basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P1

NOVEMBER 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages, a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Answer QUESTIONS 4.3 and 7.5 on the ANSWER SHEETS provided. Write your centre number and examination number in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. An information sheet with formulae is included at the end of the question paper.
11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(7-3x)(-8-x)=0$ (2)

1.1.2 $3x^2-4x=\frac{1}{3}$ (correct to TWO decimal places) (4)

1.1.3 $-x^2+16>0$ (3)

1.2 Solve for x and y if:

$x-y=1$ and $x\ +2xy+y^2=9$ (6)

1.3 The diagram below shows an RCL circuit used for voltage magnification. The formula to determine the resonant frequency (f_r) of an RCL circuit is given below.

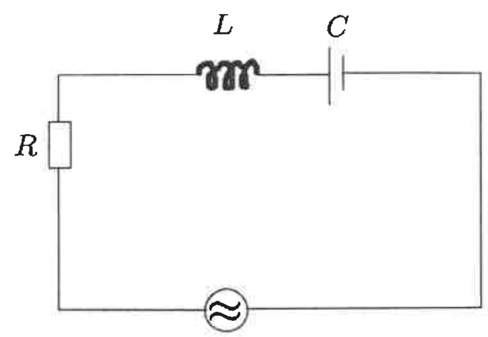


Diagram of RCL circuit

$$f_r=\frac{1}{2\pi\sqrt{LC}}$$

Where:

f_r = resonant frequency in Hz (hertz)
L = inductance in H (henry)
C = capacitance in F (farad)

1.3.1 Make L the subject of the formula. (3)

1.3.2 Hence, calculate the numerical value of L if $C=0{,}65\times10^{-6}$ F and $f_r=1{,}59$ Hz (2)

1.4 Express 24 as a binary number. (1)

1.5 Evaluate $144\div110_2$ and leave your answer as a decimal number. (2)
[23]

QUESTION 2

2.1 Given the equation: $x^2 - 4x + q = 0$

2.1.1 Determine the numerical value of the discriminant if $q = 4$ (2)

2.1.2 Hence, describe the nature of the roots of the equation. (1)

2.2 Determine the numerical value(s) of p for which the equation $x^2 - 4x + p = 0$ will have non-real roots. (3)
[6]

QUESTION 3

3.1 Simplify the following **without the use of a calculator**:

3.1.1 $\log_a a^{\frac{1}{2}}$ (1)

3.1.2 $\sqrt{5x}\left(\sqrt{45x} + 2\sqrt{80x}\right)$ (3)

3.1.3 $\left(\dfrac{4^{3n-2}}{2^{3n+2} \cdot 8^{n-3}}\right) \times 8$ (3)

3.2 Solve for x: $\log(2x-5) + \log 2 = 1$ (4)

3.3 Given the complex number: $z = 2+2i$

3.3.1 In which quadrant of the complex plane does z lie? (1)

3.3.2 Determine the value of the modulus of z. (2)

3.3.3 Hence, express z in polar form (give the angle in degrees). (3)

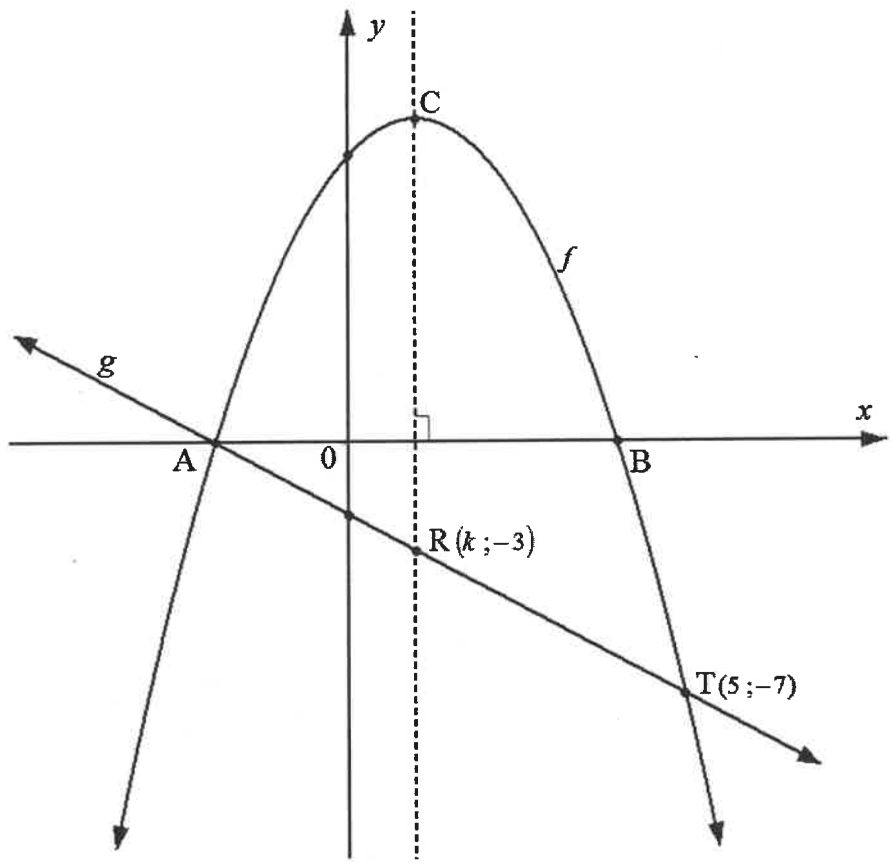
3.4 Solve for x and y if $x - 3yi = 6 + 9i$ (2)
[19]

QUESTION 4

4.1 Sketched below are the graphs of functions f and g defined by:

$f(x) = ax^2 + bx + c$ and $g(x) = -x - 2$

- A is the x-intercept of both f and g.
- B is the other x-intercept of f.
- A and $T(5\,;-7)$ are the points of intersection of f and g.
- C is the turning point of f.
- $R(k\,;-3)$ is a point on straight line g.
- CR is perpendicular to the x-axis.



4.1.1 Determine the x-coordinate of A. (2)

4.1.2 Show that $k = 1$ (1)

4.1.3 Hence, write down the x-coordinate of B. (1)

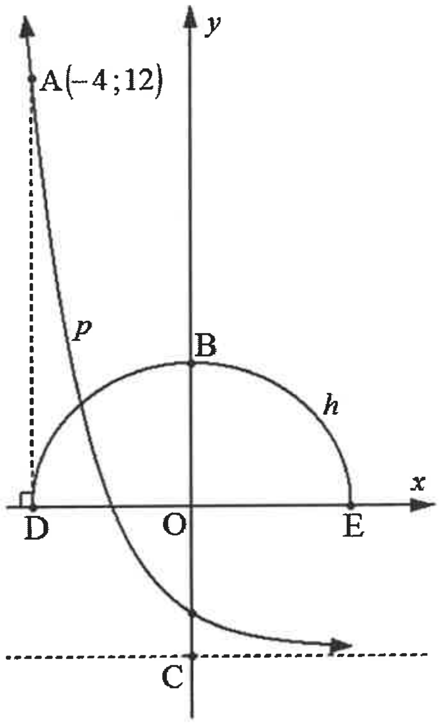
4.1.4 Show that $f(x) = -x^2 + 2x + 8$ (4)

4.1.5 Determine the range of f. (3)

4.1.6 Write down the value(s) of x for which $f(x) \geq g(x)$ (2)

4.2 The graphs below represent functions p defined by $p(x)=a^x-4$ and semicircle h defined by $h(x)=\sqrt{r^2-x^2}$

- O is the origin.
- B is the y-intercept of h.
- Function p has a horizontal asymptote passing through C.
- D and E are the x-intercepts of h.
- $A(-4;12)$ is a point on p.
- AD is perpendicular to the x-axis.



4.2.1 Write down:

(a) The length of OD (1)

(b) The defining equation of function h (2)

4.2.2 Determine the numerical value of a. (3)

4.2.3 Determine the y-intercept of p. (2)

4.2.4 A new graph defined by $f(x)=p(x)+t$ is given.

Determine the equation of the asymptote of f if $(0;0)$ is the y-intercept of f. (2)

4.3 Given: $g(x)=\frac{k}{x}+q$ where $q>0$ and $g(6)=0$

Sketch the graph of function g on the ANSWER SHEET provided. Clearly show the intercepts with the axes and the asymptotes. (3)

[26]

QUESTION 5

5.1 The nominal interest rate charged is 8% per annum, compounded monthly.

Calculate the annual effective interest rate charged. (3)

5.2 R25 000 is invested at an interest rate of $9,6\%$ per annum, compounded quarterly.

Determine the value of the investment at the end of 7 years. (4)

5.3 The temperature of a very hot metal rod is taken using a thermometer that can only take readings up to 200 °C. The temperature of the given metal rod is greater than 200 °C at room temperature. Under controlled conditions, the temperature of the metal rod decreases at a rate of $r\%$ per minute using the reducing balance method.

- After six minutes the metal rod has cooled sufficiently and its temperature measures 80 °C.
- Two minutes later the temperature drops to 50 °C.

5.3.1 Show that $r \approx 21$ (4)

5.3.2 Hence, calculate the initial temperature of the metal rod. (3)
[14]

QUESTION 6

6.1 Given: $f(x) = x - 5$

Determine $f'(x)$ using FIRST PRINCIPLES. (5)

6.2 Determine:

6.2.1 $D_x\left[-3x^9 - 7x\right]$ (2)

6.2.2 $f'(x)$ if $f(x) = \dfrac{3}{2x} + \sqrt[5]{x^{-2}}$ (4)

6.2.3 $\dfrac{dy}{dt}$ if $y^3 t^2 = 64t^{11}$ (3)

6.3 Given: $h(x) = -2x^2 + x - 5$

6.3.1 Calculate $h(1)$. (1)

6.3.2 Hence, determine the average gradient of h between the points $(1\,;h(1))$ and $(-3\,;-26)$. (3)

6.4 Determine the equation of the tangent to the curve defined by $f(x) = x^3 + 2$ at $x = 4$ (5)

[23]

QUESTION 7

Given function g defined by $g(x) = -x^3 + 5x^2 + 8x - 12$

7.1 Write down the y-intercept of g. (1)

7.2 Determine $g(-2)$. (1)

7.3 Hence, determine the x-intercepts of g. (4)

7.4 Determine the coordinates of the turning points of g. (5)

7.5 Sketch the graph of g on the ANSWER SHEET provided. Clearly show ALL intercepts with the axes as well as the turning points. (4)

7.6 Use your graph to write down the values of x for which $g(x) < 0$ (3)

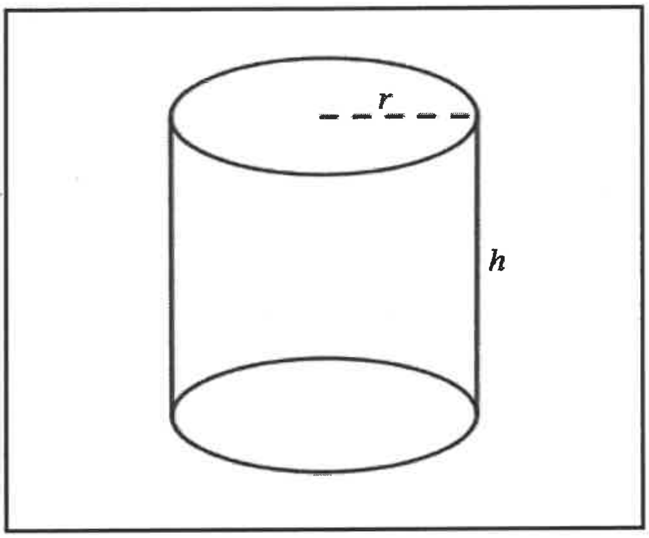
[18]

QUESTION 8

A company has been contracted to manufacture right cylindrical cans to package baked beans.

The volume of a can is 350 mℓ.

The diagram below shows a can with a radius of r cm and a height of h cm.



The following formulae may be used:

Volume = (area of the base) × height = $\pi r^2 h$

Total surface area = 2× (area of the base) + (perimeter of the base) × height
$= 2\pi r^2 + 2\pi r h$

NOTE: 1 mℓ = 1 cm³

8.1 Show that the height can be expressed as $h = \dfrac{350}{\pi r^2}$ (1)

8.2 Hence, show that the total surface area (A) can be expressed as:

$$A(r) = 2\pi r^2 + \frac{700}{r}$$
(2)

8.3 Hence, determine the dimensions of the can if the total surface area is to be a minimum. (5)

[8]

QUESTION 9

9.1 Determine the following integrals:

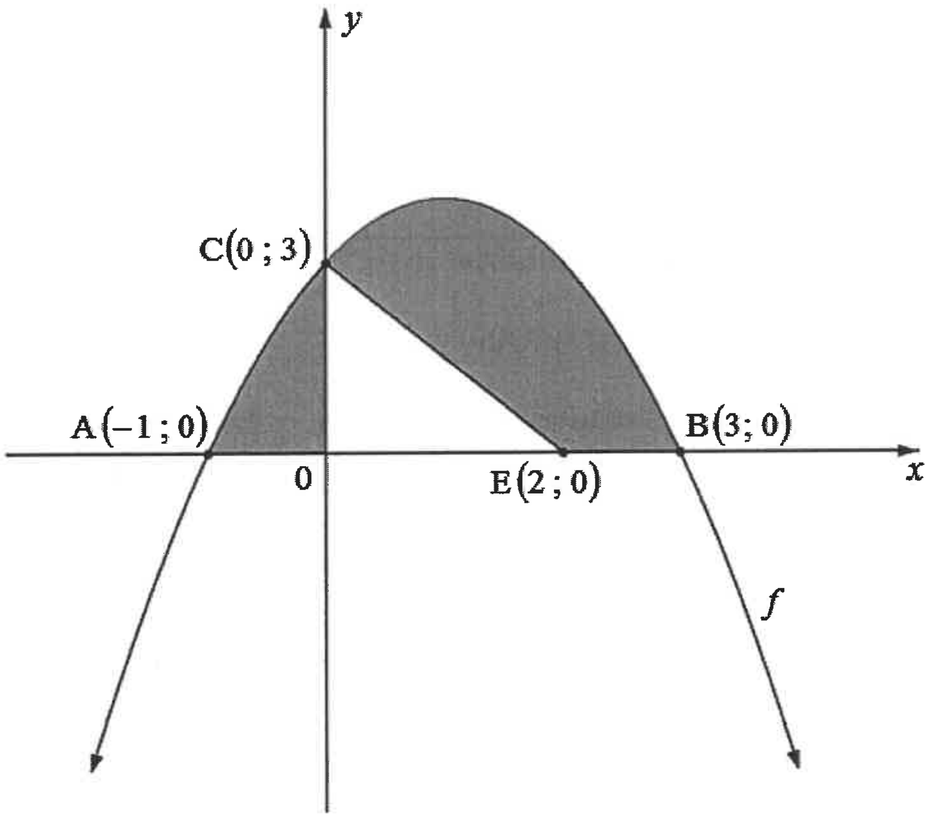
9.1.1 $\int -4 \; dt$ (2)

9.1.2 $\int x^{5}\left(x^{3}-9x^{-6}\right)dx$ (3)

9.2 The diagram below shows function f defined by $f(x)=-x^{2}+2x+3$

The graph of f cuts the x-axis at A$(-1;0)$ and B$(3;0)$ and the y-axis at point C$(0;3)$.

E$(2;0)$ is a point on the x-axis.



Determine the total shaded area represented in the diagram above.

Clearly show ALL working. (8)

[13]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \;, \; n \neq -1 \qquad \int kx^n \, dx = k \cdot \frac{x^{n+1}}{n+1} + C \;, \; n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \; x > 0 \qquad \int \frac{k}{x} dx = k \cdot \ln x + C, \; x > 0$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \;, \; a > 0 \qquad \int ka^{nx} \, dx = k \cdot \frac{a^{nx}}{n \ln a} + C \;, \; a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos A$

Area of Δ ABC $= \dfrac{1}{2} ab. \sin C$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

π rad $= 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360° n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + \ldots + m_n)$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
$o_n = n^{th}$ ordinate and n = number of ordinates

OR

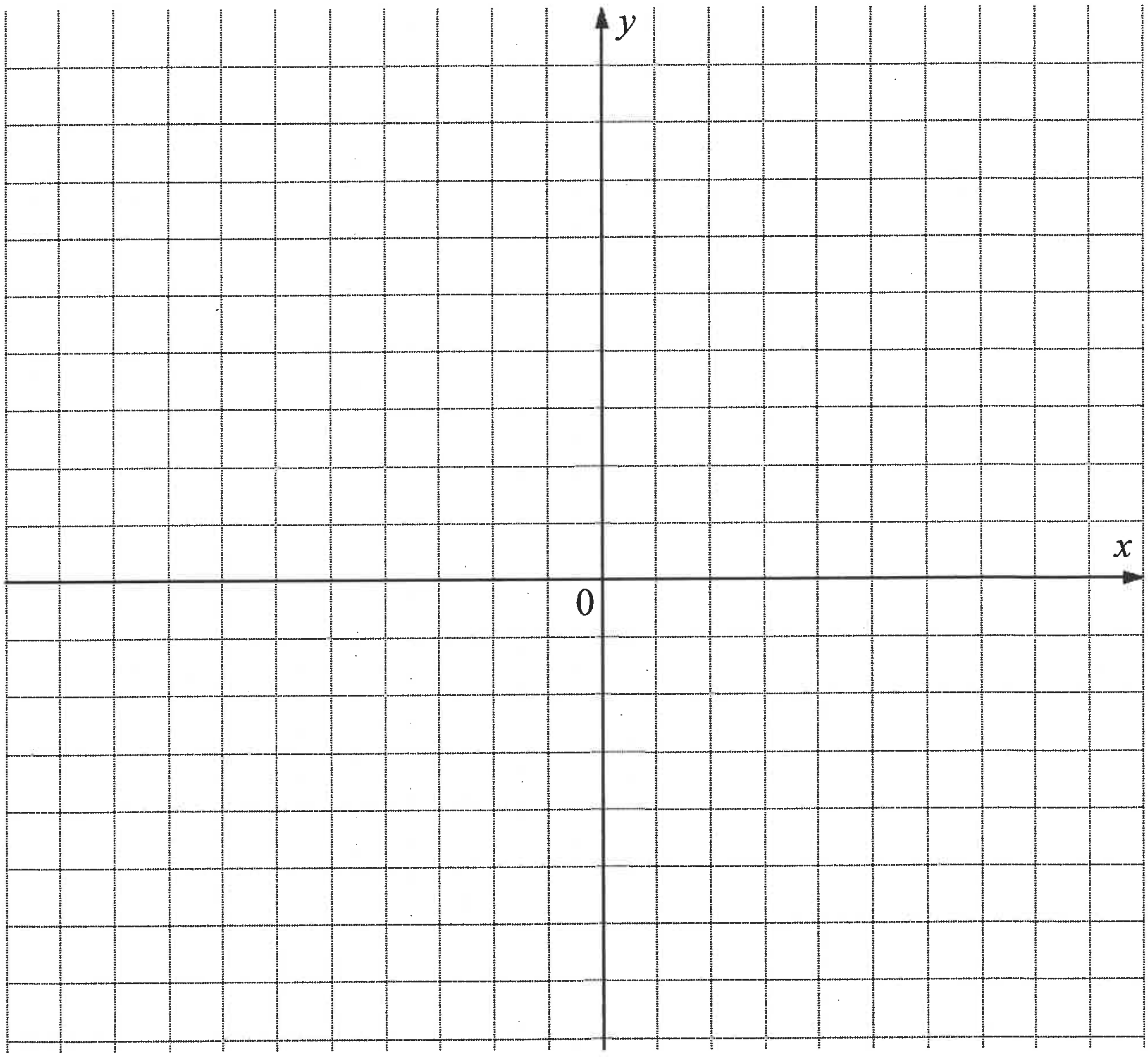
$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + \ldots + o_{n-1}\right)$ where a = width of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

ANSWER SHEET

CENTRE NUMBER							

EXAMINATION NUMBER													

QUESTION 4.3



ANSWER SHEET

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QUESTION 7.5