NASIONALE
SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

FEBRUARIE/MAART 2013

MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 21 bladsye.

NOTA:
- Indien `n kandidaat `n vraag TWEE keer beantwoord het, merk slegs die EERSTE poging.
- Indien `n kandidaat `n poging om `n vraag te beantwoord gekanselleer het en die vraag nie weer gedoen het nie, merk die gekanselleerde poging.
- Konstante akkuraatheid is van toepassing in **ALLE** aspekte van die merk memorandum.

VRAAG 1

1.1		
	Spreidiagram van wisselkoers versus olieprys	✓enige 4 punte korrek geplot ✓ enige 9 punte korrek geplot ✓ alle punte korrek geplot (3)
1.2	Soos die wisselkoers (R/\$) vermeerder verminder die olieprys (\$). **OF** Daar is `n negatiewe korrelasie tussen die wisselkoers en die olieprys.	✓✓ rede (2)
1.3	Gemiddelde $= \dfrac{852,6}{12}$ $= 71,05$	✓ 852,6 ✓ 71,05 (2)
1.4	Standaard afwyking is: $\sigma = 4,09$	✓✓ 4,09 (2)
1.5	2 standaard afwykings van die gemiddelde gemiddelde $= 71,05 + 2(4,09) = 79,23$ Die publiek sal besorg wees in Desember 2010	✓ 79,23 ✓ Des 2010 (2) **[11]**
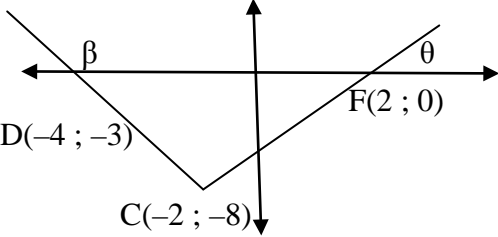
VRAAG 2

2.1	Variasiewydte van Peter se punte is 94 – 68 = 26	✓ 94 – 68 ✓ 26 (2)
2.2	Vuyani se minimum punte is 76	✓ 76 (1)
2.3	Vuyani was meer konstant gedurende die jaar want die variasiewydte van sy punte is nader om die mediaan waarde gerangskik **OF** die variasiewydte en die interkwartiel-wydte is kleiner as Peters.	✓ Vuyani ✓ rede (2) **[5]**
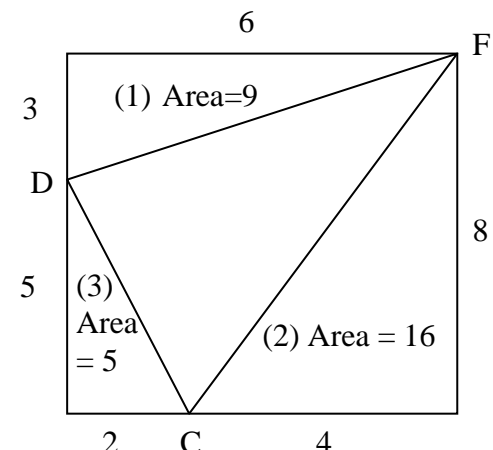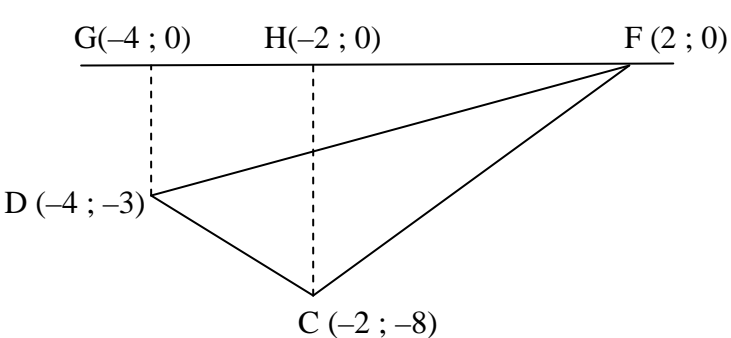
VRAAG 3

3.1		✓ plot punte by kummulatiewe frekwensie ✓ plot teen boonste limiet ✓ gegrond by (0 ; 0) ✓ gladde kurwe (4)
3.2.1	(85 ; ± 144) ± 144 leerders het punte onder 85% (Aanvaar: 144 to 146)	✓ (85 ; ± 144) ✓ ± 144 leerders (2)
3.2.2	Q_1 = 25 of 27 or 26 Q_3 = 61 of 62 or 64 Interkwartiel-wydte = 36 of 35 or 38	✓ onderste kwartiel ✓ boonste kwartiel ✓ IKW (3) **[9]**

VRAAG 4

4.1	$m_{AD} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{7 - (-3)}{1 - (-4)}$ $= 2$	✓ substitusie ✓ 2 (2)
4.2	AD//BC $m_{AD} = m_{BC} = 2$ $y - y_1 = m(x - x_1)$ $y - (-8) = 2(x - (-2))$ $\therefore y = 2x - 4$	✓ $m_{AD} = 2$ ✓ stel in formule ✓ $y = 2x - 4$ (3)
4.3	By F: $y = 0$ $0 = 2x - 4$ $x = 2$ F(2 ; 0)	✓ $y = 0$ ✓ $x = 2$ (2)
4.4	D is getransleer C volgens die reël: $D(x ; y) \rightarrow C(x + 2 ; y - 5)$ A moet ook getransleer word volgens die reël na B$^{/}$. $\therefore$ A(1 ; 7) $\rightarrow$ B$^{/}$ (3 ; 2) **OF** $x_{B^{/}} = -2 + (1 + 4) = 3$ $y_{B^{/}} = -8 + (7 + 3) = 5$	✓ $x = 3$ ✓ $y = 2$ (2) ✓ $x = 3$ ✓ $y = 2$ (2)
4.5	$m_{BC} = 2$ $\tan\theta = 2$ $\theta = 63{,}43°$ $m_{DC} = \dfrac{-8 - (-3)}{-2 - (-4)} = -\dfrac{5}{2}$ $\tan\beta = -\dfrac{5}{2}$ $\beta = 180° - 68{,}20° = 111{,}80°$ $\alpha = 111{,}80° - 63.43° = 48{,}37°$ **OF**	✓ 63,43° ✓ $\tan\beta = -\dfrac{5}{2}$ ✓ 111,8° ✓ 48,37° (4)

	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $\quad = \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $\quad = \sqrt{80}$ $DF = \sqrt{(2+4)^2 + (0+3)^2}$ $\quad = \sqrt{45}$ $\cos\alpha = \dfrac{29+80-45}{2(\sqrt{29})(\sqrt{80})}$ $\quad = 0{,}6643...$ $\alpha = 48{,}37°$ **OF** $DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $\quad = \sqrt{29}$ $DB = \sqrt{(3+4)^2 + (2+3)^2}$ $\quad = \sqrt{74}$ $BC = \sqrt{(3+2)^2 + (2+8)^2}$ $\quad = \sqrt{125}$ $\cos\alpha = \dfrac{29+125-74}{2(\sqrt{29})(\sqrt{125})}$ $\quad = 0{,}6643...$ $\alpha = 48{,}37°$	✓Subst in cos-formule ✓cos α onderwerp ✓0,6643... ✓48,37° (4) ✓Subst in cos-formule ✓cos α onderwerp ✓0,6643... ✓48,37° (4)
4.6	$DC = \sqrt{(-4+2)^2 + (-3+8)^2}$ $\quad = \sqrt{29}$ $CF = \sqrt{(-2-2)^2 + (-8-0)^2}$ $\quad = \sqrt{80}$ $Area\ \Delta DCF = \dfrac{1}{2}.DC.CF.\sin\alpha$ $\quad\quad = \dfrac{1}{2}(\sqrt{29})(\sqrt{80})\sin 48{,}37°$ $\quad\quad = 18\ \text{eenhede}^2$	✓ substitusie in formule ✓ $\sqrt{29}$ ✓ substitusie in formule ✓ $\sqrt{80}$ ✓ substitusie In die area reël ✓ 18 (6)

OF

Area ΔDCF = Area van reghoek – (1) – (2) – (3)

$$= 48 - 9 - 5 - 16$$

$$= 18 \text{ vierkante eenheid}$$

✓ reghoek en area

✓ verband tussen areas
✓ (1) = 9
✓ (2) = 16
✓ (3) = 5
✓ 18 eenhede^2
(6)

OF

Area CDF = Area CHF + Area CDGH - Area DGF

$$= \frac{1}{2} \times 4 \times 8 + 2 \times \frac{1}{2}(3 \times 8) - \frac{1}{2} \times 6 \times 3$$

$$= 16 + 11 - 9$$

$$= 18 \text{ vierkante eenheid}$$

✓ teken loodregte lyne

✓ verband tussen areas
✓ 16
✓ 11
✓ 9
✓ 18 vierkante eenheid
(6)

[19]

VRAAG 5

5.1.1	$x^2 + y^2 + 2x + 6y + 2 = 0$ $x^2 + 2x + 1 + y^2 + 6y + 9 = -2 + 10$ $(x+1)^2 + (y+3)^2 = 8$ $M(-1\,;\,-3)$	✓ $(x+1)^2 + (y+3)^2 = 8$ ✓ -1 ✓ -3 (3)
5.1.2	radius van sirkel $C_1 = \sqrt{8}$	✓ $\sqrt{8}$ (1)
5.2	$x^2 + (x-2)^2 + 2x + 6(x-2) + 2 = 0$ $x^2 + x^2 - 4x + 4 + 2x + 6x - 12 + 2 = 0$ $2x^2 + 4x - 6 = 0$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = -3 \ or \ x \neq 1$ $y = -3 - 2 = -5$ $\therefore D(-3\,;-5)$ **OF** $(x+1)^2 + (y+3)^2 = 8$ $subst. \ y = x - 2$ $(x+1)^2 + (x-2+3)^2 = 8$ $(x+1)^2 + (x+1)^2 = 8$ $x^2 + 2x - 3 = 0$ $(x+3)(x-1) = 0$ $x = -3 \ or \ x \neq 1$ $y = -3 - 2 = -5$ **OF** $(x+1)^2 + (y+3)^2 = 8$ $subst. \ y = x - 2$ $(x+1)^2 + (x-2+3)^2 = 8$ $(x+1)^2 + (x+1)^2 = 8$ $(x+1)^2 = 4$ $x + 1 = \pm 2$ $x = -3 \ or \ x \neq 1$ $y = -3 - 2 = -5$ **OF**	✓ substitusie ✓ standaard vorm ✓ faktore ✓ waarde van x ✓ waarde van y (5) ✓ substitusie ✓ standaard vorm ✓ faktore ✓ waarde van x ✓ waarde van y (5) ✓ substitusie ✓ vereenvoudiging ✓ vierkantswortel van albei kante ✓ waarde van x ✓ waarde van y (5)

	PM maak `n 45° hoek met die x-as. $\sqrt{8} = \sqrt{2^2 + 2^2}$ Dus: $x_D = x_M - 2 = -1 - 2 = -3$ $y_D = -3 - 2 = -5$	✓✓ $\sqrt{8} = \sqrt{2^2 + 2^2}$ ✓ waarde van x ✓ waarde van y (5)
5.3	$MD \perp DB$ (raaklyn $\perp$ radius) $MB^2 = MD^2 + DB^2$ (Pythagoras) $\quad\quad = (\sqrt{8})^2 + (4\sqrt{2})^2$ $\quad\quad = 40$ MB is die radius van C_2 $MB = \sqrt{40}$	✓ raaklyn $\perp$ radius ✓ substitusie in Pythagoras ✓ $\sqrt{40}$ (3)
5.4	$(x+1)^2 + (y+3)^2 = 40$	✓ LK ✓ RK (2)
5.5	Afstand van $\left(2\sqrt{5}\,;\,0\right)$ na middelpunt $= \sqrt{\left(2\sqrt{5}+1\right)^2 + \left(0+3\right)^2}$ $= 6{,}24$ $6{,}24 < 6{,}32 \left(\sqrt{40}\right)$ Afstand van $\left(2\sqrt{5}\,;\,0\right)$ na middelpunt $<$ radius van sirkel. $\left(2\sqrt{5}\,;\,0\right)$ lê binne die sirkel.	✓ substitusie in afstand formule ✓ 6,24 ✓ 6,24 < 6,32 ✓ afleiding (4) **[18]**

VRAAG 6

6.1.1	$A(-5 \,;\, 3)$ $A^{/}(-5+4 \,;\, 3-3) = (-1 \,;\, 0)$	✓ -1 ✓ 0 (2)
6.1.2	$A^{/}(-5 \,;\, -3)$	✓ -5 ✓ -3 (2)
6.2.1	Skaal faktor van vergroting is $\dfrac{K^{/}M^{/}}{KM} = \dfrac{15}{10} = \dfrac{3}{2}$ **OF** $K(-4 \,;\, 2) \rightarrow K^{/}(-6 \,;\, 3) = K^{/}\left(\dfrac{3}{2} \times -4 \,;\, \dfrac{3}{2} \times 2\right)$ Skaalfaktor is $\dfrac{3}{2}$	✓ $\dfrac{K^{/}M^{/}}{KM}$ ✓ $\dfrac{3}{2}$ ✓ $\left(\dfrac{3}{2} \times -4 \,;\, \dfrac{3}{2} \times 2\right)$ ✓ $\dfrac{3}{2}$ (2)
6.2.2	$(x \,;\, y) \rightarrow \left(\dfrac{3}{2}x \,;\, \dfrac{3}{2}y\right)$	✓ $\dfrac{3}{2}x$ ✓ $\dfrac{3}{2}y$ (2)
6.2.3	$P^{/}\left(\dfrac{3}{2} \times 3 \,;\, 2 \times \dfrac{3}{2}\right)$ $= P^{/}\left(\dfrac{9}{2} \,;\, 3\right)$	✓ $\dfrac{9}{2}$ ✓ 3 (2)
6.2.4	$a = 1$	✓✓ $a = 1$ (2)
6.2.5	$K^{//}(4 \,;\, -2)$	✓ 4 ✓ -2 (2)

6.2.6	$K^{///}K^{/} = 5$
	 $K^{/}M^{///} = 15$
	
	 $\dfrac{K^{/}K^{///}}{K^{/}M^{///}} = \dfrac{5}{15} = \dfrac{1}{3}$
		✓ $K^{///}K^{/} = 5$
		 ✓ $K^{/}M^{///} = 15$
		
		 ✓ $\dfrac{1}{3}$
		 (3)
		 [17]

VRAAG 7

7.1	$K^{/}(b\,;-a)$	✓ b ✓ $-a$ (2)

7.2	$K^{//}(b\cos\theta-a\sin\theta\,;\,-a\cos\theta-b\sin\theta)$
	
	 OF
	
	 $K^{//}(a\cos(90+\theta)+b\sin(90°+\theta)\,;\,b\cos(90°+\theta)-a\sin(90°+\theta)$
	
	 $=K^{//}(-a\sin\theta+b\cos\theta\;;-b\sin\theta-a\cos\theta)$
		✓
		 $b\cos\theta-a\sin\theta$
		 ✓
		 $-a\cos\theta-b\sin\theta$
		
		 (2)

7.3	$T^{//}(-(-4)\sin\theta+(-2)\cos\theta\,;\,-(-2)\sin\theta-(-4)\cos\theta)$
	
	 $=T^{//}(4\sin\theta-2\cos\theta\,;\,2\sin\theta+4\cos\theta)$
	
	 OF
	
	 $T^{//}(-2\cos\theta-(-4)\sin\theta\,;\,-(-4)\cos\theta-(-2)\sin\theta)$
	
	 $=T^{//}(-2\cos\theta+4\sin\theta\,;\,4\cos\theta+2\sin\theta)$
		✓ $4\sin\theta-2\cos\theta$
		 ✓ $2\sin\theta+4\cos\theta$
		
		 (2)
		
		 ✓ $4\sin\theta-2\cos\theta$
		 ✓ $2\sin\theta+4\cos\theta$
		
		 (2)

7.4	$2\sqrt{3}+1=4\sin\theta-2\cos\theta$ ....(1) $\sqrt{3}-2=2\sin\theta+4\cos\theta$ ....(2) $(2)\times 2:\;2\sqrt{3}-4=4\sin\theta+8\cos\theta$ ....(3) $(1)-(3):\;5=-10\cos\theta$ $\qquad -\dfrac{1}{2}=\cos\theta$ $\qquad\therefore\;\theta=180°-60°=120°$ **OF** $2\sqrt{3}+1=4\sin\theta-2\cos\theta$ ....(1) $\sqrt{3}-2=2\sin\theta+4\cos\theta$ ....(2) $(1)\times 2:\;4\sqrt{3}+2=8\sin\theta-4\cos\theta$ ....(3) $(2)+(3):\;5\sqrt{3}=10\sin\theta$ $\qquad \dfrac{\sqrt{3}}{2}=\sin\theta$ $\qquad\therefore\;\theta=180°-60°=120°$ **OF**	✓ substitusie om vergelyking te vorm ✓ substitusie om vergelyking te vorm ✓ $5=-10\cos\theta$ ✓ $-\dfrac{1}{2}=\cos\theta$ ✓ $120°$ (5) ✓ substitusie om vergelyking te vorm ✓ substitusie om vergelyking te vorm ✓ $5\sqrt{3}=10\sin\theta$ ✓ $\dfrac{\sqrt{3}}{2}=\sin\theta$ ✓ $120°$ (5)

$m_{OT} = \dfrac{1}{2} \Rightarrow \tan X\hat{O}T = \dfrac{1}{2}$

$X\hat{O}T = 206,565...°$

$m_{OT'} = \dfrac{\sqrt{3}-2}{2\sqrt{3}+1} \Rightarrow \tan X\hat{O}T'' = \dfrac{\sqrt{3}-2}{2\sqrt{3}+1} = -0,06....$

$X\hat{O}T = -3,434°$

$90° + \theta = 209,99...° \approx 210°$

$\theta = 120°$

	✓ $\tan X\hat{O}T = \dfrac{1}{2}$
	✓ $206.565...°$
	✓ $-0,06...$
	✓ $-3.434°$
	✓ $120°$

(5)

OF

$(TT')^2 = OT^2 + (OT')^2 - 2(OT)(OT').\cos(90° + \theta)$

$40 + 20\sqrt{3} = 40 - 40.\cos(90° + \theta)$

$\cos(90° + \theta) = -\dfrac{\sqrt{3}}{2}$

$90° + \theta = 150°$

$\theta = 60°$

	✓$(TT')^2$ $= 40 + 20\sqrt{3}$
	✓substitusie in cos-reël
	✓vereenvoudiging
	✓$150°$
	✓$60°$

(5)

[11]

VRAAG 8

8.1	$1 - \sin^2 \theta + 3 - \cos^2 \theta$ $= 4 - (\sin^2 \theta + \cos^2 \theta)$ $= 3$	✓ vereenvoudiging ✓ 3 (2)
	OF	
	$\cos^2 \theta + 3 - \cos^2 \theta$ $= 3$	✓ substitusie met identiteit ✓ 3 (2)
8.2	$\sqrt{4^{\sin 150°}.2^{3\tan 225°}}$ $= \sqrt{4^{\sin 30°}.2^{3\tan 45°}}$ $= \sqrt{\left(2^2\right)^{\frac{1}{2}}.2^3}$ $= \sqrt{16}$ $= 4$	✓ herskryf met reduksie formules ✓ stel spesiale hoeke in ✓ vereenvoudiging ✓ 4 (4)
	OF	
	$\sin 150° = \frac{1}{2}$ $\tan 225° = 1$ $\sqrt{4^{\sin 150°}2^{3\tan 225°}}$ $= \sqrt{4^{\frac{1}{2}}2^3}$ $= \sqrt{2.2^3}$ $= \sqrt{16}$ $= 4$	✓ $\sin 150° = \frac{1}{2}$ ✓ $\tan 225° = 1$ ✓ $4^{\frac{1}{2}} = 2$ ✓ 4 (4)
8.3	$LK = \dfrac{\cos^2 x(\sin^2 x + \cos^2 x)}{1 - \sin x}$ $= \dfrac{\cos^2 x.(1)}{1 - \sin x}$ $= \dfrac{(1 - \sin^2 x)}{1 - \sin x}$ $= \dfrac{(1 + \sin x)(1 - \sin x)}{1 - \sin x}$ $= 1 + \sin x$ $= RK$	✓ faktorisering ✓ 1 ✓ $1 - \sin^2 x$ ✓ faktore (4)

8.4	$\cos 3\theta$	
	$= \cos(2\theta + \theta)$	
	$= \cos 2\theta.\cos\theta - \sin 2\theta.\sin\theta$	✓uitbreiding
	$= (2\cos^2\theta - 1).\cos\theta - 2\sin\theta.\cos\theta.\sin\theta$	✓ $2\cos^2\theta - 1$
	$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta.\cos\theta$	✓ $2\sin\theta.\cos\theta$
	$= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta).\cos\theta$	✓ $1 - \cos^2\theta$
	$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$	
	$= 4\cos^3\theta - 3\cos\theta$	(4)
8.5	$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$	✓$\theta = 20°$
	$\cos 3(20°) = 4\cos^3(20°) - 3\cos(20°)$	✓$\cos 60° = \dfrac{1}{2}$
	$\dfrac{1}{2} = 4x^3 - 3x$	
	$8x^3 - 6x - 1 = 0$	(2)
		[16]

VRAAG 9

9.1	$\dfrac{\cos 160°.\tan 200°}{2\sin(-10°)}$	
	$=\dfrac{(-\cos 20°)(\tan 20°)}{2(-\sin 10°)}$	✓ $-\cos 20°$ ✓ $\tan 20°$ ✓ $-\sin 10°$
	$=\dfrac{(-\cos 20°)\left(\dfrac{\sin 20°}{\cos 20°}\right)}{-2\sin 10°}$	✓ $\dfrac{\sin 20°}{\cos 20°}$
	$=\dfrac{2\sin 10° \cos 10°}{2\sin 10°}$	✓ $2\sin 10° \cos 10°$
	$=\cos 10°$	✓ $\cos 10°$
		(6)
9.2.1	$LK = \cos(x+45°).\cos(x-45°)$	
	$= \left(\cos x.\cos 45° - \sin x \sin 45°\right)\left(\cos x \cos 45° + \sin x \sin 45°\right)$	✓ brei uit $\cos(x+45°)$
	$= \cos^2 x.\cos^2 45° - \sin^2 x.\sin^2 45°$	✓ brei uit $\cos(x-45°)$
	$=\cos^2 x\left(\dfrac{\sqrt{2}}{2}\right)^2 - \sin^2\left(\dfrac{\sqrt{2}}{2}\right)^2$ or $\left[\cos^2 x\left(\dfrac{1}{\sqrt{2}}\right)^2 - \sin^2 x\left(\dfrac{1}{\sqrt{2}}\right)^2\right]$	✓ stel in spesiale hoeke
	$=\dfrac{1}{2}\cos^2 x - \dfrac{1}{2}\sin^2 x$	
	$=\dfrac{1}{2}(\cos^2 x - \sin^2 x)$	✓ vereenvoudiging
	$=\dfrac{1}{2}\cos 2x$	
	OF	(4)
	$2\cos\alpha \cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$	
	$\cos\alpha \cos\beta = \dfrac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$	✓✓ lei identiteit af
	Let $\alpha = x+45°$ and $\beta = x-45°$	
	$\therefore \cos(x+45°)\cos(x-45°)$	
	$=\dfrac{1}{2}\left(\cos((x+45° + x-45°)) + \cos(x+45° - x+45°)\right)$	✓ substitusie
	$=\dfrac{1}{2}\left(\cos 2x + \cos 90°\right)$	✓ vereenvoudiging
	$=\dfrac{1}{2}\cos 2x$	
		(4)
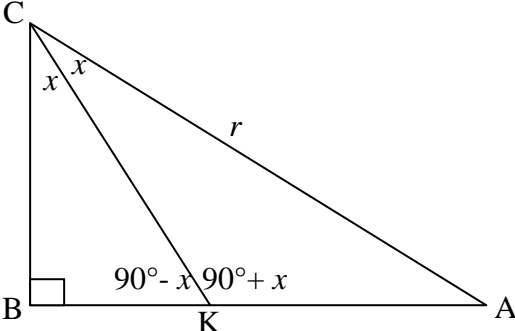
9.2.2	$\cos(x + 45°)\cos(x - 45°)$ het `n minimum waar $\dfrac{1}{2}\cos 2x$ `n minimum het.	
	Die minimum waarde van $\cos 2x$ is -1	✓ minimum waarde van -1
	$\cos 2x = -1$ $2x = 180°$ $x = 90°$	✓$2x = 180°$ ✓ $x = 90°$
		(3)
		[13]

VRAAG 10

10.1	Waardeversameling $= [-1 ; 1]$	✓✓ $[-1 ; 1]$
		(2)
10.2	$f\left(\dfrac{3}{2}x\right) = \sin 2\left(\dfrac{3}{2}x\right)$ $\qquad = \sin 3x$ $\therefore$ Periode $= \dfrac{360°}{3} = 120°$ **OF**	✓ $\sin 3x$ ✓ $120°$ (2)
	$f\left(\dfrac{3}{2}x\right) = \sin 2\left(\dfrac{3}{2}x\right)$ $\qquad = \sin 3x$ $\qquad = \sin(3x + 360°)$ $\qquad = \sin 3(x + 120°)$ $\therefore$ Periode $= 120°$	✓ $\sin 3x$ ✓ $120°$ (2)

10.3		$\checkmark$ x afsnitte $\checkmark\checkmark$ draaipunte $\checkmark$ vorm (4)
10.4	$(-180°;-90°)$ *of* $(-60°;0°)$ **OF** $-180° < x < -90°$ *of* $-60° < x < 0°$	$\checkmark$ $>-180°$ $\checkmark$ $<-90°$ $\checkmark$ $>-60°$ $\checkmark$ $<0°$ (4)
10.5	$y = \sin 2(x + 30°)$ $\therefore$ translasie van $30°$ na links	$\checkmark$ translasie $30°$ $\checkmark$ na links (2)
10.6	$\sin 2x = \cos(x - 30°)$ $\sin 2x = \sin[90° - (x - 30°)]$ $\qquad = \sin(120° - x)$ $2x = 120° - x + 360°k; k \in Z \qquad 2x = 180° - (120° - x) + 360°k$ $3x = 120° + 360°k \qquad$ **of** $2x - x = 60° + 360°k$ $x = 40° + 120°k; k \in Z \qquad\qquad x = 60° + 360°k; k \in Z$ **OF** $\sin 2x = \cos(x - 30°)$ $\cos(90° - 2x) = \cos(x - 30°)$ $90° - 2x = x - 30° + 360°k \quad$ or $\quad 90° - 2x = 360° - (x - 30°) + 360°k$ $-3x = -120° + 360°k \qquad\qquad -x = 300° + 360°k$ $\quad x = 40° - 120°k; k \in Z \qquad\qquad x = -300° - 360°k; k \in Z$ $\therefore x = 40° + 120°k \quad$ of $\quad x = 60° + 360°k \;;\; k \in Z$	$\checkmark$ gebruik ko-funksie $\checkmark$ $2x = 120° - x + 360°k$ $\checkmark$ $x = 40° + 120°k$ $\checkmark$ $2x = 180° - (120° - x)$ $+ 360°k$ $\checkmark$ $x = 60° + 360°k$ $\checkmark$ $k \in Z$ (6) $\checkmark$ $\cos(90° - x) = \cos(x - 30°)$ $\checkmark$ $90° - 2x = x - 30°$ $+ 360°k$ $\checkmark$ $x = 40° - 120°k$ $\checkmark$ $90° - 2x = 360°$ $- (x - 30°) + 360°k$ $\checkmark$ $x = -300° - 360°k$ $\checkmark$ $k \in Z$ (6) **[20]**

VRAAG 11

11.1	$\dfrac{AB}{r} = \sin 2x$ $AB = r\sin 2x$	✓ $\dfrac{AB}{r} = \sin 2x$ ✓ $AB = r\sin 2x$ (2)
11.2	$A\hat{K}C = 90° + x$	✓ $A\hat{K}C = 90° + x$ (1)

| 11.3 |

 In $\triangle AKC$:

 $\dfrac{\sin A\hat{K}C}{AC} = \dfrac{\sin A\hat{C}K}{AK}$

 $\dfrac{\sin(90° + x)}{r} = \dfrac{\sin x}{AK}$

 $AK = \dfrac{r\sin x}{\sin(90° + x)} = \dfrac{r\sin x}{\cos x}$

 $\dfrac{AK}{AB} = \dfrac{2}{3}$

 $\dfrac{\left(\dfrac{r\sin x}{\cos x}\right)}{r\sin 2x} = \dfrac{2}{3}$

 $\dfrac{\dfrac{\sin x}{\cos x}}{2\sin x\cos x} = \dfrac{2}{3}$

 $\dfrac{\sin x}{\cos x}\times\dfrac{1}{2\sin x\cos x} = \dfrac{2}{3}$

 $\dfrac{1}{2\cos^2 x} = \dfrac{2}{3}$

 $4\cos^2 x = 3$

 $\cos x = \dfrac{\sqrt{3}}{2}$

 $x = 30°$ | ✓ sin reël

 ✓ substitusie

 ✓ maak AK onderwerp van die formule
 ✓ cos x

 ✓ 2sin$x$.cos$x$

 ✓ $\dfrac{1}{2\cos^2 x}$

 ✓ $\cos x = \dfrac{\sqrt{3}}{2}$

 ✓ $x = 30°$
 (8) |
| | **OF** | |
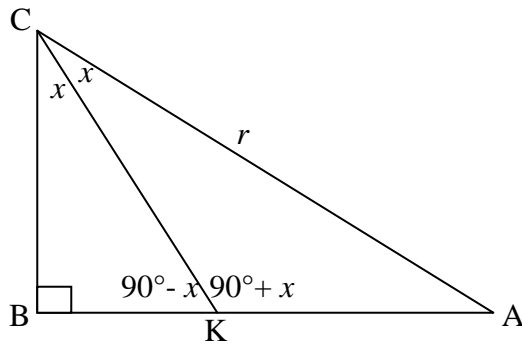
Gebruik die sin-formule in ΔCBK en ΔCKA:

$$\frac{\sin x}{BK} = \frac{\sin(90° - x)}{BC} \quad \text{en} \quad \frac{\sin x}{KA} = \frac{\sin(90° + x)}{AC}$$

$$\therefore \frac{BK}{BC} = \frac{KA}{AC}$$
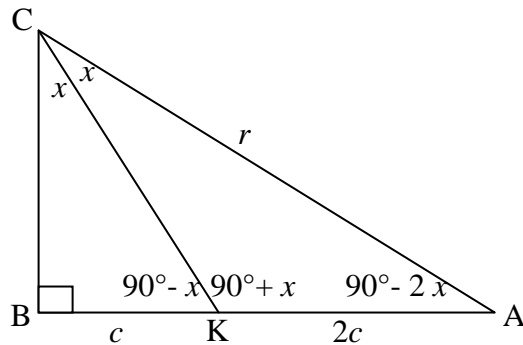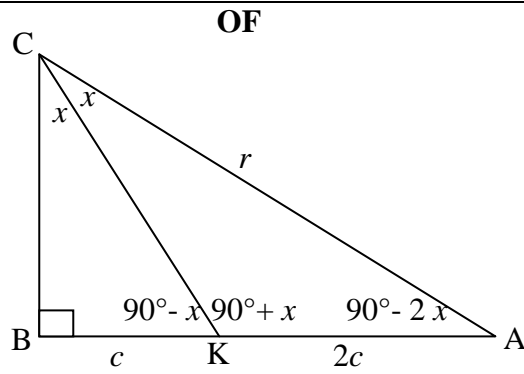
$$\therefore \frac{1}{BC} = \frac{2}{r}$$

$$\therefore BC = \frac{1}{2}r$$

$$\therefore \cos 2x = \frac{BC}{AC} = \frac{\frac{1}{2}r}{r} = \frac{1}{2}$$

$$\therefore 2x = 60°$$

$$\therefore x = 30°$$

✓ $\dfrac{\sin x}{BK} = \dfrac{\sin(90° - x)}{BC}$

✓ $\dfrac{\sin x}{KA} = \dfrac{\sin(90° + x)}{AC}$

✓ $\dfrac{BK}{BC} = \dfrac{KA}{AC}$

✓ $\dfrac{1}{BC} = \dfrac{2}{r}$

✓ $BC = \dfrac{1}{2}r$

✓ $\cos 2x = \dfrac{1}{2}$

✓ $2x = 60°$

✓ $x = 30°$

(8)

OF

ΔCBK: $\quad KC = \dfrac{c}{\sin x}$

ΔCKA: $\quad \dfrac{\sin x}{2c} = \dfrac{\sin(90° - 2x)}{KC} = \dfrac{\sin(90° - 2x).\sin x}{c}$

$$\therefore \sin(90° - 2x) = \frac{1}{2} = \sin 30°$$

$$\therefore \quad 90° - 2x = 30°$$

$$x = 30°$$

✓ $KC = \dfrac{c}{\sin x}$

✓ $\dfrac{\sin x}{2c} = \dfrac{\sin(90° - 2x)}{KC}$

✓✓ substitusie

✓✓ $\sin(90° - 2x) = \dfrac{1}{2}$

✓ $90° - 2x = 30°$

✓ $x = 30°$

OF   (8)

ΔCBK:

$$\sin 2x = \frac{3c}{r} = 2\sin x.\cos x$$

$$\therefore r\sin x = \frac{3c}{2\cos x} \quad ....(1)$$

ΔCKA:

$$\frac{2c}{\sin x} = \frac{r}{\cos x}$$

$$\therefore r\sin x = 2c\cos x ....(2)$$

Stel (1) en (2) gelyk:

$$2c.\cos x = \frac{3c}{2\cos x}$$

$$\therefore \cos^2 x = \frac{3}{4}$$

$$\therefore \cos x = \frac{\sqrt{3}}{2}$$

$$\therefore \quad x = 30°$$

OF

✓ $\sin 2x = \dfrac{3c}{r}$

✓ $2\sin x.\cos x$

✓ $r\sin x = \dfrac{3c}{2\cos x}$

✓ $\dfrac{2c}{\sin x} = \dfrac{r}{\cos x}$

✓ $r\sin x = 2c\cos x$

✓stel gelyk

✓ $\cos x = \dfrac{\sqrt{3}}{2}$

✓30°

(8)

$$\frac{AK}{KB} = \frac{2}{1} = 2$$

$$2 = \frac{\frac{1}{2}AK.BC}{\frac{1}{2}BK.BC}$$

$$= \frac{\text{area } AKC}{\text{area } ABC}$$

$$= \frac{\frac{1}{2}rCK\sin x}{\frac{1}{2}BC.CK\sin x}$$

$$= \frac{r}{BC}$$

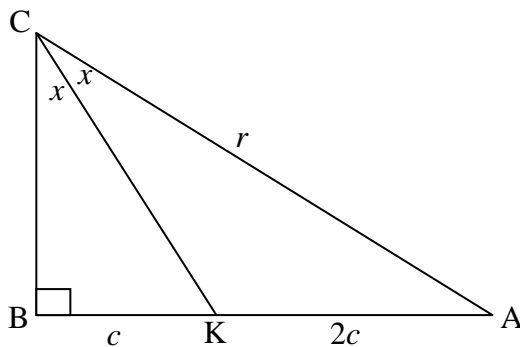$$\therefore \frac{BC}{r} = \frac{1}{2}$$

$$\therefore \cos 2x = \frac{1}{2}$$

$$\therefore 2x = 60°$$

$$\therefore x = 30°$$

✓ vermenigvuldig met $\frac{1}{2}BC$

✓ area van driehoeke

✓ area formule in driehoeke

✓ $\frac{r}{BC} = 2$

✓ $\frac{BC}{r} = \frac{1}{2}$

✓ $\cos 2x = \frac{1}{2}$

✓ $2x = 60°$

✓ $x = 30°$

(8)

OF

Deur gebruik te maak van die Interne Halveerder Stelling:

$$\frac{CB}{CA} = \frac{BK}{KA} = \frac{1}{2}$$

$$\cos 2x = \frac{1}{2}$$

$$2x = 60°$$

$$x = 30°$$

✓✓
Deur die spesifieke stelling te noem

✓✓✓ $\frac{CB}{CA} = \frac{BK}{KA} = \frac{1}{2}$

✓ $\cos 2x = \frac{1}{2}$

✓ $2x = 60°$

✓ $x = 30°$

(8)

OF
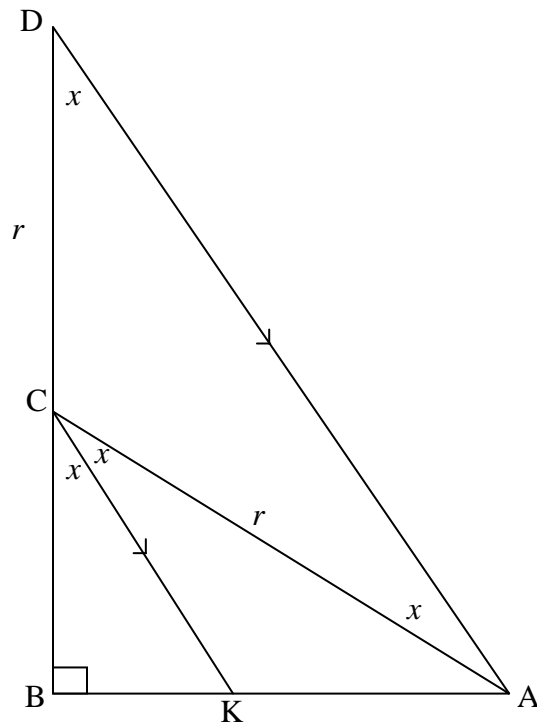
Verleng BC na D en teken CK parallel aan DA.

$C\hat{A}D = K\hat{C}A \quad and \quad B\hat{C}K = \hat{D}$

$\therefore DC = CA = r$

$\therefore \Delta BKC \;|||\; \Delta BAD$

$\therefore \dfrac{BK}{BA} = \dfrac{BC}{BD} = 3$

$\therefore BD = 3BC = BC + r$

$\therefore BC = \dfrac{1}{2}r$

$\therefore \cos 2x = \dfrac{\frac{1}{2}r}{r} = \dfrac{1}{2}$

$\therefore 2x = 60°$

$\therefore x = 30°$

✓ $DC = CA = r$

✓ $\Delta BKC \;|||\; \Delta BAD$

✓ $\dfrac{BK}{BA} = \dfrac{BC}{BD} = 3$

✓ BD = BC + r

✓ $BC = \dfrac{1}{2}r$

✓ $\cos 2x = \dfrac{1}{2}$

✓ $2x = 60°$

✓ 30°

(8)

[11]

TOTAAL: 150