These workbooks have been developed for the children of South Africa under the leadership of the Minister of Basic Education, Mrs Angie Motshekga, and the Deputy Minister of Basic Education, Mr Enver Surty.

The Rainbow Workbooks form part of the Department of Basic Education’s range of interventions aimed at improving the performance of South African learners in the first six grades. As one of the priorities of the Government’s Plan of Action, this project has been made possible by the generous funding of the National Treasury. This has enabled the Department to make these workbooks, in all the official languages, available at no cost.

We hope that teachers will find these workbooks useful in their everyday teaching and in ensuring that their learners cover the curriculum. We have taken care to guide the teacher through each of the activities by the inclusion of icons that indicate what it is that the learner should do.

We sincerely hope that children will enjoy working through the book as they grow and learn, and that you, the teacher, will share their pleasure.

We wish you and your learners every success in using these workbooks.
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Grade 9 Mathematics

Book 1

1. Revision worksheets: R1 to R16
   Key concepts from Grade 8

2. Worksheets: 1 to 64

Book 2

3. Worksheets: 65 to 144
The structure of a worksheet

Worksheet number (Revision R1 to R16, Ordinary 1 to 144)

Worksheet title

Topic introduction (Text and pictures to help you think about and discuss the topic of the worksheet.)

Question

Term indicator (There are forty worksheets per term.)

Colour code for content area

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Language colour code: Afrikaans (Red), English (Blue)

Example frame (in yellow)

Fun/challenge/problem solving activity (This is an end of worksheet activity that may include fun or challenging activities that can also be shared with parents or brothers and sisters at home.)

Teacher assessment rating, signature and date
Grade 9

Mathematics

Revision
Key concepts from Grade 7

WORKSHEETS R1 to R16

Name:
What does ‘arithmetic’ mean? Why is it important?

Arithmetic is the oldest and most elementary branch of mathematics and deals with the properties and handling of numbers. It is used by almost everyone for everyday tasks of counting and calculating through to complicated science and business calculations. It involves the study of quantity, especially as the result of combining numbers. Basic arithmetic uses the four operations of addition, subtraction, multiplication and division with integers, rational and real numbers and includes measurement and geometry.

1. Calculate and then round off your answers to the nearest ten, hundred and thousand.

   a. 78 438
      \[+19 469\]
      \[= 97 907\]

   b. 83 408
      \[–46 753\]
      \[= 36 655\]

   c. 37 489
      \[\times\ 128\]
      \[= 4 761 792\]

   d. 39 876 522

2. Use a calculator to check your answers.

3. Draw a flow diagram using the words natural numbers, whole numbers and integers.
4. Complete the following:
   a. The **commutative** property of addition and multiplication:
      i. \( a + b = \) 
      ii. \( a \times b = \) 
   b. The **associative** property of addition and multiplication:
      i. \( (a + b) + c = \) 
      ii. \( (a \times b) \times c = \) 
   c. The **distributive** property of multiplication over addition and subtraction:
      i. \( a(b + c) = \) 
      ii. \( a(b - c) = \) 
   d. 0 (zero) as the **identity** element for addition: \( \) = 
   e. 1 (one) is the **identity** element of multiplication: \( \) = 

5. Calculate the following by illustrating the properties of whole numbers:

   **Example:** \( 44 + 55 = 55 + 44 = 99 \)

   a. \( 51 + (19 + 46) = \)
   b. \( 4 \times (12 + 9) = \)
   c. \( (9 \times 64) + (9 \times 36) = \)
   d. If \( 33 + 99 = 132 \), then \( 132 = \)
   
   e. If \( 20 \times 5 = 100 \), then \( 100 = \)

**Problem solving**

Create a problem using all four basic operations. This should be an everyday example.
### Multiples and factors

#### Term 1

**Multiples**
The result of multiplying a number by an integer, e.g., $3 \times 4 = 12$. The multiples of 3 are: 3, 6, 9, ...

**Factors**
Factors are the numbers you multiply together to get a specific result, e.g., 3 and 4 are factors of 12. All the factors of 12 are 1, 2, 3, 4, 6, 12.

**LCM**
Lowest common multiple

**HCF**
Highest common factor

**Prime factors of a number** are prime numbers that divide that number exactly.

### 1. Identify the LCM.

**Example:**
Multiples of 3: {3, 6, 9, 12, 15, 18, ...}
Multiples of 4: {4, 8, 12, 16, 20, ...}
LCM = 12

**Talk about ...**

- Example: Multiples of 3: {3, 6, 9, 12, 15, 18, ...}
- Multiples of 4: {4, 8, 12, 16, 20, ...}
- LCM = 12

#### a. Multiples of:
- 7: {______________}
- 6: {______________}
LCM: _______________

#### b. Multiples of:
- 8: {______________}
- 2: {______________}
LCM: _______________

#### c. Multiples of:
- 5: {______________}
- 4: {______________}
LCM: _______________

#### d. Multiples of:
- 9: {______________}
- 6: {______________}
LCM: _______________
2. Calculate the HCF using factorisation or inspection:

Example: Factors of 192 and 216

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Factor trees of 192

192 = 2 × 2 × 2 × 2 × 2 × 2 × 3
216 = 2 × 2 × 2 × 3 × 3 × 3

Common factors are = 2, 2, 2, 3
HCF = 2 × 2 × 2 × 3 = 24

I know that 192 is divisible by 3 because 1 + 9 + 2 = 12, and 12 is divisible by 3.

Factor trees are used to break up a number into its prime factors.

a. Factors and highest common factor of 204 and 252

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204 = 2 × 2 × 3 × 17
252 = 2 × 2 × 3 × 3 × 7
HCF = 2 × 2 × 3 = 12

b. Factors and highest common factor of 208 and 234

c. Factors and highest common factor of 72 and 188

d. Factors and highest common factor of 275 and 350
Multiples and factors continued

3. Calculate the LCM using factorisation or inspection.

Example: Factors of 123 and 141

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123 = 3 × 41
141 = 3 × 47
LCM = 3 × 41 × 47 = 5781

a. Factors and lowest common multiple of 243 and 729

b. Factors and lowest common multiple of 200 and 1000
c. Factors and lowest common multiple of 225 and 675

d. Factors and lowest common multiple of 128 and 256

e. Factors and lowest common multiple of 162 and 486

f. Factors and lowest common multiple of 225 and 675

Problem solving

Explain calculating HCF using factorisation to a family member.
Revise the laws of exponents by completing the following:

\[ x^m \cdot x^n = \]  
\[ x^1 = \]

\[ x^m \div x^n = \]

\[ (x^m)^n = \]  
\[ x^0 = \] and \( x \neq 0 \)

Why should you study the laws of exponents?

---

1. Write these numbers in exponential form.

   **Example:**
   
   \[ 144 = 12 \times 12 = 12^2 \]
   
   a. 64
   
   b. 9

2. Write these numbers in exponential form.

   **Example:**
   
   \[ 81 = 3 \times 3 \times 3 \times 3 = 3^4 \]
   
   a. 27
   
   b. 8

3. Write the following in exponential form.

   **Example:**
   
   \[ 64 + 8 = 8^2 + 2^1 \]
   
   a. 125 + 25 =
   
   b. 64 + 125 =

4. Write the following in exponential form.

   **Example:**
   
   \[ 50 \times 50 \times 50 \times 50 \times 50 \times 50 \times 50 = 50^7 \]
   
   a. 30 \times 30 \times 30 \times 30 \times 30 =
   
   b. 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 \times 40 =

5. Look at the examples and calculate.

   **Example:**
   
   \[ 3^1 = 3, 25^1 = 25, m^1 = m, 9^1 = 9 \]
   
   a. \( x^1 = \)
   
   b. \( a^1 = \)

6. Answer positive or negative without calculating.

   **Example:**
   
   \((-15)^3\) will be positive,
   \((15)^2\) will be positive,
   \((-15)^3\) will be negative

   a. \((-9)^2\)
   
   b. \((18)^2\)
7. Simplify.

Example: \(a \times b \times a \times b\)  
\[= a^2 \times b^2\]
\[= b^2 \times c^2 \times c^2 \times b^2\]  
\[= b^4 \times c^4\]

a. \(g \times g \times h \times h \times h = \)

b. \(a \times a \times b \times b \times a \times a = \)

8. Revision: calculate the square root.

Example:  
\[\sqrt{64} = \]
\[= 3 \times 3\]
\[= 9\]

a. \(\sqrt{64} = \)

b. \(\sqrt{25} = \)

9. Calculate the square root using the example to guide you.

Example:  
\[\sqrt{256} = \]
\[= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}\]  
\[= 2 \times 2 \times 2 \times 2 \times 2\]
\[= 256 \text{ and } 16\]

\(\sqrt{256} = 16\)

a. \(\sqrt{324} = \)

b. \(\sqrt{1296} = \)

Test your answer:  \(16 \times 16 = 256\)

Remember this is what we call prime factorisation.

How do I know to start dividing by 2?

You should always first try the smallest prime number.

But how will I know the number is divisible by 2 or 3 or 5, etc?

You use the rules of divisibility.
Example: \( \sqrt[12]{12} \)
\( = 12 \)
a. \( \sqrt{2} \times 2 \) =

b. \( \sqrt{3} \times 3 \) =

11. Represent the square root in its simplest form.
Example: \( \sqrt{2 \times 2 \times 2} \)
\( = 2\sqrt{2} \)
a. \( \sqrt{3 \times 3 \times 3} \) =

b. \( \sqrt{6 \times 6 \times 6} \) =

12. Represent the square root in its simplest form:
Example: \( \sqrt{8} \)
\( = \sqrt{2 \times 2 \times 2} \)
\( = 2\sqrt{2} \)
a. \( \sqrt{12} \) =

b. \( \sqrt{45} \) =

13. Look at the example and complete the following:
Example: \( 3^2 = 9 \) therefore \( \sqrt{9} = 3 \)
a. \( 5^2 = \)

b. \( 9^2 = \)

14. Calculate and test your answer.
Example: \( 2^3 \times 2^2 \)
\( = 2^{3+2} \)
\( = 2^5 \)
\( = 32 \)
Test: \( 2^3 \times 2^2 \)
\( = 8 \times 4 \)
\( = 32 \)
\( \)

15. Simplify and test your answer.
Example: \( x^3 \times x^4 \)
\( = x^{3+4} \)
\( = x^7 \)
Test your answer: \( x = 2 \)
\( = 8 \times 16 \)
\( = 128 \)
\( \)
p.\( ^7 \times p^3 = \)

16. Calculate and test your answer.
Example: \( 3^5 + 3^2 \)
\( = 3^{5+2} \)
\( = 3^7 \)
\( = 27 \)
Test: \( 3^5 + 3^2 \)
\( = 243 + 9 \)
\( = 252 \)
\( = 27 \)
\( \)

\( 1^{10} + 1^{10} = \)
17. Simplify and test your answer.

Example: 
\[ x^3 ÷ x^3 = x^{3-3} = x^0 = 1 \]

Test your answer: 
\[ 2^3 ÷ 2^3 = 2^{3-3} = 2^0 = 1 \]

18. Simplify and test your answer:

Example: 
\[(2^3)^2 = 2^{3\times2} = 2^6 = 64\]

Test: 
\[(2^3)^2 = (8)^2 = 64\]

19. Simplify and test your answer:

Example: 
\[(x^3)^2 = x^{3\times2} = x^6\]

Test your answer: 
\[(3^3)^2 = (3^3)(3^3) = 3^6 = 729\]

20. Simplify:

Example: 
\[(3x^2)^3 = 3\times x^2 \times x^2 \times x^2 = 27x^6\]

21. Simplify:

Example: 
\[(a \times t)^n = a^n \times t^n\]

22. Solve using both methods.

Example: 
\[a^4 ÷ a^4 = \frac{a\times a \times a \times a}{a\times a \times a \times a} = a^4 ÷ a^4 = a^{4-4} = a^0 = 1\]

\[m^3 ÷ m^3 = \]

Why is exponent 0 = 1? Take the example of 3^0. Any number divided by itself is 1. We know that 3^2 ÷ 3^2 = 1. But 3^2 ÷ 3^2 = 3^2 ÷ 3^2 = 3^0 Therefore 3^0 = 1.

Problem solving

Add the first 10 square numbers.

Represent the square root of any four-digit number using prime factorisation.
Integers include the counting (natural) numbers \{1, 2, 3, \ldots\}, zero \{0\}, and the negative of the counting numbers \{-1, -2, -3, \ldots\}

**Commutative property:**
\[ a + b = b + a \]
\[ a \times b = b \times a \]

**Associative property:**
\[ a + (b + c) = (a + b) + c \]
\[ a \times (b \times c) = (a \times b) \times c \]

**Distributive property**
\[ a \times (b + c) = a \times b + a \times c \text{ or } (a \times b) + (a \times c) \]

1. **Identify the last term in each pattern. What is the rule?**
   
   **Example:** \(-8, -7, -6, -5, -4, -3, -2\). \(-2\) is the 7th term. The rule is + 1.

   \(-20, -18, -16, -14, -12, -10, -8\) It is the \underline{_____} term.

   The rule is \underline{_____}

2. **Write the following in ascending order:**
   \(-5, 5, 15, 55, 10, -15, -10, -55\)

3. **Fill in <, >, or =**
   
   a. \underline{4} < \underline{4} 
   b. \underline{-18} > \underline{-8} 
   c. \underline{-2} < \underline{2}

4. **Calculate the following:**
   
   **Example:** \((-7) + (5)\)
   \[ = -7 + 5 \]
   \[ = -2 \]

   a. \(({-6}) - (8) = \]
   b. \((-8) + (-4) = \]

5. **Calculate the following:**
   
   **Example:** \((-5 - 4) \times (6 - 2)\)
   \[ = -9 \times 4 \]
   \[ = -36 \]

   a. \((-2 - 3) \div (-4 - 1) \]
   b. \((5 - 6) \times (8 - 7) \]
6. Calculate the following:

Example: \((-3 + 2) + (5 - 3) \times (8 - 9)\)
\[
= (\ -1\) + (2) \times (\ -1\)
\[
= -1 + (\ -2\)
\[
= -1 - 2
\[
= -3
\]

\((-7 + 5) \times (-2 - 7) + (-5 + 3) = \_

7. Use the example to guide you to calculate the following:

Example: \(8 + (-3) = (-3) + 8 = 5 \) \(8 \times (-3) = (-3) \times 8 = -24\)

a. \(33 + (-14) = \_

b. \(7 \times (-6) = \_

8. Use subtraction to check addition or vice versa.

Example: \(8 + (-3) = 5\) then \(5 - 8 = -3\) or \(5 - (-3) = 8\)

a. \(17 + (-8) = \_

b. \(9 + (-5) = \_

9. Use the example to guide you to calculate the following:

Example: \([(-6) + 4] + (-1) = (-6) + [4 + (-1)] = (-6) + 3 = -3\)

a. \([-(-3) + 2] + (-4) = \_

b. \([-(-4) + (-10)] + 5 = \_

10. Use division to check or vice versa.

Example: \(5 \times (-6) = -30\) then \(-30 \div 5 = -6\) and \(-30 \div (-6) = 5\)

a. \(6 \times (-8) = \_

b. \(4 \times (-2) = \_

11. Complete the pattern.

Example: \((+5) \times (+5) = 25\)
\((+12) \times (+12) = \_

(-5) \times (-5) = 25
\((-12) \times (-12) = \_

(+5) \times (-5) = -25
\((+12) \times (-12) = \_

(-5) \times (+5) = -25
\((-12) \times (+12) = \_

\((-12) \times (-12) = \_

\((-12) \times (+12) = \_

\((-12) \times (-12) = \_

Problem solving

If the answer is 20 and the calculation has three operations, what could the calculation be?
Common fractions

Look at these examples and give five more examples of each.

<table>
<thead>
<tr>
<th>Proper fraction</th>
<th>Improper fraction</th>
<th>Mixed number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4}$</td>
<td>$\frac{8}{3}$</td>
<td>$1\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Improper fraction to mixed number

\[
\frac{8}{3} = 2\frac{2}{3}
\]

Mixed number to improper fraction

\[
1\frac{1}{4} = \frac{5}{4}
\]

1. Add and simplify if necessary.

Example: \[
\frac{6}{8} + \frac{4}{8} = \]

a. \[
\frac{6}{12} + \frac{8}{12} =
\]

b. \[
\frac{3}{15} + \frac{7}{15} =
\]

When we add fractions the denominators should be the same.

2. Calculate and simplify the answer if necessary.

Example: \[
\frac{2}{3} \times \frac{2}{2} + \frac{3}{6} = \]

a. \[
\frac{1}{4} - \frac{3}{8} =
\]

b. \[
\frac{3}{6} + \frac{7}{18} =
\]

3. Calculate and simplify the answer if necessary.

Example: \[
\frac{2}{3} \times \frac{4}{4} + \frac{3}{4} \times \frac{3}{3} = \]

a. \[
\frac{6}{5} + \frac{5}{6} =
\]

b. \[
\frac{3}{7} + \frac{7}{9} =
\]
4. Calculate and simplify the answer if necessary.

Example: \( \frac{2}{x} + \frac{3}{x} = \frac{2+3}{x} = \frac{5}{x} \)

a. \( \frac{6}{x} - \frac{5}{x} = \)  

b. \( \frac{1}{x} + \frac{4}{x} = \)

5. Calculate and simplify.

Example: \( \frac{5}{6} \times \frac{4}{7} = \)

a. \( \frac{5}{6} \times \frac{4}{7} = \)

b. \( \frac{6}{12} \times \frac{4}{5} = \)


Example: \( \frac{3}{x} \times \frac{x}{4} = \frac{3x}{4x} = \frac{3}{4} \)

a. \( \frac{3}{x} \times \frac{x}{12} = \)

b. \( \frac{x}{21} \times \frac{14}{x} = \)

7. Calculate and simplify the answer.

Example: \( \frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1 \frac{1}{8} \)

a. \( \frac{4}{7} \div \frac{4}{6} = \)

b. \( \frac{9}{12} \div \frac{3}{4} = \frac{1}{8} \)

### Problem solving

Name five fractions that are between two tenths and three tenths.

What is \( \frac{5}{8} + \frac{8}{5} \) in its simplest form?

Can two unit fractions give you a unit fraction if you:
- add it?
- multiply it?

What is \( \frac{3}{12} \times \frac{12}{4} \) in its simplest form?

If the answer is \( \frac{33}{49} \), what are the two fractions that have been multiplied? Is there only one answer?

If \( \frac{3}{12} \times \frac{12}{4} \) in its simplest form?

Multiply any two improper fractions and simplify your answer if necessary.
Percentages and decimal fractions

**Look at the following. What does it mean?**

\[
\frac{147}{100} = 1.47 = 147\%
\]

When in everyday life do we use:
- Decimal fractions?
- Percentages?

**Term 1**

1. **Write each of the following percentages as a fraction and a decimal fraction:**

   **Example:**
   
   \[
   18\% = \frac{18}{100} = 0.18
   \]

   a. 42%  
   b. 65.5%  

2. **Calculate.**

   **Example:**
   
   \[
   25\% \text{ of } R60 = \frac{25}{100} \times \frac{R60}{1} = \frac{R1500}{100} = R15.00
   \]

   a. 30% of R150  
   b. 65% of R125

3. **Calculate the percentage increase.**

   **Example:**
   
   Calculate the percentage increase if the price of a bus ticket of R60 is increased to R72.
   
   \[
   \frac{12}{60} \times \frac{100}{1} = \frac{1200}{60} = 20
   \]

   20% increase

   Then to work out the percentage increase we need to multiply \( \frac{12}{60} \) by 100.

   We first need to say by how much the price of the bus ticket was increased.

   The price is increased by \( \frac{12}{60} \) or by 20%.

   It was increased by R12 (R72 – R60 = R60).

   R95 to R125

   Price increase: _______
4. Calculate the percentage decrease.

Example:
Calculate the percentage decrease if the price of petrol goes down from 25 cents to 17 cents a litre. Amount decreased is 8 cents.

\[
\frac{8}{25} \times \frac{100}{1} = \frac{800}{25} = 32
\]

32% decrease

We first need to say by how much the price of petrol was decreased by.

Then to work out the percentage increase we need to multiply \( \frac{8}{25} \) by 100.

It was decreased by 8c because 17c + 8c gives you 25c.

5. Write the following in expanded notation:

Example: \( 30,405 = 30 + 0.4 + 0.005 \)

a. 39,482
b. 458,917
c. 873,002
d. 903,930

6. Calculate using both methods. Check your answer.

Example 1: \( 2.37 + 4.53 \)

\[
= (2 + 4) + (0.3 + 0.5) + (0.07 + 0.03)
\]

\[
= 6 + 0.8 + 0.1
\]

\[
= 6.9
\]

Example 2:

\[
= 2.37 + 4.53
\]

\[
= 6.90
\]

a. 89,879 – 39,999 =
b. 802,897 + 78,873 =
7. Calculate the following and check your answers with a calculator.

Example:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4 × 0.3</td>
<td>0.12</td>
</tr>
<tr>
<td>0.04 × 0.3</td>
<td>0.012</td>
</tr>
<tr>
<td>0.04 × 0.03</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

a. 0.4 × 0.5 =  

b. 0.04 × 0.5 =  

c. 0.04 × 0.05 =  

d. 0.6 × 0.3 =  

e. 0.06 × 0.3 =  

f. 0.06 × 0.03 =  

g. 0.8 × 0.7 =  

h. 0.08 × 0.7 =  

i. 0.08 × 0.07 =  

8. Calculate the following and check your answers with a calculator.

Example 1:  0.3 × 0.5 × 100  

= 0.15 × 100  

= 15

Example 2:  0.7 × 0.4 × 10  

= 0.28 × 10  

= 2.8

a. 0.9 × 0.4 × 10 =  

b. 0.7 × 0.06 × 10 =  

9. Calculate the following and check your answers with a calculator.
Round off your answers as in the example.

Example: \(4.387 \times 30\)
\[= (4 \times 30) + (0.3 \times 30) + (0.08 \times 30) + (0.007 \times 30)\]
\[= 120 + 9 + 2.4 + 0.21\]
\[= 120 + 9 + 2 + 0.4 + 0.2 + 0.01\]
\[= 131.421\]

Round off your answers to the:
Nearest unit: 131
Nearest tenth: 131,4
Nearest hundredth: 131,42

a. \(16.467 \times 40 = \)

b. \(298.999 \times 60 = \)

10. Calculate the following. Round off your answers to the nearest tenth.

Example: \(9.81 \div 9\)
\[= 1.09\] rounded off to the nearest tenth is 1.1.

a. \(5.25 \div 5 = \)

d. \(39.97 \div 7 = \)

c. \(48.48 \div 6 = \)

Problem solving

Multiply the number that is exactly between 2.71 and 2.72 by the number that equals ten times three.

You need twelve equal pieces from 144.12 m of rope. How long will each piece be?

My mother bought 77.12 m of rope. She has to divide it into eight pieces. How long will each piece be?
### Term 1

**Input and output**

What does each statement tell you? Give two more examples of each.

**Constant difference**
- e.g., –3; –7; –11; –15 “Add –4“ or “Count in –4s“ or “Add –4 to the previous pattern“.

**Constant ratio**
- e.g., –2; –4; –8; –16; –32 “Multiply the previous term by 2.”

**Not a constant difference or a ratio**
- e.g., 1; 2; 4; 7; 11; 16 “Increase the difference between consecutive terms by 1 each time.”

1. What is the constant difference between the consecutive terms?
   - a. 8, 12, 16, 20. 
   - b. 7, 14, 21, 28.

2. What is the constant ratio between the consecutive terms?
   - a. 3, 9, 27, 81
   - b. 9, –27, 81, –243

3. Does this pattern have a constant difference or ratio or neither?
   - a. 1, 4, 10, 19
   - b. 2, 4, 8, 16

4. What is the constant difference or ratio between the consecutive terms?
   - a. 5, –15, 45, –135
   - b. 6, 24, 96, 384

5. Complete the table and then state the rule.

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the term</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>n + 5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the term</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Example: Rule?
- The term + 5.

a. Complete the table

b. State the rule.

<table>
<thead>
<tr>
<th>Position</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the term</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. What will the value of the 20th term be?
6. What are the next patterns? Complete the questions.

Hexagonal number pattern:

a. What will the next pattern be? Draw it using the rule: Increase the length of each side by one match.

b. Complete this table by using the same rule.

<table>
<thead>
<tr>
<th>Hexagon</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of matches</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Complete the following table. Describe it.

Example: 8, 15, 22, 29...

<table>
<thead>
<tr>
<th>Term</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>18</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the term</td>
<td>8</td>
<td>15</td>
<td>22</td>
<td>29</td>
<td>127</td>
<td>7(n) + 1</td>
</tr>
</tbody>
</table>

- Add 7 to the value of the previous term.
- 7 \times \text{the position of the term} + 1.
- 7(n) + 1, where “n” is the position of the term.
- 7(n) + 1, where “n” is a natural number.

13, 25, 37, 49, ...

<table>
<thead>
<tr>
<th>Term</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>17</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the term</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. Complete the following:

Example:

\[
\begin{align*}
\text{t} &= p \times 2 + 3 \\
0 \times 2 + 3 &= 3 \\
2 \times 2 + 3 &= 7 \\
4 \times 2 + 3 &= 11 \\
6 \times 2 + 3 &= 15 \\
8 \times 2 + 3 &= 19
\end{align*}
\]

9. What is the rule?

Example:

\[
\begin{align*}
\text{t} &= p \times 4 - 2 \\
7 \times 4 - 2 &= 22 \\
10 \times 4 - 2 &= 38 \\
13 \times 4 - 2 &= 50 \\
16 \times 4 - 2 &= 64 \\
19 \times 4 - 2 &= 74
\end{align*}
\]

This is the rule for this flow diagram.
10. Describe the relationship between the numbers in the top row and the numbers in the bottom row of the table.

Example:

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>20</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>45</td>
<td>105</td>
<td>205</td>
<td></td>
</tr>
</tbody>
</table>

Rule is \( y = 2x + 5 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

11. Describe the relationship between the numbers in the top row and those in the bottom row of the table. Write down the values of \( m \) and \( n \).

Example:

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>m</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>30</td>
<td>27</td>
<td>n</td>
<td>21</td>
<td>18</td>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>

\( m = 1 \)
\( n = 24 \)

Rule is \( y = -3x + 24 \)

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>m</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>n</td>
</tr>
</tbody>
</table>

\( m = \) 
\( n = \)

Rule is \( y = \) 

**Problem solving**

a. If the constant ratio is \(-7\), what could a sequence be?

b. If \( t = g \times 4 - 9 \), where \( g = -8 \), what is \( t \)?

c. \( y = -x + (-2) \) is the rule. Show this in a table with \( x = -3, -2, -1, 0, 1, 2 \).
Revise the following:

Say whether if the following is an:
• expression, or an
• equation, and why.

\[x + 18 = 52\]

a) \[x + 18 = 52\]

b) \[x + 18\]

1. Calculate the following and also underline the variable in red and the constants in blue:

**Example 1:** \[3a + 4a = 7a\]

a. \[5a + 3a = \]

b. \[6m - 2m = \]

c. \[1a^2 + 2a^2 = \]

d. \[8r^2 + 5r^2 = \]

e. \[4x^2 + 2x^2 = \]

f. \[5x^2 + 5x = \]

g. \[2a \times 3a = \]

h. \[2c^2 \times 5c^2 = \]

i. \[1a \div 7a = \]

j. \[3f \div 5f = \]

**Example 2:** \[3a^2 + 4a^2 = 7a^2\]

Note: \[3a^2 + 4a^2 \text{ is not } 7a^4\]

**Example 3:** \[5x^2 + 4x^2 = 9x^2\]

**Example 4:** \[5x + 4x^2 = 5x + 4x^2\]

**Example 5:** \[3a^2 \times 4a^2 = (3a^2)(4a^2) = 12a^4\]

g. \[2a \times 3a = \]

h. \[2c^2 \times 5c^2 = \]

i. \[1a \div 7a = \]

j. \[3f \div 5f = \]

**Example 6:** \[3a^2 \div 4a^2 = \]

Note: \[\frac{3a^2}{4a^2} = \frac{3}{4}\]
2. Complete.

Example: \( 4 \times \_ = 1 \)
\[ 4 \times \frac{1}{4} = 1 \]

3. Solve for \( x \):

Example 1: \( 2x = 16 \)
\[
\frac{2x}{2} = \frac{16}{2} \\
x = 8
\]

Example 2: \( x - 2 + 3 = -5 \)
\[
x + 1 = -5 \\
x + 1 - 1 = -5 - 1 \\
x = -6
\]

Example 3: \( \frac{2x}{3} = 12 \)
\[
\frac{2x}{3} \times 3 = 12 \times 3 \\
\frac{2x}{2} = \frac{36}{2} \\
x = 18
\]

4. Calculate, if \( x = 2 \), then:

Example: \( 2x + 5 \)
\[
= 2(2) + 5 \\
= 4 + 5 \\
= 9
\]

Example: \( x^2 + 5 \)
\[
= (2)^2 + 5 \\
= 4 + 5 \\
= 9
\]
5. Solve for $x$.

**Example 1:** $-5x = 10$

\[-5x = 10\]
\[\frac{-5x}{-5} = \frac{10}{-5}\]
\[x = -2\]

**Example 2:** $2x - 6x = 16$

\[-4x = 16\]
\[\frac{-4x}{-4} = \frac{16}{-4}\]
\[x = -4\]

c. $4x - 5x = 8$
d. $8x + 4x = 4$

6. Calculate:

**Example 1:** \[\frac{x^4}{x^3}\]

\[\frac{x.x.x.x}{x.x} = x.x = x^2\]

This is a monomial – it has only one term.

**Example 2:** \[\frac{x^4 - x^3}{x^3}\]

\[\frac{x^4}{x^3} - \frac{x^3}{x^3} = x^3 - 1\]

This is a binomial – it has two terms connected by a plus or minus sign.

a. $\frac{x^2}{x}$
b. $\frac{x^3}{x^2}$
c. $\frac{x^4 - x^3}{x^3}$
d. $\frac{x^6 - x^3}{x^3}$
Problem solving

Betty has $8n$ marbles and Peter has $3n$. How many do they have altogether? Write a number sentence.

**Example 3:**

\[
\frac{x^4 - 6x^2 - 1}{x^2} = \\
\frac{x^4}{x^2} - \frac{6x^2}{x^2} - \frac{1}{x^2} = x^2 - 6 - \frac{1}{x^2}
\]

7. Revision: Simplify the following using the distributive law:

**Example 1:**

\[
2(3 + 4) = 2 \times 3 + 2 \times 4 = (2 \times 3) + (2 \times 4) = 6 + 8 = 14
\]

**Example 2:**

\[
2(x + 5) = (2 \times x) + (2 \times 5) = 2x + 10
\]

**Example 3:**

\[
2(x^2 + x + 3) = (2 \times x^2) + (2 \times x) + (2 \times 3) = 2x^2 + 2x + 6
\]

- **a.** \(2(3 + 6) = \) 
- **b.** \(4(8 + 1) = \) 
- **c.** \(2(x + 4) = \) 
- **d.** \(4(x + 7) = \) 
- **e.** \(2(x^2 + x + 4) = \) 
- **f.** \(4(3 + x + x^2) = \)
What do the graphs or words tell us about the concept?

<table>
<thead>
<tr>
<th>Linear and non-linear</th>
<th>Constant, increasing and decreasing</th>
<th>Maximum or minimum</th>
<th>Discrete or continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph 1" /></td>
<td><img src="image2" alt="Graph 2" /></td>
<td><img src="image3" alt="Graph 3" /></td>
<td><img src="image4" alt="Graph 4" /></td>
</tr>
</tbody>
</table>

1. Plot the following and write it in words.

Example: The point \((5,7)\) is 5 units along, and 7 units up.

a. \((3,7)\) is \underline{3} units along, and \underline{7} units up.
b. \((4,8)\) is \underline{4} units along, and \underline{8} units up.
c. \((5,9)\) is \underline{5} units along, and \underline{9} units up.
d. \((10,2)\) is \underline{10} units along, and \underline{2} units up.
e. \((0,6)\) is \underline{0} units along, and \underline{2} units up.

2. Complete the following:

a. The left-right \(\underline{\text{horizontal}}\) direction is called the \(x\)-axis.
b. The \(\underline{\text{vertical}}\) (vertical) direction is called the \(y\)-axis.
c. The \(y\)-axis runs vertically through the \(\underline{\text{y-intercept}}\).
d. Where the \(x\)-axis crosses the \(y\)-axis is the “\(\underline{\text{origin}}\)” point. You measure everything from here.
e. The \(x\)-axis runs horizontally through the \(\underline{\text{x-intercept}}\).
3. Complete the ordered pairs for the equations $y = x^2 + 4$ and $y = -x^2 + 4$ and the plot them on the set of axis on the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-4$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$y = (-4)^2 + 4 = 16 + 4 = 20$

$y = x^2 + 4 = x^2 + 4$

$y = x^2 + 4 = x^2 + 4$

$y = x^2 + 4 = x^2 + 4$

$y = x^2 + 4 = x^2 + 4$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-4$</th>
<th>$-3$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$y = -(-4)^2 + 4 = -16 + 4 = -12$

$y = -x^2 + 4 = -x^2 + 4$

$y = -x^2 + 4 = -x^2 + 4$

$y = -x^2 + 4 = -x^2 + 4$

$y = -x^2 + 4 = -x^2 + 4$

The first parabola has a minimum point $(___,___)$ and it opens upwards (u-shaped).

The second parabola has a maximum point $(___,___)$ and it opens downwards (n-shaped).

What happens if you throw a ball into the air?

It will arc up into the air and come down again. The ball follows the path of a parabola.

Problem solving

Describe the graph $y = x^2 + 10$
Can you remember the meaning of the following?

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>The surplus remaining after total costs are deducted from total revenue.</td>
</tr>
<tr>
<td>Loss</td>
<td>The excess of expenditure over income.</td>
</tr>
<tr>
<td>Discount</td>
<td>The amount deducted from the asking price before payment.</td>
</tr>
<tr>
<td>Interest</td>
<td>The fee charged by a lender to a borrower for the use of borrowed money, usually expressed as an annual percentage of the amount borrowed, (also called the interest rate).</td>
</tr>
<tr>
<td>VAT (Value Added Tax)</td>
<td>The tax payable on all goods and services in South Africa. The current VAT rate is 14%. Some essential foods are exempt – that means they have a 0% VAT rate.</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>The current market price for which one currency can be exchanged for another.</td>
</tr>
</tbody>
</table>

**Term 1**

Budget is the estimate of cost and revenues over a specified period.

A loan is a sum of money that an individual or a company lends to an individual or company with the objective of gaining profits when the money is paid back.

Hire purchase is a system by which a buyer pays for an asset in regular installments, while enjoying the use of it. During the repayment period, ownership of the item does not pass to the buyer. Upon the full payment of the loan, the ownership passes to the buyer.

1. **Solve the following financial problems:**

   a. Kabelo receives R120 per week pocket money. He goes ten pin bowling twice (cost R20.00 per session excluding VAT). He has coffee for R5.00 and buys R30.00 of airtime, both with VAT included. How much pocket money can he carry over to the next week?
b. You receive R400 pocket money per month for chores you do around the house. Draw up a budget in the budget column. You had the following expenses last month: Movie R60.00; Taxi R90.00; Ice Cream R5.75; New shirt R65.00; Donation to welfare R50.00; Stationery R45.00; Repairs to your bicycle R150.00. Enter these expenses in the actual amount column. You have saved R375.00. Did you save anything or will you need to use some of your savings?

<table>
<thead>
<tr>
<th>Budget</th>
<th>Actual amount</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income (Pocket money)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenses</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taxi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Movies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Donations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Savings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stationery</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net Income</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c. A total of R36 000 was invested in two accounts. One account earned 7% annual interest and the other earned 9%. The total annual interest earned was R2 920. How much was invested in each account?

d. David buys a new car on hire purchase. The car costs R75 000 (excluding VAT) and he trades in his old car (that is fully paid for) for R9 500. The car registration, documentation and licence fees are R2 000. What will his instalment be if he pays 7% p.a. in simple interest and repays the money he borrows over a period of 54 months?
e. Lindy has €45. She wants to buy jeans for $15 CAD and a T-shirt for $10 CAD. After her purchases, how much money will she have left in ZAR?

Use the exchange rates in the table below to help you solve the word problems. Show your work in the space provided.

<table>
<thead>
<tr>
<th></th>
<th>ZAR (R)</th>
<th>USD ($)</th>
<th>GBP (£)</th>
<th>CAD ($)</th>
<th>EUR (€)</th>
<th>AUD ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZAR (R)</td>
<td>1,00</td>
<td>0,15</td>
<td>0,09</td>
<td>0,15</td>
<td>0,10</td>
<td>0,14</td>
</tr>
<tr>
<td>USD ($)</td>
<td>6,76</td>
<td>1,09</td>
<td>1,09</td>
<td>1,09</td>
<td>0,69</td>
<td>1,15</td>
</tr>
<tr>
<td>GBP (£)</td>
<td>11,06</td>
<td>1,60</td>
<td>1,74</td>
<td>1,74</td>
<td>1,10</td>
<td>1,83</td>
</tr>
<tr>
<td>CAD ($)</td>
<td>6,89</td>
<td>0,92</td>
<td>0,58</td>
<td>1,00</td>
<td>0,63</td>
<td>1,05</td>
</tr>
<tr>
<td>EUR (€)</td>
<td>9,88</td>
<td>1,46</td>
<td>0,91</td>
<td>1,59</td>
<td>1,00</td>
<td>1,67</td>
</tr>
<tr>
<td>AUD ($)</td>
<td>7,17</td>
<td>0,87</td>
<td>0,55</td>
<td>0,95</td>
<td>0,60</td>
<td>1,00</td>
</tr>
</tbody>
</table>

Example: 1 ZAR (R) = 0,15 USD ($)  
1 USD ($) = 6,76 ZAR (R)

Problem solving

Make notes of the important financial tips you have learned, and share them with a family member.
Symbols you need to revise or learn.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Term 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>Angle</td>
</tr>
<tr>
<td>∆</td>
<td>&lt;</td>
</tr>
<tr>
<td>Line segments</td>
<td>Line Ray</td>
</tr>
<tr>
<td>AB</td>
<td>AB</td>
</tr>
<tr>
<td>Parallel Degrees Right angles</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Geometric figures to remember.

Geometric figures

- Triangles
  - Equilateral triangle
  - Isosceles triangle
  - Scalene triangle

- Quadrilaterals
  - Parallelogram
  - Rectangle
  - Square
  - Rhombus
  - Trapezium
  - Kite

- More polygons
  - Pentagon
  - Hexagon
  - Heptagon
  - Octagon
  - Nonagon
  - Decagon, etc.

These are also polygons

How would you calculate the total sum of the interior angles of a polygon?

Similar and congruent triangles

Similar triangles have the same shape but are not the same size. Each pair of corresponding angles is equal and the ratio of any pair of corresponding sides is the same.

Congruent triangles are triangles that have the same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.

Angles to remember.

- **Acute angle**: an angle that is less than 90°
- **Right angle**: an angle that is 90°
- **Obtuse angle**: an angle that is greater than 90° but less than 180°
- **Straight angle**: an angle that is exactly 180°
- **Reflex angle**: an angle that is greater than 180°
- **Complementary angle**: an angle in a pair of angles which add together to make 90°
1. Construct using appropriate instruments and answer the questions.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a. An angle smaller than 90°.</td>
<td>b. A polygon with more than four sides.</td>
<td>c. A triangle.</td>
</tr>
<tr>
<td>i. Name the angle.</td>
<td>i. Calculate the interior angles of the polygon.</td>
<td>i. Draw a triangle that is congruent to the triangle above. Label it.</td>
</tr>
<tr>
<td>ii. Construct another angle such that this angle and the angle above, when added together, total 90°. What do you call such a pair of angles?</td>
<td>ii. Where in everyday life will we find such a shape?</td>
<td>ii. Draw a triangle similar to the triangle above. Label it.</td>
</tr>
</tbody>
</table>

continued
2. Describe the constructions using the words below.

Vertical opposite angles
\[ a = d; b = c; \]
\[ e = h; f = g \]

Corresponding angles
\[ a = e; b = f; \]
\[ c = g; d = h \]

Alternate interior angles
\[ c = f; d = e \]

Alternate exterior angles
\[ a = h; b = g \]

Consecutive interior angles
\[ c + e = 180^\circ \]
\[ d + f = 180^\circ \]
(also called co-interior angles)
A diagonal is a straight line inside a shape that joins one vertex to another but is not an edge of that shape.

3. Can you identify any diagonals? If not draw a few.

**Problem solving**

In which job, other than that of an engineer, will people need to calculate angles. Give an example of such a person and say why the person is calculating angles.
Describe these transformations.

1. Answer the following questions:

   a. The coordinates of \( ABC \) are:

      

   b. The coordinates of \( A'B'C' \) are:

      

   c. The translation vector is:

      \((x - 13, y)\)

   d. Explain the translation vector in words.
2. Answer the following questions:

a. The coordinates of ABC are:


b. The coordinates of A'B'C' are:


c. ABC is reflected over the __________.

d. Which coordinates remain the same?


e. Which coordinates differ?


3. Answer the following questions:

a. The coordinates of ABC are:


b. The coordinates of A'B'C' are:


c. Compare the corresponding vertices.


4. Answer the following questions:

a. A'B' = __________ × AB


b. B'C' = __________ × BC


c. A'C' = __________ × AC


d. Therefore, we say that this transformation is an enlargement with scale factor = __________


Problem solving

Design a house on grid paper (top view).
Enlarge your plan by a scale factor of 2.
Reflect the house, rotate it by 90 degrees and translate it two units up and three down.
What do all these geometric objects have in common?

- cube
- icosahedron
- dodecahedron
- octahedron
- tetrahedron

What do we name this group of geometric objects?

1. Write down the name of the geometric object that each of the nets will form. How many edges, vertices and faces does each have. Complete the table below.

<table>
<thead>
<tr>
<th></th>
<th>dodecahedron</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td>b.</td>
<td>c.</td>
<td>d.</td>
</tr>
</tbody>
</table>

Describe each.

- e. 30 edges
- f. 20 vertices
- g. 12 faces
- h. 

2. Complete the following:

a. If the sides of a geometric figure are equal in length and the interior angles are equal, the geometric figure is ...

If the sides are not equal it is ...

b. What do you notice if you look at a platonic solid’s faces?

c. What do we name geometric solids if all the faces are congruent?

d. Name three geometric solids that are irregular.

3. Construct the net for a tetrahedron. We have given you the first two steps.

**Step 1:**
Construct an equilateral triangle. Label it ABC.

**Step 2:**
Construct another equilateral triangle with one base joined to base AB of the first triangle.

4. Describe the different views of the building using the drawings below.

5. Draw a cube using a 30° oblique drawing.

   **Step 1:** Draw a square.
   **Step 2**
   Draw a 30° line from the bottom right vertex.
   **Step 3**
   Draw the rest of the cube.

**Problem solving**
Make skeletons (outlines) of the platonic solids using recycled materials.
Revise these formulae:

<table>
<thead>
<tr>
<th>Perimeter of a rectangle</th>
<th>2 (l + 2b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of a rectangle: (l \times b)</td>
<td></td>
</tr>
<tr>
<td>Perimeter of a square: (4l)</td>
<td></td>
</tr>
<tr>
<td>Area of a square: (l \times l)</td>
<td></td>
</tr>
<tr>
<td>The area of a triangle is: (\frac{1}{2}b \times h)</td>
<td></td>
</tr>
</tbody>
</table>

Circumference of a circle

\(C = \pi d\) or \(2\pi r\)

Area of a circle

\(A = \pi r^2\)

Look at these conversions:

\[
\begin{align*}
1 \text{ cm} &= 10 \text{ mm} \\
1 \text{ cm}^2 &= (1 \text{ cm} \times 1 \text{ cm}) = 100 \text{ mm}^2 \quad (10 \text{ mm} \times 10 \text{ mm}) \\
1 \text{ m} &= 1000 \text{ mm} \\
1 \text{ m}^2 &= (1 \text{ m} \times 1 \text{ m}) = 1000000 \text{ mm}^2 \quad (1000 \text{ mm} \times 1000 \text{ mm}) \\
1 \text{ km} &= 1000 \text{ m} \\
1 \text{ km}^2 &= (1 \text{ km} \times 1 \text{ km}) = 100000000 \text{ m}^2 \quad (1000 \text{ m} \times 1000 \text{ m})
\end{align*}
\]

1. Calculate the perimeter and area of a square. Write your answer in mm.

Example: side 4.5 cm

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P = 4 \times l)</td>
<td>(A = l^2)</td>
</tr>
<tr>
<td>= 4 ((4.5 \text{ cm}))</td>
<td>= 4.5 cm \times 4.5 cm</td>
</tr>
<tr>
<td>= 18 cm</td>
<td>= 20.25 cm²</td>
</tr>
</tbody>
</table>

Write your answer in mm.

= 4 \((45 \text{ mm})\) = 45 mm \times 45 mm
= 180 mm = 2 025 mm²

If the area is 2 025 mm² what is the answer in cm²?

1 cm = 10 mm
1 cm² = 1 cm \times 1 cm
1 cm² = 10 mm \times 10 mm
1 cm² = 100 mm²
1 cm² = 0.01 m²

\[
\begin{align*}
\text{Convert} & \quad \text{mm} = \text{cm} = \frac{\text{mm}}{10} \\
\text{Calculate} & \quad \text{Area in cm} = \frac{2025 \text{ mm}^2}{100} \\
& \quad = 20.25 \text{ cm}^2
\end{align*}
\]

2. Calculate the area and perimeter of a rectangle. Write your answer in mm.

Example: length 3.8 cm, breadth 2.1 cm

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P = 2(l + b))</td>
<td>(A = l \times b)</td>
</tr>
<tr>
<td>= 2 ((3.8 \text{ cm} + 2.1 \text{ cm}))</td>
<td>= 3.8 cm \times 2.1 cm</td>
</tr>
<tr>
<td>= 2 ((5.9 \text{ cm}))</td>
<td>= 7.98 cm²</td>
</tr>
</tbody>
</table>

Write the area answer in mm² and m².

mm²

= 7.98 cm²

= 7.98 cm² \times 100

= 798 mm²

m²

\[
\begin{align*}
1 \text{ cm} &= 10 \text{ mm} \\
1 \text{ cm}^2 &= (1 \text{ cm} \times 1 \text{ cm}) = 100 \text{ mm}^2 \quad (10 \text{ mm} \times 10 \text{ mm}) \\
1 \text{ m} &= 1000 \text{ cm} \\
1 \text{ m}^2 &= (1 \text{ m} \times 1 \text{ m}) = 100000 \text{ cm}^2 \quad (100 \text{ cm} \times 100 \text{ cm}) \\
1 \text{ m}^2 &= 10000 \text{ mm}^2 \\
1 \text{ m}^2 &= 0.01 \text{ m}^2
\end{align*}
\]

\[
\begin{align*}
\text{Convert} & \quad \text{cm} = \text{mm} = \frac{\text{cm}}{10} \\
\text{Calculate} & \quad \text{Area in mm}^2 = \frac{798 \text{ mm}^2}{100} \\
& \quad = 7.98 \text{ mm}^2 \\
\text{Convert} & \quad \text{mm}^2 = \text{cm}^2 = \frac{\text{mm}^2}{100} \\
\text{Calculate} & \quad \text{Area in cm}^2 = \frac{798 \text{ mm}^2}{10000} \\
& \quad = 0.00798 \text{ cm}^2
\end{align*}
\]
3. Calculate the area of a triangle. Write your answer in mm.

Example:

```
<table>
<thead>
<tr>
<th>Base</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 cm</td>
<td>2.6 cm</td>
</tr>
</tbody>
</table>
```

Write your answer in mm².

3.75 cm²

(write your answer in mm²)

4. Calculate the area of the circles.

Example: Radius is 3 cm.

```
A = \pi r^2
= (3.14159) (3 cm)^2
= 28.27 cm²
```

a. Radius is 4 cm

b. Radius is 2.5 cm

Problem solving

If the area of the circle is 314.159 cm². What is the radius?
Revise the following formulae:

- The volume of a cube
  \[ v = l^3 \]
- Surface area of a prism
  \[ A = \text{the sum of the area of all the faces} \]
- The volume of a rectangular prism
  \[ v = l \times b \times h \]
- The volume of a triangular prism
  \[ v = \frac{1}{2} (b \times h) \times l \]

Revise the following:

- if 1 cm = 10 mm then 1 cm\(^3\) = 1 000 mm\(^3\)
- if 1 m = 100 cm then 1 m\(^3\) = 1 000 000 cm\(^3\).
- An object with a volume of 1 cm\(^3\) will displace exactly 1 ml of water.
- An object with a volume of 1 m\(^3\) will displace exactly 1 kl of water.

1. Calculate the volume, capacity and surface area of a cube.

Example:

<table>
<thead>
<tr>
<th>Volume</th>
<th>Capacity</th>
<th>Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v = l^3 )</td>
<td>( v = (4 \text{ cm})^3 )</td>
<td>Net of the cube. How many faces (flat surfaces) are there?</td>
</tr>
<tr>
<td>4 cm(^3)</td>
<td>64 cm(^3)</td>
<td></td>
</tr>
</tbody>
</table>

\[ v = (4 \text{ cm})^3 = 64 \text{ cm}^3 \]

Note: An object with a volume of 1 cm\(^3\) will displace 1 ml of water. Therefore an object that is 64 cm\(^3\) will displace 64 ml water or 0,064 ℓ.

Cubic mm  | Cubic cm | Cubic m | Litre
-----------|----------|---------|-------
1 000 000 000 | 1 000 000 | 1 | 1 000
1 000 000 | 1 000 | 0,001 | 1
1 000 | 1 | 0,000 001 | 0,001

Surface area = sum of the area of all the faces.
= 6 (area of a face)
= 6\(a^2\)
= 6 \(4 \text{ cm})^2\)
= 6 \times 16 \text{ cm}^2
= 96 \text{ cm}^2
2. Calculate the volume, capacity and surface area of a rectangular prism.

### Example:

<table>
<thead>
<tr>
<th>Volume</th>
<th>Capacity</th>
<th>Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 cm</td>
<td>2 cm</td>
<td>1.5 cm</td>
</tr>
</tbody>
</table>

\[
v = l \times b \times h \\
v = 4 \text{ cm} \times 1.5 \text{ cm} \times 2 \text{ cm} \\
v = 12 \text{ cm}^3
\]

Note: An object with a volume of 1 cm³ will displace 1 ml of water. 
∴ an object that is 12 cm³ will displace 12 ml.

Surface area

\[
A = 2lh + 2bh + 2(lb)
\]

= 2(4 cm \times 1.5 cm) + 2(4 cm \times 2 cm) + 2(1.5 cm \times 2 cm)

= 12 cm² + 16 cm² + 6 cm²

= 34 cm²

### Conversion Table:

<table>
<thead>
<tr>
<th>Cubic mm</th>
<th>Cubic cm</th>
<th>Cubic m</th>
<th>Litre</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 000 000 000</td>
<td>1 000 000</td>
<td>1</td>
<td>1 000</td>
</tr>
<tr>
<td>1 000 000</td>
<td>1 000</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>1 000</td>
<td>1</td>
<td>0.000001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Continued
The rectangular prism’s dimensions are: length = 4.5 cm; breadth = 3.5 cm and height 4 cm.

<table>
<thead>
<tr>
<th>Volume</th>
<th>Capacity</th>
<th>Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Calculate the volume, capacity and surface area of a triangular prism.

Example:

<table>
<thead>
<tr>
<th>Volume</th>
<th>Capacity</th>
<th>Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>v = \frac{1}{2} b \times h \times l</td>
<td>Note: An object with a volume of 1 cm³ will displace 1 ml of water. \therefore an object that is 15 cm³ will displace 15 ml of water.</td>
<td>Net of the triangular prism. How many faces (flat surfaces) are there?</td>
</tr>
<tr>
<td>v = \frac{1}{2}(5 \text{ cm}) \times 3 \text{ cm} \times 2 \text{ cm}</td>
<td>Surface area A = 2 (area of triangle) + area of 3 rectangles</td>
<td>Use Pythagoras to calculate this.</td>
</tr>
<tr>
<td>v = 2.5 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm}</td>
<td>= 2(\frac{1}{2}(5 \text{ cm}) \times 3 \text{ cm}) + 2(3.9 \text{ cm} \times 2 \text{ cm}) + 1(5 \text{ cm} \times 2 \text{ cm})</td>
<td>= 15 \text{ cm}² + 15.6 \text{ cm}² + 10 \text{ cm}²</td>
</tr>
<tr>
<td>v = 15 \text{ cm}³</td>
<td>= 40.6 \text{ cm}²</td>
<td></td>
</tr>
</tbody>
</table>
The triangular prism’s dimensions are: base of triangle 4 cm, height of triangle 2.5 cm, length of prism 5 cm, other two sides of triangle 3.2 cm each.

<table>
<thead>
<tr>
<th>Volume</th>
<th>Capacity</th>
<th>Surface area</th>
</tr>
</thead>
</table>

Problem solving

a. If the volume of a cube is 10,648 cm³, what are its dimensions in mm and m?

b. Give everyday examples of where we will use the volume, capacity and the surface area of:
   - cubes
   - rectangular prisms
   - triangular prisms
Hypothesis: grade 9 girls do better in mathematics and science than grade 9 boys.

A hypothesis is a statement or prediction for which sound evidence of its truth has to be found.

Here are some examples of hypotheses:
• Everybody in grade 9 owns a cell phone.
• All grade 9s like junk food.

1. Form your research team.

   Names of your research team:
   ____________________________
   ____________________________
   ____________________________
   ____________________________

2. What is the aim of your research?
3. What is your hypothesis?

4. Questions that might help you to plan:

   a. What data do you need?

   b. Who will you get it from?

   c. How will you collect it?

   d. How will you record it?

   e. How will you make sure the data is reliable?

7. Use the data you collected and recorded to:

a. Organise your data in a frequency table.

b. Calculate the mean, median and mode.

c. Calculate the data range.

d. Draw a stem–and–leaf display.
e. Represent your data in a graph. You may use more than one type of graph.

Problem solving

Interpret your graphs and tables and write a report, using the following headings:
1. Aim
2. Hypothesis
3. Plan
4. Data collection
5. Analysis
6. Conclusions
7. Appendices
8. References
Grade 9

Mathematics

WORKSHEETS
1 to 64

PART 2

ENGLISH

Book 1
Real numbers, rational numbers and irrational numbers

Term 1

Real number Venn diagram – a diagrammatic illustration of the real number system

\[ \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{A}_R \subset \mathbb{R} \]

(\(\subset\) = subset of)
1. Study these definitions of numbers:

**Natural** \( \mathbb{N} \) for **Natural**
Natural numbers are counting numbers (1, 2, 3, \ldots) \( (\mathbb{N}_1) \) and the positive integers of the whole numbers (0, 1, 2, 3, \ldots) \( (\mathbb{N}_0) \). Mathematicians use the term “natural” in both cases.

**Integer** \( \mathbb{Z} \) for **Zahlen** (‘numbers’ in German))
Integers are the natural or whole numbers and their negatives (… –3, –2, –1, 0, 1, 2, 3, …).

**Rational** \( \mathbb{Q} \) for **Quotient**
Rational numbers are numbers that can be expressed as a fraction of an integer (that is a ratio of an integer). Rational numbers can be added, subtracted, multiplied and divided. Eg. \( \frac{1}{2} = 0.5 \) or \( \frac{1}{3} = 0.333 \ldots \) Rational decimal expansions end or repeat.

**Real Algebraic** \( \mathbb{A}_r \) for **Algebraic**\_**Real**
A real algebraic number is defined as a number that is the root of a polynomial with rational coefficients. Real algebraic numbers may be rational or irrational. The number \( \sqrt{2} = 1.41421 \ldots \) is a real algebraic number that is irrational.

**Real** \( \mathbb{R} \) for **Real**
Real numbers are all the numbers (all the points) on the continuous, infinitely long number line with no gaps. It is a collection of every possible infinite decimal expansion. Real numbers may be rational or **irrational**, and algebraic or non–algebraic (transcendental). The numbers \( \pi = 3.14159 \ldots \) and \( e = 2.71828 \ldots \) are transcendental. A transcendental number can never be written as an exact fraction of a whole number, it requires an infinite series of terms.

**Irrational**
These numbers cannot be written as fractions of whole numbers. Irrational decimal expansions neither end nor repeat.

**Transcendental**
These are irrational numbers that cannot be constituted back as an integer through an arithmetical operation.

continued
2. Match these descriptions with the correct number line. Start at ‘Integer, Z’.

A number that can be expressed as a fraction of an integer
All the numbers
Natural numbers and their negatives
Rational or irrational numbers

<table>
<thead>
<tr>
<th>Natural, N</th>
<th>(0, 1, 2, 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integer, Z</td>
<td>(-3, -2, -1, 0, 1, 2, 3)</td>
</tr>
<tr>
<td>Rational, Q</td>
<td>(-\frac{3}{4}, -\frac{1}{3}, -\frac{1}{2}, \frac{1}{2}, +\frac{1}{3}, 2, \frac{3}{4})</td>
</tr>
<tr>
<td>Real algebraic, A_\text{a}</td>
<td>(-\sqrt{5}, -\sqrt{2}, -\frac{1}{2}, \frac{1}{2}, \sqrt{2}, \sqrt{5})</td>
</tr>
<tr>
<td>Real, R</td>
<td>(-\pi, -e, -\sqrt{2}, -\frac{1}{2}, \frac{1}{2}, \sqrt{2}, e, \pi)</td>
</tr>
</tbody>
</table>
3. What do the intervals between the integers on these number lines on the previous page mean?

I. Rational

ii. Real algebraic

iii. Real

4. Complete the table by putting ticks (✔) in the appropriate columns.

<table>
<thead>
<tr>
<th></th>
<th>Whole number</th>
<th>Natural number</th>
<th>Integer</th>
<th>Rational number</th>
<th>Irrational number</th>
<th>Real number</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>200</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td></td>
<td>✔</td>
</tr>
<tr>
<td>b</td>
<td>-29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>12/50</td>
<td>❌</td>
<td>❌</td>
<td>✔</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>0.987</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>√81</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>√5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>π</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>j</td>
<td>124.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>22/7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>√25 + 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem solving

The number e (Euler’s Number) is a famous irrational number. Why?
Factorisation

Study these methods of factorisation:

Method 1:
Ladder method.

\[
\begin{array}{c|c}
12 & 2 \\
6  & 2 \\
3  & 3 \\
1  &   \\
\end{array}
\]

In this example every factor is a prime number.

We can write it as:

\[2 \times 2 \times 3 = 12\]

or \[2^2 \times 3 = 12\]

Method 2:

What are the prime factors of 12?

Break 12 into 4 \times 3.

The prime factors of 4 are 2 and 2.
The prime factor of 3 is 3.

So the prime factors of 12 are 2, 2, 3.

We can write it as \[2 \times 2 \times 3 = 12\]
or \[2^2 \times 3 = 12\]

Method 3:

\[
\begin{array}{c|c}
12 & 3 \\
4  &   \\
2 &   \\
\end{array}
\]

Remember it is important to know your divisibility rules when working with prime numbers.

1. a. Factorise 15.

<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
<th>Method 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Factorise 72.

<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
<th>Method 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Before carrying on with questions c and d say which method you like the most and why.
Prime numbers are numbers that can be divided only by one and themselves. Show this with all the numbers between 100 and 200.

c. Factorise 95.

<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
<th>Method 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Factorise 100.

<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
<th>Method 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Prime factorisation is finding which prime numbers multiply together to make the original number. Knowing prime factorisation will help you a lot as you carry on with maths. Why? Read the comic strip. Each time a character says ‘let me try’, try and do it yourself.

A. The importance of prime numbers is that any integer can be decomposed into a product of primes.

Give me an example.

Let me try.

B. You might want to know how many different pairs of numbers can be multiplied to get 360. You can start by trying to write them down.

You will see that every composite factor of 360 is a product of a subset of the prime factors.

I hope you didn’t miss any. Now write 360 as a product of prime factors.

C. Let me try.

Problem solving

Prime numbers are numbers that can be divided only by one and themselves. Show this with all the numbers between 100 and 200.
Problems about the distance travelled in a given time can be solved using formulae.

To find distance:  
\[ \text{Distance} = \text{Speed} \times \text{Time} \]

\[ d = s \times t \]

To find time:  
\[ \text{Time} = \frac{\text{Distance}}{\text{Speed}} \]

\[ t = \frac{d}{s} \]

To find rate (speed):  
\[ \text{Speed} = \frac{\text{Distance}}{\text{Time}} \]

\[ s = \frac{d}{t} \]

When we solve problems using these formulae use ratio and proportion.

A ratio is a way of comparing the sizes of two or more quantities. So 4:7 and 8:14 are ratios.

A proportion is a statement that two ratios are equivalent. So 4:7 is proportional to 8:14 (meaning that 4 is to 7 as 8 is to 14).

A proportion can be written in two ways:
• as two equal fractions: \[ \frac{4}{7} = \frac{8}{14} \]
or
• like this: \[ 4 : 7 = 8 : 14 \]
  (or like this \[ 4 : 7 :: 8 : 14 \])

When two ratios are equal, the cross-products of the ratios are equal. So for the proportion \[ a:b::c:d \] you can multiply, \[ a \times d = b \times c \], as in this example:

\[ \frac{4}{7} = \frac{8}{14} \]

so \[ 4 \times 14 = 56 \]
and \[ 7 \times 8 = 56 \]

Example: My family travelled 300 km at a speed of 60 km per hour. For how long did they travel?

We can use a formula or work with ratios and proportion.

Formula to find time:  
\[ \text{Time} = \frac{\text{Distance}}{\text{Speed (Rate)}} \]

\[ \text{Time} = \frac{300}{60} = 5 \text{ hours} \]

Working with ratio and proportion.

\[ \frac{60 \text{ km}}{1 \text{ h}} = \frac{300 \text{ km}}{t} \]

\[ 60 \times t = 300 \times 1 \]

\[ 60t = 300 \]

\[ \frac{60t}{60} = \frac{300}{60} \]

\[ t = 5 \]

The speed (rate) “km per hour” gives distance travelled per unit of time.

What do we want to find out? The time.

Use ‘cross’ products.
1. Complete the table.

<table>
<thead>
<tr>
<th>Speed (Rate)</th>
<th>Time</th>
<th>Distance</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 90 km/h</td>
<td>?</td>
<td>11 700 km</td>
<td></td>
</tr>
<tr>
<td>b. 50 km/h</td>
<td>8 hours</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>c. 120 km/h</td>
<td>?</td>
<td>61 200 km</td>
<td></td>
</tr>
<tr>
<td>d. 500 km/h</td>
<td>2 hours 30 min</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>e. 1 000 km/h</td>
<td>?</td>
<td>20 000 m</td>
<td></td>
</tr>
</tbody>
</table>

2. A car travels 60 km in 36 minutes. At the same average speed, how far will it travel in 1 hour 12 minutes?

3. A train travelling at an average speed of 100 km/h covers a certain distance in 3 hours 36 minutes. At what average speed must the train travel to cover the same distance in 2 hours 30 minutes?

Problem solving

Write a problem using an example from you day-to-day life on speed, distance and time. Ask a family member to help you.
What is direct proportion?

Direct proportion
As one value increases (or decreases), so does the other. How do you think this will look on a graph?

While you are busy with this worksheet think about what inverse proportion could mean. We will deal with it in the next worksheet.

Using different methods to solve proportion problems
Example: 4 books cost R150. How much do 7 books cost?

<table>
<thead>
<tr>
<th>Method 1: Unitary</th>
<th>Method 2: Cross–multiply</th>
<th>Method 3: Rule of three</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find the value of 1 unit and multiply to find the value of the required number of units.</td>
<td>Align terms in correct columns; multiply 3rd term by 2nd; then divide by 1st.</td>
<td></td>
</tr>
<tr>
<td><strong>Books</strong></td>
<td><strong>Rands</strong></td>
<td><strong>Books</strong></td>
</tr>
<tr>
<td>4</td>
<td>R150</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>&lt;sup&gt;\frac{R150}{4}&lt;/sup&gt;</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>7 × R37.50</td>
<td>= R262.50</td>
</tr>
</tbody>
</table>

Draw a graph.

How does this graph show direct proportion?
1. Use the 3 methods to solve this problem and draw a graph.
5 T-shirts cost R120. How much will 9 cost?

<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
<th>Method 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Draw a graph to show this.
### Inverse proportion

As one value increases, the other value shows a matching decrease.

#### Example:

Ten people take 4 days to dig a hole, how long will it take 8 men?

<table>
<thead>
<tr>
<th>Method 1: Unitary</th>
<th>Method 2: Vedic</th>
<th>Method 3: Rule of three</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>People</strong></td>
<td><strong>Days</strong></td>
<td><strong>People</strong></td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>takes 10</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>take 10 x 4 = 40</td>
<td>40 / 8 = 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Fewer people more time.
a. If it takes 3 people to make 21 T-shirts per day, how long will it take 12 people?

<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
<th>Method 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Draw a graph.

![Graph](image)

c. How does this graph show inverse proportion?

The graph shows inverse proportion because as the number of people increases, the number of T-shirts made per day decreases.
Can you still remember what a budget is?

**Budget** is the estimate of cost and revenues over a specified period.

What are loans and interest?

A **loan** is sum of money that an individual or a company lends to an individual or company with the objective of gaining profits when the money is paid back.

**Interest** is the fee charged by a lender to a borrower for the use of borrowed money. The rate of interest is usually expressed as an annual percentage of the amount borrowed (the principal amount).

Do you know what the difference is between simple and compound interest?

**Interest can be calculated in two ways:**

- **Simple Interest**
  
  The formula for simple interest is:

  \[ \text{Principal amount} \times \frac{\text{rate of interest (\%)} \times \text{number of periods}}{100} \]

- **Compound Interest**
  
  Compound interest means that the interest will include interest calculated on interest.

  The formula for calculating the Total future amount owed is:

  \[ \text{Principal amount} \times (1 + \frac{\text{Rate of interest (\%)}}{100})^{\text{Number of periods}} \]

**Example of compound interest**

- An amount of R100 is invested for two years with interest of 10 % compounded (added) yearly.

  - The interest at the end of the first year would be: R100 x 10 % = R10

  - In the second year the interest rate of 10 % would apply not only to the R100, but also to the R10 interest of the first year.

  - In the second year the interest would be: R110 x 10 % = R11

  - Total interest earned over the two years will be: R10 (year 1) + R11 (year 2) = R21

  - Total investment after two years: R100 (principal amount) + R21 (interest) = R121

  - Using the formula: Total future amount = R100 \((1 + 0,10)^2\)

    \[= R100 \times (1,1)^2\]

    \[= R100 \times 1,21\]

    \[= R121\]
1. Palesa needs to earn R500 in interest so she will have enough to buy a used bicycle. She puts R2 000 into an account that earns 5% per year simple interest. How long will she need to leave her money in the account to have enough money for the bicycle?

2. Thabo has R500 that he invests in an account that pays 8% interest compounded yearly. How much money does Thabo have at the end of 3 years?

3. Susan has R1 000 that she invests in an account that pays 7.5% interest compounded yearly. How much money does Susan have at the end of 5 years?

4. You saved R4 750 during the last year. You decide that it will be the best to invest the money. At your local bank they have two investment options:
   Option 1: A 5 Year fixed deposit with 3.25% simple interest per year.
   Option 2: A 5 Year fixed deposit with 3.10% compound interest per year. Which 5 year investment will be the best?
Can you still remember the meaning of hire purchase?

**Hire purchase** is a system by which a buyer pays for an asset in regular installments, while enjoying the use of it.

During the repayment period, ownership of the item does not pass to the buyer. Upon the full payment of the loan, the title passes to the buyer.

Many organisations enter into hire purchase or leasing agreements to pay for and use equipment over a period of time rather than pay the full cost up front.

The repayment period is normally the same as the production life of the machine. For example: a farmer buys a tractor and pays it off over 5 years. After 5 years he typically has to replace the tractor.

1. The hire purchase price of a refrigerator is R6 500. The deposit of R500 is made and the remainder is paid in equal monthly payments of R250.

   a. Calculate the number of monthly payments that must be made.

   b. If the cash price is R4 000, express as a percentage of the cash price, the extra cost of buying on hire purchase.

   c. What is the interest rate (simple interest) charged on this transaction?
2. A new TV costs R6 900 cash. It is available on hire purchase with a deposit of 15% followed by 12 instalments of R558.50. Find the total hire purchase price and the extra amount that you would pay (on top of the cash price) using hire purchase.

3. The cash price of a bike is R220. The hire purchase price is R300. If the deposit is 10% followed by 10 equal monthly instalments, find the amount you will pay each month.

Problem solving
A DVD player costs R240 cash. It is available on hire purchase by paying a deposit of 20% followed by 12 instalments of R18.50. Find the extra amount paid by hire purchase.

Remember interest is compounded monthly. Draw a table to help you.
Do you know what exchange rate means?

An exchange rate is the current market price for which one currency can be exchanged for another.

The Rand (sign: R; code: ZAR) is the currency of South Africa.

In modern China, people use Renminbi as their money. In Chinese, “Renminbi” means “people’s money”. A unit of this currency is called the Yuan.

The symbol for the Yuan looks like this: ¥ (code: CNY)

The Canadian Dollar (sign: $; code: CAD) is the currency of Canada.

Use the exchange rates in the table to help you solve the word problems. Show your work in the space provided.

<table>
<thead>
<tr>
<th></th>
<th>ZAR (R)</th>
<th>USD ($)</th>
<th>GBP (£)</th>
<th>CAD ($)</th>
<th>EUR (€)</th>
<th>AUD ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZAR</td>
<td>1,00</td>
<td>6,76</td>
<td>11,06</td>
<td>6,89</td>
<td>9,88</td>
<td>7,17</td>
</tr>
<tr>
<td>USD</td>
<td>0,15</td>
<td>1,00</td>
<td>1,60</td>
<td>0,92</td>
<td>1,46</td>
<td>0,87</td>
</tr>
<tr>
<td>GBP</td>
<td>0,09</td>
<td>1,09</td>
<td>1,00</td>
<td>0,58</td>
<td>0,91</td>
<td>0,55</td>
</tr>
<tr>
<td>CAD</td>
<td>0,15</td>
<td>1,09</td>
<td>1,74</td>
<td>1,00</td>
<td>1,59</td>
<td>0,95</td>
</tr>
<tr>
<td>EUR</td>
<td>0,10</td>
<td>0,69</td>
<td>1,10</td>
<td>0,63</td>
<td>1,00</td>
<td>0,60</td>
</tr>
<tr>
<td>AUD</td>
<td>0,14</td>
<td>1,15</td>
<td>1,83</td>
<td>1,05</td>
<td>1,67</td>
<td>1,00</td>
</tr>
</tbody>
</table>

1. Suzanne wants to order a new CD from Germany. She has R250 in her savings account. The CD costs €5. Once she has bought the CD, how much money will she have left in ZAR?

If she can order the same CD from Canada for $7, where must she order it from for the best price provided the shipment cost is the same.
2. Reinette lives in Worcester, South Africa. Her uncle lives in Sydney, Australia. For her birthday, Reinette received $50 from her uncle. How many South African Rands (ZAR) can she buy with her birthday money?

3. Reinette takes the money she received from her uncle and orders a new computer programme from America. After she has bought the programme, she will still have R150 left. How much does the programme cost in US$?

4. Reinette wants to order another programme from England. The programme costs £15. Will Reinette have enough money to buy this programme?

Problem solving

Which currency in the table has the highest valued currency unit?
Do you know what commission means? What are rentals?

**Definition**: Commission is the fee charged by a broker or an agent for his/her service to facilitate a transaction, such as the buying or selling of goods. Rental is when an item is leased out for a specific period of time.

Many employees are paid salaries based on the number of hours they have worked over a given period of time plus a commission.

1. Andrew lives in Johannesburg. His parents are planning a vacation to Cape Town. They decide to fly to Cape Town and then rent a car. The car rental company charge R200 per day (including 200 km free) and R1.80 per km. The insurance will be 7.5% of the daily rental amount and the GPS an additional R45 per day.

What will the total cost be for the car rental if they spent 6 days in Cape Town and travelled 1650 km in total?

2. A truck rental agency charges a daily fee plus a kilometre fee. Julie was charged R460 for two days and 100 kilometres and Christina was charged R1 050 for three days and 400 kilometres. What is the agency’s daily fee and what is the kilometre fee?
3. Hertz has a processing fee of R115,00 and charges R210 per day for car rental. Avis Car Rental has a processing fee of R255,00 and charges R190 per day for a car. When will the cost of the rentals be equal?

4. Tara is a sales representative for a cosmetic company. She is paid R5,15 per hour each week plus a commission of 10% on the amount of sales over R5 000. She works 40 hours one week, and she sells R7 260 worth of cosmetics during that week. She has been offered a job at another cosmetic company that pays R5.00 per hour for a 40 hour work week plus a commission of 4% on total sales. Which job would pay more? Should she change jobs?

5. Two furniture salesmen are comparing their salaries. Gert is paid R25,00 per hour plus a 15% commission on his total sales. Ben is paid R29,00 per hour plus a 10% commission on his total sales. Suppose each has sold R5 000 worth of furniture, compare their income over various periods of time to find out when they will earn the same. What will happen after that point? Who would have earned more before that point?

Problem solving

A real estate agent received a 6% commission on the selling price of a house. If his commission was R8 650, what was the selling price of the house?
Properties of numbers

Revise: give an example of each property. Write a rule for each.

<table>
<thead>
<tr>
<th>Commutative</th>
<th>Associative</th>
<th>Distributive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero as a property of addition</td>
<td>One as a property of multiplication</td>
<td></td>
</tr>
</tbody>
</table>

1. Use the commutative property to show that the equations are equal.

Examples:

- \(a + b = b + a\)
- \(a^2 + b^2 = b^2 + a^2\)
- \(a \times b^2 = b^2 \times a\)
- \(2a + b = b + 2a\)
- \(2a \times 2b = 2b \times 2a\)

But:

- \(a \div b \neq b \div a\) and \(a - b \neq b - a\)

a. \(y^2 + x = \underline{\text{\(x + y^2\)}}\)  
   b. \(3x + y^2 = \underline{\text{\(\text{___}\)}}}\)  
   c. \(3x^2 + 5y^2 = \underline{\text{\(\text{___}\)}}}\)

d. \(2x + y = \underline{\text{\(\text{___}\)}}}\)  
   e. \(5y + x^2 = \underline{\text{\(\text{___}\)}}}\)

If \(x = 2\) and \(y = -3\), solve each of the equations in a to e.

f.  
g.  
h.  

<table>
<thead>
<tr>
<th>(y^2 + x) and (x + y^2)</th>
<th>(x + y^2)</th>
<th>(5y + x^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3)^2 + 2)  (= 9 + 2)  (= 11)</td>
<td>(2 + (-3)^2)  (= 2 + 9)  (= 11)</td>
<td></td>
</tr>
</tbody>
</table>
2. Use the associative property to show that the equations are equal.

Examples:

- \((a + b) + c = a + (b + c)\)
- \((a^2 + b^2) + c^2 = a^2 + (b^2 + c^2)\)
- \((a \times b) \times c = a \times (b \times c)\)
- \((a^2 \times b) \times c = a^2 \times (b \times c)\)

But:

- \((a - b) - c \neq a - (b - c)\)
- \((a \div b) \div c \neq a \div (b \div c)\)

a. \((3m + n) + p^2 =\)  
b. \((n^2 + p^3) + 4m^2 =\)  
c. \((m \times p) \times n^3 =\)

d. \((p^2 \times n^3) \times m^3 =\)  
e. \((n \times p^2) \times m^3 =\)

Test both sides of your equations in a to e if \(m = -4\) and \(n = 6\).

f. \(\)  
g. \(\)  
h. \(\)

i. \(\)  
j. \(\)

continued
3. Use the distributive property to show that the equations are equal.

Examples:
\[a(b + c) = a \times b + a \times c\]
\[a(b^2 + c^2) = a \times b^2 + a \times c^2\]
\[a(b - c) = a \times b - a \times c\]
\[a(b^2 - c^2) = a \times b^2 - a \times c^2\]

a. \((b^2 + c^3)d = \)

b. \((d^2 \times b^3) + (d^2 \times c^3) = \)

c. \(d \times (c + b^2) = \)

d. \(c \times (b + d^2) = \)

e. \((b^2 + d^2) \times c^3 = \)
4. Use the identity property of addition or multiplication to make the equations true.

**Example:**
\[ a + 0 = a \]  
\[ a \times 1 = a \]

a. \[ b \underline{\phantom{b}} = b \]  
   or  
   \[ b \underline{\phantom{b}} = b \]

b. \[ c^2 \underline{\phantom{c^2}} = c^2 \]  
   or  
   \[ c^2 \underline{\phantom{c^2}} = c^2 \]

c. \[ p^3 \underline{\phantom{p^3}} = p^3 \]  
   or  
   \[ p^3 \underline{\phantom{p^3}} = p^3 \]

d. \[ m^3p^2 \underline{\phantom{m^3p^2}} = m^3p^2 \]  
   or  
   \[ m^3p^2 \underline{\phantom{m^3p^2}} = m^3p^2 \]

e. \[ xx \underline{\phantom{xx}} = x^2 \]  
   or  
   \[ xx \underline{\phantom{xx}} = x^2 \]

### Problem solving

Use values \( a, b \) and \( c \) as well as the distributive property to write an equation and then solve it using the following: \( a = 2, b = 3, c = -1 \)
Addition and subtraction of fractions

Before starting this worksheet make sure you know what the following mean. Give an example of each.

- Factors
- HCF
- Multiples
- LCM
- Improper fraction to mixed number
- Mixed number to improper fraction
- To simplify a fraction

1. Show why these fractions are equivalent.

Example:
\[
\frac{3}{9} = \frac{1}{3}
\]
Factors of 3 = {1; 3}
Factors of 9 = {1; 3; 9}
HCF = 3
\[
\frac{3}{9} + \frac{3}{3} = \frac{1}{3}
\]
HCF stands for highest common factor.

a. \(\frac{4}{28} = \frac{1}{7}\)
b. \(\frac{24}{60} = \frac{2}{5}\)
c. \(\frac{25}{125} = \frac{1}{5}\)

2. Calculate and simplify fractions that are multiples of each other.

Example:
\[
\frac{1}{2} + \frac{3}{4} = \frac{1}{2} \times \frac{2}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4} = \frac{1}{4}
\]
Why did we multiply \(\frac{1}{2} \times \frac{2}{2}\)?

Can we add fractions with different denominators?

Yes, if we make the denominators the same.

a. \(\frac{2}{4} + \frac{7}{8} - \frac{1}{6} = \)
b. \(\frac{9}{10} - \frac{3-2}{5} + \frac{7}{8} = \)
c. \(\frac{2}{6} + \frac{5+1}{12} = \)
3. Calculate and simplify fractions that are not multiples of each other.

Example:

\[
\frac{2}{5} + \frac{3}{6} = \frac{11 + 3}{30} = \frac{14}{30} = \frac{7}{15}
\]

Multiples of 5 = \{5; 10; 15; 20; 25; 30; 35\}
Multiples of 6 = \{6; 12; 18; 24; 30; 36\}
LCM = 30
\[
\frac{11}{5} \times \frac{30}{30} + \frac{3}{6} \times \frac{30}{30} = \frac{11}{1} \times \frac{6}{30} + \frac{3}{1} \times \frac{5}{30} = \frac{66}{30} + \frac{15}{30} = \frac{21}{30} = \frac{7}{10}
\]

Problem solving

If the answer to a sum is \( \frac{7}{10} \), what could the sum be? Create some of your own word sums like this.

\[
d. \quad \frac{8}{10} + \frac{2}{6} - \frac{9}{12} = \]
\[
e. \quad \frac{13}{15} - \frac{8}{10} + \frac{1}{5} = \]
\[
f. \quad \frac{3}{4} - \frac{5}{6} + \frac{7}{8} = \]

\[
a. \quad 3 \frac{7}{10} - 1 \frac{8}{9} = \]
\[
b. \quad -2 \frac{2}{10} + 1 \frac{6}{7} = \]
\[
c. \quad 8 \frac{3}{4} - 6 \frac{5}{6} + \frac{1}{2} = \]

\[
d. \quad 5 \frac{4}{10} - 8 \frac{4}{5} = \]
\[
e. \quad 3 \frac{1}{2} + 2 \frac{3}{9} + \frac{3}{8} = \]
\[
f. \quad 9 \frac{7}{8} - 7 \frac{3}{7} = \]
Addition and subtraction of fractions that include squares, cubes, square roots and cube roots

Before starting this worksheet make sure you know what the following mean. Give an example of each.

Calculate a square number
Calculate a square root
Calculate a cube number
Calculate a cube root

1. Calculate the following fractions, using the example to guide you.

Example 1: \[ \frac{2^2}{2^2} + \frac{3^2}{4^2} = \frac{4}{8} + \frac{9}{16} = \frac{16}{16} + \frac{9}{16} = \frac{25}{16} = 1 \frac{9}{16} \]

Example 2: \[ \frac{1^3}{3^2} - \frac{2^3}{4^2} = \frac{1}{9} - \frac{8}{16} = \frac{16}{144} - \frac{72}{144} = \frac{56}{144} = \frac{7}{18} \]

LCM: \[3 \times 3 \times 2 \times 2 \times 2 = 144\]

HCF: \[2 \times 2 = 8\]

a. \[ \frac{8^2}{8^3} - \frac{10^2}{10^3} = \]
b. \[ \frac{2^2}{2^3} + \frac{7^2}{7^3} = \]
c. \[ \frac{4^2}{4^3} + \frac{4^2}{4^3} = \]
d. \[ \frac{5^2}{5^3} - \frac{3^2}{3^3} = \]
e. \[ \frac{1^2}{1^3} - \frac{9^2}{9^3} + \frac{11^2}{11^3} = \]
f. \[ \frac{4^2}{4^3} + \frac{15^2}{15^3} = \]

Look at example 2: Why is it important to understand LCM and HCF when we calculate fractions?
2. Calculate.

Example: \(\frac{\sqrt{9}}{\sqrt{16}} + \frac{\sqrt{8}}{\sqrt{27}}\)

\[= \frac{3}{4} + \frac{2}{3}\]

\[= \frac{9}{12} + \frac{8}{12}\] or \(\frac{9 + 8}{12}\)

\[= \frac{17}{12}\]

\[= 1 \frac{5}{12}\]

\(\text{a. } \frac{\sqrt{25}}{\sqrt{100}} + \frac{\sqrt{1331}}{\sqrt{144}} = \)

\(\text{b. } \frac{\sqrt{36}}{\sqrt{10000}} - \frac{\sqrt{64}}{\sqrt{25}} = \)

\(\text{c. } \frac{\sqrt{1}}{\sqrt{9}} + \frac{\sqrt{8}}{\sqrt{16}} = \)

\(\text{d. } \frac{\sqrt{1}}{\sqrt{10000}} - \frac{\sqrt{64}}{\sqrt{25}} = \)

\(\text{e. } \frac{\sqrt{1331}}{\sqrt{8}} + \frac{\sqrt{169}}{\sqrt{144}} = \)

\(\text{f. } \frac{\sqrt{81}}{\sqrt{10000}} - \frac{\sqrt{27}}{\sqrt{64}} = \)

Problem solving

Create your own word sums using cubes and cube roots.
What is the reciprocal of a number?

To get the reciprocal of a number divide 1 by the number.

The reciprocal of 2 is $\frac{1}{2}$, such as $3 \times \frac{1}{2} = 1$

If you multiply a number by its reciprocal you get 1.

... because $\frac{1}{0}$ is undefined.

Did you know that every number has a reciprocal except 0?

A reciprocal is also called the multiplicative inverse.

1. Calculate and simplify.

Example:

$6 \times \frac{1}{2} = 3$

a. $8 \times \frac{1}{2} =$

b. $9 \times \frac{1}{3} =$

c. $7 \times \frac{1}{14} =$

d. $5 \times \frac{2}{15} =$

e. $4 \times \frac{2}{12} =$

f. $9 \times \frac{1}{27} =$

2. Simplify.

You can simplify by finding the highest common factor (HCF) – if you cannot find the HCF straight away, keep on simplifying using smaller common factors.

Example:

$\frac{4}{8} \times \frac{7}{6} = \frac{28}{48}$

Simplify if needed:

$\frac{28}{48} \div \frac{4}{4} = \frac{7}{12}$

How did I know to simplify by dividing by 4?

Factors of 28 = {1; 2; 4; 7; 14; 28}

Factors of 48 = {1; 2; 4; 6; 8; 12; 16; 24; 48} or

Factorisation:

<table>
<thead>
<tr>
<th>Option 1</th>
<th>Option 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

48 = $2 \times 2 \times 2 \times 2 \times 3$

28 = $2 \times 2 \times 7$

HCF = $2 \times 2 = 4$
Multiplication of fractions

a. \( \frac{1}{6} \times \frac{2}{4} = \)

b. \( \frac{3}{4} \times \frac{2}{5} = \)

c. \( \frac{2}{7} \times \frac{1}{2} = \)


Example:

\[- \frac{8}{9} \times \frac{7}{10} = - \frac{8 \times 7}{9 \times 10} \]
\[- \frac{8}{9} \times \frac{7}{10} \quad \text{or} \quad - \frac{8 \times 7}{9 \times 10} \]
\[- \frac{8}{9} \times \frac{7}{10} = - \frac{56}{90} \]
\[- \frac{8}{9} \times \frac{7}{10} = - \frac{56}{90} \div \frac{2}{2} \]
\[- \frac{8}{9} \times \frac{7}{10} = - \frac{28}{45} \]

\[- \frac{8}{9} \times \frac{7}{10} = - \frac{56}{90} \div \frac{2}{2} \]
\[- \frac{8}{9} \times \frac{7}{10} = - \frac{28}{45} \]

a. \( \frac{2}{10} \times \frac{6}{8} = \)

b. \( \frac{2}{6} \times \frac{3}{7} = \)

c. \( \frac{4}{8} \times \frac{2}{2} = \)

continued
13b Multiplication of fractions continued

4. Simplify.

Example:
\[
\frac{12}{14} \times \frac{7}{8} = \frac{12 \times 7}{14 \times 8} = \frac{84}{112} = \frac{3}{4}
\]

\[
\frac{12}{14} \times \frac{8}{7} = \frac{12 \div 24}{112 \div 24} = \frac{3}{4}
\]

- a. \(\frac{3}{4} \times \frac{4}{7} = \)
- b. \(\frac{2}{9} \times \frac{3}{10} = \)
- c. \(\frac{4}{8} \times \frac{1}{6} = \)

5. Simplify and write your answers as mixed numbers (use a calculator if needed):

Example:

To convert mixed numbers to improper fractions:

\[
4 \frac{5}{6} \times 3 \frac{2}{3} = \frac{29}{6} \times \frac{11}{3}
\]

To change an improper fraction to a mixed number:

\[
\frac{319}{18} \text{ (ask how many times 18 goes into 319) } = 17 \frac{13}{18}
\]

Use a calculator if necessary.
Problem solving

A train has nine passenger wagons. Each passenger wagon has a seating capacity of 30. If these passenger wagons are replaced with wagons that have half the seating capacity, how many wagons will the train have to have to accommodate the same number of passengers?


Example:

\[-5 \frac{1}{2} \times \frac{4}{10} = -\frac{11}{2} \times \frac{4}{10} = -\frac{11 \times 4}{2 \times 10} = -\frac{44}{20} = -\frac{2}{5}\]

Do you still remember?

(positive number) × (positive number) = positive number
(positive number) × (negative number) = negative number
(negative number) × (negative number) = positive number

REVISION

a. \[-\frac{8}{9} \times -\frac{3}{4} = \]
b. \[-3 \frac{3}{8} \times \frac{1}{2} = \]
c. \[-\frac{1}{4} \times -\frac{1}{4} = \]
Revision: what does reciprocal mean?

<table>
<thead>
<tr>
<th>Number</th>
<th>Reciprocal</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1/8</td>
</tr>
</tbody>
</table>

Compare what happens if you divide and multiply $\frac{3}{4}$ and $\frac{1}{4}$.

Multiply $\frac{3}{4} \times \frac{1}{4}$

Divide $\frac{3}{4} \div \frac{1}{4}$

What do you notice?

1. Simplify.

Example:

$\frac{7}{9} + \frac{4}{12}$

$= \frac{7}{9} \times \frac{12}{4}$

$= \frac{28}{12}$

$= \frac{14}{6}$

$= 2 \frac{2}{6}$

$= 2 \frac{1}{3}$

How do I divide a fraction by another fraction?

- Turn the second fraction upside-down (this is its reciprocal).
- Multiply the first fraction by that reciprocal.
- Simplify the fraction if necessary.

a. $\frac{8}{10} \div 3 =$  
b. $\frac{2}{6} + \left( -\frac{8}{12} \right) =$  
c. $\frac{1}{4} \div 1 \frac{1}{12} =$
2. Simplify.

Example:
\[-\frac{1}{9} \div 3\frac{1}{10} = -\frac{1}{9} + \frac{31}{10} = -\frac{1}{9} \times \frac{10}{31} = -\frac{10}{279}\]

Is it possible to simplify this expression?

\[-9\frac{1}{3} \div 8\frac{3}{4} = \]

\[\text{a. } -3\frac{1}{16} \div 1\frac{1}{8} = \quad \text{b. } -7\frac{2}{5} \div 5\frac{1}{10} = \quad \text{c. } -9\frac{1}{3} \div (-8\frac{3}{4}) = \]


Example:
\[4\frac{1}{16} \div 2\frac{1}{4} = 4\frac{1}{16} \times \frac{4}{2} = 65 \cdot \frac{16}{8} = \frac{81}{8}\]

\[\text{a. } 2\frac{1}{4} \div 2 = \quad \text{b. } 4\frac{3}{4} \div 2\frac{2}{3} = \quad \text{c. } 7\frac{1}{4} \div 1 = \]

Problem solving

Ask one of your family members if they know how to divide fractions. If they don’t know or can’t remember, show them how to do it.
Percentages

1. Calculate the following:

a. What is 10% of R1 000?

b. What is 20% of R250?

c. What is 15% of R600?

What is 20% of R140?

\[
\begin{align*}
20\% \times R140 &= \frac{20}{100} \times R140 \\
&= \frac{20}{100} \times \frac{R140}{1} \\
&= \frac{R2800}{100} \\
&= R28
\end{align*}
\]
2. Complete the following:

Example: What percentage is R1,40 of R10,00?

\[
\frac{R1,40}{R10,00} \times 100\% = \frac{14}{100} = 14\%
\]

'a' of tells me it is a multiplication sum.

a. What percentage is R10,00 of R200,00?

b. What percentage is 20c of R1,95?
3. Calculate the percentage increases. Round off your answers to the nearest hundredth.

**Example:** Calculate the percentage increase in the price of petrol if it increases from R9,15 per litre to R9,50 per litre.

\[
\text{R9,50} - \text{R9,15} = \text{R0,35} \\
\frac{0,35}{9,15} \times 100\% \\
= \frac{35}{915} \times 100\% \\
= 3,83\% 
\]

**a.** Calculate the percentage increase in the price of a computer game if it increases from R450,00 to R699,00.

**b.** Calculate the percentage increase in the price of milk if it increases from R8,50 per litre to R9,25 per litre.
4. Calculate these percentage decreases. Round your answers off to the nearest hundredth.

Example: Calculate the percentage decrease in the price of maize if it decreases from R1 280 per ton to R1 275 per ton.

\[
\frac{5}{1 280} \times \frac{100}{1} \% = 0.39 \%
\]

a. Calculate the percentage decrease in the price of a laptop computer if it drops from R4 599 to R4 299.

b. Coffee goes on special at the supermarket. The price drops from R52.99 per tin to R38.99 per tin. What is the percentage decrease in price?

Problem solving

Find out what the last increase or decrease in petrol was. Calculate the percentage increase or decrease. Why do you think the price of petrol regularly increases or decreases?
1.  Write these fractions as percentages.

Example 1: \[
\frac{2}{5} \times \frac{20}{20} = \frac{40}{100} = 0.4 = 40\% 
\]

Example 2: \[
\frac{6}{8} \times \frac{125}{125} = \frac{750}{1000} = 0.75 = 75\% 
\]

We can multiply 5 by 20 to get 100, so you multiply the top (numerator) and bottom (denominator) by 20.

We can multiply 8 by 125 to get 1000, so you multiply the numerator (top) and denominator (bottom) by 125. Why did we make the denominator 1000 and not 100?

a. \( \frac{3}{4} \)  

b. \( \frac{2}{3} \)  

c. \( \frac{6}{7} \)  

d. \( \frac{1}{2} \)  

e. \( \frac{5}{7} \)  

f. \( \frac{1}{8} \)
Example 3:
There is another method for converting a fraction into a percentage. This is useful when the denominator cannot easily be multiplied by a number to get 100 or 1 000.

\[
\frac{5}{23} \times 100\% = \frac{5}{23} \times \frac{100}{1} = \frac{5}{23} \times 10 = \frac{500}{23} \approx 21.74\%
\]

Use a calculator for this.

g. \(\frac{4}{8}\)

h. \(\frac{5}{25}\)

i. \(\frac{15}{15}\)

j. \(\frac{18}{20}\)

k. \(\frac{3}{9}\)

l. \(\frac{4}{36}\)

2. Write as a percentage and as a common fraction, revision.

a. 0,6

b. 0,25

c. 0,75

d. 0,1

e. 0,530

f. 0,36

3. Write as a percentage and as a common fraction, revision.

a. 0,325

b. 0,205

c. 0,723

d. 0,825

e. 0,125

f. 0,065

Problem solving

Write 35.4\% as a common fraction and as a decimal fraction.
### Term 1

#### Addition, subtraction and rounding of decimal fractions

**1. Round off to the nearest unit, tenth and hundredth.**

**Example:**
Round off 5.9 to the nearest unit: 6  
Round off 5.91 to the nearest tenth: 5.9  
Round off 5.905 to the nearest hundredth: 5.905

<table>
<thead>
<tr>
<th>Decimal Fraction</th>
<th>Unit</th>
<th>Tenth</th>
<th>Hundredth</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 0.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. 0.123</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. 0.825</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. 0.795</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. 0.952</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f. 0.468</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Exercises:**

- What is 4.4 rounded off to the nearest unit?
- What is 2.76 rounded off to the nearest tenth?
- What is 8.469 rounded off to the nearest hundredth?
2. Calculate the following, using the expanded notation method and then the column method. Then test your answer. Round off your answer to the nearest unit, tenth and hundredth. (Use your own paper if necessary.)

Example: expanded notation method:
3,765 + 2,143
= 3 + 2 + 0.7 + 0.1 + 0.06 + 0.04 + 0.005 + 0.003
= 5 + 0.8 + 0.1 + 0.008
= 5.908

Column method: Test your answer:

\[
\begin{array}{c}
3.765 \\
+ 2.143 \\
\hline
5.908
\end{array}
\quad \begin{array}{c}
5.908 \\
- 2.143 \\
\hline
3.765
\end{array}
\]

3.765 rounded off to the nearest
Unit: 4
Tenth: 3.8
Hundredth: 3.77

a. \(2,354 + 7,265 =\)

<table>
<thead>
<tr>
<th>Expanded notation</th>
<th>Column method</th>
<th>Testing</th>
<th>Rounded off to the nearest:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Unit:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tenth:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hundredth:</td>
</tr>
</tbody>
</table>

b. \(2,686 + 1,325 =\)

<table>
<thead>
<tr>
<th>Expanded notation</th>
<th>Column method</th>
<th>Testing</th>
<th>Rounded off to the nearest:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Unit:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tenth:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hundredth:</td>
</tr>
</tbody>
</table>

c. \(8,940 – 2,355 =\)

<table>
<thead>
<tr>
<th>Expanded notation</th>
<th>Column method</th>
<th>Testing</th>
<th>Rounded off to the nearest:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Unit:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tenth:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hundredth:</td>
</tr>
</tbody>
</table>

d. \(6,725 – 4,025 =\)

<table>
<thead>
<tr>
<th>Expanded notation</th>
<th>Column method</th>
<th>Testing</th>
<th>Rounded off to the nearest:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Unit:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Tenth:</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Hundredth:</td>
</tr>
</tbody>
</table>

Problem solving

Why do we round off? Find ten examples in real life when we need to round off decimal fractions in daily life.
### Multiple operations with decimals

#### 1. Calculate the following:

**Example:**

\[(6 + 0.3) \times (7 + 0.5)\]

= \[6 + 0.3] \times 7 + [6 + 0.3] \times 0.5\]

= \[42 + 2.1 + 3.0 + 0.15\]

= \[47.25\]

- a. \((3.5 + 4.3) \times (1.2 - 0.9)\) =
- b. \(1.2 \times (1.3 + 8.6)\) =
- c. \((8.2 - 6.4) \times (5.8 - 6.2)\) =

**Example:**

\[7.3 \times 8.4\]

\[
8.4 \\
\times 7.3 \\
\hline
2.52 \\
+ 58.80 \\
\hline
61.32
\]

- a. \(6.2 \times 3.8\) =
- b. \(2.6 \times 4.9\) =
- c. \(9.5 \times 3.9\) =

### Problem solving

Choose one from questions 1, 2, 3, or 4. Write a word sum for each.

#### 3. Calculate the following:

**Example:**

\[
\begin{array}{c}
1.7 \\
\times 13.6 \\
\hline
8 \\
76 \\
\hline
24.76
\end{array}
\]

- a. \(7 \div 2.6\) =
- b. \(9 \div 29.7\) =
- c. \(6 \div 52.8\) =

#### 4. Calculate the following. Check your answer with a calculator.

**Example 1:**

\[
\begin{array}{c}
2.576 + 0.28 \\
\hline
2.856
\end{array}
\]

- a. \(1.715 + 0.35\) =
- b. \(2.756 + 0.32\) =
- c. \(3.150 + 0.24\) =

**Example 2:**

\[
\begin{array}{c}
3.150 + 0.24 \\
\hline
3.390
\end{array}
\]

- a. \(2.576 + 0.28\) =
- b. \(2.576 \times 10\) =
- c. \(2.576 + 7\) =
- d. \(368 + 4\) =
- e. \(92 + 10\) =
- f. \(9.2\) =

- a. \(1000 \div 5\) =
- b. \(100 \div 5\) =
- c. \(10 \div 5\) =
- d. \(0.1 \div 5\) =
- e. \(0.01 \div 5\) =
- f. \(0.001 \div 5\) =
Calculate squares, square roots, cubes and cube roots

Can you use a scientific calculator to calculate exponents such as 3³?

Press 3  What does ^ mean when you write exponents?  ^ means “raised to the power of”.
Press x³  Oh so 3 is the same as 3³.
Press 5
Press =

1. Estimate these squares and then calculate with a calculator.

Example: If 5² = 25 what is 5.5²?

Estimate
• 5² = 25 then 5.5² should be bigger than 25. Why?
• 6² = 36 then 5.5² should be smaller than 36. Why?

Calculate

<table>
<thead>
<tr>
<th>Calculator</th>
<th>Estimate</th>
<th>5.5²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Press 5.5</td>
<td>5, 5² = 5, 5 × 5, 5</td>
<td></td>
</tr>
<tr>
<td>Press x³</td>
<td>Use the distributive property of number.</td>
<td></td>
</tr>
<tr>
<td>Press 2</td>
<td>(5 + 0.5)² = 25 + 2.5 + 2.5 + 0.25</td>
<td></td>
</tr>
<tr>
<td>Press =</td>
<td>= 30.25</td>
<td></td>
</tr>
</tbody>
</table>

Example: If 4³ = 64 what is 4.5³?

Calculator

<table>
<thead>
<tr>
<th>Calculate</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Press 4.5</td>
<td>4³ = 64</td>
</tr>
<tr>
<td>Press x³</td>
<td>= (4 + 0.5)³</td>
</tr>
<tr>
<td>Press 3</td>
<td>= 64 + 3 + 0.75</td>
</tr>
<tr>
<td>Press =</td>
<td>= 90.125</td>
</tr>
</tbody>
</table>

Continued
4. Estimate these cube roots and then calculate with a calculator. Then show all the steps of your calculation.

Example: If \( \sqrt[3]{27} = 3 \) what is \( \sqrt[3]{50} \)

Estimate

<table>
<thead>
<tr>
<th>( \sqrt[3]{27} )</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt[3]{64} )</td>
<td>4</td>
</tr>
</tbody>
</table>

So \( \sqrt[3]{50} \) should be between 3 and 4.

Calculator

<table>
<thead>
<tr>
<th>Press</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt[3]{\phantom{0}} )</td>
<td>( \sqrt[3]{\phantom{0}} )</td>
</tr>
<tr>
<td>Press</td>
<td>3</td>
</tr>
<tr>
<td>Press</td>
<td>( \sqrt[3]{\phantom{0}} )</td>
</tr>
<tr>
<td>Press</td>
<td>68</td>
</tr>
</tbody>
</table>

\( = 3.68 \) (3.684031499)

a. If \( \sqrt[3]{64} = 4 \) what is \( \sqrt[3]{58} \)?

b. If \( \sqrt[3]{27} = 3 \) what is \( \sqrt[3]{70} \)?

c. If \( \sqrt[3]{16} = 4 \) what is \( \sqrt[3]{222} \)?

Problem solving

Give the steps you wrote down for question 1 a to c to a friend to go through and check.
20a Calculate more squares, square roots, cubes and cube roots

You need to know and revise the following:

How to calculate square roots using a calculator.
How to round off a decimal to the nearest unit, tenth or hundredth using the number lines below. Give an example of each.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Tenth</th>
<th>Hundredth</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>5.9</td>
<td>5.91</td>
</tr>
</tbody>
</table>

1. Calculate and round off to the nearest unit, tenth and hundredth.

Example: \( \sqrt{5} + \sqrt{72} \)

\[ \begin{array}{ccc}
\text{unit} & \text{tenth} & \text{hundredth} \\
6 & 5.9 & 5.91 \\
\end{array} \]

a. \( \sqrt{17} + \sqrt{24} = \)

b. \( \sqrt{65} + \sqrt{730} = \)

c. \( \sqrt{48} + \sqrt{430} = \)

2. Calculate and round off to the nearest unit, tenth and hundredth.

Example: \( 2.5^2 + 2.5^3 \)

\[ \begin{array}{ccc}
\text{unit} & \text{tenth} & \text{hundredth} \\
22 & 21.9 & 21.88 \\
\end{array} \]

a. \( 2.9^2 + 1.4^3 = \)

b. \( 1.3^3 + 11^2 = \)

c. \( 1.2^2 + 8^2 = \)

3. Calculate and round off to the nearest unit, tenth and hundredth.

Example: \( \sqrt{5} \times (\sqrt{172} + \sqrt{20}) \)

\[ \begin{array}{ccc}
\text{unit} & \text{tenth} & \text{hundredth} \\
10 & 10.4 & 10.39 \\
\end{array} \]

a. \( \sqrt{17} \times (\sqrt{3} + \sqrt{59}) = \)

b. \( \sqrt{78} - (\sqrt{500} - \sqrt{210}) = \)

c. \( \sqrt{74} - (\sqrt{70} - \sqrt{200}) = \)

Note that \( \sqrt{14} \sqrt{19} \) is the same as \( \sqrt{282} \sqrt{361} \)

4. Calculate and round off to the nearest unit, tenth and hundredth.

Example: \( 2.5^2 \times (2.5^3) \)

\[ \begin{array}{ccc}
\text{unit} & \text{tenth} & \text{hundredth} \\
98 & 97.7 & 97.66 \\
\end{array} \]

a. \( 3.5^2 \times (3.5) = \)

b. \( (1.9)^2 \times (1.9)^2 = \)

c. \( (11.2)^3 \times (11.2)^2 = \)

d. \( (6.7)^2 \times (6.7)^3 = \)

e. \( (4.8)^2 \times (4.8)^3 = \)

5. Calculate and round off to the nearest unit, tenth and hundredth.

Example: \( \sqrt{5} + (\sqrt{172} + \sqrt{20}) \)

\[ \begin{array}{ccc}
\text{unit} & \text{tenth} & \text{hundredth} \\
8 & 8.0 & 7.99 \\
\end{array} \]

a. \( \sqrt{79} - (\sqrt{17} + \sqrt{25}) = \)

b. \( \sqrt{78} - (\sqrt{500} - \sqrt{210}) = \)

c. \( \sqrt{74} - (\sqrt{70} - \sqrt{200}) = \)

continued
6. Calculate and round off to the nearest unit, tenth and hundredth.

Example: \(2.5^2 (1.5^2 + 1.2^2)\)

= \([2.5^2 \times 1.5^2] + [2.5^2 \times 1.2^2]\)

= \([6.25 \times 2.25] + [6.25 \times 1.44]\)

= 14.062 + 9

= 23.162

<table>
<thead>
<tr>
<th></th>
<th>unit</th>
<th>tenth</th>
<th>hundredth</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>3.2² ((11.6² + 7.8³))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>4.4³ ((2.8³ + 3.1³))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>8.1³ ((3.9² + 7.4³))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>11.2² ((4.2² + 5.6³))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>9.6² ((8.2³ + 10.3³))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Calculate and round off to the nearest unit, tenth and hundredth.

Example: \(\sqrt{[(\sqrt{72} + \sqrt{70})]}\)

= \([\sqrt{6} \times \sqrt{12}] + [\sqrt{6} \times \sqrt{20}]\)

= \([2.449 \times 3.464] + [2.449 \times 4.472]\)

= 8.483 + 10.952

= 19.435

<table>
<thead>
<tr>
<th></th>
<th>unit</th>
<th>tenth</th>
<th>hundredth</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(\sqrt{26} ((\sqrt{15} + \sqrt{29}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>(\sqrt{21} ((\sqrt{12} + \sqrt{16}))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>(\sqrt{26} ((\sqrt{1000} + \sqrt{13}))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Exponential form

You need to revise the following:

- **Can you remember what scientific notation is?**
  
  \[
  7.8425 = 7.8425 \times 10^3
  \]

  \[
  7.8425 = 7.8425 \times 1000 = 7.8425 \times 10^3
  \]

- **How do we write \(4.5 \times 10^9\)?**
  
  \[
  4.5 \times 1 = 4.5
  \]

1. **Revision: Compare the two numbers.**

   **Example:**
   
   \(-2)^2 = -2(-2) = 4

   \(-2)^2 = -2(-2) = -4

   a. \((-4)^2; (-4)^2\)
   b. \((-6)^2; (-6)^2\)
   c. \((-3)^2; (-3)^2\)

   d. \((-8)^2; (-8)^2\)
   e. \((-6)^2; (-6)^2\)
   f. \((-4)^2; (-4)^2\)

2. **Revision: Fill in <, > or =.**

   **Example:**
   
   \(-2)^2 > -2^2

   \(-2)^2 > -3^2

   \(-2)^2 = -2^2

   a. \((-10)^2\) \(\square\) \((-10)^2\)
   b. \((-6)^2\) \(\square\) \((-6)^2\)
   c. \((-9)^2\) \(\square\) \((-9)^2\)
   d. \((-8)^3\) \(\square\) \((-8)^3\)
   e. \((-6)^3\) \(\square\) \((-6)^3\)
   f. \((-4)^3\) \(\square\) \((-4)^3\)

3. **Convert an ordinary number to scientific notation, or scientific notation to an ordinary number.**

   **Example:**
   
   \(8740000 = 8.74 \times 10^6 = 8740000\)

   a. 256 000
   b. 790 000 000
   c. \(5 \times 10^{-6}\)

   d. \(8.1 \times 10^6\)
   e. 0.0000089
   f. \(3.12 \times 10^{-5}\)

   a. \(256 000\)
   b. \(790 000 000\)
   c. \(5 \times 10^{-6}\)

   d. \(8.1 \times 10^6\)
   e. 0.0000089
   f. \(3.12 \times 10^{-5}\)

4. **Fill in <, > or =**

   **Example:**
   
   \(4.32 \times 10^4\)

   \(4.32 \times 10^4 = 4.32 \times 10^4 = 43 200\)

   \(43 200 > 0.000432\)

   a. \(2.24 \times 10^4\) \(\square\) \(0.25 \times 10^{-4}\)
   b. \(2.5 \times 10^3\) \(\square\) \(2.5 \times 10^{-3}\)

   c. \(1.75 \times 10^{-6}\) \(\square\) \(1.75 \times 10^4\)
   d. \(1.95 \times 10^{-5}\) \(\square\) \(1.95 \times 10^4\)

   e. \(0.75 \times 10^{-5}\) \(\square\) \(0.75 \times 10^{-5}\)
   f. \(0.5 \times 10^3\) \(\square\) \(0.5 \times 10^{-3}\)

### Problem solving

**Calculate:** \(2^3 \times 2^2 = \)

Show all your calculations.
Laws of exponents: \( a^m \times a^n = a^{m+n} \)

Revise the laws of exponents and give four examples of each using numbers.

<table>
<thead>
<tr>
<th>( a^m \times a^n = a^{m+n} )</th>
<th>( \frac{a^m}{a^n} = a^{m-n} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Use the laws of exponents to simplify the following:

   Example: \( s^2 \times s^4 = s^{2+4} = s^6 \)

   a. \( a^3 \times a^4 = \)  

   b. \( b^2 \times b^3 = \)  

   c. \( 2^3 \times 2^2 = \)  

   d. \( f^3 \times f^2 = \)  

   e. \( d^3 \times d^6 = \)  

   f. \( y^3 \times y^4 = \)  

2. Calculate the following:

   Example: \( 8^5 \times 8^2 = 8^{5+2} = 8^7 = 32,768 \)

   a. \( 2^3 \times 2^1 = \)  

   b. \( 5^3 \times 5^1 = \)  

   c. \( 3^4 \times 3^2 = \)  

   d. \( 7^3 \times 7^1 = \)  

   e. \( 8^2 \times 8^1 = \)  

   f. \( 3^2 \times 3 = \)  

3. Use the laws of exponents to simplify the following:

   Example: \( y^a \times y^b = y^{a+b} \)

   a. \( a^m \times a^n = \)  

   b. \( d^e \times d^f = \)  

   c. \( v^a \times v^h = \)  

   d. \( e^i \times e^k = \)  

   e. \( x^i \times x^e = \)  

   f. \( b^p \times b^q = \)  

**Problem solving**

You need to explain to a friend who was absent from class how to do the following: multiply \( 5^4 \) by \( 5^i \) using a calculator. What will you say?
Laws of exponents: \(a^m \times a^n = a^{m+n}\)

Revise the laws of exponents and give four examples of each using variables (letters).

\[a^n \times a^m = a^{n+m}\]
\[\frac{a^n}{a^m} = a^{n-m}\]

1. Use the laws of exponents to simplify the following:

Example: \(m^5 \times m^3 = m^{5+3} = m^8\)

<table>
<thead>
<tr>
<th>a. (a^4 \times a^3 = )</th>
<th>b. (\frac{f^3}{f^2} = )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>c. (x^5 \times x^3 = )</td>
<td>d. (\frac{b^8}{b^5} = )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>e. (\frac{b^8}{b^4} = )</td>
<td>f. (h^7 \div h^3 = )</td>
</tr>
</tbody>
</table>

2. Calculate the following:

Example: \(2^4 \div 2^3 = 2^{4-3} = 4\)

<table>
<thead>
<tr>
<th>a. (\frac{2^4}{2^2} = )</th>
<th>b. (4^4 \div 4^2 = )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>c. (a^4 \div a^3 = )</td>
<td>d. (r^8 \div r^3 = )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>e. (c^8 \div c^2 = )</td>
<td>f. (j^7 \div j^3 = )</td>
</tr>
</tbody>
</table>

3. Write as a fraction and then use the laws of exponents to simplify the following:

Example: \(2^4 \div 2^3 = \frac{2^4}{2^3} = 2^{4-3} = 2^1 = 2\)

<table>
<thead>
<tr>
<th>a. (a^4 \div a^3 = )</th>
<th>b. (a^6 \div a^3 = )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>c. (x^4 \div x^3 = )</td>
<td>d. (r^4 \div r^3 = )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>e. (c^4 \div c^2 = )</td>
<td>f. (j^7 \div j^3 = )</td>
</tr>
</tbody>
</table>

4. Use the laws of exponents to simplify the following:

<table>
<thead>
<tr>
<th>a. (6^3 \times 6^3 = )</th>
<th>b. (4^2 \times 4^3 = )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>c. (2^4 \times 2^3 = )</td>
<td>d. (10^3 \times 10^2 = )</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>e. (4^3 \times 4^2 = )</td>
<td>f. (2^3 \times 2^4 = )</td>
</tr>
</tbody>
</table>

Problem solving

You need to explain to your friend who was absent how to do this: \(5^7 \times 5^3\) without using a calculator. How will you explain it?
Laws of exponents: \( a^m \div a^n = a^{m-n} \) if \( m > n \)

**Revise:** give an example using numbers and an example using variables.

\[
\begin{array}{|c|c|c|c|}
\hline
a^m \times a^n & \frac{a^m}{a^n} & \frac{a^m}{a^n} = a^{m-n} & (a^m)^n = a^{mn} \\
\hline
\end{array}
\]

1. Use the laws of exponents to calculate the following:

Example: \( \frac{w^4}{w^6} = \frac{w^4}{w^6} = \frac{1}{w^2} \) or using the laws of exponents: \( w^4 \div w^6 = w^{4-6} = w^{-2} \)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( x^2 + x^3 )</td>
<td>b.</td>
<td>( \frac{e^2}{e^3} )</td>
</tr>
<tr>
<td>d.</td>
<td>( a^3 + a^0 )</td>
<td>e.</td>
<td>( \frac{k^7}{k^2} )</td>
</tr>
</tbody>
</table>

2. Calculate the following:

Example: \( \frac{2^4}{2^2} = 2^4 \div 2^2 = 2^{4-2} = 2^2 = 4 \)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( \frac{2^3}{2^4} )</td>
<td>b.</td>
<td>( 5^{10} \div 5^{12} )</td>
</tr>
<tr>
<td>d.</td>
<td>( 10^3 \div 10^6 )</td>
<td>e.</td>
<td>( 11^9 + 11^{11} )</td>
</tr>
</tbody>
</table>

3. Use the laws of exponents to simplify the following:

Example: \( x^n + x^r = \frac{x^n}{x^r} = x^{n-r} \)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( a^n + a^r )</td>
<td>b.</td>
<td>( d^l + d^n )</td>
</tr>
</tbody>
</table>

4. Use the following laws of exponents to simplify the following:

Example: \( (a^n)^2 = (a^m)^2 = a^{2n} \)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( (m^n)^3 )</td>
<td>b.</td>
<td>( (k^r)^7 )</td>
</tr>
<tr>
<td>d.</td>
<td>( (r^p)^7 )</td>
<td>e.</td>
<td>( (s^e)^8 )</td>
</tr>
</tbody>
</table>

5. Calculate the following:

Example: \( (3^2)^3 = 3^6 = 729 \)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( (2^3)^3 )</td>
<td>b.</td>
<td>( (8^3)^3 )</td>
</tr>
<tr>
<td>d.</td>
<td>( (2^4)^7 )</td>
<td>e.</td>
<td>( (3^4)^3 )</td>
</tr>
</tbody>
</table>

6. Use the laws of exponents to simplify the following:

Example: \( (a^n)^m = a^{mn} \)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( (a^n)^m )</td>
<td>b.</td>
<td>( (d^l)^r )</td>
</tr>
<tr>
<td>d.</td>
<td>( (b^r)^p )</td>
<td>e.</td>
<td>( (c^n)^r )</td>
</tr>
</tbody>
</table>

**Problem solving**

a. What is the difference between \( x^3 \times x^2 \) and \( x^3 \times x^8 \)?
b. Solve: \( (12)^3 \)
### Laws of exponents:

- $a^0 = 1$ and $(a \times t)^n = a^m t^n$

#### Substitute numbers for the variables and exponents in each of these examples.

- $a^m \times a^n = a^{m+n}
- \frac{a^m}{a^n} = a^{m-n}
- \frac{a^m}{a^n} = a^{m-n}$ if $m < n
- (a^m)^n = a^{mn}
- (xy)^n = x^n y^n
- x^1 = x
- x^0 = 1
- a^n = \frac{1}{a^{-n}}$

#### 1. Simplify:

<table>
<thead>
<tr>
<th>a. $(b \times c)^3$</th>
<th>b. $(r \times s)^3$</th>
<th>c. $(r \times t)^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 2. Calculate the following:

Example: $(2 \times 5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100$

<table>
<thead>
<tr>
<th>a. $(2 \times 3)^2$</th>
<th>b. $(6 \times 7)^2$</th>
<th>c. $(2 \times 10)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 3. Use the laws of exponents to simplify the following:

<table>
<thead>
<tr>
<th>a. $(a \times c)^3$</th>
<th>b. $(y \times b)^3$</th>
<th>c. $(m \times p)^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $(z \times t)^3$</td>
<td>e. $(d \times f)^3$</td>
<td>f. $(q \times t)^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 4. Use the law of exponents to simplify the following:

Example: $a^2 = 1$ and $a^1 = a$

<table>
<thead>
<tr>
<th>a. $a^2$</th>
<th>b. $a^3$</th>
<th>c. $a^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $j^2$</td>
<td>e. $k^1$</td>
<td>f. $g^1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 5. Calculate the following:

Example: $12^2 = 1$ and $12^1 = 12$

<table>
<thead>
<tr>
<th>a. $4^2$</th>
<th>b. $3^1$</th>
<th>c. $10^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $5^1$</td>
<td>e. $8^0$</td>
<td>f. $11^1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 6. Use the law of exponents to simplify the following:

Example: $5^3 = \frac{1}{5}$

<table>
<thead>
<tr>
<th>a. $a^2$</th>
<th>b. $e^2$</th>
<th>c. $d^{-10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $x^{-3}$</td>
<td>e. $b^{-3}$</td>
<td>f. $g^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 7. Calculate the following:

Example: $3^3 = \frac{1}{5} = \frac{1}{125}$

<table>
<thead>
<tr>
<th>a. $3^{-2}$</th>
<th>b. $2^{-1}$</th>
<th>c. $7^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $2^{-5}$</td>
<td>e. $4^{-2}$</td>
<td>f. $3^{-1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### 8. Use the law of exponents to simplify the following:

Example: $a^{-3} = \frac{1}{a^3}$

<table>
<thead>
<tr>
<th>a. $a^3$</th>
<th>b. $d^{-1}$</th>
<th>c. $k^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. $n^{-2}$</td>
<td>e. $b^{-3}$</td>
<td>f. $r^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Problem solving**

Form a group of 4 to 6 friends and explain the laws of exponents to each other. Help each other.
Application of the law of exponents

Revise these laws.

\[
\begin{align*}
    a^n \times a^m &= a^{n+m} \\
    \frac{a^n}{a^m} &= a^{n-m} \\
    (a^n)^m &= a^{nm} \\
    (ab)^n &= a^n b^n \\
    \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \\
    a^{-n} &= \frac{1}{a^n} \\
    a^0 &= 1 \\
    a^1 &= a
\end{align*}
\]

Remember the sequence of operations

1. Use the laws of exponents to simplify the following:
   a. \((a^3 \times a^4) + (a^4 + a^3) = \)
   b. \(x^2 \times x^4 = \)
   c. \(y^3 \div y^3 = \)
   d. \(c^1 \times c^3 = \)
   e. \((e^5 \times e^8) = \)
   f. \((5^3 \times 5^4) \div 5^4 = \)

2. Use the laws of exponents to calculate the following:
   a. \(3^2 \times 3^1 = \)
   b. \(4^3 \div 2^3 = \)
   c. \(5^1 \times 5^3 = \)
   d. \(4^2 \div 2^2 = \)
   e. \((12^2 \times 5^{-3}) = \)
   f. \(7^5 \div 7^2 = \)

3. Use the laws of exponents to calculate the following:
   a. \(3a \times 9a^2 = \)
   b. \(14c \times 7c^5 = \)
   c. \(2c^3 \div 4c^3 = \)
   d. \(8d^4 \div 2c^3 = \)
   e. \(125x^3 \div 25x^1 = \)
   f. \(32d^3 \div 422d = \)

4. Revision: simplify.
   Example: \(2x^2 = 2 \times x^2 = 2 \times \frac{1}{x^2} = \frac{2}{x^2} = \)
   a. \(3y^2 = \)
   b. \(9x^{-3} = \)
   c. \(7x^3 = \)
   d. \(4y^3 = \)
   e. \(5x^{-2} = \)
   f. \(8x^{-3} = \)

5. Revision: simplify.
   Example: \(4^n = (4^n = 2 \times 2^n = (2^n = 2^n = \)
   a. \(64^n = \)
   b. \(16^n = \)
   c. \(100^n = \)
   d. \(121^n = \)
   e. \(4^n = \)
   f. \(144^n = \)
Application of the law of exponents

6. Revision: simplify.

Example: $9^2 = (3^2)^2 = 3^{2+2} = 3^4$

a. $16^3 = \quad$ b. $36^3 = \quad$ c. $121^2 = \quad$

d. $9^4 = \quad$ e. $25^5 = \quad$ f. $100^4 = \quad$

7. Simplify.

Example: $9^{2+1} = \frac{9^2}{4^1} = \frac{3^2}{2^1} = \frac{3^2}{2} = \frac{23^6}{2}$

a. $8^{2+1} = \quad$ b. $16^{3+1} = \quad$ c. $36^{4+2} = \quad$

d. $16^{1} = \quad$ e. $18^{1} = \quad$

8. Factorise.

Example: $12^2 = (12) = (2 \times 3)^2 = (2^2 \times 3^1)^2$

a. $20^4 = \quad$ b. $24^4 = \quad$ c. $54^4 = \quad$

d. $45^4 = \quad$ e. $18^4 = \quad$


Example: $9^{1+1.2} = \frac{9^1}{4^{1.2}}$

Try to get exponents with the same base.

Now we can simplify by multiplying the exponents with the same base. Use the laws of exponents to do this.

Now let us divide the exponents with the same base.

Problem solving

Write down all the laws of exponents that you used today.
Create your own sum using all these laws and solve it.
Sequences

Revision: What does each statement tell you? Give two more examples of each.

1. Describe the pattern by giving the rule and then extend it with three more terms.
   a. 2; 4; 6; 10
   b. 1; 5; 9; 13; 17
   c. –6; –8; –10; –12
   d. –30; –20; –10; 0; 10
   e. –1; 5; 11; 17
   f. 15; 12; 9; 6; 3

2. Describe the pattern by giving the rule and then extend it by three terms.
   a. 2; 4; 8; 16; 32; 64
   b. 5; –20; 80; –320; 1280
   c. 729; 81; 9; 1
   d. 25; 5; 1; 0.2; 0.04

3. Describe the pattern by giving the rule and then extend it by three terms.
   a. 2; 4; 12; 48; 240
   b. 1; 5; 13; 29; 61; 125
   c. 16; 19; 23; 28; 34
   d. 1; –5; 2; –6; 3; –7

4. Complete the table:

<table>
<thead>
<tr>
<th>Term</th>
<th>a.</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>–10</td>
<td>–8</td>
<td>–6</td>
<td>–4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>b.</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>–14</td>
<td>–12</td>
<td>–10</td>
<td>–8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>c.</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>10</th>
<th>12</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>12</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>–15</td>
<td>–12</td>
<td>–9</td>
<td>–6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Determine the 10th and nth position of the term using a table and number sentences.

<table>
<thead>
<tr>
<th>Term</th>
<th>a.</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>1</td>
<td>9</td>
<td>25</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>b.</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>1</td>
<td>4</td>
<td>16</td>
<td>64</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>c.</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>6</td>
<td>18</td>
<td>38</td>
<td>66</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>d.</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>27</td>
<td>64</td>
<td>125</td>
<td>216</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>e.</th>
<th>–5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>–5</td>
<td>0</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>–126</td>
<td>–1</td>
<td>124</td>
<td>3374</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term</th>
<th>f.</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>10</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position in sequence</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>10</td>
<td>n</td>
<td></td>
</tr>
<tr>
<td>Term</td>
<td>26</td>
<td>124</td>
<td>342</td>
<td>728</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem solving

Create your own sequences as follows:
- Constant difference between the consecutive terms
- Constant ratio between the consecutive terms
- Neither a constant difference nor a constant ratio
Geometric and numeric patterns

1. Create and complete the following geometric patterns.
   - Draw the first four terms in each of the following geometric patterns.
   - Write them in a table determining the 1st, 2nd, 3rd, 4th and n'th terms, where applicable.

   **Example: Square**
   - Position: 1st term, 2nd term, 3rd term, 4th term, n'th term
   - Value of the term: 1, 4, 9, 16, n²

<table>
<thead>
<tr>
<th>Position</th>
<th>1st term</th>
<th>2nd term</th>
<th>3rd term</th>
<th>4th term</th>
<th>n'th term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>n²</td>
</tr>
</tbody>
</table>

   **Example: Triangle**
   - Position: 1st term, 2nd term, 3rd term, 4th term, n'th term
   - Value of the term: 10

<table>
<thead>
<tr>
<th>Position</th>
<th>1st term</th>
<th>2nd term</th>
<th>3rd term</th>
<th>4th term</th>
<th>n'th term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **Example: Pentagon**
   - Position: 1st term, 2nd term, 3rd term, 4th term, n'th term
   - Value of the term: 22

<table>
<thead>
<tr>
<th>Position</th>
<th>1st term</th>
<th>2nd term</th>
<th>3rd term</th>
<th>4th term</th>
<th>n'th term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   **Example: Nonagon**
   - Position: 1st term, 2nd term, 3rd term, 4th term, n'th term
   - Value of the term: 24

<table>
<thead>
<tr>
<th>Position</th>
<th>1st term</th>
<th>2nd term</th>
<th>3rd term</th>
<th>4th term</th>
<th>n'th term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Use the rule to complete each table.

   **Example:** Rule is 2x + 1
   - Position: x, y
     | x | -2 | -1 | 0 | 1 | 2 | 5 | 10 |
     | y | -3 | -1 | 1 | 3 | 5 | 11 | 21 |

   a. Rule: y = 3x - 1
     - Position: x, y
     | x | -2 | -1 | 0 | 1 | 2 | 10 | 50 |
     | y |  -  |  2 |  4 |  6 |  8 | 14 | 22 |

   b. Rule: y = 1/2 x + 2
     - Position: x, y
     | x | 0  | 2  | 3  | 5  | 7  | 10 | 15 |
     | y |  2 |  4 |  6 |  8 | 10 | 12 | 14 |

   c. Rule: y = x - 5
     - Position: x, y
     | x | -3 | -2 | -1 | 0  | 1  | 13 | 25 |
     | y |  1 |  2 |  3 |  4 |  5 | 18 | 30 |

   d. Rule: y = 5x - 4
     - Position: x, y
     | x | 1  | 3  | 5  | 7  | 9  | 11 | 13 |
     | y |  6 | 12 | 18 | 24 | 30 | 36 | 42 |

3. Use the rule to complete each table.

   a. Rule: y = x - 2
     - Position: x, y
     | x | -2 | -1 | 0 | 1 | 2 | 5 |
     | y |  0 |  1 | 2 | 3 | 4 | 7 |

   b. Rule: y = 10 (x + 2)
     - Position: x, y
     | x | -3 | -5 | -7 | -9 | -11 | -13 | -15 |
     | y |  30 | 50 | 70 | 90 | 110 | 130 | 150 |

Problem solving

Make your own rule and give a table to a friend to solve.
Addition & subtraction of like terms

1. Revision: simplify.
   - Example: \(3a^2 + 4a^2 = 7a^2\)
   - a. \(8w^3 + 7w^3 = \)
   - b. \(5b^2 - 6b + 7b + 2b^3 = \)
   - c. \(4x^2 + 5x + 8 + 3x^2 + 6x + 4 = \)
   - d. \(4uv + 3uw^2 - 5uv + 4uv^2 = \)

2. Match column A with column B.
   - A | B
   - --- | ---
   - a. Monomial: \(3x^2 + 2x + 4x - 5\)
   - b. Binomial: \(3x^2 + 2x\)
   - c. Trinomial: \(3x^2\)
   - d. Polynomial: \(3x^2 + 2x + 5\)

3. Circle the following in each algebraic expression.
   - Example: A monomial: \(3ab + 4ab + 6b - 8\)
     - a. A binomial: \(8x^2 + 5xy + 2x + 7xy^2\)
     - b. A polynomial: \(5ab^2 + 6ab + 7a + 6ab^2\)
     - c. A trinomial: \(7a^2 + 8cd + 8ab^2 + 8cd\)
     - d. A monomial: \(9ef^3 + 4ef^2 + 5ef^3 + 5ef^3\)

4. Revision: simplify.
   - Example: \(3x^2 + 5x + 4 + 5x^2 - 2x - 1 = 8x^2 + 3x + 3\)
     - a. \(5x^2 + 3x + 4x^2 + 8x + 4 + 5 = \)
     - b. \(6a^2 + 8a + 5a^2 + 2 - 3 + 7a = \)
     - c. \(4b + 9b^2 + 7 - 5b + 6 - b^2 = \)
     - d. \(5x - 7x - 8x^2 - 2 - 3x^2 = \)
     - e. \(3 + 6a + 9a^2 + 2 + 3a^2 + 4a = \)

5. Simplify.
   - Example: \(a^3 + 4x + 5x^2 + 8 + 6 + 5x^3\)
     - a. \(7a^2 + 5x^2 + 4x + 14\)
     - b. \(4x^2 - 2x^3 + 7 - 4x^2 = \)

   - Example: \(4x^2 + 4x + 2x + 3y^2 + 5x\)
     - a. \(9x^2 + 3y^2 + 6x\)
     - b. \(8a^2 + 8a^2 - a^2 - 8b^2 - b^3 = \)

7. Simplify.
   - Example: \(3ab + 4ab + 2ab + ab + ab\)
     - a. \(5ab^2 + 6ab\)
     - b. \(5ab^4 + 7ab^3 - 9ab^2 + 6ab^4 - 3ab^2 = \)

8. Simplify.
   - Example: \(5ab^2 + 6ab + 2ab + 3ab + 6ab\)
     - a. \(7ab^2 + 9ab\)
     - b. \(4ab^4 + 5ab^2 + 7ab - 3ab^4 + 2ab^2 = \)

Problem solving

a. Create an algebraic expression with three different like terms and simplify it.

b. Write a polynomial with five terms, where two pairs are like terms. Simplify your answer.

c. If the answer is \(5x^2 + 7x^2 + 9x\) and the original sum had seven terms, what could the original sum be?

d. Write a polynomial with fifteen terms and then simplify it. Note that you should have like terms in your polynomial.
30a
The product of a monomial and binomial or trinomial

1. Revision: simplify.

Example: 2(3 + 4)

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3 + 4</td>
<td>2 × 3 + 2 × 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6 + 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>14</td>
</tr>
</tbody>
</table>

2. Revision: simplify.

Example: a(b + c)

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>b + c</td>
<td>a × b + a × c</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>ab + ac</td>
</tr>
</tbody>
</table>

3. Revision: simplify.

Example: 3(a + b)

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>a + b</td>
<td>3 × a + 3 × b</td>
</tr>
</tbody>
</table>

4. Revision: simplify.

Example: x(2 + 4)

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
<td>2 + 4</td>
<td>x × 2 + 4x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6x</td>
</tr>
</tbody>
</table>

5. Simplify.

Example: Method 1

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2x</td>
<td>(3x² - 4x + 5)</td>
<td>2x(3x² - 4x + 5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6x³ - 8x² + 10x</td>
</tr>
</tbody>
</table>

Method 2

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2x</td>
<td>(3x² - 4x + 5)</td>
<td>2x(3x² - 4x + 5)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6x³ - 8x² + 10x</td>
</tr>
</tbody>
</table>

6. Simplify using both methods.

Example: Method 1

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-x</td>
<td>(2x² + 3x + 2)</td>
<td>-(2x² + 3x + 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-2x² - 3x - 2</td>
</tr>
</tbody>
</table>

Method 2

<table>
<thead>
<tr>
<th>Term</th>
<th>Monomial</th>
<th>Binomial</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4x</td>
<td>(-3x² - 5x - 4)</td>
<td>-4x(-3x² - 5x - 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12x³ + 20x² + 16x</td>
</tr>
</tbody>
</table>

continued
### The product of a monomial & binomial or trinomial continued

<table>
<thead>
<tr>
<th>Term 1</th>
<th>30b</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. (-3x(-x^2 + 2x - 6)) =</td>
<td>d. (-2x(3x^2 + 7x + 1)) =</td>
</tr>
<tr>
<td>e. (-5x(2x^2 - 4x - 8)) =</td>
<td>f. (-6x(-3x^2 - 6x + 3)) =</td>
</tr>
</tbody>
</table>

7. If \(x = 3\), evaluate:

| a. \(4x^2 + 3x + 2\) = | b. \(5x^2 - 6x + 8\) = |
| c. \(4x(3x^2 - 2x - 2)\) = |

8. Simplify and then evaluate if \(x = -2\).

| a. \(2x(6x^2 + 3x + 5)\) = | b. \(-3x(2x^2 + 6x + 9)\) = |
| c. \(4x(3x^2 - 2x - 2)\) = |

### Problem solving

The \(a \times \text{[Times]}\) can be “distributed” across the 2 + 4 into an \(a \times 2\) plus an \(a \times 4\). What did the original sum look like?

Create your own monomial multiplied by a trinomial and simplify it.

Using the same monomial multiplied by a trinomial, simplify it through substitution.
The product of two binomials

Compare the following:

Did you know that your knowledge of map work can help you to calculate the product of two binomials? Use of the columns and rows to multiply two binomials.

| Term 1 |

1. Simplify the following:

Example: 
\[(x + 2)(x + 3)\]
\[= x^2 + 2x + 3x + 6\]
\[= x^2 + 5x + 6\]

Example:
\[(x - 2)(x - 2)\]
\[= x^2 - 2x - 2x + 4\]
\[= x^2 - 4x + 4\]

Example:
\[(x + 2)(x - 2)\]
\[= x^2 + 2x - 2x - 4\]
\[= x^2 - 4\]

a. 
\[(x + 1)(x + 2)\] 
b. 
\[(a + 2)(a + 7)\] 
c. 
\[(x + 5)(x + 4)\]

2. Simplify.

Example:
\[(x - 2)(x - 3)\]
\[= x^2 - 2x - 3x + 6\]
\[= x^2 - 5x + 6\]

Example:
\[(x + 2)(x - 3)\]
\[= x^2 + 2x - 3x - 6\]
\[= x^2 - x - 6\]

a. 
\[(x - 5)(x - 2)\] 
b. 
\[(a - 10)(a - 3)\] 
c. 
\[(x - 7)(x - 6)\]

3. Multiply:

Example:
\[(x + 2)(x - 3)\]
\[= x^2 + 2x - 3x - 6\]
\[= x^2 - x - 6\]

Example:
\[(x + 2)(x - 3)\]
\[= x^2 + 2x - 3x - 6\]
\[= x^2 - x - 6\]

a. 
\[(x + 1)(x - 4)\] 
b. 
\[(4a + 3)(a - 8)\] 
c. 
\[(2x + 3)(x - 2)\]
The product of two binomials


Example
\[(x - 2)(x + 3)\]
\[= x^2 + 3x - 2x - 6\]
\[= x^2 + x - 6\]

a. \((x - 3)(x + 4) = \)
b. \((2a - 3)(a + 1) = \)
c. \((x - 5)(x + 1) = \)

5. Multiply.

Example
\[(x ± 2)^2\]
\[= x^2 + 2x ± 2 + 2x ± 2\]
\[= x^2 + 4x + 4 \text{ and } x^2 - 4x + 4 \text{ or } x^2 ± 4x + 4\]

a. \((x ± 1)^2 = \)
b. \((a ± 6)^2 = \)


Example
\[2(x - 3)^2\]
\[= 2[(x - 3)(x - 3)]\]
\[= 2[x^2 - 3x - 3x + 9]\]
\[= 2[x^2 - 6x + 9]\]
\[= 2x^2 - 12x + 18\]

a. \(2(x + 2)^2 = \)
b. \(2(x + 7)^2 = \)

7. Problem solving: be creative

Create and solve two binomials multiplied. Use integers.

Create and solve two binomials multiplied. Use the +/- operation and coefficients.

Create and solve two binomials multiplied. Use the +/- operation.
More on the product of two binomials

Can you remember what a factor is?

Factors are numbers you multiply together to get another number.

Oh, yes the factors of 12 will be 1, 2, 3, 4, 6, and 12, since 1 x 12 = 2 x 6 = 3 x 4 = 12.

What are the factors of \( x^2 + 7x + 12 \)?

You should ask which two binomials, when multiplied together, will give you this trinomial.

- Write two brackets \( ( \quad ) \quad ) \).
- Factorise \( x^2 = (x)(x) \).
- Factorise 12 = (3)(4) and make sure that the sum of these two factors gives you 7.
- Fill in your operators \( (x + 3)(x + 4) \).

\[ \text{Example: } x^2 + 7x + 12 = (x + 3)(x + 4) \]

1. Factorise.

\[ \text{Example: } x^2 + 5x + 6 = x^2 + 5x + 6 = (x + 3)(x + 2) \]

Test: \( x^2 + 2x + 3x + 6 \)
\( x^2 + 5x + 6 \)

\[ \text{The product of the two factors gives me 6 but when added they give me 5.} \]

- a. \( x^2 + 5x + 6 = \)
- b. \( x^2 + 6x + 8 = \)
- c. \( x^2 + 9x + 14 = \)
- d. \( x^2 + 11x + 10 = \)
- e. \( x^2 + 15x + 54 = \)
- f. \( x^2 + 12x + 27 = \)

2. Factorise.

\[ \text{Example: } x^2 - 5x + 6 = x^2 - 5x + 6 = (x - 3)(x - 2) \]

\[ \text{The product of the two factors gives me 6 but when added they give me 5.} \]

- a. \( x^2 - x - 6 = \)
- b. \( x^2 + 3x - 54 = \)
- c. \( x^2 + 4x - 60 = \)
- d. \( x^2 + 5x - 14 = \)
- e. \( x^2 - x - 56 = \)
- f. \( x^2 + 7x - 8 = \)

3. Factorise.

\[ \text{Example: } x^2 - 7x + 12 = (x - 4)(x - 3) \]

\[ \text{The product of the two factors gives me 6 but when added they give me 5.} \]

- a. \( x^2 - 7x + 12 = \)
- b. \( x^2 - 13x + 42 = \)
- c. \( x^2 - 11x + 30 = \)
- d. \( x^2 - 9x + 20 = \)
- e. \( x^2 - 15x + 56 = \)
- f. \( x^2 - 8x + 15 = \)

Problem solving

Find the factors of \( x^2 + 11x + 24 \)
## Divide monomials and binomials

### Term 1

**Revise.**

<table>
<thead>
<tr>
<th>How fast can you simplify this?</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16/8 = 2</td>
<td>20/4 =</td>
<td>12/3 =</td>
<td>21/7 =</td>
</tr>
<tr>
<td>25/5 =</td>
<td>30/3 =</td>
<td>9/3 =</td>
<td>15/5 =</td>
</tr>
</tbody>
</table>

- Law of exponents
  - with variables
    \[ \frac{x^m}{x^n} = x^{m-n} \]
  - with constants
    \[ \frac{2^1}{2^2} = 2^{1-2} \]

**1. Simplify.**

**Example:** using laws of exponents

\[
\frac{8x^3}{2^2} = \frac{4x}{2^1} = \frac{3x^2}{3^1} = 3x^2
\]

- a. \( \frac{8x^3}{2^2} = \)
- b. \( \frac{16x^4}{8x} = \)
- c. \( \frac{12x^4}{3x} = \)
- d. \( \frac{20x^4}{4x} = \)
- e. \( \frac{18x^4}{9x} = \)
- f. \( \frac{21x^4}{7x^2} = \)

**2. Simplify.**

**Example:** using laws of exponents

\[
\frac{6x^3 + 8x^2}{2x} = \frac{3x^2 + 4x^2}{2x} = \frac{3x^2}{2x^2}
\]

- a. \( \frac{6x^3 + 9x^2}{3x} = \)
- b. \( \frac{16x^3 + 8x^2}{4x} = \)
- c. \( \frac{25x^2 - 15x^2}{3x} = \)
- d. \( \frac{24x^2 - 12x^2}{6x} = \)
- e. \( \frac{8x^2 + 10x^2}{2x} = \)
- f. \( \frac{30x^2 - 9x^4}{3x} = \)

**Problem solving**

Create five of your own examples of binomials divided by a monomial.
Substitution

Do you remember what substitution is? How can you use substitution to evaluate the following if \( x = -5 \)?

\[
x + 0.2 \quad x^2 \quad 5x \quad 2x^2 + x
\]

1. Revision: If \( x = 2 \), evaluate:

Example: \( x + 5 \)

\[
= 2 + 5 = 7
\]

Possible problems:

a. \( x + 9 = \)  

b. \( -x \times 2 = \)

2. If \( x = 2 \), evaluate:

Example: \( x^2 + 3x + 4 \)

\[
= 4 + 6 + 4 \\
= 14
\]

Possible problems:

a. \( x^2 + 6x + 5 = \)  

b. \( 2x^2 + 9x + 1 = \)  

c. \( x^2 + 9x + 6 = \)  

d. \( 5x^2 + 3x + 2 = \)  

e. \( 8x + x^2 - 5 = \)  

f. \( 8 - x^2 - 5x = \)

Why do these answers differ?

3. Evaluate the expression if \( x = -3 \), and if \( x = \frac{1}{3} \):

Example: \( -x^2 + 3x + 4 \)

\[
\begin{align*}
\text{If } x = -3, & \quad \text{then:} \\
& = [-(-3)]^2 + 3[-(3)] + 4 \\
& = 9 - 9 + 4 \\
& = 4 \\
\text{If } x = \frac{1}{3}, & \quad \text{then:} \\
& = \left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 4 \\
& = \frac{1}{9} + 1 + 4 \\
& = \frac{4}{9}
\end{align*}
\]

a. \( -x^2 + 4x + 2 = \)  

b. \( -x^2 + 5x + 3 = \)

c. \( 6 + 5x - 4x^2 = \)  

d. \( 7 + 2x^2 - 5x = \)

e. \( -2x^2 - x + 5 = \)  

f. \( -3 - 5x^2 + 5 = \)

Problem solving

If your answer is \(-15\), write down a possible trinomial.  
If your answer is \(15\), write down a possible trinomial.
**Factorise algebraic expressions**

How fast can you factorise the following?

Example: \(3 - 27\)

\[= 3(1 - 9)\]

\[4 + 16 = \]
\[5 - 25 = \]
\[6 + 42 = \]
\[7 + 56 = \]
\[9 + 99 = \]
\[48 - 6 = \]

1. Factorise.
   a. \(2x + 3a + 6b + 3x + b\)
   b. \(2a - b\)
   c. \(3a + b\)
   d. \(6a^2 - 4a^3\)
   e. \((2a + 3b)^2\)

2. Factorise.
   a. \((2a + b)(p - 3d) + (2a + b)(p + 3d)\)
   b. \((3x + y)(a + b) - (3x + y)(a - b)\)
   c. \((3a + b)(m) + (3a + b)(n)\)

Example 1: \(a - 4b + c\)

Example 2: \(4b - a + c\)

Example 3: \(3x^2 - 27\)

Example 4: \(ax - bx\)

Example 5: \(mx + nx\)

Example 6: \(ax + bx + 4a + 4b\)

Example 7: \(2m - 2n + 3m + 3n\)

Example 8: \(x - 2y\)

Example 9: \(2y - x\)

Example 10: \(3a^2 - 27\)

Example 11: \(5a^2 + 30\)

Example 12: \(6a^4 - 4a^2\)

Example 13: \((2a^2 - 36a + 3)\)

Example 14: \(2a^2 - (3a^2 - 4a + 3)\)

Example 15: \(2a^2 - (3a^2 - 2a^4)\)

Example 16: \(2a(3a^2 - 2)\)
90

4. Factorise.
Example: \((a + b)^2 = (a + b)(a + b)\)
   a. \((a + b)^3 = \)  
   b. \((x + y)^2 = \)  
Example: \((a + b)^2 - 5(a + b) = (a + b)(a + b - 5)\)
   c. \((x + y)^2 - 6(x + y) = \)  
   d. \((d + e)^3 - 2(d + e) = \)  

5. Factorise.
Example: \(25a^2 = [5a]^2\)
   a. \(16a^2 = \)  
   b. \(64a^2 = \)  
Note that: \(l = 3l = l + 1\)
Example: \(25a^2 - 1 = [5a]^2 - 1\)
   c. \(9a^2 - 1 = \)  
   d. \(49a^2 - 1 = \)  

6. Revision: use the example to guide your factorisation
Example: \(a^2 + b^2 = [a + b]^2\)
   a. \(x^2 + y^2 = \)  
   b. \(c^2 + d^2 = \)  
Example: \(a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)\)
   a. \(x^4 - y^4 = \)  
   b. \(c^4 - d^4 = \)  

7. Factorise.
Example: \(9(a + b)^2 - 1 = [3(a + b)]^2 - 1^2 = [3(a + b) + 1][3(a + b) - 1]\)
   a. \(64(x + y)^2 + 1 = \)  
   b. \(25(a + b)^2 - 1 = \)  

8. Simplify using factorisation.
Example: \(3x - 3y = 3(x - y)\)
   a. \(5x + 5y = \)  
   b. \(7a + 7b = \)  
Example: \(\frac{2x + 2y}{2x + 2y} = \frac{2(x + y)}{2(x + y)} = 1\)
   c. \(\frac{4x + 4y}{16x + 16y} = \)  
   d. \(\frac{5x - 5y}{10x + 10y} = \)  

Problem solving
Create an algebraic expression where the common expression is:
   a. \(4a + b\)  
   b. \([a^2 + y^2]\)  
   c. \([x + y]^2\)
Divide a trinomial and polynomial by a monomial

Give an example of each.

<table>
<thead>
<tr>
<th>trinomial</th>
<th>monomial</th>
<th>polynomial</th>
<th>monomial</th>
</tr>
</thead>
</table>

Sign: Date:

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Divide a trinomial and polynomial by a monomial

Give an example of each.

Write down a few keywords to help you remember how to:

1. Simplify the fractions using factorisation.

Example: \(3x - 3y = 3(x - y)\)

a. \(5x + 5y = \)  

b. \(7a + 7b = \)

c. \(4x + 4y = \frac{4x}{16x + 16y} \)  

d. \(5x - 5y = \frac{5x}{10x + 10y} \)

2. Simplify and factorise.

Example: Simplify: \(a + b + c \)  

Factorise: \(a + b + c \)

a. \(6x^2 - 63 = \frac{6x^2}{3x^2} \)

b. \(8x^{12} + 16x^6 = \)

c. \(9x^4 + 6x^2 + 3x = \)  

d. \(8x^4 - 4x^2 - 4x = \)

e. \(6x^3 + 4x^2 + 2x + 6 = \)

f. \(9x^4 + 6x^2 - 3x - 9 = \)

Example: Simplify: \( \frac{6x^4 - 2x^3 + 2x}{2x} \)  

Factorise: \( \frac{6x^4 - 2x^3 + 2x}{2x} \)

a. \( \frac{6x^4 - 2x^3 + 2x}{2x} = \frac{4x^2 - x}{2x} \)  

b. \( \frac{8x^2 + 16x}{4x} = \)

c. \( \frac{6x^4 - 2x^3 + 2x}{2x} = \frac{4x^2 - x}{2x} \)

Problem solving

Create a polynomial divided by a monomial. Simplify and factorise the expression.
Look at the three examples. Discuss.

1. Solve the linear equation.

   Example: \(4x = 2\)
   \[
   \frac{4x}{4} = \frac{2}{4} \\
   x = \frac{1}{2}
   \]

   a. \(6a = 3\)
   b. \(9b = 10\)

2. Solve for \(x\).

   Example: \(x = \frac{3}{x}\)
   \[
   7 \times \frac{x}{1} = \frac{3}{x} \times \frac{x}{1} \\
   7x = 3 \\
   x = \frac{3}{7}
   \]

   a. \(8 = \frac{4}{x}\)
   b. \(9 = \frac{3}{x}\)

3. Solve for \(x\).

   Example:
   \[
   \begin{align*}
   \frac{7}{x-2} & = \frac{3}{x} \\
   \frac{7}{x-2} \times \frac{x-2}{1} & = \frac{3}{x} \times \frac{x-2}{1} \\
   7 & = \frac{3}{x} \times \frac{x-2}{1} \quad \text{or} \quad 7 = \frac{3(x-2)}{x} \\
   7x & = 3(x-2) \\
   7x & = 3x - 6 \\
   7x - 3x & = 3x - 3x - 6 \\
   4x & = 6 \\
   x & = \frac{6}{4} \\
   x & = \frac{3}{2}
   \end{align*}
   \]

   a. \(\frac{8}{x+2} = \frac{4}{x}\)
   b. \(\frac{5}{x-3} = \frac{2}{x}\)

4. Solve linear equations containing fractions.

   Example:
   \[
   \frac{x}{3} = 1 \\
   \frac{x}{3} \times \frac{3}{1} = 1 \times \frac{3}{1} \\
   x = 3
   \]

   a. \(\frac{x}{2} = 1\)
   b. \(\frac{x}{5} = 1\)

A linear equation is an equation that makes a straight line when it is graphed. It has only one unknown and that is only to the power of 1.
Linear equations that contain fractions continued

Example: \( \frac{2x-1}{4} = 1 \)

\[ \frac{2x-1}{4} = \frac{4}{4} \]

\[ 2x - 1 = 4 \]

\[ 2x = 5 \]

\[ x = \frac{5}{2} \]

Example: \( \frac{x}{3} + \frac{x}{4} = 1 \)

\[ \frac{x}{3} \times \frac{4}{4} + \frac{x}{4} \times \frac{3}{3} = 1 \]

\[ \frac{4x}{12} + \frac{3x}{12} = 1 \]

\[ \frac{7x}{12} = 1 \]

\[ 7x = 12 \]

\[ x = \frac{12}{7} \]

c. \( \frac{3x+1}{5} = 1 \)

d. \( \frac{4x-2}{6} = 1 \)

e. \( \frac{x}{2} + \frac{x}{3} = 1 \)

f. \( \frac{x}{5} + \frac{x}{3} = 1 \)

g. \( \frac{x}{4} + \frac{2x+1}{2} = 1 \)

h. \( \frac{x}{5} + \frac{3x-2}{2} = 1 \)

Problem solving

Create algebraic equations that give you an answer of:

a. \( x = \frac{3}{4} \)

b. \( x = \frac{1}{2} \)

c. \( x = \frac{5}{2} \)
Solve equations of the form: a product of factors equals zero

Revise the following:

1. Factorise.
   Example: $x^2 + 5x + 6 = (x + 2)(x + 3)
   a. $x^2 - x - 6 =
   b. $x^2 + 9x + 14 =

2. Solve $x$.
   Example: $(x + 1)(x + 3) = 0$
   a. $(x + 2)(x + 3) = 0$
   b. $(x + 4)(x - 1) = 0$

3. Factorise.
   Example: $x^2 - 3x = x(x - 3)
   a. $x^2 + 2x =
   b. $x^2 + 5x =

4. Solve for $x$.
   Example: $x^2 - 3x = 0$
   a. $x^2 + 2x = 0$
   b. $x^2 - 6x = 0$

5. Factorise.
   Example: $x^2 - 25 = x^2 - 5^2$
   a. $x^2 - 36 =
   b. $x^2 - 16 =

6. Calculate the square root and use the example to show positive and negative numbers.
   Example: $\sqrt{25} = 5$ or $-5$
   a. $\sqrt{36} = 6$
   b. $\sqrt{16} = 4$ or $-4$

7. Solve for $x$.
   Example: $x^2 - 25 = 0$
   a. $x^2 - 49 = 0$
   b. $x^2 - 36 = 0$

Why is it so important to know how to factorise a polynomial?

Test:
   $(5)^2 - 25 = 0$
   $25 - 25 = 0$
   $0 = 0$

Problem solving

Create a sum where the product of factors equals zero and solve it.
39 Construct angles and polygons using a protractor

Term 2

Revise the following:

1. Construct the following with a protractor as a revision activity. Label the angles. Do this on separate piece of paper or exercise book.
   a. Obtuse angle  
   b. Acute angle  
   c. Reflex angle  
   d. Straight angle  
   e. Right angle  
   f. Revolution

2. Name all the main kinds of quadrilaterals and triangles. Label their angles.
   a. Quadrilaterals  
   b. Triangles

3. Draw the following angles and polygons. Label them.
   a. A 60° angle.  
   b. A 270° angle.  
   c. A triangle with one 45° angle and one 65° angle.  
   d. A triangle with an 80° and 35° angle.  
   e. A quadrilateral with one 70° angle and one 121° angle.  
   f. A quadrilateral with two 85° angles.

Problem solving

Construct the top view of a very modern house using a protractor.

Step 1: Draw a line segment. Label it AB.

Step 2: Place the protractor so that the origin (small hole) is over point A. Rotate the protractor so that the base line is exactly along the line AB.

Step 3: Using (in this case) the inner scale, find the angle desired - here it is 45°.

Step 4: Make a mark at this angle, and remove the protractor.

Step 5: With a ruler, draw a line from A to the mark you just made. Label this point C.

Step 6: The line drawn makes an angle BAC with a measure of 45°.
1. Draw a circle. Give an everyday example of a circle this size.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>with a radius of 1.8 cm.</td>
<td>b.</td>
</tr>
</tbody>
</table>

Everyday example:

2. Revision: Construct a perpendicular line to bisect a given line. Use the guidelines to help you.

- **Step 1**: Draw a line and mark points A and B on it. Put the compass point on A and open it so that the pencil touches point B. (So you have “measured” the length of AB with the pair of compasses.)
- **Step 2**: Leaving the compass point on A, draw an arc with the compass approximately two thirds of the line length.
- **Step 3**: With the compasses’ width the same, move the compass point to B and draw another arc which crosses the first arc at two points. Label these points C and D.
- **Step 4**: Draw a line through points C and D bisecting the line AB at E.

---

**Revision**

To draw a circle accurately, use a pair of compasses.

- Align the pencil lead with the compass point.
- Tighten the hold for the pencil so that it does not slip.
- Make sure that the hinge at the top of the compass is tightened so that it does not slip.

Set the compass to the radius of the circle. The radius is the distance between the centre and the circumference; it is half the diameter.

Press down the compass point and turn the knob at the top of the compass to draw a circle.
3. Revision: construct a 45° angle on a separate piece of paper.

**Step 1**
Follow the steps for drawing a perpendicular line.

**Step 2**
Leaving the compass point on C, draw an arc with the compass roughly halfway between C and B. Place it on B and draw an arc crossing the first one.

**Step 3**
Mark it as E and draw the line from D to E which creates two 45° angles.

4. Use your knowledge of how to construct a 45° angle to help you construct these angles.

- a. 22.5° angle
- b. 11.25° angle
- c. 135° angle
- d. 112.5° angle

**Problem solving**
Show in 4 steps how to draw a 225° angle.
Constructing triangles

Who constructs triangles in everyday life? Use some of the guidance below.

A triangle is a very strong structure. The triangle is used in structural designs to reinforce and support weight.

The right triangle is one of the most important geometrical figures and has been used in many applications for thousands of years.

1. Construct \( \triangle ABC \) in which \( AB = 5 \) cm, \( AC = 3.5 \) cm and \( BC = 4 \) cm. Follow the steps.

How to construct a triangle when three sides are given (SSS).

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>Draw ( AB = 5 ) cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2:</td>
<td>With ( A ) as centre and radius 3.5 cm, draw an arc.</td>
</tr>
<tr>
<td>Step 3:</td>
<td>With ( B ) as centre and radius 4 cm draw another arc intersecting the arc of ( C ).</td>
</tr>
<tr>
<td>Step 4:</td>
<td>Join ( AC ) and ( BC ).</td>
</tr>
</tbody>
</table>

Practise.

2. Construct \( \triangle CDE \) in which \( CD = 2.5 \) cm, \( DE = 4.2 \) cm and \( CE = 3.6 \) cm.

3. Construct \( \triangle ABC \) in which \( AB = 5.5 \) cm, \( BC = 4.5 \) cm and \( \angle ABC = 60^\circ \).

How to construct a triangle when two sides and the included angle is given (SAS).

<table>
<thead>
<tr>
<th>Step 1:</th>
<th>Draw ( AB = 5.5 ) cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2:</td>
<td>At ( B ), construct an angle ( ABX = 60^\circ ).</td>
</tr>
<tr>
<td>Step 3:</td>
<td>With ( B ) as centre and radius 4.5 cm draw an arc cutting ( BX ) at ( C ).</td>
</tr>
<tr>
<td>Step 4:</td>
<td>Join ( AC ). Then ( \triangle ABC ) is the required triangle.</td>
</tr>
</tbody>
</table>

Note: You may take \( BC = 4.5 \) cm as the base instead of \( AB \).

Practise.
4. Construct \( \triangle DEF \) in which \( DE = 3.7 \text{ cm}, \ EF = 41 \text{ mm} \) and \( \angle DEF = 55^\circ \).

5. Construct \( \triangle ABC \) in which \( \angle A = 60^\circ, \ \angle B = 45^\circ \) and \( AB = 4.5 \text{ cm} \).

How to construct a triangle when two angles and the included side are given (ASA).

- **Step 1**: Draw \( AB = 4.5 \text{ cm} \).
- **Step 2**: At \( A \), construct \( \angle BAX = 60^\circ \).
- **Step 3**: At \( B \), construct \( \angle ABY = 45^\circ \) with \( BY \) crossing \( AX \) at \( C \).

Then \( \triangle ABC \) is the required triangle.

Practise.

6. Construct \( \triangle KLM \) in which \( \angle K = 48^\circ, \ \angle L = 48^\circ \) and side \( KL = 3.9 \text{ cm} \).

7. Construct a right triangle \( \triangle ABC \), right-angled at \( B \), side \( BC = 4 \text{ cm} \) and hypotenuse \( AC = 6 \text{ cm} \).

How to construct a right triangle when its hypotenuse and a side are given.

- **Step 1**: Draw \( BC = 4 \text{ cm} \).
- **Step 2**: At \( B \), construct \( \angle CBP = 90^\circ \).
- **Step 3**: With \( C \) as centre and radius 6 cm draw an arc cutting \( BP \) at \( A \).
- **Step 4**: Join \( AC \).

Practise.

8. Construct a right triangle \( XYZ \), right-angled at \( Y \), side \( YZ = 5 \text{ cm} \) and hypotenuse \( XZ = 7 \text{ cm} \).

Practise.
Constructing quadrilaterals

How did the designers of these rooms use quadrilaterals?

1. Construct a rectangle ABCD in which AB = 4 cm and AC = 5 cm.

   **How to construct a rectangle when one of its diagonals and a side are given.**

   **Step 1:** Draw AB = 4 cm.
   **Step 2:** At B, draw $\angle ABQ = 90^\circ$.
   **Step 3:** With A as centre and radius 5 cm, draw an arc cutting BQ at C.
   **Step 4:** With C as centre and radius 4 cm, draw an arc.
   **Step 5:** With A as centre and radius = BC, draw an arc cutting the arc drawn in Step 4 at D.
   **Step 6:** Join DC and AD.

   Practise.

2. Construct a rectangle KLMN in which KL = 3.6 cm and KM = 4.5 cm.

3. Construct a square ABCD in which AB = 4.5 cm.

   **How to construct a square when its side is given.**

   **Step 1:** Draw AB = 4.5 cm.
   **Step 2:** At B, draw $\angle ABQ = 90^\circ$.
   **Step 3:** From BQ cut off BC = 4.5 cm.
   **Step 4:** From A and C, draw two arcs of radii 4.5 cm each to cut each other at D.
   **Step 5:** Join AD and CD.

   Practise.

continued
4. Construct a square GHIJ in which GH = 32 mm.

5. Construct a parallelogram in which the adjacent sides are 5 cm and 3 cm and the included angle is 60°.

How to construct a parallelogram when two adjacent sides and the included angle are given.

<table>
<thead>
<tr>
<th>Step 1: Draw AB = 5 cm.</th>
<th>Step 2: At A, construct ∠BAQ = 60°.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 3: From AQ cut off AD = 4 cm.</td>
<td>Step 4: With B and D as centres and radii equal to 4 cm and 5 cm respectively, draw two arcs cutting each other at C.</td>
</tr>
<tr>
<td>Step 5: Join CD and BC.</td>
<td></td>
</tr>
</tbody>
</table>

Practise.

6. Construct a parallelogram in which the adjacent sides are 6 cm and 3 cm and the included angle is 60°.

7. Construct a rhombus with one of its diagonals is 5 cm and the side is 3 cm.

How to construct a rhombus when one diagonal and side are given.

<table>
<thead>
<tr>
<th>Step 1: Draw AC = 5 cm.</th>
<th>Step 2: With A as centre and radius 3 cm, draw two arcs - one above AC and the other below AC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 3: With C as centre and radius 3 cm draw two arcs - one above AC and the other below AC intersecting the arcs of Step 2 in B and D respectively.</td>
<td>Step 4: Join AB, BC, CD and AD.</td>
</tr>
</tbody>
</table>

Practise.

8. Construct a rhombus, when one of its diagonals is 4 cm and the side is 3 cm.
1. Complete the table.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Total number of sides</th>
<th>Angle sizes</th>
<th>Total:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular triangle</td>
<td>3</td>
<td>60° + 60° + 60°</td>
<td>180°</td>
</tr>
<tr>
<td>Irregular triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular quadrilateral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregular quadrilateral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular pentagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregular pentagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular hexagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregular hexagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular heptagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregular heptagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular octagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregular octagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular nonagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregular nonagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Regular decagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregular decagon</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The size of the interior angles of regular polygons is a given. With irregular polygons you can give examples.

2. What is this? What polygon(s) can you identify? Describe the polygons.

3. Look at the giraffe. Identify all the regular and irregular polygons. Describe them.

4. What type of art is this? Identify all the geometric figures. Describe each.

Problem solving

Construct an irregular hexadecagon. Measure all the angles.
1. Construct a hexagon and label the vertices A to F.

a. Is this a regular or irregular hexagon? Why?

b. What is the length of the sides? How will you measure them?

c. What is the size of the angles? How will you determine it (i) without a protractor and (ii) with a protractor?

d. What is the distance from AD, FC, or BE? What is this of the circle?

e. What is the ratio between AD and AB?

2. Construct a regular hexagon with the sides equal to 3.2 cm.

Problem solving

Construct a dodecagon using a similar method to that used in this worksheet.
Constructing a pentagon

Step 1: Draw a circle around A with radius AB.

Step 2: Draw a circle around B with radius AB. Call their intersection points C and D.

Step 3: Draw a circle around D with radius DA. Circle D intersects line CD at E.

Step 4: Circle D intersects circle A at F and intersects circle B at G.

Step 5: Draw a line through FE and a line through GE. Line FE intersects circle B at H. Line GE intersects circle A at I.

Step 6: Set your pair of compasses to the length of AI. Place the compass point on I, and make a small arc above C. Then place your compass point on H and make an arc crossing the first one. Label the point of intersection J.

Step 7: All the points A, B, H, J and I are points of the pentagon. Join them.

Where will you find this pentagonal-shaped castle?

1. Construct a pentagon and label its vertices A, B, H, J and I

2. Answer the following:
   a. Complete the following: JH = _____ = _____ = _____ = _____
   b. Is the pentagon regular or irregular? Why?
   c. Describe AB, DA and DB.

3. Draw a regular pentagon with sides equal to 2.3 cm.

Problem solving

Write down step by step how you would construct a pentagon using a protractor.
Constructing an octagon

**Step 1:** Draw a circle with centre O and diameter AOB.

**Step 2:** Draw another diameter COD, perpendicular to AOB.

**Step 3:** Bisect the right angle AOC and the right angle BOD.

**Step 4:** Bisect the right angle COB and AOD.

**Step 5:** Join all the intersection points on the circumference of the circle with straight lines to make the octagon.

1. Now construct an octagon by yourself.

2. Complete the following:
   a. OA = _____ = _____ = _____ = _____ = _____ = _____
   b. What is this of the circle?
   c. AE = _____ = _____ = _____ = _____ = _____ = _____
   d. What is this of the circle?

3. Draw a regular octagon with equal radii of 2.8cm.

---

**Problem solving**

Write down step by step how you will construct an octagon using a protractor.

How are octagons used?

STOP

How are octagons used?

STOP
### Interior angles of a triangle

**Term 2**

Revise: There are special names for triangles according to:

<table>
<thead>
<tr>
<th>Sides</th>
<th>Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilateral</td>
<td>Acute: all angles are less than 90°.</td>
</tr>
<tr>
<td>Isosceles</td>
<td>Right: has a right angle (90°).</td>
</tr>
<tr>
<td>Scalene</td>
<td>Obtuse: has an angle more than 90°.</td>
</tr>
</tbody>
</table>

1. Read the following and label the triangle.

**To prove:** \( \angle A + \angle B + \angle C = 180° \)

**Construction:** through A, draw a line DAE parallel to BC.

**Proof:** Since DE is parallel to BC, and AB is a transversal

\[
\therefore \angle B = \angle DAB \text{ (pair of alternate angles)}
\]

Similarly \( \angle C = \angle EAC \text{ (pair of alternate angles)} \)

\[
\therefore \angle B + \angle C = \angle DAB + \angle EAC \text{ (two pairs of alternate angles)}
\]

Now \( \angle A + \angle DAB + \angle EAC = 180° \) (straight line)

and because \( \angle DAB + \angle EAC = \angle B + \angle C \)

\[
\therefore \angle A + \angle B + \angle C = 180°
\]

2. Prove that the sum of the three angles of a triangle is 180° for this triangle.

3. Calculate and construct these triangles. Classify the triangle.

Note that your answers and constructions could be different from those of your fellow classmates.

- a. \( \angle A + \angle B + 90° = 180° \)
- b. \( \angle A + 45° + \angle C = 180° \)
- c. \( 100° + \angle B + \angle C = 180° \)
- d. \( \angle A + 65° + \angle C = 180° \)
- e. \( \angle A + 60° + \angle C = 180° \)
- f. \( 120° + \angle B + \angle C = 180° \)

**Problem solving**

If one angle of a triangle equals 45°, what could the sizes of the other angles be? Give five different possibilities.
Triangles

1. Measure the sides of the triangles. Label the triangles and describe them.
   a. 
   b. 

2. What do these symbols mean?
   \( \Delta ABC \)
   \( AB = AC \)
   \( \angle ABC = \angle ACB \)

3. Draw an isosceles triangle and show how you would change it to be an equilateral triangle. Label your drawings with the appropriate geometric symbols.

4. What do these symbols mean?
   \( \Delta ABC \)
   \( AB \cong AC \cong BC \)
   \( \angle ABC \cong \angle ACB \cong \angle BAC \)

5. Write the symbols that show what has happened when you change a scalene triangle to a regular triangle.

6. a. Construct and label a right angle triangle.
   b. Change the irregular triangle you drew in 6a into a regular triangle.

7. Look at the diagram and complete the questions.
   a. What is this? ___________________________________________________________________

continued
b. Where will you find an example of this shape in nature? _______________________

c. What type of triangles are they? _______________________

d. Are these triangles regular or irregular? _______________________

e. Can I divide these triangles into smaller triangles? _______________________

f. What type of triangles will they form? _______________________

8. Look at the patterns in this stained glass window.

Do you see any triangles? Do you see any irregular geometric shapes?

9. Choose the correct answer and put a tick (✓) next to it:

   a. Which of the following could be the angles of a triangle?
      i. 65°, 45° and 80°
      ii. 90°, 30° and 61°
      iii. 60°, 60° and 59°
      iv. 60°, 60° and 60°

   b. The hypotenuse of a triangle is:
      i. The side opposite the right angle in a right-angled triangle.
      ii. The side next to the right angle in a right-angled triangle.
      iii. The angle of a right-angled triangle.
      iv. All three sides of a right-angled triangle.

   c. An equilateral triangle has:
      i. Two sides that are equal.
      ii. All the sides are equal but not the angles.
      iii. All the sides and the interior angles are equal.
      iv. All the angles are equal but not the sides.

   d. An isosceles triangle has:
      i. All the sides equal.
      ii. At least two sides that are equal and its base angles are equal.
      iii. At least two sides that are equal but no angles are equal.
      iv. Two angles that are equal but no sides are equal.

   e. A right-angled triangle has:
      i. No angles that are right angles.
      ii. All angles that are 60°.
      iii. Two angles that are 90°.
      iv. One angle that is a right angle.

Problem solving

Create your own stained glass pattern. You should use as many irregular triangles as you can.
Talk about this flow diagram.

**Triangles**
- Regular
  - Equilateral triangle
- Irregular
  - Scalene triangle
  - Isosceles triangle

**Quadrilaterals**
- Regular
  - Square
  - Rhombus
- Irregular
  - Rectangle
  - Parallelogram
  - Trapezium

1. Use the symbols and colours to answer the questions. Paste or draw everyday example pictures next to each or on a separate piece of paper.

   a. What is a quadrilateral? _______________________
       _______________________
       _______________________

   b. Describe a trapezium? _______________________
       _______________________
       _______________________

   c. Describe a rectangle? _______________________
       _______________________
       _______________________

   d. Describe a square? _______________________
       _______________________
       _______________________

   e. Describe a rhombus? _______________________
       _______________________
       _______________________

   f. Describe a kite? _______________________
       _______________________
       _______________________

   g. Describe a parallelogram _______________________
       _______________________
       _______________________

   In the United States of America they call it a trapezoid.

2. Construct a kite, label it and divide it into two triangles. Are these triangles regular or irregular?

3. Divide a trapezium into irregular triangles. Label it.

4. Identify and then name the regular and irregular polygons in this mosaic.

Make a mosaic (you can use old paper pieces) using different types of polygons.

Problem solving
More on polygons

What is a tangram?

The tangram is a dissection puzzle consisting of seven flat shapes, called tans, which are put together to form shapes. The objective of the puzzle is to form a specific shape using all seven pieces, which may not overlap. It was originally invented in China.

1. Make geometric shapes with all the pieces from the tangram from Cut-out 1. Draw a sketch of each in the appropriate answer block and say whether it is a regular or irregular shape. Label the shapes of its component parts. We have done the first one.
   a. Make a large square.

b. Make a rectangle.

c. Make a parallelogram.

d. Make a trapezium.

e. Make any other quadrilateral.

2. Complete the table:

<table>
<thead>
<tr>
<th>Geometric figure</th>
<th>What fraction of the square is it?</th>
<th>Name the shape</th>
<th>Is the shape regular or irregular</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. AOD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. ADB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. OGFI</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Look at the shapes below. What are the differences and similarities between the polygons?

4. Are the following shapes polygons? If they are, are they regular or irregular? Give reasons for your answers.

Problem solving
Create any other polygon using all seven tangram pieces. Draw and describe it.
Similar triangles

What is similarity?

Similar triangles have the following properties:
- Each corresponding pair of angles is equal.
- The ratio of any pair of corresponding sides is the same.
- They have the same shape but not the same size.

There are two rules to check for similar triangles. They are called the AA rule and RAR rule.

AA rule

If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

RAR rule

If the angle of one triangle is the same as the angle of another triangle and the sides containing these angles are in the same ratio, then the triangles are similar.

Solution:

Step 1: The triangles are similar because of the _____ rule.
Step 2: The ratios of the lengths are equal.
Step 3: Make use of cross multiplication to simplify.

Solution:

Step 1: The triangles are similar because of the ________ rule.
Step 2: The ratios of the lengths are equal.
Step 3: The length of a is _______.
2. Find the length of \( a \). State the rule you are using.

a. \[
\begin{array}{c}
4 \\
29^\circ \quad 40^\circ
\end{array}
\]

\[
\begin{array}{c}
8 \\
29^\circ \quad 40^\circ
\end{array}
\]

\[
\begin{array}{c}
a \\
29^\circ \quad 40^\circ
\end{array}
\]

b. \[
\begin{array}{c}
6 \\
18^\circ \quad 53^\circ
\end{array}
\]

\[
\begin{array}{c}
4 \\
18^\circ \quad 53^\circ
\end{array}
\]

\[
\begin{array}{c}
12 \\
18^\circ \quad 53^\circ
\end{array}
\]

\[
\begin{array}{c}
a \\
18^\circ \quad 53^\circ
\end{array}
\]

c. \[
\begin{array}{c}
6 \\
35^\circ \quad 12
\end{array}
\]

\[
\begin{array}{c}
8 \\
35^\circ
\end{array}
\]

d. \[
\begin{array}{c}
a \\
25^\circ \quad 32^\circ
\end{array}
\]

\[
\begin{array}{c}
2 \\
25^\circ \quad 32^\circ
\end{array}
\]

\[
\begin{array}{c}
12 \\
25^\circ \quad 32^\circ
\end{array}
\]

b. 3. Are these similar figures? Why or why not?

a.

b.

35°

3 a

35°

229°

2 cm

4 cm

4 cm

2 cm

5 cm

21 cm

12 cm

229°

12 cm

12 cm

6 cm

21 cm

15 cm

15 cm

Problem solving

Find two figures in everyday life that are similar. Construct it.
Congruent triangles are triangles that have the same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.

- The corresponding sides are: AB and DE, AC and DF and BC and EF
- The corresponding angles are: x and r, y and s, z and t.
- There are five rules to check for congruent triangles.
- These are the rules: SSS, SAS, ASA, AAS and RHS.

1. Discuss the following and draw examples:

**SSS rule (Side – Side – Side)**
If three sides of one triangle are equal to three sides of another triangle then the triangles are congruent.

- a. Draw congruent triangles using the SSS rule. Indicate the length of the sides of the triangles.

**SAS rule (Side – Angle – Side)**
If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the triangles are congruent.

- d. Draw congruent triangles using the AAS rule. Indicate the length of the sides of the triangles.

**ASA rule (Angle – Side – Angle)**
If two angles and the included side of one triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.

- b. Draw congruent triangles using the SAS rule. Indicate the length of the sides of the triangles.

**AAS rule (Angle – Angle – Side)**
If two angles and a non-included side of one triangle are equal to the corresponding two angles and a non-included side of another triangle, then the triangles are congruent.

- c. Draw congruent triangles using the ASA rule. Indicate the length of the sides of the triangles.
2. Which of the following conditions would be sufficient for these two triangles to be congruent? Give an explanation for each.

a. $a=d, x=r, b=e$

b. $a=d, y=s, z=t$

c. $c=f, y=t, b=e$

d. $a=e, y=t, z=s$

3. State whether the following pairs of triangles are congruent. If they are, give a reason for your answer using the SSS, ASA, SAS, SAA or RHS rules.

a.  

b.  

c.  

d.  

Problem solving

Find any congruent shapes in nature and make a drawing of them.
1. Name these symbols that you use when you work with angles and lines.

- \( \Delta \)
- \( \angle \)
- \( \perp \)
- \( \parallel \)
- \( \cong \)
- \( \sim \)
- \( \neq \)
- \( \bot \)
- \( \overline{AB} \)
- \( \overrightarrow{AB} \)
- \( \overleftrightarrow{AB} \)

a. Say why you will use these:

b. Say why you will not use these:

2. What helped us to draw this line?

Draw the following lines.

Give the coordinates for any other point on this line.

a. \((1,1)\) and \((3,3)\)

b. \((2,7)\) and \((5,5)\)

c. \((6,5)\) and \((7,6)\)

d. \((4,1)\) and \((7,3)\)

e. \((1,4)\) and \((3,4)\)

3. Use the graph to answer the questions.

a. Why is line \(x = 4\) a vertical line?

b. Show it on the graph.

c. Why is line \(y = 3\) a horizontal line?

d. Show it on the graph.

e. Where will these two lines be perpendicular to each other?

f. Draw a line parallel to the line in a. \((x = 4)\) and then one parallel to the line in c. \((y = 3)\). Describe it.

4. What is another name for a 180° angle?

5. Give a description of each of the following words: acute, obtuse, right and reflex. Where in everyday life do we find these angles. Which one is most commonly used?

Problem solving

Be creative and write a paragraph on what the world would be without lines and angles.
Complementary and supplementary angles

1. Draw the following angles and say if they are complementary or supplementary angles. Determine the size of the angle of unknown size.
   a. \( \angle 1 + 30^\circ = 90^\circ \)
   b. \( 48^\circ + \angle 2 = 180^\circ \)
   c. \( \angle 1 + \angle 2 = 90^\circ \)
   d. \( \angle 1 + 100^\circ = 180^\circ \)
   e. \( 36^\circ + \angle 2 = 90^\circ \)
   f. \( \angle 1 + \angle 2 = 180^\circ \)

2. Look at this picture of girders and identify and label the complementary and supplementary angles.

3. Draw five different pairs of complementary angles and label them.

4. Draw five different pairs of supplementary angles and label them.

5. Find any complementary and supplementary angles in your everyday environment. Draw and label them.

Problem solving
Can two obtuse angles be complementary? Can they be supplementary? Explain.
Transversals are straight lines that cut across other (usually parallel) straight lines. Why are many angles the same in this drawing of a transversal crossing two parallel lines?

**Parallel lines**
- **Transversal**
- **Vertically opposite angles:**
  - $a = d; \ b = c; \ e = h; \ f = g$
- **Corresponding angles:**
  - $a = e; \ f = t; \ c = g; \ d = h$
- **Alternate interior angles**
  - $c = f; \ d = e$
- **Alternate exterior angles**
  - $a = h; \ b = g$
- **Consecutive interior angles** (also known as co-interior angles)
  - $c + e = 180^\circ; \ d + f = 180^\circ$

1. Measure each angle.

<table>
<thead>
<tr>
<th>$\angle 1$</th>
<th>$\angle 2$</th>
<th>$\angle 3$</th>
<th>$\angle 4$</th>
<th>$\angle 5$</th>
<th>$\angle 6$</th>
<th>$\angle 7$</th>
<th>$\angle 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

- $\angle 1 + \angle 2 = \ldots$ and are called ________ angles.
- That will be the same for __________________________.

2. Identify and mark the vertically opposite angle.

3. Identify and mark the corresponding angle.

4. Find the co-interior angles. Write them down.

5. Why are angles 2 and 7 equal?
4. Identify and mark the alternate angle.
   a. 
   b. 
   c. 
   d. 

5. Identify all the angles that will be equal to the one shown.
   a. 
   b. 
   c. 
   d. 

6. How would you work out each angle, if angle 1 was given?

Problem solving

If $\angle 1 = 105^\circ$, what could the sizes of $\angle 2$ to $8$ be?
1. Use the knowledge learnt in previous worksheets to work out angles $\angle BCD$, $\angle CDB$, $\angle DBC$, $\angle ABD$, $\angle BDE$ and $\angle BAE$. You can work out the angles in any order you like. Triangle $BCD$ is an equiangular triangle. Angle $\angle AED$ is a right angle.

2. Make a similar ‘roller coaster problem’. Try to use all the concepts that you have learnt so far. Construct and draw or paste your picture here.

Concepts to be used when creating your problem:
- Parallel lines
- Transversal
- Vertically opposite angles
- Corresponding angles
- Alternate exterior angles
- Consecutive interior angles

Problem solving
Solve your own created problem (Question 2) with a family member.
### Application of geometric figures and lines

To be able to answer this worksheet you need to know the following concepts. Revise your knowledge of them by writing a definition for each.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Congruency</td>
<td></td>
</tr>
<tr>
<td>A line</td>
<td></td>
</tr>
<tr>
<td>Rotation</td>
<td></td>
</tr>
<tr>
<td>To plot</td>
<td></td>
</tr>
</tbody>
</table>

### Term 2

1. Complete the following

   ![Graph](image)

   - a. Plot (2,5), (6,5), (2,9) and (6,9) on the grid.
   - b. Join these points. What geometric figure does it form?
   - c. Label its vertices A, B, C, D.
   - d. Draw a line EF from (1,10) to (7,4).
   - e. What is your geometric figure now divided into?
   - f. What are the sizes of the angles?
   - g. Are the two figures congruent to each other and why?

2. Complete the following

   ![Graph](image)

   - a. Plot (1,9), (9,9) and (5,5). Join them up and label the vertices. What geometric figure does it form?
   - b. Plot (1,1). Can you form another geometric figure that is congruent to the shape in Question a, using the existing points?
   - c. Plot (5,1) and (3,3). Use those points to draw a figure similar to the ones in Questions a and b.
Problem solving

Discuss Question 3 with a family member.

3. In nature and art we often find congruent geometric figures. Identify such shapes in the pictures.

a. Write down in your own words what translation means.

b. Look at the photo of the butterfly. Identify the congruent shapes. What do you notice about the shapes if you compare them to the shapes on the snake?

c. Look at what the jewellery designer made. Identify all the congruent shapes. What type of transformations were made?

f. Plot (9,5). Draw a line from (7,3) to (9,5). How would you create a parallelogram?

e. Plot (7,7) and (7,3). Draw lines from (7,7) to (7,3), from (7,3) to (5,1) and from (7,3) to (5,5). What geometric figures do these form? Are these geometric figures congruent to any other shapes?
1. Revise the Pythagoras's theorem. About 2500 years ago, a man named Pythagoras discovered an amazing fact about triangles. Can you still remember it?

- What is the size of block A? (4²)
- What is the size of block B? (3²)
- What is the size of block C? (5²)
- What do you notice?

2. Here are the lengths of the sides of some right-angled triangles. Make drawings to show that the area of the square drawn on the longest side of each right-angled triangle is equal to the total area of the squares drawn on the other two sides. This will require some clever thinking. You will need extra paper.

<table>
<thead>
<tr>
<th>Side</th>
<th>Side</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>b.</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>c.</td>
<td>45</td>
<td>36</td>
</tr>
<tr>
<td>d.</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>e.</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

3. Write an equation for the following and calculate each side:

**Example**

```
4² + 3² = 5²
16 + 9 = 25
25 = 25
```

<table>
<thead>
<tr>
<th>Side</th>
<th>Side</th>
<th>Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>18</td>
<td>30</td>
</tr>
<tr>
<td>b.</td>
<td>130</td>
<td>78</td>
</tr>
</tbody>
</table>
4. Write an equation for each of the following:

Example
\[ a^2 + b^2 = c^2 \]

5. Find the lengths of the unknown sides in the following right-angled triangles. You may use a calculator.

Example
\[ x^2 = (3 \text{ cm})^2 + (4 \text{ cm})^2 \]
\[ x^2 = 9 \text{ cm}^2 + 16 \text{ cm}^2 \]
\[ x^2 = 25 \text{ cm}^2 \]
\[ x = \sqrt{25} \text{ cm} \]
\[ x = 5 \text{ cm} \]

Problem solving

a. Give two examples of where we can use Pythagoras in everyday life.

b. Themba walks as shown in the diagram. He moves 145 m north and 50 m west from his starting point. How far is Themba from his starting point?
1. Find the lengths of the diagonal of the rectangle.

Example

\[
x^2 = (3 \text{ cm})^2 + (4 \text{ cm})^2
\]

\[
x^2 = 9 \text{ cm}^2 + 16 \text{ cm}^2
\]

\[
x^2 = 25 \text{ cm}^2
\]

\[
x = 5 \text{ cm}
\]

Example

\[
x^2 = (5 \text{ cm})^2 + (8 \text{ cm})^2
\]

\[
x^2 = 25 \text{ cm}^2 + 64 \text{ cm}^2
\]

\[
x^2 = 89 \text{ cm}^2
\]

\[
x = 9.40 \text{ cm}
\]
3. Find the unknown side on each of these isosceles triangles.

a. 

b. 

Problem Solving

a. Lindie has put her ladder against the wall. How far up the wall does the ladder reach?

b. A triangular area is being tiled. The sides of the area are 8 cm, 12 cm and 18 cm. Is this a right-angled triangle? Explain your answer.
Perimeter of a square and rectangle, area of a square and rectangle

What do these formulae mean? Link it with the words on the right.

- \( P = 4S \) (perimeter of square)
- \( P = 2(l + w) \) or \( P = 2l + 2w \) (perimeter of rectangle)
- \( A = l^2 \) (area of square)
- \( A = l \times w \) (area of rectangle)

\( W = \) Width = Breadth = \( B \)

1. Complete the table. Give your answers in mm and cm.

<table>
<thead>
<tr>
<th>Figure</th>
<th>What formula will you use to calculate the:</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Formula:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
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<td>C</td>
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<td>D</td>
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<td></td>
</tr>
<tr>
<td>A</td>
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<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Construct and calculate the area and the perimeter of the following:

a. Rectangle ABCD where \( AB = 2.4 \text{ cm} \) and \( BC = 1.6 \text{ cm} \).

b. Square ABCD where \( AB = 3.9 \text{ cm} \).

c. Rectangle ABCD and square BEFC, where the rectangle and square share the same side BC, and side EF = 2.7 m and side AB = 4.1 m.

Problem solving

If the perimeter of a square is 24 cm, what is the length of each side?

The perimeter of a rectangular plot of land is 29.5 m. If the length is increased by 2 m and the breadth is reduced by 1 m, the area of the plot remains unchanged. Show if this is true or false.
Area of a triangle

Revise the formulas:

\[ A = \frac{1}{2} (b \times h) \]

Area of a triangle = \( \frac{1}{2} \) (base \times perpendicular height)

Every triangle has three bases (or sides), each with a related height or altitude. This height of a triangle is a line segment drawn from any vertex perpendicular to the opposite side.

1. What is the formula for calculating the area of a triangle?

2. Construct and then draw the following triangles and calculate the area by measuring the base and the perpendicular height of each triangle.
   a. An isosceles triangle.
   b. A right-angled triangle.

3. What is the area of a triangle that has a:
   a. Base of 4 cm and a height of 2.3 cm?
   b. Base of 2.8 cm and a height of 3.6 cm?
   c. Base of 34 mm and a height of 4.2 cm?

4. What is the length of the base of a triangle that has an area of 40 cm\(^2\) and a height of 4 cm?

5. Calculate the area:
   AB = 3.0 cm
   AG = 1.5 cm
   AG = ED
   CD = 2.0 cm

3. Problem solving
   If the area of a triangle is 5.635 cm\(^2\), what could the height be?
If the area of the trapezium is $39 \, \text{cm}^2$, what could the height be?

To find the area of a parallelogram, we can use a similar formula to that used for the area of a rectangle, multiplying the length of the base (length) by the perpendicular height.

To find the area of a trapezium of which the length of the parallel sides are $a$ units and $b$ units, and the perpendicular distance between them is $h$ units, use this formula:

$$A = \frac{1}{2} (a+b)h$$

1. What is the formula for calculating the:
   a. Area of a parallelogram.
2. Find the area of a trapezium of which the parallel sides are $10.5 \, \text{cm}$ and $8.2 \, \text{cm}$, and the perpendicular distance between the sides is $4 \, \text{cm}$.
3. Find the area of a parallelogram with base $6.4 \, \text{cm}$ and height $3.8 \, \text{cm}$.

Problem solving

If the area of the trapezium is $39 \, \text{cm}^2$, what could the height be?
1. What is the formula for calculating the:
   a. Area of a rhombus
   b. Area of a kite

2. Find the area of a rhombus with diagonals measuring 12.5 cm and 18.5 cm.

3. Find the area of this kite:
   a. Using the formula.
   b. Using the formula of a triangle.

Problem solving
If the area of the kite is 112 cm², what could the diagonals be?
Area of a circle

1. What is the formula for calculating the area of a circle? Test the formula.

2. Construct, label and calculate the area of circles with the following diameters:
   a. 14 cm
   b. 10.4 cm
   c. 78 cm

Problem solving

If the area of the circle is 154 cm², what will the radius be?

Area of a circle

The area of a circle is given by a formula:

\[ \text{Area} = \pi r^2 \]

where \( \pi \) is \( \approx 3.142 \) and \( r \) is the radius.

Note: The value of \( \pi \) is a decimal that goes on forever, but we usually use 3 decimal places: 3.142.

**Term 2**