education

Department:
Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2008

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages, an information sheet and 2 diagram sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. TWO diagram sheets for answering QUESTION 5.1, QUESTION 5.2, QUESTION 11.2 and QUESTION 11.3 are included at the end of this question paper. Write your examination number on these sheets in the spaces provided and hand them in together with your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

1.1 Solve for x, rounded off to TWO decimal places where necessary:

1.1.1 $x^2 = 5x - 4$ (3)

1.1.2 $x(3 - x) = -3$ (5)

1.1.3 $3 - x < 2x^2$ (5)

1.2 Determine the values of x and y if they satisfy both the following equations simultaneously:

$$2x + y = 3$$
$$x^2 + y + x = y^2$$
(8)

1.3 Given $x = 999\ 999\ 999\ 999$, determine the exact value of $\frac{x^2 - 4}{x - 2}$.

Show ALL your calculations. (3)

1.4 Explain why the equation $\frac{x^4 + 1}{x^4} = \frac{1}{2}$ has no real roots. (2)

[26]

QUESTION 2

2.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; ...$

2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)

2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)

2.2 Consider the sequence: 8 ; 18 ; 30 ; 44 ; …

2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)

2.2.2 Calculate the n^{th} term of the sequence. (6)

2.2.3 Which term of the sequence is 330? (4)

[21]

QUESTION 3

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + ...$

3.1 Determine the n^{th} term of the series. (1)

3.2 For what value(s) of x will the series converge? (3)

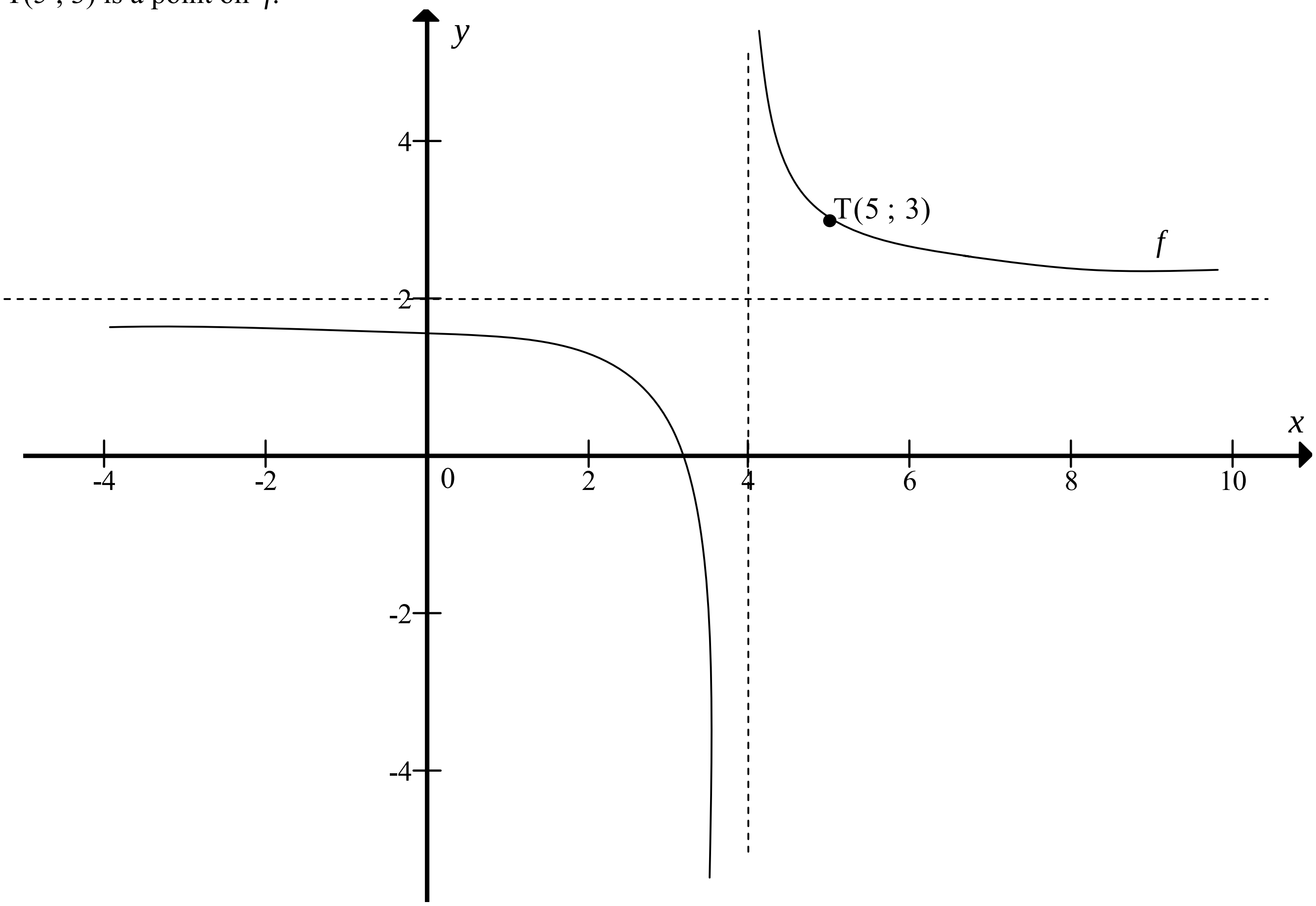
3.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)

[7]

QUESTION 4

The diagram below represents the graph of $f(x) = \dfrac{a}{x-p} + q$.

T(5 ; 3) is a point on f.



4.1 Determine the values of a, p and q. (4)

4.2 If the graph of f is reflected across the line having equation $y = -x + c$, the new graph coincides with the graph of $y = f(x)$. Determine the value of c. (3)

[7]

QUESTION 5

Given: $h(x) = 4^x$ and $f(x) = 2(x-1)^2 - 8$.

5.1 Sketch the graphs of h and f on the diagram sheet provided. Indicate ALL intercepts with the axes and any turning points. (8)

5.2 Without any further calculations, sketch the graph of $y = \log_4 x = g(x)$ on the same system of axes. (2)

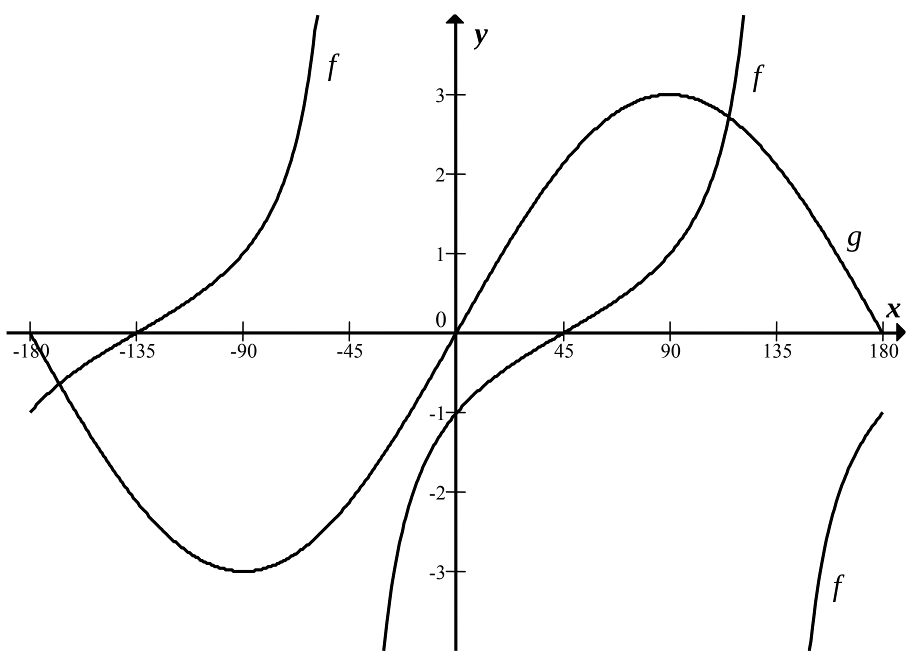
5.3 The graph of f is shifted 2 units to the LEFT. Write down the equation of the new graph. (2)

5.4 Show, algebraically, that $h\left(x + \frac{1}{2}\right) = 2h(x)$. (3)

[15]

QUESTION 6

Sketched below are the graphs of the functions $f(x) = \tan(x - 45°)$ and $g(x) = 3\sin x$ for $x \in [-180°\,;\,180°]$.



6.1 Write down the equations of the asymptotes of $y = f(x)$ for $x \in [-90°\,;\,180°]$. (2)

6.2 Describe the transformation of the graph of f to h if $h(x) = \tan(45° - x)$. (2)

6.3 The period of g is reduced to 180° and the amplitude and y-intercept remain the same. Write down the equation of the resulting function. (2)

[6]

QUESTION 7

7.1 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000? (4)

7.2 A farmer has just bought a new tractor for R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay? (3)

7.2.2
- One month after purchasing his present tractor, the farmer deposited x rands into an account that pays interest at a rate of 12% p.a., compounded monthly.
- He continued to deposit the same amount at the end of each month for a total of 60 months.
- At the end of 60 months he has exactly the amount that is needed to purchase a new tractor, after he trades in his present tractor.

Calculate the value of x. (6)

7.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will the new monthly deposit be? (4)
[17]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$. (5)

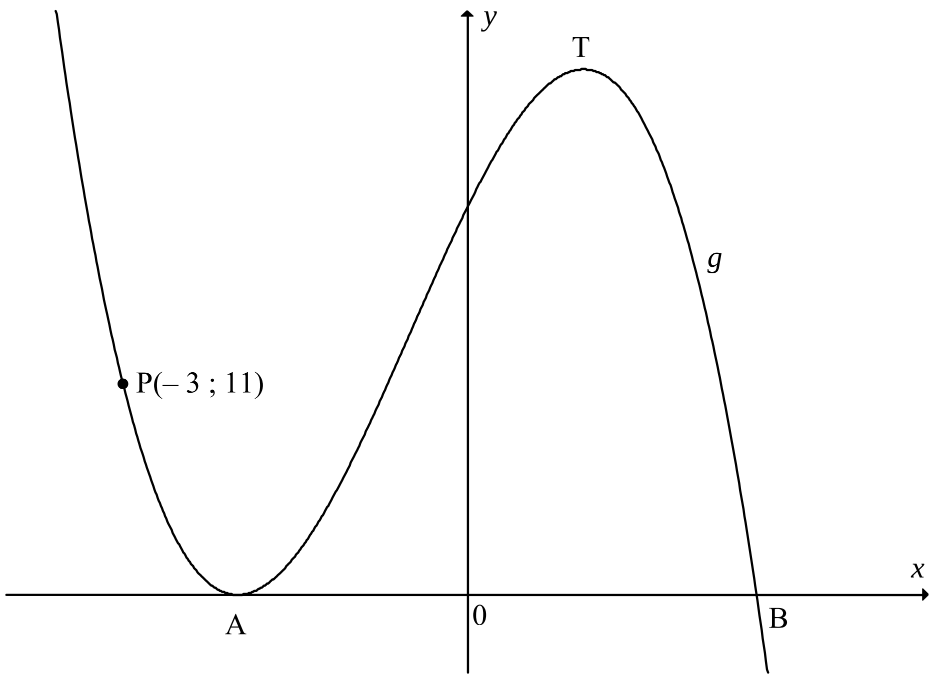
8.2 Determine, using the rules of differentiation:

$$\frac{dy}{dx} \text{ if } y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

Show ALL calculations. (3)
[8]

QUESTION 9

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x-5)(x+2)^2$
A and T are turning points of g. A and B are the x-intercepts of g.
P(– 3 ; 11) is a point on the graph.



9.1 Determine the length of AB. (2)

9.2 Determine the x-coordinate of T. (4)

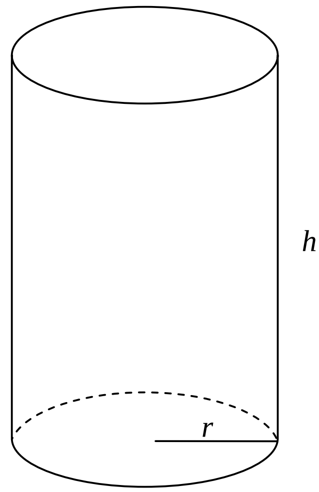
9.3 Determine the equation of the tangent to g at P(– 3 ; 11), in the form $y = \ldots$ (5)

9.4 Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots. (3)

9.5 Determine the x-coordinate of the point of inflection. (4)
[18]

QUESTION 10

A drinking glass, in the shape of a cylinder, must hold 200 mℓ of liquid when full.



10.1 Show that the height of the glass, h, can be expressed as $h = \dfrac{200}{\pi r^2}$. (2)

10.2 Show that the total surface area of the glass can be expressed as $S(r) = \pi r^2 + \dfrac{400}{r}$. (2)

10.3 Hence determine the value of r for which the total surface area of the glass is a minimum. (5)

[9]

QUESTION 11

Amina owns a small factory that manufactures two types of cellular phones, namely Acuna and Matata cellular phones.

- Each Acuna cellular phone requires 10 manhours to manufacture and each Matata cellular phone requires 8 manhours to manufacture.
- Each Acuna cellular phone requires 3 manhours in the testing department and each Matata cellular phone requires 4 manhours in the testing department.
- The manufacturing department has a maximum of 800 manhours available per week.
- The testing department has a maximum of 360 manhours available per week.
- The factory needs to manufacture at least 60 of the Matata models each week.

Let x represent the number of Acuna cellular phones manufactured in one week.
Let y represent the number of Matata cellular phones manufactured in one week.

11.1 Write down the constraints, in terms of x and y, that represent the above-mentioned information. (3)

11.2 Use the attached graph paper (DIAGRAM SHEET 2) to represent the constraints graphically. (5)

11.3 Clearly indicate the feasible region by shading it. (1)

11.4 If the profit on one Acuna cellular phone is R200 and the profit on one Matata cellular phone is R250, write down an expression that will represent the profit, P, on the cellular phones. (1)

11.5 Using a search line and your graph, determine the number of Acuna and Matata cellular phones that will give a maximum profit, assuming they are all sold out. Draw a search line on your graph. (3)

11.6 If the profit function for the factory was $P = 180x + 240y$, would there be any difference in the optimal solution? Give a reason for your answer. (3)
[16]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS
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$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$\sum_{i=1}^{n}1=n$ $\quad \sum_{i=1}^{n}i=\frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n}\left(a+(i-1)d\right)=\frac{n}{2}\left(2a+(n-1)d\right)$

$\sum_{i=1}^{n}ar^{i-1}=\frac{a\left(r^n-1\right)}{r-1}$; $r\neq 1$ $\quad \sum_{i=1}^{\infty}ar^{i-1}=\frac{a}{1-r}$; $-1<r<1$

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $\quad P=\frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$ $\quad m=\tan\theta$

$$(x-a)^2+(y-b)^2=r^2$$

In ΔABC:

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ $\quad a^2=b^2+c^2-2bc.\cos A$ $\quad area\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$$

$\sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$

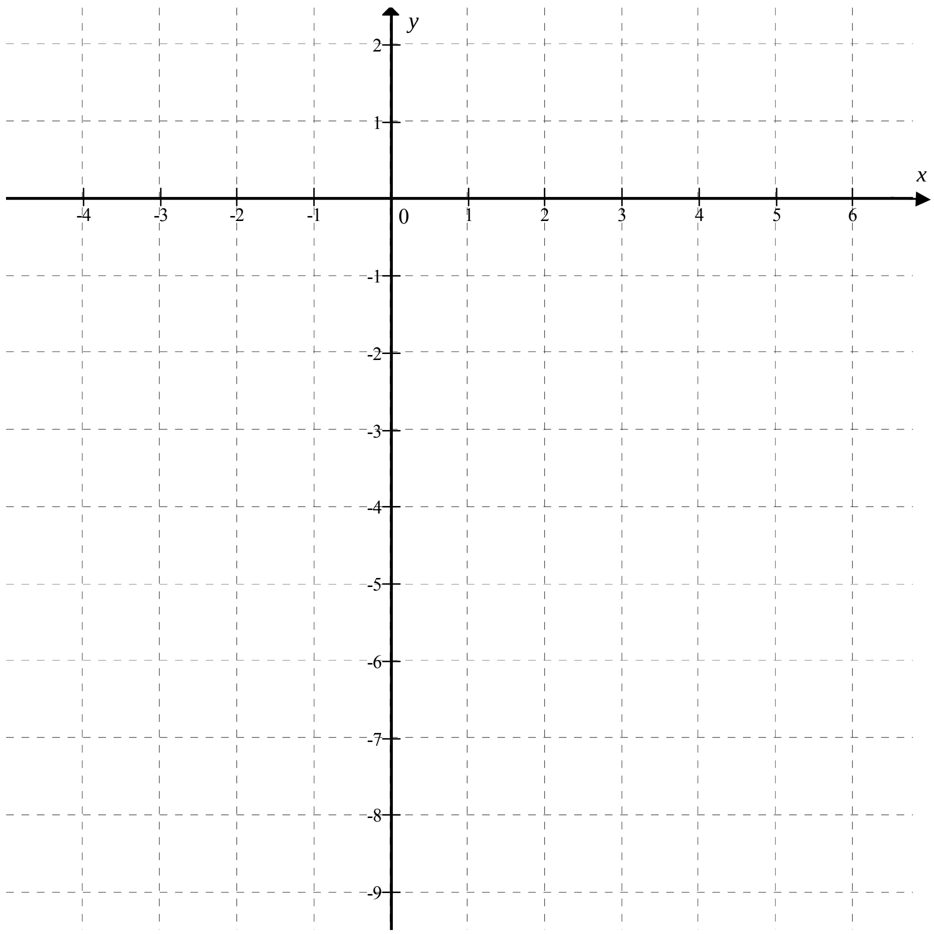
$P(A)=\frac{n(A)}{n(S)}$ $\quad P(A\ or\ B)=P(A)+P(B)-P(A\ and\ B)$

$\hat{y}=a+bx$ $\quad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTIONS 5.1 AND 5.2



EXAMINATION NUMBER:

DIAGRAM SHEET 2

QUESTIONS 11.2 AND 11.3