MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages.
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 9 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.

5. Answers only will not necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. Write neatly and legibly.
QUESTION 1

1.1 Solve for \(x\) in each of the following:

1.1.1 \(x^2 + x - 12 = 0\) \hspace{1cm} (3)

1.1.2 \(\sqrt{2x + 1} = x - 1\) \hspace{1cm} (5)

1.1.3 \(2^{x/2} = 2^{27}\) \hspace{1cm} (4)

1.1.4 \(x^2 - 2x - 8 < 0\) \hspace{1cm} (3)

1.2 Given: \(f(x) = 5x^2 + 6x - 7\)

1.2.1 Solve for \(x\) if \(f(x) = 0\) \(\) (correct to TWO decimal places). \hspace{1cm} (4)

1.2.2 Hence, or otherwise, calculate the value of \(d\) for which \(5x^2 + 6x - d = 0\) has equal roots. \hspace{1cm} (3)

1.3 Solve for \(x\) and \(y\) simultaneously:

\[x - 2y = -3 \quad \text{and} \quad xy = 20\] \hspace{1cm} (6)

[28]

QUESTION 2

2.1 Simplify, without using a calculator:

2.1.1 \[\frac{2^{n+2} \cdot 4^{n+1}}{8^{n-1}}\] \hspace{1cm} (3)

2.1.2 \[\sqrt{x + \sqrt{2x - 1}} \cdot \sqrt{x - \sqrt{2x - 1}}\] \hspace{1cm} (4)

2.2 Given: \(P = \sqrt{\frac{5}{x + 2}} + \frac{x}{3}\)

2.2.1 For what value(s) of \(x\) will \(P\) be a real number? \hspace{1cm} (2)

2.2.2 Show that \(P\) is rational if \(x = 3\). \hspace{1cm} (2)

2.3 Calculate the sum of the digits of \(2^{2015} \times 5^{2019}\). \hspace{1cm} (4)

[15]
QUESTION 3

3.1 Given the linear pattern: 5 ; -2 ; -9 ; ... ; -289

3.1.1 Write down the constant first difference. \( (1) \)

3.1.2 Write down the value of \( T_4 \). \( (1) \)

3.1.3 Calculate the number of terms in the pattern. \( (3) \)

3.2 A linear pattern has a difference of 3 between consecutive terms and its 20\(^{th}\) term is equal to 64 (that is \( T_{20} = 64 \)).

3.2.1 Determine the value of \( T_{22} \). \( (1) \)

3.2.2 Which term in the pattern will be equal to \( 3T_5 - 2 \)? \( (4) \)

3.3 Consider the quadratic pattern: 5 ; 12 ; 29 ; 56 ; ... 

3.3.1 Write down the NEXT TWO terms of the pattern. \( (2) \)

3.3.2 Prove that the first differences of this pattern will always be odd. \( (3) \) \[15\]

QUESTION 4

4.1 Consider the quadratic pattern: 3 ; 5 ; 8 ; 12 ; ...

Determine the value of \( T_{26} \). \( (6) \)

4.2 A certain quadratic pattern has the following characteristics:

- \( T_1 = p \)
- \( T_2 = 18 \)
- \( T_4 = 4T_1 \)
- \( T_3 - T_2 = 10 \)

Determine the value of \( p \). \( (6) \) \[12\]
QUESTION 5

5.1 The sketch below shows the graph of \( f(x) = \frac{-9}{x-1} - 2 \).
A is the point of intersection of the asymptotes of \( f \).

![Graph of f(x) with point A]

5.1.1 Write down the coordinates of \( A \). (2)

5.1.2 Determine the coordinates of the \( x \)- and \( y \)-intercepts of \( f \). (5)

5.1.3 Write down an equation of the axis of symmetry of \( f \) that has a negative gradient. (2)

5.1.4 Hence, or otherwise, determine the coordinates of a point that lies on \( f \) in the fourth quadrant, which is the closest to point \( A \). (5)

5.1.5 The graph of \( f \) is reflected about the \( x \)-axis to obtain the graph of \( g \). Write down the equation of \( g \) in the form \( y = ... \). (2)

5.2 Given: \( h(x) = 4(2^{-x}) + 1 \)

5.2.1 Determine the coordinates of the \( y \)-intercept of \( h \). (2)

5.2.2 Explain why \( h \) does not have an \( x \)-intercept. (2)

5.2.3 Draw a sketch graph of \( h \), clearly showing all asymptotes, intercepts with the axes and at least one other point on \( h \). (3)

5.2.4 Describe the transformation from \( h \) to \( g \) if \( g(x) = 4(2^{-x} + 2) \). (2)

[25]
QUESTION 6

The sketch below represents the graphs of two parabolas, \( f \) and \( g \).

\[
f(x) = \frac{1}{2} x^2 - 8
\]

The turning point of \( g \) is \( C(2; 9) \) and the \( y \)-intercept of \( g \) is \( A(0; 5) \). B and D are the \( x \)-intercepts of \( f \) and \( g \) respectively.

6.1 Show that \( g(x) = -x^2 + 4x + 5 \).  

6.2 Calculate the average gradient of \( g \) between A and C.  

6.3 Calculate the length of BD.  

6.4 Use the graphs to solve for \( x \), if:

6.4.1 \( f(x) \geq 0 \)  

6.4.2 \( f \) and \( g \) are both strictly increasing  

[15]
QUESTION 7

The sketch below shows the graphs of \( f(x) = 2x + 3 \) and \( g(x) = -2x^2 + 14x + k \). C is any point on \( f \) and D any point on \( g \), such that CD is parallel to the y-axis. \( k \) is a value such that C lies above D.

7.1 Write down a simplified expression for the length of CD in terms of \( x \) and \( k \). (3)

7.2 If the minimum length of CD is 5, calculate the value of \( k \). (4)

QUESTION 8

8.1 A school buys tablets at a total cost of R140 000. If the average rate of inflation is 6,1\% per annum over the next 4 years, determine the cost of replacing these tablets in 4 years' time. (3)

8.2 An investment earns interest at a rate of 7\% per annum, compounded semi-annually. Calculate the effective annual interest rate on this investment. (3)

8.3 A savings account was opened with an initial deposit of R24 000. Eighteen months later R7 000 was withdrawn from the account. Calculate how much money will be in the savings account at the end of 4 years if the interest rate was 10,5\% p.a., compounded monthly. (5)

8.4 A car costing R198 000 has a book value of R102 755,34 after 3 years. If the value of the car depreciates at \( r \% \) p.a. on a reducing balance, calculate \( r \). (5)
QUESTION 9

9.1 Given:
\[ P(A) = 0.6 \]
\[ P(B) = 0.3 \]
\[ P(A \text{ or } B) = 0.8 \]
where \( A \) and \( B \) are two different events

Are the events \( A \) and \( B \) mutually exclusive? Justify your answer with appropriate calculations and/or a diagram. (4)

9.2 The table below shows data on the monthly income of employed people in two residential areas. Representative samples were used in the collection of the data.

<table>
<thead>
<tr>
<th>MONTHLY INCOME (IN RANDS)</th>
<th>AREA 1</th>
<th>AREA 2</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x &lt; 3200 )</td>
<td>500</td>
<td>460</td>
<td>960</td>
</tr>
<tr>
<td>( 3200 \leq x &lt; 25600 )</td>
<td>1182</td>
<td>340</td>
<td>1522</td>
</tr>
<tr>
<td>( x \geq 25600 )</td>
<td>150</td>
<td>14</td>
<td>164</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>1832</td>
<td>814</td>
<td>2646</td>
</tr>
</tbody>
</table>

9.2.1 What is the probability that a person chosen randomly from the entire sample will be:

(a) From Area 1

(b) From Area 2 and earn less than R3 200 per month

(c) A person from Area 2 who earns more than or equal to R3 200

(2)

(1)

(2)

9.2.2 Prove that earning an income of less than R3 200 per month is not independent of the area in which a person resides. (5)

9.2.3 Which is more likely: a person from Area 1 earning less than R3 200 or a person from Area 2 earning less than R3 200? Show calculations to support your answer. (3)

\[ \text{[17]} \]

TOTAL: 150