education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

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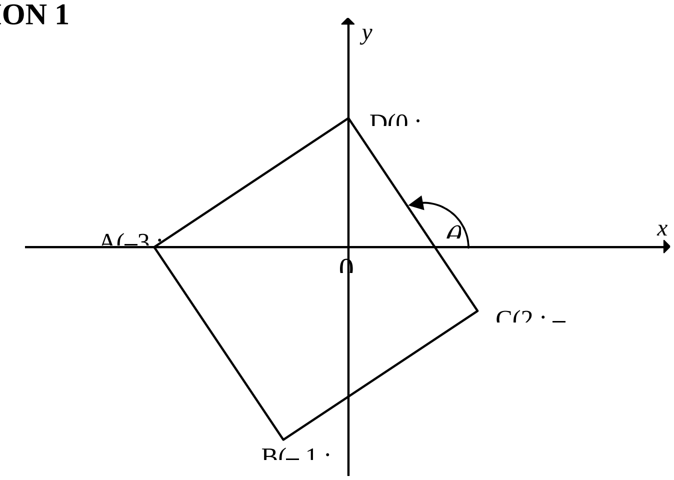
MARKS: 150

This memorandum consists of 25 pages.

- **Continued accuracy applies as a rule in the memorandum.**
- **If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.**

QUESTION 1



1.1	M $\left(\frac{2-3}{2};\frac{-1+0}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓ answer for both coordinates (2) Answer only: 1 mark per coordinate Wrong formula: 0 / 2
1.2	Midpoint BD $=\left(\frac{-1+0}{2};\frac{-3+2}{2}\right)$ $=\left(-\frac{1}{2};-\frac{1}{2}\right)$ $\therefore$ Midpoint of AC and BD are the same point therefore AC and BD bisect each other **OR**	✓ substitution into formula ✓ answer ✓ conclusion (midpoints are the same) (3)

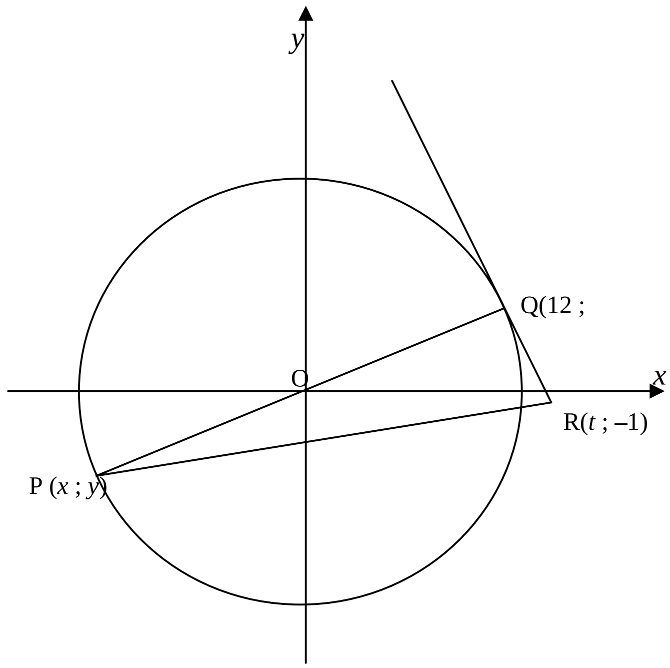
	$AM=\sqrt{\left(-3+\frac{1}{2}\right)^2+\left(0+\frac{1}{2}\right)^2}$ $AM=\sqrt{6,5}$ $CM=\sqrt{\left(2+\frac{1}{2}\right)^2+\left(-1+\frac{1}{2}\right)^2}$ $CM=\sqrt{6,5}$ $BM=\sqrt{\left(-1+\frac{1}{2}\right)^2+\left(-3+\frac{1}{2}\right)^2}$ $BM=\sqrt{6,5}$ $DM=\sqrt{\left(0+\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2}$ $DM=\sqrt{6,5}$ AC and BD bisect each other	2 / 3 for answer on the left (because candidate did not show that M is on BD)
1.3	$m_{AD}=\frac{2-0}{0+3}$ $m_{AD}=\frac{2}{3}$ $m_{CD}=\frac{-1-2}{2-0}$ $m_{CD}=-\frac{3}{2}$ $m_{AD}\times m_{CD}$ $=\frac{2}{3}\times-\frac{3}{2}$ $=-1$ $\therefore AD\perp CD$ $\therefore\ A\hat{D}C=90°$ Note: If do: $m_{AD}\times m_{CD}=-1$ $\frac{2}{3}\times-\frac{3}{2}=-1$ $-1=-1$ then **3 / 4** if calculated the gradients correctly. If $m_{AD}\times m_{CD}=-1$ and conclude AD $\perp$ CD without any working, then **1 / 4** **OR** $\tan\theta=m_{CD}$ $\tan\theta=-\frac{3}{2}$ $\theta=123,69°$ $\tan D\hat{A}C=\frac{2}{3}$ $D\hat{A}C=33,69°$ $A\hat{D}C=123,69°-33,69°$ $A\hat{D}C=90°$	✓ answer m_{AD} ✓ answer m_{CD} ✓ $m_{AD}\times m_{CD}=-1$ ✓ conclude $A\hat{D}C=90°$ (4) ✓ $\tan\theta=m_{CD}$ ✓ $\theta=123,69°$ ✓ $D\hat{A}C=33,69°$ ✓ $A\hat{D}C=90°$ (4)

	OR $AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ $AC^2 = (0-(-1))^2 + (-3-2)^2$ $AC^2 = 26$ $AD^2 + DC^2$ $= 13 + 13$ $= 26$ $= AC^2$ $\therefore$ AD $\perp$ DC $\therefore\ A\hat{D}C = 90°$	✓ $AD^2 = 13$ ✓ $DC^2 = 13$ ✓ $AC^2 = 26$ ✓ conclusion (4)
1.4	$BD = \sqrt{(2+3)^2 + (0+1)^2}$ $= \sqrt{26}$ $AC = \sqrt{(-3-2)^2 + (0+1)^2}$ $= \sqrt{26}$ diagonals are equal diagonals bisect each other (Proved in 1.2) (i.e. ABCD is a rectangle) $m_{AC}.m_{BD}$ $= \frac{1}{-5} \times \frac{5}{1}$ $= -1$ AC $\perp$ BD	✓ answer for BD ✓ answer for AC ✓diagonals are equal ✓ bisect each other ✓ $m_{AC}.m_{BD} = -1$ ✓. AC $\perp$ BD (6)
	OR $AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ The figure is a rectangle and one pair of adjacent sides are equal in length $\therefore$ it is a square. **OR**	✓ substitution ✓ answer for AD ✓ substitution ✓ answer for DC ✓✓ conclusion (6)

	$AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ $AB^2 = (-3-(-1))^2 + (0-(-3))^2$ $AB^2 = 13$ $BC^2 = (2-(-1))^2 + (-1-(-3))^2$ $BC^2 = 13$ All four sides equal and one internal angle equal to 90°	✓ answer for AD ✓ answer for AB ✓ answer for DC ✓ answer for BC ✓ all four sides are equal ✓ one internal angle equal to 90° (6)
	OR	
	The diagonals bisect one another $A\hat{D}C = 90°$ $AD^2 = (2-0)^2 + (0-(-3))^2$ $AD^2 = 13$ $DC^2 = (2-(-1))^2 + (0-2)^2$ $DC^2 = 13$ $\therefore$ adjacent sides equal in length $\therefore$ ABCD is a square	✓ diagonals bisect each other ✓ $A\hat{D}C = 90°$ ✓ substitution into distance formula ✓ answer for AD ✓ answer for DC ✓ conclusion (6)
1.5	$\tan\theta = \frac{2+1}{0-2}$ $\tan\theta = -\frac{3}{2}$ $\theta = -56,30993247\ldots + 180°$ $\theta = 123,7°$	✓ gradient of CD ✓ $\tan\theta = -\frac{3}{2}$ ✓ answer (3)
	OR	
	$\tan D\hat{A}O = \frac{2}{3}$ $D\hat{A}O = 33,7°$ $A\hat{D}C = 90°$ $\theta = 90° + 33,7°$ $\theta = 123,7°$ Penalty 1 for incorrect rounding	✓ $\theta = 90° + D\hat{A}O$ ✓ $\tan D\hat{A}O = \frac{2}{3}$ ✓ answer (3)

1.6	$OC^2 = (2 - 0)^2 + (-1 - 0)^2$ $OC^2 = 5$ $OC = 2{,}236067977$ $OC > 2$ C lies outside the circle OR $OC^2 = (2 - 0)^2 + (-1 - 0)^2$ $OC^2 = 5$ $OC^2 > 4$ C lies outside the circle OR $x^2 + y^2 = 4$ $(2)^2 + (-1)^2 = 5 > 4$ C lies outside the circle	✓ OC^2 ✓ answer (2) Answer only: 0 / 2 **[20]**

QUESTION 2

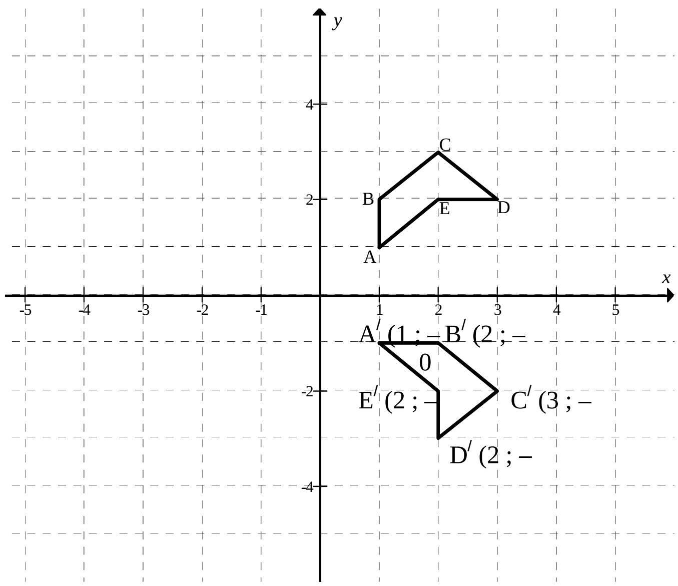


2.1	$r^2 = OQ^2$ $= (5)^2 + (12)^2$ $= 169$ $\therefore x^2 + y^2 = 169$ OR $x^2 + y^2 = (5)^2 + (12)^2 = 169$	✓ substituting (5 ; 12) into $x^2 + y^2$ ✓ 169 ✓ $x^2 + y^2 = 169$ (3) ✓ $x^2 + y^2 = r^2$ ✓ substitution coordinates ✓ 169 (3) Answer only: Full marks
2.2	$m_{PQ} = \frac{5-0}{12-0)}$ $m_{PQ} = \frac{5}{12}$ $\therefore y = \frac{5}{12}x$	✓ gradient ✓ $c = 0$ (2)

2.3	P(– 12; – 5) (By symmetry) **OR** $x^2 + y^2 = 169$ $x^2 + \left(\frac{5}{12}x\right)^2 = 169$ $144x^2 + 25x^2 = 169 \times 144 = 24336$ $169x^2 = 24336$ $x^2 = 144$ $x = \pm 12$ $x = -12$ $y = -5$	✓ $x = -12$ ✓ $y = -5$ (2)
2.4	tangent ⊥ diameter $m_{PQ} \times m_{QR} = -1$ $m_{PQ} = \frac{5}{12}$ $\therefore m_{QR} = -\frac{1}{\frac{5}{12}} = -\frac{12}{5}$ **OR** PQ ⊥ QR $m_{QR} = -\frac{12}{5}$	✓✓ $m_{PQ} \times m_{QR} = -1$ (2) ✓✓ PQ ⊥ QR (2)
2.5	$y = \frac{-12}{5}x + c$ $5 = \frac{-12}{5}(12) + c$ $c = \frac{169}{5}$ $y = -\frac{12}{5}x + \frac{169}{5}$ OR $y = -2{,}4x + 33{,}8$ **OR**	✓ $y = mx + c$ ✓substitution of gradient and (12 ; 5) ✓calculation of *c*. (3)

	$y - y_1 = m(x - x_1)$ $y - 5 = -\frac{12}{5}(x - 12)$ $5y - 25 = -12(x - 12)$ $5y = -12x + 144 + 25$ $5y = -12x + 169$ $12x + 5y - 169 = 0$ $y = -\frac{12}{5}x + \frac{169}{5}$	✓ formula ✓ substitution of gradient and (12 ; 5) ✓equation in correct form (3)
2.6	$-1 = \frac{-12}{5}(t) + \frac{169}{5}$ $12t = 174$ $t = \frac{174}{12}$ $t = 14{,}5$ **OR** $m_{QO} \times m_{QR} = -1$ $\frac{5}{12} \times \frac{-6}{t-12} = -1$ $t = 14{,}5$ **OR** $PQ^2 + QR^2 = PR^2$ $576 + 100 + (12 - t)^2 + 36 = (t + 12)^2 + 16$ $712 + 144 - 24t + t^2 = t^2 + 24t + 144 + 16$ $-48t = -696$ $t = 14{,}5$	✓substitution of $(t\,;-1)$ ✓answer (2) ✓ $\frac{5}{12} \times \frac{-6}{t-12} = -1$ ✓ answer (2) ✓ Pythagoras with substitution ✓ answer (2)
2.7	$(x - 12)^2 + (y - 5)^2 = OQ^2$ $\text{OQ}^2 = (12 - 0)^2 + (5 - 0)^2 = 169$ $(x - 12)^2 + (y - 5)^2 = 169$ **OR** $(x)^2 + (y)^2 = 169$ By translating 12 units right and 5 units up $(x - 12)^2 + (y - 5)^2 = 169$	✓ $(x-12)^2$ ✓ $(y-5)^2$ ✓169 (3) If answer only: $(x-12)^2 + (y-5)^2 = 169$: 3 / 3 **[17]**

QUESTION 3

3.1.1	$P^{/}\left(\sqrt{3};-\sqrt{2}\right)$	✓ x coordinate of $P^{/}$ ✓y-coordinate of $P^{/}$ (2)
3.1.2	$P^{/}(\sqrt{2}\,,-\sqrt{3}\,)$	✓ x coordinate of $P^{/}$ ✓ y-coordinate of $P^{/}$ (2)
3.2.1	$D^{/}(2\,;-3)$ If rotated anti-clockwise: $D^{/}(-2\,;3)$	✓ answer (1) No mark for $D^{/}(-2\,;3)$
3.2.2		✓ coordinates $A^{/}$ ✓ coordinates $B^{/}$ ✓ coordinates $C^{/}$ ✓ coordinates $E^{/}$ ✓ rotation correct (5) If all the points on the sketch are correct and labels are $A^{/}$ etc: 5 / 5 If all the points on the sketch are correct and labels at incorrect point: 4 / 5 Deduct 2 marks for anti-clockwise direction If write down coordinates correctly and did not sketch: 4 / 5

3.2.3	$D^{//}$ $(6\,;-9)$
	
	If rotated anti-clockwise: $D^{//}(-6\,;9)$
		✓ x-coordinate
		✓ y-coordinate
		(2)

3.2.4	$(x\,;y)\rightarrow(y\,;-x)$ $(y\,;-x)\rightarrow(3y\,;-3x)$ $\therefore(x\,;y)\rightarrow(3y\,;-3x)$	✓✓$(y\,;-x)$ ✓✓$(3y\,;-3x)$ (4) Answer only: 4 / 4 If answer $(ky\,;-kx)$ 3 / 4 If Answer: $3(y\,;-x)$

	If rotated anti-clockwise the answer would be: $(x\,;y)\rightarrow(-y\,;x)$ $(y\,;-x)\rightarrow(-3y\,;3x)$ $\therefore (x\,;y)\rightarrow(-3y\,;3x)$	4 / 4

3.2.5	Area ABCDE : area $A^{//}B^{//}C^{//}D^{//}E^{//}$
	$= 1^2 : 3^2$
	$= 1 : 9$
	
	OR
	
	$\dfrac{ABCDE}{A^{//}B^{//}C^{//}D^{//}E^{//}}$
	$=\dfrac{1}{9}$
		✓✓ answer
		(2)
		
		If
		$\dfrac{A^{//}B^{//}C^{//}D^{//}E^{//}}{ABCDE}$
		$=\dfrac{9}{1}$
		0 / 2
		[18]

QUESTION 4

	$x' = x\cos(-45°) - y\sin(-45°)$ $x' = 2\cos 45° + 3\sin 45°$ $x' = 2\left(\dfrac{\sqrt{2}}{2}\right) + 3\left(\dfrac{\sqrt{2}}{2}\right)$ $x' = \dfrac{5\sqrt{2}}{2}$ *or* $x' = \dfrac{5}{\sqrt{2}}$ x' = 3,54 and $y' = y\cos(-45°) + x\sin(-45°)$ $y' = 3\cos 45° - 2\sin 45°$ $y' = 3\left(\dfrac{\sqrt{2}}{2}\right) - 2\left(\dfrac{\sqrt{2}}{2}\right)$ $y' = \dfrac{\sqrt{2}}{2}$ *or* $\dfrac{1}{\sqrt{2}}$ *or* 0,71 $P^{/}\left(\dfrac{5\sqrt{2}}{2}\,;\dfrac{\sqrt{2}}{2}\right)$ A penalty of 2 marks for substituting 45° instead of $-45°$. The answer will then be $\left(-\dfrac{\sqrt{2}}{2}\,;\dfrac{5\sqrt{2}}{2}\right)$ or (– 0,71 ; 3,54)	✓ formula ✓ – 45° or 315° ✓ substitution of $\left(\dfrac{\sqrt{2}}{2}\right)$ or $\left(\dfrac{1}{\sqrt{2}}\right)$ ✓ answer for *x* ✓ formula ✓ substitution of $\left(\dfrac{\sqrt{2}}{2}\right)$ ✓ answer for *y* (7)

	OR	
	If a candidate rotates clockwise and substitutes 45° the formulae will be:	
	$x' = x\cos\theta + y\sin\theta$ $x' = 2\cos 45° + 3\sin 45°$ $x' = 2\left(\frac{\sqrt{2}}{2}\right) + 3\left(\frac{\sqrt{2}}{2}\right)$ $x' = 3{,}54$	✓ formula for x' ✓ 45° ✓ substitution ✓ answer for x'
	$y' = y\cos\theta - x\sin\theta$ $y' = 3\cos 45° - 2\sin 45°$ $y' = 3\left(\frac{\sqrt{2}}{2}\right) - 2\left(\frac{\sqrt{2}}{2}\right)$ $y' = 0{,}71$	✓ formula for y' ✓ substitution ✓ answer for y' (7)
	OR	
	Let OP = OP/ = r = $\sqrt{13}$ The x-coordinate of P = $r\cos(\theta - 45°)$ $x' = r(\cos\theta.\cos 45° + \sin\theta.\sin 45°)$ $x' = \sqrt{13}\cos\theta.\cos 45° + \sqrt{13}\sin\theta.\sin 45°$ $x' = \sqrt{13}.\frac{2}{\sqrt{13}}.\frac{\sqrt{2}}{2} + \sqrt{13}.\frac{3}{\sqrt{13}}.\frac{\sqrt{2}}{2}$ $x' = \sqrt{2} + \frac{3\sqrt{2}}{2}$ $x' = \frac{5\sqrt{2}}{2}$	✓ formula $r\cos(\theta - 45°)$ ✓ expansion ✓ substitution ✓ answer for x
	The y-coordinate of P = $r\sin(\theta - 45°)$ $y' = r(\sin\theta.\cos 45° - \cos\theta.\sin 45°)$ $y' = \sqrt{13}\sin\theta.\cos 45° - \sqrt{13}\cos\theta.\sin 45°$ $y' = \sqrt{13}.\frac{3}{\sqrt{13}}.\frac{\sqrt{2}}{2} - \sqrt{13}.\frac{2}{\sqrt{13}}.\frac{\sqrt{2}}{2}$ $y' = \frac{3\sqrt{2}}{2} - \sqrt{2}$ $y' = \frac{\sqrt{2}}{2}$ P/ $\left(\frac{5\sqrt{2}}{2}\,;\,\frac{\sqrt{2}}{2}\right)$	✓ formula $r\sin(\theta - 45°)$ ✓ expansion ✓ answer for y (7)
	OR	

	$2 = x\cos 45° - y\sin 45°$	✓ formula
	$3 = y\cos 45° + x\sin 45°$	✓ formula
	$2 = \frac{1}{\sqrt{2}}x - \frac{1}{\sqrt{2}}y$	
	$\times\sqrt{2} : 2\sqrt{2} = x - y$ --- (1)	✓ substitution
	$3 = \frac{1}{\sqrt{2}}y + \frac{1}{\sqrt{2}}x$	
	$\times\sqrt{2} : 3\sqrt{2} = x + y$ --- (2)	✓ substitution
	$(1) + (2) \quad 2x = 5\sqrt{2}$	✓ solving simultaneous
	$x = \frac{5\sqrt{2}}{2}$	✓ answer x
	$\therefore 3\sqrt{2} = \frac{5\sqrt{2}}{2} + y$	✓answer y
	$\therefore y = \frac{1}{2}\sqrt{2}$	(7)
	OR	
	$(x'; y') = (r\cos(\theta - 45°) ; r\sin(\theta - 45°))$	
	$x^2 + y^2 = r^2$	
	$2^2 + 3^2 = r^2$	
	$r = \sqrt{13}$	✓ $r = \sqrt{13}$
	$\tan\theta = \frac{3}{2}$	✓ $\tan\theta = \frac{3}{2}$
	$\theta = 56{,}30993247...°$	✓ $\theta = 56{,}30993247...°$
	$x' = r\cos(\theta - 45°)$	✓ $x' = r\cos(\theta - 45°)$
	$x' = \sqrt{13}\cos(56{,}3...° - 45°)$	
	$x' = 3{,}54$	✓ $x' = 3{,}54$
	$y' = r\sin(\theta - 45°)$	✓ $y' = r\sin(\theta - 45°)$
	$y' = \sqrt{13}\sin(56{,}3...° - 45°)$	
	$y' = 0{,}71$	✓ $y' = 0{,}71$
		(7)
		Answer only: 6 / 7
		[7]

QUESTION 5

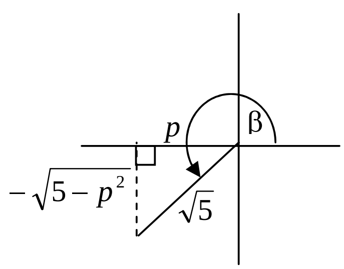
Penalise 1 mark for treating as an equation in this question.

5.1.1	$\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$ $= \frac{\tan 120°.(-\sin 60°).\cos 14°.(-\sin 45°)}{\sin 76°.(-\cos 45°)}$ $= \frac{(-\tan 60°).(-\sin 60°).\cos 14°.(-\sin 45°)}{\cos 14°.(-\cos 45°)}$ $= \frac{(-\sqrt{3})\left(-\frac{\sqrt{3}}{2}\right).\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)}$ $= \frac{3}{2}$ **OR** $\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$ $= \frac{\tan 120°.(-\sin 60°).\cos 14°.(-\sin 45°)}{\sin 76°.(-\cos 45°)}$ $= \frac{(-\tan 60°).(-\sin 60°).\sin 76°.\tan 45°}{\sin 76°}$ $= (-\sqrt{3})\left(-\frac{\sqrt{3}}{2}\right).1$ $= \frac{3}{2}$	✓ $-\sin 60°$ ✓ $-\sin 45°$ ✓ $-\cos 45°$ ✓ $-\tan 60°$ ✓ cos 14° or sin 76° ✓ substitution Penalise 1 mark for treating as an equation in this question. (6) ✓ $-\sin 60°$ ✓ $-\sin 45°$ ✓ $-\cos 45°$ ✓ $-\tan 60°$ ✓ sin 76° ✓ substitution (6)
5.1.2	$\cos 75°$ $= \cos(45° + 30°)$ $= \cos 45°.\cos 30° - \sin 45°.\sin 30°$ $= \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}.\frac{1}{2}$ $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$ $= \frac{\sqrt{2}.\sqrt{3} - \sqrt{2}}{4}$ $= \frac{\sqrt{2}(\sqrt{3} - 1)}{4}$ **OR**	✓ $\cos(45° + 30°)$ ✓ expansion ✓ substitution ✓ simplification (4)

	$\cos 75°$ $= \cos(45° + 30°)$ $= \cos 45°.\cos 30° - \sin 45°.\sin 30°$ $= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $= \frac{.\sqrt{3}-1}{2\sqrt{2}}$	✓ $\cos(45° + 30°)$ ✓ expansion ✓ substitution ✓ simplification (4)
5.2	$\cos(90° - 2x).\tan(180° + x) + \sin^2(360° - x)$ $= \sin 2x.\tan x + \sin^2 x$ $= 2\sin x.\cos x.\frac{\sin x}{\cos x} + \sin^2 x$ $= 2\sin^2 x + \sin^2 x$ $= 3\sin^2 x$	✓ sin 2x ✓ tan x ✓ $\sin^2 x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ sin 2x = 2sin x.cos x ✓ $2\sin^2 x$ (6) If uses cos 2x instead of sin 2x and then works correctly: max 3/6 **[16]**

QUESTION 6

6.1.1	$(\tan x - 1)(\sin 2x - 2\cos^2 x)$ $= \left(\dfrac{\sin x}{\cos x} - 1\right)\left(2\sin x.\cos x - 2\cos^2 x\right)$ $= \left(\dfrac{\sin x}{\cos x} - 1\right)2\cos x(\sin x - \cos x)$ $= 2(\sin x - \cos x)^2$ $= 2(\sin^2 x - 2\sin x.\cos x + \cos^2 x)$ $= 2(1 - 2\sin x.\cos x)$	✓ $\dfrac{\sin x}{\cos x} = \tan x$ ✓ $\sin 2x = 2\sin x.\cos x$ ✓ factorisation ✓ simplification ✓ $\sin^2 x + \cos^2 x = 1$ (5)
	OR	
	$(\tan x - 1)(\sin 2x - 2\cos^2 x)$ $= \left(\dfrac{\sin x}{\cos x} - 1\right)\left(2\sin x.\cos x - 2\cos^2 x\right)$ $= 2\sin^2 x - 2\sin x.\cos x - 2\sin x.\cos x + 2\cos^2 x$ $= 2(\sin^2 x - 2\sin x\cos x + \cos^2 x)$ $= 2(1 - 2\sin x.\cos x)$	✓ $\dfrac{\sin x}{\cos x} = \tan x$ ✓ $\sin 2x = 2\sin x.\cos x$ ✓ simplification ✓ factorisation ✓ $\sin^2 x + \cos^2 x = 1$
	OR	
	$2(1 - 2\sin x\cos x)$ $= 2(\sin^2 x + \cos^2 x - 2\sin x\cos x)$ $= 2(\sin x - \cos x)^2$ $= 2\cos^2 x(\dfrac{\sin x}{\cos x} - 1)^2$ $= 2\cos^2 x(\tan x - 1)(\tan x - 1)$ $= (2\cos^2 x.\tan x - 2\cos^2 x)(\tan x - 1)$ $= (2\sin x\cos x - 2\cos^2 x)(\tan x - 1)$ $= (\sin 2x - 2\cos^2 x)(\tan x - 1)$	✓ $\sin^2 x + \cos^2 x = 1$ ✓ factorisation ✓ $\dfrac{\sin x}{\cos x} = \tan x$ ✓ $\sin 2x = 2\sin x.\cos x$ ✓ simplification
	OR	
	$LHS = (\tan x - 1)(\sin 2x - \cos^2 x)$ $= \dfrac{\sin x - \cos x}{\cos x}\left(2\sin x.\cos x - \cos^2 x\right)$ $= 2\left(\sin x - \cos x\right)^2$ $RHS = 2\left(\sin^2 x + \cos^2 x - 2\sin x\cos x\right)$ $= 2(\sin x - \cos x)^2$ $= LHS$	✓ $\dfrac{\sin x}{\cos x} = \tan x$ ✓ simplification ✓ $\sin 2x = 2\sin x.\cos x$ ✓ $\sin^2 x + \cos^2 x = 1$ ✓ factorisation

6.1.2	$\frac{\tan x - 1}{2} = -3$ $\tan x - 1 = -6$ $\tan x = -5$ $x = -78{,}7° + k.180°$ $k \in Z$ **OR** $\frac{\tan x - 1}{2} = -3$ $\tan x - 1 = -6$ $\tan x = -5$ $x = 101{,}3° + k.180°$ $k \in Z$ **OR** $\frac{\tan x - 1}{2} = -3$ $\tan x - 1 = -6$ $\tan x = -5$ $x = 101{,}3° + k.360°$ *or* $x = 281{,}3° + k.360°$ $k \in Z$ **OR** If the candidate has used $\tan(x-1) = -6$ max of 2 / 5	✓ simplification ✓ simplification ✓ $-78{,}7°$ ✓ $+k.180°$ ✓ $k \in Z$ (5)
6.2.1	$\cos\beta = \frac{p}{\sqrt{5}}$ $x = p$ $r = \sqrt{5}$ $y = -\sqrt{5 - p^2}$ $\therefore \tan\beta = \frac{-\sqrt{5-p^2}}{p}$  	✓ third quadrant ✓ $y = -\sqrt{5 - p^2}$ ✓✓answer (4) If p is negative:3/4

6.2.2	$\cos 2\beta = 2\cos^2 \beta - 1$ $= 2\left(\frac{p}{\sqrt{5}}\right)^2 - 1$ $= \frac{2p^2}{5} - 1$	✓ $2\cos^2 \beta - 1$ ✓✓ $2\left(\frac{p}{\sqrt{5}}\right)^2 - 1$ or $\frac{2p^2}{5} - 1$ (3)
	OR	
	$\cos 2\beta = 1 - 2\sin^2 \beta$ $= 1 - 2\left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ $= 1 - \frac{2(5-p^2)}{5}$ $= \frac{2p^2 - 5}{5}$	✓ $1 - 2\sin^2 \beta$ ✓✓ $1 - 2\left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ or $1 - \frac{2(5-p^2)}{5}$ or $\frac{2p^2 - 5}{5}$ (3)
	OR	
	$\cos 2\beta = \cos^2 \beta - \sin^2 \beta$ $= \left(\frac{p}{\sqrt{5}}\right)^2 - \left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ $= \frac{p^2}{5} - \frac{5-p^2}{5}$ $= \frac{2p^2 - 5}{5}$	✓ $\cos^2 \beta - \sin^2 \beta$ ✓ $\left(\frac{p}{\sqrt{5}}\right)^2$ ✓ $\left(\frac{-\sqrt{5-p^2}}{\sqrt{5}}\right)^2$ or $\frac{p^2}{5} - \frac{5-p^2}{5}$ or $\frac{2p^2 - 5}{5}$ (3) **[17]**

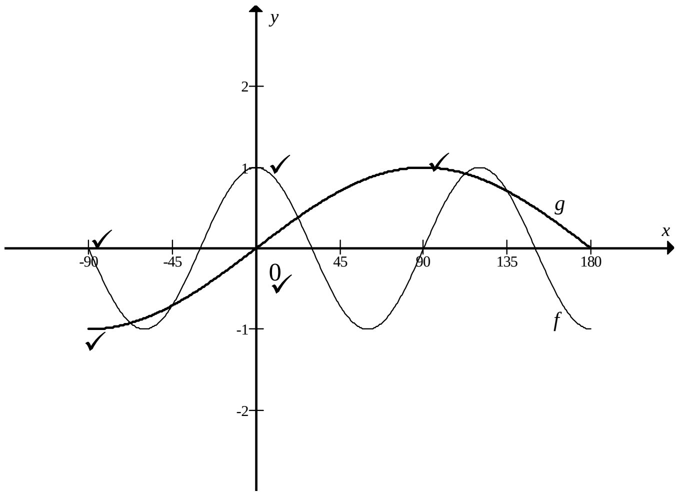
QUESTION 7

7.1	$\frac{3}{LB} = \tan 40°$ $LB = \frac{3}{\tan 40°}$ $LB = 3{,}58$ m (3,5752......) (3,5 m ; 3,57 m ; 3,6 m) **OR** $\frac{LB}{\sin 50°} = \frac{3}{\sin 40°}$ $LB = \frac{3\sin 50°}{\sin 40°}$ $LB = 3{,}58$ m (3,5752......)	✓ trig ratio ✓ answer (2) ✓ sine rule ✓ answer (2)
7.2	$AB^2 = AL^2 + BL^2 - 2.AL.BL.\cos 113°$ $AB^2 = (5.2)^2 + (3.58)^2 - 2(5.2)(3.58)\cos 113°$ $AB^2 = 54{,}40410138\text{ m}^2$ $AB = 7{,}38$ m (7,37591..........) Note: AB = 7,3 m or 7,4 m: accept	✓ use of cos rule ✓ substitution ✓ $AB^2 = 54{,}4041\ldots m^2$ ✓ answer (4) Do not penalise if units are omitted.
7.3	$Area\ of\ \Delta ABL = \frac{1}{2} AL.BL.\sin A\hat{L}B$ $= \frac{1}{2}(5.2)(3.58)\sin 113°$ $= 8.568059176$ $= 8{,}57$ m Note: Area = 8,5 or 8,6 : accept	✓ formula ✓ substitution ✓✓answer (4) If cos $A\hat{L}B$: 0/4 **[10]**

QUESTION 8

8.1	$\cos 3x = \sin x$ $\sin(90° - 3x) = \sin x$ $90° - 3x = x + k.360°$ $90° - 3x = 180° - x + k.360°$ $k \in Z$ $-4x = -90° + k.360°$ or $-2x = 90° + k.360°$ $x = 22{,}5° - k.90°$ $k \in Z$ $x = -45° - k.180°$ $k \in Z$ $x = -67{,}5°\,;\,22{,}5°\,;\,112{,}5°$ $x = -45°\,;\,135°$	✓ equating ✓ $90° - 3x = x + k.360°$ ✓ $x = 22{,}5° - k.90°$ ✓ $90° - 3x = 180° - x + k.360°$ ✓ $x = -45° - k.180°$ ✓✓✓ values of x (8)
	OR	
	$\cos 3x = \cos(90° - x)$ $3x = 90° - x + k.360°$ $3x = 360° - (90° - x) + k.360°$ $4x = 90° + k.360°$ or $2x = 270° + k.360°$ $x = 22{,}5° + k.90°$ $k \in Z$ $x = 135° + k.180°$ $k \in Z$ $x = -67{,}5°\,;\,22{,}5°\,;\,112{,}5°$ $x = -45°\,;\,135°$	✓ equating ✓ $3x = 90° - x + k.360°$ ✓ $x = 22{,}5° + k.90°$ ✓ $3x = 360° - (90° - x) + k.360°$ ✓ $x = 135° + k.180°$ ✓✓✓ values of x (8)
	OR	
	$\cos 3x = \cos(90° - x)$ $3x = 90° - x + k.360°$ $3x = -90° + x + k.360°$ $4x = 90° + k.360°$ or $2x = -90° + k.360°$ $x = 22{,}5° + k.90°$ $k \in Z$ $x = -45° - k.180°$ $k \in Z$ $x = -67{,}5°\,;\,22{,}5°\,;\,112{,}5°$ $x = -45°\,;\,135°$	✓ equating ✓ $3x = 90° - x + k.360°$ ✓ $x = 22{,}5° + k.90°$ ✓ $3x = -90° + x + k.360°$ ✓ $x = -45° - k.180°$ ✓✓✓ values of x (8) Note: If not all 5 values for x is given, the following applies 4 or 3 values : 2 marks 2 values : 1 mark 1 value : 0 marks

8.2



(6)
Penalise with -1 going beyond the domain.

8.3	$-67{,}5° \leq x \leq -45°$ **OR** $x \in [-67{,}5°; -45°]$ **OR** From $-67{,}5°$ up to and including $-45°$	✓✓ critical values ✓ notation (3) Note: If $-67{,}5° < x < -45°$: 2/3 Half of the inequality: 1/3 If $x = -67{,}5°$ or $x = -45°$: 0/3 If answer is $22{,}5° \leq x \leq 112{,}5°$ then 2 / 3 If answer is $135° \leq x \leq 180°$ then 2 / 3 **[17]**

QUESTION 9

<table>
<tr><td>9.1</td><td>Mean = $\frac{220}{10} = 22$ minutes</td><td>✓ $\frac{\text{sum of minutes}}{\text{number of runners}}$
✓answer
(2)

Answer only: 2/2</td></tr>
<tr><td>9.2</td><td>

Time taken	$(x - \bar{x})$	$(x_i - \bar{x})^2$
18	-4	16
21	-1	1
16	-6	36
24	2	4
28	6	36
20	-2	4
22	0	0
29	7	49
19	-3	9
23	1	1
Sum		156

$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}} = \sqrt{\frac{156}{10}} = 3{,}95$

(If the candidate used a calculator to answer QUESTION 9.1 and QUESTION 9.2, award full marks if answers are correct.)

If only one mistake in the calculation: 3 / 4

Answer only: 4 / 4

If candidate uses $n - 1$ in the formula, the answer</td><td>✓✓setting up of table and correct values in column of $(x_i - \bar{x})^2$

✓ substitution in formula
✓answer
(4)</td></tr>
<tr><td>9.3</td><td>One standard deviation of the mean is in the interval (22 - 3,95 ; 22 + 3,95) which is (18,05 ; 25,95)

$\therefore$ 6 runners completed the race within one standard deviation of the mean.
(List of times: 21, 24, 20, 22, 19, 23)

If candidate used $\sigma = 4{,}16$, then the interval is (17,84 ; 26,16) and the answer is 7 runners.</td><td>✓✓ answer
(2)
Answer only : 2 / 2
[8]</td></tr>
</table>

QUESTION 10

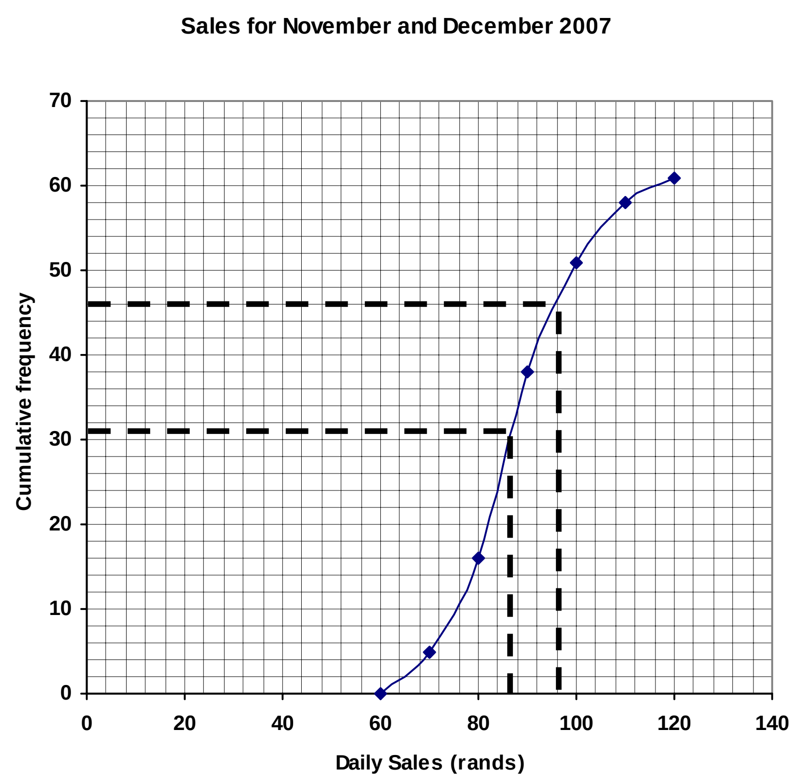
10.1

Daily Sales (in Rand)	Frequency	Cumulative Frequency
$60 \leq$ rand < 70	5	5
$70 \leq$ rand < 80	11	16
$80 \leq$ rand < 90	22	38
$90 \leq$ rand < 100	13	51
$100 \leq$ rand < 110	7	58
$110 \leq$ rand < 120	3	61

✓ Frequency Column
✓✓ cumulative frequencies
(3)

If one wrong in the frequency column, deduct 1 mark.

10.2



✓cumulative totals
✓points at upper limits of intervals
✓shape
(3)

If the ogive is NOT grounded, no penalty.

If plotted as the midpoint of the interval and the cumulative frequency:
2 / 3

10.3

Median = R 87
(Accept answers between 84 and 90)

✓correctly read off ogive
(1)

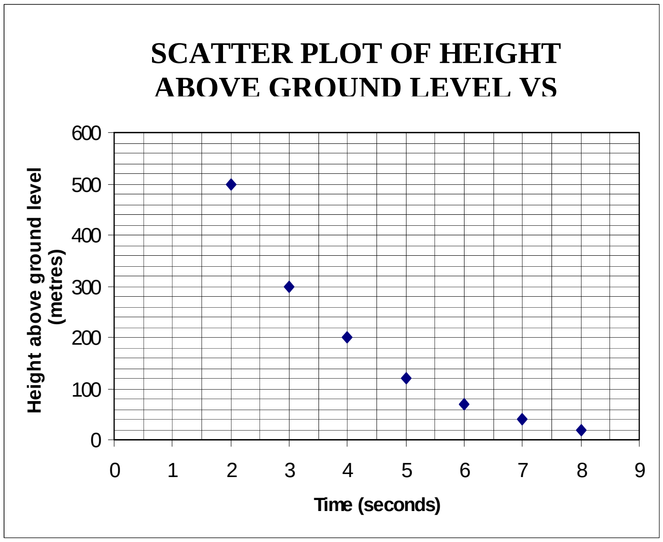
10.4

R 96 $\leq$ sales $\leq$ R 120

✓✓correctly read off ogive
(2)
[9]

QUESTION 11

11.1



✓✓ all points plotted correctly.
(2)

No penalty if the points are joined.

11.2	Exponential OR Quadratic OR Hyperbola OR Decreasing steeply then gradually. (Applicable descriptions are acceptable)	✓ answer (1) Straight line : 0 / 1
11.3	Approximately 90 m	✓ answer (1) **[4]**

QUESTION 12

12.1	The median, the maximum scores, IQR Note: Any two statements that are valid in the context of the problem apply.	✓✓ any two of the list (2)
12.2	IQR = 90 – 72 = 18.	✓ formula ✓ answer (2) Answer only: 2 / 2
12.3	No. In the calculation of the median only the value in the middle of an ordered data set is of importance. The extreme values are not taken into account. In this case, 25% of the learners in Class A had a score of less than 66 marks. The minimum mark in Class B is 66 marks. Hence the performance of the two classes differ significantly. OR No. The one is skewed to the left and the other is skewed to the right. The extreme values are not taken into account. OR No. The lower quartile of Class A is below the minimum of Class B. The extreme values are not taken into account. OR No. The left whisker of Class A is much longer than the left whisker of Class B. The extreme values are not taken into account.	✓ No ✓ extreme values not taken into account ✓ minimum marks different (3) **[7]**

TOTAL: 150 marks