**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

TECHNICAL MATHEMATICS P2

MARKING GUIDELINES
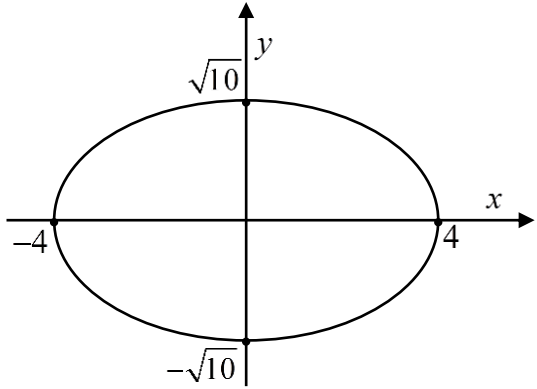
EXEMPLAR 2018

MARKS: 150

These marking guidelines consist of 14 pages

QUESTION 1

1.1	$\text{AD} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5-1)^2 + (-2-4)^2}$ $= \sqrt{52}$ $= 2\sqrt{13}$	✓Substitution in correct formula ✓Simplification ✓ $2\sqrt{13}$ simplified surd (3)
1.2	$\text{M}\left(\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2}\right)$ $\text{M}\left(\dfrac{1+5}{2}; \dfrac{4-2}{2}\right)$ $\text{M}(3; 1)$	✓Substitution ✓M(3; 1) (2)
1.3	$m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{1-4}{-3-1}$ $= \dfrac{3}{4}$ AB ‖ MC $m_{MC} = \dfrac{3}{4}$ $y - y_1 = m(x - x_1)$ or $y = mx + c$ $y - 1 = \dfrac{3}{4}(x-3)$ $1 = \dfrac{3}{4}(3) + c$ $-3x + 4y + 5 = 0$ $c = \dfrac{4-9}{4} = -\dfrac{5}{4}$ or $3x + -4y - 5 = 0$ $y = \dfrac{3}{4}x - \dfrac{5}{4}$ $-3x + 4y + 5 = 0$	✓Substitution in correct formula ✓Gradient of MC ✓Substitution in correct formula ✓Simplification ✓Correct answer in correct form (5)
1.4	$\tan \alpha = \dfrac{3}{4}$ $\alpha \approx 36{,}87°$	✓ $\tan \alpha = \dfrac{3}{4}$ ✓ $36{,}87°$ (2)

1.5	$m_{AD} = \dfrac{y_2 - y_1}{x_2 - x_1}$	
	$= \dfrac{-2 - 4}{5 - 1}$	$\checkmark \dfrac{-3}{2}$
	$= \dfrac{-3}{2}$	
	$\tan \beta = \dfrac{-3}{2}$	$\checkmark \beta = 123,69°$
	$\beta \approx 123,69°$	$\checkmark 123,69° - 36,87°$
	$\hat{BAD} = 123,69° - 36,87°$ *ext $\angle$ of Δ*	$\checkmark 86,82°$
	$= 86,82°$	(4)
		[16]

QUESTION 2

2.1.1	$x^2 + y^2 = r^2$ $r^2 = (10)^2 + (-4)^2$ $\therefore r = \sqrt{116} = 2\sqrt{29}$	✓Substitution. ✓ $\sqrt{116} = 2\sqrt{29}$ (2)
2.1.2	$m_{OH} = m_{OF} = \dfrac{0-(-4)}{0-10}$ $= -\dfrac{4}{10}$ $= -\dfrac{2}{5}$	✓Substitution. ✓ $-\dfrac{2}{5}$ (2)
2.1.3	$m_{GH} = \dfrac{5}{2} \qquad (m_{GH} \times m_{OH} = -1)$ By symmetry $H(-10 ; 4)$ $y - y_1 = m(x - x_1) \qquad or \qquad y = mx + c$ $y - 4 = \dfrac{5}{2}(x+10) \qquad\qquad 4 = \dfrac{5}{2}(-10) + c$ $y = \dfrac{5}{2}x + 29 \qquad\qquad\qquad c = 29$ $\qquad\qquad\qquad\qquad\qquad y = \dfrac{5}{2}x + 29$	✓ m_{GH} ✓ $H(-10 ; 4)$ ✓Substitution of H ✓Equation (4)
2.2		✓Both x-intercepts ✓Both y-intercepts ✓Shape (3)
		[11]

QUESTION 3

3.1.1	$OP^2 = (12)^2 + (-5)^2$ $\therefore OP = 13$ units	✓ $OP = 13$ units (1)
3.1.2	$5\cot\theta - 13\cos\theta$ $= 5\left(\dfrac{12}{-5}\right) - 13\left(\dfrac{12}{13}\right)$ $= -24$	✓ $\dfrac{12}{-5}$ ✓ $\dfrac{12}{13}$ ✓ Simplification (3)
3.1.3	$\cosec^2 x - 1$ $= \left(\dfrac{13}{-5}\right)^2 - 1$ $= \dfrac{144}{25}$ $or \ \cosec^2 x - 1 = \cot^2 x$ $= \left(\dfrac{12}{-5}\right)^2$ $= \dfrac{144}{25}$	✓ $\left(\dfrac{13}{-5}\right)^2 \ or \ \left(\dfrac{12}{-5}\right)^2$ ✓ Simplification (2)
3.2	$\sec(a-b)$ $= \sec(2,659 - 1,112)$ $= \sec 1,547$ $= \dfrac{1}{\cos 1,547}$ $\approx 42,03$	✓ Substitution ✓ Reciprocal ✓ Simplification using radians (3)
		[**9**]

QUESTION 4

4.1	$\dfrac{\sin(360° - x).\cos(180° - x).\tan 120°}{\cos^2 x.\sin \dfrac{5\pi}{6}}$	$\checkmark -\sin x$
		$\checkmark -\cos x$
		$\checkmark -\tan 60°$
	$= \dfrac{(-\sin x).(-\cos x).(-\tan 60°)}{\cos^2 x.\sin \dfrac{\pi}{6}}$	
		$\checkmark \sin \dfrac{\pi}{6}$
	$= \dfrac{-\sin x.\cos x.-\sqrt{3}}{\cos^2 x \cdot \dfrac{1}{2}}$	$\checkmark -\sqrt{3}$ and $\dfrac{1}{2}$
	$= \dfrac{2\sqrt{3}\sin x}{\cos x}$	$\checkmark$ simplification
	$= 2\sqrt{3}\tan x$	$\checkmark \tan x$
		(7)
4.2	$\cos^2 3x$	$\checkmark \cos^2 3x$
		(1)
4.3	$LHS = \dfrac{\sin x}{\cos x}(\sin x)$	
	$= \dfrac{\sin^2 x}{\cos x}$	$\checkmark \dfrac{\sin x}{\cos x}$
	$RHS = \sec x - \cos x$	
	$= \dfrac{1}{\cos x} - \cos x$	$\checkmark \dfrac{1}{\cos x}$
	$= \dfrac{1 - \cos^2 x}{\cos x}$	$\checkmark \dfrac{1 - \cos^2 x}{\cos x}$
	$= \dfrac{\sin^2 x}{\cos x}$	$\checkmark \sin^2 x$
	$= LHS$	(4)
4.4	$\operatorname{cosec} 2x = 2{,}114$	
	$\sin 2x = \dfrac{1}{2{,}114}$	$\checkmark$ using reciprocal
	$2x = \sin^{-1}\left(\dfrac{1}{2{,}114}\right)$	$\checkmark$ inverse
	$2x = \sin^{-1}\left(\dfrac{1}{2{,}114}\right)$	
	$2x \approx 28{,}23°$ _or_ $2x \approx 180° - 28{,}23°$	$\checkmark$ value of x
	$x \approx 14{,}12°$ _or_ $x \approx 75{,}89°$	$\checkmark$ another value of x
		(4)
		[16]

QUESTION 5

5.1	$\sin 55^\circ = \dfrac{50}{AC}$ $AC = \dfrac{50}{\sin 55^\circ}$ $\approx 61\,m$	✓Definition ✓AC subject ✓ 61 m (3)
5.2	$AD \approx 61$ m $DC^2 = AC^2 + AD^2 - 2AC.AD\cos 65^\circ$ $DC^2 = (61)^2 + (61)^2 - 2(61)(61)\cos 65^\circ$ $DC \approx 66\,m$	✓AD $\approx$ 61 m ✓Using cosine rule ✓Substitution ✓ 66 m (4)
5.3	$BD = \sqrt{AD^2 - AB^2}$ $\qquad \tan 55^\circ = \dfrac{50}{BD}$ $= \sqrt{(61)^2 - (50)^2}$ OR $\quad BD = \dfrac{50}{\tan 55^\circ}$ $\approx 35\,m$ $\qquad\qquad\qquad = 35\,m$ $\therefore area\ of\ \Delta BDC = \frac{1}{2}(35)(66)\sin B\hat{D}C = 563$ $\sin B\hat{D}C = \dfrac{563}{\frac{1}{2}(35)(66)}$ $\therefore\ B\hat{D}C = \sin^{-1}\left(\dfrac{563}{\frac{1}{2}(35)(66)}\right)$ $= 29,17^\circ$	✓Using Pythagoras theorem or tan ✓BD = 35 m ✓Substitution in area formula ✓Area = 563 ✓ $\sin B\hat{D}C = \dfrac{563}{\frac{1}{2}(35)(66)}$ ✓Simplification (6)
		[13]

QUESTION 6

6.1		*f(x)* ✓*x*-intercepts ✓*y*-intercept ✓Shape *g(x)* ✓*y*-intercept ✓Turning points ✓Shape (6)
6.2	2	✓2 (1)
6.3	360°	✓360° (1)
6.4	$60° < x < 240°$	✓✓Answer with correct notation (2)
		[10]

QUESTION 7

7.1	the angle in the alternate segment	✓Correct statement (1)
7.2.1	$\hat{R}_2 = \hat{S}_1 = 38°$ $\quad$ tan – chord theorem	✓Statement ✓Reason (2)
7.2.2	$\hat{M}_1 = 2\hat{R}_2 = 76°$ $\quad$ $\angle$ at center $= 2\angle$ at circum	✓Statement ✓Reason (2)
7.2.3.	$\hat{S}_2 = 90° - 38° = 52°$ $\quad$ tan $\perp$ radius	✓Statement ✓Reason (2)
7.2.4	$\hat{R}_1 = 90°$ $\qquad\qquad\qquad$ $\angle$ in a semi – circle $\hat{Q}_1 = \hat{S}_1 = 38°$ $\qquad\qquad$ tan– chord theorem $\therefore \hat{Q}_2 = 180° - (17° + 90° + 38°)$ $\quad$ sum of $\angle$'s of a Δ $\qquad = 35°$	✓Statement ✓Reason ✓Reason ✓Statement ✓Statement (5)
7.2.5	$\hat{R}_2 = 38°$ $\quad$ and $\hat{P}_1 = 17°$ $\therefore \hat{R}_2 \neq \hat{P}_1$ $\quad$ (alternate angles are not equal)	✓Alternate angles are not equal *or* $\hat{R}_2 \neq \hat{P}_1$ *or* $\hat{R}_2 = 38°$ $\quad$ and $\hat{P}_1 = 17°$ (1)
		[13]

QUESTION 8

8.1	divides the other two sides proportionally	✓Answer (1)
8.2.1	$\dfrac{x}{8} = \dfrac{4}{10}$ $10x = 32$ $x = 3,2$	✓Prop. ✓Simplification ✓Value of x (3)
8.2.2	RTSP is a parallelogram (both pairs of opposite sides of a quadrilateral are parallel)	✓Statement ✓Reason (2)
8.2.3	$\dfrac{y}{9} = \dfrac{3,2}{8}$ Prop. Theorem ; TS∥MP $y = 3,6$	✓Statement ✓Reason ✓Value of y (3)
8.2.4	$\dfrac{\text{MR}}{\text{TS}} = \dfrac{10}{4} = 2,5$ $\dfrac{\text{RT}}{\text{SN}} = \dfrac{9}{3,6} = 2,5$ $\dfrac{\text{MT}}{\text{TN}} = \dfrac{8}{3,2} = 2,5$ $\Delta\text{MRT}\,\|\|\|\,\Delta\text{TSN}$ sides of triangle are in the same proportion	✓Ratio ✓Ratio ✓Ratio ✓Reason (4)
		[13]

QUESTION 9

9.1	$\hat{K}_1 = L\hat{H}F$ ext $\angle$ of cyclic quad $= G\hat{F}K$ corres. angles LK // GF	✓ Statement ✓ Reason ✓ Statement/Reason (3)

9.2	$M\hat{F}G = K_1 = 104°$ *corres* $\angle s$; FG ⫽ KL
	
	 $\hat{G} + 104° + 20° = 180°$ *angles of* Δ
	
	      $\hat{G} = 56°$
	
	 $M\hat{F}G = \hat{K}_1 = 104°$ *corres.* $\angle s$; FG // KL
	
	 $\hat{G} + 104° + 20° = 180°$ *angles of* Δ
	
	      $\hat{G} = 56°$
	
	 OR
	
	 $M\hat{L}K + 104° + 20° = 180°$ *angles of* Δ $\angle s$; FG // KL
	
	 $M\hat{L}K = 56°$
	
	 $M\hat{L}K = \hat{G}$ *corresp* $\angle s$; FG // KL
	
	    $\hat{G} = 56°$
		✓ Statement
		 ✓ Statement
		 ✓ Reason
		
		 ✓ Statement
		
		 ✓ Statement
		 ✓ Reason
		
		 (3)

9.3.1	$\dfrac{10}{30} = \dfrac{12}{MG}$ (Prop. Theorem; KL // FG) $MG = \dfrac{360}{10} = 36$ units	✓ Statement ✓ Reason ✓ 36 units (3)
9.3.2	$M\hat{H}F = G\hat{F}M = 104°$ proved in question 9.1. $\hat{M}$ is common $\hat{G} = \hat{F}_2$ sum of angles of a Δ $\Delta MFH \mid\mid\mid \Delta MGF$ $\angle; \angle; \angle$	✓ Statement ✓ Statement and reason ✓ Reason (3)
9.3.3	$\Delta MFH \mid\mid\mid \Delta MGF \mid\mid\mid \Delta MLK$	✓ ΔMLK (1)
		[13]

QUESTION 10

10.1	$x^2 - 4dh + 4h^2 = 0$ $x^2 - (4 \times 220\,mm \times 60\,mm) + 4(60\,mm)^2 = 0$ $x^2 = \sqrt{67200\,mm^2}$ $\therefore\ x = 259,23\,mm$	✓Formula ✓Substitution. ✓Simplifying ✓Length (4)
10.2.1	$v = \pi Dn$ $= \pi(18\,m)(\dfrac{225}{60\,s})$ $\approx 212,06\,m/s$	✓Correct formula ✓Correct diameter ✓Substitution ✓212,06 m/s (4)
10.2.2	$\omega = 2\pi n$ $= 2\pi(\dfrac{225}{60\,s})$ $\approx 23,56\,rad/s$	✓Correct formula ✓Substitution ✓23,56 rad/s (3)
10.3.1	$\overset{\wedge}{\text{LBA}} = 180° - 70° = 110°$ co-int.angles; AK // BL	✓Statement ✓Reason (2)
10.3.2	$\overset{\wedge}{\text{KAD}} = 360^O - 140^O = 220^O$ $\qquad \approx 3,84\text{rad}$ $s_1 = r\theta = 50(3,84)$ $\qquad \approx 192\,cm$ $s_2 = 48,8\,cm$ Length of belt $= 110 + 192 + 110 + 48,80$ $\qquad\qquad\qquad = 460,80\,cm$	✓Size of $\overset{\wedge}{\text{KAD}}$ ✓Conversion to radians ✓Correct substitution in arc length formula ✓Arc length s_1 ✓Length of DE = 110 ✓Length of belt (6)
		[19]

QUESTION 11

11.1.1	Volume of pyramid $=\dfrac{1}{3}(area\ of\ base)(height)$ $=\dfrac{1}{3}(4m\times 4m)(1,1m)$ $=5,87m^3$ Volume of Cube $=(l)(b)(h)$ $=(4m)(4m)(4m)$ $=64m^3$ Total Volume $=5,87\ m^3+64\ m^3$ $\qquad\qquad\approx 69,87m^3$	✓Substitution in correct formula ✓Height ✓5,87m^3 ✓Substitution in correct formula ✓64m^3 ✓69,87m^3 (6)
11.1.2	total surface area $=$ surface area of cube base $+$ surface area of pyramid $= 4\ (\text{side}\times\text{side})+4\left(\dfrac{1}{2}\times\text{base}\times\text{slant h}eight\right)$ $=4(4m\times 4m)+4(\dfrac{1}{2}\times 4m\times\sqrt{1,1+2^2}\ m)$ $=64m^2+4(2m\times\sqrt{5,1}\ m)$ $=82,07m^2$	✓Slant height ✓Correct substitution into area of a cube ✓Correct substitution into area of pyramid ✓Simplification ✓Total area (5)
11.1.3	Cost of paint $=82,07\times$ R30,50 $\qquad\qquad\qquad=$ R2503,14	✓82,07 $\times$ R30,50 ✓R2503,14 (2)

11.2	$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_2 + o_3 + \ldots\ldots\, o_{n-1}\right)$	✓Formula
	$= 4\left(\dfrac{6,2 + 2}{2} + y + 5,1 + 4,9\right)$	✓Substitution
	$= 4(14,1 + y)$	
	$= 56,4 + 4y$	
	$\therefore 72 = 56,4 + 4y$	✓Simplification
	$\Rightarrow y = \dfrac{15,6}{4}$	
	$= 3,9\,m$	✓ $3,9m$
	OR	**OR**
	$A_T = a\left(m_1 + m_2 + m_3 + \ldots\ldots\, m_n\right)$	✓Formula
	$72 = 4\left(\dfrac{6,2 + y}{2} + \dfrac{y + 5,1}{2} + \dfrac{51 + 4,9}{2} + \dfrac{4,9 + 2}{2}\right)$	✓Substitution
	$18 = \dfrac{11,3 + 2y}{2} + 8,45$	
	$9,55 = \dfrac{11,3 + 2y}{2}$	✓Simplification
	$19,1 = 11,3 + 2y$	
	$\therefore y = 3,9\,m$	✓ $3,9m$
		(4)
		[17]

TOTAL: 150