basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

TECHNICAL MATHEMATICS

EXEMPLAR 2017

MEMORANDUM

MARKS: 150

This memorandum consists of 12 pages.

QUESTION 1

1.1	$M_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4-(2)}{-6-(2)}$ $= \frac{2}{-8} = -\frac{1}{4}$	✓Formula ✓Subst ✓Answer (3)
1.2	$D\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $D\left(\frac{-6-6}{2}; \frac{4-2}{2}\right)$ $D(-6; 1)$	✓formula ✓Subst ✓answer (3)
1.3	$M_{BD} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{1}{8}$ $y = \frac{1}{8}x + c \quad (1)$ $subt.(2;2)\text{ into }(1)$ $2 = (\frac{1}{8})(2) + c$ $2 = \frac{1}{4} + c$ $c = \frac{7}{4}$ $y = \frac{1}{8}x + \frac{7}{4}$	✓grad. of BD ✓correct subst. in formula ✓Value of c ✓Answer (4)
1.4	$D_{BC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-6-2)^2 + (-2-2)^2}$ $= \sqrt{(-8)^2 + (-4)^2}$ $= \sqrt{80}$	✓formula ✓substitution ✓answer (3)

1.5	$M_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2-2}{-6-2}$ $= \frac{-4}{-8}$ M $= \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26{,}57^0$	✓Subst. ✓$\frac{1}{2}$ ✓angle (3)
1.6	$\theta + 90^0 + C = 180^0$ *OR* $\hat{C} + 90^0 = 180^0$ $\hat{C} = 90^0 - 26{,}57^0$ $\hat{C} = 90^0 - \theta$ $= 63{,}43^0$ $= 63{,}43^0$	✓✓ *using sum of* $\angle^s$ *of* Δ ✓ -26.57^0 ✓Angle (4)
1.7	$y = mx + c$ $y = -\frac{1}{4}x + c$.....(1) *subst*$(-6;1)$ into (1) $1 = (-\frac{1}{4})(-6) + c$ Gradiënt of the line $= -\frac{1}{4}$ $1 = \frac{3}{2} + c$ $c = -\frac{1}{2}$ $y = -\frac{1}{4}x - \frac{1}{2}$	✓subst. of $-\frac{1}{4}$ ✓subst. ✓value of c ✓Answer (4) **[24]**

QUESTION 2

2.1.1	$\tan\theta = \frac{y}{x} = \frac{3}{1} = 3$ $\theta = \tan^{-1}(3)$ $\theta = 71.57^0$	✓ $\tan\theta$ ✓answer (2)
2.1.2	$OP^2 = (1)^2 + (3)^2$ $OP = \sqrt{10}$	✓ OP^2 ✓answer (2)
2.1.3 (a)	$\sin\theta = \frac{3}{\sqrt{10}}$	✓answer (1)
2.1.3 (b)	$\cos(180^0 + \theta) = -\cos\theta$ $= -\frac{1}{\sqrt{10}}$ Answer only : Full marks	✓ $-\cos\theta$ ✓answer (2)
2.2	$\tan(\theta - 60^0) = 4$ $\theta - 60^0 = \tan^{-1}(4)$ $\theta = \tan^{-1}(4) + 60^0$ $\theta = 135,96^0$	✓ $\frac{1}{\tan x}$ ✓ $\tan^{-1}$ ✓ answer (3)
2.3	$\frac{\cos(360^0 - x).\sin(180^0 - x).\tan 135^0}{\cos^2(180^0 + x).\sin 240^0}$ $= \frac{\cos x.\sin x.(-\tan 45)}{\cos^2 x.(-\sin 60^0)}$ $= \frac{\sin x.(-1)}{\cos x.\left(-\frac{\sqrt{3}}{2}\right)}$ $= \frac{2}{\sqrt{3}}\tan x$	✓ $\sin x$ ✓-$\sin x$ ✓ $\tan 45^0$ ✓ $\cos^2 x$ ✓ $-\sin 60^0$ ✓ $\frac{-\sin x}{-\frac{\sqrt{3}}{2}\cos x}$ ✓ $\frac{2\sqrt{3}}{3}\tan x$ (7)

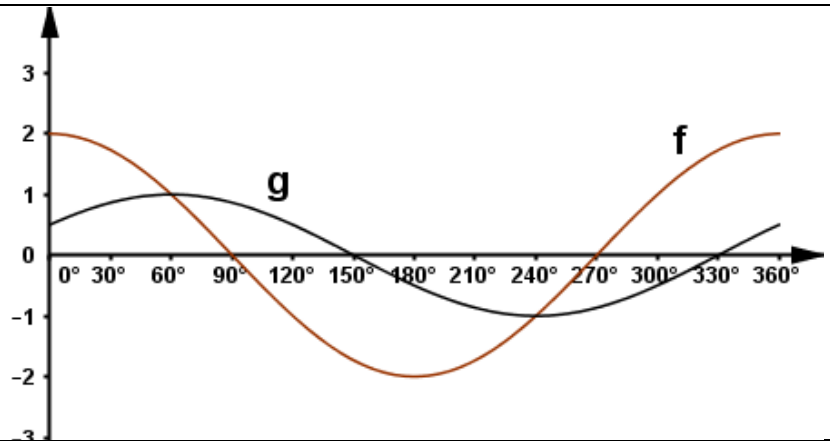
2.4	$\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}=\frac{2}{\sin x}$ LHS $\frac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$ $=\frac{\sin^2 x+1+2\cos x+\cos^2 x}{\sin x(1+\cos x)}$ $=\frac{2+2\cos x}{\sin x(1+\cos x)}$ $=\frac{2(1+\cos x)}{\sin x(1+\cos x)}$ $=\frac{2}{\sin x}=RHS$	✓numerator ✓denominator ✓expansion ✓simplification ✓factorisation (5) **[22]**

QUESTION 3

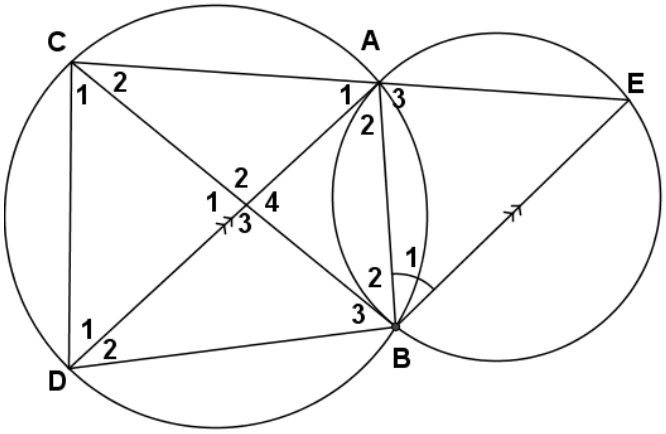
3.1	$D\hat{E}C=40^0$ $ALT\angle s\ AC\ //\ BE$	✓answer (1)
3.2	$\tan 40^0=\frac{CD}{100}$ $\therefore CD=100\times\tan 40^0$ $CD=83{,}91m$	✓Using tan and substitution ✓isolating CD ✓answer (3)
3.3	$A\hat{E}C=40^0-25^0$ $ext.\angle of\ a\Delta=sum\ of\ \text{int}opp\angle s$ $=15^0$	✓✓answer (2)
3.4	$\cos 40^0=\frac{100}{CE}$ $CE=\frac{100}{\cos 40^0}$ $\therefore CE=130{,}54m$ Or $CE^2=83{,}91^2+100^2$ (Pythagoras theorem) $=1703{,}921$ $\therefore CE=130{,}54m$	✓using cos and substitution ✓answer ✓substituting into Pythagoras ✓answer (2)

3.5	$\frac{AC}{\sin 15^0} = \frac{130,54}{\sin 25^0}$ $AC = \frac{130,54 \times \sin 15^0}{\sin 25^0}$ $\therefore AC = 79,95m$ Or In ΔABE, $A\hat{E}B = 25^0$ and $AB = 83,91$ Let $BD = x$, then $BE = (x+100)m$ $\tan 25^0 = \frac{83,91}{x+100}$ $x+100 = \frac{83,91}{\tan 25^0}$ $\therefore x = BD = 79,95m$	✓using sine rule ✓✓substitution ✓isolating AC ✓answer OR ✓ $BE = (x+100)m$ ✓✓ $\tan 25^0 = \frac{83,91}{x+100}$ ✓isolating x ✓answer (5)
		[13]

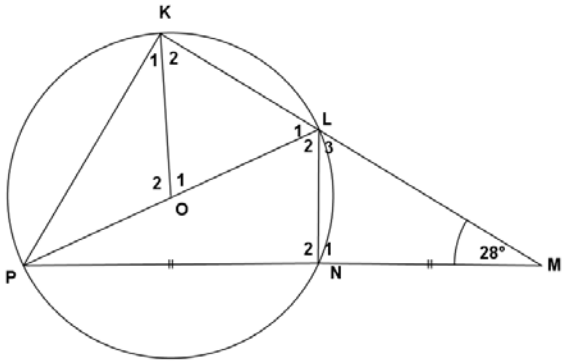
QUESTION 4

4.1		✓y-int. of g ✓shape of g ✓x intercepts of f ✓y intercept of f ✓shape of f (5)
4.2	2	✓ answer (1)
4.3	360^o	✓ answer (1)
4.4	$0^o < x \leq 60^o$ $180^0 < x \leq 360^0$	✓critical values ✓notation (2)
4.5	$60^0 < x < 240^0$	✓✓ answer with correct notation (2)
		[11]

QUESTION 5

5.1	 $\widehat{C}_2 = 40^0$ (tan *chord theorem*) $\widehat{D}_2 = 40^0$ (tan *chord theorem*) $\widehat{A}_2 = 40^0$ ($alt\angle s; AD//BE$) $\widehat{A}_1 = \widehat{A}_2 = 40^0$ (*given AD* bisects $C\widehat{A}B$) $\widehat{B}_3 = A_1 = 40^0$ ($\angle s$ *in same segment*) $\widehat{C}_1 = \widehat{A}_2 = 40^0$ ($\angle s$ *in same segment* $\widehat{E} = \widehat{A}_1 = 40$ ($corresp\angle s; AD//EB$	✓ angle ✓ reason ✓ angle ✓ reason ✓ angle ✓ reason ✓ angle ✓ reason ✓ angle ✓ reason ✓ angle ✓ reason ✓ angle ✓ reason (14)
5.2	$180^0 - (\widehat{E} + B_1) = sum\ of\ \angle s\ of\ \Delta$ $\widehat{A}_3 = 180^0 - (40^0 + 40^0)$ $= 180^0 - 80^0$ $= 100^0$	✓ *sum of* $\angle s$ *in* Δ is 180^0 ✓ answer (2)
		[16]

QUESTION 6

6.1	 *In* Δ *LPN and* Δ *LMN* 1) $PN = NM$ *given* 2) $N_2 = 90^0$ $\angle$ *In semi circle* $\hat{N}_1 = \hat{N}_2$ *adj.*supl *angles* $= 90^0$ 3)*LN is common* $\Rightarrow \Delta LPN = \Delta LMN$ (SAS) $\Rightarrow L\hat{P}N = 28^0$	✓given ✓ $\angle$*s in semi / circle* ✓ $\angle$*s on a str line* ✓common side ✓ *SAS* ✓ Answer (6)
6.2	$\hat{K}_1 + \hat{K}_2 = 90^0$ $K\hat{P}M = 180^0 - (90^0 + 28^0)$ *sum of* $\angle$*'s of* Δ $= 180^0 - 118^0$ $= 62^0$ $\Rightarrow K\hat{P}0 = 62^0 - 28^0$ $= 34^0$ $\Rightarrow K\hat{O}P = 180^0 - (34^0 + 34^0)$ *sum of* $\angle$*'s of* Δ $= 180^0 - 68^0$ $= 112^0$ *OR* $\hat{L}_1 = 28^0 + 28^0$ *ext.*$\angle$ = *sum of opp.*int.$\angle$ $K\hat{O}P = \hat{L}_1 + \hat{K}_2$ *ext.*$\angle$ = *sum of opp.*int.$\angle$ $K\hat{O}P = 56^0 + 56^0$ $= 112^0$	✓ $\angle$*s in semi / circle* ✓ *sum of* $\angle$*s in* Δ ✓ 34^0 ✓ *sum of* $\angle$*s in* Δ ✓answer ✓✓statement and reason ✓✓ statement and reason ✓answer (5)
		[11]

QUESTION 7

7.1.1	$OB = EB = x + 8$ *radii*	✓answer (1)
7.1.2	*In* ΔOBD $O\hat{D}B = 90^0$ line from centre to midpoint of the chord $OD^2 + DB^2 = OB^2$ Pythagoras theorem $x^2 + 12^2 = (x+8)^2$ $144 = 16x + 64$ $16x = 80 \Rightarrow x = 5$ $\therefore OB = 5 + 8 = 13cm$	✓90^0 ✓formula ✓subst. ✓answer (4)
7.2.1	$x = 180^0 - (68^0 + 68^0)$ $= 180^0 - 136^0$ $= 44^0$	✓ ✓ *sum of* $\angle s$ *of* Δ ✓Answer (3)
7.2.2	$\hat{B}_1 = 32^0$ tan *chord theorem* $y = 180^0 - (36^0 + 32^0)$ $= 180^0 - 68^0$ $= 112^0$	✓tan chord theorem ✓ *sum of* $\angle s$ *of* Δ ✓Answer (3)
7.2.3	$\hat{E} = 68^0$ $BD^2 = ED^2 + BE^2 - 2ED.BE.\cos\hat{E}$ $= (11)^2 + (11)^2 - 2(11)(11)\cos 68^0$ $BD = 12,3\,cm$ *OR* $\therefore \frac{BD}{\sin\hat{E}} = \frac{EB}{\sin B\hat{D}E}$ $\therefore BD = \frac{11 \times \sin 68^0}{\sin 56^0}$ $= 12,3cm$	✓value of angle E ✓cosine rule ✓subst. ✓answer ✓sine rule ✓value of angle E ✓value of $B\hat{D}E$ ✓answer (4)
		[15]

QUESTION 8

8.1	$\theta = 50^0,\ d = 52cm\ ,\ r = 26cm$ $s = r\theta$ $= 26 \times 50^0 \times \frac{\pi}{180^0}$ $= \frac{65}{9}\pi = 22{,}69cm$	✓substitution into correct formula ✓answer (2)
8.2.1	$38rev.\min^{-1} = \frac{38rev}{1 \times 60s} = 0{,}63rev.s^{-1}$	✓answer (1)
8.2.2	$n = 0{,}63rev.s^{-1}$ $\omega = 2\pi n$ $= 2 \times \pi \times 0{,}63$ $= \frac{63}{50}\pi = 3{,}96rad.s^{-1}$	✓correct formula ✓substitution ✓answer (3)
8.3.1	$s = \frac{2}{3}\pi d$ $= \frac{2}{3} \times \pi \times 8$ $= 16{,}76\text{ km}$ $v = \frac{s}{t}$ $= \frac{16{,}76km}{\frac{40}{60}h}$ $= 25{,}14\text{ km/h}$	✓substitution ✓value of s ✓Substitution into $v = \frac{s}{t}$ ✓answer (4)
8.3.2	$v = \pi dn$ $25{,}14 = \pi \times 8 \times n$ $n = \frac{25{,}14}{\pi \times 8}$ $= 1\text{ rev/h}$ $\therefore t = \frac{4}{1}hours$ $= 4\text{ hours}$ OR $v = \frac{s}{t}$ $t = \frac{s}{v}$ $= \frac{4\pi \times 8}{25{,}14km/h}$ $= 4hours$	✓substitution into correct formula ✓answer (2)
		[12]

QUESTION 9

9.1.1	$s = 2{,}1m \quad \theta = 1{,}8$ $s = r\theta$ $2{,}1 = r \times 1{,}8$ $r = 1{,}17\text{m}$	✓substitution ✓answer (2)
9.1.2	Area of a sector $= \frac{1}{2}r^2\theta$ $= \frac{1}{2} \times (1{,}17)^2 \times 1{,}8$ $= 1{,}23m^2$	✓correct formula ✓substitution ✓answer (3)
9.2	$x = 200mm = 20cm \quad d = 36cm$ $4h^2 - 4dh + x^2 = 0 \qquad \frac{x^2}{4h} + h = d$ $4h^2 - 4(36)h + 20^2 = 0 \quad$ ***OR*** $\quad \frac{20^2}{4h} + h = 36$ $h^2 - 36h + 100 = 0$ If $ah^2 + bh + c = 0$ Then $h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-36) \pm \sqrt{(-36)^2 - 4(1)(100)}}{2(1)}$ $= \frac{36 \pm \sqrt{896}}{2}$ $\therefore h = 32{,}97cm \quad$ or $\quad h = 3{,}03cm$	✓formula ✓substitution ✓standard form ✓substitution into quadratic formula ✓answers (5)
		[10]

QUESTION 10		
10.1.1	$Volume = l \times b \times h$ $= 25 \times 14 \times 6$ $= 2100m^3$ $= 2100000\,liter$	✓subst. into correct formula ✓conversion ✓answer (3)
10.1.2	$Surface\ Area = 2(L \times d) + (l \times b) + 2(d \times b)$ $Surface\ Area = 2(25 \times 6) + (25 \times 14) + 2(6 \times 14)$ $= 300 + 350 + 168$ $= 818m^2$ $Cost\ of\ p\text{aint} = R50 \times 818$ $= R40900$	✓Formula ✓Subst. ✓answer ✓Answer with unit (4)
10.2	$Surface\ area\ of\ the\ hemisphere + area\ of\ cylinder$ $= (\frac{1}{2})(4\pi r^2) + 2\pi r^2 + 2\pi rh$ $= (\frac{1}{2})(4)(\pi)(4)^2 + 2\pi(4)^2 + 2\pi(4)(16)$ $= 603,18cm^2$	✓formula ✓✓subst. ✓ conversion ✓answer (5)
10.3	$Area = (4)[(\frac{4+3,6}{2} + 4,5 + 4,7 + 4,9)]$ $= 4(3.8 + 14,1)$ $= 4(17,9)$ $= 71,6m^2$ **OR** $Area = 4\left(\frac{4+4.5}{2} + \frac{4,5+4,7}{2} + \frac{4,7+4,9}{2} + \frac{4,9+3,6}{2}\right)$ $= 4(17,9)$ $= 71,6m^2$	✓Formula ✓subst. ✓simplify ✓answer Or ✓Formula ✓subst. ✓simplify ✓answer (4) **[16]**
		TOTAL: 150