



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 12**

**TECHNICAL MATHEMATICS P2**

**NOVEMBER 2019**

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 15 pages including and a 2-page information sheet.**

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

**QUESTION 1**

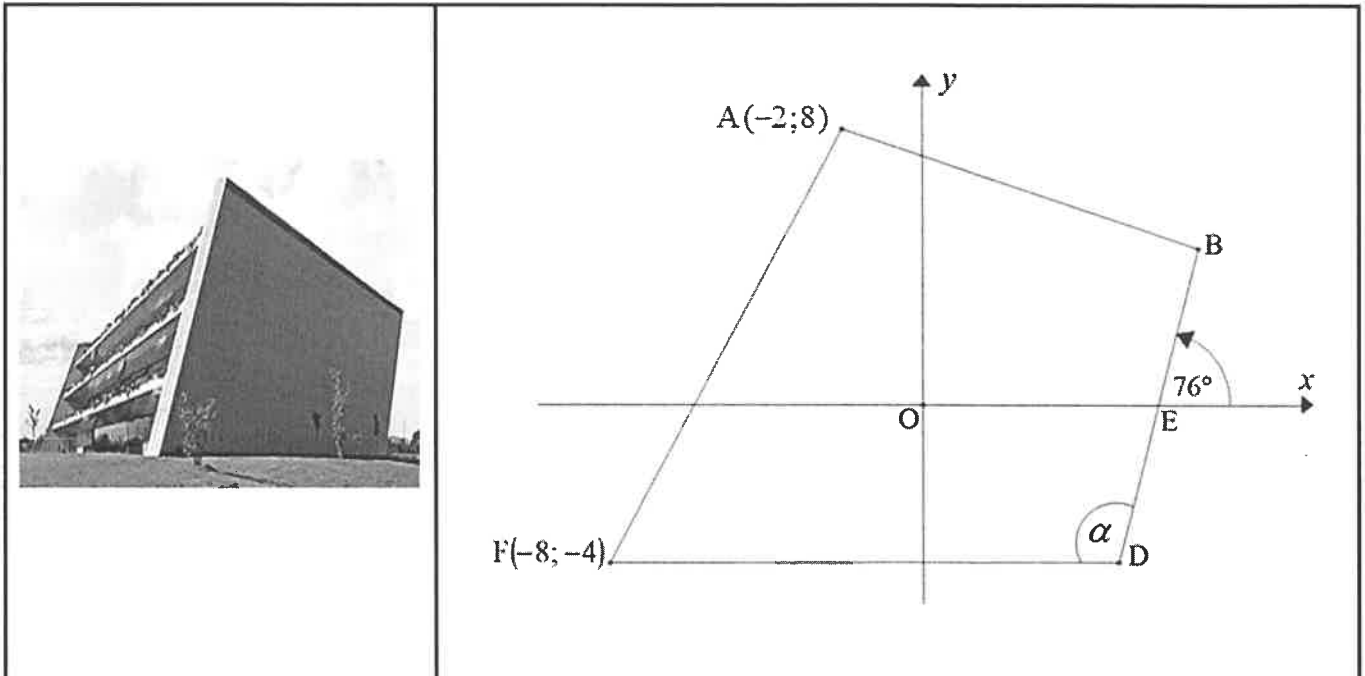
The building shown in the picture below, has sides in the form of quadrilaterals.

Quadrilateral  $ABDF$  models a side view of the building, in the Cartesian plane, with vertices  $A(-2; 8)$ ,  $B$ ,  $D$  and  $F(-8; -4)$ .

The acute angle formed by the  $x$ -axis and  $BD$  is  $76^\circ$  and  $\hat{D} = \alpha$

$FD$  is parallel to the  $x$ -axis.

(The diagram is NOT drawn to scale.)

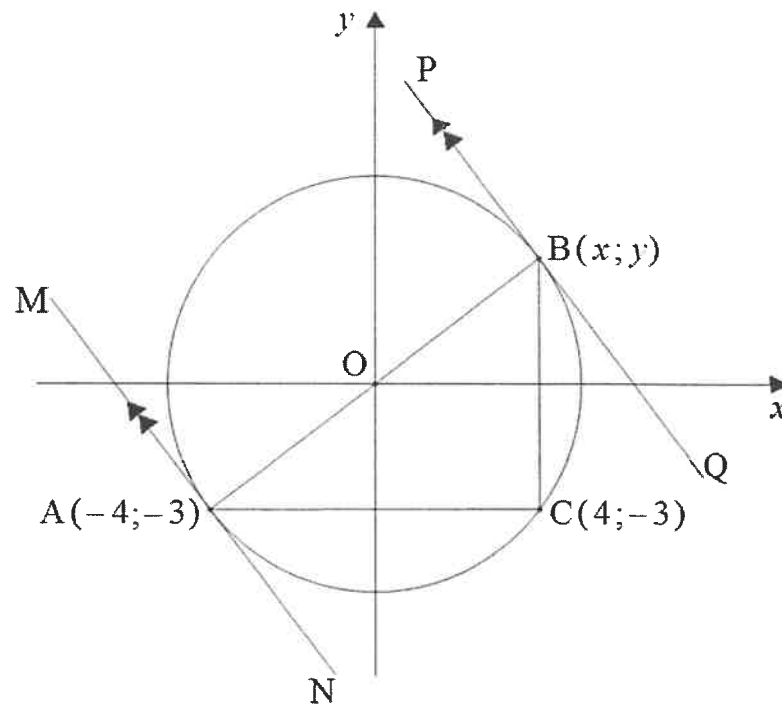


Determine:

- 1.1 The size of  $\alpha$  (1)
  - 1.2 The length of  $AF$  (leave answer in simplified surd form) (2)
  - 1.3 The gradient of  $BD$  (rounded off to the nearest integer) (2)
  - 1.4 The coordinates of the midpoint of  $AF$  (2)
  - 1.5 Hence, the equation of the perpendicular bisector of  $AF$  in the form  $y = \dots$  (5)
- [12]**

**QUESTION 2**

- 2.1 In the diagram below, O (0 ; 0) is the centre of circle ABC with A(-4; -3) and C(4; -3).  
Tangents PQ and MN touch the circle at B and A respectively.  
The equation of tangent MN is given by  $y = -\frac{4}{3}x - \frac{25}{3}$   
MN || PQ



- 2.1.1 Determine the equation of the circle. (2)
- 2.1.2 Write down:
- (a) The coordinates of B (1)
  - (b) The gradient of PQ (1)
- 2.1.3 Hence, determine the equation of tangent PQ in the form  $y = \dots$  (3)

2.2 Given the graph defined by the equation:

$$x^2 + 8y^2 - 32 = 0$$

- 2.2.1 Express the equation in the form:  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$   
Leave your answer in simplified surd form. (2)
- 2.2.2 Hence, sketch the graph on the set of axes provided. Clearly show ALL the intercepts with the axes. (3)

[12]

**QUESTION 3**

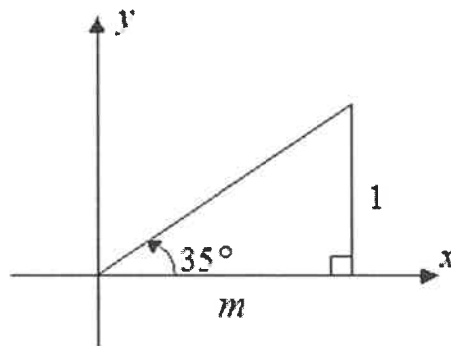
3.1 Given:  $\theta = 20^\circ$  and  $\alpha = 32^\circ$

Calculate the numerical value of:

3.1.1  $\sin 3\alpha$  (1)

3.1.2  $\frac{\sec^2 \theta - 1}{\tan \alpha}$  (2)

3.2 Given the sketch below:



Determine the following in terms of  $m$ :

3.2.1  $\sin 35^\circ$  (2)

3.2.2  $\left( \cos \frac{29}{36} \pi \right) \left( \tan \frac{7}{36} \pi \right)$  (5)

3.3 Given:  $2 \cos \theta + \sin \theta = 0$  for  $\theta \in [0^\circ; 360^\circ]$

3.3.1 Show that the above equation can be expressed as  $\tan \theta = -2$  (2)

3.3.2 Hence, or otherwise, determine the value(s) of  $\theta$ . (4)

[16]

**QUESTION 4**

4.1 Simplify:

4.1.1  $\cot^2 2\beta - \operatorname{cosec}^2 2\beta$  (1)

4.1.2  $\tan^2 A \cdot \operatorname{cosec}^2 A - \cos 2\pi$  (5)

4.2 Given the identity:

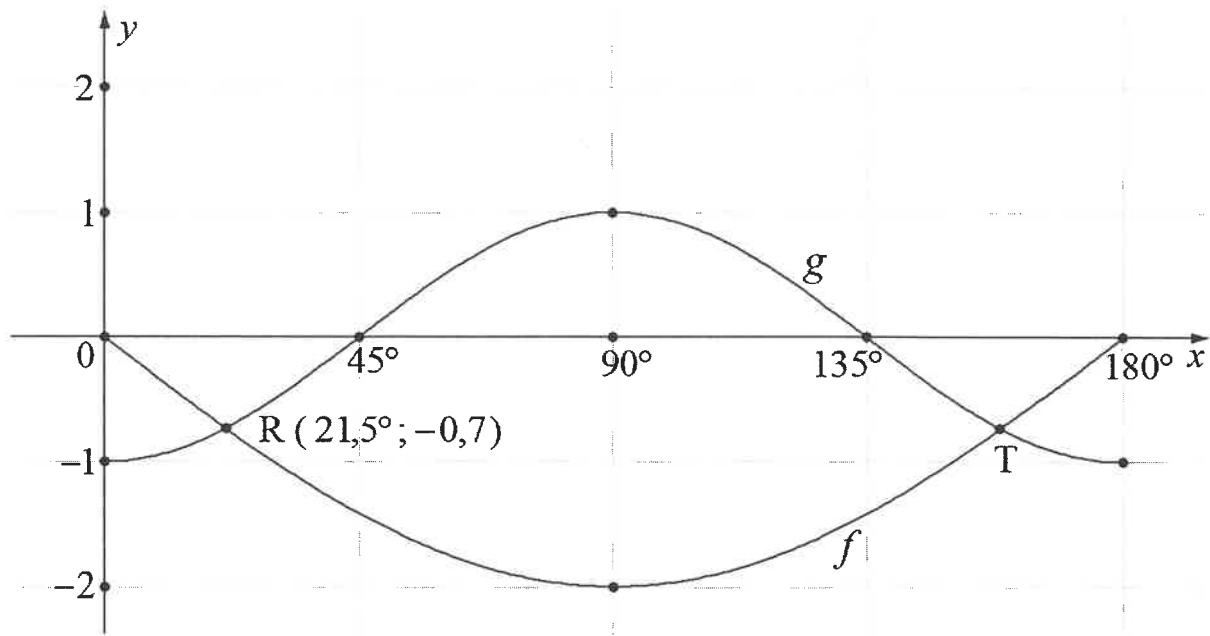
$$\operatorname{cosec}(180^\circ + \theta) \cdot \sin(360^\circ - \theta) - [\sin(180^\circ + \theta)]^{\sec 60^\circ} = \cos^2 \theta$$

4.2.1 Write down the numerical value of  $\sec 60^\circ$ . (1)4.2.2 Hence, prove the identity. (5)  
[12]

**QUESTION 5**

The graphs below represent the curves of functions  $f$  and  $g$  defined by  $f(x) = a \sin x$  and  $g(x) = -\cos bx$  respectively for  $x \in [0^\circ; 180^\circ]$ .

$R(21,5^\circ; -0,7)$  and  $T$  are the points of intersection of the curves of  $f$  and  $g$ .



Use the graphs above to answer the following questions.

- 5.1 Give the period of  $f$ . (1)
- 5.2 Determine the numerical values of  $a$  and  $b$ . (2)
- 5.3 Write down the coordinates of  $T$ . (2)
- 5.4 Determine the value(s) of  $x$  for which:
  - 5.4.1  $g(x) \cdot f(x) > 0$  for  $x \in [90^\circ; 180^\circ]$  (2)
  - 5.4.2  $\frac{f(x)}{g(x)}$  will be undefined (2)

[9]

**QUESTION 6**

In the diagram below, AB represents a vertical tower. Mpolokeng is standing at point C which is 150 m away from base B of the tower.

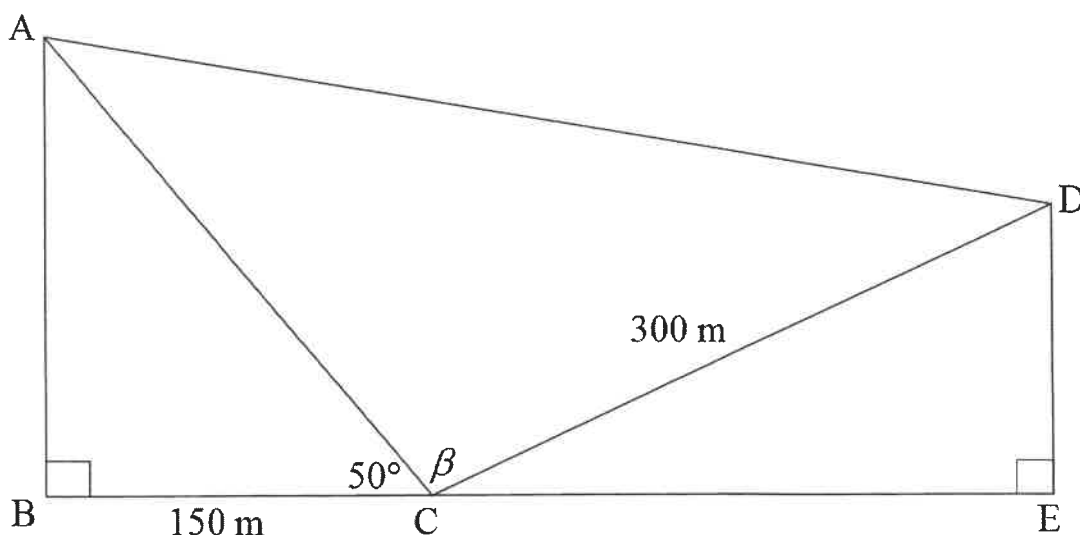
The angle of elevation of A from C is  $50^\circ$ .

Mpolokeng then walks 300 m up an inclined road to point D.

BC is produced to E such that  $BE \perp DE$ .

Points A, B, C, D and E lie in the same vertical plane.

$$\hat{ACD} = \beta > 90^\circ$$



- 6.1 Calculate the distance of AC. (3)
- 6.2 Hence, determine the size of  $\beta$ , if the area of  $\triangle ACD = 3,3648 \times 10^4 \text{ m}^2$ . (5)
- 6.3 Hence, determine the distance of AD. (4)

[12]



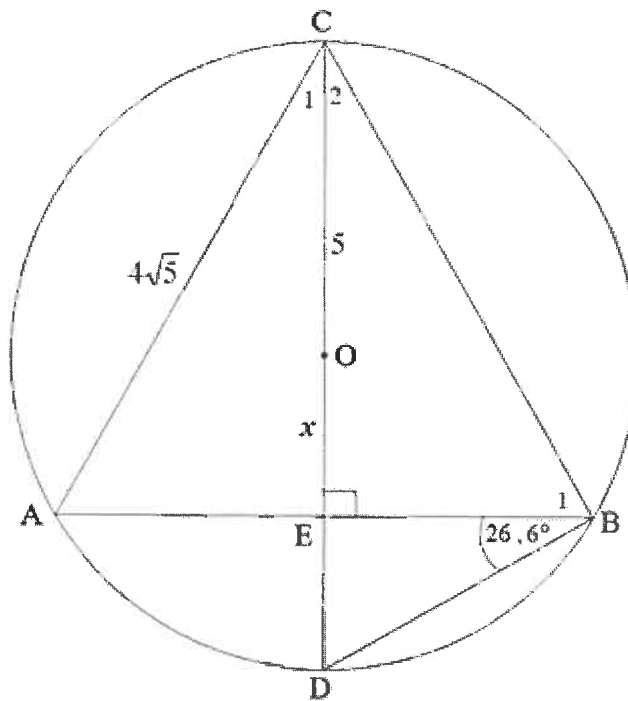
**QUESTION 7**

7.1 Complete the following theorem:

The line drawn from the centre of a circle to the midpoint of a chord is ... (1)

7.2 In the diagram below, O is the centre of circle CADB.  
CD is the diameter of the circle.  
 $CE \perp AB$  with E on AB.

$AB = 8$  units,  $OC = 5$  units,  $AC = 4\sqrt{5}$  units,  $EO = x$  units and  $\hat{DBE} = 26,6^\circ$



7.2.1 Determine, with reasons, the size of each of the following angles:

(a)  $\hat{C}_1$  (2)

(b)  $\hat{A}$  (1)

(c)  $\hat{B}_1$  (2)

7.2.2 Write down, without giving reasons, the length of:

(a) AE (1)

(b) ED in terms of x (1)

7.2.3 Hence, or otherwise, determine the numerical value of x. (4)

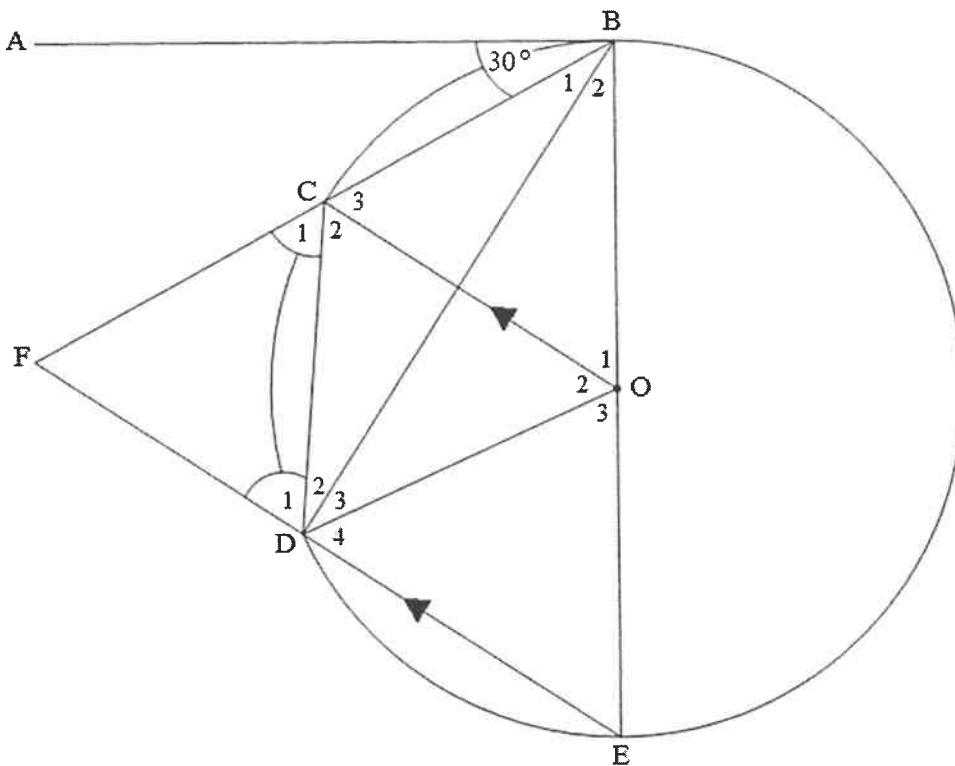
[12]

**QUESTION 8**

8.1 Complete the following theorem:

The angle subtended by the diameter at the circumference of the circle is ... (1)

8.2 In the diagram below, O is the centre of circle BCDE with diameter BE.  
 AB is a tangent to the circle at B.  
 BC and ED are produced to meet at F.  
 CO || FE  
 $\hat{A}BF = 30^\circ$



8.2.1 Determine, with reasons, the size of each of the following angles:

(a)  $\hat{CBO}$  (2)

(b)  $\hat{D}_2$  (2)

(c)  $\hat{O}_1$  (2)

(d)  $\hat{O}_2$  (3)

8.2.2 Show, with reasons, that  $FC = FD$  (4)

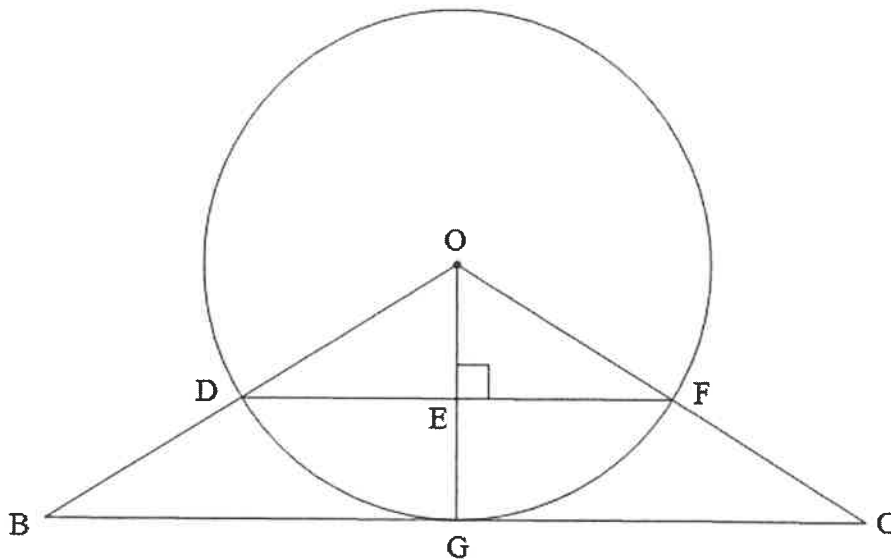
[14]

**QUESTION 9**

9.1 Complete the following theorem:

If a line divides two sides of a triangle in the same proportion, then the line is ... (1)

9.2 The diagram below shows circle DGF with centre O having a radius of 6 units. OD and OF are produced to B and C respectively. BC is a tangent to the circle at G. OG and DF intersect at E such that  $OG \perp DF$ .  $OD : OB = 3 : 5$



9.2.1 Show, with reasons, that  $DF \parallel BC$ . (3)

9.2.2 Determine:

(a) The ratio of  $BC : DF$  (2)

(b) The length of  $EG$  (3)

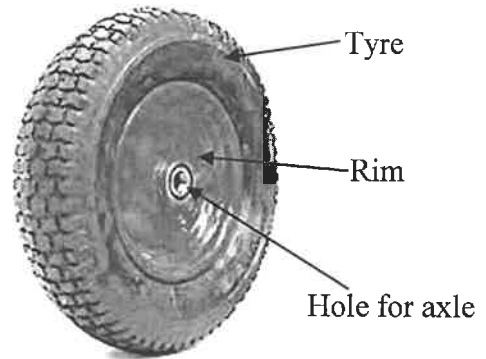
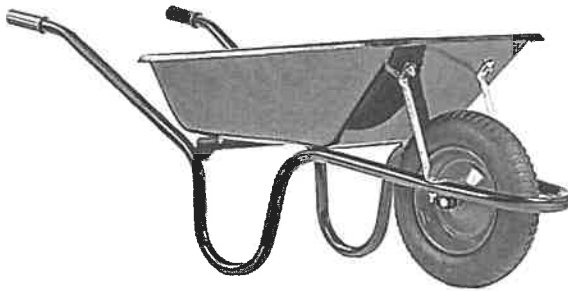
(c) The numerical value of  $\frac{\text{Area } \triangle OBG}{\text{Area } \triangle ODE}$  (3)

9.3 Show, with reasons, that  $\triangle DOE \parallel \triangle BOG$ . (3)  
**[15]**

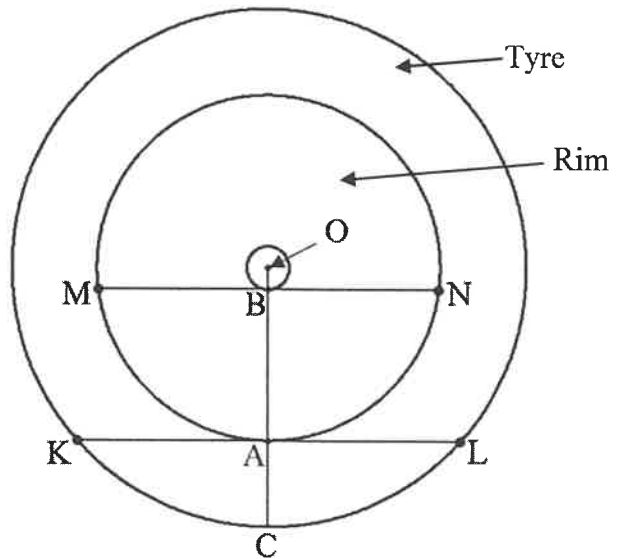
**QUESTION 10**

10.1

The pictures below show a wheelbarrow and an enlargement of the wheel of the wheelbarrow. The wheel consists of a tyre and a rim with a circular hole in the centre of the rim for the axle.



The diagram alongside shows the wheel of the wheelbarrow.  $O$  is the common centre of the hole, the rim and the tyre. Chord  $MN$  of the rim is also a tangent to the hole at point  $B$ . Chord  $KL$  of the tyre is the tangent of the rim at point  $A$ .  $OBAC$  is a straight line with  $C$  a point on the outer circle.

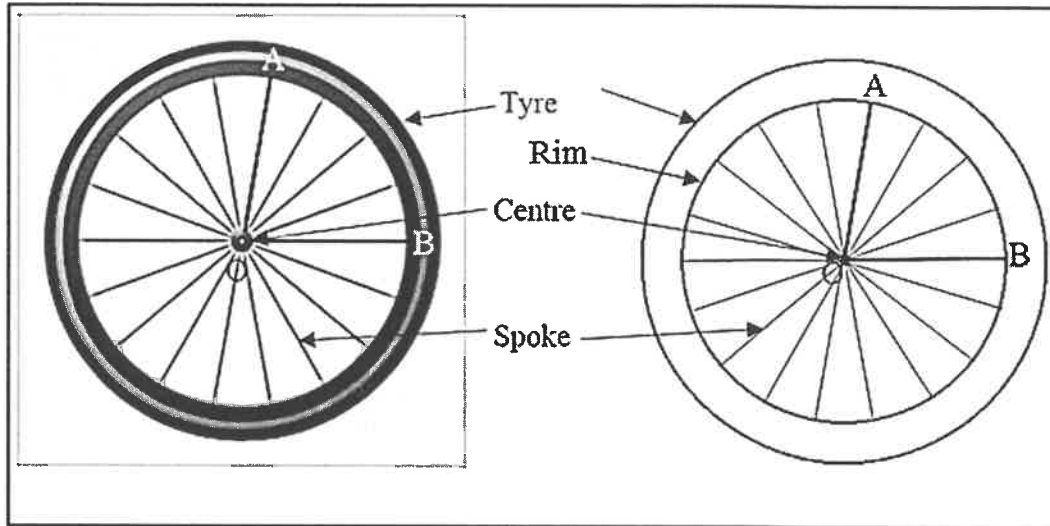


The diameter of the wheel is 40 cm.  
The angular velocity of the rotating wheel, when the wheelbarrow is pushed, is  $64\pi$  revolutions per minute.  
The length of the radius of the hole is 1,5 cm.

- 10.1.1 Give the length of  $BC$ . (1)
- 10.1.2 Hence, determine the length of  $AB$  if the length of chord  $KL$  is 32 cm. (5)
- 10.1.3 Calculate the rotational frequency ( $n$ ) of the rotating wheel. (3)
- 10.1.4 Hence, determine the circumferential velocity of the rotating wheel. (4)

10.2 The picture below shows a wheel consisting of a rubber tyre and a rim with equally spaced spokes.

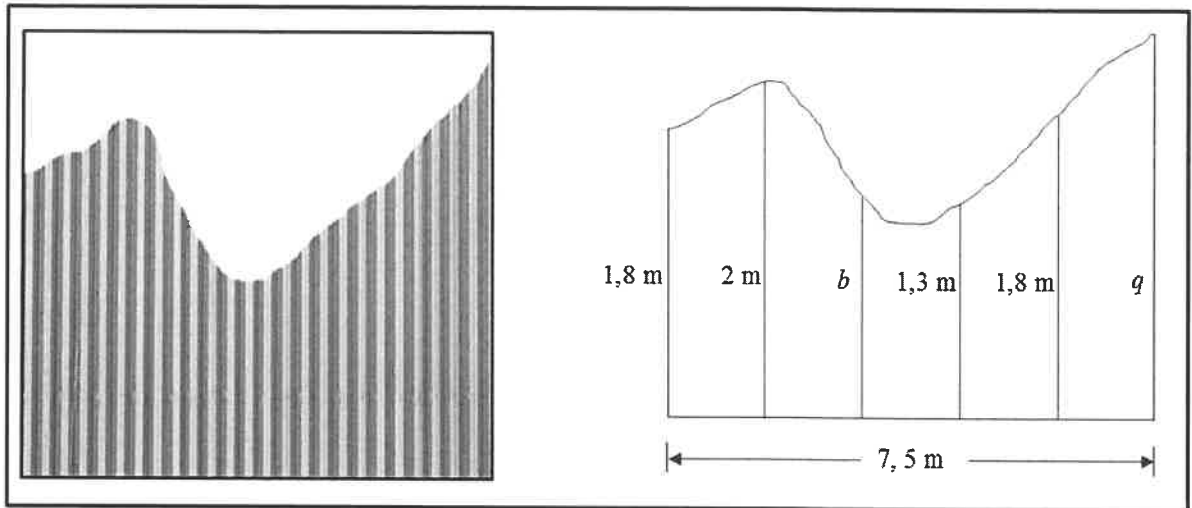
The diagram models the wheel with  $O$  the centre.  $OA$  and  $OB$  are two radii (spokes) of the rim. The length of a spoke is 5,2 cm.



- 10.2.1 Determine the size, in radians, of acute angle  $\hat{AOB}$ . (3)
  - 10.2.2 Hence, or otherwise, determine (correct to ONE decimal place) the length of minor arc  $AB$ . (3)
  - 10.2.3 Hence, or otherwise, determine the area (to the nearest  $\text{cm}^2$ ) of minor sector  $AOB$ . (3)
- [22]

**QUESTION 11**

- 11.1 An irregularly shaped portion, as shown in the picture below, was cut from a rectangular iron sheet which has an area of  $19,125 \text{ m}^2$ . The diagram alongside the picture models the irregular shaped portion which has one straight side,  $7,5 \text{ m}$  long, which is divided into 5 equal parts.

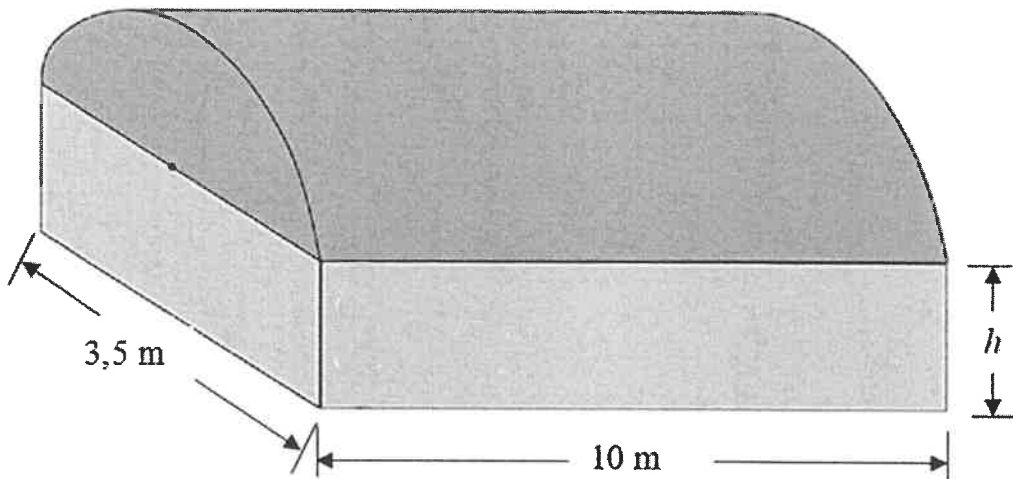


The six ordinates dividing the equal parts:  $1,8 \text{ m}$  ;  $2 \text{ m}$  ;  $b$  ;  $1,3 \text{ m}$  ;  $1,8 \text{ m}$  and  $q$

- 11.1.1 Write down the length (correct to ONE decimal place) of  $b$ , if  $b$  is the square root of the length of the second ordinate from the left. (1)
- 11.1.2 Hence, determine the length (correct to ONE decimal place) of  $q$ , if the area of the irregular shaped portion is two thirds of the area of the iron sheet. (5)

11.2

A greenhouse structure, as shown in the diagram below, has a shape made up of a combination of a rectangular right prism and half of a right cylinder. The length, breadth and height of the rectangular part of the greenhouse is 3,5 m, 10 m and  $h$  respectively. The volume of the rectangular part of the greenhouse is  $70 \text{ m}^3$ .



The following formulae may be used:

$$\text{Total surface area of a right cylinder} = 2\pi r^2 + 2\pi r h$$

$$\text{Area of a rectangle} = \text{length} \times \text{breadth}$$

$$\text{Volume of a right prism} = (\text{area of the base}) \times \text{height}$$

- 11.2.1 Determine the height ( $h$ ) of the rectangular prism. (1)
- 11.2.2 Write down the length of the radius of the half cylinder. (1)
- 11.2.3 Show whether the total external surface area of the greenhouse is less than  $120 \text{ m}^2$ . (6)
- [14]**

**TOTAL: 150**

**INFORMATION SHEET: TECHNICAL MATHEMATICS**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}$$

$$y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$i_{\text{eff}} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{Area of } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$



$$\pi \text{ rad} = 180^\circ$$

Angular velocity  $= \omega = 2\pi n = 360^\circ n$       where  $n =$  rotation frequency

Circumferential velocity  $= v = \pi Dn$       where  $D =$  diameter and  $n =$  rotation frequency

$s = r\theta$       where  $r =$  radius and  $\theta =$  central angle in radians

Area of a sector  $= \frac{rs}{2} = \frac{r^2\theta}{2}$       where  $r =$  radius,  $s =$  arc length and  
 $\theta =$  central angle in radians

$4h^2 - 4dh + x^2 = 0$       where  $h =$  height of segment,       $d =$  diameter of circle and  
 $x =$  length of chord

$A_T = a(m_1 + m_2 + m_3 + \dots + m_n)$       where  $a =$  equal parts,  $m_1 = \frac{o_1 + o_2}{2}$  and  
 $n =$  number of ordinates

**OR**

$A_T = a \left( \frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$       where  $a =$  equal parts,  $o_i = i^{\text{th}}$  ordinate and  
 $n =$  number of ordinates