



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P1

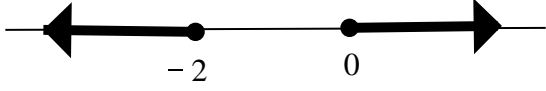
MARKING GUIDELINES

EXEMPLAR 2018

MARKS: 150

These marking guidelines consist of 12 pages.

QUESTION 1

<p>1.1.1</p>	$x(x+2) = 0$ $\therefore x = 0 \text{ or } x = -2$	$\checkmark x = 0$ $\checkmark x = -2$ <p style="text-align: right;">(2)</p>
<p>1.1.2</p>	$x(x+2) \geq 0$ $\therefore x \leq -2 \quad \text{OR} \quad x \geq 0$ 	$\checkmark x \leq -2 \quad \checkmark x \geq 0$ $\checkmark \text{ OR}$ $\checkmark \text{ Graphical representation}$ <p style="text-align: right;">(4)</p>
<p>1.2</p>	$5x^2 = 2 + x$ $5x^2 - x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2(5)} = \frac{1 \pm \sqrt{41}}{10}$ $\therefore x \approx 0,74 \text{ or } x \approx -0,54$	$\checkmark \text{ Standard form}$ $\checkmark \text{ Substitution into the quadratic formula}$ $\checkmark x \approx 0,74 \quad \checkmark x \approx -0,54$ <p style="text-align: right;">(4)</p>
<p>1.3</p>	$m - t - 1 = 0$ $m = t + 1$ $m^2 + t^2 = 5$ $(t+1)^2 + t^2 = 5$ $t^2 + 2t + 1 + t^2 - 5 = 0$ $2t^2 + 2t - 4 = 0$ $t^2 + t - 2 = 0$ $(t+2)(t-1) = 0$ $\therefore t = -2 \text{ or } t = 1$ $m = -2 + 1 = -1 \quad \text{or} \quad m = 1 + 1 = 2$ <p>OR</p> $m - t - 1 = 0$ $t = m - 1$ $m^2 + t^2 = 9$ $m^2 + (m-1)^2 = 5$ $m^2 + m^2 - 2m + 1 - 5 = 0$ $2m^2 - 2m - 4 = 0$ $m^2 - m - 2 = 0$ $(m-2)(m+1) = 0$ $\therefore m = 2 \text{ or } m = -1$ $t = 2 - 1 = 1 \text{ or } t = -1 - 1 = -2$	$\checkmark \text{ Making } m \text{ the subject}$ $\checkmark \text{ Substitution}$ $\checkmark \text{ Simplification}$ $\checkmark \text{ Factors}$ $\checkmark \text{ Both values of } t$ $\checkmark \text{ Both values of } m$ <p>OR</p> $\checkmark \text{ Making } t \text{ the subject}$ $\checkmark \text{ Substitution}$ $\checkmark \text{ Simplification}$ $\checkmark \text{ Factors}$ $\checkmark \text{ Both values of } m$ $\checkmark \text{ Both values of } t$ <p style="text-align: right;">(6)</p>

1.4.1	$\varepsilon = \frac{L_2 - L_1}{L_1}$ $\varepsilon L_1 = L_2 - L_1$ $\varepsilon L_1 + L_1 = L_2$ $L_1(\varepsilon + 1) = L_2$ $L_1 = \frac{L_2}{(\varepsilon + 1)}$	$\varepsilon = \frac{L_2}{L_1} - 1$ $\varepsilon + 1 = \frac{L_2}{L_1}$ $L_1(\varepsilon + 1) = L_2$ $\therefore L_1 = \frac{L_2}{(\varepsilon + 1)}$	✓ multiply with LCD ✓ common factor ✓ divide by factor (3)
1.4.2	$L_1 = \frac{L_2}{\varepsilon + 1}$ $= \frac{18}{1 + 0,8} \text{ cm}$ $= 10 \text{ cm}$		✓ Substitution ✓ Simplification (2)
1.4.3	$10 = 8 + 2 = 2^3 + 2$ $= 1010_2$		✓ $2^3 + 2$ ✓ 1010_2 , (2)
1.5	$12 \times 0,00361$ $= 0,04332$ $= 4,332 \times 10^{-2}$		✓ 0,04332 ✓ $4,332 \times 10^{-2}$ [25]

QUESTION 2

2.1.1	$p = -1$		✓ $p = -1$ (1)
2.1.2	$9 - 3p < 0$ $9 < 3p$ $\therefore p > 3$		✓ $9 - 3p < 0$ ✓ $p > 3$ (2)
2.1.3	$0 \text{ OR } 3$		✓ $0 \text{ OR } 3$ (1)
2.2	$x^2 - 4x + (k - 1) = 0$ For equal roots, $\Delta = b^2 - 4ac = 0$ $(-4)^2 - 4(1)(k - 1) = 0$ $16 - 4k + 4 = 0$ $-4k = -20$ $\therefore k = 5$		✓ For equal roots, $\Delta = 0$ ✓ Substitution ✓ Simplification ✓ Value of k (4) [8]

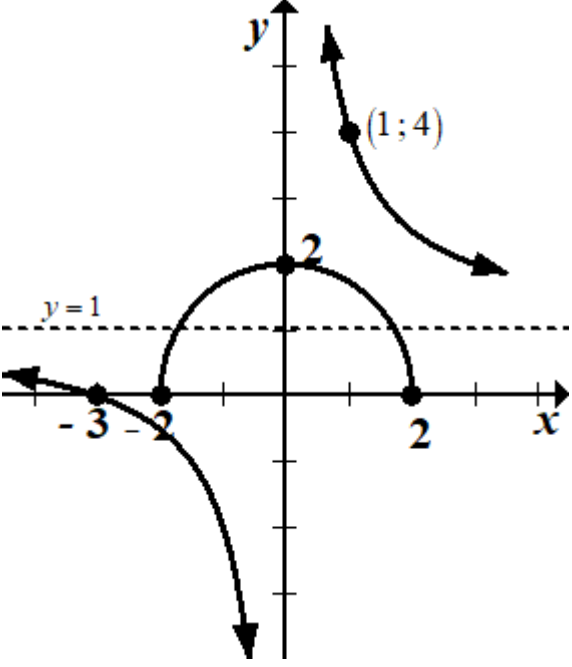
QUESTION 3

<p>3.1.1</p>	$\frac{5 \times 2^{n-1} - 2^n}{2^n}$ $= \frac{2^n (5 \times 2^{-1} - 1)}{2^n}$ $= 5 \times \frac{1}{2} - 1 = \frac{3}{2}$ <p>OR</p> $\frac{5 \times 2^{n-1} - 2^n}{2^n}$ $= \frac{5 \times 2^{n-1}}{2^n} - \frac{2^n}{2^n} = 5 \times 2^{-1} - 1$ $= 2 \frac{1}{2} - 1 = \frac{3}{2}$	<p>✓✓ Common factor</p> <p>✓ Simplification</p> <p>✓✓ Dividing each term by the denominator</p> <p>✓ Simplification</p> <p style="text-align: right;">(3)</p>
<p>3.1.2</p>	$\sqrt{64+16} - \sqrt{20}$ $= \sqrt{80} - \sqrt{4 \times 5}$ $= 4\sqrt{5} - 2\sqrt{5}$ $= 2\sqrt{5}$	<p>✓ Addition</p> <p>✓ Simplified surd</p> <p>✓ Simplified surd</p> <p>✓ Simplification</p> <p style="text-align: right;">(4)</p>
<p>3.1.3</p>	$\log_6 216 \times \log 0,001$ $= \log_6 6^3 \times \log \frac{1}{1000}$ $= \log_6 6^3 \times \log 10^{-3}$ $= 3 \log_6 6 \times (-3 \log 10)$ $= 3(1) \times (-3)(1)$ $= -9$	<p>✓ $\log_6 6^3$ ✓ $\log 10^{-3}$</p> <p>✓ $3 \log_6 6 - 3 \log 10$</p> <p>✓ Simplification</p> <p style="text-align: right;">(4)</p>
<p>3.2.2</p>	$\log(x+18) - \log x = 1$ $\log \frac{(x+18)}{x} = 1$ $\frac{(x+18)}{x} = 10$ $10x = x+18$ $9x = 18$ $\therefore x = 2$	<p>✓ Apply log property</p> <p>✓ Change from log form to exp. Form</p> <p>✓ Simplification</p> <p>✓ Value of x</p> <p style="text-align: right;">(4)</p>

<p>3.3</p>	$z = 3 + \sqrt{3}i$ $ z = r = \sqrt{x^2 + y^2}$ $= \sqrt{(3)^2 + (\sqrt{3})^2} = \sqrt{12}$ $\tan \theta = \frac{\sqrt{3}}{3}$ $\theta = 30^\circ$ $z = \sqrt{12} \operatorname{cis}(30^\circ) \quad \text{OR} \quad z = \sqrt{12} [\cos 30^\circ + i \sin 30^\circ]$	<p>✓ Calculating the modulus</p> <p>✓ Simplification</p> <p>✓ $\tan \theta = \frac{\sqrt{3}}{3}$</p> <p>✓ Argument</p> <p>✓ Correct polar form</p> <p style="text-align: right;">(5)</p>
<p>3.4</p>	$x + yi = (3 + 5i)(2 - 7i)$ $x + yi = 6 - 11i - 35i^2$ $x + yi = 6 - 11i - 35(-1)$ $x + yi = 6 - 11i + 35$ $x + yi = 41 - 11i$ $\therefore x = 41 \text{ and } y = -11$	<p>✓ $6 - 11i - 35i^2$</p> <p>✓ $i^2 = -1$</p> <p>✓ $x = 41$ ✓ $y = -11$</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;">[24]</p>

QUESTION 4

4.1.1	x – intercepts, $f(x) = 0$ $2x^2 + 4x - 6 = 0$ $2(x+3)(x-1) = 0$ OR $(x+3)(2x-21) = 0$ $\therefore x = -3$ or $x = 1$ $\therefore B(1; 0)$	✓ Finding other factor ✓ Coordinates of B. (2)
4.1.2	$f(x) = 2x^2 + 4x - 6$ $\left(\frac{-b}{2a}; \frac{4ac - b^2}{4a}\right) = \left(\frac{-4}{2(2)}; \frac{4(2)(-6) - (4)^2}{4(2)}\right)$ $\therefore D(-1; -8)$ OR $x = \frac{-b}{2a} = \frac{-4}{2(2)}$ $\therefore x = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$ OR $x_D = \frac{-3+1}{2} = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$ OR $f(x) = 2x^2 + 4x - 6$ $f'(x) = 4x + 4 = 0$ $\therefore x = -1$ $f(-1) = 2(-1)^2 + 4(-1) - 6 = -8$ $\therefore D(-1; -8)$	✓✓ Substitution in formula ✓ Coordinates of D OR ✓ Substitution in formula ✓ Substitution to find y ✓ Coordinates of D OR ✓ Using x -intercepts ✓ Substitution to find y ✓ Coordinates of D OR ✓ Using the derivative ✓ Substitution to find y ✓ Coordinates of D (3)
4.1.3	$g(x) = k^x + q$ $10 = k^2 + 6$ $k^2 = 4$ $\therefore k = 2$	✓ Substituting coordinates of Q ✓ Simplified equation ✓ Correct value of k . (3)
4.1.4	$y = 6$	✓ $y = 6$ (1)
4.1.5	$-3 < x < 1$	✓ Correct critical values ✓ Correct notation (2)

4.2.1	$x = 0$ and $y = 1$	✓ $x = 0$ ✓ $y = 1$ (2)
4.2.2	$h(x) = \frac{3}{x} + 1$ $0 = \frac{3}{x} + 1$ $-1 = \frac{3}{x}$ $\therefore x = -3$	✓ Substituting coordinates of Q ✓ Value of x (2)
4.2.3	$r = 2$	✓ $r = 2$ (1)
4.2.4		✓ Shape of h ✓ Asymptote ✓ x -intercept ✓ Any other point on the graph of h ✓ Shape of g ✓ x -intercepts of g ✓ y -intercept of g (7)
4.2.5	$0 \leq y \leq 2$	✓ $0 \leq y$ ✓ $y \leq 2$ (2) [25]

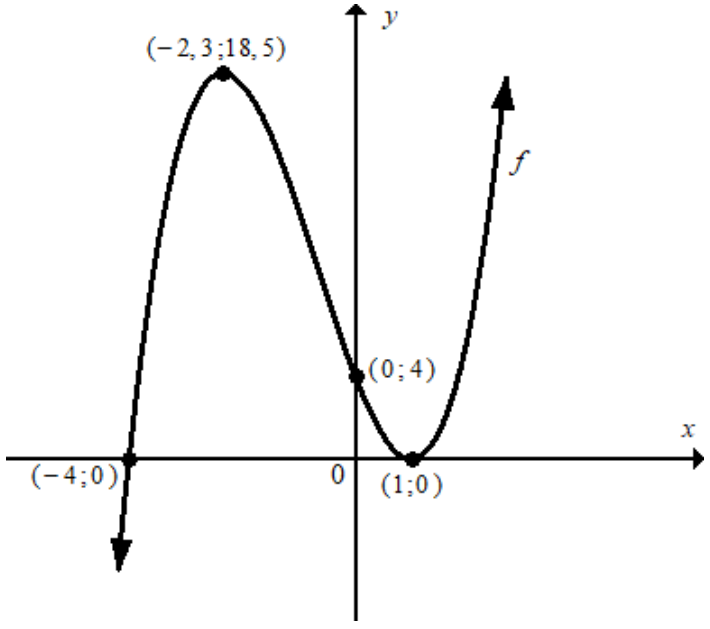
QUESTION 5

<p>5.1</p>	$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$ $i_{eff} = \left(1 + \frac{0,072}{2}\right)^2 - 1$ $\approx 0,073296$ <p>\therefore annual effective interest rate is 7,33%</p>	<ul style="list-style-type: none"> ✓ Correct substitution ✓ Simplification ✓ Effective rate as % <p style="text-align: right;">(3)</p>
<p>5.2</p>	$A = P(1 - i)^n$ $70 = 220(1 - 0,08)^n$ $\frac{7}{22} = (0,92)^n$ $n = \log_{0,92} \frac{7}{22}$ <p>$\therefore n \approx 13,73363166$</p> <p>It will take approximately 14 minutes.</p>	<ul style="list-style-type: none"> ✓ Correct formula ✓ Correct substitution ✓ Simplified power form ✓ Using logarithms ✓ Nearest minute <p style="text-align: right;">(5)</p>
<p>5.2.2</p>	<p>Value of A after 3 years:</p> $A = P(1 + i)^n$ $A = R150000 \left(1 + \frac{10,5\%}{4}\right)^{3 \times 4}$ $= R204705,40$ <p>Value of P after withdrawal:</p> $P = R204705,40 - R30000 = R174705,40$ <p>Amount received at the end of the investment period:</p> $A = R174705,40 \left(1 + \frac{10,5\%}{4}\right)^{2 \times 4}$ <p>$\therefore A = R214947,15$</p>	<ul style="list-style-type: none"> ✓ Correct formula ✓ Correct substitution ✓ R204705,40 ✓ P = R174705,40 ✓ Correct substitution ✓ Final amount <p style="text-align: right;">(6)</p> <p style="text-align: right;">[14]</p>

QUESTION 6

<p>6.1</p>	$f(x) = 2x^2 - 3$ <p>Average gradient = $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$</p> $= \frac{[2(1)^2 - 3] - [2(-2)^2 - 3]}{1 - (-2)}$ $= \frac{-1 - 5}{3}$ $= -2$	<p>✓ Corresponding y-value</p> <p>✓ Corresponding y-value</p> <p>✓ Substitution in formula</p> <p>✓ Simplification</p> <p style="text-align: right;">(4)</p>
<p>6.2</p>	$f(x) = 4 - 3x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{[4 - 3(x+h)] - (4 - 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4 - 3x - 3h - 4 + 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{-3h}{h}$ $= \lim_{h \rightarrow 0} (-3)$ $= -3$	<p>✓ Definition</p> <p>✓ Substitution in the definition</p> <p>✓ Simplification (removing brackets)</p> <p>✓ Simplification (division)</p> <p>✓ Simplification</p> <p style="text-align: right;">(5)</p>
<p>6.3</p>	$y = \frac{2}{x^3} + \sqrt{x}$ $y = 2x^{-3} + x^{\frac{1}{2}}$ $\frac{dy}{dx} = -6x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$	<p>✓ $2x^{-3}$ ✓ $x^{\frac{1}{2}}$</p> <p>✓ $-6x^{-4}$ ✓ $\frac{1}{2}x^{-\frac{1}{2}}$</p> <p style="text-align: right;">(4)</p>
<p>6.4</p>	$g(x) = -x^2 - x$ $g(2) = -(2)^2 - 2 = -6$ <p>The point of contact is (2; -6)</p> $g'(x) = -2x - 1$ $\therefore m_{\tan} = g'(2) = -2(2) - 1 = -5$ $y = mx + c \quad \text{OR} \quad y - y_1 = m(x - x_1)$ $-6 = -5(2) + c \quad \text{OR} \quad y - (-6) = -5(x - 2)$ $c = 4 \quad \text{OR} \quad y + 6 = -5x + 10$ $\therefore y = -5x + 4$	<p>✓ value of y</p> <p>✓ $m_{\tan} = -5$</p> <p>✓ Correct substitution</p> <p>✓ Value of c (simplification)</p> <p>✓ Equation (any form)</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">[18]</p>

QUESTION 7

<p>7.1</p>	$f(x) = x^3 + 2x^2 - 7x + 4$ $f(1) = (1)^3 + 2(1)^2 - 7(1) + 4$ $\therefore f(1) = 0$ $\therefore x - 1 \text{ is a factor of } f$	<p>✓ Substitution ✓ 0</p> <p style="text-align: right;">(2)</p>
<p>7.2</p>	<p><i>x</i>-intercepts: $f(x) = 0$ $x^3 + 2x^2 - 7x + 4 = 0$ $(x - 1)(x^2 + 3x - 4) = 0$ $(x - 1)(x - 1)(x + 4) = 0$ $x = 1$ or $x = -4$</p>	<p>✓ $(x^2 + 3x - 4)$ (quadratic) ✓ $(x - 1)(x - 1)(x + 4)$ (linear) ✓ <i>x</i>-intercepts</p> <p style="text-align: right;">(3)</p>
<p>7.3</p>	$f(x) = x^3 + 2x^2 - 7x + 4$ $f'(x) = 3x^2 + 4x - 7$ $f'(x) = 0$ $\therefore 3x^2 + 4x - 7 = 0$ $(3x + 7)(x - 1) = 0$ $\therefore x = -\frac{7}{3} \text{ or } x = 1$ $(-2, 3 ; 18, 5) \text{ and } (1; 0)$	<p>✓ Derivative ✓ $f'(x) = 0$</p> <p>✓ Factorisation</p> <p>✓ Both values of <i>x</i></p> <p>✓ Coordinates of the turnings</p> <p style="text-align: right;">(5)</p>
<p>7.4</p>		<p>✓ Shape ✓ Intercepts with <i>x</i>-axis ✓ <i>y</i>-intercept ✓ Turning points</p> <p style="text-align: right;">(4) [14]</p>

QUESTION 8

8.1.1	After 2 hrs $D(2) = 4 + 0,5(2)^2 - 0,25(2)^3 \text{ m}$ $= 4\text{m}$	✓Substituting 2 ✓Simplification (2)
8.1.2	$D = 4 + 0,5t^2 - 0,25t^3$ $D'(t) = t - 0,75t^2$ At 12:00 (3 hours later): $D'(3) = (3) - 0,75(3)^2$ $= -3,75 \text{ m.h}^{-1}$ \therefore	✓Derivative ✓Substitution of 3 ✓Simplified rate (3)
8.2.1	$P = -3v^2 + 30v$ Neither profit nor loss at $P = 0$ $-3v^2 + 30v = 0$ $-3v(v - 10) = 0$ $\therefore v = 0 \text{ or } v = 10$ $v = 10 \text{ km.h}^{-1}$	✓ $P = 0$ ✓Factors ✓Correct value of v (3)
8.2.2	$P = -3v^2 + 30v$ $\frac{dP}{dv} = -6v + 30 = 0$ $\therefore v = 5 \text{ km.h}^{-1}$	✓Derivative ✓Equating to 0 ✓Value of v (3)
8.2.3	$P_{\text{max}} \text{ (in R1000)} = -3(5)^2 + 30(5) = 75$ OR R75 000	✓Substitution ✓Profit in R1 000 (2) [13]

QUESTION 9

<p>9.1</p>	$\int \left(x^{-4} + \frac{7}{x} - 1 \right) dx$ $= \int x^{-4} dx + 7 \int \frac{1}{x} dx - \int dx$ $= \frac{x^{-3}}{-3} + 7 \ln x - x + C$	$\checkmark \frac{x^{-5}}{-5}$ $\checkmark 7 \ln x$ $\checkmark -x$ $\checkmark C$ <p style="text-align: right;">(4)</p>
<p>9.2</p>	$h(x) = -2x^2 - 6x$ $\int_{-3}^0 (-2x^2 - 6x) dx$ $= \left[-\frac{2x^3}{3} - 3x^2 \right]_{-3}^0$ $= \left[\left(-\frac{2(0)^3}{3} - 3(0)^2 \right) - \left(-\frac{2(-3)^3}{3} - 3(-3)^2 \right) \right]$ $= -18 + 27$ $= 9 \text{ units square}$	$\checkmark -\frac{2x^3}{3}; \quad \checkmark -3x^2$ $\checkmark \text{Substituting } 0$ $\checkmark \text{Substituting } -3$ $\checkmark \text{Simplification}$ <p style="text-align: right;">(5) [9]</p>

TOTAL: 150