basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P1/*WISKUNDE V1*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2014

MEMORANDUM

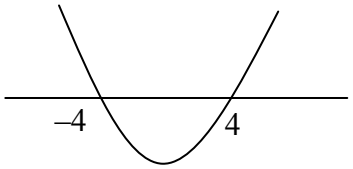
MARKS/*PUNTE*: 150

This memorandum consists of 17 pages.
Hierdie memorandum bestaan uit 17 bladsye.

NOTE:
- If a candidate answered a question TWICE, mark only the first attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out question.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, merk slegs die eerste poging.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, merk die deurgehaalde antwoord.*
- *Volgehoue akkuraatheid is DEURGAANS in ALLE aspekte van die memorandum van toepassing.*
- *Aanvaarding van waardes/antwoorde om 'n problem op te los, is onaanvaarbaar.*

QUESTION/*VRAAG* 1

1.1.1	$x^2 - 2x - 35 = 0$ $(x-7)(x+5) = 0$ $x = 7$ or $x = -5$	✓factors ✓answer ✓ answer (3)
1.1.2	$x^2 - 16 \geq 0$ $(x-4)(x+4) \geq 0$ + 0 − 0 + (–4, 4) or  $x \leq -4$ or $x \geq 4$ **OR** $x \in (-\infty ; -4]$ or $x \in [4 ; \infty)$ **OR** $x \in (-\infty ; -4] \cup [4 ; \infty)$	✓ factors ✓ critical values ✓ $x \leq -4$ ✓ $x \geq 4$ (4)
1.1.3	$9.2^{x-1} = 2.3^x$ $3^2.2^{x-1} = 2.3^x$ $2^{x-2} = 3^{x-2}$ $\left(\frac{2}{3}\right)^{x-2} = 1$ $\left(\frac{2}{3}\right)^{x-2} = \left(\frac{2}{3}\right)^0$ $x - 2 = 0$ $x = 2$ **OR**	✓ $2^{x-2} = 3^{x-2}$ ✓ $x - 2 = 0$ ✓answer (3)

	$9.2^{x-1} = 2.3^x$ $\frac{9.2^x}{2} = 2.3^x$ $9.2^x = 4.3^x$ $\frac{2^x}{3^x} = \frac{4}{9}$ $\left(\frac{2}{3}\right)^x = \frac{4}{9}$ $\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^2$ $x = 2$	✓ $\frac{9.2^x}{2}$ ✓ $\left(\frac{2}{3}\right)^x = \frac{4}{9}$ ✓answer (3)
1.2	$f(x) = x^2 - 5x + c$ $x = \frac{5 \pm \sqrt{25 - 4(1)(c)}}{2}$ $25 - 4c = 41$ $-4c = 16$ $c = -4$ **OR** $x = \frac{5 \pm \sqrt{41}}{2}$ $2x - 5 = \pm\sqrt{41}$ $(2x-5)^2 = 41$ $4x^2 - 20x + 25 = 41$ $4x^2 - 20x - 16 = 0$ $x^2 - 5x - 4 = 0$ $\therefore c = -4$	✓substitution into correct formula ✓ $25 - 4c = 41$ ✓ $c = -4$ (3) ✓squaring both sides ✓ $4x^2 - 20x - 16 = 0$ ✓ $c = -4$ (3)
1.3	$3^{x-10} = 3^{3x}$ $x - 10 = 3x$ $2x = -10$ $x = -5$ $y^2 + x = 20$ $y^2 - 5 = 20$ $y^2 = 25$ $y = -5$ or $y = 5$	✓ equating of exponents ✓ x-value ✓ $20 = y^2 - 5$ ✓✓ y-values (5) **[18]**

QUESTION 2/*VRAAG* 2

2.1.1	$T_3 = 20$ and $T_4 = 40$ $r = \frac{T_4}{T_3} = 2$	✓ answer (1)
2.1.2	$T_n = ar^{n-1}$ $20 = a.2^{3-1}$ $a = 5$ $T_n = 5.2^{n-1}$ **OR** $40 = a.2^{4-1}$ $a = 5$ $T_n = 5.2^{n-1}$	✓subs into correct formula ✓ $a = 5$ ✓answer (3) ✓subs into correct formula ✓ $a = 5$ ✓answer (3)
2.2.1	$\frac{-7}{125}$	✓ answer (1)
2.2.2	$T_n = \frac{2 + (n-1)(-3)}{(1).5^{n-1}}$ $T_n = \frac{5-3n}{5^{n-1}}$	✓ 5 ✓ 5^{n-1} ✓ $-3n$ (3)
2.2.3	$T_n = \frac{5-3n}{5^{n-1}}$ $T_{500} = \frac{5-3(500)}{5^{499}}$ $= \frac{-1495}{5^{499}}$	✓ numerator ✓ denominator (2)
2.2.4	$5-3n < -59$ $-3n < -64$ $n > 21{,}333...$ $n = 22$	✓ $5-3n < -59$ ✓ $n > 21{,}333...$ ✓ $n = 22$ (3) **[13]**

QUESTION/*VRAAG* 3

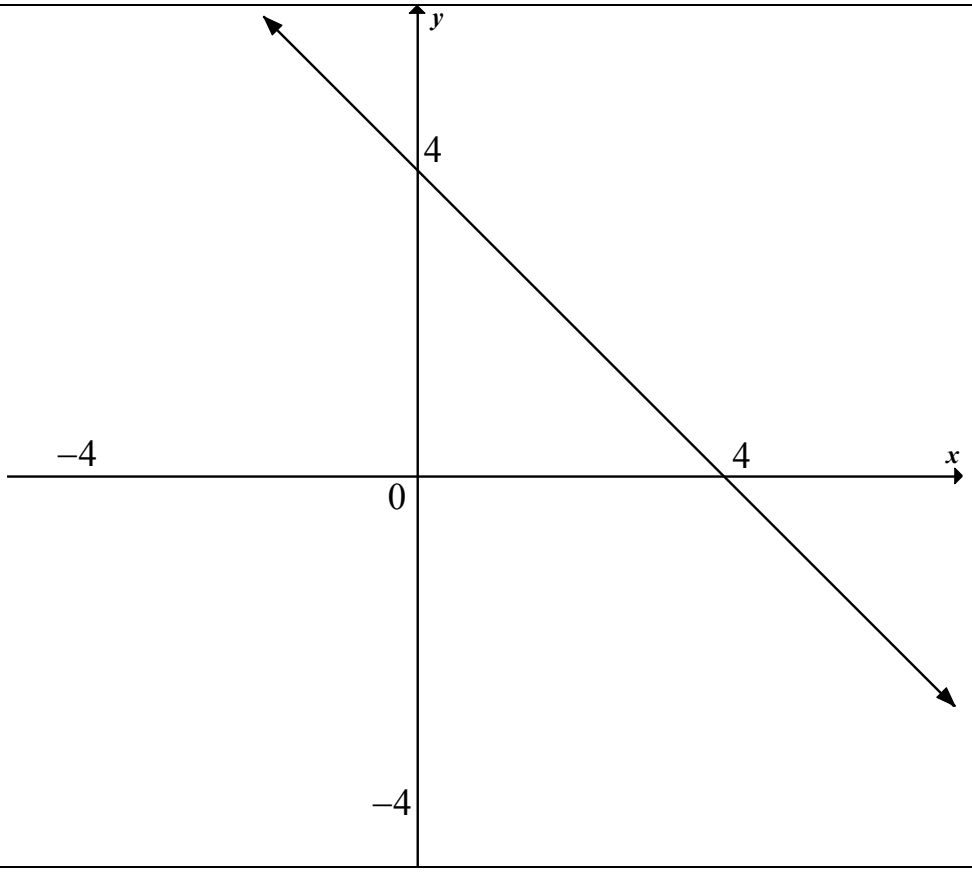
3.1.1	$w-3;\ 2w-4;\ 23-w$ $(2w-4)-(w-3)=(23-w)-(2w-4)$ $w-1=27-3w$ $4w=28$ $w=7$	✓ $(2w-4)-(w-3)$ $=(23-w)-(2w-4)$ ✓ $w=7$ (2)
3.1.2	Sequence is: 4 ; 10 ; 16 First difference / *Eerste verskil* = 6 **OR** $d=w-1$ $=6$	✓ answer (1) ✓ answer (1)
3.2	$T_{50}=3+(4+10+16+\ldots \text{ to 49 terms})$ $T_{50}=3+\frac{49}{2}[2(4)+(49-1)(6)]$ $=3+7252$ $=7255$ **OR** $2a=6$ $a=3$ $3a+b=4$ $3(3)+b=4$ $b=-5$ $a+b+c=3$ $3-5+c=3$ $c=5$ $T_n=3n^2-5n+5$ $T_{50}=3(50)^2-5(50)+5$ $=7255$	✓ $T_{50}=3+$ sum of 49 linear terms ✓ $a=4$ ✓ $n=49$ ✓ 7252(sum of 49 terms) ✓ answer (5) ✓ $a=3$ ✓ $b=-5$ ✓ $c=5$ ✓ substitution 50 ✓ answer (5) **[8]**

QUESTION/*VRAAG* 4

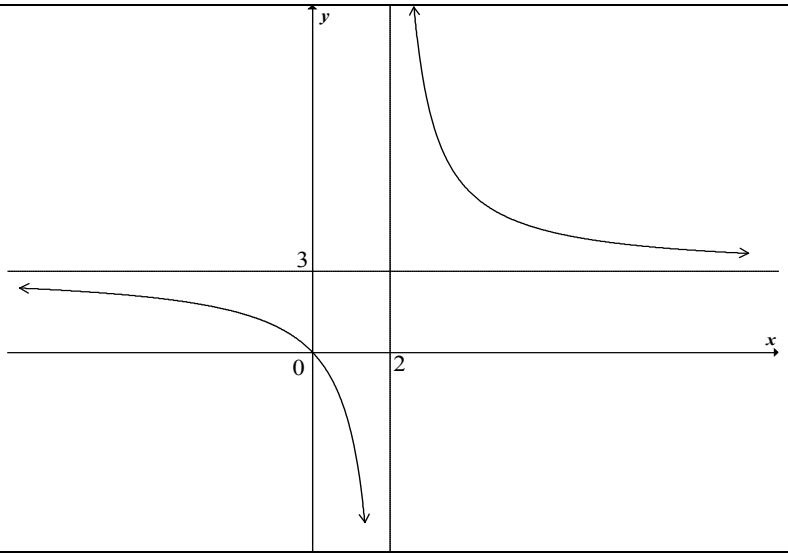
4.1	$S_n = p\left(1-\left(\frac{1}{2}\right)^n\right)$ $a = p\left[1-\left(\frac{1}{2}\right)^1\right]$ $= \frac{p}{2}$ $r = \frac{1}{2}$ $\therefore 10 = \frac{\frac{p}{2}}{1-\frac{1}{2}}$ $5 = \frac{p}{2}$ $p = 10$ **OR** $\left(\frac{1}{2}\right)^n \to 0$ as $n \to \infty$ $\therefore S_\infty = p$ $p = 10$	✓ $a = \frac{p}{2}$ ✓ $r = \frac{1}{2}$ ✓ substitute in correct formula ✓ answer (4)
4.2	$r = \frac{1}{2}$ $\frac{a}{1-\frac{1}{2}} = 10$ $a = 5$ $T_2 = ar = \frac{5}{2}$ **OR**	✓ $r = \frac{1}{2}$ ✓ substitution ✓ $a = 5$ ✓ answer

	$S_n = 10 - 10.2^{-n}$ $a = T_1$ $= S_1$ $= 10 - 10.2^{-1}$ $= 10 - \frac{10}{2}$ $= 5$ $T_2 = S_2 - T_1$ $= 10 - 10.2^{-2} - 5$ $= 10 - \frac{10}{4} - 5$ $= \frac{5}{2}$	✓ $S_1 = 5$ ✓ $a = 5$ ✓ $T_2 = S_2 - T_1$ ✓ answer (4)
	OR $T_2 = S_2 - S_1$ $= p\left(1 - \left(\frac{1}{2}\right)^2\right) - p\left(1 - \frac{1}{2}\right)$ $= \frac{p}{4}$ $= \frac{10}{4}$ $= \frac{5}{2}$	✓ $T_2 = S_2 - S_1$ ✓ substitution ✓ $\frac{p}{4}$ ✓ answer (4) **[8]**

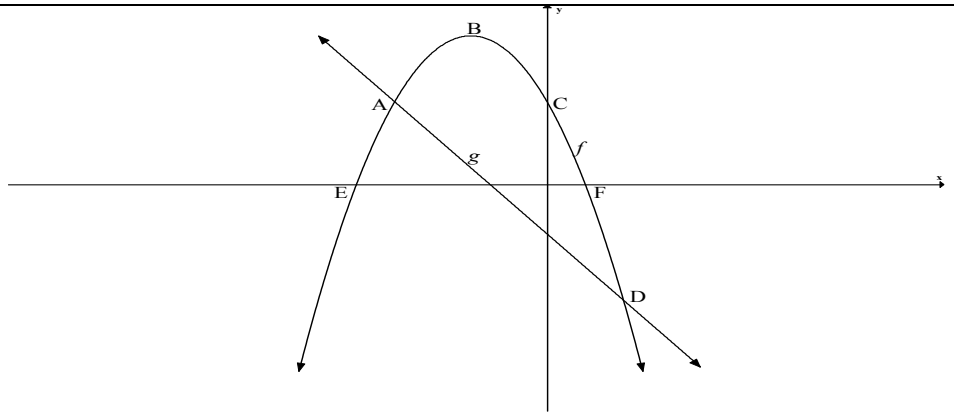
QUESTION/*VRAAG* 5

5.1		✓ *x*-intercept of straight line ✓ *y*-intercept of straight line ✓✓ circle (4)
5.2	Points of intersection are (0 ; 4) and (4 ; 0)	✓ (0 ; 4) ✓ (4 ; 0) (2) **[6]**

QUESTION/*VRAAG* 6

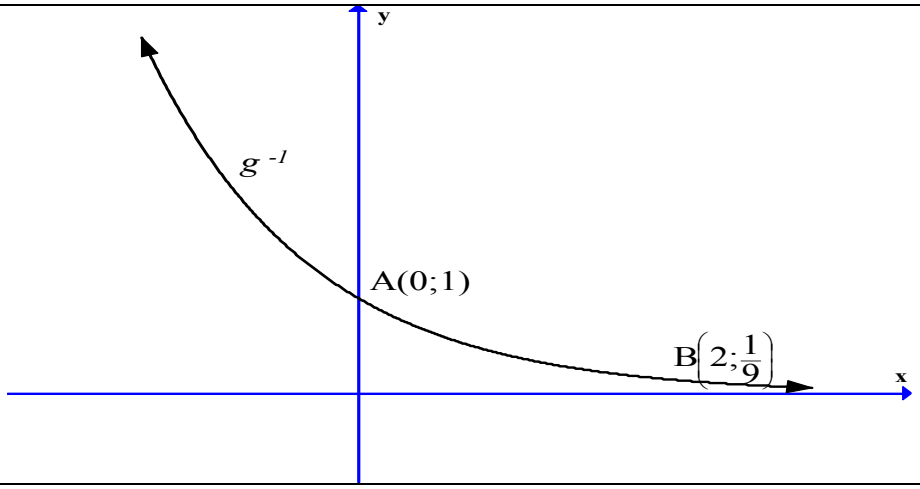
6.1	$x = 2$ $y = 3$	✓ $x = 2$ ✓ $y = 3$ (2)
6.2	**R** ; $x \neq 2$ **OR** $(-\infty\ ;\ 2) \cup (2\ ;\ \infty)$ **OR** **R** $-\{2\}$	✓ answer (1)
6.3		✓ shape ✓ intercept at origin ✓✓asymptotes (4)
6.4	$y = x + 3$ and $y = -x + 1$ $x + 3 = -x + 1$ $2x = -2$ $x = -1$ $y = -1 + 3$ $= 2$ Point of intersection of asymptotes: $(-1\ ;\ 2)$ *Die snypunt van die asimptote:* The transformation is a translation 3 units left and 1 unit down *Die transformasie is 'n translasie van 3 eenhede na links en 1 eenheid na onder* **OR** The transformation is $(x\ ;\ y) \rightarrow (x - 3\ ;\ y - 1)$	✓ $x + 3 = -x + 1$ ✓$x = -1$ ✓ $y = 2$ ✓ transformation (4) **[11]**

QUESTION/*VRAAG* 7

7.1	 C(0 ; 5)	✓ C(0 ; 5) (1)
7.2	$x = \frac{-4+0}{2}$ $= -2$	✓ $\frac{-4+0}{2}$ (1)
7.3	$p = -2$ $y = a(x+2)^2 + q$ $5 = a(0+2)^2 + q$ $\quad 0 = a(1+2)^2 + q$ $5 = 4a + q$ $\quad 0 = 9a + q$ $5a = -5$ $a = -1$ $q = 9$ $f(x) = -(x+2)^2 + 9$ **OR** $p = -2$ $f(x) = a(x-1)(x+5)$ $5 = a(-1)(5)$ $a = -1$ $q = f(-2)$ $= -1(-3)(3)$ $= 9$	✓ $p = -2$ ✓ $5 = 4a + q$ ✓ $0 = 9a + q$ ✓ simultaneous equation ✓ $a = -1$ ✓ $q = 9$ (6) ✓ $p = -2$ ✓ $f(x) = a(x-1)(x+5)$ ✓substitution ✓ $a = -1$ ✓ substitution ✓ $q = 9$ (6)
7.4	$-x^2 - 4x + 5 = -2x - 3$ $x^2 + 2x - 8 = 0$ $(x+4)(x-2) = 0$ $x = -4$ or $x = 2$ $\therefore$ $x-$coordinate of D is 2	✓ equating ✓standard form ✓ factors ✓ 2 (4)

7.5	(–2 ; –9)	✓2 ✓ – 9 (2) **[14]**

QUESTION/*VRAAG* 8

8.1	A(1; 0)	✓ answer (1)
8.2		✓shape ✓ A(0 ; 1) ✓ $B\left(2;\frac{1}{9}\right)$ (3)
8.3	**R**	✓ answer (1) **[5]**

QUESTION/*VRAAG* 9

9.1	$1+i_{eff}=\left(1+\frac{i_{nom}}{m}\right)^{m}$ $1+i_{eff}=\left(1+\frac{0,07}{12}\right)^{12}$ $i_{eff}=0,07229008$ $i_{eff}=7,23\%$	✓ substitution into correct formula ✓ $\frac{0,07}{12}$ ✓ answer (3)

9.2	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $350\,000 = \frac{6300[1-\left(1+\frac{0.07}{12}\right)^{-n}}{\frac{0,07}{12}}$ $\frac{73}{108} = \left(1+\frac{0.07}{12}\right)^{-n}$ $\log\frac{73}{108} = -n\log\left(1+\frac{0.07}{12}\right)$ $n = 67{,}33938079$ $n = 67{,}34$ months	✓ $i = \frac{0.07}{12}$ ✓ substitution in the correct formula ✓ simplification ✓ use of logs ✓ answer (5)
9.3	$P_v = \frac{x[1-(1+i)^{-n}]}{i}(1+i)$ $P_v = \frac{6\,300[1-(1+\frac{0,07}{12})^{-0,3393....}]}{\frac{0,07}{12}}\left(1+\frac{0,07}{12}\right)$ $P_v = \text{R } 2\,142{,}21$ **OR** Balance outstanding: $= \left[350\,000\left(1+\frac{0,07}{12}\right)^{67} - \frac{6\,300[(1+\frac{0,07}{12})^{67.}-1]}{\frac{0,07}{12}}\right]\left(1+\frac{0,07}{12}\right)$ $= \text{R } 2\,142{,}21$	✓ $n = -0{,}3393...$ ✓ $i = \frac{0,07}{12}$ ✓ substitution in the correct formula ✓ multiplication by $\left(1+\frac{0,07}{12}\right)$ ✓ answer (5) ✓ $n = -0{,}3393...$ ✓ $i = \frac{0,07}{12}$ ✓ substitution in the correct formula ✓ multiplication by ✓ answer (5)
9.4	$252\,000 = 350\,000(1-i)^3$ $(1-i)^3 = \frac{252\,000}{350\,000}$ $i = 1-\sqrt[3]{\frac{252}{350}}$ $i = 10{,}37\%$	✓ $n = 3$ ✓ substitution in the correct formula ✓ answer (3) **[16]**

QUESTION/*VRAAG* 10

10.1.1

$$f(x) = -\frac{2}{x}$$

$$f(x+h) = -\frac{2}{(x+h)}$$

$$f(x+h) - f(x) = -\frac{2}{(x+h)} - \left(-\frac{2}{x}\right)$$

$$= \frac{-2x + 2(x+h)}{x(x+h)}$$

$$= \frac{-2x + 2x + 2h}{x(x+h)}$$

$$= \frac{2h}{x(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \left(\frac{2}{x^2 + xh}\right)$$

$$= \frac{2}{x^2}$$

✓ substitution
✓ simplification
✓ formula
✓ common factor
✓ answer (5)

OR

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left[-\frac{2}{(x+h)}\right] - \left(-\frac{2}{x}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-2x + 2(x+h)}{x(x+h)}}{h}$$

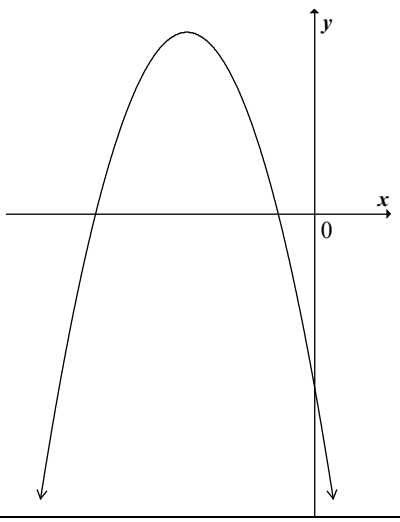
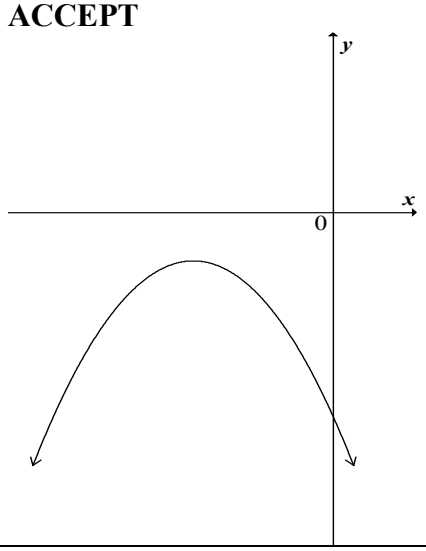
$$= \lim_{h \to 0} \frac{\frac{-2x + 2x + 2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{2h}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \left(\frac{2}{x^2 + xh}\right)$$

$$= \frac{2}{x^2}$$

✓ formula
✓ substitution
✓ simplification
✓ common factor
✓ answer (5)

10.1.2	$f'(x)=\frac{2}{x^2}$ $x^2 \geq 0$ for $x \in R$ $f'(x)>0$ for $x \in R; x \neq 0$	✓ $x^2 \geq 0$ or $\frac{2}{x^2} \geq 0$ for $x \in R$ ✓ $f'(x)>0$ for $x \in R; x \neq 0$ (2)
10.2	$y=\frac{1}{4}x^2-2x$ $\frac{dy}{dx}=\frac{1}{2}x-2$	✓ $\frac{1}{2}x$ ✓ -2 (2)
10.3	$y=4\left(\sqrt[3]{x^2}\right)$ $y=4x^{\frac{2}{3}}$ and $x=w^{-3}$ $y=4(w^{-3})^{\frac{2}{3}}$ $=4w^{-2}$ $\frac{dy}{dw}=-8w^{-3}$ $=-\frac{8}{w^3}$	✓ $y=4x^{\frac{2}{3}}$ ✓ subs: $4(w^{-3})^{\frac{2}{3}}$ ✓simplification ✓ answer (4)
10.4	$f'(x)=3ax^2+2bx+c$ $a<0$ shape (max TP) $c<0$ y - intercept is negative b < 0 axis of symmetry on LHS of y - axis **ACCEPT**   	✓ $f'(x)=3ax^2+2$ ✓ shape (max TP) ✓ axis of symmetry on LHS if y-axis ✓y – intercept is below x-axis (4) **[17]**

QUESTION/*VRAAG* 11

11.1	$f(x) = -(x-1)(x-2)(x-4)$ $f(x) = -(x^2-3x+2)(x-4)$ $f(x) = -x^3+7x^2-14x+8$	✓ $-(x-1)(x-2)(x-4)$ ✓ $a=7$ ✓ $b=-14$ ✓ $c=8$ (4)
11.2	$f(x) = -x^3+7x^2-14x+8$ $f'(x) = 0$ $-3x^2+14x-14=0$ $3x^2-14x+14=0$ $x = \frac{14 \pm \sqrt{14^2-4(3)(14)}}{2(3)}$ $= \frac{14 \pm \sqrt{28}}{6}$ $= \frac{7 \pm \sqrt{7}}{3}$ $x = 1{,}45$ or $x = 3{,}22$	✓ $f'(x)=0$ ✓ $-3x^2+14x-14=0$ ✓subs into formula ✓x –value ✓x – value (5)
11.3	$x < 1{,}45$ or $x > 3{,}22$	✓critical values ✓✓notation (3) **[12]**

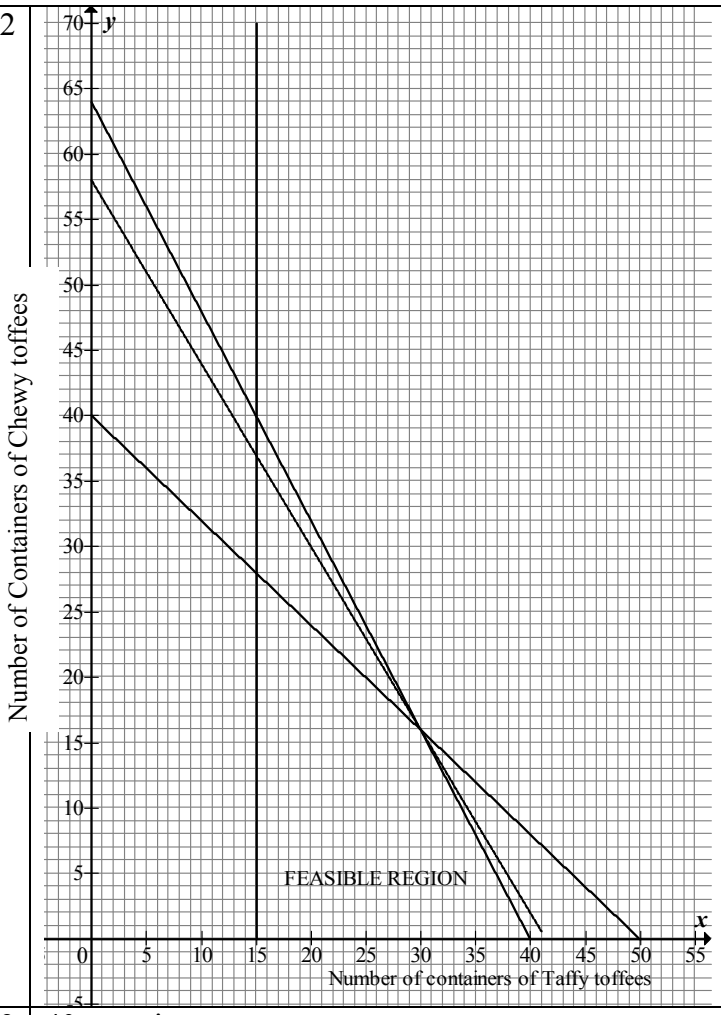
QUESTION/*VRAAG* 12

12.1	$40-x$	✓answer (1)
12.2	$P(x) = (40-x)(144+4x)$ $= 4(40-x)(36+x)$ $= 5\,760+16x-4x^2$	✓concept of multiplication ✓ $(144+4x)$ ✓ answer (3)
12.3	$P'(x) = 16-8x$ $P'(x) = 0$ $16-8x=0$ $8x=16$ $x=2$ Cost = 144 + 4 (2) = R 152 **OR** Max at $x = \frac{40-36}{2} = 2$ Cost = 144 + 4 (2) = R 152	✓ $P'(x) = 16-8x$ ✓ $P'(x)=0$ ✓ $x=2$ ✓answer (4) ✓ $x = 40$ & 36 are solutions to $P(x) = 0$ ✓✓ $x = \frac{40-36}{2} = 2$ ✓ answer (4)

	OR Number of watches Cost Income Year 0: 40 144 5 760 Year 1: 39 148 5 772 Year 2: 38 152 5 776 Year 3: 37 156 5 772 Max Income at $x = 2$ Max cost = R 152	✓✓ explanation ✓ $x = 2$ ✓ R 152 (4) **[8]**

QUESTION/*VRAAG* 13

13.1	$40x + 50y \le 2000$ $64x + 40y \le 2560$ $x \ge 15$ **OR** $y \le -\frac{4}{5}x + 40$ $y \le -\frac{8}{5}x + 64$ $x \ge 15$ **OR** $\frac{y}{40} + \frac{x}{50} \le 1$ $\frac{y}{64} + \frac{x}{40} \le 1$ $x \ge 15$	✓✓ $40x + 50y \le 2000$ ✓✓ $64x + 40y \le 2560$ ✓ $x \ge 15$ (5) ✓✓ $y \le -\frac{4}{5}x + 40$ ✓✓ $y \le -\frac{8}{5}x + 64$ ✓ $x \ge 15$ (5) ✓✓ $\frac{y}{40} + \frac{x}{50} \le 1$ ✓✓ $\frac{y}{64} + \frac{x}{40} \le 1$ ✓ $x \ge 15$ (5)

13.2		✓ feasible region ✓ $40x + 50y \leq 2000$ ✓ $64x + 40y \leq 2560$ ✓ $x \geq 15$ (4)
13.3	40 containers	✓✓ answer (2)
13.4	$P = 1400x + 1000y$ $m = -\frac{14}{10}$ $m = -\frac{7}{5}$ Using the search line : Maximum achieved at (30; 16)	✓ $P = 1400x + 1000y$ ✓ search line ✓Max at (30 ; 16) (3) **[14]**

TOTAL/*TOTAAL*: 150