



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

WISKUNDE V2

FEBRUARIE/MAART 2012

MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 18 bladsye.

VRAAG 1

1.1	<p>Gemiddeld</p> $\frac{\sum_1^n x_1}{n} = \frac{102100}{9}$ $= R11\,344,44$	<p>✓ 102100</p> <p>✓ antwoord (2)</p>
1.2	<p>Standaardafwyking</p> $\sqrt{\frac{\sum_1^n (x_1 - \bar{x})^2}{n}} = R4\,460,97$	<p>✓✓ antwoord (2)</p>
1.3	<p>Waarde van een standaardafwyking bo gemiddeld</p> $= R11\,344,44 + R4\,460,97$ $= R15\,805,41$ <p>Slegs een persoon het kommissie bo R 15 805,41 verdien. Dus het slegs 1 persoon 'n gradering van 'goed' ontvang.</p>	<p>✓ tel gemiddeld en std. af. by</p> <p>✓ afleiding (2)</p> <p>[6]</p>

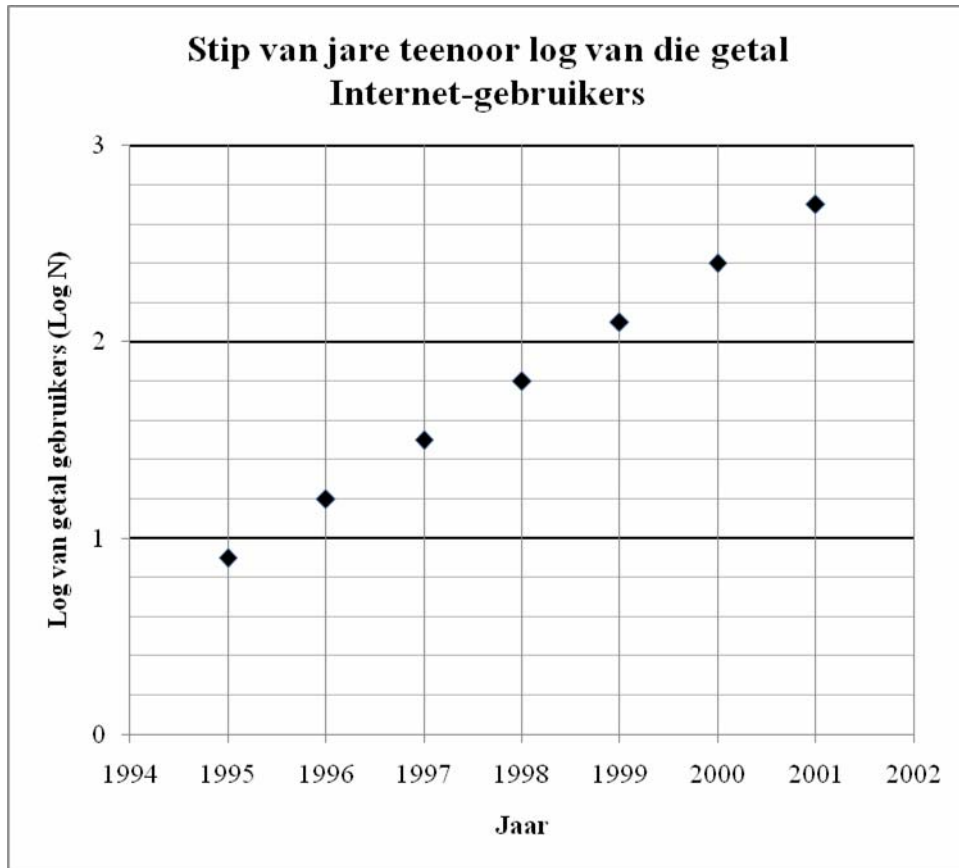
VRAAG 2

2.1	<p style="text-align: center;">Stip van jare teenoor log van die getal Internet-gebruikers</p> <table border="1"> <caption>Data points from the scatter plot</caption> <thead> <tr> <th>Jaar</th> <th>Getal (N) gebruikers (in Miljoene)</th> </tr> </thead> <tbody> <tr><td>1995</td><td>~10</td></tr> <tr><td>1996</td><td>~20</td></tr> <tr><td>1997</td><td>~35</td></tr> <tr><td>1998</td><td>~70</td></tr> <tr><td>1999</td><td>~140</td></tr> <tr><td>2000</td><td>~280</td></tr> <tr><td>2001</td><td>~560</td></tr> </tbody> </table>	Jaar	Getal (N) gebruikers (in Miljoene)	1995	~10	1996	~20	1997	~35	1998	~70	1999	~140	2000	~280	2001	~560	<p>✓ ten minste vier punte korrek</p> <p>✓ al die punte korrek (2)</p>
Jaar	Getal (N) gebruikers (in Miljoene)																	
1995	~10																	
1996	~20																	
1997	~35																	
1998	~70																	
1999	~140																	
2000	~280																	
2001	~560																	
2.2	<p>Eksponeensieel, want die toename in groei wys amper 'n verdubbeling elke jaar.</p>	<p>✓ eksponensieel</p> <p>✓ rede (2)</p>																

2.3	<table border="1"> <thead> <tr> <th>JAAR</th> <th>1995</th> <th>1996</th> <th>1997</th> <th>1998</th> <th>1999</th> <th>2000</th> <th>2001</th> </tr> </thead> <tbody> <tr> <td>N (Getal in miljoene)</td> <td>8</td> <td>17</td> <td>34</td> <td>67</td> <td>135</td> <td>281</td> <td>552</td> </tr> <tr> <td>Log N (korrek tot EEN desimale plek)</td> <td>6,9</td> <td>7,2</td> <td>7,5</td> <td>7,8</td> <td>8,1</td> <td>8,4</td> <td>8,7</td> </tr> </tbody> </table>	JAAR	1995	1996	1997	1998	1999	2000	2001	N (Getal in miljoene)	8	17	34	67	135	281	552	Log N (korrek tot EEN desimale plek)	6,9	7,2	7,5	7,8	8,1	8,4	8,7	✓ ten minste 4 punte korrek ✓ alle punte korrek (2)
	JAAR	1995	1996	1997	1998	1999	2000	2001																		
N (Getal in miljoene)	8	17	34	67	135	281	552																			
Log N (korrek tot EEN desimale plek)	6,9	7,2	7,5	7,8	8,1	8,4	8,7																			
OF (indien slegs logwaardes in table in berekening gebring is)																										
	<table border="1"> <thead> <tr> <th>JAAR</th> <th>1995</th> <th>1996</th> <th>1997</th> <th>1998</th> <th>1999</th> <th>2000</th> <th>2001</th> </tr> </thead> <tbody> <tr> <td>N (Getal in miljoene)</td> <td>8</td> <td>17</td> <td>34</td> <td>67</td> <td>135</td> <td>281</td> <td>552</td> </tr> <tr> <td>Log N (korrek tot EEN desimale plek)</td> <td>0,9</td> <td>1,2</td> <td>1,5</td> <td>1,8</td> <td>2,1</td> <td>2,4</td> <td>2,7</td> </tr> </tbody> </table>	JAAR	1995	1996	1997	1998	1999	2000	2001	N (Getal in miljoene)	8	17	34	67	135	281	552	Log N (korrek tot EEN desimale plek)	0,9	1,2	1,5	1,8	2,1	2,4	2,7	✓ ten minste 4 punte korrek ✓ alle punte korrek (2)
JAAR	1995	1996	1997	1998	1999	2000	2001																			
N (Getal in miljoene)	8	17	34	67	135	281	552																			
Log N (korrek tot EEN desimale plek)	0,9	1,2	1,5	1,8	2,1	2,4	2,7																			

2.4	<p style="text-align: center;">Stip van jare teenoor log van die getal Internet-gebruikers</p> <table border="1"> <caption>Data for Stip van jare teenoor log van die getal Internet-gebruikers</caption> <thead> <tr> <th>Jaar</th> <th>Log van getal gebruikers (Log N)</th> </tr> </thead> <tbody> <tr> <td>1995</td> <td>6,9</td> </tr> <tr> <td>1996</td> <td>7,2</td> </tr> <tr> <td>1997</td> <td>7,5</td> </tr> <tr> <td>1998</td> <td>7,8</td> </tr> <tr> <td>1999</td> <td>8,1</td> </tr> <tr> <td>2000</td> <td>8,4</td> </tr> <tr> <td>2001</td> <td>8,7</td> </tr> </tbody> </table>	Jaar	Log van getal gebruikers (Log N)	1995	6,9	1996	7,2	1997	7,5	1998	7,8	1999	8,1	2000	8,4	2001	8,7	✓ ten minste 4 punte korrek gestip ✓ alle punte korrek (2)
Jaar	Log van getal gebruikers (Log N)																	
1995	6,9																	
1996	7,2																	
1997	7,5																	
1998	7,8																	
1999	8,1																	
2000	8,4																	
2001	8,7																	

OF (indien slegs logwaardes in table in berekening gebring is)



✓ ten minste 4 punte
 korrek gestip
 ✓ alle punte korrek

(2)

2.5 Lineêr, want die punte lê baie naby aan 'n reguitlyn

✓ lineêr
 ✓ rede (2)
[9]

VRAAG 3

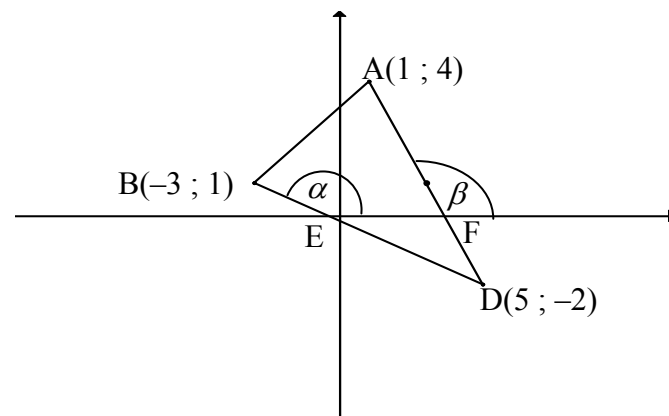
3.1	40	✓ 40 (1)												
3.2	<table border="1"> <thead> <tr> <th>Tyd, t, in minute</th> <th>Frekwensie</th> </tr> </thead> <tbody> <tr> <td>$0 \leq t < 5$</td> <td>3</td> </tr> <tr> <td>$5 \leq t < 10$</td> <td>5</td> </tr> <tr> <td>$10 \leq t < 15$</td> <td>10</td> </tr> <tr> <td>$15 \leq t < 20$</td> <td>15</td> </tr> <tr> <td>$20 \leq t < 25$</td> <td>7</td> </tr> </tbody> </table>	Tyd, t, in minute	Frekwensie	$0 \leq t < 5$	3	$5 \leq t < 10$	5	$10 \leq t < 15$	10	$15 \leq t < 20$	15	$20 \leq t < 25$	7	✓ vir intervalle in tabel ✓ vir eerste drie korrekte frekwensies ✓ vir laaste twee korrekte frekwensies (3)
Tyd, t, in minute	Frekwensie													
$0 \leq t < 5$	3													
$5 \leq t < 10$	5													
$10 \leq t < 15$	10													
$15 \leq t < 20$	15													
$20 \leq t < 25$	7													
3.3		✓ korrekte frekwensies ✓ middelpuntwaardes ✓ geen spasies tussen stawe (3) [7]												

VRAAG 4

$a = 7$	$b = 15$	$c = 17$	$d = 23$	$e = 34$	$f = 37$	$g = 42$	✓ elke korrekte antwoord (7)
OF							
$g = 42 ; a = 7 ; d = 23 ; f = 37 ; b = 15$ $\frac{42 + 7 + 23 + 37 + 15 + 3c}{7} = 25$ $3c = 51$ $c = 17$ $e = 34$							✓ g ✓ a ✓ d ✓ f ✓ b ✓ c ✓ e (7) [7]

VRAAG 5

5.1	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2 - 4}{5 - 1}$ $= -\frac{6}{4} = -\frac{3}{2}$	✓ vir substitusie ✓ vir antwoord (2)
5.2	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - 1)^2 + (-2 - 4)^2}$ $= \sqrt{16 + 36}$ $= \sqrt{52}$	✓ vir substitusie ✓ $\sqrt{52}$ (2)
5.3	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ $M = \left(\frac{1 + 5}{2}, \frac{4 - 2}{2} \right)$ $M = (3; 1)$	✓ x-waarde ✓ y-waarde (2)
5.4	$m_{BC} = m_{AD}$ $= -\frac{3}{2}$ $y - y_1 = m(x - x_1)$ $y - 1 = -\frac{3}{2}(x + 3)$ $2y - 2 = -3x - 9$ $3x + 2y + 7 = 0$ <p style="text-align: center;">OF</p> $y = -\frac{3}{2}x + c$ $1 = -\frac{3}{2}(-3) + c$ $c = -\frac{7}{2}$ $y = -\frac{3}{2}x - \frac{7}{2}$ $3x + 2y + 7 = 0$	lyne is parallel ✓ waarde m_{BC} ✓ subst $(-3; 1)$ ✓ vergelyking (3) ✓ waarde m_{BC} ✓ subst $(-3; 1)$ ✓ vergelyking (3)

<p>5.5.1</p>	$m_{AD} = -\frac{3}{2}$ $\tan \beta = -\frac{3}{2}$ $\beta = 180^\circ - 56,31^\circ$ $\beta = 123,69^\circ$ 	<p>✓ $\tan \beta = m_{AD}$</p> <p>✓ $123,69^\circ$</p> <p>(2)</p>
<p>5.5.2</p>	$m_{BD} = \frac{-2-1}{5-(-3)} = \frac{-3}{8}$ $\tan \alpha = -\frac{3}{8}$ $\alpha = 180^\circ - 20,56^\circ$ $\alpha = 159,44^\circ$ $\widehat{FED} = 180^\circ - 159,44^\circ = 20,56^\circ$ $\widehat{EFD} = 123,69^\circ$ $\widehat{FDE} = 180^\circ - (20,56^\circ + 123,69^\circ) = 35,75^\circ$	<p>✓ $m_{BD} = \frac{-3}{8}$</p> <p>✓ $159,44^\circ$</p> <p>✓ $20,56^\circ$</p> <p>✓ $123,69^\circ$</p> <p>✓ $35,75^\circ$</p> <p>(5)</p>
<p>5.6</p>	<p>Koördinate van middelpunt M (3 ; 1) Radius van sirkel: $\frac{1}{2}$ of AD = $\frac{1}{2} (2\sqrt{13}) = \sqrt{13} = \frac{1}{2}\sqrt{52}$ Vergelyking van die sirkel is: $(x-3)^2 + (y-1)^2 = 13$</p> <p style="text-align: center;">OF</p> <p>$r^2 = (3-1)^2 + (1-4)^2 = 13$ Vergelyking van die sirkel is: $(x-3)^2 + (y-1)^2 = 13$</p>	<p>✓ waarde van radius</p> <p>✓ substitusie in vergelyking vir sirkelmiddelpunt-vorm (2)</p> <p>✓ waarde van r^2</p> <p>✓ substitusie in vergelyking in sirkelmiddelpunt vorm (2)</p>
<p>5.7</p>	<p>M(3 ; 1) B(-3 ; 1) $MB = \sqrt{(3+3)^2 + (1-1)^2}$ $MB = 6$ Punt B lê buite die sirkel want $MB >$ radius</p> <p style="text-align: center;">OF</p> <p>M(3 ; 1) B(-3 ; 1) $MB = 3 + 3 = 6$ Radius van die sirkel = $\sqrt{13} < 6$ Punt B lê buite die sirkel want $MB >$ radius</p>	<p>✓ substitusie</p> <p>✓ buite (2)</p> <p>✓ substitusie</p> <p>✓ buite (2)</p> <p>[20]</p>

VRAAG 6

6.1	Koördinate van middelpunt M $(-2 ; 1)$ $(1+2)^2 + (-2-1)^2 = 18 = r^2$ Radius = $\sqrt{18}$ of $3\sqrt{2}$	✓✓ koördinate van middelpunt ✓ berekening ✓ waarde (4)
6.2	$m_{MS} = \frac{-3}{3} = -1$ $m_{MS} \times m_{RS} = -1 \quad \text{OF} \quad \text{raaklyn} \perp \text{radius}$ $m_{RS} = 1$ $y - y_1 = m(x - x_1)$ $y + 2 = 1(x - 1)$ $y = x - 3$ <p style="text-align: center;">OF</p> $m_{MS} = \frac{-3}{3} = -1$ $m_{MS} \times m_{RS} = -1$ $m_{RS} = 1$ $y = x + c$ $-2 = 1 + c$ $c = -3$ $y = x - 3$	✓ gradiënt MS ✓ gradiënt RS ✓ subst $(1 ; -2)$ ✓ vergelyking (4)
6.3	$\frac{MS}{MP} = \frac{1}{3}$ $\therefore MP = 3MS$ $MP^2 = 9MS^2$ $(a+2)^2 + (b-1)^2 = 9(3^2 + 3^2) = 162 \quad (1)$ $MS \perp SR \text{ en } PS \perp SR \quad \therefore m_{PS} = m_{MS}$ $\frac{b+2}{a-1} = \frac{3}{-3} = -1$ $b+2 = -a+1$ $b = -a-1 \quad (2)$ <p>Vervang (2) in (1)</p>	✓ MP = 3MS ✓ vergelyking ✓ dieselfde gradiënte ✓ gradiënt ✓ $b = -a - 1$

	$(a+2)^2 + (-a-1-1)^2 = 162$ $(a+2)^2 + (a+2)^2 = 162$ $2(a+2)^2 = 162$ $(a+2)^2 = 81$ $a+2 = 9 \text{ of } -9$ $a = 7 \text{ of } -11$ $b = -a - 1 = -8$ $P(7; -8)$ <p>OF</p> $\frac{MS}{MP} = \frac{1}{3}$ $\therefore MP = 3MS$ $MP^2 = 9MS^2$ $(a+2)^2 + (b-1)^2 = 9(3^2 + 3^2) = 162 \quad (1)$ $MS \perp SR \text{ en } PS \perp SR \quad \therefore m_{PS} = m_{MS}$ $\frac{b+2}{a-1} = \frac{3}{-3} = -1$ $b+2 = -a+1$ $b = -a-1 \quad (2)$ <p>Vervangt (2) in (1)</p> $a^2 + 4a + 4 + a^2 + 4a + 4 = 162$ $2a^2 + 8a - 154 = 0$ $a^2 + 4a - 77 = 0$ $(a+11)(a-7) = 0$ $a = 7 \text{ of } -11$ <p>Maar $a > 0$</p> $\therefore a = 7$ $b = -a - 1 = -8$ $P(7; -8)$ <p>OF</p>	<p>✓ substitusie</p> <p>✓ $a = 7$ ✓ $b = -8$</p> <p>(8)</p> <p>✓ $MP = 3MS$</p> <p>✓ vergelyking</p> <p>✓ dieselfde gradiënte</p> <p>✓ gradiënt</p> <p>✓ $b = -a - 1$</p> <p>✓ substitusie</p> <p>✓ $a = 7$ ✓ $b = -8$</p> <p>(8)</p>
--	--	---

P(a ; b)
MSP is 'n reguitlyn (MS ⊥ SR)

$$m_{PM} = -1$$

$$\frac{b-1}{a+2} = -1$$

$$b-1 = -a-2$$

$$b = -a-1 \dots\dots(1)$$

$$PS = 2MS = 2\sqrt{9+9} = 2\sqrt{18}$$

$$PS^2 = 4(18) = 72$$

$$(a-1)^2 + (b+2)^2 = 72 \dots\dots(2)$$

$$(a-1)^2 + (-a-1+2)^2 = 72$$

$$2a^2 - 4a - 70 = 0$$

$$a^2 - 2a - 35 = 0$$

$$(a-7)(a+5) = 0$$

$$a = 7 \text{ of } a = -5$$

$$b = -7-1 = -8$$

$$P(7 ; -8)$$

OF

$$2(a-1)^2 = 72$$

$$(a-1)^2 = 36$$

$$a-1 = 6 \text{ of } -6$$

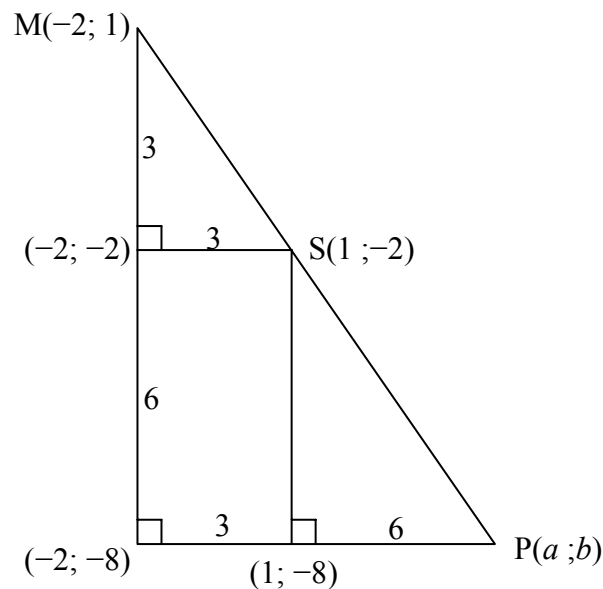
$$a = 7 \text{ of } -5$$

$$a = 7$$

$$b = -8$$

$$P(7 ; -8)$$

OF



✓ MSP is 'n reguitlyn
✓ $m_{PM} = -1$

✓ $\frac{b-1}{a+2}$

✓ vergelyking 1

✓ vergelyking 2

✓ substitusie van vergelyking 1 in vergelyking 2

✓✓ koördinate

(8)

✓✓ diagram

✓✓ (-2; -8)

✓ (-2; -2)

✓ (1; -8)

✓✓ P(7 ; -8)

(8)

	<p>P(a ; b)</p> $\frac{x_S - x_M}{x_P - x_M} = \frac{y_S - y_M}{y_P - y_M} = \frac{1}{3}$ $\frac{-3}{b-1} = \frac{3}{a+2} = \frac{1}{3}$ $-9 = b-1$ $b = -8$ $9 = a+2$ $a = 7$ <p>P(7;-8)</p>	<p>✓✓ verdeling van 'n lynstuk in 'n gegewe verhouding</p> <p>✓✓ substitusie</p> <p>✓ vergelyking</p> <p>✓ vergelyking</p> <p>✓ koördinate</p> <p style="text-align: right;">(8) [16]</p>
--	---	---

VRAAG 7

7.1		<p>Vir korrekte koördinate en benoeming van elke beeld:</p> <p>✓ K'</p> <p>✓ L'</p> <p>✓ M'</p> <p>✓ N'</p> <p style="text-align: right;">(4)</p>
7.2.1	Transformasie is nie rigied (star) nie, want die area het verander as gevolg van die vergroting.	<p>✓ nie rigied nie</p> <p>✓ grootte nie behou nie</p> <p style="text-align: right;">(2)</p>
7.2.2	N''(-2; -2)	<p>✓✓ koördinate van N''</p> <p style="text-align: right;">(2)</p>
7.3	(x ; y) → (-y ; x) → (-2y ; 2x)	<p>✓ -y</p> <p>✓ x</p> <p>✓ -2y</p> <p>✓ 2x</p> <p style="text-align: right;">(4)</p>
7.4	Area van KLMN : area van K''L''M''N'' = 1 : 4	<p>✓✓ antwoord</p> <p style="text-align: right;">(2)</p>
7.5	<p>Indien die verste punt vanaf die oorsprong in die sirkel gedruk/gestuur word, dan sal die hele vierhoek in die sirkel gedruk wees. K is die verste weg.</p> $KO = \sqrt{3^2 + 3^2} = \sqrt{18}$	<p>✓ K – verste</p> <p>✓ KO = $\sqrt{18}$</p> <p>✓ antwoord</p> <p style="text-align: right;">(3)</p>

$$p.KO = 1, p = \frac{1}{\sqrt{18}}$$

[17]

VRAAG 8

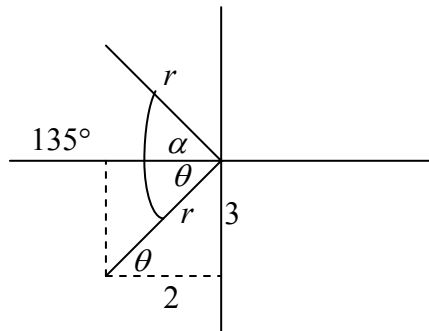
8.	$x_Q = x \cos \theta + y \sin \theta$ $x_Q = -2 \cos 135^\circ + (-3) \sin 135^\circ$ $x_Q = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \text{ of } \frac{-\sqrt{2}}{2} \text{ or } -0,71$ $y_Q = y \cos \theta - x \sin \theta$ $y_Q = -3 \cos 135^\circ - (-2) \sin 135^\circ$ $y_Q = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 3,54$ $Q\left(\frac{-1}{\sqrt{2}}; \frac{5}{\sqrt{2}}\right)$ <p style="text-align: center;">OF</p> $x_Q = x \cos \theta - y \sin \theta$ $x_Q = -2 \cos(-135^\circ) - (-3) \sin(-135^\circ)$ $x_Q = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{-1}{\sqrt{2}} \text{ of } \frac{-\sqrt{2}}{2} \text{ or } -0,71$ $y_Q = y \cos \theta + x \sin \theta$ $y_Q = -3 \cos(-135^\circ) + (-2) \sin(-135^\circ)$ $y_Q = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} = 3,54$ $Q\left(\frac{-1}{\sqrt{2}}; \frac{5}{\sqrt{2}}\right)$ <p style="text-align: center;">OF</p> $x' = x \cos \theta - y \sin \theta$ $-2 = x \cos 135^\circ - y \sin 135^\circ$ $-2 = \frac{-x}{\sqrt{2}} - \frac{y}{\sqrt{2}}$ $-2\sqrt{2} = -x - y \quad (1)$ $y' = y \cos \theta + x \sin \theta$ $-3 = y \cos 135^\circ + x \sin 135^\circ$ $-3 = \frac{-y}{\sqrt{2}} + \frac{x}{\sqrt{2}}$ $-3\sqrt{2} = x - y \quad (2)$ <p>Los (1) en (2) gelyktydig op:</p>	<ul style="list-style-type: none"> ✓ subst -2 en -3 in korrekte formule vir x_Q ✓ gebruik 135° ✓ x-koördinate (in enige formaat) ✓ subst -2 en -3 in korrekte formule vir y_Q ✓ vir y-koördinate (in enige formaat) (5) ✓ subst -2 en -3 in korrekte formule vir x_Q ✓ gebruik -135° ✓ x-koördinate (in enige formaat) ✓ subst -2 en -3 in korrekte formule vir y_Q ✓ vir y-koördinate (in enige formaat) (5) ✓ subst -2 en 135° in korrekte formule vir x' ✓ vereenvoudiging ✓ subst -2 en 135° in korrekte formule vir y' ✓ y-koördinaat ✓ x-koördinaat (5)
----	---	--

$$-5\sqrt{2} = -2y$$

$$y = \frac{5}{\sqrt{2}} \quad \text{en} \quad x = \frac{-1}{\sqrt{2}}$$

OF

Gebruik eerste beginsels: $Q = (-r \cos \alpha; r \sin \alpha)$



$$Q' = (-2; -3)$$

$$\tan \theta = \frac{3}{2}$$

$$r = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\theta = 56,31^\circ$$

$$\therefore \alpha = 135^\circ - 56,31^\circ = 78,69^\circ$$

$$Q = (-r \cos \alpha; r \sin \alpha)$$

$$= (-0,71; 3,54)$$

$$\checkmark \tan \theta = \frac{3}{2}$$

$$\checkmark r = \sqrt{13}$$

$$\checkmark \theta = 56,31^\circ$$

✓

$$Q = (-r \cos \alpha; r \sin \alpha)$$

✓ antwoord

(5)

[5]

VRAAG 9

9.1.1	$r = 13$ $\cos \alpha = \frac{12}{13}$	$\checkmark 13$ $\checkmark \frac{12}{13}$ (2)
9.1.2	$\widehat{Q\hat{O}R} = 180^\circ - (90^\circ + \alpha)$ $= 90^\circ - \alpha$	$\checkmark 180^\circ - (90^\circ + \alpha)$ $\checkmark 90^\circ - \alpha$ (2)
9.1.3	$\cos \widehat{Q\hat{O}R} = \frac{QR}{OQ}$ $\cos(90^\circ - \alpha) = \frac{7,5}{OQ}$ $OQ = \frac{7,5}{\cos(90^\circ - \alpha)}$ $OQ = \frac{7,5}{\sin \alpha}$ $OQ = \frac{7,5}{\frac{5}{13}}$ $OQ = 19,5$ <p style="text-align: center;">OF</p> $\sin(\widehat{R\hat{Q}O}) = \frac{7,5}{OQ}$ $\therefore OQ = \frac{7,5}{\sin \alpha}$ $OQ = \frac{7,5}{\frac{5}{13}}$ $OQ = 19,5$	\checkmark $\cos(90^\circ - \alpha) = \frac{7,5}{OQ}$ $\checkmark \frac{7,5}{\sin \alpha}$ $\checkmark \frac{5}{13}$ $\checkmark 19,5$ (4) \checkmark $\sin(\widehat{R\hat{Q}O}) = \frac{7,5}{OQ}$ $\checkmark \frac{7,5}{\sin \alpha}$ $\checkmark \frac{5}{13}$ $\checkmark 19,5$ (4)
9.2	$LK = \frac{\cos x \cdot \cos x(-\tan x)}{-\cos x}$ $= \cos x \cdot \frac{\sin x}{\cos x}$ $= \sin x$ $= RK$	$\checkmark \cos x$ $\checkmark -\tan x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark \text{antwoord}$ (4) [12]

VRAAG 10

10.1	Periode = 120°	✓ 120° (1)
10.2	$\sin 3x = -1$ $x = -30^\circ$ of $x = 90^\circ$	✓ -30° ✓ 90° (2)
10.3	Maksimum waarde van $f(x)$ is 1 \therefore Maksimum waarde van $h(x)$ is 0	✓ maks van $f(x)$ ✓ antwoord (2)
10.4		✓ -90° ; 90° ✓ $(0^\circ; 3)$ ✓ $(180^\circ; -3)$ (3)
10.5	$\frac{\sin 3x}{3} - \cos x = 0$ $\sin 3x - 3 \cos x = 0$ $\therefore \sin 3x = 3 \cos x$ <p>Daar is 2 oplossings waar die grafieke van f en g gelyk is</p>	✓ $\sin 3x = 3 \cos x$ ✓ antwoord (2)
10.6	$f(x) \cdot g(x) < 0$ $x \in (-60^\circ; 0^\circ)$ of $(60^\circ; 90^\circ)$ of $(120^\circ; 180^\circ)$ <p style="text-align: center;">OF</p> $-60^\circ < x < 0^\circ$ of $60^\circ < x < 90^\circ$ of $120^\circ < x < 180^\circ$	✓✓✓ vir elke interval ✓ korrekte hakies of korrekte simbole (4) [14]

VRAAG 11

11.1.1	$\sin 61^\circ = \sqrt{p}$ $\sin 241^\circ = \sin (180^\circ + 61^\circ)$ $= -\sin 61^\circ$ $= -\sqrt{p}$		✓ $-\sin 61^\circ$ ✓ antwoord	(2)
11.1.2	$\cos 61^\circ = \sqrt{1 - \sin^2 61^\circ}$ $= \sqrt{1 - p}$		✓ identiteit ✓ antwoord	(2)
11.1.3	$\cos 122^\circ = \cos 2(61^\circ)$ $= 2\cos^2 61^\circ - 1$ $= 2(\sqrt{1-p})^2 - 1$ $= 2(1-p) - 1$ $= 2 - 2p - 1$ $= 1 - 2p$		✓ dubbelhoek ✓ uitbreiding ✓ antwoord	(3)
11.1.4	$\cos 73^\circ \cos 15^\circ + \sin 73^\circ \sin 15^\circ$ $= \cos(73^\circ - 15^\circ)$ $= \cos 58^\circ = (\cos 180^\circ - 122^\circ)$ $= -(\cos 122^\circ)$ $= -(1 - 2p)$ $= 2p - 1$		✓ $\cos(73^\circ - 15^\circ)$ ✓ $-(\cos 122^\circ)$ ✓ antwoord	(3)
11.2.1	$LK = \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{\cos^2 x + 2\cos x \sin x + \sin^2 x - (\cos^2 x - 2\sin x \cos x + \sin^2 x)}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \frac{4\cos x \sin x}{\cos^2 x - \sin^2 x}$ $= \frac{2\sin 2x}{\cos 2x}$ $= 2\tan x$ $= RK$		✓ $\frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ ✓ teller ✓ $4\cos x \sin x$ ✓ $\cos^2 x - \sin^2 x$ ✓ $2\sin 2x$ ✓ $\cos 2x$	(6)
11.2.2	$x = 45^\circ; 135^\circ$		✓ 45° ✓ 135°	(2)
11.3.1	$\sin x = \cos 2x - 1$ $\sin x = 1 - 2\sin^2 x - 1$ $\sin x = -2\sin^2 x$ $2\sin^2 x + \sin x = 0$		✓ $1 - 2\sin^2 x$	(1)

<p>11.3.2</p>	$\sin x = \cos 2x - 1$ $2 \sin^2 x + \sin x = 0$ $\sin x (2 \sin x + 1) = 0$ $\sin x = 0 \text{ of } \sin x = -\frac{1}{2}$ $\therefore x = 0^\circ + 180^\circ k; k \in \mathbb{Z} \text{ of } x = \{210^\circ \text{ or } 330^\circ\} + 360^\circ k; k \in \mathbb{Z}$ <p style="text-align: center;">OF</p> $x = n \cdot 180^\circ$ $x = n \cdot 360^\circ - 30^\circ$ $x = (2n + 1) \cdot 180^\circ + 30^\circ, n \in \mathbb{Z}$	$\checkmark \sin x (2 \sin x + 1) = 0$ $\checkmark \sin x = 0 \text{ of } \sin x = -\frac{1}{2}$ $\checkmark 0^\circ + 180^\circ k$ $\checkmark 210^\circ$ $\checkmark 330^\circ$ $\checkmark + 360^\circ k; k \in \mathbb{Z}$ <p style="text-align: right;">(6)</p>
---------------	--	--

<p>11.4</p>	$\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \times \tan 4^\circ \times \dots \times \tan 87^\circ \times \tan 88^\circ \times \tan 89^\circ$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin 88^\circ}{\cos 88^\circ}\right) \left(\frac{\sin 89^\circ}{\cos 89^\circ}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\sin(90^\circ - 2^\circ)}{\cos(90^\circ - 2^\circ)}\right) \left(\frac{\sin(90^\circ - 1^\circ)}{\cos(90^\circ - 1^\circ)}\right)$ $= \left(\frac{\sin 1^\circ}{\cos 1^\circ}\right) \left(\frac{\sin 2^\circ}{\cos 2^\circ}\right) \dots \left(\frac{\sin 45^\circ}{\cos 45^\circ}\right) \dots \left(\frac{\cos 2^\circ}{\sin 2^\circ}\right) \left(\frac{\cos 1^\circ}{\sin 1^\circ}\right)$ $= \tan 45^\circ$ $= 1$ <p style="text-align: center;">OF</p> $\tan 89^\circ = \cot 1^\circ \quad \tan 88^\circ = \cot 2^\circ \dots$ $\therefore \text{produk is } (\tan 1^\circ \cdot \cot 1^\circ)(\tan 2^\circ \cdot \cot 2^\circ) \dots (\tan 44^\circ \cdot \cot 44^\circ) \cdot \tan 45^\circ$ $= 1 \times 1 \times 1 \times \dots \times 1 = 1$	$\checkmark \text{ identiteit}$ $\checkmark \text{ ko-verhoudings}$ $\checkmark \text{ vereenvoudiging}$ $\checkmark \text{ vir antwoord}$ <p style="text-align: right;">(4)</p> $\checkmark \text{ identiteit}$ $\checkmark \text{ ko-verhoudings}$ $\checkmark \text{ vereenvoudiging}$ $\checkmark \text{ vir antwoord}$ <p style="text-align: right;">(4) [29]</p>
-------------	--	--

VRAAG 12

12	<p>By $\triangle CBG$ en $\triangle CDH$:</p> $CG^2 = x^2 + y^2 \quad \text{Pythagoras}$ $CH^2 = x^2 + y^2 \quad \text{Pythagoras}$ <p>By $\triangle FAE$</p> $AE^2 = x^2 + x^2$ $= 2x^2$ $= GH^2$ <p>By $\triangle CGH$</p> $GH^2 = CG^2 + CH^2 - 2 CG \cdot CH \cdot \cos GCH$ $\cos \hat{GCH} = \frac{CG^2 + CH^2 - GH^2}{2CG \cdot CH}$ $\cos \hat{GCH} = \frac{x^2 + y^2 + x^2 + y^2 - 2x^2}{2\sqrt{x^2 + y^2} \cdot \sqrt{x^2 + y^2}}$ $\cos \hat{GCH} = \frac{2y^2}{2(x^2 + y^2)}$ $\cos \hat{GCH} = \frac{y^2}{x^2 + y^2}$	<p>✓ CG^2</p> <p>✓ CH^2</p> <p>✓ AE^2</p> <p>✓ $AE^2 = GH^2$</p> <p>✓ gebruik van cos-reël</p> <p>✓ manipulatie van formule</p> <p>✓ substitusie</p> <p>✓ $\cos \hat{GCH} = \frac{2y^2}{2(x^2 + y^2)}$</p> <p style="text-align: right;">(8) [8]</p>
----	---	--

TOTAAL: 150