



**education**

Department:  
Education  
REPUBLIC OF SOUTH AFRICA

# **NATIONAL CURRICULUM STATEMENT GRADES 10-12 (GENERAL)**

## **SUBJECT ASSESSMENT GUIDELINES**

### **MATHEMATICS**

**JANUARY 2008**



## **PREFACE TO SUBJECT ASSESSMENT GUIDELINES**

The Department of Education has developed and published Subject Assessment Guidelines for all 29 subjects of the National Curriculum Statement (NCS). These Assessment Guidelines should be read in conjunction with the relevant Subject Statements and Learning Programme Guidelines.

Writing Teams established from nominees of the nine provincial education departments and the teacher unions formulated the Subject Assessment Guidelines. The draft copies of the Subject Assessment Guidelines developed by the Writing Teams were sent to a wide range of readers, whose advice and suggestions were considered in refining these Guidelines. In addition, the Department of Education field-tested the Subject Assessment Guidelines in 2006 and asked for the comments and advice of teachers and subject specialists.

The Subject Assessment Guidelines are intended to provide clear guidance on assessment in Grades 10 to 12 from 2008.

The Department of Education wishes you success in the teaching of the National Curriculum Statement.



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## **1. PURPOSE OF THE SUBJECT ASSESSMENT GUIDELINES**

This document provides guidelines for assessment in the National Curriculum Statement Grades 10 - 12 (General). The guidelines must be read in conjunction with *The National Senior Certificate: A Qualification at Level 4 on the National Qualifications Framework (NQF)* and the relevant Subject Statements. The Subject Assessment Guidelines will be applicable for Grades 10 to 12 from 2008.

The Department of Education encourages teachers to use these guidelines as they prepare to teach the National Curriculum Statement. Teachers should also use every available opportunity to hone their assessment skills. These skills relate both to the setting and marking of assessment tasks.

## **2. ASSESSMENT IN THE NATIONAL CURRICULUM STATEMENT**

### **2.1 Introduction**

Assessment in the National Curriculum Statement is an integral part of teaching and learning. For this reason, assessment should be part of every lesson and teachers should plan assessment activities to complement learning activities. In addition, teachers should plan a formal year-long Programme of Assessment. Together the informal daily assessment and the formal Programme of Assessment should be used to monitor learner progress through the school year.

Continuous assessment through informal daily assessment and the formal Programme of Assessment should be used to:

- develop learners' knowledge, skills and values
- assess learners' strengths and weaknesses
- provide additional support to learners
- revisit or revise certain sections of the curriculum and
- motivate and encourage learners.

In Grades 10 and 11 all assessment of the National Curriculum Statement is internal. In Grade 12 the formal Programme of Assessment which counts 25% is internally set and marked and externally moderated. The remaining 75% of the final mark for certification in Grade 12 is externally set, marked and moderated. In Life Orientation however, all assessment is internal and makes up 100% of the final mark for promotion and certification.

### **2.2 Continuous assessment**

Continuous assessment involves assessment activities that are undertaken throughout the year, using various assessment forms, methods and tools. In Grades 10-12 continuous assessment comprises two different but related activities: informal daily assessment and a formal Programme of Assessment.

### **2.2.1 Daily assessment**

The daily assessment tasks are the planned teaching and learning activities that take place in the subject classroom. Learner progress should be monitored during learning activities. This informal daily monitoring of progress can be done through question and answer sessions; short assessment tasks completed during the lesson by individuals, pairs or groups or homework exercises.

Individual learners, groups of learners or teachers can mark these assessment tasks. Self-assessment, peer assessment and group assessment actively involves learners in assessment. This is important as it allows learners to learn from and reflect on their own performance.

The results of the informal daily assessment tasks are not formally recorded unless the teacher wishes to do so. In such instances, a simple checklist may be used to record this assessment. However, teachers may use the learners' performance in these assessment tasks to provide verbal or written feedback to learners, the School Management Team and parents. This is particularly important if barriers to learning or poor levels of participation are encountered.

The results of these assessment tasks are not taken into account for promotion and certification purposes.

### **2.2.2 Programme of Assessment**

In addition to daily assessment, teachers should develop a year-long formal Programme of Assessment for each subject and grade. In Grades 10 and 11 the Programme of Assessment consists of tasks undertaken during the school year and an end-of-year examination. The marks allocated to assessment tasks completed during the school year will be 25%, and the end-of-year examination mark will be 75% of the total mark. This excludes Life Orientation.

In Grade 12, the Programme of Assessment consists of tasks undertaken during the school year and counts 25% of the final Grade 12 mark. The other 75% is made up of externally set assessment tasks. This excludes Life Orientation where the internal assessment component counts 100% of the final assessment mark.

The marks achieved in each assessment task in the formal Programme of Assessment must be recorded and included in formal reports to parents and School Management Teams. These marks will determine if the learners in Grades 10 and 11 are promoted. In Grade 12, these marks will be submitted as the internal continuous assessment mark. Section 3 of this document provides details on the weighting of the tasks for promotion purposes.



### 2.2.2.1 Number and forms of assessment required for Programmes of Assessment in Grades 10 and 11

The requirements for the formal Programme of Assessment for Grades 10 and 11 are summarised in Table 2.1. The teacher must provide the Programme of Assessment to the subject head and School Management Team before the start of the school year. This will be used to draw up a school assessment plan for each of the subjects in each grade. The proposed school assessment plan should be provided to learners and parents in the first week of the first term.

**Table 2.1: Number of assessment tasks which make up the Programme of Assessment by subject in Grades 10 and 11**

SUBJECTS		TERM 1	TERM 2	TERM 3	TERM 4	TOTAL
Language 1: Home Language		4	4*	4	4*	16
Language 2: Choice of HL or FAL	HL	4	4*	4	4*	16
	FAL	4	4*	4	4*	16
Life Orientation		1	1*	1	2*	5
Mathematics or Maths Literacy		2	2*	2	2*	8
Subject choice 1**		2	2*	2	1*	7
Subject choice 2**		2	2*	2	1*	7
Subject choice 3		2	2*	2	1*	7

Note:

\* One of these tasks must be an examination

\*\* If one or two of the subjects chosen for subject choices 1, 2 or 3 include a Language, the number of tasks indicated for Languages 1 and 2 at Home Language (HL) and First Additional Language (FAL) are still applicable. Learners who opt for a Second Additional Language are required to complete 13 tasks in total: 4 tasks in term 1 and 3 tasks in each of terms 2, 3 and 4.

Two of the assessment tasks for each subject must be examinations. In Grades 10 and 11 these examinations should be administered in mid-year and November. These examinations should take account of the requirements set out in Section 3 of this document. They should be carefully designed and weighted to cover all the Learning Outcomes of the subject.

Two of the assessment tasks for all subjects, excluding Life Orientation, should be tests written under controlled conditions at a specified time. The tests should be written in the first and third terms of the year.

The remainder of the assessment tasks should not be tests or examinations. They should be carefully designed tasks, which give learners opportunities to research and explore the subject in exciting and varied ways. Examples of assessment forms are debates, presentations, projects, simulations, written reports, practical tasks, performances, exhibitions and research projects. The most appropriate forms of assessment for each subject are set out in Section 3. Care should be taken to ensure that learners cover a variety of assessment forms in the three grades.

The weighting of the tasks for each subject is set out in Section 3.

### 2.2.2.2 Number and forms of assessment required for Programme of Assessment in Grade 12

In Grade 12 all subjects include an internal assessment component, which is 25% of the final assessment mark. The requirements of the internal Programme of Assessment for Grade 12 are summarised in Table 2.2. The teacher must provide the Programme of Assessment to the subject head and School Management Team before the start of the school year. This will be used to draw up a school assessment plan for each of the subjects in each grade. The proposed school assessment plan should be provided to learners and parents in the first week of the first term.

**Table 2.2: Number of assessment tasks which make up the Programme of Assessment by subject in Grade 12**

SUBJECTS	TERM 1	TERM 2	TERM 3	TERM 4	TOTAL
Language 1: Home Language	5	5*	4*		14
Language 2: Choice of HL or FAL	HL	5	5*	4*	14
	FAL	5	5*	4*	14
Life Orientation	1	2*	2*		5
Mathematics or Maths Literacy	3	2*	2*		7
Subject choice 1**	2	2*	(2*) 3*		(6 <sup>#</sup> ) 7
Subject choice 2**	2	2*	(2*) 3*		(6 <sup>#</sup> ) 7
Subject choice 3	2	2*	(2*) 3*		(6 <sup>#</sup> ) 7

Note:

\* One of these tasks in Term 2 and/or Term 3 must be an examination

\*\* If one or two of the subjects chosen for subject choices 1, 2 or 3 include a Language, the number of tasks indicated for Languages 1 and 2 at Home Language (HL) and First Additional Language (FAL) are still applicable. Learners who opt for a Second Additional Language are required to complete 12 tasks in total: 5 tasks in term 1, 4 tasks in term 2 and 3 tasks in term 3.

# The number of internal tasks per subject differs from 6 to 7 as specified in Section 3 of this document.

Schools can choose to write one or two internal examinations in Grade 12. Should a school choose to write only one internal examination in Grade 12, a scheduled test should be written at the end of the term to replace the other examination. Internal examinations should conform to the requirements set out in Section 3 of this document. They should be carefully designed and weighted to cover all the Learning Outcomes of the subject.

Two of the assessment tasks for all subjects, excluding Life Orientation, should be tests written under controlled conditions at a specified time.

The remainder of the assessment tasks should not be tests or examinations. They should be carefully designed tasks, which give learners opportunities to research and explore the subject in exciting and focused ways. Examples of assessment forms are debates, presentations, projects, simulations, assignments, case studies, essays, practical tasks, performances, exhibitions and research projects. The most appropriate forms of assessment for each subject are set out in Section 3.

### 2.3 External assessment in Grade 12

External assessment is only applicable to Grade 12 and applies to the final end-of-year examination. This makes up 75% of the final mark for Grade 12. This excludes Life Orientation which is not externally examined.

The external examinations are set externally, administered at schools under conditions specified in the *National policy on the conduct, administration and management of the assessment of the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF)* and marked externally.

In some subjects the external assessment includes practical or performance tasks that are externally set, internally assessed and externally moderated. These performance tasks account for one third of the end-of-year external examination mark in Grade 12 (that is 25% of the final mark). Details of these tasks are provided in Section 3.

Guidelines for the external examinations are provided in Section 3.

### 2.4 Recording and reporting on the Programme of Assessment

The Programme of Assessment should be recorded in the teacher's portfolio of assessment. The following should be included in the teacher's portfolio:

- a contents page;
- the formal Programme of Assessment;
- the requirements of each of the assessment tasks;
- the tools used for assessment for each task; and
- record sheets for each class.

Teachers must report regularly and timeously to learners and parents on the progress of learners. Schools will determine the reporting mechanism but it could include written reports, parent-teacher interviews and parents' days. Schools are required to provide written reports to parents once per term on the Programme of Assessment using a formal reporting tool. This report must indicate the percentage achieved per subject and include the following seven-point scale.

RATING CODE	RATING	MARKS %
7	Outstanding achievement	80 – 100
6	Meritorious achievement	70 – 79
5	Substantial achievement	60 – 69
4	Adequate achievement	50 – 59
3	Moderate achievement	40 – 49
2	Elementary achievement	30 – 39
1	Not achieved	0 – 29

## 2.5 Moderation of the assessment tasks in the Programme of Assessment

Moderation of the assessment tasks should take place at three levels.

LEVEL	MODERATION REQUIREMENTS
School	The Programme of Assessment should be submitted to the subject head and School Management Team before the start of the academic year for moderation purposes. Each task which is to be used as part of the Programme of Assessment should be submitted to the subject head for moderation before learners attempt the task. Teacher portfolios and evidence of learner performance should be moderated twice a year by the head of the subject or her/his delegate.
Cluster/ district/ region	Teacher portfolios and a sample of evidence of learner performance must be moderated twice during the first three terms.
Provincial/ national	Teacher portfolios and a sample of evidence of learner performance must be moderated once a year.

### **3. ASSESSMENT OF MATHEMATICS IN GRADES 10 - 12**

#### **3.1 Introduction**

Assessment in Mathematics should focus on collecting reliable information regarding learners' mathematical growth and competence. Assessment includes informal assessment, formal internal assessment and external assessment. Informal or daily assessment informs the teacher about how learners are progressing towards achieving the Assessment Standards with the purpose of enhancing teaching and learning. Formal internal assessment tools should provide the teacher with a means to differentiate between learners on the scale indicated in Section 2.4. External assessment occurs in the Grade 12 National Senior Certificate examinations.

The Learning Outcomes and Assessment Standards of the Mathematics National Curriculum Statement have been divided into Core Assessment Standards (Appendix 1) and Optional Assessment Standards (Appendix 2). The Core Assessment Standards will be examined by means of two compulsory papers: Paper 1 (LO1 and LO2) and Paper 2 (LO3 and LO4). The Optional Assessment Standards will be examined by means of a single paper: Paper 3. Paper 3 will be optional to all learners in Grade 12 from 2008-2010. It is anticipated that those Assessment Standards identified as Optional will with time (after 2010) become compulsory.

For the benefit of their learners, teachers are strongly encouraged to work towards teaching the content and reasoning required by the Optional Assessment Standards as soon as possible.

Examples of examinations for Grade 10 and Grade 12 can be found at <http://www.thutong.org.za>. Teachers are encouraged to access and use them in their teaching and assessment preparation for 2006, 2007 and 2008.

#### **3.2 Daily assessment in Grades 10, 11 and 12**

Daily or informal assessment is used by teachers to make decisions about teaching and to determine how learners are progressing towards achieving the Learning Outcomes. Learner performance in such tasks is not formally recorded. The purpose of daily assessment is to evaluate the performance of individuals and the class on a certain part of the Mathematics curriculum. Therefore, the assessment tools used should tell the teacher about the strengths and weaknesses of individual learners and the class so that he or she can determine who needs more help and what kind of help is required.

Appendix 3 contains four examples of daily assessment tasks. The assessment tasks can be done in class, at home, individually or in groups. The purpose of each assessment task is to enhance learning and teaching.

### 3.3 Formal Programme of Assessment in Grades 10 and 11

The Programme of Assessment for Mathematics in Grades 10 and 11 comprises eight tasks which are internally assessed (school-based assessment). As indicated in table 3.1, the seven tasks completed during the school year make up 25% of the total mark for Mathematics, while the end-of-year examination makes up the remaining 75%. Assessment should be ongoing and spread across the school year.

Table 3.1 illustrates the forms of assessment and the weighting that should be used to compile learners' promotion mark. Table 3.1 also suggests when the assessment tasks should be given. This table applies to learners preparing for the Core Assessment Standards and to those learners preparing for both the Core and Optional Assessment Standards. In other words no additional assessment tasks are required of those learners preparing for the optional Assessment Standards.

The following table provides **an example** of a Programme of Assessment. The **order** of the tasks is not prescribed.

**Table 3.1: Example of a Programme of Assessment for Grades 10 and 11**

	GRADE 10		GRADE 11	
	TASKS	WEIGHT (%)	TASKS	WEIGHT (%)
<b>Term 1</b>	Test	10	Test	10
	Investigation	10	Investigation	10
<b>Term 2</b>	Investigation	10	Investigation	10
	Examination	30	Examination	30
<b>Term 3</b>	Project	20	Project	20
	Test	10	Test	10
<b>Term 4</b>	Assignment	10	Assignment	10
<b>Programme of Assessment mark</b>		100		100
<b>Programme of Assessment mark (as % of promotion mark)</b>		25%		25%
<b>End-of-year examinations</b>		75%		75%
<b>Promotion mark</b>		100%		100%

A minimum of 50% of the assessment mark must be obtained from assessment in Learning Outcomes 1 and 2. A minimum of 30% of the mark must be obtained from assessment in Learning Outcomes 3 and 4. These minimums apply irrespective of whether the learners are working with the Optional Assessment Standards or not.

#### 3.3.1 Examples of assessment tasks and tools

The Programme of Assessment requires assessment tasks that provide the learner with the opportunity to demonstrate his or her mathematical competence. Although not expected within each task, collectively the assessment tasks should enable the teacher to differentiate between various

levels of performance and learner competence. The most commonly used forms of assessment in Mathematics are described in Table 3.2.

**Table 3.2: Forms of assessment and suitable assessment tools**

FORM OF ASSESSMENT	ASSESSMENT TOOLS
<b>Tests</b>	Marking memorandum
<b>Exams</b>	Marking memorandum
<b>Investigations</b>	Rubric
<b>Assignment</b>	<ul style="list-style-type: none"> <li>• Class assignment – rubric or marking memorandum</li> <li>• Home assignment – rubric or marking memorandum (including an accountability test or observation sheet)</li> </ul>
<b>Project</b>	<p>Shorter projects:</p> <ul style="list-style-type: none"> <li>• An instruction sheet outlining the task, resources and allocating some class time to the project are useful when completing a short (2-5 days) project.</li> <li>• A generic rubric can be used for assessment of the project.</li> </ul> <p>Longer project:</p> <ul style="list-style-type: none"> <li>• An instruction sheet should be given to the learners well in advance to allow enough time for research.</li> <li>• A class checklist to monitor the process helps learners stay on track with their project work.</li> <li>• A rubric can be used for assessment of the project.</li> </ul>

### 3.3.2 Examinations

Examinations in Grades 10 and 11 should, where possible, have a distribution of marks similar to that of Grade 12 given in Tables 3.6 and 3.7. Grade 10 and 11 examinations should also include problem-solving questions in preparation for such questions in the National Senior Certificate Mathematics examination. The end-of-year examination in Grade 11 should be representative of the style, nature and complexity of a National Senior Certificate Mathematics examination.

Guidance regarding the number of papers, duration of each paper and timing of the papers is provided in table 3.3. The Core Assessment Standards (Appendix 1) examined through Papers 1 and 2 are compulsory. The Optional Assessment Standards are examined through Paper 3. Learners who write Paper 3 will have an endorsement on their National Senior Certificate. Teachers are encouraged to prepare themselves for the teaching of the Optional Assessment Standards and, in at least Grades 10 and 11, to teach this content as soon as they are confident to do so.

**Table 3.3: Suggested number of examination papers and times in Grades 10 and 11**

	<b>GRADE 10</b>	<b>GRADE 11</b>
<b>TERM 2</b>	Paper 1: 2 hours (100 marks)	Paper 1: 2 hours (100 marks)  Paper 2: 2 hours (100 marks)  Paper 3: 2 hours (100 marks)
<b>TERM 4</b>	Paper 1: 2 hours (100 marks)  Paper 2: 2 hours (100 marks)  Paper 3: minimum of 1 hour if offered (50 marks at least)	Paper 1: 3 hours (150 marks)  Paper 2: 3 hours (150 marks)  Paper 3: 2 hours if offered (100 marks)

The seven levels of performance described in Section 2.4 should be used to set a differentiated exam. It is recommended that teachers use the taxonomy presented in Section 3.4.2. This taxonomy has four categories of mathematical task or question that ensures an assessment capable of differentiating learner performance and classifying the performance according to the competence descriptors of the seven rating levels.

### **3.4 Assessment in Grade 12**

In Grade 12, assessment consists of two components: a Programme of Assessment which makes up 25% of the National Senior Certificate mark for Mathematics and an external examination which makes up the remaining 75%. The Programme of Assessment for Mathematics in Grade 12 consists of seven tasks which are internally assessed. The external examination is externally set and moderated.

#### **3.4.1 Programme of Assessment**

Table 3.4 illustrates the forms of assessment and their weighting that should be used in the compilation of the learners' internal Programme of Assessment mark. Table 3.4 also suggests when these assessment items should be given. This table applies to learners preparing for only the Core Assessment Standards and to learners preparing for the Core and Optional Assessment Standards. In other words, no additional assessment tasks are required of those learners working with the optional Assessment Standards.



**Table 3.4: Forms of assessment in the annual internal Programme of Assessment for Grade 12**

TERM	TASKS	WEIGHT (%)
Term 1	Test	10
	Investigation or project	20
	Assignment	10
Term 2	Assignment	10
	Examination	15
Term 3	Test	10
	Examination	25
<b>Internal assessment mark</b>		<b>100</b>

In Grade 12 one of the tasks in Term 2 and/or Term 3 must be an internal examination. In instances where only one of the two internal examinations is written in Grade 12, the other examination should be replaced by a test at the end of the term.

A minimum of 50% of the internal assessment mark must be obtained from assessment in Learning Outcomes 1 and 2. A minimum of 50% of the internal assessment mark must be obtained from assessment in Learning Outcomes 3 and 4. These minimums apply irrespective of whether the learners are working with the Optional Assessment Standards or not.

The term 2 and 3 examinations should reflect the duration, mark allocation and structure of the external National Senior Certificate Mathematics examination.

### 3.4.2 External assessment

The National Senior Certification process includes a formal external assessment at the end of Grade 12. The formal external Mathematics assessment assesses the Assessment Standards of Grades 11 and 12. However, the linear nature of Mathematics is such that work done in Grade 10 and earlier is not excluded. The assessment will consist of two compulsory papers (Paper 1 and Paper 2) and one optional paper (Paper 3). The structure, time allocation and marks of the Grade 12 National Mathematics examinations are illustrated in Table 3.5.

**Table 3.5: Summary of the National Senior Certificate external Grade 12 assessment**

EXAM PAPER	LEARNING OUTCOMES	TIME ALLOCATION	TOTAL MARKS
Paper 1	LO1 and LO2 (see Appendix 1)	3 hours	150 marks
Paper 2	LO3 and LO4 (see Appendix 1)	3 hours	150 marks
Paper 3	LO3 and LO4 (see Appendix 2)	2 hours	100 marks

The National Senior Certificate Mathematics examinations will be structured in line with the weightings indicated in Table 3.6. The level of complexity of

the mathematical questions in the examinations will be in line with the taxonomical categories given in Table 3.7. Appendix 4 gives a detailed explanation and examples of taxonomical categories of mathematical demand.

**Table 3.6: Suggested distribution of marks for Grade 12 question papers**

PAPER 1			PAPER 2			OPTIONAL PAPER		
Bookwork: max of 6 marks			Bookwork: 0 marks			Bookwork: max of 15 marks		
LO1:	Patterns and sequences	± 30	LO3:	Coordinate geometry	±40	LO1:	Recursive sequences	±5*
LO1:	Annuities and finance	±15	LO3:	Transformation	±25	LO3:	Geometry	±40
LO2:	Functions and graphs (see note below)	±35	LO3:	Trigonometry (see note below)	±60	LO4:	Descriptive statistics and interpretation	±20
LO2:	Algebra and equations	±20	LO4:	Data handling	±25	LO4:	Probability	±20
LO2:	Calculus	±35				LO4:	Bivariate data	±15
LO2:	Linear programming	±15						
<b>Total</b>		<b>150</b>	<b>Total</b>		<b>150</b>	<b>Total</b>		<b>100</b>

\* Sequences defined recursively will occasionally be used to replace some of the marks of bivariate data questions.

NOTE:

The trigonometric graphs listed in LO 10.2.2 and LO 11.2.2 can be examined in both papers 1 and 2. In Paper 1 they are incorporated into the function and graphs. In Paper 2 they may be examined in the trigonometry section.

Modelling as a process should be included in all papers.

**Table 3.7: Taxonomical differentiation of questions on Grade 12 question papers**

TAXONOMICAL CATEGORIES	APPROXIMATE PROPORTION OF THE PAPER		
	%	150 mark paper	150 mark paper
Knowledge	± 25	25 – 35	25 – 35
Performing routine procedures	± 30	30 – 40	30 - 40
Performing complex procedures	± 30	30 – 40	30 - 40
Problem Solving	± 15	15 – 25	15 – 25

The above taxonomical categories are based on the 1999 TIMSS Mathematics survey.

The taxonomy of categories of mathematical demand as suggested in the 1999 TIMMS Mathematics survey includes four cognitive levels. The four categories of cognitive levels and their related skills are provided below.

COGNITIVE LEVELS	EXPLANATION OF SKILLS TO BE DEMONSTRATED
KNOWLEDGE (25%)	<ul style="list-style-type: none"> <li>• Algorithms</li> <li>• Estimation; appropriate rounding of numbers</li> <li>• Theorems</li> <li>• Straight recall</li> <li>• Identifying from data sheet</li> <li>• Simple mathematical facts</li> <li>• Know and use of appropriate vocabulary</li> <li>• Knowledge and use of formulae</li> </ul> <p>All of the above will be based on known knowledge.</p>
ROUTINE PROCEDURES (30%)	<ul style="list-style-type: none"> <li>• Problems are not necessarily unfamiliar and can involve the integration of different LOs</li> <li>• Perform well-known procedures</li> <li>• Simple applications and calculations which must have many steps and may require interpretation from given information</li> <li>• Identifying and manipulating of formulae</li> </ul> <p>All of the above will be based on known procedures.</p>
COMPLEX PROCEDURES (30%)	<ul style="list-style-type: none"> <li>• Problems are mainly unfamiliar and learners are expected to solve by integrating different LOs</li> <li>• Problems do not have a direct route to the solution but involve:               <ul style="list-style-type: none"> <li>▪ using higher level calculation skills and reasoning to solve problems</li> <li>▪ mathematical reasoning processes</li> </ul> </li> <li>• These problems are not necessarily based on real world contexts and may be abstract requiring fairly complex procedures in finding the solutions.</li> </ul>
SOLVING PROBLEMS (15%)	<ul style="list-style-type: none"> <li>• Solving non-routine, unseen problems by demonstrating higher level understanding and cognitive processes</li> <li>• Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts</li> <li>• Using higher level cognitive skills and reasoning to solve non-routine problems</li> <li>• Being able to break down a problem into its constituent parts – identifying what is required to be solved and then using appropriate methods in solving the problem</li> <li>• Non-routine problems be based on real contexts</li> </ul>

### 3.5 Promotion

A learner must achieve a minimum of 30% (Level 2: Elementary achievement) in Mathematics for promotion at the end of Grades 10 and 11 and for certification at the end of Grade 12.

### 3.6 Moderation of internal assessment

Moderation of assessment tasks will take place at schools in Grades 10, 11 and 12. In addition, moderation of assessment in Grade 12 will also take place at cluster, district or region level as well as at provincial and national levels.

***School moderation***

The Programme of Assessment should be submitted to the head of department or subject head and School Management Team before the start of the academic year together with the Learning Programme for moderation purposes. Each task that will be used for the Programme of Assessment should be submitted to the head of department or subject head for moderation before the learners are given the work to do. The learner tasks should be moderated at least once per term by the head of department, subject head, or his or her delegate.

***Cluster, district or region moderation***

Teacher portfolios and a sample of learner tasks will be moderated twice during the first three terms. This can be done by cluster co-ordinators or district subject co-ordinators.

# APPENDIX 1: MATHEMATICS CORE ASSESSMENT STANDARDS FOR EXAMINATION IN GRADE 12 IN 2008, 2009 and 2010

## Learning Outcome 1: Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:
10.1.1 Identify rational numbers and convert between terminating or recurring decimals and the form: $\frac{a}{b}; a, b \in \mathbb{Z}; b \neq 0$	11.1.1 Understand that not all numbers are real. (This requires the recognition but not the study of non-real numbers.)	
10.1.2 (a) Simplify expressions using the laws of exponents for integral exponents. (b) Establish between which two integers any simple surd lies. (c) Round rational and irrational numbers to an appropriate degree of accuracy.	11.1.2 (a) Simplify expressions using the laws of exponents for rational exponents. (b) Add, subtract, multiply and divide simple surds (e.g. $\sqrt{3} + \sqrt{12} = 3\sqrt{3}$ and $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ ) (c) Demonstrate an understanding of error margins.	12.1.2 Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems (e.g. growth and decay see 12.1.4(a)).
10.1.3 Investigate number patterns (including but not limited to those where there is a constant difference between consecutive terms in a number pattern, and the general term is therefore linear) and hence: (a) make conjectures and generalisations (b) provide explanations and justifications and attempt to prove conjectures.	11.1.3 Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence: (a) make conjectures and generalisations (b) provide explanations and justifications and attempt to prove conjectures.	12.1.3 (a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series. (b) Correctly interpret sigma notation. (c) Prove and correctly select the formula for and calculate the sum of series, including: $\sum_{i=1}^n 1 = n$ $\sum_{i=1}^n i = \frac{n(n+1)}{2}$ $\sum_{i=1}^n a + (i-1)d = \frac{n}{2} [2a + (n-1)d]$

		$\sum_{i=1}^n a \times r^{i-1} = \frac{a(r^n - 1)}{r - 1}; r \neq 1$ $\sum_{i=1}^{\infty} a \times r^{i-1} = \frac{a}{1 - r} \text{ for } -1 < r < 1$
10.1.4 Use simple and compound growth formulae ( $A = P(1 + ni)$ and $A = P(1 + i)^n$ ) to solve problems, including interest, hire-purchase, inflation, population growth and other real-life problems.	11.1.4 Use simple and compound decay formulae ( $A = P(1 - ni)$ and $A = P(1 - i)^n$ ) to solve problems (including straight line depreciation and depreciation on a reducing balance) ( <i>link to Learning Outcome 2</i> ).	12.1.4 (a) Calculate the value of n in the formulae: $A = P(1 \pm i)^n$ (b) Apply knowledge of geometric series to solving annuity, bond repayment and sinking fund problems, with or without the use of the formulae: $F = \frac{x[(1 + i)^n - 1]}{i} \text{ and } P = \frac{x[1 - (1 + i)^{-n}]}{i}$
10.1.5 Demonstrate an understanding of the implications of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).	11.1.5 Demonstrate an understanding of different periods of compounding growth and decay (including effective compounding growth and decay and including effective and nominal interest rates).	12.1.5 Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).
10.1.6 Solve non-routine, unseen problems.	11.1.6 Solve non-routine, unseen problems.	12.1.6 Solve non-routine, unseen problems.

## Learning Outcome 2: Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:
<p>10.2.1</p> <p>(a) Demonstrate the ability to work with various types of functions, including those listed in the following Assessment Standard.</p> <p>(b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).</p>	<p>11.2.1</p> <p>(a) Demonstrate the ability to work with various types of functions including those listed in the following Assessment Standard.</p> <p>(b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).</p>	<p>12.2.1</p> <p>(a) Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard.</p> <p>(b) Demonstrate knowledge of the formal definition of a function</p>
<p>10.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generalise the effects of the parameters <math>a</math> and <math>q</math> on the graphs of functions including:</p> <p><math>y = ax + q</math></p> <p><math>y = ax^2 + q</math></p> <p><math>y = \frac{a}{x} + q</math></p> <p><math>y = ab^x + q; b &gt; 0</math></p> <p><math>y = a \sin(x) + q</math></p> <p><math>y = a \cos(x) + q</math></p> <p><math>y = a \tan(x) + q</math></p>	<p>11.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters <math>k, p, a</math> and <math>q</math> for functions including:</p> <p><math>y = \sin(kx)</math></p> <p><math>y = \cos(kx)</math></p> <p><math>y = \tan(kx)</math></p> <p><math>y = \sin(x + p)</math></p> <p><math>y = \cos(x + p)</math></p> <p><math>y = \tan(x + p)</math></p> <p><math>y = a(x + p)^2 + q</math></p> <p><math>y = \frac{a}{x + p} + q</math></p> <p><math>y = ab^{x+p} + q; b &gt; 0</math></p>	<p>12.2.2</p> <p>(a) Investigate and generate graphs of the inverse relations of functions, in particular the inverses of:</p> <p><math>y = ax + q</math></p> <p><math>y = ax^2</math></p> <p><math>y = a^x; a &gt; 0</math></p> <p>(b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.</p>

<p>10.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed in 10.2.2 above:</p> <ul style="list-style-type: none"> <li>(a) domain and range</li> <li>(b) intercepts with the axes</li> <li>(c) turning points, minima and maxima</li> <li>(d) asymptotes</li> <li>(e) shape and symmetry</li> <li>(f) periodicity and amplitude</li> <li>(g) average gradient (average rate of change)</li> <li>(h) intervals on which the function increases/decreases</li> <li>(i) the discrete or continuous nature of the graph.</li> </ul>	<p>11.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed above:</p> <ul style="list-style-type: none"> <li>a) domain and range;</li> <li>b) intercepts with the axes;</li> <li>c) turning points, minima and maxima;</li> <li>d) asymptotes;</li> <li>e) shape and symmetry;</li> <li>f) periodicity and amplitude;</li> <li>g) average gradient (average rate of change);</li> <li>h) intervals on which the function increases/decreases;</li> <li>i) the discrete or continuous nature of the graph.</li> </ul>	<p>12.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above:</p> <ul style="list-style-type: none"> <li>(a) domain and range;</li> <li>(b) intercepts with the axes;</li> <li>(c) turning points, minima and maxima;</li> <li>(d) asymptotes;</li> <li>(e) shape and symmetry;</li> <li>(f) average gradient (average rate of change);</li> <li>(g) intervals on which the function increases/decreases.</li> </ul>
<p>10.2.4 Manipulate algebraic expressions:</p> <ul style="list-style-type: none"> <li>(a) multiplying a binomial by a trinomial</li> <li>(b) factorising trinomials</li> <li>(c) factorising by grouping in pairs</li> <li>(d) simplifying algebraic fractions with monomial denominators</li> </ul>	<p>11.2.4 Manipulate algebraic expressions:</p> <ul style="list-style-type: none"> <li>(a) by completing the square;</li> <li>(b) simplifying algebraic fractions with binomial denominators.</li> </ul>	<p>12.2.4 Factorise third degree polynomials (including examples which require the factor theorem).</p>
<p>10.2.5 Solve:</p> <ul style="list-style-type: none"> <li>(a) linear equations</li> <li>(b) quadratic equations by factorisation</li> <li>(c) exponential equations of the form <math>ka^{x+p} = m</math> (including examples solved by trial and error)</li> <li>(d) linear inequalities in one variable and illustrate the solution graphically</li> <li>(e) linear equations in two variables simultaneously (numerically, algebraically and graphically)</li> </ul>	<p>11.2.5 Solve:</p> <ul style="list-style-type: none"> <li>(a) quadratic equations (by factorisation, by completing the square, and by using the quadratic formula) and quadratic inequalities in one variable and interpret the solution graphically;</li> <li>(b) equations in two unknowns, one of which is linear and one of which is quadratic, algebraically or graphically.</li> </ul>	
<p>10.2.6 Use mathematical models to investigate problems that arise in real-life contexts:</p> <ul style="list-style-type: none"> <li>a) making conjectures, demonstrating and explaining their validity;</li> <li>b) expressing and justifying mathematical generalisations of situations;</li> <li>c) using the various representations to interpolate and extrapolate;</li> <li>d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and features.</li> </ul> <p>(Examples should include issues related to health, social, economic, cultural, political and environmental matters).</p>	<p>11.2.6 Use mathematical models to investigate problems that arise in real-life contexts:</p> <ul style="list-style-type: none"> <li>(a) making conjectures, demonstrating and explaining their validity;</li> <li>(b) expressing and justifying mathematical generalisations of situations;</li> <li>(c) using various representations to interpolate and extrapolate;</li> <li>(d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and pertinent features.</li> </ul> <p>(Examples should include issues related to health, social, economic, cultural, political and environmental matters.)</p>	



<p>10.2.7 Investigate the average rate of change of a function between two values of the independent variable, demonstrating an intuitive understanding of average rate of change over different intervals (e.g. investigate water consumption by calculating the average rate of change over different time intervals and compare results with the graph of the relationship).</p>	<p>11.2.7 Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point.</p>	<p>12.2.7</p> <p>(a) Investigate and use instantaneous rate of change of a variable when interpreting models of situations:</p> <ul style="list-style-type: none"> <li>demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient of a function at point;</li> <li>establishing the derivatives of the following functions from first principles:           <math display="block">f(x) = b</math> <math display="block">f(x) = x</math> <math display="block">f(x) = x^2</math> <math display="block">f(x) = x^3</math> <math display="block">f(x) = \frac{1}{x}</math>           and then generalise to the derivative of           <math display="block">f(x) = x^n</math> </li> </ul> <p>(b) Use the following rules of differentiation:</p> $\frac{d}{dx} [f(x) \pm g(x)] = \frac{d}{dx} [f(x)] \pm \frac{d}{dx} [g(x)]$ $\frac{d}{dx} [kf(x)] = k \frac{d}{dx} [f(x)]$ <p>(c) Determine the equations of the tangents to graphs.</p> <p>(d) Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the x-axis.</p> <p>(e) Solve practical problems involving optimisation and rates of change.</p>
	<p>11.2.8</p> <p>a) Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region.</p> <p>b) Solve a system of linear equations to find the co-ordinates of the vertices of the feasible region.</p>	<p>12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.</p>

### Learning Outcome 3: Space, Shape and Measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:
10.3.1 Understand and determine the effect on the volume and surface area of right prisms and cylinders, of multiplying any dimension by a constant factor $k$ .	11.3.1 Use the formulae for surface area and volume of right pyramids, right cones, spheres and combinations of these geometric objects.	
10.3.2 (a) Through investigations, produce conjectures and generalisations related to triangles, quadrilaterals and other polygons, and attempt to validate, justify, explain or prove them, using any logical method (Euclidean, co-ordinate and/or transformation).		
10.3.3 Represent geometric figures on a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1; y_1)$ and $(x_2; y_2)$ , a formula for calculating: (a) the distance between the two points (b) the gradient of the line segment joining the points (c) the co-ordinates of the mid-point of the line segment joining the points.	11.3.3 Use a Cartesian co-ordinate system to derive and apply: (a) the equation of a line through two given points (b) the equation of a line through one point and parallel or perpendicular to a given line (c) the inclination of a line.	12.3.3 Use a two-dimensional Cartesian co-ordinate system to derive and apply: (a) the equation of a circle (any centre); (b) the equation of a tangent to a circle given a point on the circle.  NOTE learners are expected to know and be able to use as an axiom: “the tangent to a circle is perpendicular to the radius drawn to the point of contact”
10.3.4 Investigate, generalise and apply the effect of the following transformations of the point $(x; y)$ : a) a translation of $p$ units horizontally and $q$ units vertically b) a reflection in the $x$ -axis, the $y$ -axis or the line $y = x$ .	11.3.4 Investigate, generalise and apply the effect on the co-ordinates of: (a) the point $(x; y)$ after rotation around the origin through an angle of $90^\circ$ or $180^\circ$ ; (b) the vertices $(x_1; y_1), (x_2; y_2), \dots, (x_n; y_n)$ of a polygon after enlargement through the origin, by a constant factor $k$ .	12.3.4 (a) Use the compound angle identities to generalise the effect on the co-ordinates of the point $(x; y)$ after rotation about the origin through an angle. (b) Demonstrate the knowledge that rigid transformations (translations, reflections, rotations and glide reflections) preserve shape and size, and that enlargement preserves shape but not size.
10.3.5 Understand that the similarity of triangles is fundamental to the trigonometric functions $\sin \theta$ , $\cos \theta$ , and $\tan \theta$ , and is able to define and use the functions.	11.3.5 (a) Derive and use the values of the trigonometric functions (in surd form where applicable) of $30^\circ$ , $45^\circ$ and $60^\circ$ . (b) Derive and use the following identities: • $\tan \theta = \frac{\sin \theta}{\cos \theta}$ • $\sin^2 \theta + \cos^2 \theta = 1$ (c) Derive the reduction formulae for:	12.3.5 Derive and use the following compound angle identities (without derivation): (a) $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$ (b) $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ (c) $\sin 2\alpha = 2 \sin \alpha \cos \alpha$

	$\sin(90^\circ \pm \theta)$ , $\cos(90^\circ \pm \theta)$ , $\sin(180^\circ \pm \theta)$ , $\cos(180^\circ \pm \theta)$ , $\tan(180^\circ \pm \theta)$ , $\sin(360^\circ \pm \theta)$ , $\cos(360^\circ \pm \theta)$ , $\tan(360^\circ \pm \theta)$ , $\sin(-\theta)$ , $\cos(-\theta)$ , $\tan(-\theta)$ (d) Determine the general solution of trigonometric equations (e) Determine the general solution of trigonometric equations (f) Establish and apply the sine, cosine and area rules.	$(d) \cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \\ 1 - 2 \sin^2 \alpha \end{cases}$
10.3.6 Solve problems in two dimensions by using the trigonometric functions ( $\sin \theta$ , $\cos \theta$ , and $\tan \theta$ ) in right-angled triangles and by constructing and interpreting geometric and trigonometric models (examples to include scale drawings, maps and building plans).	11.3.6 Solve problems in two dimensions by using the sine, cosine and area rules; and by constructing and interpreting geometric and trigonometric models.	12.3.6 Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models.
10.3.7 (Not to be examined, can be assessed internally by means of a project) Demonstrate an appreciation of the contributions to the history of the development and use of geometry and trigonometry by various cultures through a project.	11.3.7 (Not to be examined, can be assessed internally by means of a project) Demonstrate an appreciation of the contributions to the history of the development and use of geometry and trigonometry by various cultures through educative forms of assessment (e.g. an investigative project).	12.3.7 (Not to be examined, can be assessed internally.) Demonstrate a basic understanding of the development and uses of geometry through history and some familiarity with other geometries (e.g. spherical geometry, taxi-cab geometry, and fractals).

## Learning Outcome 4: Data Handling and Probability

*The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.*

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:
<p>10.4.1</p> <p>(a) Collect, organise and interpret univariate numerical data in order to determine:</p> <ul style="list-style-type: none"> <li>• measures of central tendency (mean, median, mode) of grouped and ungrouped data, and know which is the most appropriate under given conditions;</li> <li>• measures of dispersion: range, percentiles, quartiles, inter-quartile and semi-inter-quartile range.</li> </ul> <p>(b) Represent data effectively, choosing appropriately from:</p> <ul style="list-style-type: none"> <li>• bar and compound bar graphs;</li> <li>• histograms (grouped data);</li> <li>• frequency polygons;</li> <li>• pie charts;</li> <li>• line and broken line graphs.</li> </ul>	<p>11.4.1</p> <p>(a) Calculate and represent measures of central tendency and dispersion in univariate numerical data by:</p> <ul style="list-style-type: none"> <li>• five number summary (maximum, minimum and quartiles);</li> <li>• box and whisker diagrams;</li> <li>• ogives;</li> <li>• calculating the variance and standard deviation of sets of data manually (for small sets of data) and using available technology (for larger sets of data), and representing results graphically using histograms and frequency polygons.</li> </ul> <p>(b) Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health, social, economic, cultural, political and environmental issues).</p>	

## APPENDIX 2: OPTIONAL MATHEMATICS ASSESSMENT STANDARDS FOR EXAMINATION IN GRADE 12 IN 2008, 2009 and 2010

### Learning Outcome 1: Number and Number Relationships

*When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.*

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:
		12.1.3 (d) Correctly interpret recursive formulae: (e.g. $T_{n+1} = T_n + T_{n-1}$ )

### Learning Outcome 2: Functions and Algebra

*The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.*

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:

### Learning Outcome 3: Space, Shape and Measurement

*The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.*

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:
10.3.2 (a) Disprove false conjectures by producing counter-examples. (b) Investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangle, the kite, parallelogram, rectangle, rhombus and square).	11.3.2 (a) Investigate necessary and sufficient conditions for polygons to be similar. (b) Prove and use (accepting results established in earlier grades): <ul style="list-style-type: none"> <li>• that a line drawn parallel to one side of a triangle divides the other two sides proportionally (the Mid-point Theorem as a special case of this theorem);</li> <li>• that equiangular triangles are similar;</li> <li>• that triangles with sides in proportion are</li> </ul>	12.3.2 (a) Accept the following as axioms: <ul style="list-style-type: none"> <li>• results established in earlier grades</li> <li>• a tangent is perpendicular to the radius, drawn at the point of contact with the circle, and then investigate and prove the theorems of the geometry of circles:</li> <li>• the line drawn from the centre of a circle, perpendicular to a chord, bisects the chord and its converse</li> <li>• the perpendicular bisector of a chord passes</li> </ul>

	<p>similar;</p> <ul style="list-style-type: none"> <li>the Pythagorean Theorem by similar triangles.</li> </ul>	<p>through the centre of the circle</p> <ul style="list-style-type: none"> <li>the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle</li> <li>angles subtended by a chord at the circle on the same side of the chord are equal and its converse</li> <li>the opposite angles of a cyclic quadrilateral are supplementary and its converse</li> <li>two tangents drawn to a circle from the same point outside the circle are equal in length</li> <li>the angles between a tangent and a chord, drawn to the point of contact of the chord, are equal to the angles which the chord subtends in the alternate chord segments and its converse.</li> </ul> <p>(b) Use the theorems listed above to:</p> <ul style="list-style-type: none"> <li>make and prove or disprove conjectures</li> <li>prove riders.</li> </ul>
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## Learning Outcome 4: Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:	Grade 12 We know this when the learner is able to:
		12.4.1 (a) Demonstrate the ability to draw a suitable sample from a population and understand the importance of sample size in predicting the mean and standard deviation of a population. (b) Use available technology to calculate the regression function which best fits a given set of bivariate numerical data. (c) Use available technology to calculate the correlation co-efficient of a set of bivariate numerical data to make relevant deductions.
10.4.2 (a) Use probability models for comparing the relative frequency of an outcome with the probability of an outcome (understanding, for example, that it takes a very large number of trials before the relative frequency of throwing a head approaches the probability of throwing a head). (b) Use Venn diagrams as an aid to solving probability problems, appreciating and correctly identifying: <ul style="list-style-type: none"> <li>• the sample space of a random experiment;</li> <li>• an event of the random experiment as a subset of the sample space;</li> <li>• the union and intersection of two or more subsets of the sample space;</li> <li>• <math>P(S) = 1</math> (where S is the sample space);</li> <li>• <math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math> (where A and B are events within a sample space);</li> <li>• disjoint (mutually exclusive) events, and is therefore able to calculate the probability of either of the events occurring by applying the addition rule for disjoint events: <math>P(A \text{ or } B) = P(A) + P(B)</math>;</li> <li>• complementary events, and is therefore able to calculate the probability of an event not occurring:</li> </ul>	11.4.2 (a) Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events: $P(A \text{ and } B) = P(A).P(B)$ . (b) Use tree and Venn diagrams to solve probability problems (where events are not necessarily independent).	12.4.2 Generalise the fundamental counting principle (successive choices from $m_1$ then $m_2$ then $m_3 \dots$ options create $m_1 \times m_2 \times m_3 \dots$ different combined options) and solve problems using the fundamental counting principle.

	$P(\text{not } A) = 1 - P(A)$ .	
10.4.3 (a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here). (b) Effectively communicate conclusions and predictions that can be made from the analysis of data.	11.4.3 (a) Identify potential sources of bias, error in measurement, potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here). (b) Effectively communicate conclusions and predictions that can be made from the analysis of data.	12.4.3 (a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here). (b) Effectively communicate conclusions and predictions that can be made from the analysis of data.
	11.4.4 Differentiate between symmetric and skewed data and make relevant deductions.	12.4.4 Identify data which is normally distributed about a mean by investigating appropriate histograms and frequency polygons.
10.4.5 (Not to be assessed in an examination, can be assessed internally by means of a project) Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).	11.4.5 (Not to be assessed in an examination, can be assessed internally by means of a project) Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).	12.4.5 (Not to be assessed in an examination, can be assessed internally.) Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).



## APPENDIX 3: EXAMPLES OF TASKS THAT CAN BE USED FOR DAILY ASSESSMENT TO MONITOR PROGRESS

### TASK 1

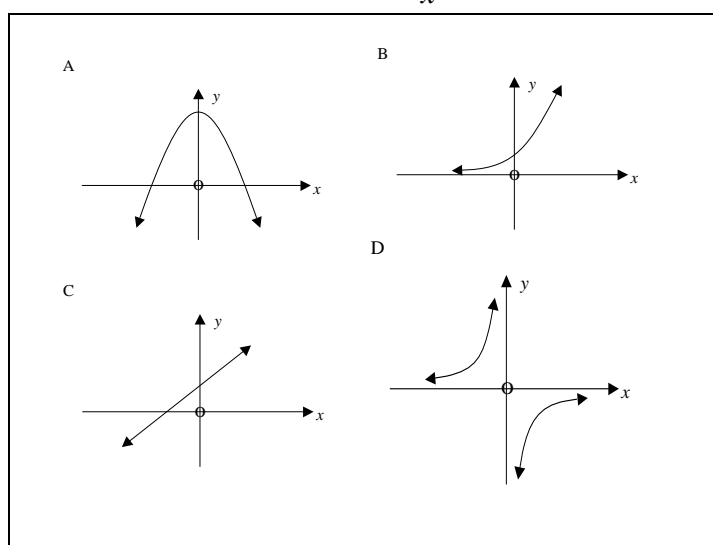
Grade: 10

LO2: Functions and algebra

AS 10.2.2: Understanding the effects of a and q on different graphs

Given: A card with four graphs representing the following equations:

$$y = ax + q; \quad y = ax^2 + q; \quad y = \frac{a}{x} + q; \quad y = ab^x + q \quad (b > 0)$$



#### Questions:

1. Choose which one of the above equations suits each graph best. Give a reason for your choice.
2. State in each case if  $a > 0$  or  $a < 0$ . Motivate your answer.
3. State in each case if  $q > 0$ ;  $q < 0$  or  $q = 0$ . Motivate your answer.
4. What is the effect on each of the graphs if  $q$  increases? Why?
5. If  $a = 0$  in any of the equations, sketch an example of the graph that may arise.

#### Notes:

1. The class work can be done in groups or individually.
2. Communicating within a group and eventually as a group with the rest of the class will enhance mathematical learning.
3. Teachers can learn a lot about learners' grasp of Mathematics and their ability to communicate mathematically by observing learners describe their mathematical ideas in words.
4. Teachers can have a few cards with different graphs for practice.

## **TASK 2**

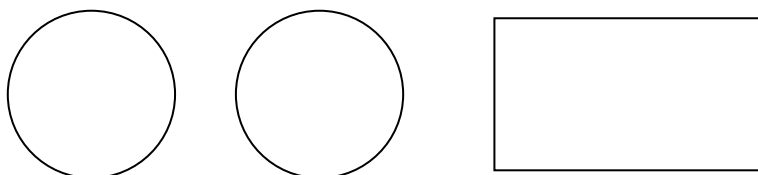
**Grade:** 10

**LO 3:** Measurement

**AS 10.3.1:** Calculation of Volume, Surface Area and scale factors of objects

### **Assignment:**

1. Take any tin (of cooldrink, peas, mushrooms, soup, etc.)
2. Measure the diameter of the top or base and the height of the tin.
3. Write the dimensions of each part on the figures below:



4. Determine the following in  $\text{cm}^2$  (rounding off to 2 decimal places):
  - a) the area of the circles;
  - b) the area of the rectangle; and
  - c) the surface area of the tin.
5. Determine the volume of the tin in  $\text{cm}^3$  (rounding off to 2 decimal places).
6. Read the volume of the tin's contents from the label.
7. Calculate the volume of air in the tin when it contains the product.
8. Why do you think there needs to be space for air in the tin?
9. If the metal to make the tin costs 0,13 cents/ $\text{cm}^2$ , calculate the cost of making the tin.
10. The manufacturer of the tin wants to double the volume of the tin, but keep the radius the same. By what factor must she increase the height?
11. If the radius is doubled, but the height stays the same, by what scale factor will:
  - a) the area of the base of the tin increase?
  - b) the area of the side surface increase?

### **TASK 3**

Grade: 10

LO1: Numbers and number relationships

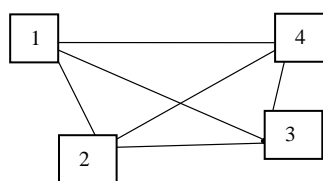
AS10.1.3: Number patterns

#### **Assignment:**

Thabiseng is currently researching her family history. She needs to draw a picture to show all possible relationships between the members in her family. You decide to assist her in this task.

1. As a group complete the table below in search of a pattern

<b>Number of family members</b>	<b>Number of relationships</b>
<b>2</b>	<b>1</b>
<b>3</b>	<b>3</b>
<b>4</b>	<b>6</b>
<b>5</b>	
<b>6</b>	
<b>7</b>	
<b>8</b>	
<b>9</b>	
<b>10</b>	



The number of relationships between the four family members in the diagram above is indicated by the number of lines which connect them. In this instance it takes six lines to show all the relationships between the four members.

2. Use graph paper to plot the relationship between the number of family members (x-axis) and the number of relationships (y-axis).
3. What type of graph is represented? Explain your answer.
4. Is this a continuous graph or not? Explain your answer.
5. Use the table and the graph to work out a formula that relates the number of family members to the number of relationships.
6. What will the number of relationships be among 50 family members?

**Comment:** Compare the groups' answers and let them discuss their differences.

**TASK 4**

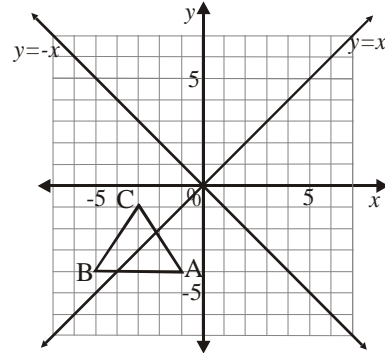
**Grade:** 10

**LO3:** Numbers and number relationships

**AS 10.3.4:** To assess the effect of the following transformations of the point  $(x;y)$ :  
 (a) a translation of  $p$  units horizontally and  $q$  units vertically;  
 (b) a reflection in the  $x$ -axis, the  $y$ -axis or the line  $y = x$ .

**Assignment:**

1. The shape ABC is given on graph paper.

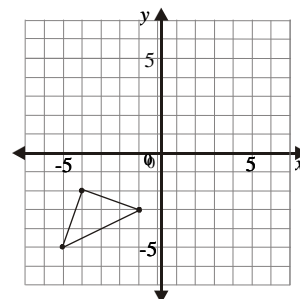
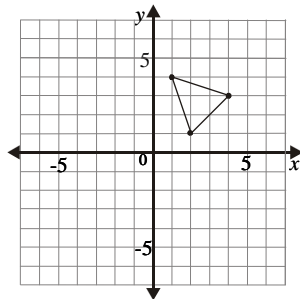


- a) Write down the co-ordinates of A, B and C on the graph. ....  
 .....  
 .....
- b) Transform ABC to A'B'C' so that the x-co-ordinates of A'B'C' are 6 more than the x-co-ordinates of ABC but the y-co-ordinates are unchanged.  
 c) Describe the transformation. ....  
 .....  
 .....
- d) Transform ABC to A''B''C'' so that the y-co-ordinates of A''B''C'' are 7 more than the y-co-ordinates of ABC but the x-co-ordinates are unchanged.  
 e) Describe the transformation. ....  
 .....  
 .....
- f) Perform the transformation of ABC to A'''B'''C''' that changes the signs of all the x and y-co-ordinates.

2. Draw the image of these shapes according to the rules given and identify the type of transformation.

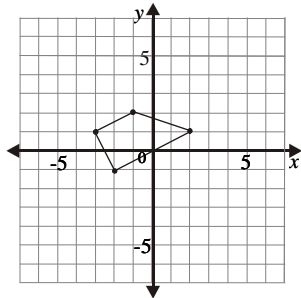
a)  $(x; y) \rightarrow (x - 4; y)$

b)  $(x; y) \rightarrow (-x; -y)$



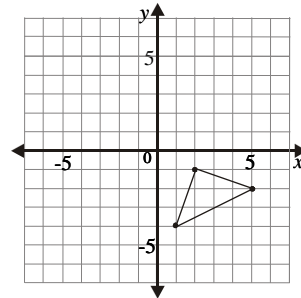
c)  $(x; y) \rightarrow (x + 3; y - 3)$

.....  
 .....



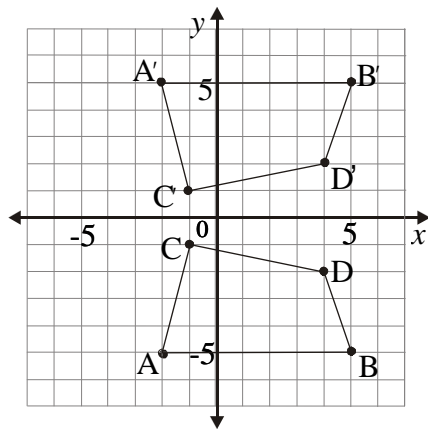
d)  $(x; y) \rightarrow (y; x)$

.....  
 .....



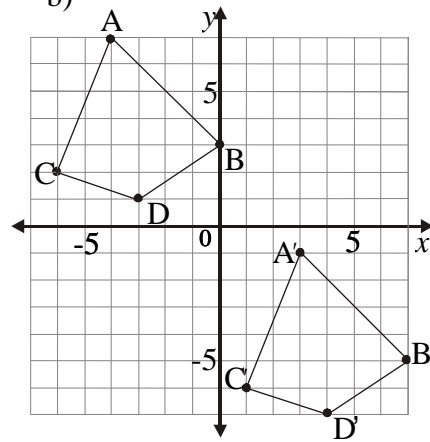
3. Write a rule for the transformation of shape ABCD to A'B'C'D' in each case below:

a)



.....  
 .....

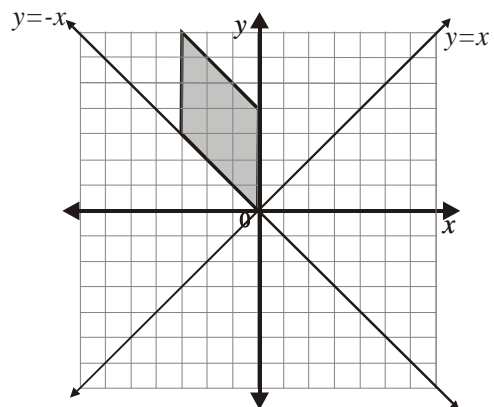
b)



.....  
 .....

4. Create a stylised flower using the given shape by:

- a) reflecting it in the line  $y = x$ ; and
- b) repeatedly reflecting the image created in question (a) in the  $x$ - and  $y$ -axis.



## APPENDIX 4: TAXONOMY OF CATEGORIES OF MATHEMATICAL DEMAND

The following tables provide examples of the types of questions that can be set for each of the four categories of mathematical demand in Grades 10, 11 and 12. The examples included are not comprehensive in nature and do not cover all areas of content.

KNOWLEDGE		
Algorithms; estimation; appropriate rounding of numbers; theorems; simple recall of mathematical facts; identifying from data sheet; use of formulae.		
GRADE 10	GRADE 11	GRADE 12
<ul style="list-style-type: none"> <li>Know that a number pattern with a constant difference has a linear formula.</li> <li>Solve for x: <math>2x + 1 = 5</math></li> <li>Draw the graph of <math>y = \sin x</math></li> <li>Reflect a point (x; y) in the x-axis.</li> <li>Factorise a trinomial like <math>x^2 + 3x + 2</math></li> </ul>	<ul style="list-style-type: none"> <li>Simplify: <math>\sin(180^\circ + x)</math></li> <li>Simplify <math>\sqrt{3} + \sqrt{12}</math></li> <li>Rotate a point (x; y) around the origin through an angle of <math>180^\circ</math></li> <li>Give the domain of a polynomial function</li> <li>Find the y-intercept of a function</li> </ul>	<ul style="list-style-type: none"> <li>Determine the 25th term of the sequence 7, 11, 15, ...</li> <li>Find <math>f'(x)</math> if <math>f(x) = 2x^2 - x + 3</math></li> <li>Determine the mode</li> <li>Find asymptotes of a rational function</li> </ul> <p>Optional:</p> <ul style="list-style-type: none"> <li>Prove that opposite angles in a cyclic quadrilateral are supplementary</li> </ul>

ROUTINE PROCEDURES		
Problems are not necessarily unfamiliar and can involve the integration of different LOs. Perform well-known procedures. Simple applications and calculations which must have many steps and may require interpretation from given information Identifying and manipulating of formulae.		
GRADE 10	GRADE 11	GRADE 12
<ul style="list-style-type: none"> <li>Solve for x: <math>x^2 + 3x + 1 = 5</math></li> <li>Draw the graph of <math>y = 3\sin(x)</math></li> <li>Combine a reflection and a translation of a point (x; y)</li> <li>Determine the formula for a sequence with a first order difference</li> </ul>	<ul style="list-style-type: none"> <li>Solve for x: <math>7 \cos x = 2</math></li> <li>Find the mean and standard deviation using a calculator</li> </ul> <p>Simplify:</p> <ul style="list-style-type: none"> <li><math>(x^2 - 4) = 3x - 9</math></li> <li>Draw the graph of <math>y = \sin(3x)</math></li> <li>Draw the graph of <math>y = 3(x - 2)^2 + 1</math></li> <li>Rotate (x; y) about the origin through an angle of <math>90^\circ</math></li> <li>Produce a box-and-whisker plot</li> <li>Find asymptotes</li> <li>Calculating reducing balance depreciation.</li> </ul> <p>Optional:</p> <ul style="list-style-type: none"> <li>Fill in values (not all of which are given) on a given Venn diagram</li> </ul>	<ul style="list-style-type: none"> <li>If the 5<sup>th</sup> term of an arithmetic sequence is 12 and the 12<sup>th</sup> term is 33, determine the first term and the constant difference</li> <li>Change between effective and nominal interest rates</li> <li>Find <math>f'(x)</math> if <math>f(x) = 4x^{-1} + \sqrt{x}</math></li> <li>Rotating (x; y) about the origin through an angle of <math>50^\circ</math></li> <li>Calculate n in <math>A = P(1+i)^n</math></li> <li>Solve the value of an annuity</li> </ul> <p>Optional:</p> <ul style="list-style-type: none"> <li>Find the correlation between two sets of data using a calculator</li> </ul>

**COMPLEX PROCEDURES**

Problems are mainly unfamiliar and learners are expected to solve by integrating different LOs. Problems do not have a direct route to the solution but require higher level calculation skills and reasoning to solve problems. These problems are not necessarily based on real world contexts and may be abstract requiring fairly complex procedures in finding the solutions.

GRADE 10	GRADE 11	GRADE 12
<ul style="list-style-type: none"> <li>• Solve for x: <math>5 \cdot 2^{x+1} - 8 = 32</math></li> <li>• Factorise <math>4x^5 - 3x^4 - 4x + 3</math></li> <li>• Draw the graph of <math>y = 2\sin(x) + 1</math></li> <li>• Determine the median of grouped data</li> </ul>	<ul style="list-style-type: none"> <li>• Complete the square to solve a quadratic equation</li> <li>• Solve 3-D trig problems</li> <li>• Solve x: <math>\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} = 1</math></li> <li>• Determine a formula for a sequence with a second order difference</li> <li>• Sketch <math>y = 2 \cos (x + 30^\circ)</math></li> <li>• Calculate the standard deviation</li> </ul> <p>Optional:</p> <ul style="list-style-type: none"> <li>• Create a Venn diagram to solve a probability question</li> </ul>	<ul style="list-style-type: none"> <li>• Determine the value of a sinking fund and the required monthly deposits</li> <li>• Solve <math>x \in [-180^\circ; 180^\circ]</math> <math>\cos x = \frac{1}{2} - \sin x</math></li> <li>• Max and min calculus problems</li> <li>• Rotate a quadrilateral about the origin through an angle of <math>35^\circ</math></li> <li>• Calculate n in <math>F = \frac{x[(1+i)^n - 1]}{i}</math></li> </ul> <p>Optional:</p> <ul style="list-style-type: none"> <li>• Solve riders where a number of steps are required to connect the required parts (e.g. prove a line is a tangent to a circle)</li> </ul>

### SOLVING PROBLEMS

Solving non-routine, unseen problems by demonstrating higher level understanding and cognitive processes.  
 Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts.  
 Using higher level cognitive skills and reasoning to solve non-routine problems.  
 Analysing problems to identify what is to be solved and then use appropriate methods to solve the problems.  
 Solve non-routine problems based on real life context.

GRADE 10	GRADE 11	GRADE 12
<ul style="list-style-type: none"> <li>• Determine the units digit of the product of the first 100 prime numbers</li> <li>• Solve for x:  <math>2^x = 17</math> (trial and error is expected)</li> <li>• Express:  <math>0.\bar{1} + 0.0\bar{1} + 0.000\bar{1}</math>                      as a simplified fraction</li> <li>• Open mathematical modelling tasks</li> <li>• Non-routine problems</li> </ul>	<ul style="list-style-type: none"> <li>• Connect (0,0) to (5,3) with a line segment, where it goes through 7 unit squares. If you connect (0,0) to (p,q) where p and q are positive whole numbers, how many squares does the line go through?</li> <li>• Patterning tasks set in a context that requires conjecture before a relationship can be found</li> <li>• Open mathematical modelling tasks.</li> <li>• Non-routine problems</li> </ul> <p>Optional:</p> <ul style="list-style-type: none"> <li>• In the figure AE=6, EB=7 and BC=5. What is the area of EBCD?</li> </ul> <div style="text-align: center;"> </div>	<ul style="list-style-type: none"> <li>• Non-routine problems</li> <li>• Determine the maximum value of:  <math display="block">\frac{5 - 3 \cos x}{3 + 2 \cos x}</math></li> <li>• Compare higher purchase, micro-lending and bank loan options</li> <li>• Given that <math>\sqrt{-1} = i</math></li> <li>• Write the roots of <math>x^2 + 2x + 5 = 0</math> in terms of i</li> <li>• Determine <math>i^{2003}</math></li> </ul> <p>Optional:</p> <ul style="list-style-type: none"> <li>• In how many ways can the letters of SOCCER be arranged so that the arrangements start with C and end with S?</li> </ul>