



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

FEBRUARY/MARCH/FEBRUARIE/MAART 2015

MEMORANDUM

MARKS/PUNTE: 150

**This memorandum consists of 24 pages.
*Hierdie memorandum bestaan uit 24 bladsye.***

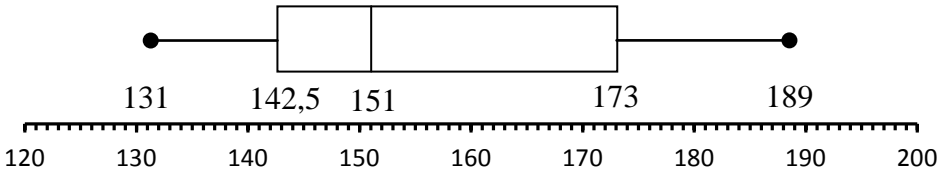
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEEKEER beantwoord, merk slegs die EERSTE poging.
- As 'n kandidaat 'n poging om die vraag te beantwoord, doodgetrek het en nie dit oorgedoen het nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas.
- Aanvaarding van antwoorde/waardes om 'n probleem op te los, is ONaanvaarbaar.

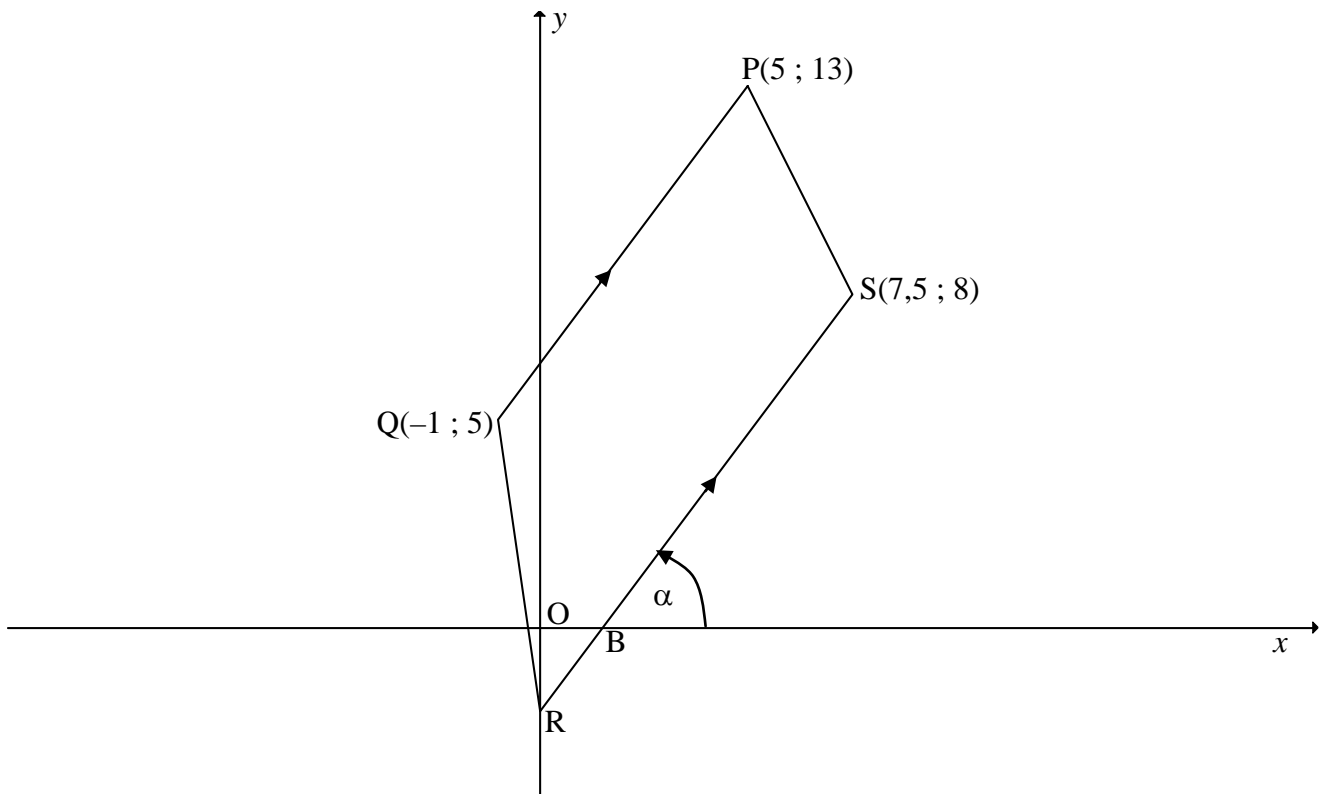
QUESTION/VRAAG 1

1.1	$\bar{x} = \frac{3310}{21}$ $= 157,62$	Answer only: Full marks <i>slegs antw: volpunte</i>	$\checkmark \frac{3310}{21}$ $\checkmark 157,62$ (2)
1.2	(131 ; 142,5 ; 151 ; 173 ; 189)		$\checkmark 131 \text{ and/}$ $\text{en } 189$ $\checkmark 142,5$ $\checkmark 173$ $\checkmark 151$ (4)
1.3			$\checkmark \text{box/mond}$ $\checkmark \text{whiskers/}$ snor (2)
1.4	positively skewed/ <i>positief skeef</i> OR/OF skewed to the right/ <i>skeef na regs</i>		$\checkmark \text{answer/}$ antwoord (1)
1.5	$\sigma = 17,27$		$\checkmark \checkmark \text{answer/}$ antwoord (2)
1.6.1	$\bar{x} = 157,62 + p$		$\checkmark \text{answer}$ (1)
1.6.2	$\sigma = 17,27$		$\checkmark \text{answer/}$ antwoord (1) [13]

QUESTION/VRAAG 2

2.1	<p>As the temperature increases, the sales of ice-creams increase/<i>Soos die temperatuur styg, neem die verkope toe.</i></p> <p style="text-align: center;">OR/OF</p> <p>As the temperature decreases, the sales of ice-creams decrease/<i>Soos die temperatuur daal, neem die verkope af.</i></p>	<p>✓ reason/<i>rede</i> (1)</p> <p>✓ reason/<i>rede</i> (1)</p>
2.2	<p>The liveable temperature cannot keep on increasing/<i>Die leefbare temperatuur kan nie aanhou styg nie.</i></p>	<p>✓ reason/<i>rede</i> (1)</p>
2.3	<p>$a = -460,35$ $b = 30,09$ $\hat{y} = 30,09x - 460,35$ OR/OF $\hat{y} = -460,35 + 30,09x$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Answer only: Full marks <i>slegs antw: volpunte</i></p> </div>	<p>✓✓ $-460,35$ ✓ $30,09$ ✓ equation/<i>vgl</i> (4)</p>
2.4	<p>$r = 0,96$</p>	<p>✓ $0,96$ (1)</p>
2.5	<p>There is a <u>very strong</u> positive relationship (correlation)/<i>Daar is 'n baie sterk positiewe verband (korrelasie).</i></p>	<p>✓ very strong/<i>baie sterk</i> (1)</p> <p style="text-align: right;">[8]</p>

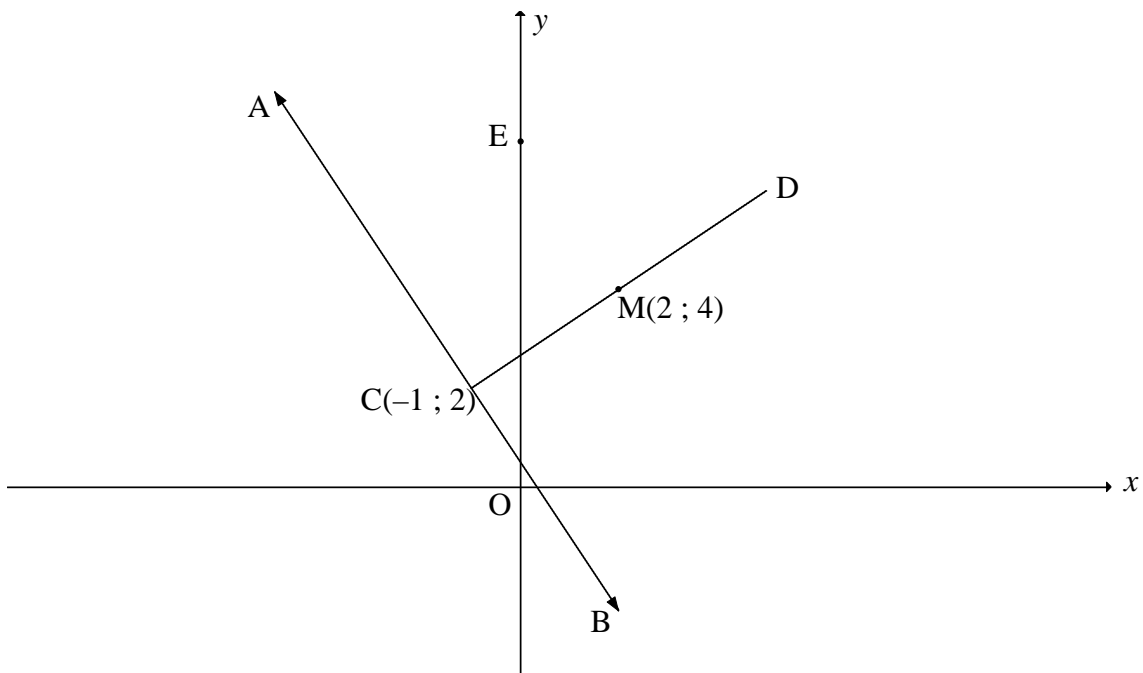
QUESTION/VRAAG 3



<p>3.1</p>	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 + 1)^2 + (13 - 5)^2}$ $= 10$	<ul style="list-style-type: none"> ✓ use of distance formula/gebruik afstandformule ✓ correct subst into form/korrekte subst in formule ✓ 10 <p style="text-align: right;">(3)</p>
<p>3.2</p>	$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{13 - 5}{5 - (-1)}$ $= \frac{8}{6} = \frac{4}{3}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Answer only: Full marks slegs antw: volpunte</p> </div>	<ul style="list-style-type: none"> ✓ correct subst into gradient formula/korrekte subst in gradiëntformule ✓ gradient/gradiënt <p style="text-align: right;">(2)</p>

$m_{QB} = \frac{5-0}{-1-1,5} = \frac{5}{-2,5} = -2$ $m_{PS} = \frac{13-8}{5-7,5} = \frac{5}{-2,5} = -2$ $m_{QB} = m_{PS}$ $\therefore QB \parallel PS$ $PQ \parallel BS$ <p>PQBS = parallelogram (both pairs opp sides //beide pr tos sye //)</p> <p style="text-align: center;">OR/OF</p> $BS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left(\frac{15}{2} - \frac{3}{2}\right)^2 + (8-0)^2} \quad \therefore PQ = BS$ $= 10$ $QB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-1-1,5)^2 + (5-0)^2} = \sqrt{(2,5)^2 + (5)^2} = \frac{5\sqrt{5}}{2} \text{ or } 5,59$ $PS = \sqrt{(5-7,5)^2 + (13-8)^2} = \sqrt{(2,5)^2 + (5)^2} = \frac{\sqrt{125}}{2} \text{ or } 5,59$ $QB = PS$ <p>PQBS = parallelogram (both pairs opp sides =/ beide pr tos sye =)</p>	<p>✓ m_{QB}</p> <p>✓ m_{PS}</p> <p>✓ $QB \parallel PS$</p> <p>✓ reason/rede (4)</p> <p>✓ correct subst into form/korrekte subst in formule</p> <p>✓ $PQ = 10$</p> <p>✓ $QB = PS$</p> <p>✓ reason/rede (4)</p> <p style="text-align: right;">[18]</p>
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QUESTION/VRAAG 4



<p>4.1.1</p>	<p>Radius = $\sqrt{(2+1)^2 + (4-2)^2}$ $r = \sqrt{13}$ Equation of circle/vgl van sirkel: $(x-2)^2 + (y-4)^2 = 13$</p> <p style="text-align: center;">OR/OF</p> <p>$(x-2)^2 + (y-4)^2 = r^2$ $(-1-2)^2 + (2-4)^2 = r^2$ $r^2 = 13$ $\therefore (x-2)^2 + (y-4)^2 = 13$</p>	<p>$\checkmark \sqrt{(2+1)^2 + (4-2)^2}$ or/of $\checkmark \sqrt{13}$ $\checkmark (x-2)^2 + (y-4)^2$ $\checkmark 13$ (3)</p> <p>$\checkmark (x-2)^2 + (y-4)^2$ $\checkmark (-1-2)^2 + (2-4)^2$ $\checkmark 13$ (3)</p>
<p>4.1.2</p>	<p>At/by D: $\frac{-1+x_D}{2} = 2$ $\frac{2+y_D}{2} = 4$ $-1+x_D = 4$ and/en $2+y_D = 8$ $x_D = 5$ $y_D = 6$ D(5 ; 6)</p> <p style="text-align: center;">OR/OF</p> <p>By inspection/deur inspeksie: D(5 ; 6)</p>	<p>$\checkmark x - \text{value/waarde}$ $\checkmark y - \text{value/waarde}$ (2)</p> <p>$\checkmark x - \text{value/waarde}$ $\checkmark y - \text{value/waarde}$ (2)</p>

<p>4.1.3</p>	$m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $m_{AB} \times m_{MC} = -1 \quad (\text{Tangent } \perp \text{ radius/raaklyn } \perp \text{ radius})$ $m_{AB} = -\frac{3}{2}$ $y - y_1 = m(x - x_1) \quad \text{OR/OF} \quad y = mx + c$ $y - 2 = -\frac{3}{2}(x + 1)$ $y = -\frac{3}{2}x + \frac{1}{2}$	$\checkmark m_{MC} = \frac{4-2}{2+1} = \frac{2}{3}$ $\checkmark m_{AB} \times m_{MC} = -1$ $\checkmark m_{AB} = -\frac{3}{2}$ <p>\checkmark subst m and $(-1 ; 2)$ into eq /subst m en $(-1 ; 2)$ in vgl</p> <p>\checkmark eq in standard form/ vgl in st vorm</p> <p>(5)</p>
<p>4.1.4</p>	<p>At/by E:</p> $(0 - 2)^2 + (y - 4)^2 = 13$ $(y - 4)^2 = 9$ $y - 4 = \pm 3$ $y = 7 \text{ or } y = 1$ <p>E(0 ; 7)</p> <p style="text-align: center;">OR/OF</p> <p>At/by E:</p> $(0 - 2)^2 + (y - 4)^2 = 13$ $4 + y^2 - 8y + 16 = 13$ $y^2 - 8y + 7 = 0$ $(y - 7)(y - 1) = 0$ $y = 7 \text{ or } y = 1$ <p>E(0 ; 7)</p>	<p>$\checkmark x = 0$</p> <p>\checkmark simplification/ vereenvoudiging</p> <p>\checkmark y - values/waardes</p> <p>\checkmark E(0 ; 7)</p> <p>(4)</p> <p>$\checkmark x = 0$</p> <p>\checkmark simplification/ vereenvoudiging</p> <p>\checkmark y - values/waardes</p> <p>\checkmark E(0 ; 7)</p> <p>(4)</p>
<p>4.1.5</p>	$m_{EM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4 - 7}{2 - 0}$ $= -\frac{3}{2}$ $m_{AB} = -\frac{3}{2}$ <p>$\therefore EM \parallel AB \quad (m_{EM} = m_{AB})$</p>	$\checkmark m_{EM} = -\frac{3}{2}$ <p>\checkmark reason/rede</p> <p>(2)</p>

4.2	<p>The centres of the circles are / <i>Die middelpunte van die sirkels is</i> P(-2 ; 4) and / <i>en</i> Q(5 ; -1)</p> $QP^2 = (-2 - 5)^2 + (4 - (-1))^2$ $QP = \sqrt{74} \approx 8,60 \text{ units}$ $r_M + r_P = 5 + 3$ $= 8$ $\therefore r_M + r_P < QP$ $\therefore \text{The two circles do not intersect/} \textit{Die twee sirkels sny nie}$	<p>✓ both centres/<i>albei Midpte</i> ✓ QP ✓ correct subst into form/<i>korrekte subst in formule</i> ✓ distance between 2 centres/<i>afstand tussen 2 midpte</i></p> <p>✓✓ $r_M + r_P < QP$</p> <p>(6) [22]</p>
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QUESTION/VRAAG 5

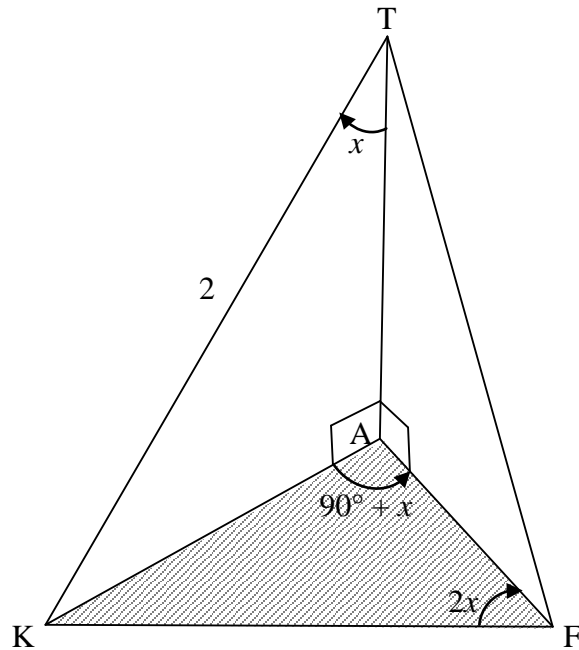
<p>5.1</p>	$x^2 + y^2$ $= (3 \sin \theta)^2 + (3 \cos \theta)^2$ $= 9 \sin^2 \theta + 9 \cos^2 \theta$ $= 9(\sin^2 \theta + \cos^2 \theta)$ $= 9(1)$ $= 9$	<p>✓ simpl/vereenv ✓ CF/GF = 9 ✓ answer/antw (3)</p>
<p>5.2</p>	$\sin(540^\circ - x) \cdot \sin(-x) - \cos(180^\circ - x) \cdot \sin(90^\circ + x)$ $\sin(180^\circ - x) \cdot \sin(-x) - \cos(180^\circ - x) \cdot \sin(90^\circ + x)$ $= (\sin x)(-\sin x) - (-\cos x)(\cos x)$ $= -\sin^2 x + \cos^2 x$ $= \cos 2x$	<p>✓ $\sin(540^\circ - x) = \sin x$ ✓ $\sin(-x) = -\sin x$ ✓ $\cos(180^\circ - x) = -\cos x$ ✓ $\sin(90^\circ + x) = \cos x$ ✓ $-\sin^2 x + \cos^2 x$ ✓ $\cos 2x$ (6)</p>
<p>5.3.1</p>	$OT = \sqrt{x^2 + p^2}$ $\sin \alpha = \frac{y_T}{OT}$ $= \frac{p}{\sqrt{x^2 + p^2}}$ $\frac{p}{\sqrt{x^2 + p^2}} = \frac{p}{\sqrt{1 + p^2}}$ $x^2 = 1$ $x = -1$ <p style="text-align: center;">OR/OF (P lies in 3rd quadrant)</p> $x^2 + y^2 = r^2$ $x^2 + p^2 = (\sqrt{1 + p^2})^2$ $x^2 + p^2 = 1 + p^2$ $x^2 = 1$ $x = -1$ <p style="text-align: center;">(P lies in 3rd quadrant)</p>	<p>✓ $OT = \sqrt{x^2 + p^2}$ ✓ $\sin \alpha = \frac{y_T}{OT}$ ✓ $x^2 = 1$ (3)</p> <p>✓ $x^2 + y^2 = r^2$ ✓ subst ✓ $x^2 = 1$ (3)</p>
<p>5.3.2</p>	$\cos(180^\circ + \alpha)$ $= -\cos \alpha$ $= -\left(\frac{-1}{\sqrt{1 + p^2}}\right)$ $= \frac{1}{\sqrt{1 + p^2}}$	<p>✓ $-\cos \alpha$ ✓ answer/antw (2)</p>

<p>5.3.3</p>	$\begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= \left(\frac{-1}{\sqrt{1+p^2}} \right)^2 - \left(\frac{p}{\sqrt{1+p^2}} \right)^2 \\ &= \frac{1}{1+p^2} - \frac{p^2}{1+p^2} \\ &= \frac{1-p^2}{1+p^2} \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} \cos 2\alpha &= 1 - 2\sin^2 \alpha \\ &= 1 - 2 \left(\frac{p}{\sqrt{1+p^2}} \right)^2 \\ &= 1 - 2 \left(\frac{p^2}{1+p^2} \right) \\ &= 1 - \frac{2p^2}{1+p^2} \\ &= \frac{1+p^2 - 2p^2}{1+p^2} \\ &= \frac{1-p^2}{1+p^2} \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} \cos 2\alpha &= 2\cos^2 \alpha - 1 \\ &= 2 \left(\frac{-1}{\sqrt{1+p^2}} \right)^2 - 1 \\ &= 2 \left(\frac{1}{1+p^2} \right) - 1 \\ &= \frac{2}{1+p^2} - 1 \\ &= \frac{2-1-p^2}{1+p^2} \\ &= \frac{1-p^2}{1+p^2} \end{aligned}$	<p>✓ expansion/ uitbreiding</p> <p>✓✓ squaring each term/kwadreer elke term (3)</p> <p>✓ expansion/ uitbreiding</p> <p>✓squaring/kwadreering</p> <p>✓ writing as single fraction/skryf as enkelterm (3)</p> <p>✓ expansion/ uitbreiding</p> <p>✓squaring/kwadreering</p> <p>✓ writing as single fraction/skryf as enkelterm (3)</p>
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<p>5.4.1</p>	<p>The identity is undefined for/<i>die identiteit is ongedefinieerd as:</i> $2\sin^2 x = 0$ $\therefore \sin x = 0: x = 0^\circ ; 180^\circ$ <i>or/of</i> $\tan x = \infty: x = 90^\circ$ $\therefore x = 0^\circ ; 90^\circ ; 180^\circ$</p>	<p>✓ $x = 0^\circ$ ✓ $x = 90^\circ$ ✓ $x = 180^\circ$ (3)</p>
<p>5.4.2</p>	<p>LHS/LK = $\frac{2 \tan x - \sin 2x}{2 \sin^2 x}$ $= \frac{2\left(\frac{\sin x}{\cos x}\right) - 2 \sin x \cos x}{2 \sin^2 x}$ $= \left(\frac{2 \sin x - 2 \sin x \cos^2 x}{\cos x}\right) \times \frac{1}{2 \sin^2 x}$ $= \frac{2 \sin x(1 - \cos^2 x)}{\cos x} \times \frac{1}{2 \sin^2 x}$ $= \frac{2 \sin x(\sin^2 x)}{\cos x} \times \frac{1}{2 \sin^2 x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ $= \text{RHS/RK}$ <p style="text-align: center;">OR/OF</p> <p>LHS/LK = $\frac{2 \tan x - \sin 2x}{2 \sin^2 x}$ $= \frac{2\left(\frac{\sin x}{\cos x}\right) - 2 \sin x \cos x}{2 \sin^2 x} \times \frac{\cos x}{\cos x}$ $= \frac{2 \sin x - 2 \sin x \cos^2 x}{2 \sin^2 x \cos x}$ $= \frac{2 \sin x(1 - \cos^2 x)}{2 \sin^2 x \cos x}$ $= \frac{2 \sin x \cdot \sin^2 x}{2 \sin^2 x \cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ $= \text{RHS/RK}$</p> </p>	<p>✓ $\frac{\sin x}{\cos x}$ ✓ $2 \sin x \cdot \cos x$ ✓ simplify numerator/ <i>vereenv teller</i> ✓ factorising/<i>fakt</i> ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to/<i>vereenv</i> <i>na</i> $\frac{\sin x}{\cos x}$ (6)</p> <p>✓ $\frac{\sin x}{\cos x}$ ✓ $2 \sin x \cdot \cos x$ ✓ simpl/<i>vereenv</i> ✓ factorising/<i>fakt</i> ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplify to /<i>vereenv</i> <i>na</i> $\frac{\sin x}{\cos x}$ (6) [26]</p>

QUESTION/VRAAG 6

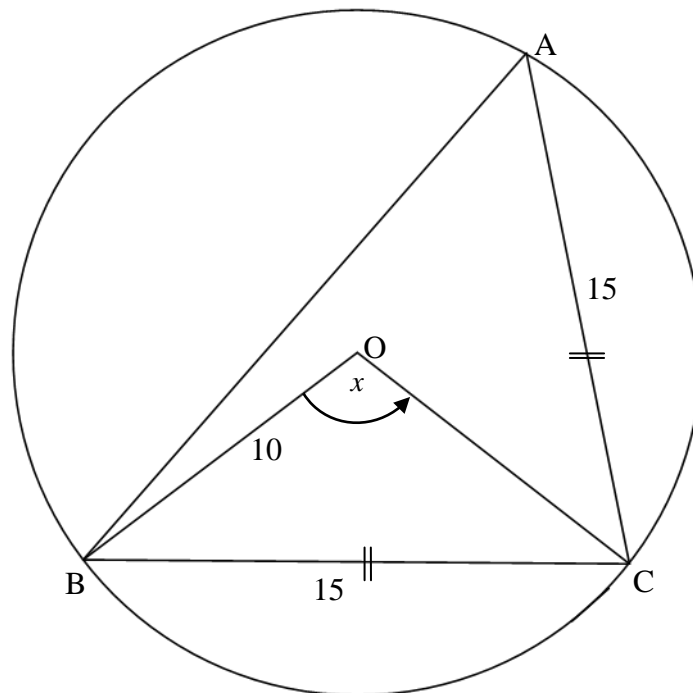
6.1



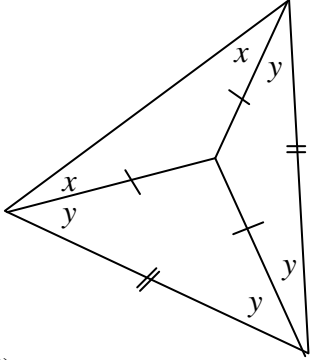
<p>6.1.1</p>	<p>In ΔTAK:</p> $\frac{AK}{KT} = \sin \hat{KTA}$ $AK = KT \cdot \sin x$ $= 2 \sin x$ <p style="text-align: center;">OR/OF</p> $\frac{\sin \hat{KTA}}{AK} = \frac{\sin \hat{KAT}}{KT}$ $\frac{\sin 90^\circ}{2} = \frac{\sin x}{AK}$ $AK = 2 \sin x$	<p>✓ correct trig ratio/ korrekte trigverh.</p> <p>✓ answer/antw (2)</p> <p>✓ correct subst into sine rule/korrekte subst in sin-reël</p> <p>✓ answer/antw (2)</p>
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<p>6.1.2</p>	<p>In $\triangle AKF$:</p> $\frac{KF}{\sin \hat{K}AF} = \frac{AK}{\sin \hat{A}FK}$ $\frac{KF}{\sin(90^\circ + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK \cdot \sin(90^\circ + x)}{\sin 2x}$ $= \frac{2 \sin x \cdot \cos x}{2 \sin x \cdot \cos x}$ $= 1$ <p style="text-align: center;">OR/OF</p> <p>In $\triangle AKF$:</p> $\frac{KF}{\sin \hat{K}AF} = \frac{AK}{\sin \hat{A}FK}$ $\frac{KF}{\sin(90^\circ + x)} = \frac{AK}{\sin 2x}$ $KF = \frac{AK \cdot \sin(90^\circ + x)}{\sin 2x}$ $= \frac{AT \cdot \tan x \cdot \cos x}{2 \sin x \cdot \cos x}$ $= \frac{2 \cos x \cdot \frac{\sin x}{\cos x} \cdot \cos x}{2 \sin x \cdot \cos x}$ $= 1$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\cos x = \frac{AT}{2}$ $\therefore AT = 2 \cos x$ </div>	<ul style="list-style-type: none"> ✓ using sine rule/ <i>gebruik sin-reël</i> ✓ correct subst into sine rule/<i>korrekte subst in sin-reël</i> ✓ $\sin(90^\circ + x) = \cos x$ ✓ $2 \sin x \cdot \cos x$ ✓ 1 <p style="text-align: right;">(5)</p> <ul style="list-style-type: none"> ✓ using sine rule/ <i>gebruik sin-reël</i> ✓ correct subst into sine rule/<i>korrekte subst in sin-reël</i> ✓ $\sin(90^\circ + x) = \cos x$ ✓ $2 \sin x \cdot \cos x$ ✓ 1 <p style="text-align: right;">(5)</p>
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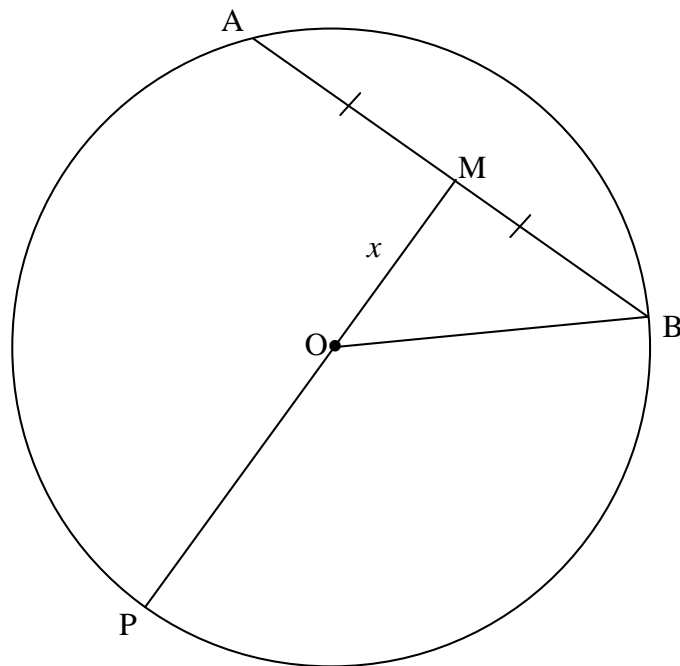
6.2



<p>6.2.1</p>	<p>In $\triangle BOC$: $BC^2 = BO^2 + CO^2 - 2 \cdot BO \cdot CO \cdot \cos x$ $15^2 = 10^2 + 10^2 - 2(10)(10) \cdot \cos x$ $200 \cos x = -25$ $\cos x = -0,125$ $x = 180^\circ - 82,82^\circ$ $= 97,18^\circ$</p> <p style="text-align: center;">OR/OF</p> <p>Draw a line $OD \perp BC$: $BD = DC$ (line from centre \perp on chord) $\triangle OBD \cong \triangle OCD$ (90°; h; s) $\sin \frac{x}{2} = \frac{7,5}{10}$ $\frac{x}{2} = 48,59^\circ$ $\therefore x = 97,18^\circ$</p>	<p>✓ using cosine rule/ <i>gebruik cos-reël</i> ✓ correct subst/ <i>korrekte subst</i> ✓ $\cos x = -0,125$ ✓ $97,18^\circ$ (4)</p> <p>✓ S/R ✓ correct ratio/ <i>korrekte verh</i> ✓ value of/waarde van $\frac{x}{2}$ ✓ $97,18^\circ$ (4)</p>
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<p>6.2.2</p>	<p> $\widehat{BAC} = 48,59^\circ$ (\angle at centre = $2 \times \angle$ at circ / \angle by midpt = $2 \times \angle$ omt) $\widehat{ABC} = \widehat{BAC} = 48,59^\circ$ (\angle's opp equal sides / \anglee teenoor = sye) $\therefore \widehat{ACB} = 82,82^\circ$ (sum of \angles of Δ / som van \anglee van Δ) </p> <p style="text-align: center;">OR/OF</p> <p> $\widehat{ACB} = \frac{1}{2} \widehat{AOB}$ (\angle at centre = $2 \times \angle$ at circle) (\angle by midpt = $2 \times \angle$ omt) $= \frac{1}{2} [360^\circ - 2(97,18^\circ)]$ $= 82,82^\circ$ </p> <p style="text-align: center;">OR/OF</p> <p> $\widehat{OCB} = \frac{1}{2} (180^\circ - 97,18^\circ)$ (\angle's opp equal sides; sum of \angles of Δ) (\anglee teenoor = sye; som van \anglee van Δ) $= 41,41^\circ$ </p> <div style="text-align: center;">  </div> <p> $\widehat{ACB} = 2(41,41^\circ)$ $= 82,82^\circ$ </p>	<p> \checkmark S \checkmark S $\checkmark 82,82^\circ$ (3) </p> <p> \checkmark S \checkmark S $\checkmark 82,82^\circ$ (3) </p> <p> \checkmark S $\checkmark 82,82^\circ$ (3) </p>
<p>6.2.3</p>	<p>Area/Oppervlakte ΔABC</p> <p> $= \frac{1}{2} (BC)(AC) \sin \widehat{ACB}$ $= \frac{1}{2} (15)(15)(\sin 82,82^\circ)$ $= 111,62 \text{ cm}^2$ </p>	<p> \checkmark correct subst into area rule / korrekte subst in opp-reël $\checkmark 111,62 \text{ cm}^2$ (2) [16] </p>

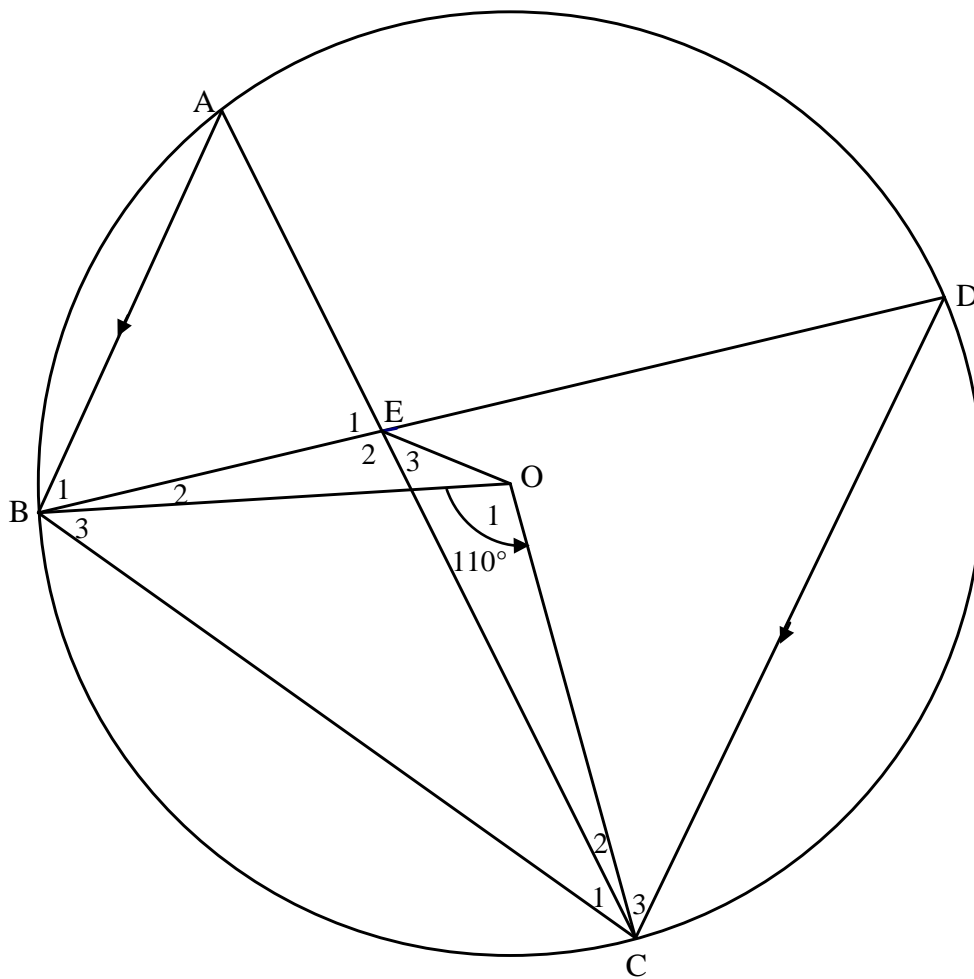
QUESTION/VRAAG 7



7.1	MB = 10 cm	✓ answer/antw (1)
7.2	line from centre to midpoint of chord is perpendicular to chord/ <i>lyn vanaf midpt na midpt van koord is loodreg op koord</i> OR/OF line from centre bisects chord/ <i>lyn vanaf midpt halveer koord</i>	✓ answer/antw (1) ✓ answer/antw (1)
7.3	$\frac{MP}{OM} = \frac{5}{2}$ $\frac{x + OP}{x} = \frac{5}{2}$ $2x + 2OP = 5x$ $OP = \frac{3x}{2}$ OR/OF $\frac{OP}{OM} = \frac{3}{2}$ $OP = \frac{3x}{2}$	$\checkmark \frac{x + OP}{x} = \frac{5}{2}$ $\checkmark OP = \frac{3x}{2}$ (2) $\checkmark \frac{OP}{OM} = \frac{3}{2}$ $\checkmark OP = \frac{3x}{2}$ (2)

7.4	$OM^2 + MB^2 = OB^2$ $x^2 + 10^2 = \left(\frac{3x}{2}\right)^2$ $4x^2 + 400 = 9x^2$ $5x^2 = 400$ $x^2 = 80$ $x = 8,94 \text{ or } 4\sqrt{5} \text{ or } \sqrt{80}$	✓ subst into/subst Pythagoras ✓ $4x^2 + 400 = 9x^2$ ✓ answer/antw (3) [7]
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QUESTION/VRAAG 8

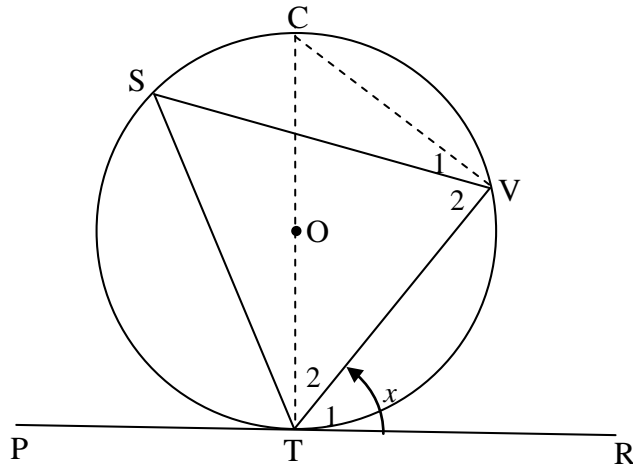


8.1.1	$\hat{D} = \frac{1}{2} \hat{O}_1 = 55^\circ$ (\angle at centre = $2 \times \angle$ at circ / \angle by midpt = $2 \times \angle$ by omt)	✓S ✓R (2)
8.1.2	$\hat{A} = \frac{1}{2} \hat{O}_1 = 55^\circ$ (\angle at centre = $2 \times \angle$ at circ / \angle by midpt = $2 \times \angle$ by omt)	✓S ✓R (2)
	OR/OF	
	$\hat{A} = \hat{D} = 55^\circ$ (\angle s in same segment / \angle e in dieselfde segment)	✓S ✓R (2)
8.1.3	$\hat{B}_1 = \hat{D} = 55^\circ$ (alternate \angle s / verwiss \angle e; $AB \parallel DC$) $\hat{E}_2 = \hat{B}_1 + \hat{A}$ (ext \angle of Δ = sum of opp \angle s / buite \angle v Δ = som v tos \angle e) $= 55^\circ + 55^\circ$ $\hat{E}_2 = 110^\circ$	✓S ✓R ✓R ✓ answer/antw (4)
8.2	$\hat{E}_2 = \hat{O}_1 = 110^\circ$ (proven in / bewys in 8.1.3) BEOC is a cyclic quadrilateral (equal \angle s subtended by line / gelyke \angle e onderspan deur lyn)	✓S ✓R (2) [10]

QUESTION/VRAAG 9

9.1	the interior opposite angle/ <i>die teenoorstaande binnehoek.</i>	✓ answer/antw (1)
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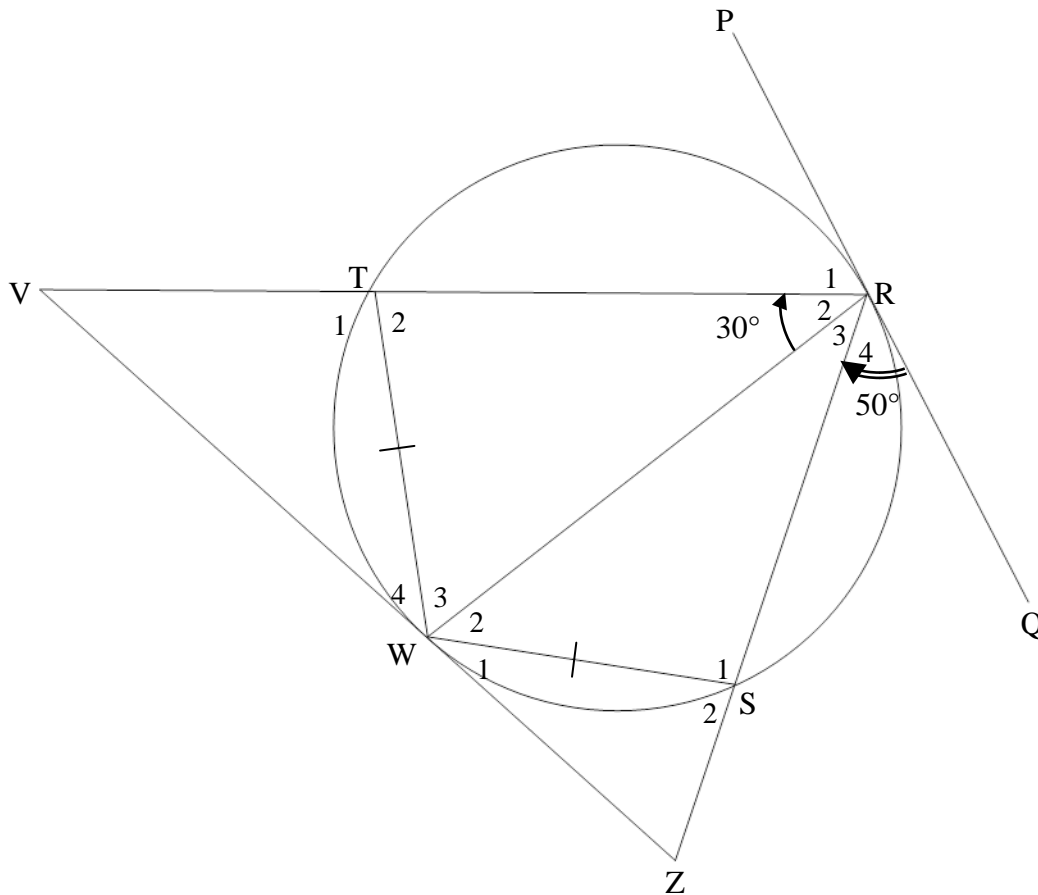
9.2



Construction: Draw diameter CT and join CV.
Konstruksie: Trek middellyn CT en verbind CV.

$\hat{V}_1 + \hat{V}_2 = 90^\circ$	\angle in semi-circle/ \angle in halfsirkel	✓ S ✓ R
$\hat{T}_2 = 90^\circ - x$	Tangent \perp diameter/radius/raaklyn \perp middellyn/radius	✓ R
$\therefore \hat{C} = x$	Sum of the angles of triangle/Som van die hoeke van 'n driehoek	✓ S
$\therefore \hat{S} = x$	\angle 's same segment/ \angle in dieselfde segment	✓ R
$\therefore \hat{VTR} = \hat{S}$		(5)

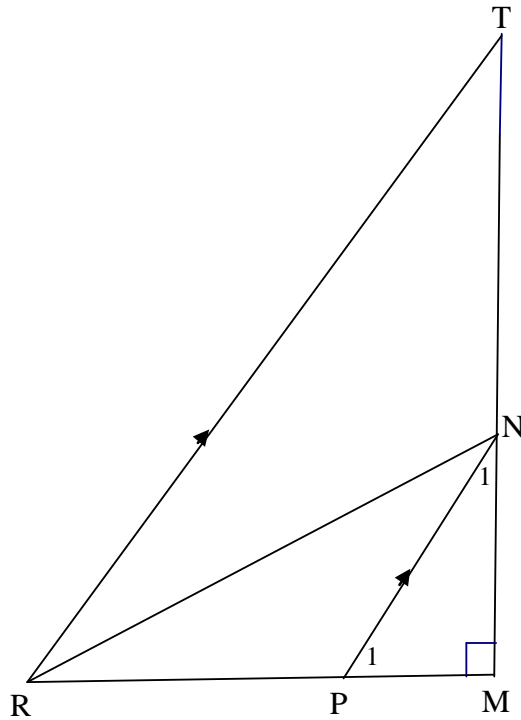
9.3



9.3.1	Equal chords subtend equal \angle s/ <i>Gelyke koorde onderspan gelyke \anglee</i>	✓ R (1)
9.3.2	$\hat{W}_4 = 30^\circ$ (tan chord theorem/ <i>rkl-koordst</i>) $\hat{W}_1 = 30^\circ$	✓ answer/ <i>antw</i> ✓ reason/ <i>rede</i> ✓ answer/ <i>antw</i> (3)
9.3.3(a)	$\hat{R}_4 = \hat{W}_2 = 50^\circ$ (tan chord theorem/ <i>rkl-koordst</i>) $\hat{S}_2 = \hat{R}_3 + \hat{W}_2$ (ext \angle of Δ / <i>buite \angle v Δ</i>) $\therefore \hat{S}_2 = 80^\circ$ OR/OF $\hat{R}_2 = \hat{R}_3 = 30^\circ$ (= chords subtend = \angle s / = <i>kde onderspan = \anglee</i>) $\hat{R}_4 = \hat{W}_2 = 50^\circ$ (tan chord theorem/ <i>rkl-koordst</i>) $\therefore \hat{S}_2 = 80^\circ$	✓ S ✓ R ✓ S (3) ✓ S ✓ R ✓ S (3)

9.3.3(b)	$\hat{T}_2 = \hat{S}_2 = 80^\circ$ (ext \angle of cyclic quad/ <i>buite</i> \angle van koordevh) $V + \hat{W}_4 = \hat{T}_2$ (ext \angle of Δ / <i>buite</i> \angle van Δ) $\therefore \hat{V} = 50^\circ$	\checkmark S \checkmark R \checkmark S \checkmark S (4)
9.3.4	<p>In ΔRVW and/en ΔRWS:</p> <p>$\hat{R}_2 = \hat{R}_3 = 30^\circ$ (proven/<i>bewys</i> in 9.3.1) $\hat{V} = \hat{W}_2 = 50^\circ$ (proven/<i>bewys</i> in 9.3.3) $V\hat{W}R = \hat{S}_1$ (3rd \angle in Δ) $\therefore \Delta RVW \parallel \Delta RWS$ ($\angle\angle\angle$)</p> <p>$\therefore \frac{WR}{RV} = \frac{RS}{WR}$ ($\Delta RVW \parallel \Delta RWS$) $\therefore WR^2 = RV \cdot RS$</p>	\checkmark using the correct Δ s/ <i>gebruik korrekte Δe</i> \checkmark S \checkmark S \checkmark R (3rd \angle in Δ) or ($\angle\angle\angle$) \checkmark S (5) [22]

QUESTION/VRAAG 10



10.1.1	corresponding \angle s/ooreenkomstige \angle e; $PN \parallel RT$	✓ answer/antw (1)
10.1.2	\angle ; \angle ; \angle OR/OF \angle ; \angle	✓ answer/antw (1)
10.2	$\frac{PM}{RM} = \frac{PN}{RT} \quad (\Delta PNM \parallel \Delta RTM)$ $= \frac{PN}{3PN}$ $= \frac{1}{3}$	✓ S ✓ S (2)
10.3	$\frac{PM}{RM} = \frac{1}{3} \quad \therefore \frac{RP}{RM} = \frac{2}{3}$ $RN^2 - PN^2 = (RM^2 + NM^2) - (PM^2 + NM^2) \quad (\text{Pyth})$ $= RM^2 - PM^2$ $= \left(\frac{3}{2}RP\right)^2 - \left(\frac{1}{2}RP\right)^2$ $= \frac{9}{4}RP^2 - \frac{1}{4}RP^2$ $= 2RP^2$ <p style="text-align: center;">OR/OF</p>	✓ Use of Pyth. for RN^2 and PN^2 ✓ $RM = \frac{3}{2}RP$ ✓ $PM = \frac{1}{2}RP$ ✓ $\frac{9}{4}RP^2$ & $\frac{1}{4}RP^2$ (4)

$\begin{aligned} RN^2 - PN^2 &= (RM^2 + NM^2) - (PM^2 + NM^2) \quad (\text{Pyth}) \\ &= RM^2 - PM^2 \\ &= (3PM)^2 - PM^2 \\ &= 8PM^2 \\ &= 2(2PM)^2 \\ &= 2RP^2 \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} RN^2 - PN^2 &= (RM^2 + NM^2) - (PM^2 + NM^2) \quad (\text{Pyth}) \\ &= RM^2 - PM^2 \\ &= (RP + PM)^2 - PM^2 \\ &= RP^2 + 2RP \cdot PM + PM^2 - PM^2 \\ &= RP^2 + 2RP \cdot \frac{1}{2}RP \\ &= 2RP^2 \end{aligned}$	\checkmark Use of Pyth. for RN^2 and PN^2 \checkmark $RM = RP + PM$ \checkmark $(3PM)^2 - PM^2$ \checkmark $RP = 2PM$ (4)
	\checkmark Use of Pyth. for RN^2 and PN^2 \checkmark $RM = RP + PM$ \checkmark expansion/ <i>uitbreiding</i> \checkmark $PM = \frac{1}{2}RP$ (4) [8]

TOTAL/TOTAAL: 150