MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 information sheet and an answer book of 27 pages.
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.
QUESTION 1

An organisation decided that it would set up blood donor clinics at various colleges. Students would donate blood over a period of 10 days. The number of units of blood donated per day by students of college X is shown in the table below.

<table>
<thead>
<tr>
<th>DAYS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>UNITS OF BLOOD</td>
<td>45</td>
<td>59</td>
<td>65</td>
<td>73</td>
<td>79</td>
<td>82</td>
<td>91</td>
<td>99</td>
<td>101</td>
<td>106</td>
</tr>
</tbody>
</table>

1.1 Calculate:

1.1.1 The mean of the units of blood donated per day over the period of 10 days (2)

1.1.2 The standard deviation of the data (2)

1.1.3 How many days is the number of units of blood donated at college X outside one standard deviation from the mean? (3)

1.2 The number of units of blood donated by the students of college X is represented in the box and whisker diagram below.

![Box and Whisker Diagram]

1.2.1 Describe the skewness of the data. (1)

1.2.2 Write down the values of A and B, the lower quartile and the upper quartile respectively, of the data set. (2)

1.3 It was discovered that there was an error in counting the number of units of blood donated by college X each day. The correct mean of the data is 95 units of blood. How many units of blood were NOT counted over the ten days? (1)

[11]
QUESTION 2

The table below shows the number of hours that a sales representative of a company spent with each of his nine clients in one year and the value of the sales (in thousands of rands) for that client.

<table>
<thead>
<tr>
<th>NUMBER OF HOURS</th>
<th>30</th>
<th>50</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>150</th>
<th>190</th>
<th>220</th>
<th>260</th>
</tr>
</thead>
<tbody>
<tr>
<td>VALUE OF SALES</td>
<td>270</td>
<td>275</td>
<td>376</td>
<td>100</td>
<td>420</td>
<td>602</td>
<td>684</td>
<td>800</td>
<td>820</td>
</tr>
<tr>
<td>(IN THOUSANDS OF RANDS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SCATTER PLOT

2.1 Identify an outlier in the data above.

2.2 Calculate the equation of the least squares regression line of the data.

2.3 The sales representative forgot to record the sales of one of his clients. Predict the value of this client's sales (in thousands of rands) if he spent 240 hours with him during the year.

2.4 What is the expected increase in sales for EACH additional hour spent with a client?
QUESTION 3

In the diagram, P, Q(−7 ; −2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. PTO = 71,57° and SRN = θ.

Determine:

3.1 The equation of SR

3.2 The gradient of QP to the nearest integer

3.3 The equation of QP in the form \( y = mx + c \)

3.4 The length of QR. Leave your answer in surd form.

3.5 \( \tan(90° - \theta) \)

3.6 The area of ΔRSN, without using a calculator
QUESTION 4

In the diagram, PKT is a common tangent to both circles at $K(a \, b)$. The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at $O$ is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at $M$.

4.1 Write down the length of OK in surd form. \hspace{1cm} (1)

4.2 Show that $K$ is the point $(-12 ; -6)$. \hspace{1cm} (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ \hspace{1cm} (3)

4.3.2 The coordinates of $M$ \hspace{1cm} (6)

4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ \hspace{1cm} (2)

4.4 For which value(s) of $r$ will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at $M$ at two distinct points? \hspace{1cm} (3)

4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre $M(-16 ; -8)$. \hspace{1cm} (5)

[24]
QUESTION 5

5.1 If \( \cos 2\theta = -\frac{5}{6} \), where \( 2\theta \in [180^\circ;270^\circ] \), calculate, without using a calculator, the values in simplest form of:

5.1.1 \( \sin 2\theta \) \hspace{1cm} (4)
5.1.2 \( \sin^2 \theta \) \hspace{1cm} (3)

5.2 Simplify \( \sin(180^\circ - x).\cos(-x) + \cos(90^\circ + x).\cos(x-180^\circ) \) to a single trigonometric ratio. \hspace{1cm} (6)

5.3 Determine the value of \( \sin 3x.\cos y + \cos 3x.\sin y \) if \( 3x + y = 270^\circ \). \hspace{1cm} (2)

5.4 Given: \( 2\cos x = 3\tan x \)

5.4.1 Show that the equation can be rewritten as \( 2\sin^2 x + 3\sin x - 2 = 0. \) \hspace{1cm} (3)
5.4.2 Determine the general solution of \( x \) if \( 2\cos x = 3\tan x. \) \hspace{1cm} (5)
5.4.3 Hence, determine two values of \( y, \ 144^\circ \le y \le 216^\circ, \) that are solutions of \( 2\cos 5y = 3\tan 5y. \) \hspace{1cm} (4)

5.5 Consider: \( g(x) = -4\cos(x + 30^\circ) \)

5.5.1 Write down the maximum value of \( g(x). \) \hspace{1cm} (1)
5.5.2 Determine the range of \( g(x) + 1. \) \hspace{1cm} (2)
5.5.3 The graph of \( g \) is shifted \( 60^\circ \) to the left and then reflected about the \( x \)-axis to form a new graph \( h. \) Determine the equation of \( h \) in its simplest form. \hspace{1cm} (3) \[33\]
QUESTION 6

PQ and AB are two vertical towers. From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are $\theta$ and $2\theta$ respectively. 

$AQR = 90^\circ + \theta$, $QAR = \theta$ and $QR = x$.

6.1 Determine in terms of $x$ and $\theta$:

6.1.1 $QP$ (2)

6.1.2 $AR$ (2)

6.2 Show that $AB = 2x\cos^2 \theta$ (4)

6.3 Determine $\frac{AB}{QP}$ if $\theta = 12^\circ$. (2)

[10]
QUESTION 7

In the diagram, PQRT is a cyclic quadrilateral in a circle such that PT = TR. PT and QR are produced to meet in S. TQ is drawn. SQP = 70°

7.1 Calculate, with reasons, the size of:

7.1.1 \( \hat{T}_1 \)  

7.1.2 \( \hat{Q}_1 \)  

(2)  

7.2 If it is further given that PQ \parallel TR:

7.2.1 Calculate, with reasons, the size of \( \hat{T}_2 \)  

(2)  

7.2.2 Prove that \( \frac{TR}{TS} = \frac{RQ}{RS} \)  

(2) [8]
QUESTION 8

In the diagram, PR is a diameter of the circle with centre O. ST is a tangent to the circle at T and meets RP produced at S. \( \hat{S}T = x \) and \( \hat{S} = y \).

Determine, with reasons, \( y \) in terms of \( x \). [6]
QUESTION 9

In the diagram, DEFG is a quadrilateral with DE = 45 and GF = 80. The diagonals GE and DF meet in H. \(G\hat{E}D = F\hat{E}G\) and \(D\hat{G}E = E\hat{F}G\).

9.1 Give a reason why \(\triangle DEG \parallel \triangle EGF\). (1)

9.2 Calculate the length of \(GE\). (3)

9.3 Prove that \(\triangle DEH \parallel \triangle FGH\). (3)

9.4 Hence, calculate the length of \(GH\). (3)
QUESTION 10

10.1 In the diagram, O is the centre of the circle with A, B and C drawn on the circle.

Prove the theorem which states that \( \angle BOC = 2\angle \). (5)
10.2 In the diagram, the circle with centre \( F \) is drawn. Points A, B, C and D lie on the circle. Chords AC and BD intersect at E such that EC = ED. K is the midpoint of chord BD. FK, AB, CD, AF, FE and FD are drawn. Let \( \hat{B} = x \).

10.2.1 Determine, with reasons, the size of EACH of the following in terms of \( x \):

(a) \( \hat{F}_1 \) \hspace{2cm} (2)

(b) \( \hat{C} \) \hspace{2cm} (2)

10.2.2 Prove, with reasons, that AFED is a cyclic quadrilateral. \hspace{2cm} (4)

10.2.3 Prove, with reasons, that \( \hat{F}_3 = x \). \hspace{2cm} (6)

10.2.4 If area \( \triangle AEB = 6.25 \times \text{area} \triangle DEC \), calculate \( \frac{AE}{ED} \). \hspace{2cm} (5)

[24]

TOTAL: 150
INFORMATION SHEET: MATHEMATICS

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ A = P(1 + mi), \quad A = P(1 - mi), \quad A = P(1 - i)^n, \quad A = P(1 + i)^n \]
\[ T_n = a + (n-1)d, \quad S_n = \frac{n}{2}[2a + (n-1)d] \]
\[ T_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1 \]
\[ S_n = \frac{a}{1-r}; \quad -1 < r < 1 \]
\[ F_i = \frac{x[(1+i)^n - 1]}{i}, \quad P = \frac{x[1-(1+i)^n]}{i} \]
\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \]
\[ y = mx + c, \quad y - y_1 = m(x - x_1), \quad m = \frac{y_2 - y_1}{x_2 - x_1}, \quad m = \tan \theta \]
\[ (x - a)^2 + (y - b)^2 = r^2 \]

\[
\begin{align*}
\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\
\sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\
\sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\
\cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\
\cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\
\cos 2\alpha &= 1 - 2\sin^2 \alpha \\
2\cos^2 \alpha - 1 \\
\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\
\text{area } \triangle ABC &= \frac{1}{2} ab \sin C \\
\sin^2 \alpha + \cos^2 \alpha &= 1 \\
\end{align*}
\]

\[ \bar{x} = \frac{\sum x}{n} \]
\[ \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} \]
\[ P(A) = \frac{n(A)}{n(S)} \]
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]
\[ \hat{y} = a + bx \]
\[ b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2} \]

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