basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2011

POSSIBLE ANSWERS

MARKS: 150

This memorandum consists of 28 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in all aspects of the marking memorandum.

QUESTION 1

1.1.1	$x(x+1)=6$ $x^2+x=6$ $x^2+x-6=0$ $(x+3)(x-2)=0$ $x=-3 \text{ or } 2$ **OR** $x^2+x-6=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-1\pm\sqrt{1^2-4(1)(-6)}}{2(1)}$ $x=-3 \text{ or } 2$	**Note:** Answers by inspection: award 3/3 marks **Note:** Answer only of $x=2$: award 1/3 marks **Note:** If candidate converts equation to linear: award 0/3 marks	✓ standard form ✓ factors ✓ answers (3) ✓ standard form ✓ substitution into correct formula ✓ answers (3)
1.1.2	$3x^2-4x=8$ $3x^2-4x-8=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(-4)\pm\sqrt{(-4)^2-4(3)(-8)}}{2(3)}$ $=\frac{4\pm\sqrt{16+96}}{6}$ $=\frac{4\pm\sqrt{112}}{6}$ $=\frac{2\pm2\sqrt{7}}{3}$ $=2{,}43 \text{ or } -1{,}10$ **Note:** Penalise 1 mark for inaccurate rounding off to ANY number of decimal places if candidate gives decimal answers.	**Note:** If candidate uses incorrect formula: maximum 1/4 marks (for standard form) **Note:** If an error in subs and gets: $\frac{4\pm\sqrt{-80}}{6}$ and states "no solution": maximum 3/4 marks If doesn't conclude with "no solution": maximum 2/4 marks	✓ standard form ✓ substitution into correct formula ✓ $\sqrt{112}$ ✓ $\frac{4\pm\sqrt{112}}{6}$ or decimal answer (4)

OR

$$3x^2 - 4x = 8$$
$$3x^2 - 4x - 8 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$$
$$= 2{,}43 \text{ or } -1{,}10$$

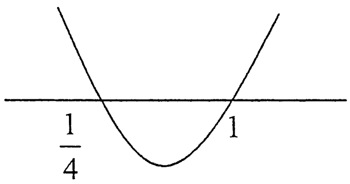
Note: Penalise 1 mark for inaccurate rounding off to ANY number of decimal places if candidate gives decimal answers

✓ standard form
✓ substitution into correct formula
✓ answer
✓ answer
(4)

1.1.3

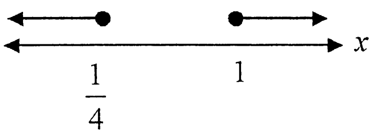
$$4x^2 + 1 \geq 5x$$
$$4x^2 - 5x + 1 \geq 0$$
$$(4x-1)(x-1) \geq 0$$

+	0	−	0	+
	$\frac{1}{4}$		1	

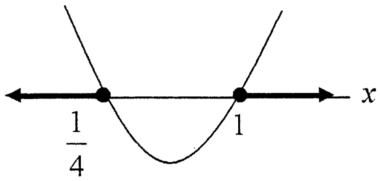


$x \leq \frac{1}{4}$ or $x \geq 1$ **OR** $\left(-\infty; \frac{1}{4}\right] \cup [1; \infty)$

OR



OR



Note: If candidate gives either of these correct graphical solutions but writes down the incorrect intervals or uses AND: max 3/4 marks

✓ factors
✓ both critical values of $\frac{1}{4}$ and 1
✓ or **OR** $\cup$
✓ answer
(4)

NOTES:

If a candidate gives an answer of $1 \leq x \leq \frac{1}{4}$ then max 3/4 marks.

If a candidate gives an answer of $\frac{1}{4} \leq x \leq 1$ then max 2/4 marks.

If a candidate gives an answer of $x \leq \frac{1}{4}$ **and** $x \geq 1$ then max 3/4 marks.

If the candidate leaves out the equality of the notation then penalty of 1 mark.

If a candidate gives an answer of $x \leq \frac{1}{4}$; $x \geq 1$ then max 3/4 marks.

If candidate gives $x \geq \frac{1}{4}$ and/or $x \geq 1$, BREAKDOWN: max 2/4 marks.

If candidate gives : award 3/4 marks

+	0	−	0	+
	$\frac{1}{4}$		1	

1.2.1	$x^2+5xy+6y^2=0$ $(x+3y)(x+2y)=0$ $x+3y=0$ OR $x+2y=0$ $x=-3y$ OR $x=-2y$ $\frac{x}{y}=-3$ $\frac{x}{y}=-2$ **Note:** If a candidate gives $-\frac{x}{y}=3$ or $-\frac{x}{y}=2$ award 2/3 marks	✓ factors ✓✓ answers (3)
	OR Let $k=\frac{x}{y}$ $x^2+5xy+6y^2=0$ $\left(\frac{x}{y}\right)^2+5\left(\frac{x}{y}\right)+6=0$ $k^2+5k+6=0$ $(k+3)(k+2)=0$ $k=-3$ or $k=-2$ $\frac{x}{y}=-3$ or $\frac{x}{y}=-2$	✓ factors ✓✓ answers (3)
	OR $x^2+5xy+6y^2=0$ $x=\frac{-5y\pm\sqrt{(5y)^2-4(1)(6y^2)}}{2(1)}$ $x=\frac{-5y\pm\sqrt{y^2}}{2}$ $x=\frac{-5y\pm y}{2}$ $x=-3y$ $x=-2y$ $\frac{x}{y}=-3$ or $\frac{x}{y}=-2$	✓ substitutes correctly into correct formula ✓✓ answers (3)
	OR $x^2+5xy+6y^2=0$ $x^2+5xy+\left(\frac{5}{2}y\right)^2=-6y^2+\left(\frac{5}{2}y\right)^2$ $\left(x+\frac{5}{2}y\right)^2=\frac{1}{4}y^2$ $x+\frac{5}{2}y=\pm\frac{1}{2}y$ $x=-\frac{5}{2}y\pm\frac{1}{2}y$	✓ completing the square

	$x=-3y \qquad x=-2y$ $\frac{x}{y}=-3$ or $\frac{x}{y}=-2$ **OR** Let $k=\frac{x}{y}$ $x=ky$ $x^2+5xy+6y^2=0$ $(ky)^2+5y(ky)+6y^2=0$ $k^2y^2+5y^2k+6y^2=0$ $y^2(k^2+5k+6)=0$ $(k^2+5k+6)=0$ $(k+3)(k+2)=0$ $k=-3$ or $k=-2$ $\frac{x}{y}=-3$ or $\frac{x}{y}=-2$ Note: $(x;y)=(0;0)$ is also a solution, but in this case $\frac{x}{y}$ is undefined **OR** Let $y=1$, $x^2+5x+6=0$ $(x+2)(x+3)=0$ $x=-2$ or $x=-3$ $\frac{x}{y}=-2$ or $\frac{x}{y}=-3$	✓✓ answers (3) ✓ factors ✓✓ answers (3) ✓ factors ✓✓ answers (3)
1.2.2	$x+y=8 \qquad x+y=8$ $-3y+y=8 \qquad -2y+y=8$ $-2y=8$ OR $-y=8$ $y=-4 \qquad y=-8$ $x=12 \qquad x=16$ **OR** $\frac{8-y}{y}=-3$ OR $\frac{8-y}{y}=-2$ $8-y=-3y \qquad 8-y=-2y$ $8=-2y \qquad 8=-y$ $y=-4 \qquad y=-8$ $x=12 \qquad x=16$	✓ substitution $x=-3y$ ✓ subs $x=-2y$ ✓✓ y values ✓ both x values correct (5) ✓ $x=8-y$ ✓ substitution ✓✓ y values ✓ both correct x values (5)

	OR $x+y=8$ $y=8-x$ $\frac{x}{8-x}=-3$ OR $\frac{x}{8-x}=-2$ $x=-3(8-x)$ $x=-2(8-x)$ $x=-24+3x$ $x=-16+2x$ $-2x=-24$ $-x=-16$ $x=12$ $x=16$ $y=-4$ $y=-8$	✓ $y=8-x$ ✓ substitution ✓✓ x values correct ✓ both y values correct (5)
	OR $(x+2y)(x+3y)=0$ $x+y=8$ $x=8-y$ $(y+8)(2y+8)=0$ $y=-8$ or $y=-4$ $x=16$ $x=12$	✓ $x=8-y$ ✓ substitution ✓✓ y values correct ✓ both x values correct (5)
	OR $x=8-y$ $(8-y)^2+5(8-y)y+6y^2=0$ $64-16y+y^2+40y-5y^2+6y^2=0$ $2y^2+24y+64=0$ $y^2+12y+32=0$ $(y+8)(y+4)=0$ $y=-8$ or $y=-4$ $x=16$ $x=12$ **OR**	✓ $x=8-y$ ✓ substitution ✓factors ✓ both y values correct ✓ both x values correct (5)

	OR	
	$x = 8 - y$	✓ $x = 8 - y$
	$(8-y)^2 + 5(8-y)y + 6y^2 = 0$	✓ substitution
	$64 - 16y + y^2 + 40y - 5y^2 + 6y^2 = 0$	
	$2y^2 + 24y + 64 = 0$	
	$y^2 + 12y + 32 = 0$	
	$y = \dfrac{-12 \pm \sqrt{12^2 - 4(1)(32)}}{2(1)}$	✓ substitutes into correct formula
	$= \dfrac{-12 \pm \sqrt{16}}{2}$	✓ both y values correct
	$y = -8$ or $y = -4$	✓ both x values correct
	$x = 16 \qquad x = 12$	
	Note: If a candidate uses the formula and replaces x for y and then answers are swapped: maximum 4/5 marks	(5)
	OR	
	$y = 8 - x$	✓ $y = 8 - x$
	$x^2 + 5x(8-x) + 6(8-x)^2 = 0$	✓ substitution
	$x^2 + 40x - 5x^2 + 6(64 - 16x + x^2) = 0$	
	$2x^2 - 56x + 384 = 0$	✓factors
	$x^2 - 28x + 192 = 0$	
	$(x-16)(x-12) = 0$	✓ both x values correct
	$x = 12$ or $x = 16$	✓ both y values correct
	$y = -4 \qquad y = -8$	(5)
	OR	
	$y = 8 - x$	✓ $y = 8 - x$
	$x^2 + 5x(8-x) + 6(8-x)^2 = 0$	✓ substitution
	$x^2 + 40x - 5x^2 + 6(64 - 16x + x^2) = 0$	
	$2x^2 - 56x + 384 = 0$	
	$x^2 - 28x + 192 = 0$	
	$x = \dfrac{-(-28) \pm \sqrt{(-28)^2 - 4(1)(192)}}{2(1)}$	✓ substitutes into correct formula
	$= \dfrac{28 \pm \sqrt{416}}{2}$	✓ both x values correct
	$x = 12$ or $x = 16$	✓ both correct y values
	$y = -4 \qquad y = -8$	(5)
		[19]

QUESTION 2

2.1.1	$x-4=32-x$ $2x=36$ $x=18$ **Note:** If answer only: award 2/2 marks **OR** $a=4$ $a+2d=32$ $2d=28$ $d=14$ $x=14+4$ $x=18$ **Note:** If candidate writes $x-4 \quad 32-x$ only (i.e. omits equality) : 0/2 marks **OR** $x=\frac{4+32}{2}=18$	✓ $T_2-T_1=T_3-T_2$ ✓ answer (2) ✓ $a+2d=32$ and $a=4$ ✓ answer (2) ✓ substitutes correctly into arithmetic mean formula i.e. $\frac{4+32}{2}$ ✓ answers (2)
2.1.2	$\frac{x}{4}=\frac{32}{x}$ $x^2=128$ $x=\pm\sqrt{128}$ $x=\pm 8\sqrt{2}$ OR $x=\pm 11{,}31$ OR $x=\pm 2^{\frac{7}{2}}$ **Note:** If candidate writes $\frac{x}{4} \quad \frac{32}{x}$ only (i.e. omits equality) : 0/2 marks **OR** $a=4$ $r=\frac{x}{4}$ $ar^2=4\left(\frac{x}{4}\right)^2$ $32=4\left(\frac{x}{4}\right)^2$ $x^2=128$ $x=\pm\sqrt{128}$ $x=\pm 8\sqrt{2}$ or $x=\pm 11{,}31$ or $x=\pm 2^{\frac{7}{2}}$ **Note:** If only $x=\sqrt{128}$ then penalty 1 mark **OR** $x=\pm\sqrt{4\times 32}$ $x=\pm\sqrt{128}$ or $x=\pm 8\sqrt{2}$ or $x=\pm 11{,}31$ or $x=\pm 2^{\frac{7}{2}}$	✓ $\frac{T_2}{T_1}=\frac{T_3}{T_2}$ ✓ $x^2=128$ ✓ both answers (surd or decimal or exponential form) (3) ✓ $32=4\left(\frac{x}{4}\right)^2$ ✓ $x^2=128$ ✓ both answers (surd or decimal or exponential form) (3) ✓✓ substitutes correctly into geometric mean formula i.e. $\pm\sqrt{4\times 32}$ ✓ both answers (surd or decimal or exponential form) (3)

2.2	$P = \sum_{k=1}^{13} 3^{k-5}$ $= 3^{1-5} + 3^{2-5} + 3^{3-5} + \ldots + 3^{13-5}$ $= 3^{-4} + 3^{-3} + 3^{-2} + \ldots + 3^{8}$ $= \dfrac{3^{-4}(3^{13}-1)}{3-1}$ $= 9841{,}49$ or $9841\frac{40}{81}$ or $\frac{797161}{81}$ **Note:** Correct answer only: 1/4 marks only	✓ $a = 3^{-4}$ or $\frac{1}{81}$ ✓ $r = 3$ ✓ subs into correct formula ✓ answer (4)
	OR $P = \sum_{k=1}^{13} 3^{k-5}$ $= 3^{1-5} + 3^{2-5} + 3^{3-5} + \ldots + 3^{13-5}$ $= 3^{-4} + 3^{-3} + 3^{-2} + \ldots + 3^{8}$ $= \frac{1}{81} + \frac{1}{27} + \frac{1}{9} + \ldots + 6561$ $= 9841{,}49$ or $9841\frac{40}{81}$ or $\frac{797161}{81}$ **Note:** If the candidate rounds off and gets 9841,46 (i.e. correct to one decimal place): DO NOT penalise for the rounding off.	✓✓ expand the sum ✓ 13 terms in expansion ✓ answer (4)
2.3	$S_n = a + [a+d] + [a+2d] + \ldots + [a+(n-2)d] + [a+(n-1)d]$ $S_n = [a+(n-1)d] + [a+(n-2)d] + \ldots + [a+d] + a$ $2S_n = [2a+(n-1)d] + [2a+(n-1)d] + \ldots + [2a+(n-1)d] + [2a+(n-1)d]$ $= n[2a+(n-1)d]$ $S_n = \frac{n}{2}[2a+(n-1)d]$	✓writing out S_n ✓ "reversing" S_n ✓expressing $2S_n$ ✓ grouping to get $2S_n = n[2a+(n-1)d]$ (4)
	OR $S_n = a + [a+d] + [a+2d] + \ldots + (T_n - d) + T_n$ $S_n = T_n + (T_n - d) + \ldots + [a+d] + a$ $2S_n = a + T_n + a + T_n + a + T_n + \ldots + a + T_n$ $= n[a + a + (n-1)d]$ $= [2a+(n-1)d]$ $S_n = \frac{n}{2}[2a+(n-1)d]$ **Note:** If a candidate uses a circular argument (eg $S_{n+1} = S_n + T_n$): max 1/4 marks (for writing out S_n) **Note:** If a candidate uses a specific linear sequence, then NO marks.	✓writing out S_n ✓ "reversing" S_n ✓expressing $2S_n$ ✓ grouping to get $2S_n = n[a+a+(n-1)d]$ (4) **[13]**

QUESTION 3

3.1	21; 24 **Note:** If candidate writes $T_8 = 21$ $T_7 = 24$: award 1/2 marks	✓ 21 ✓ 24 (2)
3.2	$T_{2k} = 3.2^{k-1}$ and so $T_{52} = 3.2^{26-1} = 100663296$ $T_{2k-1} = 3 + 6(k-1) = 6k - 3$ and so $T_{51} = 6(26) - 3 = 153$ $T_{52} - T_{51} = 100663296 - 153$ $= 100663143$ **Note:** If candidate writes out all 52 terms **and** gets correct answer: award 5/5 marks **Note:** If candidate used $k = 52$: max 2/5	✓ 3.2^{k-1} ✓ T_{52} ✓ $6k - 3$ ✓ T_{51} ✓ answer (5)
	OR Consider sequence P: 3 ; 6 ; 12 … $P_n = 3.2^{n-1}$ $P_{26} = 3.2^{26-1} = 100663296$ Consider sequence Q: 3 ; 9 ; 15 … $Q_n = 6n - 3$ $Q_{26} = 6(26) - 3 = 153$ $T_{52} - T_{51} = P_{26} - Q_{26}$ $= 100663296 - 153$ $= 100663143$ **Note:** if candidate interchanges order i.e. does $T_{51} - T_{52}$: max 4/5 marks **Note:** writes out all 52 terms and subtracts $T_{51} - T_{52}$: max 4/5 marks	✓ $P_n = 3.2^{n-1}$ ✓ P_{26} ✓ $Q_n = 6n - 3$ ✓ Q_{26} ✓ answer (5)

3.3	For all $n \in \mathbf{N}$, $n = 2k$ or $n = 2k-1$ for some $k \in \mathbf{N}$ If $n = 2k$: $T_n = T_{2k} = 3.2^{k-1}$ If $n = 2k-1$: $T_n = T_{2k-1}$ $= 6k-3$ $= 3(2k-1)$ In either case, T_n has a factor of 3, so is divisible by 3. **Note:** If a candidate only illustrates divisibility by 3 with a specific finite part of the sequence, not the general term: 0/2 marks **OR** $P_n = 3.2^{n-1}$ Which is a multiple of 3 $Q_n = 6n-3$ $= 3(2n-1)$ Which is also a multiple of 3 Since $T_n = Q_{2k-1}$ or $T_n = P_{2k}$ for all $n \in \mathbf{N}$, T_n is always divisible by 3 **OR** The odd terms are odd multiples of 3 and the even terms are 3 times a power of 2. This means that all the terms are multiples of 3 and are therefore divisible by 3.	✓ factors 3.2^{k-1} ✓ factors $3(2k-1)$ (2) ✓ factors 3.2^{n-1} ✓ factors $3(2n-1)$ (2) ✓ odd multiples of 3 ✓ 3 times a power of 2 (2) **[9]**

QUESTION 4

4.1 The second, third, fourth and fifth terms are 1 ; – 6 ; T_4 and – 14

First differences are: -7 ; T_4+6 ; $-14-T_4$
So $T_4+6+7=-14-2T_4-6$
$T_4=-11$
$d=-11+6+7=2$ or $-14+22-6=2$

> **Note:** Answer only (i.e. d = 2) with no working: 3 marks

> **Note:** Candidate gives $T_4=-11$ and $d=2$ only: award 5/5 marks

✓ -7
✓ T_4+6
✓ $-14-T_4$
✓ setting up equation
✓ answer
(5)

OR

T_2		T_3		T_4		T_5
1		-6				-14
	-7		$-7+d$		$-7+2d$	
		d		d		

$$T_5-T_2=(T_5-T_4)+(T_4-T_3)+(T_3-T_2)$$
$$-15=(-7+2d)+(-7+d)+-7$$
$$-15=-21+3d$$
$$6=3d$$
$$d=2$$

> **Note:** Candidate uses trial and error **and** shows this: award 5/5 marks

✓ -7
✓ $-7+d$
✓ $-7+2d$
✓ setting up equation $T_5-T_2=(T_5-T_4)+(T_4-T_3)+(T_3-T_2)$
✓ answer
(5)

OR

$$4a+2b+c=1$$
$$9a+3b+c=-6$$
$$5a+b=-7$$

$$25a+5b+c=-14$$
$$16a+2b=-8$$
$$10a+2b=-14$$
$$6a=6$$
$$a=1$$
$$d=2a=2$$

✓ $4a+2b+c=1$
✓ $9a+3b+c=-6$
✓ $25a+5b+c=-14$
✓ solved simultaneously
✓ answer
(5)

OR

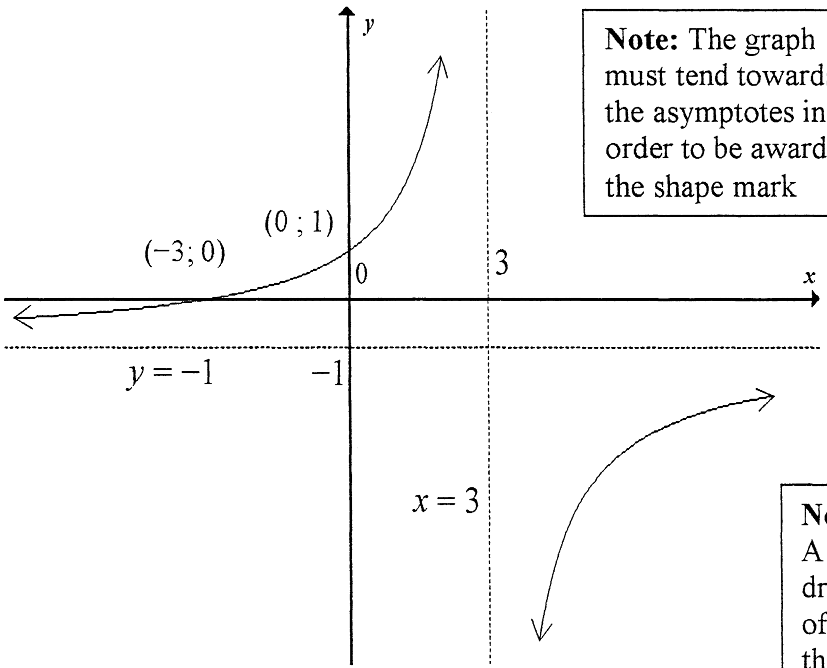
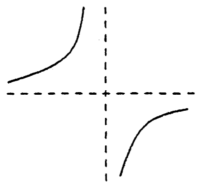
T_1		1		-6		T_4		-14
	$1-T_1$		-7		T_4+6		$-14-T_4$	
		T_1-8		T_4+13		$-20-2T_4$		

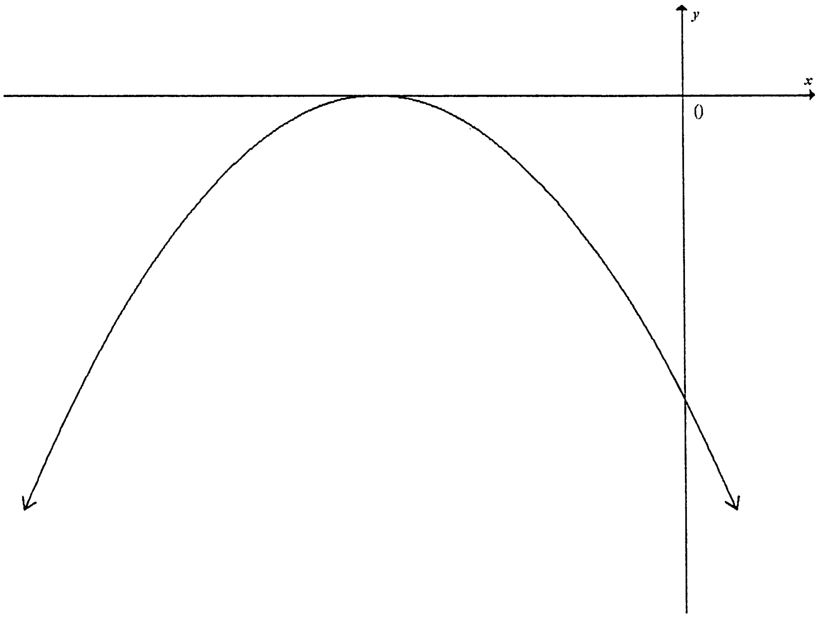
$$T_4+13=-20-2T_4$$
$$3T_4=-33$$
$$T_4=-11$$
$$d=-11+13$$
$$d=2$$

✓ -7
✓ T_4+6
✓ $-14-T_4$
✓ setting up equation
✓ answer
(5)

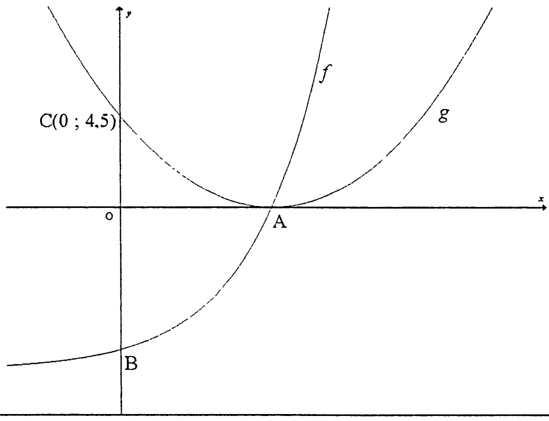
	OR $\begin{array}{ccccccccc} T_1 & & T_2 & & T_3 & & T_4 & & T_5 \\ x & & 1 & & -6 & & y & & -14 \\ & 1-x & & -7 & & y+6 & & -14-y & \\ & & -8+x & & y+13 & & -20-2y & & \end{array}$ $y+13=-20-2y$ $3y=-33$ $y=-11$ Second difference = $y+13=-11+13=2$	✓ -7 ✓ $y+6$ ✓ $-14-y$ ✓ setting up equation ✓ answer (5)
4.2	$\begin{array}{ccccc} T_1 & & 1 & & -6 \\ & -9 & & -7 & \\ & & 2 & & \end{array}$ $T_1 = 10$ **Note:** Answer only: award 2/2 marks **Note:** If incorrect d in 4.1, 2/2 CA marks for $T_1 = d+8$ (since $1-T_1=-7-d$)	✓ method ✓ $T_1 = 10$ (2)
	OR $a=1$ $5a+b=-7$ $5(1)+b=-7$ $b=-12$ $a+b+c=1$ $4(1)+2(-12)+c=1$ $c=21$ $T_n = n^2-12n+21$ $T_1=(1)^2-12(1)+21$ $=10$	✓ method ✓ $T_1 = 10$ (2)
	OR $T_4+13=-8+T_1$ **OR** $y+13=-8+x$ $-11+13=-8+T_1$ **OR** $-11+13=-8+x$ $T_1=10$ **OR** $x=10$	✓ method ✓ $T_1 = 10$ (2) [7]

QUESTION 5

5.1.1	$y = f(0)$ $= \frac{-6}{0-3} - 1$ $= 1$ $(0\ ;\ 1)$ **OR** $x = 0$ and $y = 1$ **Note:** Mark 5.1.1 and 5.1.2 as a single question. If the intercepts are interchanged: max 3/5 marks	✓ $y = 1$ ✓ $x = 0$ (2)
5.1.2	$0 = \frac{-6}{x-3} - 1$ $1 = \frac{-6}{x-3}$ $x - 3 = -6$ $x = -3$ $(-3\ ;\ 0)$	✓ $y = 0$ ✓ $x - 3 = -6$ ✓ answer (3)
5.1.3	 **Note:** The graph must tend towards the asymptotes in order to be awarded the shape mark **Note:** A candidate who draws only one 'arm' of the hyperbola loses the 'shape' mark i.e. max 3/4 marks	✓ shape  ✓ both intercepts correct ✓ horizontal asymptote ✓ vertical asymptote (4)
5.1.4	$-3 < x < 3$ **OR** $(-3; 3)$ **OR** $-3 < x$ and $x < 3$ **Note**: if candidate writes $-3 < x$ only: 1/2 marks **Note**: if candidate writes $x < 3$ only: 1/2 marks	✓-3 and 3 ✓inequality OR interval notation (2)

5.1.5	$y = \dfrac{-6}{-2-3} - 1$ $= \dfrac{1}{5}$ $m = \dfrac{1-\frac{1}{5}}{0-(-2)}$ $= \dfrac{2}{5}$ **OR** $m = \dfrac{f(0)-f(-2)}{0-(-2)}$ $= \dfrac{1-\frac{1}{5}}{0+2}$ $= \dfrac{2}{5}$	✓ $\frac{1}{5}$ ✓ formula ✓ substitution ✓ answer (4) ✓ formula ✓ $f(-2) = \frac{1}{5}$ ✓ substitution ✓ answer (4)
5.2	$x = -\dfrac{b}{2a} < 0$ since $b < 0$ and $a < 0$  	✓ y-intercept negative ✓ turning point on the x axis ✓ turning point on the left of the y axis ✓ **maximum** TP and quadratic shape (4) **[19]**

QUESTION 6



6.1	$0 = 2^x - 8$ $8 = 2^x$ $2^3 = 2^x$ $x = 3$ A(3 ; 0) $f(0) = 2^0 - 8$ $= 1 - 8$ $= -7$ B(0 ; –7)	✓ $y = 0$ ✓ answer for A ✓ $x = 0$ ✓ answer for B (4)
6.2	$y = -8$ **OR** $y + 8 = 0$ **Note**: no CA marks	✓ answer (1)
6.3	$h(x) = f(2x) + 8$ $= \left(2^{2x} - 8\right) + 8$ $= 4^x$ or 2^{2x} **Note**: answer only: award 2/2 marks	✓ $\left(2^{2x} - 8\right)$ ✓ answer of $h(x) = 4^x$ or 2^{2x} (2)
6.4	$x = 4^y$ $y = \log_4 x$ **OR** $y = \dfrac{\log x}{\log 4}$ **OR** $x = 2^{2y}$ $2y = \log_2 x$ $y = \dfrac{1}{2}\log_2 x$ **OR** $y = \log_2 \sqrt{x}$ **Note**: answer only award 2/2 marks **Note**: candidate works out f^{-1} and gets $y = \log_2(x+8)$ award 1/2 marks	✓ switch x and y ✓ answer in the form $y = ...$ (2)
6.5	$p(x) = -\log_4 x$ **OR** $p(x) = \log_{\frac{1}{4}} x$ **OR** $p(x) = \log_4 \dfrac{1}{x}$ **OR** $p(x) = -\dfrac{1}{2}\log_2 x$ **OR** $y = -\log_2 \sqrt{x}$	✓answer (1)

6.6	$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ $x = 3$ is the axis of symmetry of g $\therefore$ by symmetry $g(2) = g(4)$ and $g(1) = g(5)$ Answer = $g(0) + g(3)$ $= 4{,}5 + 0$ $= 4{,}5$	✓ $= g(0) + g(1) + g(2) + g(3) - g(4) - g(5)$ ✓ $g(2) = g(4)$ and $g(1) = g(5)$ ✓ $g(0) + g(3)$ ✓ answer (4)
	OR	
	$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $\sum_{k=0}^{3} g(k) = g(0) + g(1) + g(2) + g(3)$ $\sum_{k=4}^{5} g(k) = g(4) + g(5)$ $x = 3$ is the axis of symmetry of g $\therefore$ by symmetry $g(4) = g(2)$ $g(5) = g(1)$ $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $= g(0) + g(3)$ $= 4{,}5 + 0$ $= 4{,}5$	✓ expansion ✓ $g(2) = g(4)$ and $g(1) = g(5)$ ✓ $g(0) + g(3)$ ✓ answer (4)
	OR	
	$g(x) = a(x-3)^2 + 0$ $4{,}5 = a(0-3)^2 + 0$ $4{,}5 = 9a$ $a = \frac{1}{2}$ $g(x) = \frac{1}{2}(x-3)^2$ $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $\sum_{k=0}^{3} g(k) = g(0) + g(1) + g(2) + g(3)$ $= 4{,}5 + 2 + 0{,}5 + 0$ $= 7$	✓ $g(x) = \frac{1}{2}(x-3)^2$ ✓ expansion

	$\sum_{k=4}^{5} g(k) = g(4) + g(5)$ $= 0,5 + 2$ $= 2,5$ $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$ $= 7 - 2,5$ $= 4,5$	✓ $7 - 2,5$ ✓ answer (4)
	OR	
	$g(x) = ax^2 + bx + c$ $g(k) = ak^2 + bk + c$ $g(0) = c$ $g(1) = a + b + c$ $g(2) = 4a + 2b + c$ $g(3) = 9a + 3b + c$ $\sum_{k=0}^{3} g(k) = 14a + 6b + 4c$ $g(4) = 16a + 4b + c$ $g(5) = 25a + 9b + c$ $\sum_{k=4}^{5} g(k) = 41a + 9b + 2c$ $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k) = -27a - 3b + 2c$	✓✓ $-27a - 3b + 2c$
	$g(x) = a(x-3)^2 + 0$ $4,5 = a(0-3)^2 + 0$ $4,5 = 9a$ $a = \frac{1}{2}$ $g(x) = \frac{1}{2}(x-3)^2$ $= \frac{1}{2}x^2 - 3x + \frac{9}{2}$	✓ $g(x) = \frac{1}{2}(x-3)^2$
	$\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k) = -27a - 3b + 2c$ $= -27\left(\frac{1}{2}\right) - 3(-3) + 2\left(\frac{9}{2}\right)$ $= 4,5$	✓ answer (4) **[14]**

QUESTION 7

7.1	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log\frac{1}{2} = n\log 0{,}93$ $n = \frac{\log\frac{1}{2}}{\log 0{,}93}$ $= 9{,}55$ years **OR** $A = P(1-i)^n$ $\frac{P}{2} = P(1-0{,}07)^n$ $\frac{1}{2} = 0{,}93^n$ $\log_{0,93}\frac{1}{2} = n$ $n = 9{,}55$ years **Note:** If candidate interchanges A and P i.e. uses $P = \frac{A}{2}$: max 2/4 marks **Note:** If candidate uses incorrect formula: max 1/4 marks for $A = \frac{P}{2}$	✓ $A = \frac{P}{2}$ ✓ subs into correct formula ✓ log ✓ answer (4)

7.2	**Radesh:** $A = P(1+in)$ $= 6\,000(1+0{,}085\times 5)$ $= 8\,550$ OR $A = 6\,000 + 8{,}5\% \text{ of } 6000\times 5$ $= 6000 + 510\times 5$ $= 6000 + 2550$ $= 8\,550$ $\text{Bonus} = 0{,}05\times 6\,000$ $= 300$ $\text{Received} = 8\,550 + 300$ $= \text{R}8\,850$ **Thandi:** $A = P(1+i)^n$ $= 6\,000\left(1+\frac{0{,}08}{4}\right)^{20}$ $= \text{R}8\,915{,}68$ Thandi's investment is bigger.	✓ 8 550 ✓ **R**8 850 ✓ $n = 20$ ✓ $i = \frac{0{,}08}{4}$ ✓ answer ✓ choice made (6)
7.3	F_v = initial deposit with interest + annuity $= 1\,000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18} - 1}{\frac{0{,}15}{12}}\right)$ $= 1\,250{,}58 + 14\,032{,}33$ $= \text{R}15\,282{,}91$ **OR** F_v = initial deposit with interest + annuity $= 1\,000\left(1+\frac{0{,}15}{12}\right)^{18} + 700\left(\frac{1-\left(1+\frac{0{,}15}{12}\right)^{-18}}{\frac{0{,}15}{12}}\right)\left(1+\frac{0.15}{12}\right)^{18}$ $= 1\,250{,}58 + 11220{,}68\left(1+\frac{0.15}{12}\right)^{18}$ $= 1\,250{,}58 + 14\,032{,}33$ $= \text{R}15\,282{,}91$	✓ $i = \frac{0{,}15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 18$ ✓ $n = 18$ ✓ $1\,000\left(1+\frac{0{,}15}{12}\right)^{18}$ ✓ $700\left(\frac{\left(1+\frac{0{,}15}{12}\right)^{18} - 1}{\frac{0{,}15}{12}}\right)$ ✓ answer (6) ✓ $i = \frac{0{,}15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 18$ ✓ $n = 18$ ✓ $1\,000\left(1+\frac{0{,}15}{12}\right)^{18}$ ✓ $700\left(\frac{1-\left(1+\frac{0.15}{12}\right)^{-18}}{\frac{0.15}{12}}\right)\left(1+\frac{0.15}{12}\right)^{18}$ ✓ answer 4 (6)

	OR $F_v = 300\left(1+\frac{0,15}{12}\right)^{18} + 700\left(\frac{\left(1+\frac{0,15}{12}\right)^{19}-1}{\frac{0,15}{12}}\right)$ $= 375,17 + 14\ 907,74$ $= \text{R}15\ 282,91$	✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125 ✓ $n = 19$ (corresponding to 700) ✓ $n = 18$ (corresponding to 300) ✓ $300\left(1+\frac{0,15}{12}\right)^{18}$ ✓ $700\left(\frac{\left(1+\frac{0,15}{12}\right)^{19}-1}{\frac{0,15}{12}}\right)$ ✓ answer (6) **[16]**

QUESTION 8

8.1	$f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $= \lim\limits_{h\to 0}\frac{-4(x+h)^2-(-4x^2)}{h}$ $= \lim\limits_{h\to 0}\frac{-4(x^2+2xh+h^2)+4x^2}{h}$ $= \lim\limits_{h\to 0}\frac{-4x^2-8xh-4h^2+4x^2}{h}$ $= \lim\limits_{h\to 0}\frac{-8xh-4h^2}{h}$ $= \lim\limits_{h\to 0}\frac{h(-8x-4h)}{h}$ $= \lim\limits_{h\to 0}(-8x-4h)$ $= -8x$ **OR**	**Note:** Incorrect notation: no lim written: penalty 2 marks lim written before equals sign: penalty 1 mark **Note:** A candidate who gives $-8x$ only: 0/5 marks **Note:** A candidate who omits brackets in the line $\lim\limits_{h\to 0}(-8x-4h)$: NO penalty	✓ formula ✓ substitution ✓ expansion ✓ $-8x-4h$ ✓ answer (5)

	$f(x) = -4x^2$ $f(x+h) = -4(x+h)^2$ $= -4x^2 - 8xh - 4h^2$ $f(x+h) - f(x) = -8xh - 4h^2$ $f'(x) = \lim\limits_{h\to 0} \dfrac{-8xh - 4h^2}{h}$ $= \lim\limits_{h\to 0} \dfrac{h(-8x - 4h)}{h}$ $= \lim\limits_{h\to 0} (-8x - 4h)$ $= -8x$	✓ substitution ✓ expansion ✓ formula ✓ $-8x - 4h$ ✓ answer (5)
8.2.1	$y = \dfrac{3}{2x} - \dfrac{x^2}{2}$ $= \dfrac{3}{2}x^{-1} - \dfrac{1}{2}x^2$ $\dfrac{dy}{dx} = -\dfrac{3}{2}x^{-2} - x$ $= -\dfrac{3}{2x^2} - x$ **Note:** Incorrect notation in 8.2.1 and/or 8.2.2: Penalise 1 mark	✓ $\dfrac{3}{2}x^{-1}$ ✓ $-\dfrac{3}{2}x^{-2}$ ✓ $-x$ (3)
8.2.2	$f(x) = (7x+1)^2$ $= 49x^2 + 14x + 1$ $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$ **OR** $f(x) = (7x+1)^2$ $f'(x) = 2(7x+1).(7)$ By the chain rule $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$	✓multiplication ✓ $98x$ ✓14 ✓ answer (4) ✓✓ chain rule ✓✓ answer (4) **[12]**

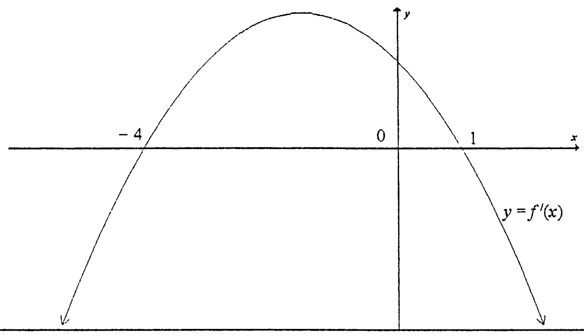
QUESTION 9

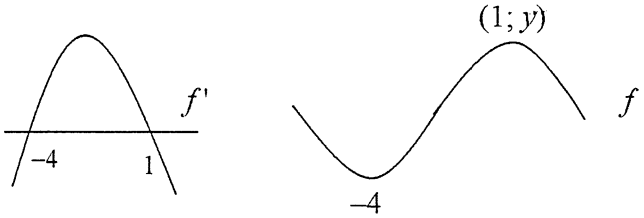
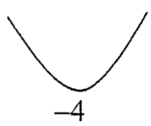
9.1	$f(x)=-2x^3+ax^2+bx+c$ $f'(x)=-6x^2+2ax+b$ $=-6(x-5)(x-2)$ $=-6(x^2-7x+10)$ $=-6x^2+42x-60$ $2a=42$ $a=21$ $b=-60$ **Note:** A candidate who substitutes the values of a, b and c and then checks (by substitution) that $T(2;-9)$ and $S(5;18)$ lie on the curve: award max 2/7 marks $f(5)=-2(5)^3+21(5)^2-60(5)+c$ $18=-25+c$ $c=43$ OR $f(2)=-2(2)^3+21(2)^2-60(2)+c$ $-9=-52+c$ $c=43$ $a=21$; $b=-60$; $c=43$	✓ $f'(x)=-6x^2+2ax+b$ ✓✓ $-6(x-5)(x-2)$ ✓ $b=-60$ ✓ $2a=42$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c=43$ (7)
	Note: A candidate who substitutes the values of a, b and c into the function i.e. gets $f(x)=-2x^3-21x^2-60x+43$ and then shows by substitution that $T(2;-9)$ and $S(5;18)$ are on the curve **and** works out the derivative i.e. gets $f'(x)=-6x^2-42x-60$ **and** shows (by substitution into the derivative) that the turning points are at $x=2$ and $x=5$ (assuming what s/he sets out to prove and proving what is given): **award max 4/7 marks** as follows: ✓ $x=2$ from $f'(x)=0$ OR subs $x=2$ into the derivative and gets 0 ✓ $x=5$ from $f'(x)=0$ OR subs $x=5$ into the derivative and gets 0 ✓substitution of $x=2$ in f and gets -9 ✓substitution of $x=5$ in f and gets 18	
	OR	
	$f'(x)=-6x^2+2ax+b$ $f'(2)=-6(2)^2+2a(2)+b$ $0=-24+4a+b$ $b=24-4a$ $f'(5)=-6(5)^2+2a(5)+b$ $0=-150+10a+b$ $0=-150+10a+(24-4a)$ $0=-126+6a$ $6a=126$ $a=21$ $b=-60$ **Note:** If derivative equal to zero is not written: penalize once only $f(5)=-2(5)^3+21(5)^2-60(5)+c$ $18=-25+c$ $c=43$ OR $f(2)=-2(2)^3+21(2)^2-60(2)+c$ $-9=-52+c$ $c=43$ $a=21$; $b=-60$; $c=43$	✓ $f'(x)=-6x^2+2ax+b$ ✓ $f'(2)=0$ ✓ $f'(5)=0$ ✓ $6a=126$ ✓ $b=-60$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c=43$ (7)

	OR $f(2)=-9$ i.e. $-16+4a+2b+c=-9$ $4a+2b+c=7$ $f(5)=18$ i.e. $-250+25a+5b+c=18$ $25a+5b+c=268$ $21a+3b=261$ $f'(x)=-6x^2+2ax+b$ and $f'(2)=0$ OR $f'(5)=0$ $4a+b=24$ $\quad$ $10a+b=150$ $12a+3b=72$ $\quad$ OR $\quad$ $30a+3b=450$ $9a=189$ $\quad$ $9a=189$ $a=\frac{189}{9}$ $\quad$ $a=\frac{189}{9}$ $a=21$ $\quad$ $a=21$ $12(21)+3b=72$ $3b=-180$ $b=-60$ $4a+2b+c=7$ $\quad$ OR $\quad$ $25a+5b+c=268$ $4(21)+2(-60)+c=7$ $\quad$ $25(21)+5(-60)+c=268$ $c=43$ $\quad$ $c=43$	✓ $-16+4a+2b+c=-9$ and $-250+25a+5b+c=18$ ✓ $f'(x)=-6x^2+2ax+b$ ✓ $f'(2)=0$ or $f'(5)=0$ ✓ $9a=189$ ✓ $b=-60$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c=43$ (7)
9.2	$f'(x)=-6x^2+42x-60$ $m_{\tan}=-6(1)^2+42(1)-60$ $=-24$ $f(1)=-2(1)^3+21(1)^2-60(1)+43$ $=2$ Point of contact is (1 ; 2) $y-2=-24(x-1)$ $y=-24x+26$ **OR** $y=-24x+c$ $2=-24(1)+c$ $c=26$ $y=-24x+26$	✓ $f'(x)=-6x^2+42x-60$ ✓ subs $f'(1)$ ✓ $m_{\tan}=-24$ ✓ $f(1)=2$ ✓ $y-2=-24(x-1)$ OR $y=-24x+26$ (5)
9.3	$f'(x)=-6x^2+42x-60$ $f''(x)=-12x+42$ $0=-12x+42$ $x=\frac{7}{2}$ **OR**	✓ $f''(x)=-12x+42$ ✓ $x=\frac{7}{2}$ (2) ✓ $x=\frac{2+5}{2}$

	$x = \dfrac{2+5}{2}$ $x = \dfrac{7}{2}$ **OR** $x = \dfrac{-21}{3(-2)}$ $= \dfrac{7}{2}$	✓ $x = \dfrac{7}{2}$ (2) ✓ $x = \dfrac{-21}{3(-2)}$ ✓ $x = \dfrac{7}{2}$ (2) **[14]**

QUESTION 10



10.1	x-value of turning point: $x = \dfrac{-4+1}{2}$ $= -\dfrac{3}{2}$ $\therefore x > -\dfrac{3}{2}$ OR $\therefore x \in \left(-\dfrac{3}{2}; \infty\right)$	✓ $x > -\dfrac{3}{2}$ OR $\left(-\dfrac{3}{2}; \infty\right)$ (1)
10.2	f has a local minimum at $x = -4$ because:  **OR** $f'(x) < 0$ for $x < -4$, so f is decreasing for $x < -4$. $f'(x) > 0$ for $-4 < x < 1$, so f is increasing for $-4 < x < 1$. i.e.  $\therefore f$ has a local minimum at $x = -4$ **OR**	✓ $x = -4$ ✓✓ graph (3) ✓ $x = -4$ ✓ $f'(x) < 0$ for $x < -4$ ✓ $f'(x) > 0$ for $-4 < x < 1$ (3)

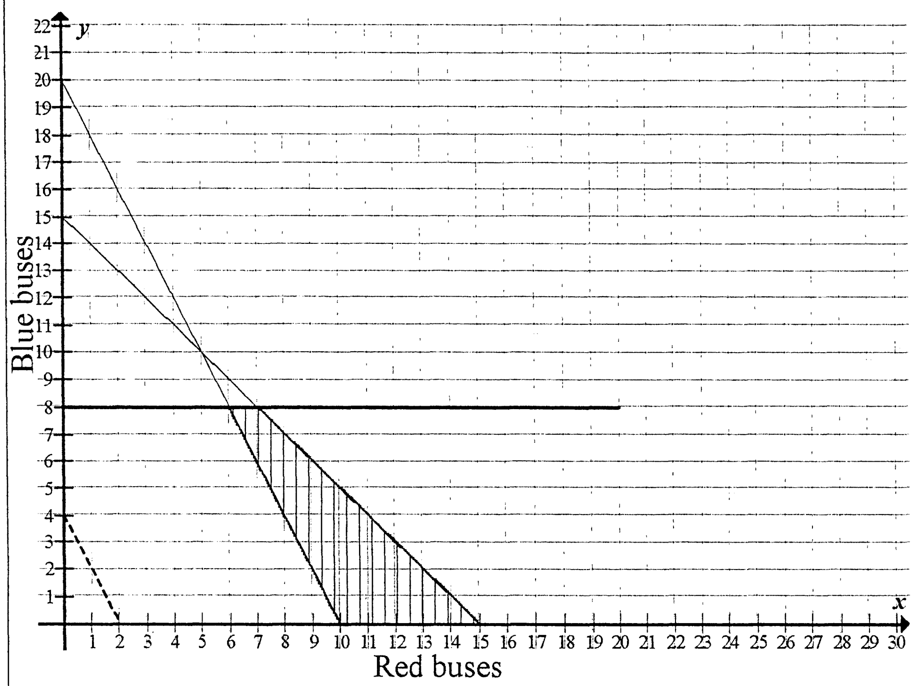
	OR Gradient of f changes from negative to positive at $x = -4$ **OR** $f'(-4) = 0$ $f''(-4) > 0$ so graph is concave up at $x = -4$, so f has a local minimum at $x = -4$.	✓ $x = -4$ ✓gradient negative for $x < -4$ ✓ gradient positive for $-4 < x < 1$ (3) ✓ $f'(-4) = 0$ ✓ $f''(-4) > 0$ ✓ $x = -4$ (3) **[4]**

QUESTION 11

11.1	$V(0) = 100 - 4(0)$ $= 100$ litres	✓ answer (1)
11.2	Rate in – rate out $= 5 - k \quad l/\min$ $V'(t) = -4 \quad l/\min$	✓ $5 - k$ ✓ -4 ✓ units stated once (3)
11.3	$5 - k = -4$ $k = 9 \quad l/\min$ **Note:** Answer only: award 2/2 marks **OR** Volume at any time t = initial volume + incoming total – outgoing total $100 + 5t - kt = 100 - 4t$ $5t - kt = -4t$ $9t - kt = 0$ $t(9 - k) = 0$ At 1 minute from start, $t = 1$, $9 - k = 0$, so $k = 9$ **OR** Since $\frac{dV}{dt} = -4$, the volume of water in the tank is decreasing by 4 litres every minute. So k is greater than 5 by 4, that is, $k = 9$.	✓ $5 - k = -4$ ✓ $k = 9$ (2) ✓ $100 + 5t - kt = 100 - 4t$ ✓ $k = 9$ (2) ✓✓ $k = 9$ (2) **[6]**

QUESTION 12

Note: If the wrong inequality $50x + 25y \leq 500$ is used, candidate wrongly says that there are more learners than available seats. Maximum of 10 marks.

12.1	$x, y \in \mathbf{N}$ $x + y \leq 15$ OR $y \leq -x + 15$ $50x + 25y \geq 500$ OR $y \geq -2x + 20$ $y \leq 8$ OR $y \leq 8$ **Note:** If candidate gives $50x + 25y = 500$: max 5/6 marks **Note:** for the inequality's marks to be awarded, the LHS and the RHS must be correct	✓✓ $x + y \leq 15$ ✓✓ $y \leq 8$ ✓✓ $50x + 25y \geq 500$ (6)
12.2		✓ $x + y \leq 15$ ✓ $50x + 25y \geq 500$ ✓ $y \leq 8$ ✓ feasible region (4)
12.3	$C = 600x + 300y$	✓ answer (1)
12.4.1	(6 ; 8) ; (7 ; 6) ; (8 ; 4) ; (9 ; 2) and (10 ; 0) **NOTE:** The gradient of the search line is $m = -\frac{2}{1}$	3 marks for all correct solutions 2 marks if only 3 or 4 correct solutions 1 mark if only 1 or 2 correct solutions (3)
12.4.2	$C = 6(600) + 8(300) = R6\,000$ or $C = 7(600) + 6(300) = R6\,000$ or $C = 8(600) + 4(300) = R6\,000$ or $C = 9(600) + 2(300) = R6\,000$ or $C = 10(600) + 0(300) = R6\,000$	✓ subs ✓ answer (2)
12.5	8 red ; 4 blue	✓ answer (1) **[17]**

TOTAL: 150

QUESTION 12.2