basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/ NATIONAL SENIOR CERTIFICATE EXAMINATIONS

TECHNICAL MATHEMATICS P2

2021

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages and a 2-page information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used to determine your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise.

6. Diagrams are NOT necessarily drawn to scale.

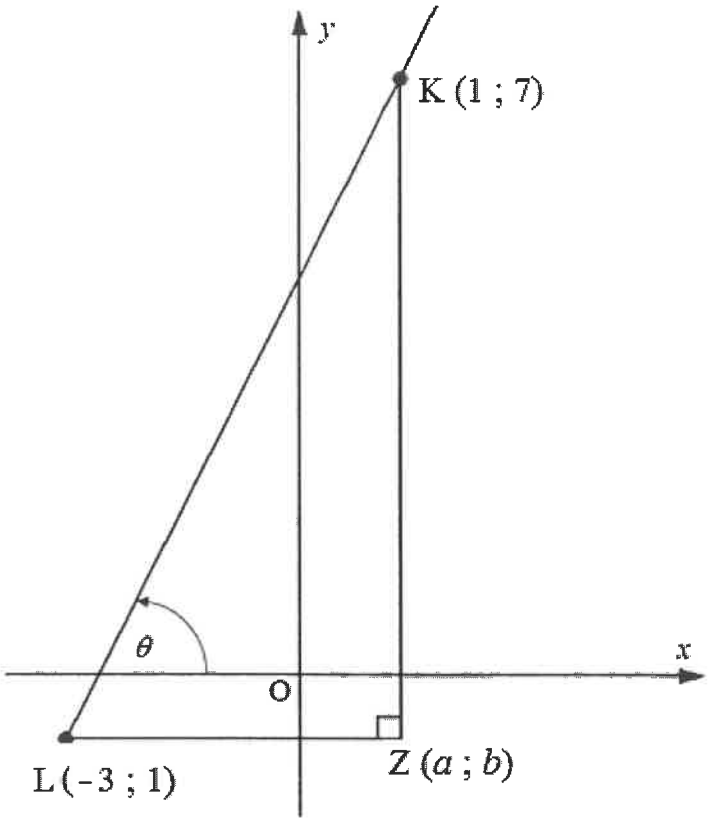
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

The diagram below shows a side view of a slanted ladder KL against a vertical wall KZ.
K, L and Z lie in the same vertical plane.
The vertices of the right-angled triangle are K(1 ; 7), L(−3 ;−1) and Z(a ; b).
The angle formed by KL and the x-axis is θ.



Determine:

1.1 The numerical values of a and b (2)

1.2 The length of KL (2)

1.3 The coordinates of the midpoint of KL (2)

1.4 The gradient of KL (2)

1.5 The size of θ (rounded off to ONE decimal place) (2)

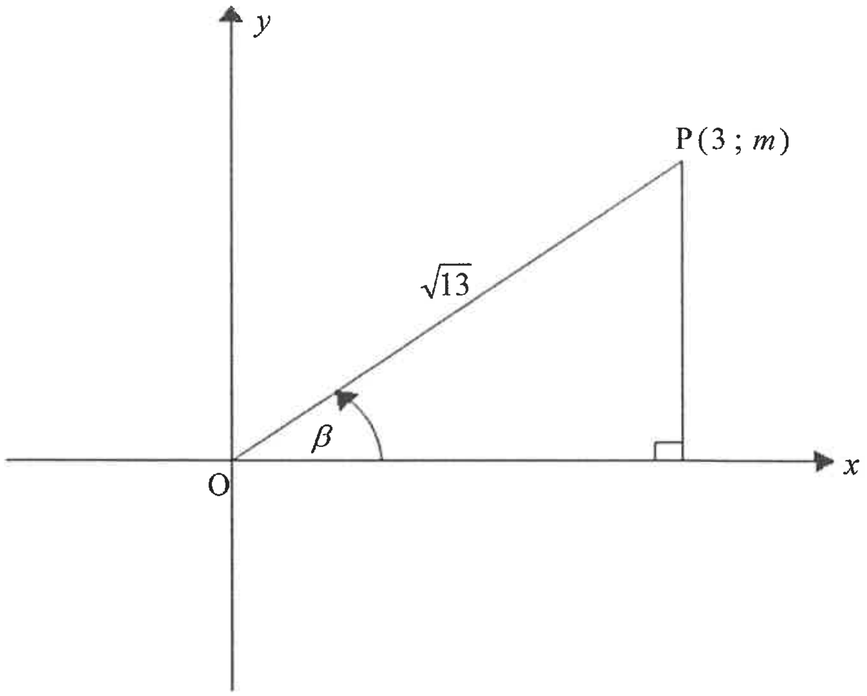
1.6 The equation of the straight line parallel to KL and passing through the point (− 5 ; 1).
Write the equation in the form $y = \ldots$ (3)

1.7 Whether point (− 4 ; − 2) lies on straight line parallel to KL (2)

[15]

QUESTION 3

3.1 In the diagram below, $P(3\,;\,m)$ is a point in a Cartesian plane with $OP = \sqrt{13}$
β is an acute angle.



Determine, WITHOUT using a calculator, the numerical value of:

3.1.1 m (1)

3.1.2 $\sec^2\beta + \tan^2\beta$ (4)

3.2 If $\cos\theta = \dfrac{1}{2}$ where $0° \leq \theta \leq 90°$ and $\tan\alpha = -1$ where $0° \leq \alpha \leq 180°$

Calculate:

3.2.1 The size of θ (1)

3.2.2 The size of α (3)

3.2.3 The value of $\cos(\alpha - \theta)$ (2)

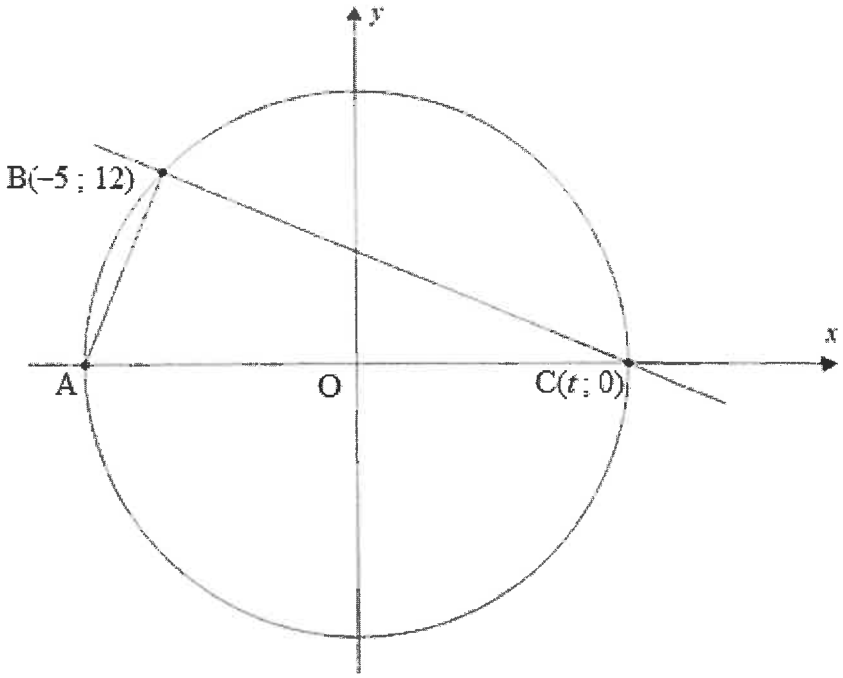
3.3 Solve for x:

$2\tan x + 0{,}924 = 0$ for $x \in [0°\,;\,360°]$ (4)

[15]

QUESTION 2

2.1 In the diagram below, A, B $(-5\,;\,12)$ and C $(t\,;0)$ lie on a circle with centre O at the origin. A line is drawn to intersect the circle at B and C.
A is on the x-axis and AB is drawn.



Determine:

2.1.1 The equation of the circle (2)

2.1.2 The numerical value of t (1)

2.1.3 The equation of the tangent to the circle at B in the form $y = \ldots$ (4)

2.2 Draw, on the grid provided in the ANSWER BOOK, the graph defined by:

$$\frac{x^2}{16} + \frac{y^2}{35} = 1$$

Clearly show ALL the intercepts with the axes. (3)

[10]

QUESTION 4

Simplify the following to a single trigonometric ratio:

4.1 $\cos\theta\,(\tan\theta + \cot\theta)$ (5)

4.2 $\dfrac{\sin^2(180° + B)\cdot\text{cosec}(\pi - B)}{\sec(2\pi - B)\cdot\cos(180° - B)}$ (7)

[12]

QUESTION 5

Given: $f(x) = \tan x$ and $g(x) = \cos(x - 45°)$ for $x \in [0°; 360°]$

5.1 Draw sketch graphs of f and g on the same set of axes on the grid provided in the ANSWER BOOK. Clearly indicate ALL turning points, end points, asymptotes and intercepts with the axes. (8)

5.2 Use your graphs to write down the value(s) of x for which:

5.2.1 f is undefined (2)

5.2.2 $f(x)\cdot g(x) \leq 0$ where $x \in [90°\,;\,180°]$ (2)

[12]

QUESTION 6

The diagram below shows farmland in the form of a cyclic quadrilateral PQRS.
The land has the following dimensions:

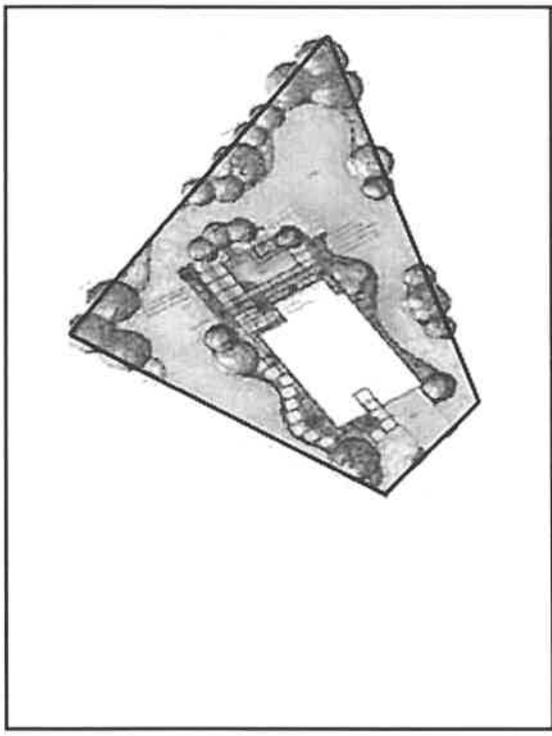
$PQ = 1\ 200\,\text{m}$

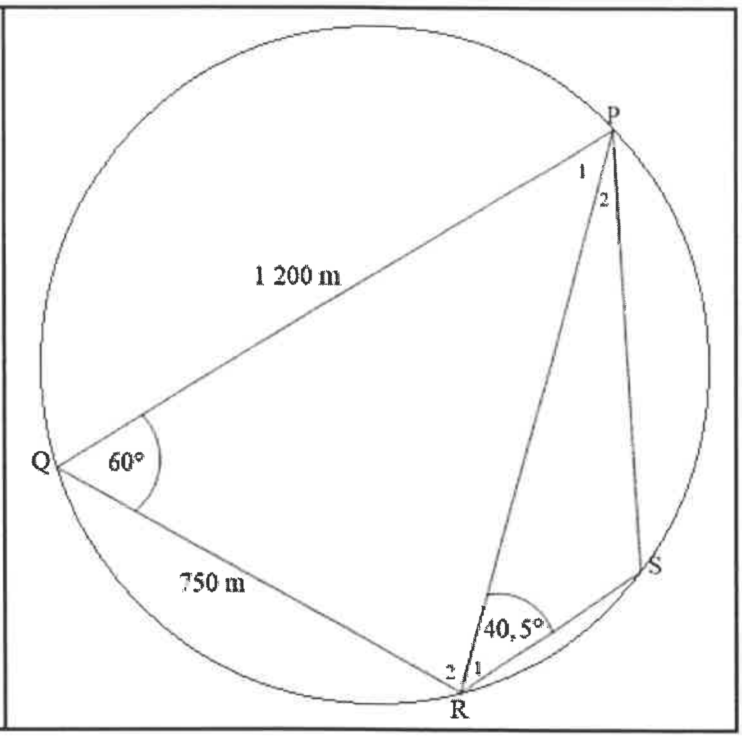
$QR = 750\,\text{m}$

$\hat{Q} = 60°$

$\hat{R}_1 = 40,5°$

P, Q, R and S lie on the same horizontal plane.





Determine:

6.1 The length of PR (3)

6.2 The size of $\hat{S}$ (1)

6.3 The length of PS (3)

6.4 The area of ΔQPR (3)

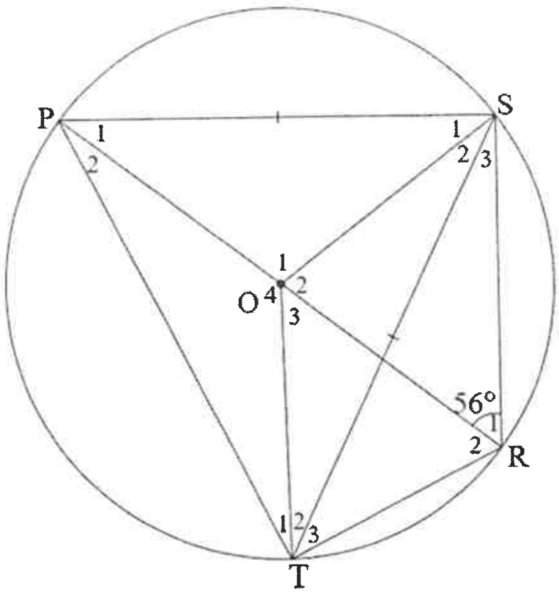
[10]

QUESTION 7

7.1 Complete the following theorem statement:

Angles subtended by a chord of the circle, on the same side of the chord … (1)

7.2 In the diagram below, circle PTRS, with centre O, is given such that PS = TS.
POR is a diameter, OT and OS are radii.
$\hat{R}_1 = 56°$



7.2.1 Determine, stating reasons:

(a) Three other angles each equal to 56° (5)

(b) The size of $\hat{P}_1$ (3)

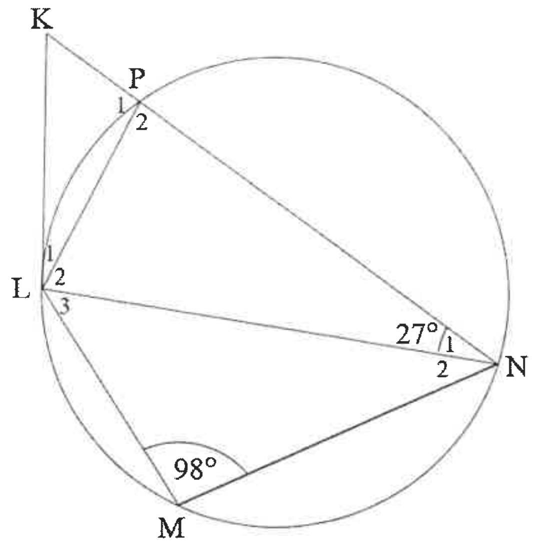
(c) The size of $\hat{S}_3$ (3)

7.2.2 Prove, stating reasons, that OT is NOT parallel to SR. (3)
[15]

QUESTION 8

The diagram below shows circle LMNP with KL a tangent to the circle at L.
LN and NPK are straight lines.

$\hat{N}_1 = 27°$ and $\hat{M} = 98°$



8.1 Determine, giving reasons, whether line LN is a diameter. (2)

8.2 Determine, stating reasons, the size of:

8.2.1 $\hat{P}_2$ (2)

8.2.2 $\hat{P}_1$ (2)

8.2.3 $\hat{L}_1$ (2)

8.3 Prove, stating reasons, that:

8.3.1 $\Delta KLP \parallel\!| \Delta KNL$ (3)

8.3.2 $KL^2 = KN \cdot KP$ (2)

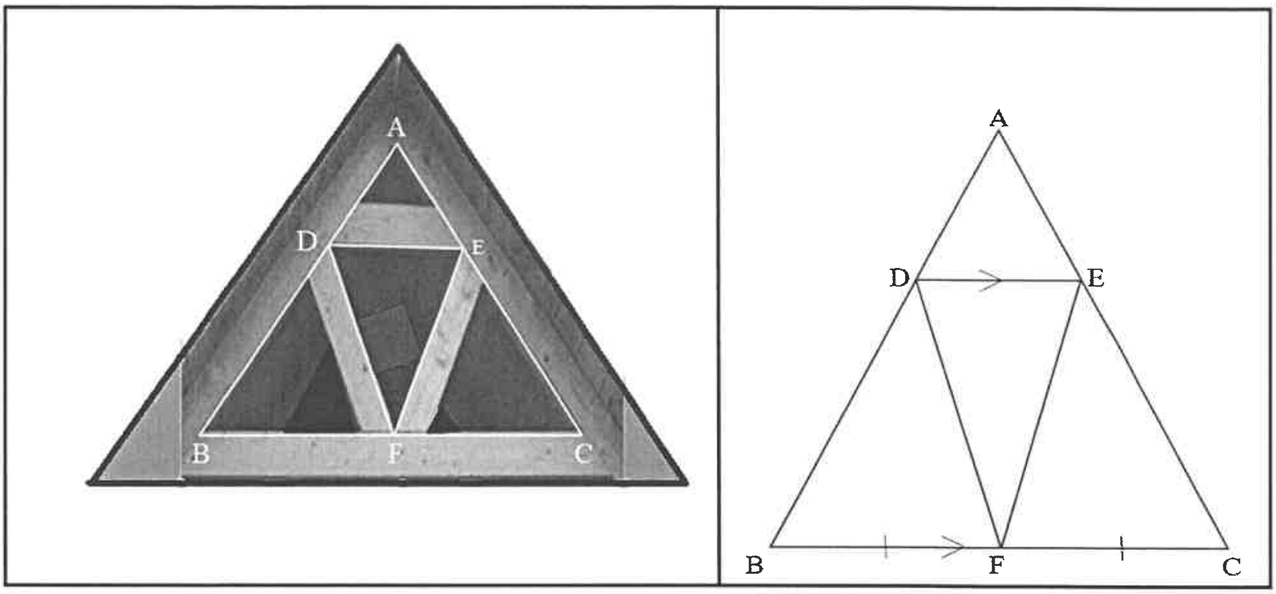
8.4 Hence, determine the length of KP if it is further given that KL = 6 units and KN = 13 units. (2)

8.5 Determine, giving reasons, whether KLMN is a cyclic quadrilateral. (3)

[18]

QUESTION 9

The diagram below is a picture of a triangular roof truss, as shown.
Triangle ABC has AB = AC.
DE || BC and F is the midpoint of BC.
AE : EC = 1 : 2 and AB = 1,8 m.



9.1 Determine the length of:

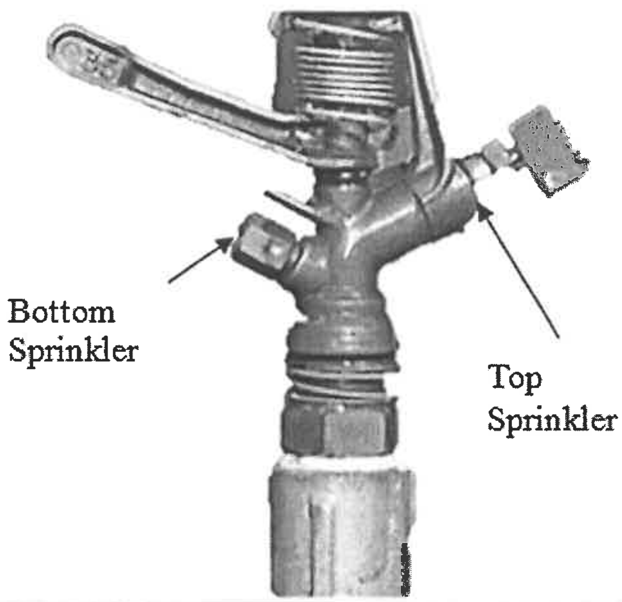
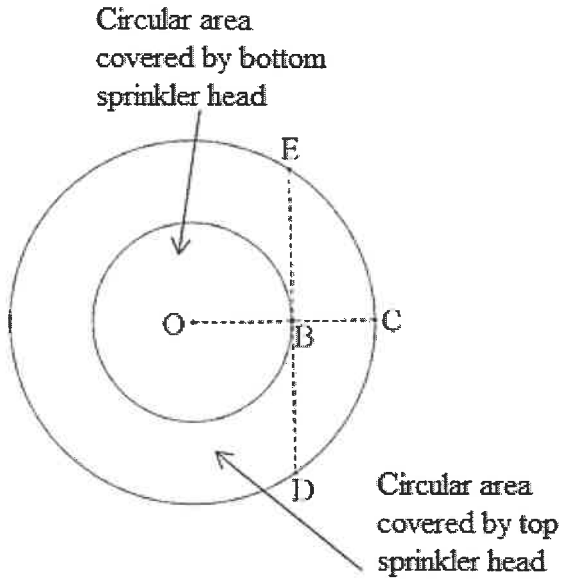
9.1.1 DB, giving reason(s) (2)

9.1.2 DF if $DF = \frac{3}{2} AD$ (2)

9.2 Determine, giving reasons, whether EF is parallel to AB. (3)
[7]

QUESTION 10

10.1 A double-headed rotating sprinkler (as shown in the picture below) is used to irrigate a circular vegetable garden. The diagram below shows the circular areas covered by the sprinkler heads.
The top sprinkler head covers a circular area with a radius of 6,95 m.
The bottom sprinkler head covers a circular area with a radius of 4 m.
O represents the location of the rotating sprinkler.
OC represents the radius of the larger circular area and OB is the radius of the smaller circular area.
Chord EBD is a tangent to the smaller circle at B.

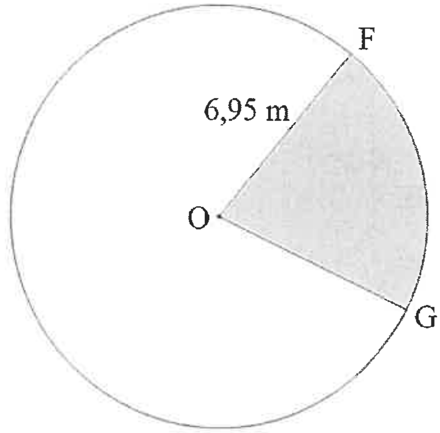
DOUBLE-HEADED SPRINKLER	**TOP VIEW**
	

10.1.1 Determine:

(a) The length of BC (1)

(b) The length of chord ED (4)

10.1.2 The bottom sprinkler head is set to irrigate only a sector which is 20% of the circular garden, as shown in the shaded part of the diagram below.



Calculate:

(a) The size of acute angle $F\hat{O}G$ in radians (3)

(b) The area of the sector that is irrigated (3)

10.2 A Ferris wheel with a radius of 10 m rotates at 18 revolutions per hour.

10.2.1 Determine the rotational frequency of the rotating wheel in radians per second. (2)

10.2.2 Calculate the circumferencial velocity (in metres per second) of the rotating wheel. (3)

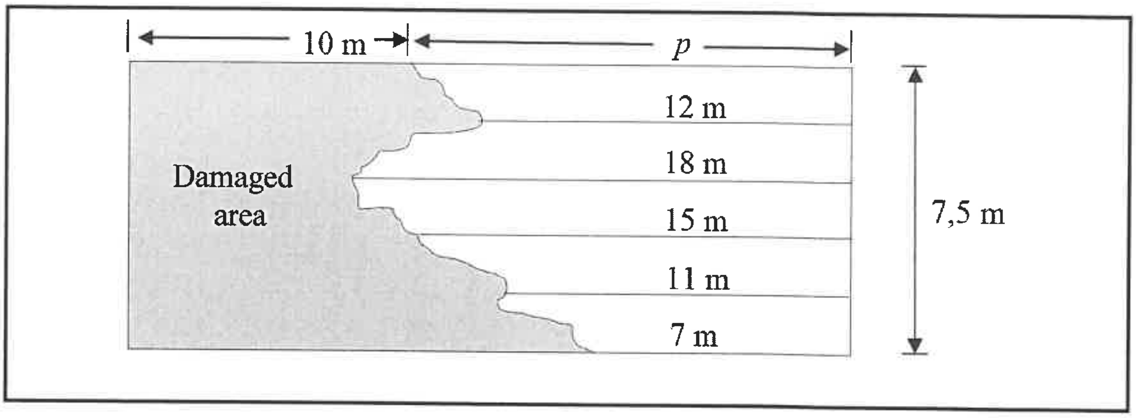
10.2.3 Calculate the angular velocity of the rotating chairs (in radians per second). (3)
[19]

QUESTION 11

11.1 A rectangular carpet was damaged by a chemical substance. The diagram below shows the damaged irregular surface of the carpet. The total surface area of the rectangular carpet is $187,5 \text{ m}^2$.

The ordinates of the undamaged part of the carpet are: p ; 12 m ; 18 m ; 15 m; 11 m and 7 m.

The breadth of the rectangular carpet is 7,5 m.



11.1.1 Determine the length of the rectangular carpet. (2)

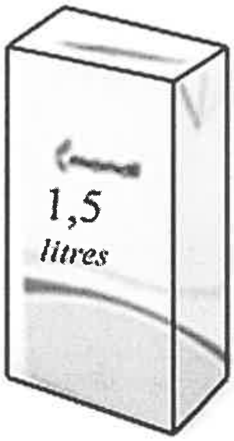
11.1.2 Determine the value of p. (1)

11.1.3 It takes 0,25 hours to repair 1 m^2. Calculate the minimum time required to repair the damaged area.
HINT: Use the mid-ordinate rule. (6)

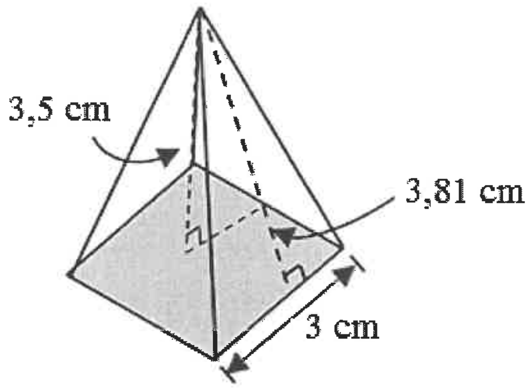
11.2 A rectangular container with a volume of 1,5 ℓ is filled with liquid. The liquid in the rectangular container must be poured into a number of identical, small, square-based, right, pyramid-shaped containers. The interior measurements of a pyramid, as in the diagram below, are as follows:

- The perpendicular height is 3,5 cm
- The length of the sides of the square base is 3 cm
- The slant height is 3,81 cm

CONTAINER



INTERIOR MEASUREMENTS OF THE PYRAMID-SHAPED CONTAINER



The following formula may be used:

Volume of a rectangular prism = length × breadth × height

Volume of a right pyramid $= \frac{1}{3} \times (\text{area of the base}) \times \text{height}$

Total surface area of a

square-based pyramid $= 4 \times \left[\frac{1}{2}(\text{side length of base}) \times (\text{slant height})\right] + (\text{side length})^2$

11.2.1 Determine the volume of the rectangular container in cm^3 if $1\ \ell = 1\ 000\ \text{cm}^3$. (1)

11.2.2 Calculate the total surface area of the pyramid-shaped container. (3)

11.2.3 Determine how much milk (in cm^3) will be poured into the last pyramid container. (4)

[17]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=-\frac{b}{2a}$ $y=\frac{4ac-b^2}{4a}$

$a^x=b\Leftrightarrow x=\log_a b$, $a>0$, $a\neq 1$ and $b>0$

$A=P(1+ni)$ $A=P(1-ni)$ $A=P(1-i)^n$ $A=P(1+i)^n$

$i_{eff}=\left(1+\frac{i}{m}\right)^m-1$

$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\int x^n dx=\frac{x^{n+1}}{n+1}+C$, $n\neq -1$

$\int \frac{1}{x}dx=\ln x+C,$ $x>0$ $\int a^x dx=\frac{a^x}{\ln a}+C$, $a>0$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $y-y_1=m(x-x_1)$ $m=\frac{y_2-y_1}{x_2-x_1}$ $m=\tan\theta$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $a^2=b^2+c^2-2bc\cdot\cos A$

Area of $\Delta ABC=\frac{1}{2}ab\cdot\sin C$

$\sin^2\theta+\cos^2\theta=1$ $1+\tan^2\theta=\sec^2\theta$ $\cot^2\theta+1=\mathrm{cosec}^2\theta$

$\pi rad = 180°$

Angular velocity $= \omega = 2\pi n = 360° n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

$s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \frac{rs}{2} = \frac{r^2\theta}{2}$ where r = radius, s = arc length and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = equal parts, $m_1 = \frac{o_1 + o_2}{2}$ and n = number of ordinates

OR

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + ... + o_{n-1}\right)$ where a = equal parts, $o_i = i^{th}$ ordinate and n = number of ordinates