and concepts selections for the lesson,
  - The assessment tasks to be used in the lesson,
  - The resources needed for the lessons, and
  - Integration opportunities;
- The actual dates over which the Lesson Plan will stretch;
- Conceptual links to previous and future Lesson Plans;
- Details and sequencing of the teaching, learning and assessment activities that will make up the Lesson Plan;
- Any particular teaching approach and method to be used; and
- Special and important notes regarding the needs of the learners in the class for whom the teacher is preparing the Lesson Plan.

Individual teachers will prepare their own Lesson Plans to support teaching, learning and assessment in their particular classrooms.

The figure below indicates the relationship among the three different stages of planning.

**Figure 1: Relationship among the three different stages of planning.**




**NOTE:** *In the above diagram Lesson Plans are only indicated for Grade 7, but would also appear alongside Grades 8 and 9 in the final column when the Learning Programme has been planned in its entirety.*

## 1.5 ISSUES TO BE CONSIDERED WHEN DEVELOPING LEARNING PROGRAMMES, WORK SCHEDULES AND LESSON PLANS

The aim of a Learning Programme is to design and sequence teaching, learning and assessment activities that will result in meaningful and relevant learning. Teachers need to find ways of making the planning process a manageable one, so that the process of planning is facilitative rather than being a tedious task. For example, there is not much point in rewriting Learning Outcomes and Assessment Standards each time an activity is developed. Numbering the Assessment Standards and then referring to the numbered Assessment Standards may be easier.

To achieve the aim of Learning Programmes, Work Schedules and Lesson Plans, the following aspects have to be considered during planning:

### 1.5.1 Philosophy and Policy

- The RNCS is an embodiment of the nation's social values, and its expectations of roles, rights and responsibilities of the democratic South African citizen as expressed in the Constitution. Full discussion on this section is not included here as it is required that teachers read the RNCS for the discussion and detail on the philosophy and policy underpinning the RNCS.
- Outcomes-based education (OBE) philosophy and practice with the Critical and Developmental Outcomes is the underlying educational philosophy.
- Other national and local policies also impact on effective delivery.

### 1.5.2 Principles Underpinning the Curriculum

The RNCS is underpinned by principles that are crucial for working towards the aims of the education system. These are, amongst others:

- Social Justice
- a Healthy Environment
- Human Rights
- Inclusivity

In simple terms **social justice** refers to one's responsibility to care for others to the common good of society. Social justice serves to remind all humanity (government and civil society) that the needs of all individuals and societies should be met within the constraints imposed by the biosphere, and that all should have equal opportunity to improve their living conditions.

A **healthy environment** cannot be attained independent of people, their lifestyles and choices, their rights and social justice. Environment includes the social, political, economic and biophysical dimensions of all life and life-support systems (air, water and soil).

**Human rights** and their infringement are grounded in the daily experiences of people within their local environments. They are an inextricable part of our lives – so much so that we often take for granted the protection they offer us.

**Inclusivity** deals with a number of social justice and human rights issues, and at the same time taps into the rich diversity of our learners and communities for effective and meaningful decision-making and functioning for a healthy environment. Schools are encouraged to create cultures and practices that ensure the full participation of all learners irrespective of their cultures, race, language, economic background and ability. All learners come with their own experiences, interests, strengths and barriers to learning which need to be accommodated.

In developing Learning Programmes, educators and other curriculum developers will need to pay attention to these principles and to find ways of developing teaching, learning and assessment activities and providing Learning and Teaching Support Materials that offer learners opportunities to explore these principles.

### 1.5.3. Time Allocation and Weighting

- The RNCS overview document details the time allocated to each Learning Area in both the Intermediate and Senior phases, and to Learning Programmes in the Foundation Phase.
- These Guidelines also make recommendations with respect to how Learning Outcomes should be weighted with respect to each other.

To be able to develop Learning Programmes, teachers need to be aware of these allocations and weightings, as well as how these translate into hours and periods in the school(s) for which they are developing the Learning Programme.

### 1.5.4 Integration

Integrated learning is central to outcomes-based education. The historically fragmented nature of knowledge can be overcome if attention is paid to relevant integration both within Learning Areas, and across Learning Areas.

Teachers need to have a clear understanding of the role of integration within their Learning Programmes. The key, however, is the balance to be struck between integration and conceptual progression. That is, integration must support conceptual development rather than being introduced for its own sake. Teachers must therefore be aware of and look for opportunities for integration both within and across Learning Areas.

### 1.5.5 Resources

Different Learning Areas, and in turn different Learning Programmes, will rely on different resources for their success. Teachers will have to be familiar with the resources needed and the resources available as they develop their programmes. Care should be taken not to develop Learning Programmes where lack of access to resources will discriminate against learners. Teachers must also be sensitive to the limitations of learners who experience barriers to learning and how their progress may be affected by availability of resources.

### 1.5.6 Inclusivity and Barriers to Learning

The RNCS assumes an inclusive approach to teaching, learning and assessment. Learning Programmes need to address any barriers that learners for whom the programme is being developed may experience. Teachers need to be aware of the social, emotional, physical and other needs of the learners as they develop their Learning Programmes. For ensuring that matters of Inclusivity are addressed, teachers need to consider any particular barriers to learning and/or assessment that exist in different Learning Areas and make provision for these when developing Learning Programmes.

### 1.5.7 Differences between Learning Areas and Learning Area Statements

While each of the Learning Area Statements has been developed according to the same framework and philosophy, careful examination will show that subtle differences exist between them. These differences are a natural consequence of the peculiarities of each of the Learning Areas. The implication of the differences between Learning Areas and Learning Area Statements for Learning Programme, Work Schedule and Lesson Plan development is that such development in each Learning Area will have to take note of these peculiarities. Furthermore, as teachers in one Learning Area look for integration opportunities with other Learning Areas, they should be aware of the peculiarities of those other Learning Areas.

Some of the most striking differences are the following:

**Natural Sciences** has a separate chapter (chapter 5) in the Learning Area Statement that lists "Core Knowledge and Concepts" – these provide the context in which at least 70% of teaching, learning and assessment should take place, the other 30% can come from local contexts. The Core Knowledge and Concepts are presented by phase and organised into four main content areas or knowledge strands:

- *Life and Living*
  - Living Processes and Healthy Living
  - Interactions in Environments
  - Biodiversity, Change and Continuity
- *Energy and Change*
  - Energy Transfers and Systems
  - Energy and Development in South Africa

- *Planet Earth and Beyond*
  - Our Place in Space
  - Atmosphere and Weather
  - The Changing Earth
- *Matter and Materials*
  - Properties and Uses of Materials
  - Structures, Reactions and Changes of Materials

**Technology** does not have a separate chapter listing knowledge focus or contexts, but Learning Outcome 2 (Technological Knowledge and Understanding) identifies three core knowledge areas for the Learning Area and organises the Assessment Standards for the Learning Outcome accordingly:

- *Structures*
- *Processing*
- *Systems and Control*

It should also be noted that **Natural Sciences** and **Technology** have the same Learning Outcome 3 (Technology: Technology, Society and the Environment, and Natural Sciences: Science, Society and the Environment). This is a deliberate design feature of these two Learning Areas intended to facilitate integration between the Learning Areas, both short-term integration as well as the combining of the Learning Areas into one Learning Programme in the Intermediate Phase.

**Social Sciences** has a separate chapter (chapter 5) in the Learning Area Statement that lists the "Knowledge Focus Framework." This is grade-by-grade specific and provides knowledge/topics for both History and Geography.

While **Arts and Culture** also does not have a separate chapter on knowledge, the Assessment Standards for each Learning Outcome are classified under the following "art forms":

- *Dance*
- *Drama*
- *Music*
- *Visual Arts*
- *Composite*—only for some Learning Outcomes and only in some Grades

The Arts and Culture Learning Area Statement lists, furthermore, on pages 7 and 8 organising principles for each grade in each phase and for each Learning Outcome in each grade, a further organising principle for the Assessment Standards.

**Languages** do not have a list of knowledge contexts in the same way that some of the Learning Areas already mentioned do. However, lists of *recommended texts* are provided by grade in Chapters 2, 3 and 4 of the Learning Area Statement.

### 1.5.8 Clustering of Assessment Standards

Teachers, when planning assessment activities, recording learner performance and reporting on learner progress will look to the Assessment Standards for descriptions of the level at which learners should demonstrate their achievement of the various Learning Outcomes. Having selected the Learning Outcomes and when planning teaching, learning and assessment, teachers may find that certain Assessment Standards can be grouped or **clustered** together quite naturally.

In **some Learning Area**s (certainly not all), it would not be practical to teach to each and every Assessment Standard for each Learning Outcome. Firstly, the Assessment Standards in those Learning Areas do not stand alone, and secondly, there are simply too many Assessment Standards per Learning Outcome for the teacher to be able to deal with them individually. In such cases, the teacher on examining the Assessment Standards, may realise that they group quite naturally into **clusters** of Assessment Standards. These clusters can in turn be used for planning.

For example, in Mathematics in the Intermediate Phase, there are some eleven Assessment Standards for Learning Outcome 1 (Numbers, Operations and Relationships). An examination of these Assessment Standards suggests that they can quite naturally be organised into the following Assessment Standards clusters:

- Recognising, classifying and representing numbers
- Applications of numbers to problems
- Calculation types involving numbers
- Properties of numbers

The Mathematics Learning Area statement neither clusters nor suggests clustering. While the Mathematics Guideline does suggest clustering and even recommends possible clusters, it is up to the teacher to decide whether or not to cluster the Assessment Standards.

While the clustering of Assessment Standards is something that teachers may choose to do, the following should be noted when clustering Assessment Standards:

- Clustering of Assessment Standards should not occur across Learning Outcomes. Recording and reporting needs to be against Learning Outcomes and the selected Assessment Standards. Clustering Assessment Standards across Learning Outcomes would make reporting and recording impossible.
- Learning Outcomes are never clustered. While we may develop Lesson Plans with more than one Learning Outcome, we would consider this to be an example of integration and not clustering.
- When clustering Assessment Standards, it is not allowed that *new* Assessment Standards are written as a result of the clustering.
- While clustering of Assessment Standards is possible for planning the teaching, learning and assessment activities, teachers record learner performance against the individual Assessment Standards in that cluster.

Guidelines on how to deal with the Learning Outcomes and Assessment Standards of each Learning Area are provided in the Learning Area specific section of each Learning Area's Guideline.

## 1.6 DEVELOPMENT PROCESS

While the development process suggested in this document may appear tightly sequenced and ordered, teachers will, in practice, find themselves going back and forth between steps.

### 1.6.1 Developing a Learning Programme

Once teachers have taken all the philosophy, policy and other issues already described into account, the following steps are suggested as a more detailed guide for this task:

- *Select the Learning Outcomes*
  The Learning Outcomes (and how they are attended to) are what drive the development process. It is important that teachers decide which Learning Outcomes are to be focused on at a particular time and how they are packaged together. The Learning Area specific section that follows will indicate how Learning Outcomes can be packaged or explored.

- *Identify Assessment Standards*
  Teachers need to identify the Assessment Standards (or at least clusters of standards) for each Learning Outcome that will be targeted at a particular time within the Learning Programme. Assessment is planned to ensure that evidence is provided of how learners are performing against the Assessment Standards. When *recording* learner performance, teachers will show how each learner is meeting the Assessment Standard(s) and at what level the Learning Outcomes are being attained. Teachers will then *report* on every learner's performance and progress against the Learning Outcomes.

  More detail on the assessment programme, forms of assessment, and recording and reporting processes for each Learning Area is provided in the Assessment Guidelines for the different Learning Areas.

- *Determine the teaching, learning and assessment context(s) and/or core knowledge and concepts*
  Two main kinds of contexts have been identified for inclusion in Learning Programme development, and where appropriate teachers need to be explicit about these.

  One level is the broad consideration of the social, economic, cultural and environmental contexts of the learners. This can also include the local needs of the learners, of the school and the surrounding community.

  The other level is the Learning Area with contexts unique to the Learning Area and the specifics required by the Learning Area (see 1.5.7 above). Such contexts are reflected in the kinds of examples used, the types of projects given, the language used, the barriers being addressed, and the teaching, learning and assessment activities. Context must make specific provision for learners with disabilities.

When dealing with core knowledge and concepts, teachers must select core knowledge and concepts that address the identified Learning Outcomes and Assessment Standards. In Learning Areas where this information is not provided, teachers need to determine their own.

- *Allocate time*
  Teachers need to allocate appropriate weighting and allocation of time to each Learning Outcome and its associated Assessment Standards – as per the weightings discussed in the Learning Area specific sections of the document. It is also important to check that the time allocated to the Learning Programme is consistent with the time allocations of each Learning Area within the phase.

  After this process, it is recommended that teachers should stand back and examine the Learning Programmes in terms of the various features discussed in this chapter. It is also important to analyse all the Learning Programmes for a phase so that implications of one programme on another in terms of learner work load can be resolved. In this way, it is imagined that the time allocation for each programme will be modified and finalised through continued reflection and refinement.

  Learners who experience barriers to learning must be accommodated through flexibility in terms of time allocated to complete activities. Additional time may be given or alternatively learners may be allowed to complete their tasks at a later stage. There must be recognition of the fact that completing only part of the task also has value. These arrangements are planned as part of the individual support for each learner who has a barrier to learning.

- *Integration and resources*
  Integration and the selection and use of resources have already been discussed in detail in 1.5.4 and 1.5.5 above. Teachers will also need to consider integration and resourcing when planning a Learning Programme. While they may only show the details regarding resources and integration in the Work Schedules, they must apply their minds to these issues at the time of Learning Programme development.

### 1.6.2 Developing a Work Schedule

A Work Schedule must be developed for each year in the Learning Programme. A Work Schedule gives a greater level of detail for each aspect or element of the Learning Programme and adds further detail with respect to other aspects.

It should be emphasized that the process of developing a Work Schedule should not be seen as a process that occurs in a linear way, but as a holistic and integrated process. The following should be considered when developing a Work Schedule:

- *Details from the Learning Programme*
  In developing the Learning Programme decisions have already been taken about the sequencing of Lesson Plans, the Learning Outcomes and Assessment Standards that will be focused on by each Lesson Plan, the selection of contexts and/or knowledge and the time allocation to the Lesson Plans. If necessary the teacher(s) developing the Work Schedule may want to amplify these details.

- *Assessment tasks*
  The Department of Education has developed Assessment Guidelines for each Learning Area. Among other details, these documents spell out the forms of assessment to be completed by each learner in each grade. At the time of planning the Work Schedule, the teacher(s) should decide when to use each of the assessment forms to ensure both their most appropriate application and to spread the assessment demands on the learners evenly across the year.

- *Resources required*
  In developing the Work Schedules, teachers will need to consider in detail the resources that will be required for each Lesson Plan and may need to re-sequence units according to the availability of the resources.

- *Integration*
  In developing the Work Schedule, teachers will have to consider in greater detail, matters of integration. In the case of integration across Learning Areas, this may include meeting with the teachers from the other Learning Area(s) to ensure that the anticipated integration is workable in terms of their respective Work Schedules.

### 1.6.3 Developing a Lesson Plan

Lesson Plans are developed from the yearlong Work Schedule by individual teachers. A Lesson Plan is assumed to be a complete and coherent series of teaching, learning and assessment activities. It can consist of a single activity or several activities spread over a few days or a number of weeks.

In as much as Learning Programme and Work Schedule design is influenced by philosophy, policy and several other factors already discussed in 1.5, Lesson Plan development is further informed by the classroom realities of the teacher's class.

Realities of the classroom that have an impact on planning a Lesson Plan include:

- *Learning styles*
  Since different learners have particular and preferred learning styles, every class is certain to contain groups of learners who assimilate information and develop understanding in different ways. Before a teacher is able to develop a Lesson Plan s/he must have a clear sense of the different learning styles of the learners in the class. S/he must also have a sense of those activities that are likely to succeed with particular individuals or groups and those that are unlikely to, and must plan to accommodate all learners in the class.

- *Teaching approach and methodology*
  Teachers must decide how they will approach their teaching and what methods they will use. The nature of the Learning Area often determines what approach and which methods will best support the teaching, learning and assessment activities in the particular Learning Area.

- *Barriers to learning*
  While it is possible to list many different types of barriers to learning in general, not all of these will apply in every class. Similarly there may be barriers to learning that are particular

to individual learners only. When developing a Lesson Plan the teachers must have a clear sense of barriers to learning that exist in the class so that they can overcome these through the way in which they structure activities and also through the activities that they select.

- *Resources available to the school and class*
  Different schools have access to different types of resources, and so while a particular Lesson Plan may work well in one school, it may fail in another because of a difference in the available resources — both types and quantity — available to teachers and their classes.

- *What learners already know*
  It is important to be aware of the prior learning that is both required for different Lesson Plans and the levels of this prior learning present in the class for whom the teacher is developing a Lesson Plan. Learners could demonstrate different levels of knowledge and concept development from the same learning experience. What learners already know becomes an important point of departure for planning what will happen next in an activity.

  At times teachers may wish to perform some form of baseline assessment to be able to establish the level of prior learning and accordingly plan appropriate support for the learners.

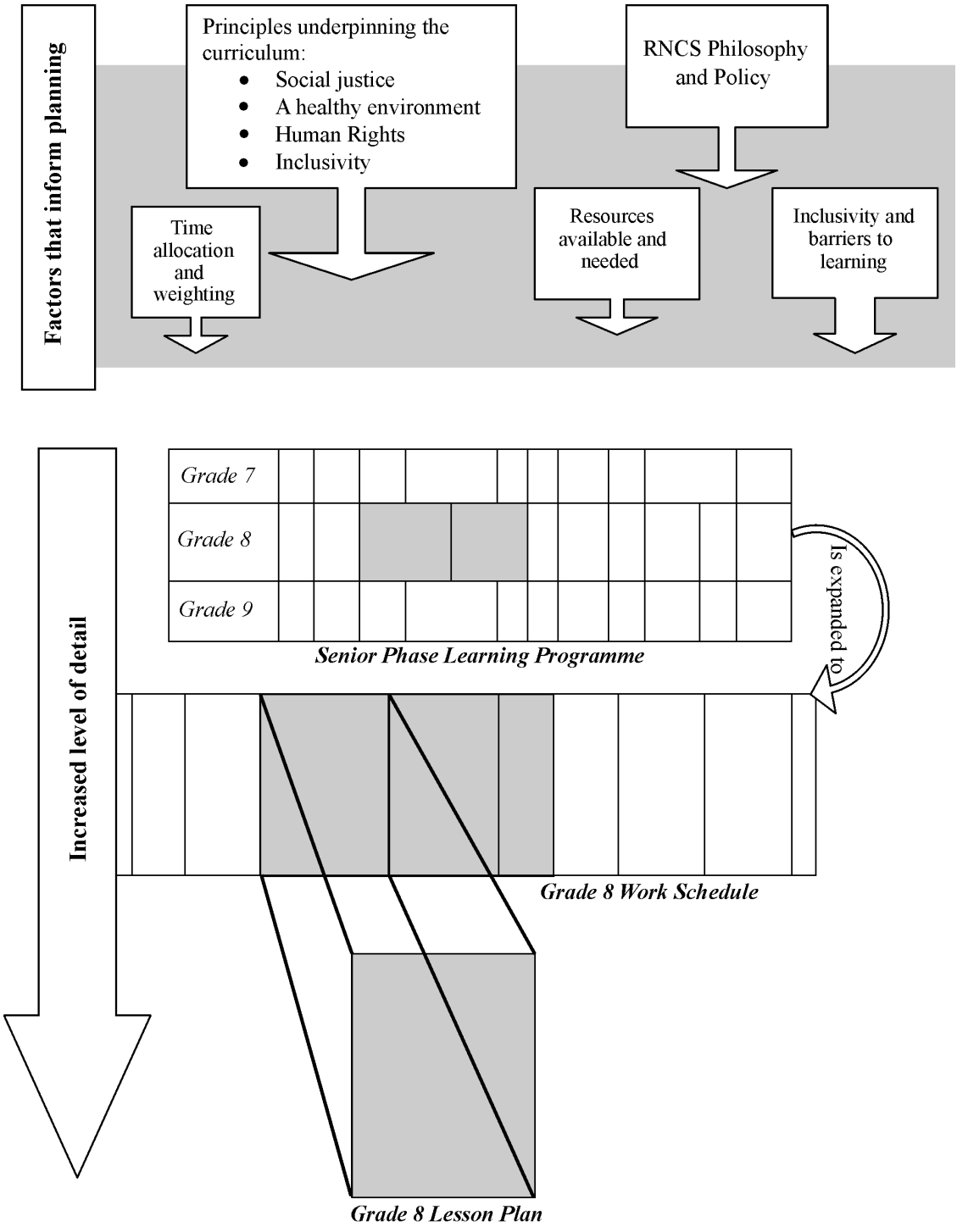
- *School policies*
  In the same way that national education policy will impact on Learning Programme design, so too will the policies of the school impact on both the design of the Lesson Plan and its execution.

  When the above issues have been considered, the teacher is finally ready to develop the Lesson Plan in detail. Within the planning, the teacher has to:
  - Develop and/or source teaching, learning and assessment activities;
  - Identify the role of outcomes and concepts from other Learning Areas;
  - Decide on assessment strategies and select or develop instruments to be used; and
  - Plan how to support learners who experience barriers to learning.

  After a Lesson Plan has been developed, the execution remains. In the execution of a Lesson Plan the teacher will become aware of issues that may not have been anticipated. These will need to be incorporated and should, in turn, be considered when planning the next Lesson Plan(s). Like Learning Programme and Work Schedule development, Lesson Plan development is not a linear process, but rather one of continual modification, reflection, revision and refinement.

The figure below indicates the factors that inform the development of Learning Programmes, Work Schedules and Lesson Plans. It also shows how the levels of planning in a Learning Programme proceed from the RNCS to the Learning Programme to the Work Schedule to the Lesson Plan.

**Figure 2: The planning process and factors taken into account when developing Learning Programmes.**



# 1.7 ASSESSMENT

## 1.7.1 Nature of Assessment

The assessment requirements of the curriculum policy have presented strong challenges to most educators. This section is therefore provided to support the implementation of sound assessment practices.

The assessment practices that are encouraged through the RNCS for Grades R-9 (Schools) are continuous, planned and integrated processes of gathering information about the performance of learners measured against the Learning Outcomes. The level at which the learner is to be assessed is provided by the Assessment Standards which are progressive from grade to grade. A Learning Programme, Work Schedule and Lesson Plan design should ensure that assessment is an integral part of teaching, learning and assessment.

Planning assessment to include the assessment of learners who experience barriers to learning is important. It is likely that in every classroom there would be some learners who experience barriers to learning. However, these barriers will not always be the same and could be situated in the learning context, i.e. inflexible methodology, lack of resources or in the learners themselves, i.e. sensory, physical, intellectual disabilities or disease/illness. They can also arise from the social context, i.e. poverty, violence or difficult home conditions. When planning an assessment activity, the teacher should have a clear sense of the wide range of barriers that may inhibit learning and the achievement of the Learning Outcomes and how to address them. The key is to determine what exactly is being assessed, (i.e. concepts, application, skill) and to develop assessment tasks in such a way that learners have a variety of options to demonstrate their learning with respect to the Learning Outcomes and Assessment Standards as outlined in the RNCS. (For more details on alternative methods of assessment, please refer to *Curriculum 2005: Assessment Guidelines for Inclusion, May 2002.)*

Assessment should:

- enhance individual growth and development, monitor the progress of learners and facilitate learning;
- find out what a learner knows, understands and/or can do;
- make judgements based on valid and appropriate evidence – these judgements should then enable us to make well informed decisions about what a learner needs to learn next;
- give an indication of the success of the programme of learning including how appropriate resources have been;
- include a variety of techniques;
- encourage learners to go beyond simple recall of data or facts;
- close the gap between the classroom and the real world;
- include opportunities for learners to perform tasks and solve problems; and
- make provision for adaptive methods of assessment.

### 1.7.2 Planning for Assessment

Assessment cannot be neutral with respect to what is taught and learned. Any assessment is an expression of values on teaching, learning and assessment. We need to view assessment as a critical and integrated part of the teaching-learning process. As planning for teaching, learning and assessment activities begins with a Learning Programme, planning for assessment should also be integrated in these plans.

When planning for assessment the following documents should provide the framework for planning:

- The Assessment Policy for the General Education and Training Band, Grades R-9 and ABET (December 1998);
- The RNCS (The Overview and the Learning Area Statements);
- Assessment Guidelines for each Learning Area; and
- Assessment Guidelines for Inclusion.

The planning for assessment in the Learning Programme should give schools an indication of resources and time needed for assessment in that phase. To do this teachers need to know what knowledge, skills, attitudes and values the learners are expected to possess so that they are able to integrate the assessment programme within teaching and learning activities.

In a ***Learning Programme*** teachers need to:

- Mention all the **possible forms of assessment** they are likely to use in determining the achievement of the Learning Outcomes. In doing this also take the Assessment Standards into consideration;
- Mention the **resources** they are likely to need (including assistive devices);
- Take the **context and core knowledge and concepts** into consideration; and
- Indicate the **time** that will be needed.

In the ***Work Schedule*** planning for assessment focuses on a grade. When planning a Work Schedule considerations should be given to the following:

- Learning Outcomes give guidance by indicating **what** should be assessed;
- Assessment Standards indicate the **level** at which the Learning Outcome should be assessed;
- Indicate the **assessment strategies** or different forms of assessment teachers plan to use;
- Indicate the **resources** teachers will use; and
- Take into consideration the **diverse needs** of the learners.

In a ***Lesson Plan*** teachers should:

- Indicate **how** the Learning Outcomes would be assessed;
- Consider the **level** at which the Learning Outcomes would be assessed using the Assessment Standards;
- Also consider the **context,** the availability of **resources** and the **diverse needs** of learners; and
- Give a detailed description of how they plan to use the various **assessment strategy(ies)** and/or different form(s) of assessment, how these will be integrated within teaching and learning, and what will be recorded.

For each level of planning in the Learning Programme, the Work Schedule and the Lesson Plan, teachers need to describe the following clearly:

- **When** they are going to assess;
- **How** they are going to assess;
- What **resources** they are going to use; and
- How they are going to support the **diverse needs** of learners.

### 1.7.3 Assessment Strategies

A wide range of assessment strategies may be used to measure learner performance. Teachers can select these depending on the purpose of assessment. These will also depend on a specific Learning Area. The forms/types chosen must provide a range of opportunities for learners to demonstrate attainment of knowledge, skills, values and attitudes. The following are some of the various forms/types of assessment that could be used by the teachers to assess learner achievement:

a) Tests
b) Performance-based assessment
c) Interviews
d) Questionnaires
e) Structured questions
f) Assignments
g) Case studies
h) Practical exercises/demonstrations
i) Projects
j) Role-plays
k) Simulations
l) Aural/Oral Questions
m) Observations
n) Self-report assessment

These assessment strategies and the different forms of assessment for each of the Learning Areas are discussed at length in the Assessment Guidelines for each Learning Area.

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# SECTION 2
# THE MATHEMATICS LEARNING PROGRAMME

## 2.1 SYNOPSIS

We live in an increasingly numerical world, a world in which numbers and quantitative methods are used to describe the world and are used in attempts to exercise control over the world, nature, risk and even life itself. Data, graphs and tables, bombard our senses through television, newspapers and other media. Political, economic and social decisions are informed by and based on numbers. The learner of today needs to be equipped to participate in and contribute to this increasingly numerical world. In South Africa, as in other countries, being at ease with numbers is a prerequisite for full and meaningful participation in society — the study of Mathematics is therefore a human right in itself.

### 2.1.1 Description of the Learning Area

Mathematics:

- Is a powerful tool for describing the world.
- Is the product of investigation by different cultures throughout history.
- Has a beauty that derives from, among other things, its logic, its internal consistency and its coherence.
- Is an important GET Learning Area that develops foundational skills for other study and further study in Mathematics. Skills such as: reasoning, decision making, problem solving, managing resources, interpreting information, understanding systems, applying technology etc., are developed through the study of Mathematics.

### 2.1.2 Learning Area Context

Mathematics has been and can also be misused. Mathematics can be used to misrepresent situations and to justify decisions that are socially or morally unsound. Society uses Mathematics, in preference to any other Learning Area, as a measuring rod to judge the educability/intelligence of its members. Aware of these dangers learners need to learn of the potential for the misuse of Mathematics and teachers need to ensure that the mathematical knowledge, skills and values taught, serve the needs of the learner and society.

### 2.1.3 Learning Outcomes

The Learning Outcomes (LOs) and the Assessment Standards (ASs) in the Mathematics Learning Area statement have been selected with needs of the learner as citizen in mind. Compared with earlier curricula, there is, among other changes:

- An increased focus on data and data handling.
- A shift in the focus of early algebra from algebra as an exercise in manipulation to algebra as a tool for describing situations in order to understand them and make predictions about them.

- A shift in the study of space and shape (geometry) from Euclidean geometry to transformational geometry — the geometry of position and movement.

Knowledge, skills and values are organised in five Learning Outcomes in the Mathematics Learning Area Statement.

### Learning Outcome 1: Numbers, Operations and Relationships

**The learner will be able to recognise, describe and represent numbers and their relationships and to count, estimate, calculate and check with competence and confidence in solving problems.**

### Learning Outcome 2: Patterns, Functions and Algebra

**The learner will be able to recognise, describe and represent patterns and relationships, as well as to solve problems using algebraic language and skills.**

### Learning Outcome 3: Space and Shape (Geometry)

**The learner will be able to describe and represent characteristics and relationships between 2-D shapes and 3-D objects in a variety of orientations and positions.**

### Learning Outcome 4: Measurement

**The learner will be able to use appropriate measuring units, instruments and formulae in a variety of contexts.**

### Learning Outcome 5: Data Handling

**The learner will be able to collect, summarise, display and critically analyse data in order to draw conclusions and make predictions, and to interpret and determine chance variation.**

## 2.2 RELATIONSHIP BETWEEN OUTCOMES

It is important not to think of the Learning Outcomes as independent of each other. It is, for example, impossible to study measurement without having an understanding of numbers and operations involving numbers. Furthermore the learning of Mathematics is developmental, hierarchical and dependent — learners must first be familiar with and be able to use positive whole numbers before they can deal with fractions or negative numbers and these must in turn be internalized before the learner begins to use irrational numbers. Similarly one cannot study compound events involving probability without having an understanding of simple events. Teachers need to be familiar with the interrelationships of concepts both within Learning Outcomes and across Learning Outcomes to ensure that learning and assessment opportunities are structured to account for the interrelated and interdependent nature of mathematical knowledge, skills and values.

Time allocation to the different Learning Outcomes is an important consideration when planning a Mathematics Learning Programme. While it is impossible to be definitive and describe time allocation in terms of hours and minutes the table below is intended to suggest a ratio between the different Learning Outcomes.

**Table 1: Time allocation per Learning Outcome.**

| Time Allocation | Foundation Phase | Intermediate Phase | Senior Phase 7 → 9 |
|---|---|---|---|
| LO 1 | 55% | 40% | 25% → 10% |
| LO 2 | 7 ½ % | 15% | 25% → 35% |
| LO 3 | 30% | 30% | 25% → 20% |
| LO 4 | | | 10% → 10% |
| LO 5 | 7 ½ % | 15% | 15% → 25% |

This table is meant to signal:

- Shifts in focus through the years. The allocation of time to Number, Operations and Relationships (Learning Outcome 1) drops through the years while the allocation to Patterns, Functions and Algebra (Learning Outcome 2) increases. This shift must be seen in the interrelated nature of these two Learning Outcomes, while learners in the early years are developing numerical knowledge and skills, in the later years they use those skills in developing the more generalized language of Mathematics: Algebra. Similarly while there seems to be a large allocation of time to Space and Shape and Measurement (together) in the Foundation Phase, Measurement is a rich context for the development of Numbers, Operations and Relationships and Space and Shape a context for developing the early Algebra skills of pattern recognition.
- The relative importance of the different Learning Outcomes in the development of the Mathematics learner has been described earlier. Notice how Data Handling gains prominence through the years as the learner is increasingly able to deal with more complex data representations, more complex data types and the concept of chance (probability). Data Handling should also be seen as the context where early ideas in graphing are developed — ideas that inform the understanding of algebraic graphs in grade 9.

Another thing that should be considered when planning a Learning Programme is that in exactly the same way that the Learning Outcomes are not conceptually independent, so the time allocation should also not be independent. Time should not be allocated to the Learning Outcomes on a once a year basis but rather a number of time allocations per year as the knowledge and skills developed in one Learning Outcome complement the knowledge and skills to be developed in another.

## 2.3 ASSESSMENT STANDARDS

The Mathematics Learning Area Statement lists Assessment Standards per Learning Outcome per grade. These Assessment Standards are minimum Assessment Standards — that is, they show the minimum that a learner should be able to demonstrate at each grade level. However, minimum should not be interpreted as average — if learners are performing at the minimum level for their grade then their performance is age appropriate. In light of the remarks about deep conceptual understanding, it is far more important for learners to develop deep and meaningful understanding than to be rushed "ahead."

The Learning Outcomes and their Assessment Standards are cognitively dependent and supportive of each other, for example important Number development (Learning Outcome 1) can take place in Measurement (Learning Outcome 4) and Data Handling (Learning Outcome 5) contexts. These cognitive links are reflected in Assessment Standards that sometimes stay the same across one or more grades. Progression in such Assessment Standards should be interpreted in terms of increased knowledge and skills developed between grades in other Learning Outcomes/Assessment Standards. Assessment of such Assessment Standards should take place in the increasingly sophisticated contexts in which learners can work as they progress from one grade to the next.

Within each Learning Outcome it is possible to organise the Assessment Standards into a number of clusters. These clusters can be used to guide the planning of teaching, learning and assessment.

**Table 2: Assessment Standard clusters per Learning Outcome.**

| | LO 1 | LO 2 | LO 3 | LO 4 | LO 5 |
|---|---|---|---|---|---|
| **Intermediate Phase** | Recognising, classifying and representing numbers | Patterns | Shapes and Objects | Time | Collecting and Organising Data |
| | Applications of numbers to problems | Equations | Transformations | Units and Instruments | Representing and Interpreting Data |
| | Calculation types involving numbers | Equivalent representations | Position | Perimeter, Area and Volume | Chance |
| | Properties of numbers | | | | |
| **Senior Phase** | Recognising, classifying and representing numbers | Patterns | Shapes and Objects | Measurement | Collecting and Organising Data |
| | Applications of numbers to problems | Equations | Transformations | | Representing and Interpreting Data |
| | Calculation types involving numbers | Graphs | Position | | Probability |
| | Properties of numbers | Equivalent representations | Straight Line Geometry (8 & 9 only) | | |
| | | Algebraic conventions | | | |

In developing the Assessment Standards for the Learning Area Statement there were several issues that received attention. Some of these (listed in the Overview to the National Curriculum Statement) include issues of: a healthy environment, social justice, human rights and inclusivity as well as indigenous knowledge. There are several Assessment Standards in each Learning Outcome that aim to address these issues. Some examples of such Assessment Standards are listed below:

- Describes and illustrates various ways of counting in different cultures (including local) throughout history (Learning Outcome 1 Intermediate Phase)
- Constructs mathematical models that represent, describe and provide solutions to problem situations, showing responsibility toward the environment and the health of others (including problems within human rights, social, economic, cultural and environmental contexts) (Learning Outcome 2 Senior Phase)
- Recognises and describes natural and cultural 2-D shapes, 3-D objects and patterns in terms of geometric properties (Learning Outcome 3 Intermediate Phase)
- Describes and illustrates the development of measuring instruments and conventions in different cultures throughout history (Learning Outcome 4 Senior Phase)
- Critically reads and interprets data with awareness of sources of error, and manipulation to draw conclusions and make predictions about:
  - social, environmental and political issues (i.e. crime, national expenditure, conservation, HIV/AIDS)
  - characteristics of target groups (i.e. age, gender, race, socio-economic groups)
  - attitudes or opinions of people on issues (smoking, tourism, sport)
  - any other human rights and inclusivity issues (Learning Outcome 5 Senior Phase)

  It is critical that teachers take note of the way in which Assessment Standards such as these have been included in the Learning Area Statement and in turn ensure that they form an integral component of learning, teaching and assessment activities.

## 2.4 TEACHING AND LEARNING IN MATHEMATICS

Believing that all learners can learn Mathematics is critical for the teacher if he/she is to create an environment in which Mathematics is well taught and effectively learnt.

For the most part Mathematical concepts are abstract. The use of concrete objects and apparatus in the early years — indeed in all years — can contribute to the development of understanding and must therefore be encouraged. The use of contexts for learning activities, that is contexts relevant to the lives of the learners can also contribute to understanding and is similarly encouraged. However, it is important that the teacher also recognises that learners eventually need to develop understanding in the absence of concrete objects and contexts. If learners are to develop rich mathematical understanding then they need to be able to "see" mathematical concepts as objects themselves. While the number 2 can be used to denote the number of bottle tops in a pile, in the statement 2 + 5, the number 2 is an object itself — independent of the situation that gave meaning to it. Implications for the teaching, learning and assessment of Mathematics are vast and include for the teacher:

- The recognition that the learning of Mathematics cannot be rushed.
- That learners all learn at a different pace and through different opportunities.
- That the frequent and repeated use of mathematical concepts is critical to learning.

- That understanding develops over time and through the use of Mathematics.

Effective teaching relies on an understanding of Mathematics and an understanding of what learners know, what they need to know and structuring learning opportunities appropriate to the needs of the particular learners that will support and encourage their learning. Different learners learn in different ways and teachers need to be aware of this as they plan teaching activities. The teacher of Mathematics needs to have available, a wide repertoire of teaching strategies that he/she can use effectively to ensure successful learning by all learners (including especially learners with who experience barriers to learning). Among these strategies are:

- Problem posing and problem solving
- Investigation
- Observation
- Modelling
- Reading
- Group work
- Drill and practice
- Following worked examples

There is no hierarchy among these and each has a place in the Mathematics classroom. What is inappropriate is to use one approach to the exclusion of all others. While drill and practice can be used to consolidate the learning of concepts and can lead to mastery of various skills, other approaches such as problem posing and problem solving and investigation are needed to develop understanding.

Quite apart from how learning opportunities are structured in the classroom, learning will only be effective if:

- Learners engage with worthwhile and challenging mathematical tasks. It is important that learners can see the value of the tasks that they are doing. They may be solving problems related to their lives or problems of a purely mathematical nature, either way the value of the Mathematics (the value of the task) must be evident.
- Learners are given opportunities to develop a deep and coherent conceptual understanding of Mathematics. Given its hierarchical nature (and the process-product duality already described) it is critical that learners have an understanding of what they are doing. Performing operations by rote or following a recipe simply will not help the development of understanding and hence mathematical knowledge, skills or values. One of the implications for developing understanding is that learners must have opportunities to negotiate meaning — that is they must be able to discuss their understanding of concepts with each other and their teacher. In the absence of discussion, it is not possible to develop deep conceptual meaning and understanding.
- Learners are able to see the interrelatedness of the Mathematics they learn. Learners should for example recognise that the associative, distributive and commutative laws for numbers are exactly the same for algebraic expressions (i.e. they should recognise that algebraic expressions are simply generalized numbers sentences), that words, number sentences, algebraic expressions, tables and graphs can be equivalent ways of representing a situation, and that the properties of 2-D shapes and 3-D objects are unchanged by transformation, even if the view (perspective) of the figure or object has changed.

### 2.4.1 Integration within Mathematics and across Learning Areas

Integration provides a way for learners to make sense of and consolidate their Mathematics. There are three different types of integration that the teacher should consider:

- Integration between Learning Outcomes within Mathematics
- Integration of Mathematics in other Learning Areas
- Integration of other Learning Areas within Mathematics.

Integration between Learning Outcomes within Mathematics occurs when we use one of the Learning Outcomes as a context for consolidating the knowledge and skills learnt in another Learning Outcome. Measurement (Learning Outcome 4) is a good example of a context in which the learner in the early years is able to consolidate their understanding of Numbers, Operations and Relationships (Learning Outcome 1). Shape and Shape (Learning Outcome 3) and Measurement (Learning Outcome 4) in the Intermediate Phase similarly provide several opportunities for integration that should be explored to strengthen the conceptual development.

When Mathematics is integrated in other Learning Areas, Mathematics becomes a tool for exploring and understanding aspects of the other Learning Area.

***Mathematics in Arts and Culture***

In the Arts and Culture Learning Area Statement we read:

**Learning Outcome 1:** Creating, Interpreting and Presenting
The learner will be able to create, interpret and present work in each of the art forms:

*Grade 5 Assessment Standards (pg 41)*

We know this when the learner:

- Improvises and creates dance sequences that use the concept of contrast, while making clear transitions from one movement or shape to another, focusing on:
  - space (high/low, large/small, forward/sideward/backward, near/far, narrow/wide);
  - time (fast/slow, regular/irregular);
  - force (strong/light, smooth/percussive)
- Improvises and creates dance sequences that explore:
  - the movement range of each body part;
  - geometric concepts such as parallel, symmetry, distance, volume and mass, rectangles, pentagon, hexagon, octagon.

When the Arts and Culture teacher develops a Lesson Plan with this Learning Outcome and these Assessment Standards as a focus they are drawing on Mathematics. In particular they are drawing on the Mathematical knowledge, skills and values developed in Learning Outcome 3 (the Shapes and Objects, Transformation and Position Assessment Standards clusters) and Learning Outcome 4 (the Perimeter, Area and Volume Assessment Standards cluster).

When other Learning Areas are integrated into the Mathematics Learning Programme, the other Learning Area invariably provides a context for making sense of Mathematics.

***Social Sciences in Mathematics***

When the Mathematics teacher is developing a unit of work with Learning Outcome 5 (Data Handling) as the focus Learning Outcome useful contexts can be taken from Social Sciences. The Learning Outcome and associated knowledge from Social Science below illustrates the wide range of data used in that Learning Area which can form the basis of activities in Mathematics work in Learning Outcome 5

**Social Science (Geography)**

**Learning Outcome 3:** Exploring Issues

The learner will be able to make informed decisions about social and environmental issues and problems

**Knowledge Focus Framework for Grade 7 (pg89):**

- Population growth and change:
  - Factors affecting population growth and change (i.e. age and gender structures, population movement, life expectancy, mortality, fertility, aging populations)
  - processes affecting population growth and change (i.e. disease, poverty, attitudes to birth and death, conflict and war, genocide, forced migrations, rural-urban migration, cause-and-effect relationships on different scales, focus on the impact of HIV/AIDS)

Notwithstanding the importance and power of integration, the following should be remembered when planning for integrated activities:

- Conceptual development must not be compromised by integration; integration should support conceptual development by providing a context or opportunity to practice using the concept.
- Learning Outcomes must drive integration and not contexts (or themes). The Learning Programme design question should be "What other Learning Areas/Learning Outcomes will support the conceptual development of 'measuring using informal units'?" rather than "Given the context/theme: Camping, what Mathematics can we include?"

Integration should go beyond the types and examples listed above. Integration also requires collaboration between teachers of different Learning Areas at both the planning and implementation of teaching, learning and assessment. Success with integration will rely on integrated assessment and this needs to be carefully planned for.

### 2.4.2 Skills across Learning Outcomes in Mathematics

In addition to the knowledge, skills and values explicitly listed in the Learning Outcomes and Assessment Standards of the Mathematics Learning Area Statement, there are a number of other skills that need to be developed in learners through the study of Mathematics. These skills are:

- Problem Solving
- Reasoning
- Communication

These skills are overarching and should be developed throughout the phases and across the Learning Outcomes. In designing a Learning Programme attention needs to be given to ensuring that these skills are actively developed at the same time as the knowledge, skills and values listed in the Mathematics Learning Area Statement.

**Problem solving in Mathematics**

Problem solving is a very important skill to be developed in all learners. Problem solving is:

- One of the Critical Outcomes of the GET phase.
- An important skill across all Learning Areas.
- At the heart of Mathematics — Mathematics being a problem solving activity.

Teachers need to recognise that a Foundation Phase problem solver will have different skills and problem solving abilities to a Senior Phase problem solver. However, teachers need to ensure that all learners have a large number of opportunities to solve problems appropriate to their skills and mathematical sophistication. Learners also need to develop the understanding that solving a problem is as important as finding the most efficient and aesthetically appealing solution.

In order to promote the development of problem solving skills, Learning Programmes should include activities that involve:

- Routine problems;
- Non-routine problems;
- Problems that can be resolved through multiple approaches; and
- Open-ended problems

**Routine problems**

Routine problems are problems to which the approach to be followed by the learner is obvious to the learner (these problems are often not even referred to as problems but rather as exercises). Routine problems are often presented for consolidation of concepts — drill-and-practice — when learners need to develop proficiency in the use of a concept. Routine problems contribute to the development of problem solving skills in learners by helping them to recognise that there are situations/problems when the solution method is known and simply needs to be executed.

### Non-routine problems

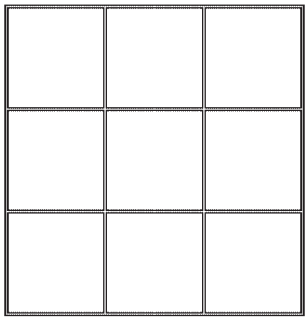
Non-routine problems are problems in which the approach to solving the problem is not obvious from the context. The problem usually has a well-defined solution.

When tackling non-routine problems, learners should employ one or more of the following steps/strategies:

- Understand the problem and identify their objective.
- Compare the problem with more familiar problems in order to select an initial approach.
- Consider simpler cases of the problem.
- Develop conjectures with regard to the solution.
- Use trial and improvement in approaching the problem.
- Evaluate their approaches and solutions with respect to the objective.

**A good example of a non-routine problem is the 3 x 3 magic square:**

Place the numbers: 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the square below so that the numbers in each row, each column and each diagonal add up to the same total.



### Multiple approach problems

Multiple approach problems generally have a single well-defined solution but the problem lends itself to being solved in more than one way — ways that are generally well known to the learner. Learners may also be able to use different forms of knowledge and skills in solving the problem. Problems of this type may, for example, be resolved using both geometry and algebra, or using a graphical approach versus a tabular approach versus an algebraic approach. Problem solving techniques can include: guess and check versus looking for a pattern and/or using a formula.

Teachers should use problems with multiple solution approaches to help learners develop a sense of the interrelatedness of Mathematics. Through doing such problems, learners will also come to understand that the strengths of different approaches are related to the context of a problem and what they are hoping to achieve.

It is important that teachers encourage learners to do problems with multiple solution approaches in a number of the different possible ways.

### Open-ended problems

Open-ended problems are characterised by having neither a well-defined solution nor an obvious approach to solving them. Many of the problems that people experience in their daily lives are open-ended in nature.

When tackling open-ended problems, learners should:

- Ask "what if?" type questions.
- Explore the role and importance of the different variables in the problem.
- Vary their assumptions about the problem to see how this impacts on the problem.

- Be comfortable with taking risks in attempting to solve the problem.
- Be comfortable with investigating different conjectures and possibilities with several of these not bearing fruit.
- Reflect on and evaluate their approaches and progress.

**Reasoning in Mathematics**

The ability to reason or argue a case is not only important in Mathematics but is also an important life-skill in its own right. Mathematical reasoning teaches reasoning skills in general, but mathematical reasoning also has conventions that are peculiar to Mathematics and learners need to recognise this.

Some of the skills related to mathematical reasoning include:

- Analysing
- Selecting
- Synthesising
- Generalising
- Conjecturing

The ability to reason, and in particular to reason mathematically develops with time and experience. The following are typical stages in the development of reasoning skills:

- *Explaining*
  The learner is able to report on what he/she has done, witnessed or considered.
- *Justifying*
  The learner is able to explain why he/she chose a particular course of action or did what he/she did.
- *Convincing*
  The learner is able to convince his/her peers or their teacher that he/she has either understood a situation or performed the most appropriate action for the situation.
- *Proving*
  The learner is able do more than simply convince his/her audience based on his/her actions or understanding. The learner uses mathematical conventions, rules and techniques to build an argument that cannot be refuted.
- *Evaluate*
  The learner is able to evaluate the justification, argument and or proof as presented by his/her peers or somebody else.

Depending on the age of the learner, Learning Programmes need to provide many opportunities for learners to engage in reasoning as described above.

**Communication in Mathematics**

Communication is one of the critical skills to be developed throughout the GET phase. Learning Programmes need to ensure ample opportunities for learners to practice communicating. Within the Mathematics Learning Area learners need to develop the ability to:

- Talk, read and write about Mathematics with understanding
- Listen to and interpret discussions about and involving Mathematics

- Use mathematical code such as symbols and language that is peculiar to Mathematics
- Recognise the use of communication techniques (especially mathematical techniques) to misrepresent situations both mathematical situations and others.

## 2.5 ASSESSMENT IN MATHEMATICS

Assessment is at the heart of and an integral component of teaching and learning in Mathematics. It supports teaching and learning by providing both the teacher and learner with insights on what the learner understands and what the learner still needs to understand or is ready to learn. As with teaching, there is no "right way" to assess. Assessment, however, must:

- Reveal what learners know and are able to do;
- Support learning;
- Be transparent; and
- Cater for the special needs of all learners.

Different assessment strategies will reveal different things about learners. A well-structured multiple-choice item may reveal that a learner understands certain concepts or can perform certain skills with reliability while an extended response answer to an open ended question may reveal that a learner is able to process information and present a logical argument.

Assessment strategies need to be varied not only to assess different things but also to account for differences among learners — differences in terms of talented learners and those who struggle; learners who are learning in their mother tongue language and those who are not, learners in large classes and those in small ones, learners in rural settings and those in urban settings, learners who experience barriers to learning and those who do not experience barriers to learning.

Teachers need to be familiar with the differences between and the role of different assessment types such as:

- Formative assessment
- Summative assessment
- Baseline assessment
- Diagnostic assessment

The Mathematics Learning Area Statement lists Assessment Standards per Learning Outcome per grade. It is important that the teacher recognises that these Assessment Standards are not independent of each other. The Assessment Standards should not be seen as a "checklist" with assessment planned on an Assessment Standard by Assessment Standard basis. Instead, teachers need to cluster the standards in logical ways (see p.37) to ensure that more than one concept, skill or value is assessed at the same time. Consider the following Learning Outcome 3 Assessment Standards in Grade 4:

- Recognises, visualizes and names 2-D shapes and 3-D objects in the environment including:·
  - rectangular prisms, spheres, cylinders, and other shapes
  - prisms and pyramids
  - circles and rectangles
  - polygons in terms of the number of sides up to 8-sided figures

- Recognises and describes natural and cultural 2-D shapes, 3-D objects and patterns in terms of geometric properties
- Describes changes in the view of an object held in different positions

Although these are listed as three separate Assessment Standards, clustered together it is possible to create an assessment task that covers all three.

It is important to see the role of assessment as developmental and contributing to the learning of Mathematics. At the same time we also need to recognise that our learners are increasingly being subjected to systemic assessments by outside agencies i.e. Grade 3, 6 and 9 countrywide testing in Mathematics. Such assessments take on particular forms and it is important that learners also have sufficient experience with such assessment to ensure that they are able to communicate their understanding of concepts and their ability to use different skills.

## 2.6 OVERCOMING BARRIERS TO TEACHING, LEARNING AND ASSESSMENT

Quite apart from general barriers to learning and other issues of inclusive education discussed in the *Teacher's Guide to the South African Classroom* (DoE), there are several Mathematics specific issues that are discussed here. Teachers and Learning Programme developers need to be aware of these and to address them both in the development and delivery of Learning Programmes.

There is several **stereotypes** associated with Mathematics that can act as barriers to the learning of Mathematics. Such stereotypes include among others **gender stereotypes** and **general social expectations** with respect to performance in Mathematics. Some of these stereotypes are that:

- It is acceptable to under-perform in Mathematics because Mathematics is "difficult." This stereotype sometimes leads to a fear of Mathematics among learners.
- Certain groups of people (defined by class, race, and/or rural vs. urban schooling) will be more or less successful in the study of Mathematics
- Girls and boys have different aptitudes for and/or interests in Mathematics and hence perform differently.

There is no basis for any of these stereotypes and teachers and Learning Programme developers need to address these. Some techniques that can be used to challenge stereotypes in the teaching and learning of mathematics include:

- Consciously involving all learners in all learning activities especially any learners who risk being marginalised.
- Ensuring that all contexts are accessible to and interesting to all learners.
- Avoiding gender stereotypes in examples.
- Identifying, referring to and celebrating role models (in terms of careers and mathematical success) from different racial, cultural, socio-economic and gender backgrounds.

**Language** in terms of both the **Language of Learning and Teaching (LOLT)** as well as the **language of Mathematics** can act as a barrier to learning. Everyday language can often have quite a different meaning in Mathematics for example while volume may refer to the setting on a radio in everyday usage, the term is limited to describing capacity in Mathematics. Furthermore, Mathematics (especially algebra) has very specific terminology of its own that can confuse and hence act as a barrier to learning, for

example learners can struggle with the difference in meaning between *algebraic expression* and *equation.* In addition to difficulties with language for learners learning Mathematics in the mother tongue, learners who must learn in a language of instruction that is not their mother tongue will experience still further difficulties. Part of what makes learning Mathematics in non-mother tongue instruction difficult is that there are very poorly developed Mathematical lexicons for most of South Africa's indigenous languages. Another source of difficulty arising from language is the transition that some learners experience in Grade 4 when they must move from learning in their mother tongue to learning in a language of instruction that is not their mother tongue. Some techniques that can be used to address issues of language include:

- Teacher awareness of and empathy toward this barrier. In particular teachers should be aware of which words are likely to cause difficulties.
- Teacher knowledge of appropriate and comparable mother tongue vocabulary.
- Care in the use of language in "word-problems." It is possible to obscure the Mathematics in a problem through the words used resulting in learners being unable to demonstrate their understanding.
- Pacing: teachers should pace lessons to allow for opportunities to check for and negotiate understanding of meaning.
- Code switching: teachers should allow learners to switch codes as they discuss their understanding and Mathematics between each other.

The role of **parents** is important in the study of Mathematics. While parents should play a role in encouraging their children to learn Mathematics — a role that should not be influenced by their own success of lack thereof in Mathematics — they can also play an important role in setting expectations and supporting learning. Schools need to make parents aware that they have an important role to play in the success of their children in learning Mathematics. Parents should:

- Recognise the importance of Mathematics for their children and communicate this to them.
- Expect and encourage their children to succeed in Mathematics.
- Oversee and/or structure opportunities (where possible) for daily practice of Mathematics.

Teachers should also recognise that different parents will have different capacities in terms of their ability and confidence to help their children in their study of Mathematics. Such recognition should influence the nature of mathematical tasks that are set for homework, expectations with respect to homework, and may include creating alternative opportunities for doing homework.

Since **learners learn in different ways** and all **learners learn at a different pace to each other** teachers need to provide different learning opportunities for the different learners in their class. In the same way that learners learn in different ways and at a different pace to each other and teaching must be organized accordingly, so **assessment** must also provide opportunities for all learners to demonstrate their mathematical knowledge, skills and values. Teaching and assessing in one way for all is a significant barrier to learning.

**Learning support materials** can also be thought of as barriers to learning. The absence of learning support materials (in particular the absence of any form of textbook in a school) is a clear barrier to learning. However, it is also important that in those cases where textbooks do exist and are used that they be used coherently, the random selection of topics from different textbook and/or haphazard selection of

activities from one textbook, can hamper conceptual development and as such can be a barrier to learning as well. In those cases where the exercise book of the learner becomes the "text" or record of learning it is as important that this is also organized in a coherent manner. Finally when the Assessment Standards speak of concrete objects such as counters etc. it is important that the teacher recognises that there are many resources in the immediate environment that can substitute for commercial objects. The inability to buy commercial counters need not prevent learners from using counters in their study of Mathematics.

## 2.7 ISSUES IN DESIGNING A LEARNING PROGRAMME, WORK SCHEDULE AND LESSON PLAN FOR MATHEMATICS

### 2.7.1 Issues in Designing a Learning Programme

There are several points to be considered when developing a Learning Programme for Mathematics:

- The time allocation to Mathematics in each phase is determined in the National Curriculum Statement overview document
- Time allocation for each Learning Outcome is discussed in section 2.2 of this document. A first step in developing a Learning Programme may well be the allocation of available time to each Learning Outcome.
- Having allocated time per Learning Outcome, Learning Programme developers must decide on how to further divide the time available into units of work over the years. The choice of both the number of units and the length of these units are reasonably arbitrary and should be informed by the local conditions for which the Learning Programme is being developed.
- The sequencing of the units is at one level arbitrary in that it does not necessarily matter if the year starts with a unit that focuses on Learning Outcome 1 or a unit that focuses on Learning Outcome 3. However, it is important to recognise that many units will be conceptually interdependent in terms of prerequisite knowledge, skills and values and this will influence the sequencing of units.
- The level of detail at which a Learning Programme is developed is something for the developer to decide on. However, it is thought that it is not possible to sequence units effectively unless attention is given to the Learning Outcomes and Assessments Standard clusters that will be targeted by each unit.
- While a Learning Programme is developed at some point in time, it is not fixed for all time. The Learning Programme will have to be subject to revision and review in terms of the actual time that units take to complete versus the anticipated time and the sequencing of the units in terms of the knowledge, skills and values that the learners have versus those that were anticipated in the initial planning of the Learning Programme.

### 2.7.2 Mathematics Issues in Developing a Work Schedule

Developing Work Schedules from Learning Programmes involves increasing the level of detail for each unit in the Learning Programme. It is important that this increased level of detail is planned at the start of the year, as it is through such planning that the teacher can ensure a spread and range of among other things:

- Types of assessment;

- Contexts, especially: human rights, social justice, healthy environment, inclusivity and indigenous knowledge;
- Resources to be used;
- Integration opportunities, both within Mathematics and with other Learning Areas.

### 2.7.3 Mathematics Issues in Developing a Lesson Plan

As the teacher develops lesson plans, it is important that the ideas in this guideline document as well as the Mathematics Learning Area Statement translate into reality. In particular it is important that lesson plans should:

- Provide for a range of teaching, learning and assessment activities;
- Ensure that the focus of the activities within a unit are on the formation and development of concepts;
- Be worthwhile and challenging mathematically, and that learners should see the value of the tasks that they do; and
- Enable learners to see the Mathematics of the lessons in relation to the Mathematics of the other lessons in a year.

### 2.7.4 Learning Outcome Focus by Phase

In addition to the various general issues raised in the paragraphs above, it is also important that the teacher recognises shifts in focus with respect to the different Learning Outcomes in the different phases. The paragraphs below discuss the focus of teaching, learning and assessment per Learning Outcome in each phase.

#### Learning Outcome 1 focus (Numbers, Operations and Relationships)

**Intermediate Phase focus**

The Learning Outcome 1 Assessment Standards for this phase can be arranged into four clusters, namely Assessment Standards that deal with:

- Recognising, classifying and representing numbers;
- Applications of numbers to problems;
- Different calculation types involving numbers;
- Properties of numbers.

This clustering is useful in planning teaching, learning and assessment activities.

Many of the Assessment Standards for this Learning Outcome consist of a main (stem) sentence with a list of numbers or calculation types that follow. It is important that in reading the Assessment Standards the teacher considers the main (stem) sentence. For example while *"whole numbers to at least 9 digit numbers"* are mentioned, they are mentioned in Grade 6 under the stem sentence which includes *"recognising and representing … in order to describe and compare"* 9 digit numbers are, therefore, **not** the minimum standard for say addition and subtraction.

Mental calculations and the use of the calculator are both mentioned in the Assessment Standards.

There is no contradiction here. It is important that the Intermediate Phase Mathematics learner develops strong mental abilities. Being able to perform mental calculations is important in estimation, judging the reasonableness of both the selection of operations and the quality of a solution, and in developing a strong number sense. By contrast the calculator can be used as a powerful tool of learning. Learners should use calculators to explore patterns and relationships, as well as in verifying solutions and confirming the correctness of their work. Learning Programmes should develop mental and calculator skills in a complimentary manner as opposed to giving either a higher status than the other.

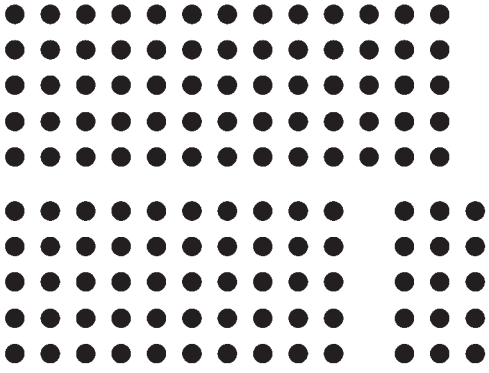
In terms of fractions, the reader of the Assessment Standards will observe that no special mention is made of improper, proper or mixed fractions. This is by design. These classifications of fractions are not seen as a form of progression. Learners should use all of these fraction types from the start and without concern for their different classification.

While the Assessment Standards expect that learners should add and subtract fractions in a contexts this would imply that the fractions added and subtracted in such contexts are fractions that either relate to each other (i.e. ½ and ¼) or come naturally from the contexts. For example it would be considered unusual for a learner to be able to divide a cake into seven equal parts and hence the use of $^1/_7$ would be unusual.

A final comment regarding fractions is to stress that fraction work in the Intermediate Phase should be characterised by work that is concrete in nature. Learners should have constant and easy access to objects such a counters, fraction cards and fraction walls as they work with fractions even in the Assessment Standards that speaks about *"selecting and using operations appropriate to solving problems that involve … ."*

The Assessment Standards state that learners should be able to *"recognise, describe and use"* various properties of numbers i.e. the commutative, associative and distributive properties. In the note that accompanies this Assessment Standard it is further stated that *"the expectation is that learners should be able to use the properties and not necessarily know their names."* It is suggested that learners should have opportunities to develop an understanding of these properties by describing and building patterns that use them. For example:

The students in Miss Moyane's class have been asked to determine how many counters there are in the following pile.



Zakhele writes down:

$5 \times 13 = 65$

Salama rearranges the counters and writes down:

$5 \times 13 = 5 \times (10 + 3)$
$= 5 \times 10 + 5 \times 3$
$= 50 + 15$
$= 65$

Discuss whether or not these are both valid and the advantage of each method.

While ratio and rate are mentioned in the Assessment Standard that deals with *"solve problems that involve,"* it must be stressed that ratio and rate are not important topics as such. That is, there is no expectation that learners will study ratio and rate as such, only that they will use their number skills in problems that involve ratio and rate. The reader will also notice that this Assessment Standard does not show progression in itself from one grade to the next. Progression with respect to this Assessment Standard should rather be seen in problems becoming more complex through the years as a result of changing contexts etc.

There is an Assessment Standard for each grade that deals with *"describes, appreciates, respects and illustrates"* and then addresses one or more aspect of number systems other than our own. It must be understood that this is not an activity that stands alone but rather one that supports learning about our own number system. For example to understand how our own number system works in terms of say place value or base 10, learners should examine number systems that do not use the same conventions.

**Senior Phase focus**

The Learning Outcome 1 Assessment Standards for this phase can be arranged into four clusters, namely Assessment Standards that deal with:

- Recognising, classifying and representing numbers
- Applications of numbers to problems
- Different calculation types involving numbers
- Properties of numbers

This clustering is useful in planning teaching, learning and assessment activities.

Learning Outcome 1 in the Senior Phase needs to be seen as being closely related to Learning Outcome 2 in that number knowledge plays a profound role in the development of algebra. Algebra, in turn, contributes to an understanding of operations involving numbers. The challenge of Learning Outcome 1 in the Senior Phase is the dramatically increased level of abstractness that the learner is expected to deal with in comparison to the Intermediate and Foundation Phases. Unlike the earlier phases the numbers that are introduced in this phase in particular the negative integers have no foundation in concrete or measurement contexts. Not only are negative integers abstract in the sense that they are mathematical constructions but operations involving negative numbers can be as confusing. Teachers and Learning Programme developers need to recognise the huge cognitive hurdle presented by these numbers and operations involving them and hence to ensure that great care is taken in ensuring a careful development of knowledge and skills involving them.

The Senior Phase is also the phase in which learners come into contact with irrational numbers. It is thought that learners should deal with irrational numbers as they arise from a measurement context only as opposed to studying irrational numbers and operations involving them as a separate topic. Great care should also be taken in the introduction of the symbols associated with irrational numbers (i.e. $\pi$, $\sqrt{\ }$ , $\sqrt[3]{\ }$ etc). With respect to calculations involving irrational numbers it is only expected that learners will determine irrational approximations (to an appropriate number of decimal places) using their calculators and then to use these accordingly.

While irrational numbers should arise and be used in a measurement context, it is also important that their historical development be understood. For example, it would be a pity not to use the opportunity to explore the development of $\pi$ or square and cube roots through history.

## Learning Outcome 2 focus (Patterns, Functions and Algebra)

**Intermediate Phase focus**

The Learning Outcomes Assessment Standards for this phase can be arranged into three clusters, namely Assessment Standards that deal with:

- Patterns
- Equations
- Equivalent representations

This clustering is useful in planning teaching, learning and assessment activities.

The importance of Learning Outcome 2 (Patterns, Functions and Algebra) in the Intermediate Phase is in laying the foundation for the study of formal algebra in the Senior Phase while at the same time developing important mathematical thinking skills. The teacher and Learning Programme developers should guard against attempting to introduce formal algebraic techniques or language. The focus should be on the important ideas and skills that learners of algebra need. It is important that learners develop the ability to observe and describe situations and to be able to make predictions about them. This is developed through the study of many different kinds of patterns.

It is important that patterns studied are not limited to patterns of a "skip-counting" nature. Learners need to work with a large range of patterns and pattern types to develop an awareness of:

- Different kinds of change within patterns (i.e. patterns with a constant difference and patterns with a constant ratio as well as patterns where consecutive differences change as well as other pattern types involving triangular, square and Fibonaci numbers etc.)
- The value as well as limitations of patterns in being able to describe and predict (i.e. while it is possible to predict say the time of the sunrise for consecutive days based on a number of recorded times, rainfall next year cannot be predicted based on patterns of rainfall this year and there is no arithmetical relationship between foot size and age)
- Different pattern types (learners need to be able to recognise say that the size of consecutive terms in a pattern with constant difference increase more slowly than the size of consecutive terms in patterns with constant ratio etc.)

Consider the pattern above.
The pattern is made up of a sequence of drawings. Each drawing in the sequence is made of squares and each square is made using lines of the same length.
By considering the pattern in different ways the learner can understand how different (though equivalent) descriptions of the number of lines needed for each pattern in the sequence are possible.

Considering the pattern in this way suggests that the basis of the patterns is a square (4 lines) and to get to subsequent drawings in the pattern three lines are added. This leads to a pattern of the form:

*number of lines = 4 + 3 × (drawing number – 1)*

However, considering the pattern in this way suggests that the basis of the patterns is a single line and to get the first and each subsequent drawing three lines are added.

*number of lines = 1 + 3 × drawing number*

The Assessment Standards mention both numeric and geometric patterns. In the Intermediate Phase geometric patterns are taken to be both pictorial and concrete representations of number patterns as well as patterns in artistic and visual designs. The study of the structure of number patterns is enhanced through concrete representations of such patterns since the concrete representations can often highlight and/or make sense of the numerical structure of the pattern.

While one of the clusters of Assessment Standards has been labelled equations, there is no expectation that learners in the Intermediate Phase will write equations in a formal algebraic way. Equations in the Intermediate Phase are taken to mean descriptions that can take a number of forms such as verbal descriptions, flow diagrams and tables. In terms of using these descriptions to determine input and/or output values, Grade 4 and 5 learners should be able to determine both output values for given input values and input values for given output values for the representations mentioned (verbal descriptions, flow diagrams and tables). The Grade 6 learner should also be able to determine an unknown operation in a number sentence if both the input and output are provided. It is stressed that all Intermediate Phase learners should use and value trial and improvement as the predominant technique when solving these types of problems. Rather than being encouraged to adopt or use algebraic techniques, learners should be encouraged to become more agile in their ability to predict using trial and improvement.

One of the most important mathematical skills that needs to be developed in learners in the GET band is the ability to recognise different representations of a situation as being equivalent. The foundations for this are developed through the study and description of patterns in the Intermediate Phase. Learners need to appreciate that two learners can use different verbal descriptions to describe the same pattern — verbal descriptions that are not only different in words but also in structure (i.e. while one learner may say "I multiplied the number by three," another may say "I added the number to itself and then added the same number to the answer."). This is also the phase where learners develop the awareness that a flow diagram, a table and a verbal description can all be used to describe the same pattern or situation. This is not an easy concept and much time needs to be devoted to helping learners develop this awareness.

**Senior Phase focus**

The Learning Outcome 2 Assessment Standards for this phase can be arranged into five clusters, namely Assessment Standards that deal with:

- Patterns
- Equations
- Graphs
- Equivalent representations
- Algebraic conventions (Grades 8 and 9 only)

This clustering is useful in planning teaching, learning and assessment activities.

While Learning Outcome 2 in Grade 7 is very much an extension of the Intermediate Phase in that a foundation for algebra is being developed through the study of patterns, Grades 8 and 9 are seen as the years in which learners are introduced to Algebra (the language of Mathematics). Teaching and learning activities in Grades 8 and 9 should still involve the study of patterns and these should still be based on concrete experiences, what changes is the expectation that learners should use algebra and algebraic processes in their description of these patterns.

In addition to using algebra to describe patterns, situations and problems, the learner needs to be formally introduced to algebra, algebraic conventions and operations involving algebraic expressions. In particular the learner needs to be comfortable with vocabulary such as variable, relationship, expression, formulae, equation etc. It is, however, important that algebra should not be seen as more than generalised arithmetic and algebraic conventions should be acquired and understood in an arithmetic manner.

In solving equations it is expected that learners should continue to use trial and improvement as an important technique and that algebraic processes should compliment this approach. Learners should become comfortable with the possibility that equations can have more than one solution and that there are also statements with no solutions (false statements) and statements that are true for any number (identities). While learners in the Intermediate Phase are expected to check their solutions to equations as a matter of course, this should continue in the Senior Phase. However learners in the Senior Phase also need to become sensitive to the possibility that their solutions are not valid, for example if these lead to a zero denominator.

The Senior Phase marks the first time that learners begin drawing graphs other than data graphs,

the shift is from graphs of categorical information (data graphs) to graphs describing relationships between variables. The Assessment Standards do not deal with specific algebraic graphs but rather graphs depicting relationships. The Grade 7 learner should deal with graphs that for example show the relationship between electricity usage and time of day or the height of the sun as a function of time of day etc. By Grade 9 the learner should be able to draw graphs for formulae and equations as well as determining formulae and equations from graphs (using tables if needed).

One of the most important skills that the learner can develop in this phase is the ability to convert between different representations of the same situation i.e. from a situation to a table to an equation and/or a graph etc.

## Learning Outcome 3 focus (Geometry)

**Intermediate Phase focus**

The Learning Outcome 3 Assessment Standards for this phase can be arranged into three clusters, namely Assessment Standards that deal with:

- Shapes and Objects
- Transformations
- Position

This clustering is useful in planning teaching, learning and assessment activities.

Learning Outcomes is deliberately called space and shape as opposed to shape. The outcome deals with much more than geometric shapes and objects and their properties, it deals with these from different perspectives and in different positions. In this way it is important to recognise that the Assessment Standard clusters listed above are interrelated and should be taught in an integrated way.

In the Intermediate Phase the study of shapes and objects moves from 3-D shapes to 2-D objects. It is important that learners get the chance to work with concrete shapes and objects as they study them.

The focus of the study of shapes and objects moves from the naming of shapes and objects (in the Foundation Phase) to the properties of these shapes and objects as well as their classification. Properties should be established through investigation and it is anticipated that the learner will investigate these properties through construction and measurement as well as through transformations. For example it is expected that learners should physically place one shape over another in trying to compare them in terms of being exactly the same (congruent) etc.

Learning Outcome 3 is very closely related to Learning Outcome 4 (Measurement) for example to be able to *"describe and classify 2-D shapes and 3 - D objects in terms of properties including: … angle size of corners"* (Learning Outcome 3 - Grade 6) relies on a learner's ability to *"recognise and describe angles in 2-D shapes, 3-D objects and the environment in terms of: right angles; angles smaller that right angles; and angles greater that right angles"* (Learning Outcome 4 - Grade 6). Learning Programmes should recognise the interrelatedness of these Learning Outcomes and Assessment Standards should be structured in such a way that ensures

that these Learning Outcomes are studied in a complimentary manner.

It is also important to stress that space and shape should be studied on an ongoing basis throughout the year. Learning Programmes should be structured in such a way that the study of Learning Outcome 3 and Learning Outcome 4 compliment each other and in such a way that the two are re-visited on several occasions throughout the year.

**Senior Phase focus**

The Learning Outcomes Assessment Standards for this phase can be arranged into four clusters, namely Assessment Standards that deal with:

- Shapes and Objects
- Transformations, Congruence and Similarity
- Position
- Straight Line Geometry (Grades 8 & 9 only)

This clustering is useful in planning teaching, learning and assessment activities.

In the Senior Phase we see a shift in the study of space and shape from investigating geometric figures and solids to establishing their properties and justifying, explaining and in some cases proving these properties. Justifications, explanations and proofs are based on transformations, similarity, congruence and the geometry of straight lines. It is still important that the learner works with concrete objects and builds and constructs these where necessary. The shift is however from using measurement as a basis for determining properties to the interrelationship between figures and solids and their properties.

A strong focus of space and shape in the Senior Phase is on developing mathematical arguments. It is not so much expected that learners will all structure their arguments in a particular way as it is expected that they will become sensitive to the characteristics of a sound argument. Learners need to recognise the difference between a property "measured" in one situation to being able to argue that the property will exist in all situations where certain criteria exist.

Learning Outcome 3 is related to Learning Outcome 4 in terms of angles and angles sizes and Learning Outcome 2 in terms of the study of position and the Cartesian plane.

Teachers and Learning Programme developers should make a determined effort to demonstrate the interrelated nature of the Assessment Standards in this Learning Outcome. For example it is possible to develop the congruence axioms through transformations.

## Learning Outcome 4 focus (Measurement)

**Intermediate Phase focus**

The Learning Outcomes Assessment Standards for this phase can be arranged into three clusters, namely Assessment Standards that deal with:

- Time
- Units and instruments
- Perimeter, Area and Volume

This clustering is useful in planning teaching, learning and assessment activities.

Learning Outcome 4 is closely related to Learning Outcome 3 in terms of the interdependence of the Assessment Standards and to Learning Outcome 1 in terms of the range of numbers that the learner can deal with. For example the precision with which measurements can be made and reported is a function of the context, the instrument being used and the range of numbers that have been dealt with under Learning Outcome 1. It is critical that the developers of Learning Programmes recognise these interrelationships in the Learning Programme design.

As with Learning Outcome 3, Measurement should be dealt with in a very practical manner in the Intermediate Phase. Learners are expected to use measuring instruments to measure dimensions of shapes and objects as well as measuring time. In the study of area and volume, progressions should be seen in terms of being able to approximate area (using square grids and tiling) and volume (by packing or filling) of more complex shapes and objects and in improving counting strategies rather than trying to move towards formulae for these. It is important that the learner develops the notion of what area and volume are and knows ways of estimating each rather than being able to calculate them at this stage.

**Senior Phase focus**

The Learning Outcome 4 Assessment Standards for this phase can be arranged into a single cluster, namely Assessment Standards that deal with:

- Measurement

This clustering is useful in planning teaching, learning and assessment activities.

Learning Outcome 4 in the Senior Phase is very closely linked to both Learning Outcome 1 and Learning Outcome 2. The irrational numbers of Learning Outcome 1 arise from measuring contexts while the numbers studies in Learning Outcome 1 inform the types of rate and ratio problems that can be worked on in Learning Outcome 4. The algebraic conventions and skills learnt in Learning Outcome 2 in turn inform the development and use of formulae used in measurement for the first time in the Senior Phase. This interrelatedness of Learning Outcome 4 suggests that the Learning Outcome will seldom be studied in its own right but rather in conjunction with the other related Learning Outcomes.

Precision and accuracy — important ideas in Senior Phase measurement — are also related to Learning Outcome 1 in terms of the use of significant places and what these mean. A lot of attention needs to be given to these ideas, as learners should become sensitive to their importance in reporting and problems.

## Learning Outcome 5 focus (Data Handling)

**Intermediate Phase focus**

The Learning Outcome 5 Assessment Standards for this phase can be arranged into three clusters, namely Assessment Standards that deal with:

- Collecting and Organising Data
- Representing and interpreting Data
- Chance

This clustering is useful in planning teaching, learning and assessment activities.

There are two important roles that Data Handling (Learning Outcome 5) plays in the Intermediate Phase. Firstly, it lays the foundation for the study of Data Handling in the Senior Phase, and secondly it is the single Learning Outcome that deals with use and misuse of numbers/mathematics. The learner must learn from an early age that data is not neutral — they should be aware that different questions will lead to different answers which in turn leads to different possible interpretations of the data. For example the questions: "What is your favourite television programme?" and "What television programme do you watch most?" are not the same and will certainly give different answers. While a person's favourite television programme — say Ally McBeal — may be flighted only once a week, since they watch sports highlights every day their answers to the two questions would have been: Ally McBeal and sports highlights respectively.

As important as it is to realise that different questions give different answers and hence different interpretations and/or predictions so learners also need to learn that different representations will highlight different aspects of the data. That different representations can also contribute to the misrepresentation of data, is as important as realising that different questions give different answers. So while it is important to learn how to draw the data graphs listed in the Assessment Standards because these graphs are the foundation for algebraic graphs that will follow in the Senior Phase, it is as important that learners develop a sensitivity to the role of representation in shaping the possible interpretations of that data.

Learning Programmes in the Intermediate Phase need to recognise that Learning Outcome 5 is as much (if not more so) about developing sensitivity to the power of data and its possible misuse and misrepresentation as it is about learning the skills of collecting, organising and representing data.

The study of chance (probability) in the Intermediate Phase is very much about recognising that different events can have a different likelihood of occurring. Learners need to work with events that have both a finite number of different possible outcomes and these outcomes need to range in likelihood from impossible to very likely. Absolutely no attempt should be made to develop a formal understanding of probability in this phase. Learners should rather be involved in performing simple experiments, listing the frequency of the different possible outcomes and using these findings to make predictions about repeated experiments. For example learners should be able to make predictions about, say, how many of 100 drawing pins dropped on the table will land point up after having studied a number of experiments involving only 10 drawing pins.

Arising from this is the importance of developing the understanding that while we may predict what fraction of 100 drawing pins will land point-up after collecting a lot of data, we cannot do so with reliability after dropping a single drawing pin on the table. Similarly learners should realise that (with fair die) the outcome of one experiment has no bearing on the next outcome and that there is no difference in the likelihood of rolling a "six" or of rolling a "one." In other words misconceptions such as rolling a double six is "harder" than rolling a double two with two die need to be challenged.

**Senior Phase focus**

The Learning Outcome 5 Assessment Standards for this phase can be arranged into three clusters, namely Assessment Standards that deal with:

- Collecting and Organising Data
- Representing and interpreting Data
- Probability

This clustering is useful in planning teaching, learning and assessment activities.

The Senior Phase learner being more independent is expected to demonstrate a greater role in and responsibility for decisions in the data collection, organisation, summarising, representation and interpretation cycle. That is, the learner is expected to pose their own questions, select their own data sources, choose their own grouping criteria, mode of display and finally to draw justifiable conclusions. The learner is also able to deal with problems of a wider and with a more significant social impact than problems in which they are directly involved. Much attention needs to be given to the interpretation of data in particular the learner needs to demonstrate sensitivity to the role of the setting and techniques for data gathering, the selection of representation and the conclusions that are and can be drawn. New skills that are acquired in the Senior Phase deal with the grouping of data for organisation purposes and the summarising of data in terms of central tendencies etc.

Probability takes on a greater significance in the Senior Phase with a big shift being the introduction of numerical values for probability. Learners should be able to calculate the probability of different outcomes for events, determine though experimentation the relative frequency of outcomes for the same event and be able to compare them. Learners should understand the relationship between relative frequency and probability and the power of each with respect to the other.

# SECTION 3
# INTERMEDIATE PHASE

## 3.1 THE INTERMEDIATE PHASE LEARNER

The Intermediate Phase is important in that it provides for a specific group of learners in the approximate age group 10-12 in grades 4-6. In this phase, learners are:

- Becoming more sensitive to how their actions affect others;
- Beginning to consider the needs, desires and points of view of others;
- Able to function co-operatively in the completion of group tasks with increasing ease;
- Enjoying the challenge of tackling independent tasks;
- Beginning to reveal the desire to take control of their own learning;
- Attempting to satisfy their curiosity about the world around them through active participation and critical enquiry in the learning process;
- Beginning to seek more order; while still manifesting spontaneity and creativity;
- Becoming more deliberate and methodical in their approach;
- Increasingly able to apply acquired methods in new contexts;
- Increasingly able to access, record and manipulate information; and
- Increasingly able to investigate, compare and assess critically.

### 3.1.1 Implications for Teaching, Learning and Assessment in Mathematics

With respect to the study of Mathematics the following characteristics of the Intermediate Phase learner are of great importance:

- The Intermediate Phase learner is starting to think more abstractly
- The Intermediate Phase learner is in the process of establishing and developing moral and social identities. At the same time the learner is also becoming more critical in terms of his/her thinking.
- The Intermediate Phase learner is becoming more independent as a learner and is increasingly able to evaluate his/her own decisions.
- The Intermediate Phase learner is experiencing a shift from being an individual learner to being able to learn as a member of a team.

Recognising these attributes, the Mathematics teacher needs to provide opportunities that take into account the learner's development. The following are strategies that the teacher can employ:

- Considering the learner's ability to think more abstractly, the learner should be expected to engage in abstract thinking. That said, the learner should still be expected to make extensive use of concrete objects as a basis for abstraction.
- In terms of reasoning, the learner should be moving from describing actions and behaviours to justifying these.
- Contexts used in the study of Mathematics need to recognise that the learner is developing moral and social identities and accordingly need to increasingly address issues of concern to the learner. Issues that involve human rights and that require judgements of "fairness" are

particularly appropriate here.

- Given that learners are becoming more independent as learners, they can increasingly manage tasks that: take an extended amount of time to complete, require individual and unsupervised work and which expect the learner to reflect on his/her work rather than the teacher doing all the critiquing.
- Increasingly able to work as a member of a group or team, the learner should be expected to do so. Skills such as turn taking and listening need to be developed while group work still needs to be closely monitored to ensure that all learners feel safe.

Finally it is very important to remember that there are learners who make a transition to non-mother tongue instruction in the Intermediate Phase. This has profound implications for teaching and Learning Programmes need to take special care in providing for such learners.

## 3.2 LEARNING PROGRAMME OPTIONS IN THE INTERMEDIATE PHASE

This section provides guidelines for the development of Learning Programmes in the Intermediate Phase. Three options on how to develop a Learning Programme are discussed in this section. When designing a Learning Programme, Work Schedule or Learning Unit, it is important to remember that Assessment tasks and the recording of learner performance are planned together as part of the activities being developed.

Even though the Department of Education provides guidelines at a national level, provinces will develop further guidelines where necessary in order to accommodate diversity. Schools could propose the number and nature of the integrated Learning Programmes based on their own school contexts. These integrated Learning Programmes must ensure that the prescribed outcomes for each Learning Area are dealt with effectively and comprehensively.

The purpose of developing integrated Learning Programmes is to address the following:

- Even though there are eight Learning Areas, with their own knowledge domains, it is important to remember that knowledge does not exist in isolation. There are natural connections between and across Learning Areas, and knowledge in one Learning Area is relevant and can be used to achieve outcomes in another Learning Area.
- When planning, it is useful to consider using the Learning Outcomes in one Learning Area to enrich another Learning Area. The linkage though should reflect a natural connectedness, and should not be a forced link for the sake of linking with another Learning Area.
- Using the option for integration allows for effective management of available staff and issues of overload in the diverse school contexts that exist. At Intermediate Phase, it may be that the following situations prevail:
  - 1 teacher is responsible for teaching all Learning Areas in a grade
  - 1 teacher is responsible for teaching more than one Learning Area in different grades
  - 1 teacher teaches across Learning Areas and across grades

  (There may be other arrangements in different schools.)

  The options presented are meant to address these situations especially as teachers will develop the Learning Programmes themselves.

It is understood that eight Learning Areas MUST be implemented in the Intermediate Phase. Also Languages and Mathematics will be distinct Learning Programmes. This is important for reporting and recording purposes. Irrespective of how Learning Areas are integrated, reporting is done against the Learning Outcomes of each of the Learning Areas. Teachers will record learner performance against the Learning Outcomes and Assessment Standards selected for developing an activity.

### 3.2.1 What the Integration of Learning Areas means

Integrating Learning Areas should enhance the knowledge, skills, attitude and values embedded in the Learning Outcomes of each Learning Area. Learning Outcomes have been translated into the Assessment Standards relevant to each grade and although planning starts with Learning Outcomes, it is how Assessment Standards are integrated that is important. Some Assessment Standards can stand-alone while others may be clustered with Assessment Standards from other Learning Areas.

The Learning Area content, concepts or themes are NOT the starting point when planning integration. However, they are important vehicles for achieving the outcomes and are to be considered as part of the planning. Each Learning Area has its own concepts and knowledge domain, but achieving knowledge on its own without developing appropriate skills is not what we strive for in an outcomes-based curriculum.

The integration of Learning Areas into Learning Programmes will have implications for planning.

### 3.2.2 Implications for Planning

When planning it is important to take into consideration:

- How well the teacher knows the other Learning Areas in order to be able to integrate and assess effectively.
- The integrity of the Learning Areas must be maintained and learning in each Learning Area must not be compromised.
- Coverage of all Learning Outcomes and Assessment Standards in each LA must be ensured.
- That the planning for Assessment is done as part of the planning for activities. This is important when integrating across Learning Areas as well as when integrating within a Learning Area. It is also important for avoiding overload in teaching and learning, and teachers feeling that they are doing "too much" Assessment.
- That the planning needs to involve all teachers at school and/or at a grade level.
- That the time utilisation must be in line with national policy.
- Different approaches to teaching and learning can be used to support classroom practice. For example, the investigative and practical approach to teaching Natural Sciences will best suit the Learning Outcomes.
- That the effective use of available resources must be planned, and that resources that are not available but needed must be identified and provided.

### 3.2.3 Options to be Considered

The following options are suggested for integrating (or combining) Learning Areas into Learning Programmes, remembering that Languages and Mathematics are distinct Learning Programmes. Planning an integrated Learning Programme always starts with identifying and combining the relevant Learning Outcomes of those Learning Areas being integrated. Even when planning from a Learning Programme to a Learning Unit, teachers start with integrating the Learning Outcomes and Assessment Standards.

***Option 1: Learning Programmes are derived directly from Learning Areas.***

This implies that the Learning Areas themselves are the Learning Programmes.

This does not mean there are no opportunities within the Learning Programme to integrate, especially if it is to enrich teaching, learning and assessment. In this option teachers may want to cluster Learning Outcomes and Assessment Standards within the Learning Area.

Teachers Assess learner performance in line with the Assessment Standards and report against the Learning Outcomes.

***Option 2: One Learning Area is integrated with another Learning Area.***

For example:
- Natural Sciences and Technology, or
- Economic and Management Sciences and Technology, or
- Economic and Management Sciences and Social Sciences, or
- Life Orientation and Arts & Culture, or any other combination of Learning Areas.

In this option, Learning Outcomes of both Learning Areas are matched, and the appropriate Assessment Standards are clustered. There are instances within an integrated Learning Programme where some Learning Outcomes can be dealt with separately.

Teachers are NOT to write new Learning Outcomes or new Assessment Standards.

They Assess against the clusters of Assessment Standards and record against these Assessment Standards as defined in the activity or task. They report against Learning Outcomes.

***Option 3: Short term integration with other Learning Areas***

In this option there is short-term integration across Learning Areas for enrichment where there are natural connections with other Learning Outcomes and Assessment Standards.

For example:
- Life Orientation with SS and A&C
- EMS with TECH and Life Orientation
- EMS with SS (Geography)
- NS with SS (Geography)

- Tech with A&C

(Or any other combinations)

Teachers assess and record against the Learning Outcomes and Assessment Standards of each of the Learning Areas that have been integrated. Reporting is done against the Learning Outcomes.

The illustrations that follow provide examples of how these options are planned in the different Learning Areas.

## 3.3 ILLUSTRATION OF A LEARNING PROGRAMME

The following is an example of one possible phase long programme in Mathematics for the Intermediate Phase. It is important to recognise that this is only one possible phase programme. It is also important to recognise that the level of detail could vary from one phase long programme to another.

**Intermediate Phase Learning Programme in Mathematics.**

<table>
<tr><th>Week</th><th>Grade 4</th><th>Grade 5</th><th>Grade 6</th></tr>
<tr><td>1</td><td rowspan="3"></td><td rowspan="2"></td><td rowspan="3"></td></tr>
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<tr><td>3</td><td rowspan="3"></td></tr>
<tr><td>4</td><td rowspan="2"></td><td rowspan="3">Whole Numbers / Time<br>LO1 & LO4</td></tr>
<tr><td>5</td></tr>
<tr><td>6</td><td rowspan="4">Data Handling I<br>LO5</td><td rowspan="2">Symmetry and Transformations<br>LO3</td></tr>
<tr><td>7</td><td rowspan="3">Common Fractions<br>LO1</td></tr>
<tr><td>8</td><td rowspan="3">Patterns and Functions I<br>LO2</td></tr>
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| 33 | | | |
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| 38 | | | |

**Intermediate phase Learning Programme in Mathematics showing Assessment Standards clusters in Grade 5.**

| Week | Grade 4 | Grade 5 | | Grade 6 |
|---|---|---|---|---|
| 1 | | Whole Numbers I<br>LO1 | • All LO1 assessment standard clusters | |
| 2 | | | | |
| 3 | | Shapes and Objects<br>LO3 & LO4 | • Shapes and objects – LO3<br>• Units & instruments – LO4<br>• Perimeter, Area and Volume – LO4 | |
| 4 | | | | |
| 5 | | | | |
| 6 | | Symmetry and<br>Transformations<br>LO3 | • Transformations | |
| 7 | | | | |
| 8 | | Patterns and Functions I<br>LO2 | • Patterns<br>• Equations<br>• Equivalent representations | |
| 9 | | | | |
| 10 | | | | |
| 11 | | Common Fractions I<br>LO1 | • All LO1 assessment standard clusters | |
| 12 | | | | |
| 13 | | | | |

## 3.4 ILLUSTRATION OF A WORK SCHEDULE

As the Learning Programme is converted to three separate Work Schedules so the level of detail is increased. The illustration below, shows the increased detail for weeks 3 to 10.

**Illustration of Grade 5 Work Schedule details for week 3 to 10.**

| Weeks | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|
| Unit title | Shapes and Objects | | | Symmetry & Transformations (Introduction to different transformations) | | Patterns and functions involving whole numbers | | |
| Learning Outcomes & Assessment Standard clusters | LO3<br>• Shapes and objects<br>LO4<br>• Units & instruments<br>• Perimeter, Area and Volume | | | LO3<br>• Transformations | | LO2<br>• Patterns | | |
| Related Learning Outcomes & Assessment Standard clusters | LO1<br>• Applications of numbers to problems<br>• Calculation types involving numbers | | | None | | LO1<br>• Recognising, classifying and representing numbers<br>• Applications of numbers to problems | | |
| Teaching, Learning and Assessment contexts | • Packaging in daily life<br>• Environmental issues e.g. volumes of garbage by type<br>• Indigenous art form | | | • Introduction to rotational symmetry and the different transformations | | • Review and extension of Grade 4 learning<br>• Contexts involving increased number ranges compared with<br>• Grade 4 | | |
| Forms of assessment | • Homework<br>• Project | | | • Investigation<br>• Classwork | | • Classwork<br>• Test | | |

## 3.5 ILLUSTRATION OF A LESSON PLAN

In developing a unit of learning, the teacher increases the level of detail still further and now gives thought to the selection and sequencing of teaching, learning and assessment activities.

**Illustration of a Grade 5 Lesson Plan.**

| Weeks | 6 | 7 |
|---|---|---|
| Unit title | Symmetry & Transformations (Introduction to different transformations) | |
| Learning Outcomes & Assessment Standard clusters | LO3<br>• Transformations | |
| Related Learning Outcomes & Assessment Standard clusters | None | |
| Teaching, Learning and Assessment contexts | • Introduction the different transformations | |
| Forms of assessment | • Investigation<br>• Classwork | |

| Weeks | 6 | 7 |
|---|---|---|
| Unit title | Symmetry & Transformations<br>(Introduction to different transformations) | |
| Learning Outcomes & Assessment Standard clusters | LO3<br>• Transformations | |
| Related Learning Outcomes & Assessment Standard clusters | None | |
| Teaching, Learning and Assessment contexts | • Introduction the different transformations | |
| Forms of assessment | • Investigation<br>• Classwork | |
| Unit development | • Teacher describes and demonstrates the three transformations. Learners work with own examples using a range of everyday shapes and objects. Focus attention on the correct use of the language of transformations.<br>• Working in pairs learners match objects to images in terms of the transformations between them. First use worksheets with pictures of objects/shapes and their images. Secondly have one learners create objects/shapes and their images for each other and let the second learner identify the transformations.<br>• Investigation:<br>▸ Part 1: Learners are given different pictures of geometric patterns which they must describe in terms of transformations (orally).<br>▸ Part 2: Learners look for and identify geometric patterns in home, school and other environments, sketch the patterns and describe them in terms of transformations<br>▸ Part 3: Learners create their own geometric patterns using at least two different shapes and at least two different transformations (supply dotty paper) | |
| Notes | • Have dotty paper available for use by learners in the different activities<br>• Resources:<br>▸ Pictures with geometric patterns that involve transformations<br>▸ Cut out shapes and models of objects<br>▸ Remember that it is important that learners realise that while a shape/object may have been transformed its properties remain unchanged. | |

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WORKING TOGETHER

# SECTION 4
# SENIOR PHASE

## 4.1 THE SENIOR PHASE LEARNER

In this phase learners should be provided with opportunities to acquire, develop and apply a range of more advanced knowledge, understanding and skills. Breadth, depth, access and entitlement are particularly important to ensure that learners are given a sound basis from which to take advantage of choices at the FET phase. Learners should know enough about the nature of the options to ensure their decisions about future choices are informed ones.

The phase suggests that the essence of the curriculum at the Senior Phase is transitional, to inform choice and to enable independence on the part of the learner. The Senior Phase is there to bridge the gap between consolidation and extension at the Intermediate Phase and choice at the Further Education and Training (FET) Phase. In this phase, it is important to remember that:

- Learners are becoming more independent.
- They are becoming clearer about own interests.
- They mature physically and sexually.
- They mature cognitively and socially and use lateral reasoning.
- They have some understanding of probability, correlation, combinations, prepositional reasoning and other higher level cognitive skills.
- They have the ability to perform controlled experimentation, keeping all but one factor constant.
- They have the ability to hypothesise variables before experimentation to reverse direction between reality and possibility.
- They can also use inter-propositional operations, combining propositions by conjunction, disjunction, negation and implication.
- They continually anticipate the reactions of others to their appearance and behaviour.
- Peer influence plays a major role in their social development.
- They believe that one must be sensitive about infringing on the right of others and always avoid violating rules made by their peers.
- They respect ideas and values of others, but rely on their own intellect and values in making personal decisions.
- Learners further develop abstract thought. They concentrate on thinking in abstract terms and hypothesise and use lateral reasoning. At this level sophistication of thought processes really begins and with appropriate support, the learner can analyse events and have some understanding of probability, correlations, combinations, prepositional reasoning and other higher-level cognitive skills.
- Their capability for abstract thinking influences moral judgement and decisions. They still concentrate on social responsibilities, but are moving towards independent morality.

It is important during this phase to get them focused on critical and creative thinking skills, attitude development and the understanding of their role in society.

With respect to the study of Mathematics the following characteristics of the Senior Phase learner are of great importance:

- The Senior Phase learner is capable of abstract thought, though the level of such abstractness will vary a great deal across learners within the phase.
- The Senior Phase learner has established a moral and social identity even if this will change with time and as a result of interaction with peers. The learner is also able to engage critically with ideas and concepts including ideas beyond his/her immediate environment.
- The Senior Phase learner is becoming more independent as a learner and is increasingly able to evaluate his/her own decisions.
- The Senior Phase learner is able to learn as a member of a team.
- The Senior Phase learner is giving thought to career options and is thinking of making choices with respect to which subjects to study including what form of Mathematics.

Recognising these attributes the Mathematics teacher needs to provide opportunities that take into account the learner's development. The following are strategies that the teacher can employ:

- Considering the learner's ability to think more abstractly, the learner should be expected to engage in abstract thinking as well as tasks involving:
  - Analysis
  - Hypothesising
  - Critical and creative thinking
  - Reflection
- In terms of reasoning the learner should be moving from *justifying* actions and behaviours to *proving* their findings.
- Contexts used in the study of Mathematics need to be selection recognising that the learner has established moral and social identities — irrespective of how dynamic these are. Contexts should also move beyond those that directly involve the learner.
- Given that learners are becoming more independent as learners they can increasingly manage tasks that: take an extended amount of time to complete, require individual and unsupervised work and which expect the learner to reflect on his/her work rather than the teacher doing all the critiquing.
- Increasingly able to work as a member of a group or team, the learner should be expected to do so.

In the Senior Phase it may be useful, when planning a Learning Programme, to think of Grades 8 & 9 as a two-year programme rather than two distinct years. Such an approach would ensure a balance of new concepts per year and also allow for consolidation of concepts in Grade 9. It must also be remembered that for many learners Grade 8 is the year in which they start high school. At some schools the Grade 8s come from many different primary schools with different experiences of Mathematics. As such the Grade 8 learning needs to make provision for revisiting some of the knowledge, skills and values learnt in Grade 7.

## 4.2 ILLUSTRATIONS OF LEARNING PROGRAMMES

The following is an example of one possible phase long programme in Mathematics for the Senior Phase that has been developed using the points raised in section 4.1 of this document. It is important to recognise that this is only one possible phase programme. It is also important to recognise that the level of detail can vary.

**Senior Phase Learning Programme in Mathematics.**

<table>
<tr><th></th><th>Grade 7</th><th>Grade 8</th><th>Grade 9</th></tr>
<tr><td>1</td><td rowspan="4"></td><td rowspan="3"></td><td rowspan="3"></td></tr>
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<tr><td>26</td></tr>
<tr><td>27</td><td rowspan="3">Patterns and Functions II<br>LO2</td></tr>
<tr><td>28</td><td rowspan="3">Patterns and Functions II<br>LO2</td></tr>
<tr><td>29</td><td rowspan="3">Coordinates<br>LO3</td></tr>
<tr><td>30</td><td rowspan="3">Probability<br>LO5</td></tr>
<tr><td>31</td><td rowspan="4">Probability<br>LO5</td></tr>
<tr><td>32</td><td rowspan="4">Congruence, Similarity & Transformations<br>LO3</td></tr>
<tr><td>33</td><td rowspan="3">Data Handling (extended investigation)<br>LO5</td></tr>
<tr><td>34</td></tr>
<tr><td>35</td><td rowspan="3"></td></tr>
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</table>

**Senior Phase Learning Programme in Mathematics showing Assessment Standards clusters in Grade 8.**

| | Grade 7 | Grade 8 | | Grade 9 |
|---|---|---|---|---|
| | | | | |
| 23 | | Perimeter, Area, Volume and Pythagoras LO4 | • Measurement | |
| 24 | | | | |
| 25 | | | | |
| 26 | | | | |
| 27 | | Patterns and Functions II LO2 | • Patterns<br>• Graphs<br>• Equivalent representations | |
| 28 | | | | |
| 29 | | | | |
| 30 | | Probability LO5 | • Probability | |
| 31 | | | | |
| 32 | | | | |
| 33 | | Data Handling (extended investigation) LO5 | • Collecting and organising data<br>• Representing and interpreting data<br>• Probability | |
| 34 | | | | |
| 35 | | | | |
| 36 | | Geometry of Straight Lines LO3 | • Straight line geometry | |
| 37 | | | | |
| 38 | | | | |

## 4.3 ILLUSTRATION OF A WORK SCHEDULE

**Illustration of Grade 8 Work Schedule details for week 24 to 33.**

| Weeks | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|
| Unit title | Perimeter, Area. Volume and Pythagoras | | | | Patterns and Functions (Algebra and Graphs) | | | Probability (Experimental and Theoretical Probability) | | |
| Learning Outcomes & Assessment Standard clusters | LO4<br>• Measurement<br><br>LO1 | | | | LO2<br>• Patterns<br>• Graphs<br>• Equivalent Representations | | | LO5<br>• Probability<br><br>LO5 | | |
| Related Learning Outcomes & Assessment Standard clusters | • Applications of numbers to problems<br>• Calculation types involving numbers<br>LO3<br>• Shapes and objects | | | | LO1<br>• Applications of numbers to problems<br>LO3<br>• Position | | | • Collecting and organizing data<br>LO2<br>• Equations | | |
| Teaching, Learning and Assessment contexts | • Historical development of Pythagoras<br>• Natural Science contexts especially environmental<br>• Technology contexts | | | | • Global graphs with social relevance<br>• Relationship between features of graphs and features of the related algebraic expressions | | | • Games of chance | | |
| Forms of assessment | • Classwork<br>• Project in groups | | | | • Test<br>• Assignment | | | • Investigation<br>• Homework | | |

As the Learning Programme is converted to three separate Work Schedules so the level of detail is increased.

## 4.4 ILLUSTRATION OF A LESSON PLAN

In developing a lesson plan, the teacher increases the level of detail still further and now gives thought to the selection and sequencing of teaching, learning and assessment activities.

**Illustration of a Grade 8 Lesson Plan.**

| **Weeks** | **31** | **32** | **33** |
|---|---|---|---|
| Unit title | Probability (Experimental and Theoretical Probability) | | |
| Learning Outcomes & Assessment Standard clusters | LO5<br>• Probability | | |
| Related Learning Outcomes & Assessment Standard clusters | LO5<br>• Collecting and organizing data<br>LO2<br>• Equations | | |
| Teaching, Learning and Assessment contexts | • Games of chance | | |
| Forms of assessment | • Investigation<br>• Homework | | |
| Unit development | The lesson series is based on the three fairground games found in the *Mathematics Teacher* (March 1992)<br>• Activity 1:<br>▸ Play game 1 in groups<br>▸ Summarise the class results for game 1<br>▸ Interpret results for game 1: Is the game fair or not<br>▸ Analyse game 1 in terms of its features and predict a theoretical win: lose ratio<br>• Activity 2: Repeat Activity 1 for game 2<br>• Activity 3: Repeat Activity 1 for game 3<br>• Activity 4 (working in pairs):<br>▸ Develop own game<br>▸ Predict win: lose ratio based on the structure of the game (theoretical probability)<br>▸ Play game and explain the relationship between the theoretical and experimental probability<br>▸ Write up report on the investigation individually | | |
| Notes | • Learners must supply coins and die for the games<br>• Own games should use spinners, coins, or die only<br>• Use this topic to reinforce fractions and ratios dealt with in weeks 20 to 22<br>• Watch Ken as he does not play games well — he wants to win at all costs | | |

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# SECTION 5
# LEARNING AND TEACHING SUPPORT MATERIALS (LTSMs)

Learning and Teaching Support Material plays an important role in the teaching, learning and assessment of Mathematics. This has been covered in an earlier section, but the following points are highlighted:

- Classroom-based activities can be supported by relevant and appropriate Learning and Teaching Support Material
- Learning and Teaching Support Material can ensure that the direct needs of a particular learner or group of learners are considered.
- Learner-centredness can be reinforced through the use of appropriate Learning and Teaching Support Materials
- Expanded opportunities for enrichment and remediation can be included, and alternative opportunities for learners with disabilities can be included.

There is a range of Learning and Teaching Support Materials in Mathematics. These include:

- *Manipulatives which include:*
  - Cuisenaire rods
  - Counters
  - Shapes and Objects
  - Number charts
  - Posters
  - Diene's blocks
- *Maths Instruments:*
  - Chalkboard
  - Personal
- *Technology:*
  - Computer software
  - Calculators
  - Educational videos
- *Worksheets*
- *Teacher (maths) journals*
- *Textbooks*

Teachers can generate their own Learning and Teaching Support Materials. However, teacher-generated Learning and Teaching Support Materials are not the only Learning and Teaching Support Materials than can be used effectively in a classroom. A range of Learning and Teaching Support Materials is commercially available. Where teachers are unable to purchase Learning and Teaching Support Materials, many of the above can be made. What is important in mathematics teaching is to allow learners to develop from the concrete to the abstract. Learning and Teaching Support Material should play a role in providing the concrete. The teacher is strongly advised to have a supply of different publications for personal use, recognising that all books have

strengths and weaknesses. Publications need to lend themselves to a wide variety of contextual factors. The publications need to be suitable for the learners within the school and for the context within which they will be learning. Not all publications will necessarily be suitable for your specific school with its own peculiar set of contextual factors.

Finally, it is important to note that the provision of a supply of suitable Learning and Teaching Support Materials will not automatically lead to an improvement in the teaching, learning and assessment of the Mathematics. The teacher needs to devise and implement a system of the effective use of these support materials.

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