basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

NOVEMBER 2023

MARKS: 150

TIME: 3 hours

This question paper consists of 16 pages and a 2-page information sheet.

INSTRUCTIONS AND INFORMATION

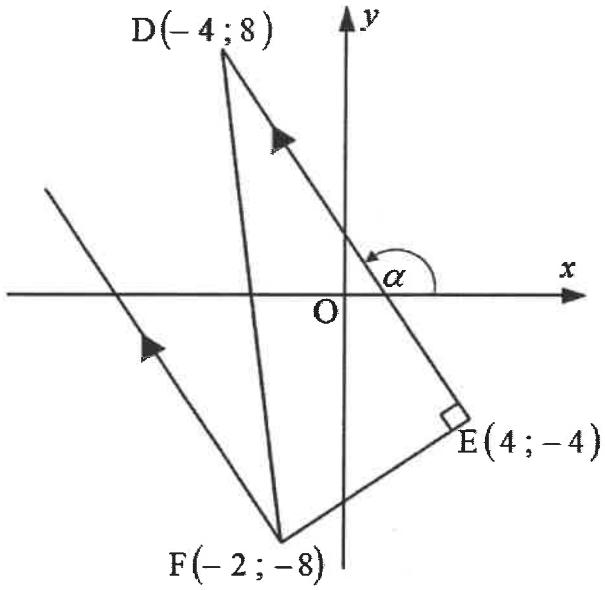
Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The diagram below shows ΔDEF with vertices $D(-4;8)$, $E(4;-4)$ and $F(-2;-8)$.
The angle of inclination of DE with the positive x-axis is α.

$\hat{E} = 90°$



1.1 Determine the gradient of DE. (2)

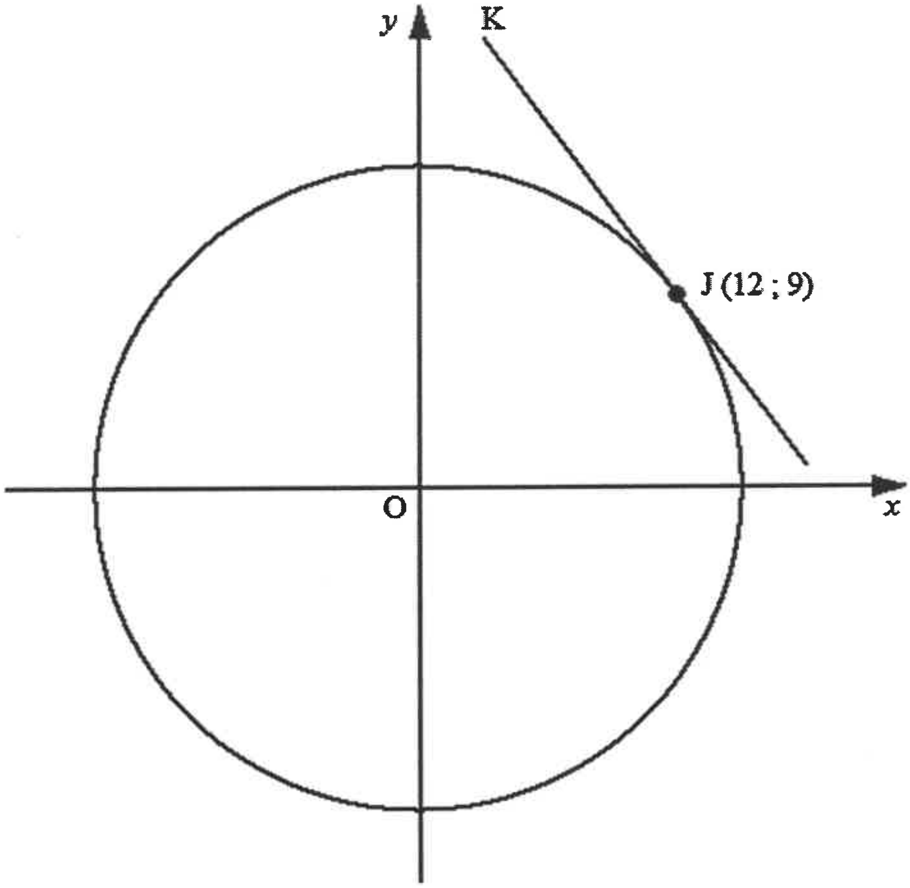
1.2 Determine the size of angle α. (3)

1.3 Determine whether the line parallel to DE, passing through F, also passes through point $(-10;5)$. (4)

1.4 Calculate the area of ΔDEF. (5)
[14]

QUESTION 2

2.1 In the diagram below, O is the centre of the circle.
JK is a tangent to the circle at point $J(12\ ;\ 9)$.



2.1.1 Determine the equation of the circle passing through J. (2)

2.1.2 Complete the following:

$m_{OJ} \times m_{JK} = \ldots$ (1)

2.1.3 Determine the equation of JK in the form $y = \ldots$ (4)

2.2 Given: $\dfrac{x^2}{11} + \dfrac{y^2}{64} = 1$

2.2.1 Express the equation in the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (1)

2.2.2 Hence, sketch the graph defined by $\dfrac{x^2}{11} + \dfrac{y^2}{64} = 1$ (2)

[10]

QUESTION 3

3.1 Given: $x = 152{,}4°$ and $y = 24{,}8°$

Determine the following:

3.1.1 $\sin(x - y)$ (2)

3.1.2 $\frac{1}{2}\sec\left(\frac{x}{2} + 80°\right)$ (2)

3.2 Given: $\sin\beta = -\frac{4}{5}$ and $\beta \in (90°\,;\,270°)$

Determine the following **without the use of a calculator**:

3.2.1 $\text{cosec}\,\beta$ (1)

3.2.2 $\tan\beta + \cos\beta$ (5)

3.3 Determine the value(s) of x if $\cos x = -\sin 56{,}7°$ and $x \in (0°\,;\,360°)$ (4)

[14]

QUESTION 4

4.1 Complete the following:

4.1.1 $\text{cosec A} = \frac{\ldots}{\ldots}$ (1)

4.1.2 $\cos(2\pi + \text{A}) = \ldots$ (1)

4.1.3 $\text{cosec}(180° + \text{A}) = \ldots$ (1)

4.2 Simplify the following:

$\sin(180° + \text{A})\cdot\cot(360° - \text{A})\cdot\cos(2\pi - \text{A}) + \sin^2(360° - \text{A})$ (7)

4.3 Given: $\frac{\text{cosec}\,x - \text{cosec}\,x \cdot \sec x}{\sec x - (\tan^2 x + 1)} = \cot x$

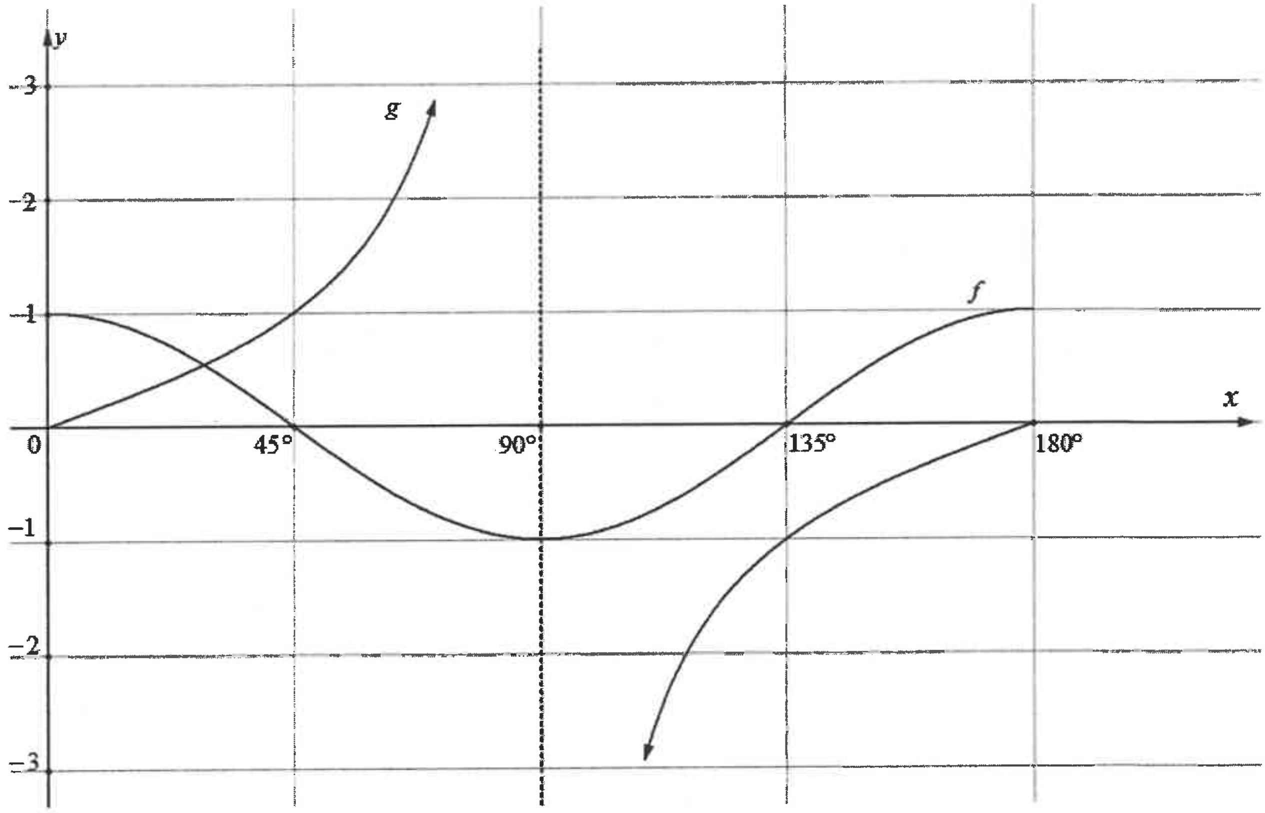
4.3.1 Factorise: $\sec x - \sec^2 x$ (1)

4.3.2 Hence, prove the identity: $\frac{\text{cosec}\,x - \text{cosec}\,x \cdot \sec x}{\sec x - (\tan^2 x + 1)} = \cot x$ (4)

[15]

QUESTION 5

The graphs below represent the functions defined by $f(x) = \cos ax$ and $g(x) = \tan x$ for $x \in [0°\,;180°]$



Use the graphs above to answer the following:

5.1 Write down:

5.1.1 The value of a (1)

5.1.2 The period of g (1)

5.1.3 The value of x for which $-\tan x + 1 = 0$ (2)

5.1.4 The range of g (1)

5.1.5 The value(s) of x for which $f(x) < 0$ (2)

5.2 Determine $g(180°) - f(180°)$ (2)

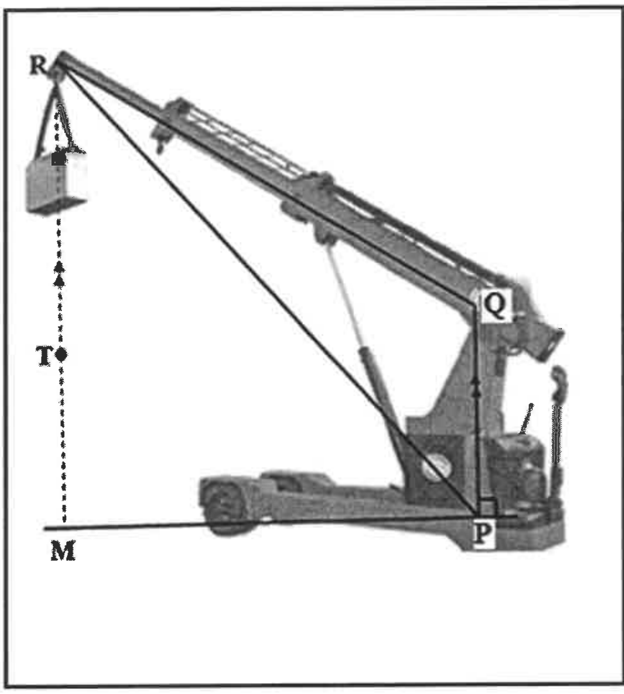
5.3 Write down the value(s) of x for which f is decreasing. (2)

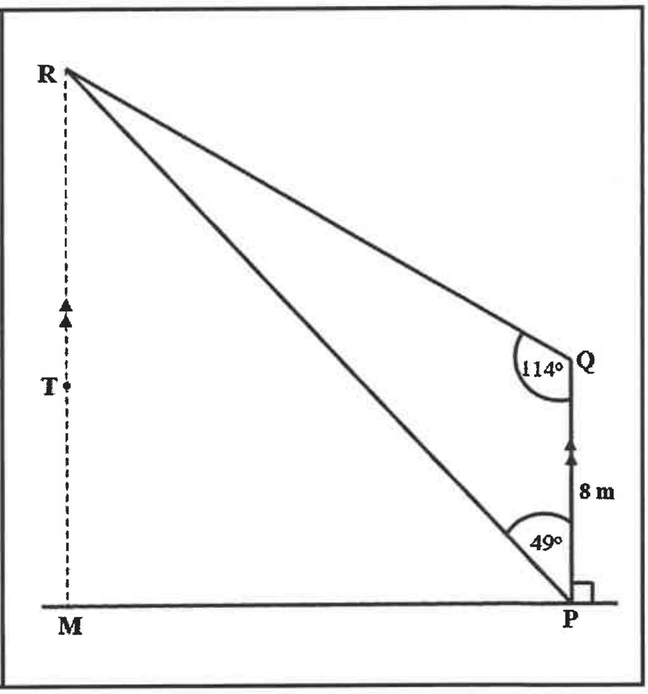
[11]

QUESTION 6

The picture and diagram below show a crane PQR lifting a box from point M to point R. PQ and MR are perpendicular to the ground level MP, such that PQRM lies in the same vertical plane.
T is a point on MR.

PQ = 8 m; $P\hat{Q}R = 114°$ and $Q\hat{P}R = 49°$





6.1 Determine the length of PR. (4)

6.2 Write down the size of $R\hat{P}M$ (1)

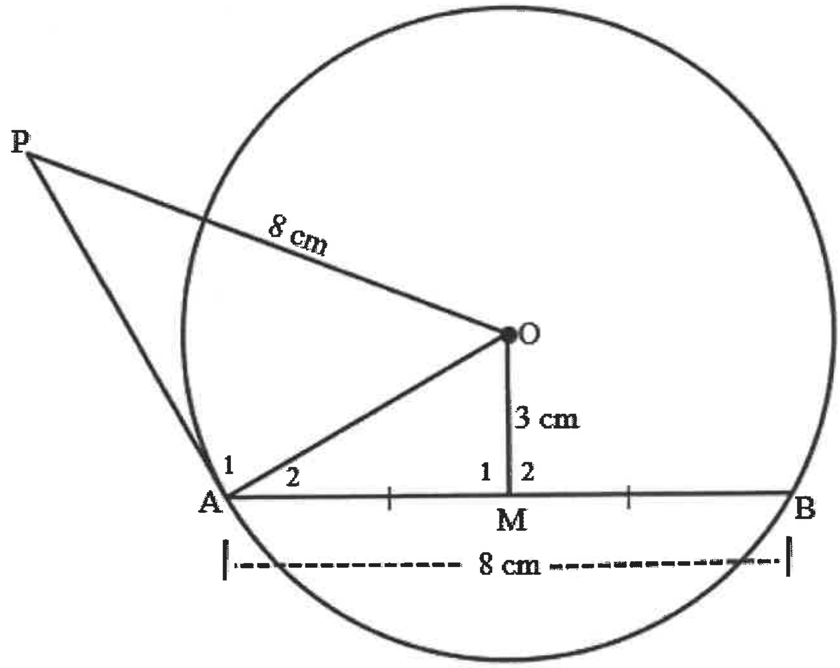
6.3 Complete the following ratio with respect to ΔRPM: $\sin R\hat{P}M = \frac{\dots}{\dots}$ (1)

6.4 If TR = 5 m, determine MT. (3)
[9]

Give reasons for your statements in QUESTIONS 7, 8 and 9.

QUESTION 7

In the diagram below, O is the centre of the circle.
M is the midpoint of chord AB and OM = 3 cm
AP is a tangent to the circle at A.
AB = OP = 8 cm



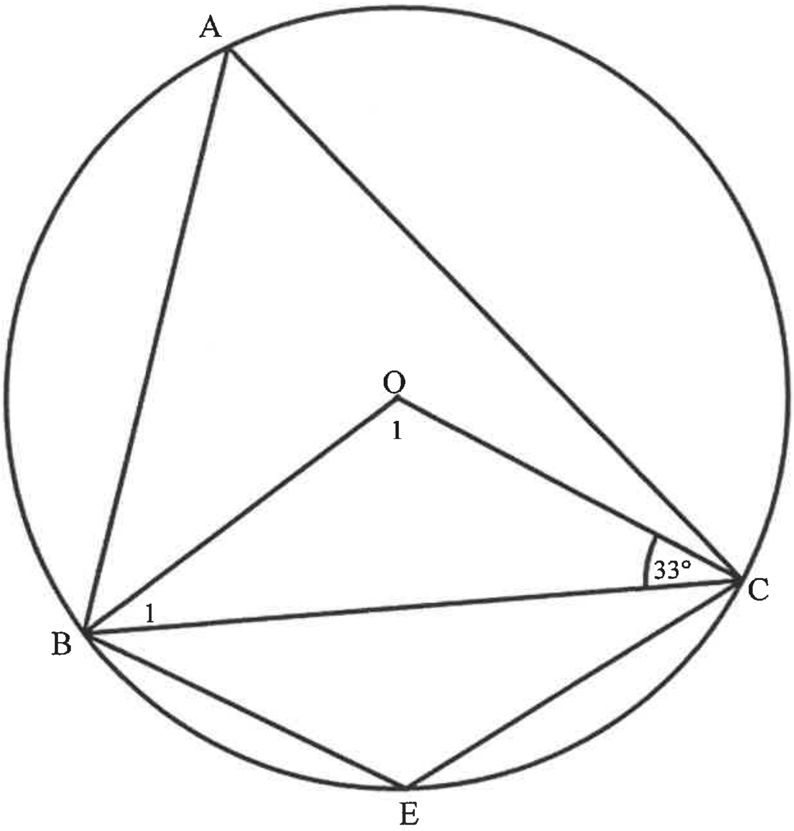
7.1 Write down, stating a reason, the size of $\hat{M}_1$. (2)

7.2 Give a reason why $\hat{A}_1 = 90°$ (1)

7.3 Determine the length of AP. (3)
[6]

QUESTION 8

8.1 In the diagram below, A, B, E and C are points on the circle with centre O.
$\hat{OCB} = 33°$



Determine, stating reasons, the size of the following angles:

8.1.1 $\hat{B}_1$ (2)

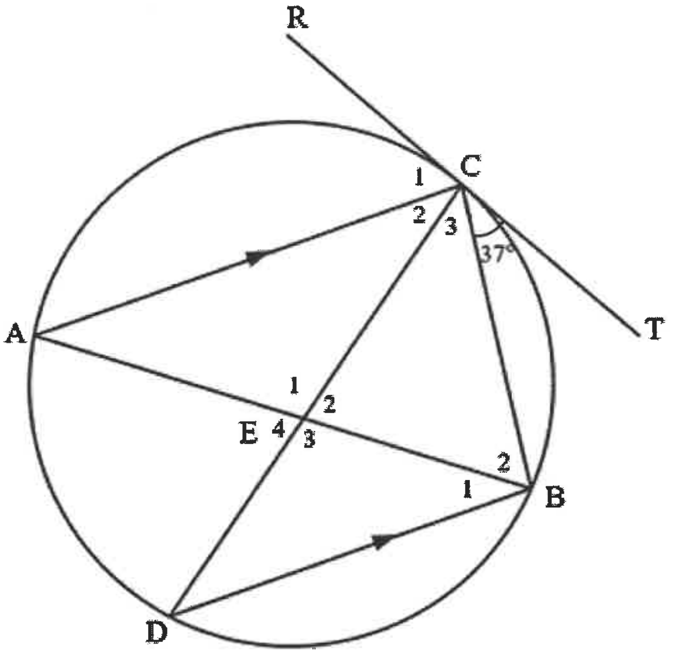
8.1.2 $\hat{O}_1$ (2)

8.1.3 $\hat{E}$ (4)

8.2 In the diagram below, RT is a tangent to circle ADBC at point C such that

$T\hat{C}B = 37°$

$AC \parallel DB$



8.2.1 Write down, stating reasons, FOUR other angles equal to 37° (6)

8.2.2 Hence, show that Δ AEC ||| Δ BED (2)

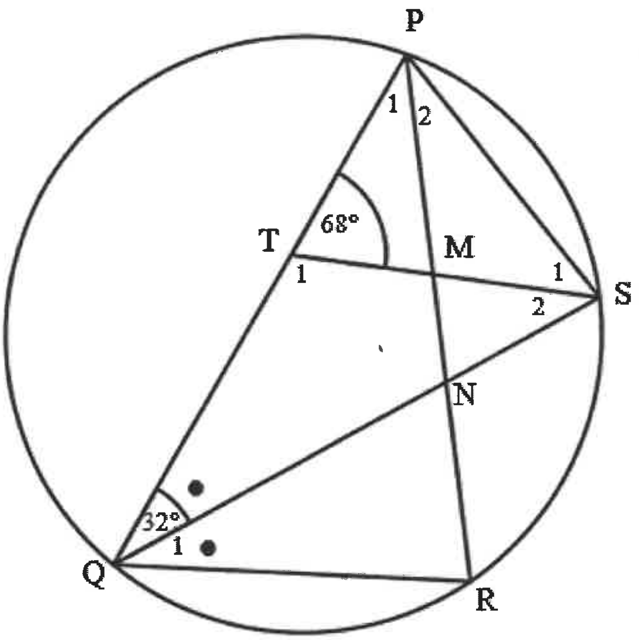
8.2.3 Hence, complete the statement $AE \times ED = \ldots \times \ldots$ (2)

8.3 In the diagram below, R, Q, P and S are points on the circle.
T is a point on PQ.

$S\hat{Q}P = 32°$ and $S\hat{T}P = 68°$

SQ bisects $\hat{Q}$.

PS = ST



8.3.1 Write down the size of the following angles:

(a) $\hat{Q}_1$ (1)

(b) $\hat{P}_2$ (2)

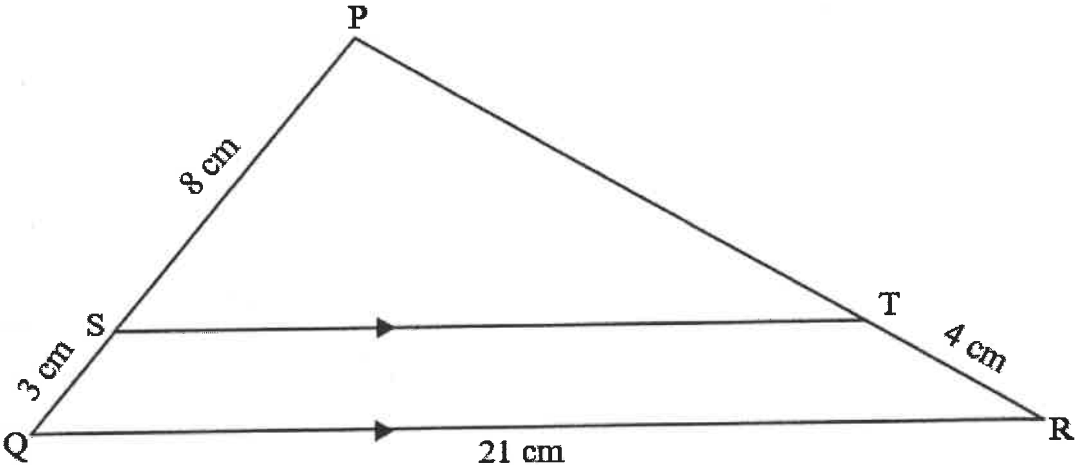
8.3.2 Hence, show that $\hat{P}_1 = \hat{S}_2$ (5)

[26]

QUESTION 9

The diagram below shows ΔPQR with $ST \parallel QR$.

$PS = 8$ cm, $SQ = 3$ cm, $RT = 4$ cm and $QR = 21$ cm.



9.1 Give the correct reason for the statement: $\frac{PT}{TR} = \frac{PS}{SQ}$ (...) (1)

9.2 Hence, calculate the length of PT. (2)

9.3 Complete the statement and give the correct reason:

$$\frac{ST}{QR} = \frac{PS}{...} \quad (...)$$
(2)

9.4 Hence, calculate the length of ST. (2)
[7]

QUESTION 10

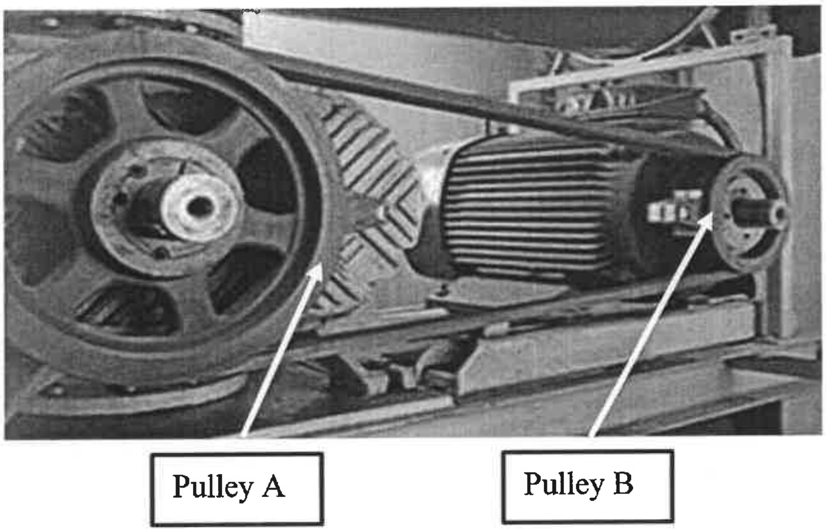
10.1 The picture and diagram below show two circular pulleys, A and B, connected with a belt and moving anti-clockwise.

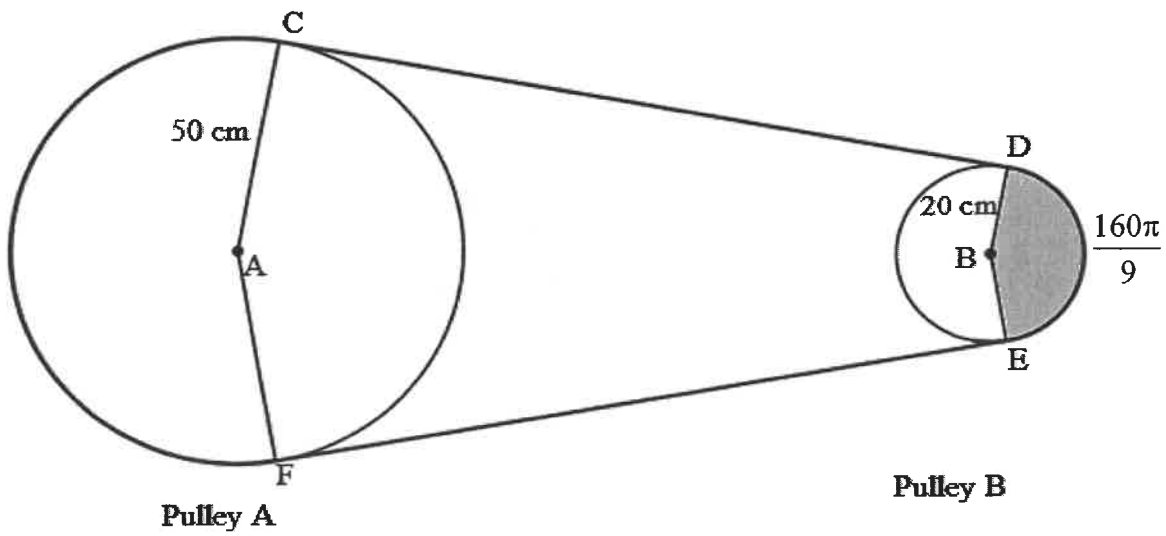
Pulley A has a radius of 50 cm and pulley B has a radius of 20 cm.

The belt covers $\frac{5}{9}$ of the arch length of pulley A.

The belt forms tangents to the pulleys at points C, D, E and F.

The arch length of DE = $\frac{160\pi}{9}$ cm.





10.1.1 Show that reflex $C\hat{A}F = 200°$ (1)

10.1.2 Convert reflex $C\hat{A}F = 200°$ to radians. (1)

10.1.3 Hence, determine the major arc length of CF. (3)

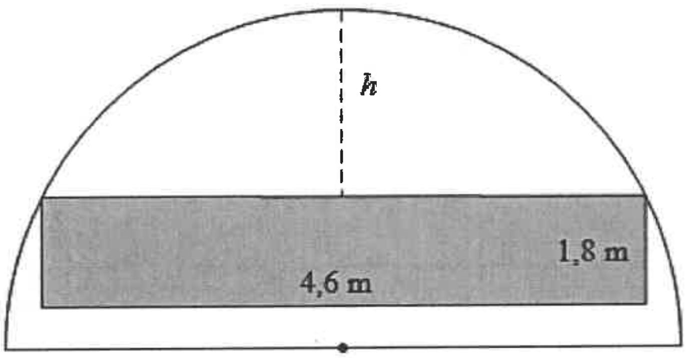
10.1.4 Pulley A rotates at 500 revolutions per minute (r/min).

(a) Calculate the circumferential velocity (cm/min) of a particle on the belt at point F. (3)

(b) Hence, determine, in revolutions per second, the rotational frequency of pulley B. (4)

10.1.5 Determine the area of the shaded minor sector DBE. (3)

10.2 A rectangular advertisement board with a length of 4,6 metres and a breadth of 1,8 metres will be positioned on the semicircular wall as indicated in the diagram below. The height h from the top part of the rectangular board to the top part of the semicircular wall is 0,72 metres longer than the breadth of the rectangular advertisement board.

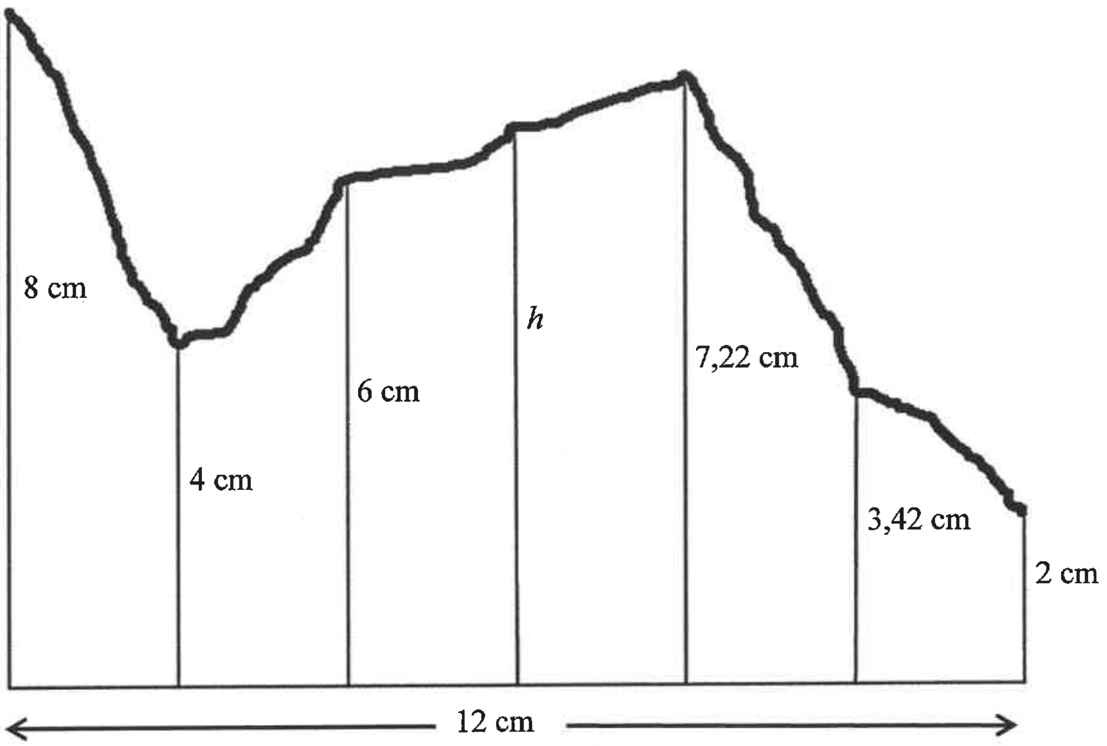


10.2.1 Determine the value of height h. (1)

10.2.2 Hence, calculate the length of the diameter of the semicircular wall. (4)
[20]

QUESTION 11

11.1 The irregular figure below has a horizontal straight side, 12 cm long, which is divided into 6 equal parts.
The ordinates dividing the parts are 8 cm, 4 cm, 6 cm, h, 7,22 cm, 3,42 cm and 2 cm.
The length of h is the average of the third and fifth ordinates.



11.1.1 Write down the width of each of the equal parts. (1)

11.1.2 Determine the value of h. (2)

11.1.3 Hence, determine the area of the irregular figure by using the mid-ordinate rule. (3)

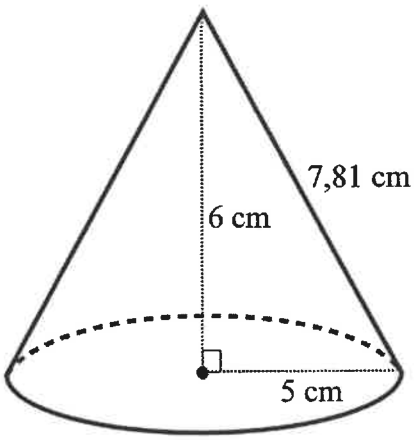
11.2 The pictures below show spherical soccer balls. Picture A represents ball A with radius = 11 cm and picture B represents a smaller ball B with radius = x cm.

Soccer ball A with radius $r = 11\text{ cm}$	Soccer ball B with radius $r = x\text{ cm}$
	
Picture A	Picture B

Calculate x, the radius of ball B, if the volume of ball B is half the volume of ball A.

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ (5)

11.3 The diagram below shows a closed cone. The radius of the cone is 5 cm. It has a height of 6 cm and a slant height (l) of 7,81 cm.



11.3.1 Calculate the surface area of the cone, where:
Surface area $= \pi r^2 + \pi r l$ (2)

11.3.2 The radius of the cone is increased by 20% and the height of the cone is decreased by 10%.

Determine whether the new surface area is greater than the surface area of the original cone. (5)
[18]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f^{/}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \qquad \int kx^n\, dx = k.\frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln x + C, \quad x > 0 \qquad \int \frac{k}{x} dx = k.\ln x + C, \quad x > 0$$

$$\int a^x\, dx = \frac{a^x}{\ln a} + C, \quad a > 0 \qquad \int k a^{nx}\, dx = k.\frac{a^{nx}}{n \ln a} + C, \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

Area of Δ ABC $= \dfrac{1}{2} ab.\sin C$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

π rad $= 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360° n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{rs}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2\theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = width of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$
$o_n = n^{th}$ ordinate and n = number of ordinates

OR

$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = width of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates