basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

NOVEMBER 2012

MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 29 bladsye.

NOTA:

- Indien `n kandidaat `n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.
- Indien `n kandidaat `n poging van `n vraag gekanselleer het en nie die vraag weer gedoen het nie, merk die gekanselleerde weergawe.
- Volgehoue akkuraatheid is van toepassing in ALLE aspekte van die memorandum tensy anders aangedui.

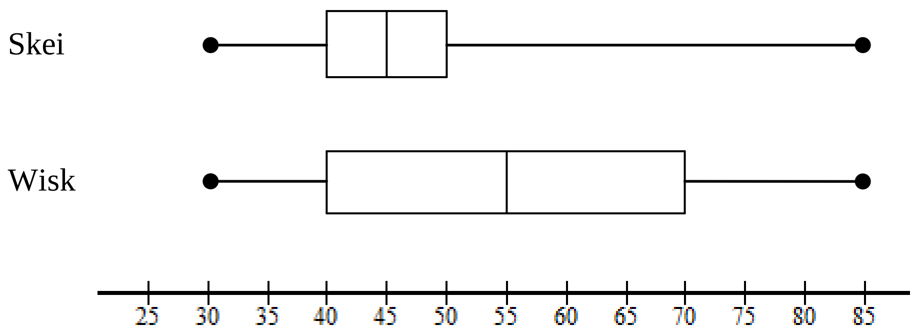
VRAAG 1

1.1	Ongeveer 121cm (Aanvaar 120 – 122)	✓antwoord (1)
1.2	Soos die ouderdom toeneem sal die lengte toeneem **OF** Elke jaar die lengte sal toeneem met ongeveer 6,2 cm **OF** Reguit lyn (linieêr) met positiewe gradient **OF** Toename in lengte: toename in ouderdom is `n konstante **OF** Sterk positiewe korrelasie	✓beskrywing (1)
1.3	Geskatte toename in gemiddelde lengte $= \frac{169-88}{15-2}$ $= 6{,}23$ Interval vir noemer (87 – 89 ; 167 – 170) (Aanvaar enige antwoord tussen 6 – 6,4 cm)	✓ lees van grafiek af ✓ noemer ✓antwoord (3)
1.4	Kinders hou op groei as hulle volwassenheid bereik. **OF** Indien die neiging voortduur sal die seuns onmoontlike lengtes bereik **OF** Die neiging sal ‘n konstante waarde nader. **OF** Mense kan nie onbeperk groei nie.	✓opmerking (1) **[6]**

VRAAG 2

2.1	Gemiddelde aantal lopies $\bar{x}=\frac{\sum x}{n}=\frac{128}{8}=16$	✓ 128 ✓16 (2)
2.2	Standaard afwyking = 7,55 **NOTA**: Penalisering van 1 punt vir nie-korrekte afronding	✓✓ 7,55 (2)
2.3	Standaard afwyking = 9,71 Standaard afwyking vermeerder. **OF** 2 en 35 is ver van die gemiddelde, naamlik 16. Die standaard afwyking hang af van hoe ver die data punte vanaf die gemiddelde is, en daarom word dit verwag dat die standaard afwyking sal toeneem	✓ 9,71 ✓ vermeerder (2) ✓2 en 35 ver van gemiddelde ✓vermeerder (2)
2.4	Totale aantal lopies benodig is 20 x 16 = 320 Totale aantal lopies aangeteken tydens die laaste vyf wedstryde = 320 – 59 – 128 = 133 Gemiddelde aantal lopies vir die laaste wedstryd is $\frac{133}{5}=26{,}6$ **OF** $\frac{128+59+x}{16}=20$ $187+x=320$ $\therefore x=133$ $\therefore \frac{133}{5}=26{,}6$ **OF** $\frac{128+59+5x}{16}=20$ $5x=133$ $\therefore x=26{,}6$	✓ 320 ✓ 133 ✓ 26,6 (3) ✓ 320 ✓ 133 ✓ 26,6 (3) ✓ 320 ✓ 133 ✓ 26,6 (3) **[9]**

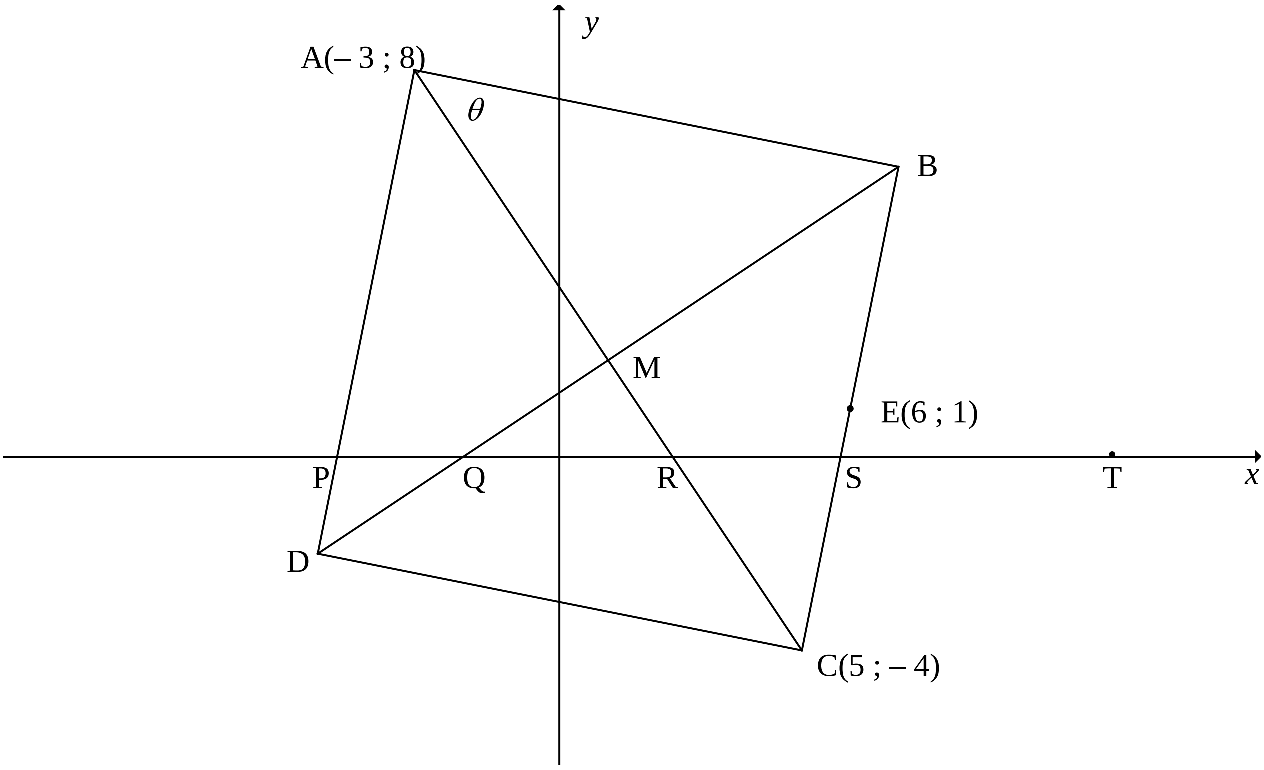
VRAAG 3

3.1	Omvang (Variasiewydte) = 85 – 30 = 55	✓ 55 (1)
3.2		✓ maks 85 ✓ $Q_3 = 70$ ✓ $Q_1 = 40$ ✓ Mediaan = 55 (4)
3.3	Vanuit die inligting oor Wiskunde, die waarde van die derde kwartiel is 70%. Dus sal 75% van die leerder se punte onder 70% wees. Verwagte aantal leerders minder as 70% is $\frac{75}{100} \times 60 = \frac{3}{4} \times 60 = 45$ leerders	✓ 75% van leerders ✓ 45 leerders (2)
3.4	Nee, Joe se stelling is nie geldig nie. 50% van die leerders het tussen 30% en 45% presteer in Skeinat. 50% van die leerders het tussen 30% en 55% presteer in Wiskunde. Daarom is die aantal leerders dieselfde **OF** Nee, Joe se stelling is nie geldig nie. Selfde aantal leerders (tussen min en mediaan)	✓nie geldig nie/no ✓ mediaan represents 50% of leerders (2) **[9]**

VRAAG 4

4.1	Modale interval(klas) is $50 \leq x < 60$ **OF** $50 < x \leq 60$ **OF** 50 tot 60	✓korrekte interval (1)
4.2	Mediaan posisie is 15 leerders (gegroepeerde data). Geskatte gewig is omtrent 53 kg. (Aanvaar 52 kg - 54 kg)	✓ 53 kg (1)
4.3	30 – 23 = 7 leerders het meer as 60 kg versamel.	✓ ✓ 7 leerders (2) **[4]**

VRAAG 5



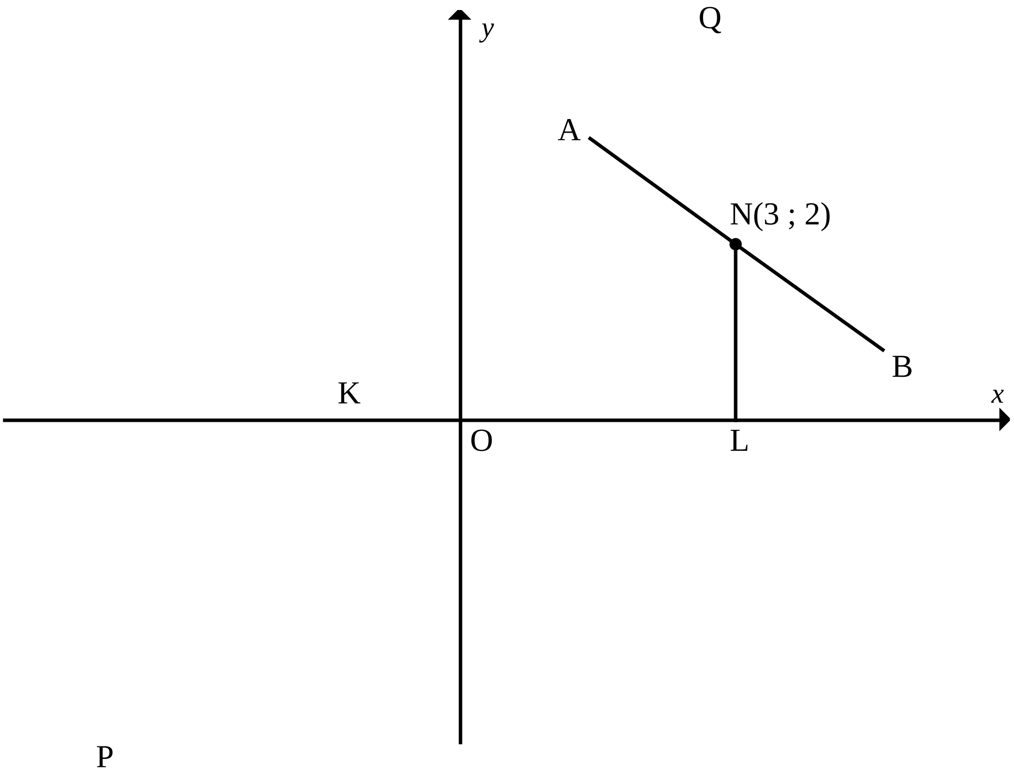
5.1	Hoeklyne halveer mekaar by M: $x_M = \frac{-3+5}{2} = 1$; $y_M = \frac{8+(-4)}{2} = 2$ M(1 ; 2)	✓ $x_M = 1$ ✓ $y_M = 2$ (2)
5.2	$m_{BC} = \frac{1+4}{6-5}$ $m_{BC} = 5$ **OF** $m_{BC} = \frac{-4-1}{5-6}$ $m_{BC} = 5$	✓ vervanging in gradient formule ✓ 5 (2) ✓ $m_{BC} = \frac{-4-1}{5-6}$ ✓ 5 (2)
5.3	$y - y_1 = m(x - x_1)$ $y - 8 = m(x+3)$ $m_{AD} = m_{BC} = 5$ Ewewydige lyne $y - 8 = 5(x+3)$ $y = 5x + 23$	✓ vervanging (–3 ; 8) ✓ gradiente gelyk ✓ vergelyking (3)

	OF $m_{AD} = m_{BC}$ Ewewydige lyne $m_{AD} = 5$ $y = 5x + c$ $8 = 5(-3) + c$ $c = 23$ $y = 5x + 23$	✓ gradiente gelyk ✓ vervanging (–3 ; 8) ✓ vergelyking (3)
5.4	ABCD is `n ruit, daarom AB = BC $\theta = B\hat{C}A = A\hat{R}S - R\hat{S}C$ $= A\hat{R}S - B\hat{S}T$ $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5}$ $\tan A\hat{R}S = -\frac{3}{2}$ $A\hat{R}S = 180° - 56,3099...$ $A\hat{R}S = 123,69°$ $\tan B\hat{S}T = m_{BC} = 5$ $B\hat{S}T = 78,69°$ $A\hat{R}S = B\hat{C}A + B\hat{S}T$ $B\hat{C}A = 123,69° - 78,69°$ $B\hat{C}A = 45°$ **NOTA:** Indien `n leerder aanvaar dat E die middelpunt van BC is dan max 3/6 **OF** AB is loodreg op BC dan max 1/6  **OF**  $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $A\hat{R}S = 123,69°$ $\tan A\hat{P}R = m_{AD} = 5$ $A\hat{P}R = 78,69°$ $P\hat{A}R = A\hat{R}S - A\hat{P}R$ Buitehoek van `n driehoek $= 123,69° - 78,69°$ $= 45°$ $\theta = P\hat{A}R$ Hoeklyne van `n ruit halveer teenoorstaande hoeke $= 45°$	✓ $\theta = B\hat{C}A$ ✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan B\hat{S}T = m_{BC} = 5$ ✓ 78,69° ✓ $\theta = 45°$ (6) ✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan A\hat{P}R = m_{AD} = 5$ ✓ 78,69° ✓ $P\hat{A}R = 45°$ ✓ $\theta = 45°$ (6)

	OF $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $A\hat{R}S = 123{,}69°$ $\tan A\hat{P}R = 5$ $A\hat{P}R = 78{,}69°$ $\theta = P\hat{A}R$ Hoeklyne van `n ruit halveer teenoorstaande hoeke $\theta = A\hat{R}S - A\hat{P}R$ Buite hoek van driehoek $\theta = 123{,}69° - 78{,}69°$ $\theta = 45°$	✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan A\hat{P}R = m_{AD} = 5$ ✓ 78,69° ✓ $\theta = P\hat{A}R$ ✓ $\theta = 45°$ (6)
	OF $\tan A\hat{R}S = m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $A\hat{R}S = 123{,}69°$ $\tan B\hat{S}T = 5$ $B\hat{S}T = 78{,}69°$ $\theta = R\hat{C}S$ BA=BC $R\hat{C}S + B\hat{S}T = R\hat{C}S + R\hat{S}C$ $= A\hat{R}S$ $\theta = A\hat{R}S - B\hat{S}T$ $= 123{,}69° - 78{,}69°$ $= 45°$	✓ $\tan A\hat{R}S = -\frac{3}{2}$ ✓ 123,69° ✓ $\tan B\hat{S}T = 5$ ✓ 78,69° ✓ $\theta = R\hat{C}S$ ✓ $\theta = 45°$ (6)
	OF ABCD is `n ruit, daarom AB = BC $\therefore A\hat{C}B = B\hat{A}C$ $\tan\theta = \tan A\hat{C}B$ $= \tan(A\hat{R}S - B\hat{S}T)$ $= \frac{\tan A\hat{R}S - \tan B\hat{S}T}{1 + \tan A\hat{R}S . \tan B\hat{S}T}$ $= \frac{\left(\frac{12}{-8}\right) - \left(\frac{-5}{-1}\right)}{1 + \left(\frac{12}{8}\right)\left(\frac{5}{1}\right)}$ $= 1$ $\theta = 45°$	✓ $A\hat{C}B = B\hat{A}C$ ✓ $\tan\theta = \tan A\hat{C}B$ ✓ formule ✓ vervanging ✓ $\tan\theta = 1$ ✓ $\theta = 45°$ (6)

	OF	
	Uit 5.1, M se koordinate is (1 ; 2) Verbind ME $m_{ME} = \frac{2-1}{1-6} = -\frac{1}{5}$	✓gradient van ME
	Uit 5.2 $m_{BC} = 5$	✓gradient van BC
	$\therefore m_{ME} \times .m_{BC} = -1$ $\therefore M\hat{E}C = 90°$	✓ $M\hat{E}C = 90°$
	$ME = \sqrt{(1-6)^2 + (2-1)^2} = \sqrt{26}$	✓ $ME = \sqrt{26}$
	$EC = \sqrt{(5-6)^2 + (-4-1)^2} = \sqrt{26}$	✓ $EC = \sqrt{26}$
	$\therefore$MEC is ‘n reghoekige driehoek. $E\hat{C}M = 45°$	✓ $E\hat{C}M = 45°$
	ABCD is ‘n ruit, dus AB = BC $\therefore \theta = B\hat{C}M = 45°$	(6)
	OF	
	$AM = \sqrt{(-3-1)^2 + (8-2)^2} = 2\sqrt{13}$	✓ $AM = 2\sqrt{13}$
	Berekeninge om die koordinate van B te bepaal $m_{AC} = \frac{8+4}{-3-5} = -\frac{3}{2}$ $m_{BD} \times m_{AC} = -1$ $m_{BD} = \frac{2}{3}$ Hoeklyne halveer reghoekig	
	Vergelyking van BD is $y = \frac{2}{3}x + \frac{4}{3}$	✓ $y = \frac{2}{3}x + \frac{4}{3}$
	Vergelyking van BC is $y = 5x - 29$	✓ $y = 5x - 29$
	BD en BC ontmoet in B. Los gelyktydig op om B(7 ; 6) te kry.	✓ B(7 ; 6)
	$\therefore BM = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{52} = 2\sqrt{13}$	✓ $BM = 2\sqrt{13}$
	$\therefore BM = AM$ Omdat $A\hat{M}B = 90°$ $\tan\theta = \frac{BM}{AM}$	
	$\therefore \tan\theta = 1$ $\theta = 45°$	✓ 45° (6) **[13]**

VRAAG 6



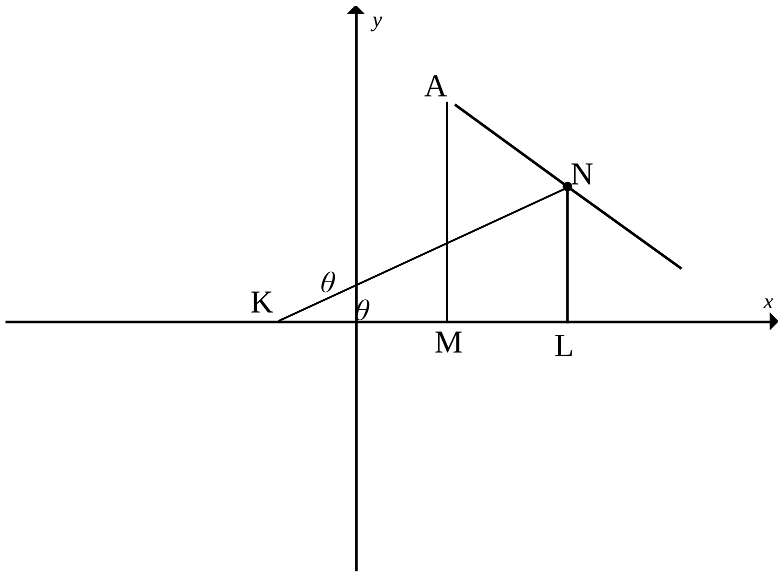
6.1	Die radius (NL) van `n sirkel is loodreg op die raaklyn (OL) by die kontakpunt	✓ radius ⊥ raaklyn (1)
6.2	L(3 ; 0)	✓ (3 ; 0) (1)
6.3	Middelpunt N (3 ; 2) en r = NL = 2 Vergelyking van die sirkel N: $(x-a)^2+(y-b)^2=r^2$ $(x-3)^2+(y-2)^2=4$	✓ $r = 2$ ✓ $(x-3)^2+(y-2)^2$ ✓ 4 (3)
6.4	koordinates van K. K is die x-afsnit van die raaklyn. $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ KL = 3 – (–1) of KL = 3 + 1 KL = 4	✓ vervanging $y = 0$ in vergelyking van raaklyn ✓ $x = -1$ ✓ $KL = 4$ (3)

	OF $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ $KL=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $KL=\sqrt{(3+1)^2+(0-0)^2}$ $KL=\sqrt{16}$ $KL=4$	✓ vervanging $y = 0$ in vergelyking van raaklyn ✓ $x = -1$ ✓ $KL = 4$ (3)
	OF Vir AK, m = $\frac{4}{3}$, c = $\frac{4}{3}$ $\frac{\frac{4}{3}}{OK}=\tan A\hat{K}O=\frac{4}{3}$ $OK=1$ $\therefore KL=4$	✓ $\frac{\frac{4}{3}}{OK}=\frac{4}{3}$ ✓ OK = 1 ✓ $KL = 4$ (3)
	OF $y=\frac{4}{3}x+\frac{4}{3}$ $0=\frac{4}{3}x+\frac{4}{3}$ $0=4x+4$ $4x=-4$ $x=-1$ $K(-1;0)$ $KN^2 = NL^2 + KL^2$ Pythagoras se stelling $(-1-3)^2+(0-2)^2=4+KL^2$ $20 = 4 + KL^2$ $16 = KL^2$ $KL = 4$	✓ $x = -1$ ✓ $KN^2 = NL^2 + KL^2$ ✓ $KL = 4$ (3)

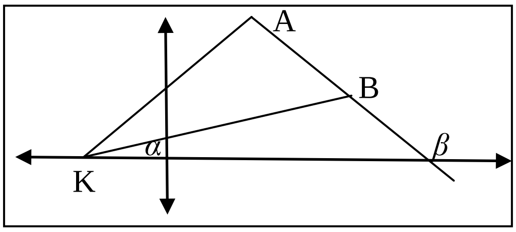
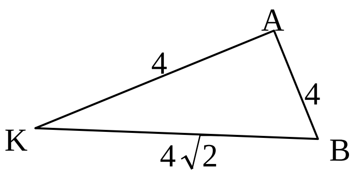
6.5	$m_{AB} \times m_{AK} = -1$ raaklyn ⊥ radius $m_{AK} = \frac{4}{3}$ $\therefore m_{AB} = -\frac{3}{4}$ $y - y_1 = m(x - x_1)$ $y - 2 = -\frac{3}{4}(x-3)$ $y = -\frac{3}{4}x + \frac{9}{4} + \frac{8}{4}$ $y = -\frac{3}{4}x + \frac{17}{4}$	✓ $m_{AK} = \frac{4}{3}$ ✓ $m_{AB} = -\frac{3}{4}$ ✓ vervanging van punt (3;2) in vergelyking ✓ vergelyking (4)
	OF	
	$m_{AB} \times m_{AK} = -1$ raaklyn ⊥ radius $m_{AK} = \frac{4}{3}$ $\therefore m_{AB} = -\frac{3}{4}$ $y = -\frac{3}{4}x + c$ $2 = \left(-\frac{3}{4}\right)(3) + c$ $c = \frac{8}{4} + \frac{9}{4}$ $c = \frac{17}{4}$ $y = -\frac{3}{4}x + \frac{17}{4}$	✓ $m_{AK} = \frac{4}{3}$ ✓ $m_{AB} = -\frac{3}{4}$ ✓ vervanging van punt (3;2) in vergelyking ✓ vergelyking (4)

6.6	Punt A is op PQ en AB. Dus $\frac{4}{3}x+\frac{4}{3}=-\frac{3}{4}x+\frac{17}{4}$ $16x+16=-9x+51$ $25x = 35$ $x = \frac{7}{5}$ $y = -\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$ $y = \frac{16}{5}$ $A\left(\frac{7}{5}\ ;\ \frac{16}{5}\right)$ **OF** Punt A is op PQ en die sirkel. Dus $(x-3)^2+(\frac{4}{3}x+\frac{4}{3}-2)^2=4$ $(x-3)^2+(\frac{4}{3}x-\frac{2}{3})^2=4$ $25x^2-70x+49=0$ $(5x-7)^2=0$ $x=\frac{7}{5}$ $y = -\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$ $y = \frac{16}{5}$ **OF**	✓ vergelyking ✓ $25x = 35$ ✓ vervanging van x (3) ✓ vergelyking ✓ $(5x-7)^2 = 0$ 3) ✓ substitusie van x

	Punt A lê op die sirkel en op lyn AB $(x-3)^2+(y-2)^2=4$ ---------- (1) $y=-\frac{3}{4}x+\frac{17}{4}$ ---------- (2) Subs (2) in (1) : $x^2-6x+9+(-\frac{3}{4}x+\frac{17}{4}-2)^2=4$ $x^2-6x+9+(-\frac{3}{4}x+\frac{9}{4})^2=4$ $25x^2-150x+161=0$ $(5x-23)(5x-7)=0$ $x=\frac{7}{5}$ $y=-\frac{3}{4}\left(\frac{7}{5}\right)+\frac{17}{4}$ $y=\frac{16}{5}$	✓ vergelyking ✓ $(5x-23)(5x-7)=0$ ✓vervanging van x (3)
	OF Gebruik rotasie: Stel $\theta=A\hat{K}N=L\hat{K}N$ Skuif die diagram met 1 eenheid regs. Dan is $L^{/}$ geroteer deur 2θ die punt $A^{/}$. $\tan\theta=\frac{AN}{KA}=\frac{2}{4}=\frac{1}{2}$ $\therefore \sin 2\theta=2\sin\theta\cos\theta=2(\frac{1}{\sqrt{5}})(\frac{2}{\sqrt{5}})=\frac{4}{5}$ $\cos 2\theta=\cos^2\theta-\sin^2\theta=(\frac{2}{\sqrt{5}})^2-(\frac{1}{\sqrt{5}})^2=\frac{3}{5}$ $\therefore x_{A''}=x_{L'}\cos 2\theta-y_{L'}\sin 2\theta=4(\frac{3}{5})-(0)(\frac{4}{5})=\frac{12}{5}$ $y_{A''}=x_{L'}\sin 2\theta+y_{L'}\cos 2\theta=4(\frac{4}{5})-(0)(\frac{3}{5})=\frac{16}{5}$ $A'(\frac{12}{5}\ ;\ \frac{16}{5})$ Om terug te keer na A, skuif 1 eenheid links. $\therefore A(\frac{7}{5}\ ;\ \frac{16}{5})$	✓ waardes van sin 2θ en cos 2θ ✓vervanging in die rotasie-formules ✓ $A'(\frac{12}{5}\ ;\ \frac{16}{5})$ (3)
	OF	

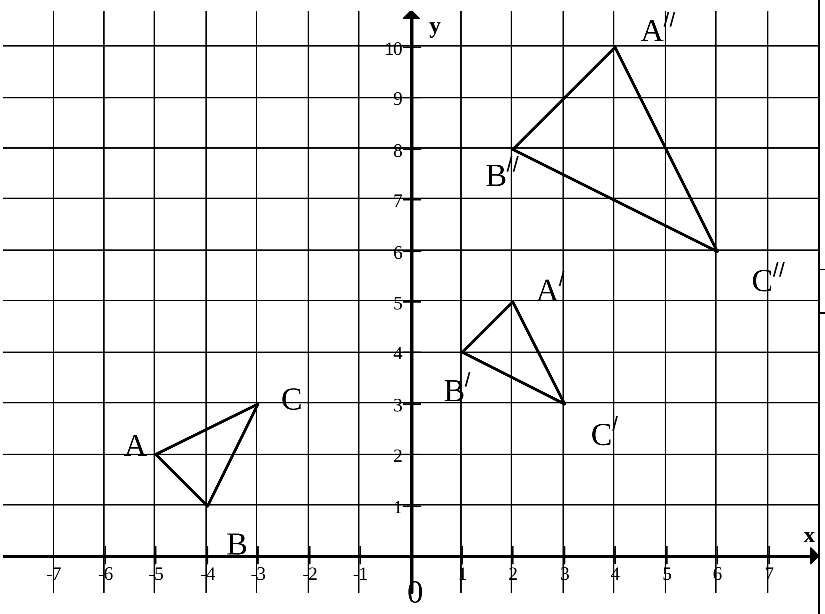
		
	Stel $N\hat{K}L = \theta$. Dan sal $\tan\theta = \frac{NL}{KN} = \frac{2}{4} = \frac{1}{2}$. Dus: $\sin\theta = \frac{1}{\sqrt{5}}$ en $\cos\theta = \frac{2}{\sqrt{5}}$	✓$\tan\ \theta = \frac{1}{2}$
	Laat $AM \perp x-as$ met M op $x-as$ $\Delta NAK \equiv \Delta NLK$ $A\hat{K}N = N\hat{K}L = \theta$ $\therefore A\hat{K}L = 2\theta$	
	$y_A = AM = AK\sin 2\theta = KL\sin 2\theta = 4\sin 2\theta$ $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$ $y_A = 4\left(\frac{4}{5}\right) = \frac{16}{5}$	✓ $\sin 2\theta = \frac{4}{5}$ ✓ los op vir *x* en *y*
	$x_A = OL - NA\sin M\hat{A}N$ $= 3 - 2\sin\left(90° - M\hat{A}K\right)$ $= 3 - 2\sin 2\theta$ $= 3 - \frac{8}{5}$ $= \frac{7}{5}$	(3)

6.7	$KA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{7}{5}+1\right)^2 + \left(\frac{16}{5}-0\right)^2}$ $= 4$ **OF** $KN = \sqrt{4^2 + 2^2} = \sqrt{20}$ $KA^2 = KN^2 - AN^2$ $= 20 - 4$ $= 16$ $KA = 4$ **OF** KA = KL Raaklyne van dieselfde punt af is gelyk KA = 4	✓ afstandformule ✓ vervanging ✓ 4 (3) ✓ $KN = \sqrt{20}$ ✓ $KA^2 = KN^2 - AN^2$ ✓ 4 (3) ✓ KA=KL ✓ rede ✓ 4 (3)
6.8	AN = NL Radii is gelyk KA = KL ∴KLNA is a kite twee paar aangrensde sye is gelyk.	✓ AN = NL ✓ KA = KL (2)
6.9	AB = AN + NB = 2 + 2 = 4 AK = 4= AB $K\hat{A}B = 90°$ raaklyn ⊥ radius $\therefore \Delta AKB$ is `n reghoekige gelykbenige driehoek $A\hat{K}B + A\hat{B}K = 90°$ $2A\hat{B}K = 90°$ $\therefore A\hat{B}K = 45°$ **OF**	✓ AB = 4 ✓ AK = AB ✓ $K\hat{A}B = 90°$ (3)

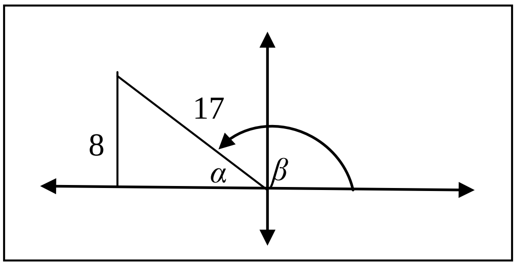
	N is die middelpunt van AB Veronderstel B is $\left(x_B ; y_B\right)$ $\dfrac{x_B + \frac{7}{5}}{2} = 3 \qquad \dfrac{y_B + \frac{16}{5}}{2} = 2$ $\therefore x_B = \dfrac{23}{5} \qquad \therefore y_B = \dfrac{4}{5}$ $\therefore B\left(\dfrac{23}{5};\dfrac{4}{5}\right)$  $\tan\beta = m_{AB} = -\dfrac{3}{4}$ $\beta = 180° - 36{,}87°$ $\beta = 143{,}13°$ $\tan\alpha = m_{KB} = \dfrac{\frac{4}{5} - 0}{\frac{23}{5} + 1} = \dfrac{1}{7}$ $\alpha = 8{,}13°$ $A\hat{B}K = \alpha + (180° - \beta)$ $= 8{,}13° + 36{,}87°$ $= 45°$	✓ $143{,}13°$ ✓ $8{,}13°$ ✓ $A\hat{B}K = \alpha + (180° - \beta)$ (3)
	OF N is die middelpunt van AB Veronderstel B is $\left(x_B ; y_B\right)$ $\dfrac{x_B + \frac{7}{5}}{2} = 3 \qquad \dfrac{y_B + \frac{16}{5}}{2} = 2$ $\therefore x_B = \dfrac{23}{5} \qquad \therefore y_B = \dfrac{4}{5}$ $\therefore \mathrm{B}(\dfrac{23}{5}\ ;\dfrac{4}{5})$  $KB = \sqrt{(\dfrac{23}{5}+1)^2 + (\dfrac{4}{5})^2} = 4\sqrt{2}$ $4^2 = 4^2 + (\sqrt{32})^2 - 2(4)(\sqrt{32})\cos\theta$ $\cos\theta = \dfrac{\sqrt{2}}{2}$ $\therefore \theta = 45°$	✓ $4\sqrt{2}$ ✓ vervanging in cos-formule ✓ $\cos\theta = \dfrac{\sqrt{2}}{2}$ (3)
6.10	$N^{/}(3;-2)$	✓ $N^{/}(3;-2)$ (1) **[24]**

VRAAG 7

NOTA: CA nie van toepassing in hierdie VRAAG

7.1	Rotasie om oorsprong deur 90° in `n kloksgewyse rigting.  **OF** Rotasie om oorsprong deur 270° in `n anti-kloksgewyse rigting. **OF** Rotasie om oorsprong deur -90°.	✓ rotasie van 90° ✓ kloksgewyse rigting (2) ✓ rotasie of 270° ✓ anti-kloksgewyse rigting (2) ✓✓stelling (2)
7.2	$(x;y) \to (y;-x)$	✓ ✓(beide) $(x;y) \to (y;-x)$ (2)
7.3		✓ een punt korrek ✓ alle punte korrek en driehoek geteken (2)
7.4	$(x;y) \to (2x;2y)$	✓ $(2x;2y)$ (1)
7.5.1	$A(-5;2) \to (-5;-2) \to D(5;-2)$	✓ 5 ✓– 2 (2)
7.5.2	$(x;y) \to (x;-y) \to (-x;-y)$	✓ $(x;-y)$ ✓$(-x;-y)$ (2)
7.5.3	Rotasie deur 180° om die oorsprong in beide rigtings. **OF** Refleksie in die oorsprong.	✓ rotasie ✓ 180° (2) ✓ refleksie ✓ oorsprong (2) **[13]**

VRAAG 8
Geen sakrekenaar toegelaat in hierdie VRAAG

8.1.1	OT = *k*, PT = 8 en OP = 17 $k^2 + 8^2 = 17^2$ $k^2 = 289 - 64$ $k^2 = 225$ $k = \pm 15$ $k > 0$ $k = 15$ **OF** $k^2 = 17^2 - 8^2$ $k^2 = (17-8)(17+8)$ $= 25 \times 9$ $= 225$ $k = \pm 15$ $k > 0$ $k = 15$	✓ vervanging in Pythagoras ✓ $k = 15$ (2) ✓ vervanging in Pythagoras ✓ $k = 15$ (2)
8.1.2	$\cos\alpha = \frac{15}{17}$	✓ $\frac{15}{17}$ (1)
8.1.3	$\alpha + \beta = 180°$ $\beta = 180° - \alpha$ $\therefore \cos\beta = \cos(180° - \alpha)$ $= -\cos\alpha$ $= -\frac{15}{17}$ **OF**  $\therefore \cos\beta = \cos(180° - \alpha)$ $= -\cos\alpha$ $= -\frac{15}{17}$	✓ $\cos(180° - \alpha)$ or – cos α ✓ $-\frac{15}{17}$ (2) ✓ $\cos(180° - \alpha)$ or – cos α ✓ $-\frac{15}{17}$ (2)

8.1.4	$\sin(\beta-\alpha)$ $=\sin\beta\cos\alpha-\cos\beta\sin\alpha$ $=\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)-\left(-\frac{15}{17}\right)\left(\frac{8}{17}\right)$ $=\frac{120}{289}+\frac{120}{289}$ $=\frac{240}{289}$ **OF** $\beta-\alpha=(180°-\alpha)-\alpha$ $=180°-2\alpha$ $\sin(\beta-\alpha)=\sin(180°-2\alpha)$ $=\sin 2\alpha$ $=2\sin\alpha.\cos\alpha$ $=2\left(\frac{8}{17}\right)\left(\frac{15}{17}\right)$ $=\frac{240}{289}$	✓ uitbreiding ✓ $\sin\beta=\frac{8}{17}$ ✓ $\sin\alpha=\frac{8}{17}$ ✓ $\frac{240}{289}$ (4) ✓ vervanging β ✓ *2*sinαcosα ✓ $\sin\alpha=\frac{8}{17}$ ✓ $\frac{240}{289}$ (4)
8.2.1	$LK=\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\frac{1-(1-2\sin^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\frac{\sin x}{\cos x}$ $=\tan x$ $=RK$ **OF**	✓ $1-2\sin^2 x$ ✓ $2\sin x\cos x$ ✓ of $\sin x(2\sin x-1)$ of $\cos x(2\sin x-1)$ ✓ $\frac{\sin x}{\cos x}$ (4)

	$LK=\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\frac{1-(2\cos^2 x-1)-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{2-2\cos^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{2(1-\cos^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\frac{\sin x}{\cos x}$ $=\tan x$ $=RK$	✓ $2\cos^2 x-1$ ✓ $2\sin x\cos x$ ✓ $\sin x(2\sin x-1)$ of $\cos x(2\sin x-1)$ ✓ $\frac{\sin x}{\cos x}$ (4)
	OF	
	$LK=\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ $=\frac{1-(\cos^2 x-\sin^2 x)-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{1-\cos^2 x+\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{2\sin^2 x-\sin x}{2\sin x\cos x-\cos x}$ $=\frac{\sin x(2\sin x-1)}{\cos x(2\sin x-1)}$ $=\frac{\sin x}{\cos x}$ $=\tan x$ $=RK$	✓ $\cos^2 x-\sin^2 x$ ✓ $2\sin x\cos x$ ✓ of $\sin x(2\sin x-1)$ of $\cos x(2\sin x-1)$ ✓ $\frac{\sin x}{\cos x}$ (4)

8.2.2	$\sin 2x - \cos x = 0$ $2\sin x\cos x - \cos x = 0$ $\cos x(2\sin x - 1) = 0$ $\cos x = 0$ $x = 90° + 360°k$ of $x = 270° + 360°k$ $k \in Z$ of $\sin x = \frac{1}{2}$ $x = 30° + 360°k$ of $x = 150° + 360°k$ $x = 90°$ of $x = 270°$ of $x = 30°$ of $x = 150°$	✓ $2\sin x\cos x$ ✓ $\cos x = 0$ en $\sin x = \frac{1}{2}$ ✓ vir twee korrekte antwoorde ✓ vir vier korrekte antwoorde (4)
	OF $\sin 2x = \cos x$ $\sin 2x = \sin(90° - x)$ $2x = 90° - x + 360°k; k \in Z$ of $2x = 180° - (90° - x) + 360°k$ $3x = 90° + 360°k$ $2x = 90° + x + 360°k$ $x = 30° + 120°k$ $x = 90° + 360°k$ $x = 30°$ of $x = 150°$ of $x = 270°$ of $x = 90°$	✓ $\sin(90° - x)$ ✓ $x = 30° + 120°.k$ en $x = 90° + 360°.k$ ✓ vir twee korrekte antwoorde ✓ vir vier korrekte antwoorde (4) **[17]**

VRAAG 9

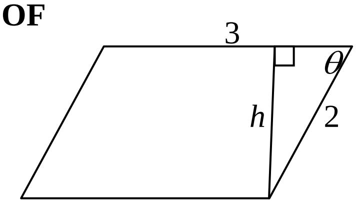
9.1	$\dfrac{\sin^2\theta}{\sin(180°-\theta).\cos(90°+\theta)+\tan 45°}$ $=\dfrac{\sin^2\theta}{(\sin\theta)(-\sin\theta)+1}$ $=\dfrac{\sin^2\theta}{-\sin^2\theta+1}$ $=\dfrac{\sin^2\theta}{\cos^2\theta}$ $=\tan^2\theta$	✓ $\sin\theta$ ✓ $-\sin\theta$ ✓ 1 ✓ $\cos^2\theta$ ✓ $\tan^2\theta$ (5)
9.2	$\dfrac{\sin 104°(2\cos^2 15°-1)}{\tan 38°\sin^2 412°}$ $=\dfrac{\sin 76°.\cos 30°}{\tan 38°.(\sin 52°)^2}$ $=\dfrac{2\sin 38°\cos 38°\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sin 38°}{\cos 38°}\right)(\cos 38°)^2}$ $=\dfrac{\sqrt{3}\sin 38°\cos 38°}{\sin 38°\cos 38°}$ $=\sqrt{3}$ **NOTA:** • Indien cos 30° nie getoon word nie: -1 • Slegs antwoord: 0/8 **OF** $\dfrac{\sin 104°(2\cos^2 15°-1)}{\tan 38°\sin^2 412°}$ $=\dfrac{\sin 2(52°).(2\cos^2 15°-1)}{\frac{\sin 38°}{\cos 38°}.(\sin 52°)^2}$ $=\dfrac{2\sin 52°\cos 52°.\cos 30°}{\left(\frac{\cos 52°}{\sin 52°}\right)(\sin 52°)^2}$ $=2\cos 30°$ $=2.\dfrac{\sqrt{3}}{2}$ $=\sqrt{3}$	✓ sin 76° ✓cos30° ✓ $\dfrac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓2sin38°cos38° ✓ $\dfrac{\sqrt{3}}{2}$ ✓ sin52°=cos38° ✓ $\sqrt{3}$ (8) ✓ sin2(52°) ✓ $\dfrac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓2sin52°cos52° ✓cos30° ✓ cos52°=sin38° en sin52°=cos38° ✓ $\dfrac{\sqrt{3}}{2}$ ✓ $\sqrt{3}$ (8)

	OF $\frac{\sin 104°(2\cos^2 15° - 1)}{\tan 38° \sin^2 412°}$ $= \frac{\cos 14° . \cos 30°}{\left(\frac{\sin 38°}{\cos 38°}\right)(\sin 52°)^2}$ $= \frac{\cos 14°\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sin 38°}{\cos 38°}\right)(\cos^2 38)}$ $= \frac{\sqrt{3}\cos 14°}{2\sin 38° \cos 38°}$ $= \frac{\sqrt{3}\cos 14°}{\sin 76°}$ $= \frac{\sqrt{3}\cos 14°}{\cos 14°}$ $= \sqrt{3}$	✓cos30° ✓ $\frac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓cos 38° ✓ $\frac{\sqrt{3}}{2}$ ✓✓sin76° ✓ $\sqrt{3}$ (8)
	OF $\frac{\sin 104°(2\cos^2 15° - 1)}{\tan 38° \sin^2 412°}$ $= \frac{\sin 104° . \cos 30°}{\frac{\sin 38°}{\cos 38°}.(\sin 52°)^2}$ $= \frac{\sin 104° . \frac{\sqrt{3}}{2}}{\left(\frac{\cos 52°}{\sin 52°}\right)(\sin 52°)^2}$ $= \frac{\sin 104° . \frac{\sqrt{3}}{2}}{\cos 52°(\sin 52°)}$ $= \frac{\sin 104° . \frac{\sqrt{3}}{2}}{\frac{1}{2}\sin 104°}$ $= \sqrt{3}$	✓ cos30° ✓ $\frac{\sin 38°}{\cos 38°}$ ✓ sin52° ✓ $\frac{\sqrt{3}}{2}$ ✓ cos52°=sin38° en sin52°=cos38° ✓ $\cos 52^0 . \sin 52^0$ ✓ $\frac{1}{2}\sin 104°$ ✓ $\sqrt{3}$ (8) **[13]**

VRAAG 10

10.1	$f(0)-g(0)=0{,}5-(-2)=2{,}5$	✓ 2,5 (1)
10.2	$\sin(x+30°)=-2\cos x$ $\sin x.\cos 30°+\cos x.\sin 30°=-2\cos x$ $\left(\frac{\sqrt{3}}{2}\right)\sin x+\left(\frac{1}{2}\right)\cos x=-2\cos x$ $\sqrt{3}\sin x+\cos x=-4\cos x$ $\sqrt{3}\sin x=-5\cos x$ $\tan x=-\frac{5}{\sqrt{3}}$ $x=109{,}11°+180°.k\ ;\ k\in Z$ $x_P=-70{,}89°\ en\ x_Q=109{,}11°$ **OF** $\sin(x+30°)=-2\cos x$ $\cos(90°-x-30°)=-2\cos x$ $\cos(60°-x)=-2\cos x$ $\cos 60°\cos x+\sin 60°\sin x=-2\cos x$ $\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=-2\cos x$ $\cos x+\sqrt{3}\sin x=-4\cos x$ $\sqrt{3}\sin x=-5\cos x$ $\tan x=-\frac{5}{\sqrt{3}}$ $x=109{,}11°+180°.k\ ;\ k\in Z$ $x_P=-70{,}89°\ and\ x_Q=109{,}11°$	✓ vergelyking ✓ uitbreiding sin(*x*+30°) ✓ vervanging van spesiale hoeke ✓ vereenvoudiging ✓ $\tan x=-\frac{5}{\sqrt{3}}$ ✓ $x_P=-70{,}89°$ ✓ $x_Q=109{,}11°$ (7) ✓ vergelyking ✓ uitbreiding van cos(60° – *x*) ✓ vervanging van spesiale hoeke ✓ vereenvoudiging ✓ $\tan x=-\frac{5}{\sqrt{3}}$ ✓ $x_P=-70{,}89°$ ✓ $x_Q=109{,}11°$ (7)
10.3	$-70{,}89°\le x\le 109{,}11°$ **OF** $[-70{,}89°\ ;\ 109{,}11°]$ **OF** $x_P\le x\le x_Q$	✓ hoeke ✓ korrekte interval (2)
10.4	$h(x)=2\sin(x+60°+30°)=2\sin(x+90°)=2\cos x$ *h* is die refleksie van *g* in die *x*-as. **OF** *f* word na links geskuif deur 60° en dan verdubbel. ∴ *h* is die refleksie van *g* in die *x*-as.	✓✓ refleksie in die *x*-as of lyn *y* = 0 (2) ✓✓ refleksie iIn die *x*-as of lyn *y* = 0 (2) **[12]**

VRAAG 11

11.1	Area parallelogram ABCD $= 2 \times$ Area ΔABC $= 2\left[\left(\frac{1}{2}\right)(3)(2)\sin\theta\right]$ $= 6\sin\theta$ **NOTA:** Geen bewerkings dan 0/3	✓✓ 2area ΔABC ✓ vervanging in area reel (3)
	OF  $\frac{h}{2} = \sin\theta$ $h = 2\sin\theta$ $\therefore$ Area $ABCD = b.h = 3.2\sin\theta = 6\sin\theta$	✓ $\frac{h}{2} = \sin\theta$ ✓ $h = 2\sin\theta$ ✓ $b.h$ (3)
	OF Area van parallelogram ABCD = area of ΔABC + area of ΔADC $= \left(\frac{1}{2}\right)(3)(2)\sin\theta + \left(\frac{1}{2}\right)(3)(2)\sin\theta$ = 6 sin θ	✓ som van areas ✓✓gelyke sye en gelyke hoeke (3)

	OF
	Area = $\frac{1}{2}$(sum of // sides)× h
	$= \frac{1}{2}(3 + 3) \times 2\sin\theta$
	= 6 sin θ
		✓ formule
		✓ $h = 2\sin\theta$
		✓vervanging
		(3)

11.2	Area van parallelogram ABCD = $3\sqrt{3}$ $6\sin\theta = 3\sqrt{3}$ $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = 60°$ **NOTA:** `n Penalisasie van 1 punt indien beide 60° en 120°	✓ $6\sin\theta = 3\sqrt{3}$ ✓ $\sin\theta = \frac{\sqrt{3}}{2}$ ✓ 60° (3)
	OF 6 sin 60° = $3\sqrt{3}$ $\therefore$ θ = 60°	✓✓ $6\sin\theta = 3\sqrt{3}$ ✓60° (3)
11.3	Maksimum area van parallelogram verkry as sin θ = 1, dit is waar $\theta = 90°$	✓ sin θ = 1 ✓ $\theta = 90°$ (2) **[8]**

VRAAG 12

<table>
<tr>
<td>12.1</td>
<td>

$\frac{\text{CB}}{\text{sin D}} = \frac{\text{CD}}{\text{sin B}}$

$\frac{\text{CB}}{\sin 2x} = \frac{k}{\sin(90° - x)}$

$CB = \frac{k.\sin 2x}{\sin(90° - x)}$

$\text{CB} = \frac{k.2\sin x \cos x}{\cos x}$

$= 2k \sin x$

</td>
<td>

✓ Gebruik die sin reel in driehoek CBD

✓ $\frac{CB}{\sin 2x} = \frac{k}{\sin(90° - x)}$

✓ $\frac{k.\sin 2x}{\sin(90° - x)}$

✓ 2sin*x*.cos*x*

✓ cos *x*

(5)

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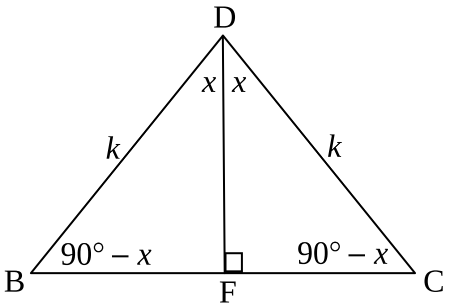
OF

$D\hat{C}B = 180° - (90° - x + 2x) = 90° - x$

$\therefore DC = DB = k$



Trek DF ⊥ BC

$\frac{CF}{CD} = \sin x$

$CF = k \sin x$

$CB = 2CF$

$CB = 2k \sin x$

</td>
<td>

✓ $D\hat{C}B = D\hat{B}C = 90° - x$

✓ $DC = DB = k$

✓ $C\hat{D}F = x$

✓ $CF = k \sin x$

✓ *CB=2 CF*

(5)

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OF

$D\hat{C}B = 180° - (90° - x + 2x) = 90° - x$

$\therefore DC = DB = k$

$\text{CB}^2 = \text{CD}^2 + \text{BD}^2 - 2.\text{CD.BD.cos}2x$

$CB^2 = k^2 + k^2 - 2k^2 \cos 2x$

$= 2k^2(1 - \cos 2x)$

$= 2k^2(1 - (1 - 2\sin^2 x))$

$= 2k^2(2\sin^2 x)$

$= 4k^2 \sin^2 x$

$= (2k \sin x)^2$

$CB = 2k \sin x$

</td>
<td>

✓ $D\hat{C}B = D\hat{B}C = 90° - x$

✓ $DC = DB = k$

✓ gebruik cos reel in driehoek CDB

✓ faktore

✓ vereenvoudiging

(5)

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</table>

12.2	$\cos x = \dfrac{\text{BC}}{\text{HC}}$ $\text{HC} = \dfrac{\text{BC}}{\cos x}$ $= \dfrac{2k\sin x}{\cos x}$ $= 2k\tan x$ **OF** $\dfrac{HC}{\sin 90°} = \dfrac{BC}{\sin(90° - x)}$ $HC = \dfrac{BC}{\sin(90° - x)}$ $= \dfrac{2k\sin x}{\cos x}$ $= 2k\tan x$	✓ $\cos x = \dfrac{BC}{HC}$ ✓ $HC = \dfrac{BC}{\cos x}$ ✓ vervanging van BC (3) ✓ $HC = \dfrac{BC}{\sin(90° - x)}$ ✓ vervanging van BC ✓ $\sin(90° - x) = \cos x$ (3)
12.3	HC = $2k\tan x$ = 2(40).tan(23°) = 33,9579... In ΔHCD: $CD^2 = HC^2 + HD^2 - 2HC.HD.\cos\theta$ $\cos\theta = \dfrac{HC^2 + HD^2 - CD^2}{2HC.HD}$ $= \dfrac{(33{,}9579...)^2 + 31{,}8^2 - 40^2}{2(33{,}9579...)(31{,}8)}$ $\cos\theta = 0{,}2613...$ $\therefore \theta = 74{,}85°$	✓waarde van HC ✓vervanging in cos formule ✓cos θ = 0,2613... ✓ 74,85° (4) **[12]**

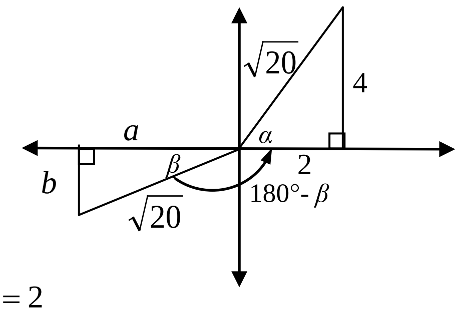
VRAAG 13

<table>
<tr><td>13.1</td><td>Hoek waardeur minuut wyser beweeg is:
$\frac{37}{60}\times 360°$
$= 222°$

P is geroteer deur 138° in `n anti-kloksgewyse rigting:
$a = 2\cos 138° - 4\sin 138°$ en $b = 4\cos 138° + 2\sin 138°$
$= -4{,}16$ $= -1{,}63$</td><td>✓✓ $\frac{37}{60}\times 360°$
✓ 222°
✓ vervanging van 138^0 in formule vir x en y
✓ −4,16
✓ −1,63
(6)</td></tr>
<tr><td></td><td>OF

Hoek waardeur minuut wyser beweeg is:
$\frac{37}{60}\times 360°$
$= 222°$

P is geroteer deur 222° in `n kloksgewyse rigting:
$a = 2\cos 222° + 4\sin 222°$ en $b = 4\cos 222° - 2\sin 222°$
$= -4{,}16$ $= -1{,}63$</td><td>✓✓ $\frac{37}{60}\times 360°$
✓ 222°
✓ vervanging van 222^0 in formule vir x en y
✓ −4,16
✓ −1,63
(6)</td></tr>
<tr><td></td><td>OF



$\tan\alpha = 2$
$\alpha = 63{,}43°$

$\alpha + 180° - \beta = 222°$
$\beta = 63{,}43° + 180° - 222°$
$= 21{,}43°$
$\therefore a = -\sqrt{20}\cos 21{,}43° = -4{,}16$
$b = -\sqrt{20}\sin 21{,}43° = -1{,}63$</td><td>✓ $\tan\alpha = 2$
✓ $\alpha = 63{,}43°$
✓ $\alpha + 180° - \beta = 222°$
✓ $\beta = 21{,}43°$
✓ −4,16
✓ −1,63
(6)</td></tr>
</table>

13.2	Die minuut-wyser beweeg deur 360° in 60 minute. Die uur-wyser beweeg deur 30° in 60 minute, dus `n $\frac{1}{12}$ van die minuut-wyser. Daarom as die minuut-wyser deur 222° beweeg, beweeg die uur-wyser deur $\frac{222°}{12} = 18{,}5°$	✓ 360° ✓ 30° ✓ $\frac{1}{12}$ ✓ 18,5° (4)
	OF	
	Uur-wyser beweeg deur $\frac{360°}{12} = 30°$ grade in 60 minute. $\therefore$ 37 minute: $\frac{37}{60} \times 30° = 18{,}5°$	✓ 360° ✓ 30° ✓ $\frac{37}{60} \times 30°$ ✓ 18,5° (4) **[10]**

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