basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V1

FEBRUARIE/MAART 2013

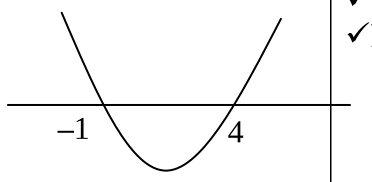
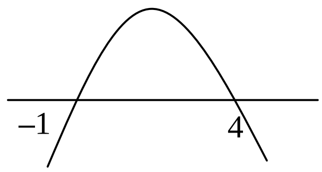
MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 19 bladsye.

VRAAG 1

1.1.1	$(x^2-9)(2x+1)=0$ $(x-3)(x+3)(2x+1)=0$ $x=\pm 3$ of $x=-\frac{1}{2}$ **OF** $(x^2-9)(2x+1)=0$ $x=\pm 3$ of $x=-\frac{1}{2}$	✓ $(x-3)(x+3)$ ✓ ± 3 ✓ $-\frac{1}{2}$ (3) ✓ -3 ✓ 3 ✓ $-\frac{1}{2}$ (3)
1.1.2	$x^2+x-13=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-1\pm\sqrt{1-4(1)(-13)}}{2}$ $=\frac{-1\pm\sqrt{53}}{2}$ $x=3{,}14$ of $x=-4{,}14$	✓subs in formule ✓ $\sqrt{53}$ ✓antwoord ✓antwoord (4)
1.1.3	$2\cdot 3^x=81-3^x$ $2\cdot 3^x+3^x=81$ $3^x(2+1)=81$ $3^x=27$ $3^x=3^3$ $x=3$ **OF** $2.3^x=81-3^x$ $2.3^x+3^x=81$ $3^x(2+1)=81$ $3^{x+1}=3^4$ $x+1=4$ $x=3$	✓ $2\cdot 3^x+3^x=81$ ✓ 3^x as gemeenskaplike faktor ✓vereenvoudiging ✓antwoord (4) ✓ $2\cdot 3^x+3^x=81$ ✓ 3^x as gemeenskaplike faktor ✓ $3^{x+1}=3^4$ ✓antwoord (4)

1.1.4	$(x+1)(4-x)>0$ $(x+1)(x-4)<0$ + 0 − 0 + –1 4 of  $-1<x<4$ **OF** $(x+1)(4-x)>0$ − 0 + 0 − –1 4  $-1<x<4$	✓verandering van teken ✓beide kritieke waardes ✓korrekte ongelykheidsteken (3) ✓metode ✓beide kritieke waardes ✓korrekte ongelykheidsteken (3)
1.2.1	$2^x+2^{x+2}=-5y+20$ $2^x(1+2^2)=-5y+20$ $2^x=\frac{-5y+20}{5}$ **OF** $2^x=-y+4$	✓ 2^x gemeenskaplike faktor ✓antwoord (2)
1.2.2	As $y=-4$, $2^x+2^{x+2}=-5y+20$ $2^x+2^{x+2}=40$ $2^x(1+2^2)=40$ $2^x=8$ $2^x=2^3$ $x=3$	✓substitusie ✓antwoord (2)
1.2.3	$-y+4>0$ $y<4$ Grootste heelgetalwaarde van y is 3 $2^x=-3+4$ $2^x=1$ $x=0$	✓ $-y+4>0$ ✓ $y=3$ ✓ $x=0$ (3) **[21]**

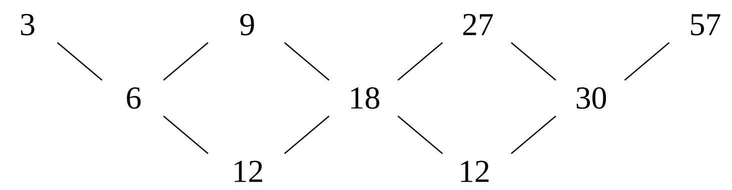
VRAAG 2

2.1.1	$r=-\frac{32}{64}=-\frac{1}{2}$ $p=256\left(-\frac{1}{2}\right)$ $p=-128$	✓ $-\frac{1}{2}$ ✓substitusie ✓antwoord (3)
	OF	
	$\frac{p}{256}=\frac{64}{p}$ $p^2=16384$ $p=\pm 128$ $p=-128$	✓ $\frac{p}{256}=\frac{64}{p}$ ✓ $p=\pm 128$ ✓antwoord (3)
	OF	
	$\frac{p}{256}=\frac{-32}{64}$ $64p=8192$ $p=-128$	✓ $\frac{p}{256}=\frac{-32}{64}$ ✓vereenvoudiging ✓antwoord (3)
	OF	
	$\frac{1}{r}=\frac{64}{-32}=-2$ $p=-2\times 64$ $p=-128$	✓ $\frac{1}{r}=\frac{64}{-32}=-2$ ✓vereenvoudiging ✓antwoord (3)
2.1.2	$S_n=\frac{a\left[1-r^n\right]}{1-r}$ $S_8=\frac{256\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $=\frac{512}{3}\left(\frac{255}{256}\right)$ $=170$	✓formule ✓substitusie ✓antwoord (3)
	OF	

	$S_n = \frac{a\left[1-r^n\right]}{1-r}$ $S_8 = \frac{2^8\left[1-\left(-\frac{1}{2}\right)^8\right]}{1+\frac{1}{2}}$ $= \frac{2^9}{3}\left(\frac{255}{2^8}\right)$ $= 170$	✓formule ✓substitusie ✓antwoord (3)
2.1.3	$-1 < r < 1$ **OF** Die gemeenskaplike verhouding is $-\frac{1}{2}$ wat tussen – 1 en 1 is. **OF** $-1 < -\frac{1}{2} < 1$	✓antwoord (1) ✓antwoord (1) ✓antwoord (1)
2.1.4	$S_\infty = \frac{a}{1-r}$ $= \frac{256}{1-\left(-\frac{1}{2}\right)}$ $= \frac{512}{3}$ $= 170,67$	✓formule ✓substitusie ✓antwoord (3)

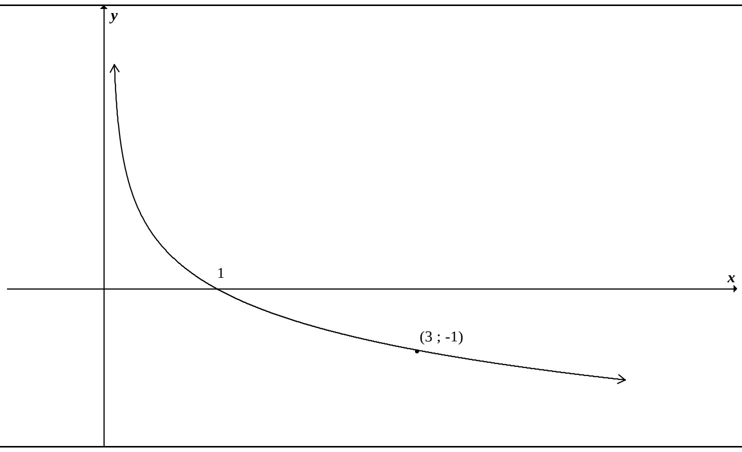
2.2.1	16	✓antwoord (1)
2.2.2	$T_n = -8 + 6(n-1)$ $148 = 6n - 14$ $6n = 162$ $n = 27$	✓substitusie in vergelyking ✓$T_n = 148$ ✓antwoord (3)
2.2.3	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $\frac{n}{2}\left[2(-8) + (n-1)(6)\right] > 10\,140$ $3n^2 - 11n > 10\,140$ $3n^2 - 11n - 10\,140 > 0$ $(3n+169)(n-60) > 0$ Indien n = 60, S_n = 10 140 Kleinste n = 61	✓ $\frac{n}{2}\left[2(-8)+(n-1)(6)\right]$ ✓ $3n^2 - 11n > 10\,140$ ✓faktore ✓n = 60 ✓antwoord (5)
2.3	$\sum_{k=1}^{30}(3k+5)$ $a = 8 \quad n = 30 \quad d = 3$ $\sum_{k=1}^{30}(3k+5) = \frac{30}{2}[2(8) + 29(3)]$ $= 15(103)$ $= 1545$	✓n = 30 ✓substitusie in korrekte formule ✓antwoord (3) **[22]**

VRAAG 3

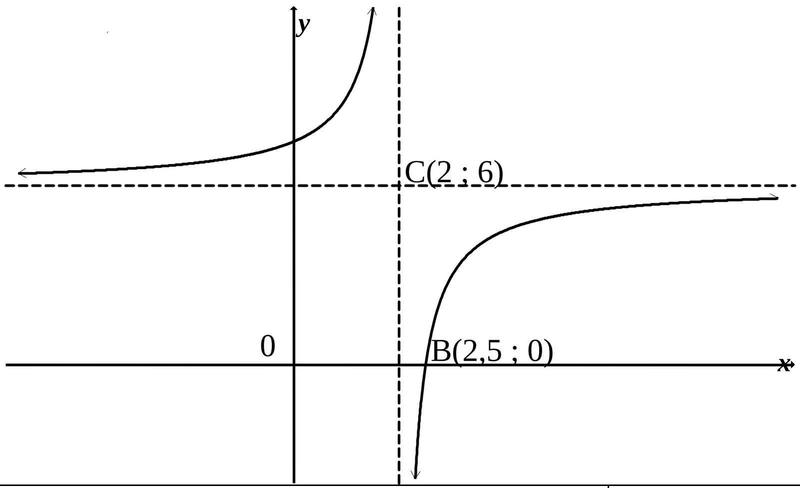
3.1	Jakob het uitgewerk dat die ry meetkundig of eksponensieel is. Vusi het die ry as kwadraties uitgewerk. **OF** Jakob vermenigvuldig elke term met 3 om die volgende term te kry. Vusi sien dit as 'n ry met 'n konstante tweede verskil. **OF** Jakob het uitgewerk dat die ry meetkundig of eksponensieel is. Vusi het uitgewerk dat die ry 'n kombinasie van eksponensieel en derde mag is.	✓Jakob (meetkundig/ eksponensieel) ✓Vusi(kwadraties) (2) ✓ Jakob (vermenigvuldig elke term met 3) ✓Vusi(konstante tweede verskil) (2) ✓Jakob (meetkundig/ eksponensieel) ✓Vusi(eksponensieel en derde mag gekombineerd) (2)
3.2.1	$T_n = 3^n$ **OF** $T_n = 3.3^{n-1}$	✓antwoord (1) ✓antwoord (1)
3.2.2	 $2a = 12$ $3a + b = 6$ $a + b + c = 3$ $a = 6$ $18 + b = 6$ $6 - 12 + c = 3$ $b = -12$ $c = 9$ $T_n = 6n^2 - 12n + 9$ **OF** $2a = 12$ $a = 6$ $T_0 = c = 9$ $T_n = an^2 + bn + 9$ $3 = 6(1)^2 + b(1) + 9$ $b = -12$ $T_n = 6n^2 - 12n + 9$ **OF**	✓$a = 6$ ✓metode ✓$b = -12$ ✓$c = 9$ (4) ✓$a = 6$ ✓$c = 9$ ✓metode ✓$b = -12$ (4)

	$2a=12$ $a=6$ $T_n=6n^2+bn+c$ $T_1=3=6(1)^2+b(1)+c$ dus $3=6+b+c$ $T_2=9=6(2)^2+b(2)+c$ dus $9=24+2b+c$ $6=18+b$ $b=-12$ $c=9$ $T_n=6n^2-12n+9$	✓$a = 6$ ✓metode ✓$b = -12$ ✓$c = 9$ (4)
	OF $T_n=3^n+k(n-1)(n-2)(n-3)$ $57=3^4+k(3)(2)(1)$ $6k=-24$ $k=-4$ $T_n=3^n-4(n-1)(n-2)(n-3)$	✓✓ $T_n=3^n+k(n-1)(n-2)(n-3)$ ✓substitusie ✓antwoord (4) **[7]**

VRAAG 4

4.1	**R OF** $(-\infty;\infty)$	✓antwoord (1)
4.2	$y=0$	✓ $y=0$ (1)
4.3	$x=\left(\frac{1}{3}\right)^y$ $y=\log_{\frac{1}{3}} x$ **OF** $x=\left(\frac{1}{3}\right)^y$ $x=3^{-y}$ $-y=\log_3 x$ $y=-\log_3 x$	✓ $x=\left(\frac{1}{3}\right)^y$ ✓ $y=\log_{\frac{1}{3}} x$ (2) ✓ $x=\left(\frac{1}{3}\right)^y$ ✓ $y=-\log_3 x$ (2)
4.4		✓vorm ✓afsnit by (1 ; 0) ✓enige ander korrekte punt (3)
4.5	$x=-2$	✓✓ $x=-2$ (2)
4.6	$\text{LK}=[f(x)]^2-[f(-x)]^2$ $=\left[\left(\frac{1}{3}\right)^x\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]^2$ $=3^{-2x}-3^{2x}$ $\text{RK}=f(2x)-f(-2x)$ $=\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ $=3^{-2x}-3^{2x}$ $\therefore$ LK = RK $[f(x)]^2-[f(-x)]^2=f(2x)-f(-2x)$	✓ $\left[\left(\frac{1}{3}\right)^x\right]^2-\left[\left(\frac{1}{3}\right)^{-x}\right]$ ✓ $3^{-2x}-3^{2x}$ ✓ $\left(\frac{1}{3}\right)^{2x}-\left(\frac{1}{3}\right)^{-2x}$ (3) **[12]**

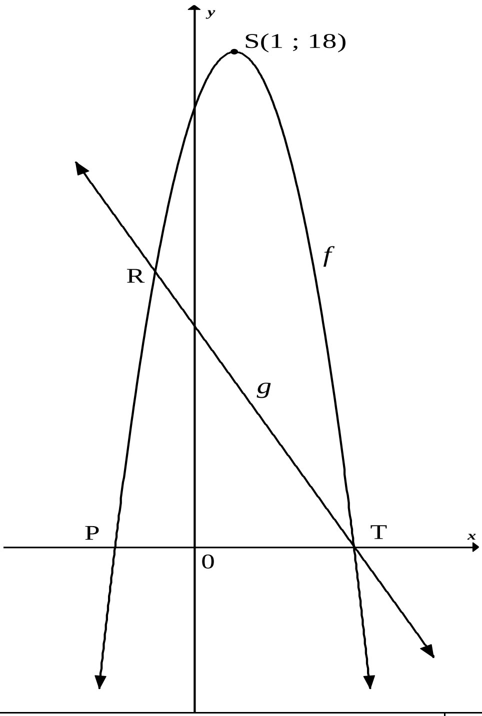
VRAAG 5



5.1	$g(x)=\frac{a}{x-2}+6$ $0=\frac{a}{2{,}5-2}+6$ $0=2a+6$ $a=-3$ $g(x)=\frac{-3}{x-2}+6$	✓ $p=2$ ✓ $q=6$ ✓vervang B(2,5 ; 0) ✓ $a=-3$ (4)
5.2	$x_f=2-\frac{1}{2}$ $x_f=\frac{3}{2}$ $y_f=6+6$ $y_f=12$ $\text{F}\left(\frac{3}{2};12\right)$	✓*x*-koördinaat ✓*y*-koördinaat (2) **[6]**

VRAAG 6

$f(x) = ax^2 + bx + c$
$g(x) = -2x + 8$



6.1	$0 = -2x + 8$ $2x = 8$ $x = 4$ T(4 ; 0)	✓ $y = 0$ ✓ $x = 4$ (2)
6.2	Deur simmetrie, P(– 2 ; 0) $f(x) = a(x+2)(x-4)$ $18 = a(1+2)(1-4)$ $a = -2$ $f(x) = -2(x+2)(x-4)$ $= -2(x^2 - 2x - 8)$ $= -2x^2 + 4x + 16$ **OF** $f(x) = a(x-1)^2 + 18$ $0 = a(4-1)^2 + 18$ $a = -2$ $f(x) = -2(x-1)^2 + 18$ $= -2(x^2 - 2x + 1) + 18$ $= -2x^2 + 4x + 16$	✓ $f(x) = a(x+2)(x-4)$ ✓vervang S(1 ; 18) ✓ $a = -2$ ✓vermenigvuldig korrek en kry $-2x^2 + 4x + 16$ (4) ✓ $f(x) = a(x-1)^2 + 18$ ✓vervang T(4 ; 0) ✓ $a = -2$ ✓vermenigvuldig korrek en kry $-2x^2 + 4x + 16$ (4)

6.3	$-2x+8=-2x^2+4x+16$ $2x^2-6x-8=0$ $x^2-3x-4=0$ $(x-4)(x+1)=0$ $x=4$ or $x=-1$ by R is $y=-2(-1)+8=10$ dus R(– 1; 10)	✓ $-2x+8=-2x^2+4x+16$ ✓ $2x^2-6x-8=0$ ✓ $x=-1$ ✓ $y=10$ (4)
6.4.1	$-1\le x\le 4$	✓ $-1\le x$ ✓ $x\le 4$ (2)
6.4.2	$-2x^2+4x-2<0$ $-2x^2+4x-2+18<18$ $-2x^2+4x+16<18$ $f(x)<18$ $(-\infty\,;1)\cup(1\,;\infty)$ **OF** $-2x^2+4x-2<0$ $-2x^2+4x-2+18<18$ $-2x^2+4x+16<18$ $f(x)<18$ $x\in\mathbf{R}\,;\ x\ne 1$	✓ $-2x^2+4x-2+18<18$ ✓ $-2x^2+4x+16<18$ ✓ $f(x)<18$ ✓ $(-\infty\,;1)\cup(1\,;\infty)$ (4) ✓ $-2x^2+4x-2+18<18$ ✓ $-2x^2+4x+16<18$ ✓ $f(x)<18$ ✓ $x\in\mathbf{R}\,;\ x\ne 1$ (4) **[16]**

VRAAG 7

7.1	$F = P(1+i)^n$ $= 4\,000\,000(1+0{,}06)^3$ $= \text{R}4\ 764\ 064$	✓formule ✓substitusie ✓antwoord (3)
7.2.1	$4\,000\,000 = \dfrac{30\,000\left[1-\left(1+\frac{0{,}06}{12}\right)^{-n}\right]}{\frac{0{,}06}{12}}$ $\dfrac{4000000\times\left(\frac{0{,}06}{12}\right)}{30000} = 1-\left(1+\frac{0{,}06}{12}\right)^{-n}$ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ $\log_{\left(1+\frac{0{,}06}{12}\right)}\frac{1}{3} = -n$ $n = 220{,}27$ Sy sal dus 220 onttrekkings van R30 000 maak.	✓formule ✓ $i = \frac{0{,}06}{12}$ ✓substitusie in korrekte formule ✓ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ ✓korrekte gebruik van logs ✓antwoord van 220 onttrekkings (6)
	OF $4\,000\,000 = \dfrac{30\,000\left[1-\left(1+\frac{0{,}06}{12}\right)^{-n}\right]}{\frac{0{,}06}{12}}$ $\dfrac{4000000\times\left(\frac{0{,}06}{12}\right)}{30000} = 1-\left(1+\frac{0{,}06}{12}\right)^{-n}$ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ $\log\frac{1}{3} = -n\log\left(1+\frac{0{,}06}{12}\right)$ $n = 220{,}27$ Sy sal dus 220 onttrekkings van R30 000 maak.	✓formule ✓ $i = \frac{0{,}06}{12}$ ✓substitusie in korrekte formule ✓ $\frac{1}{3} = \left(1+\frac{0{,}06}{12}\right)^{-n}$ ✓korrekte gebruik van logs ✓antwoord van 220 onttrekkings (6)

7.2.2	$4\,000\,000=\dfrac{20\,000\left[1-\left(1+\dfrac{0,06}{12}\right)^{-n}\right]}{\dfrac{0,06}{12}}$ $0=\left(1+\dfrac{0,06}{12}\right)^{-n}$ Sy kan soveel onttrekkings maak as wat sy wil.	✓ $4\,000\,000=\dfrac{20\,000\left[1-\left(1+\dfrac{0,06}{12}\right)^{-n}\right]}{\dfrac{0,06}{12}}$ ✓ $0=\left(1+\dfrac{0,06}{12}\right)^{-n}$ ✓gevolgtrekking (3) **[12]**

VRAAG 8

	$\left(1+\dfrac{0,08}{12}\right)^{12}=\left(1+\dfrac{r}{2}\right)^{2}$ $\dfrac{r}{2}=0,040672622$ $r=8,13452446\%$ $r=8,13\%$	✓ $\left(1+\dfrac{0,08}{12}\right)^{12}$ ✓ $\left(1+\dfrac{i}{2}\right)^{2}$ ✓antwoord **[3]**

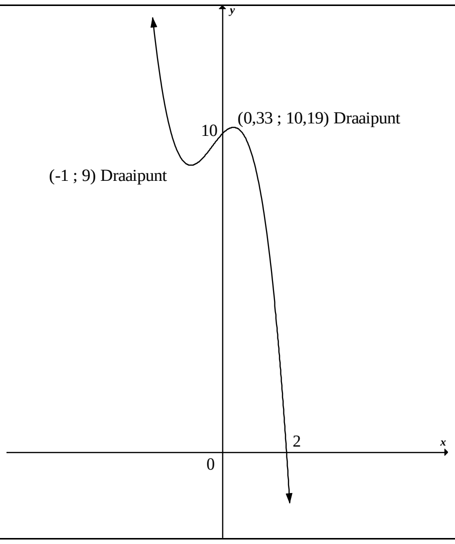
VRAAG 9

9.1	$f(x) = 2x^3$ $f(x+h) = 2(x+h)^3$ $= 2(x^3 + 3x^2h + 3xh^2 + h^3)$ $= 2x^3 + 6x^2h + 6xh^2 + 2h^3$ $f(x+h) - f(x) = 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3$ $= 6x^2h + 6xh^2 + 2h^3$ $f'(x) = \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$ $= \lim_{h \to 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$ $= \lim_{h \to 0}(6x^2 + 6xh + 2h^2)$ $f'(x) = 6x^2$ **OF** $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{2(x+h)^3 - 2x^3}{h}$ $= \lim_{h \to 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - 2x^3}{h}$ $= \lim_{h \to 0} \frac{6x^2h + 6xh^2 + 2h^3}{h}$ $= \lim_{h \to 0} \frac{h(6x^2 + 6xh + 2h^2)}{h}$ $= \lim_{h \to 0}(6x^2 + 6xh + 2h^2)$ $f'(x) = 6x^2$	✓substitusie ✓uitbreiding ✓formule ✓ $6x^2 + 6xh + 2h^2$ ✓antwoord (5) ✓formule ✓substitusie ✓uitbreiding ✓ $6x^2 + 6xh + 2h^2$ ✓antwoord (5)
9.2	$y = \frac{2\sqrt{x} + 1}{x^2}$ $= 2x^{-\frac{3}{2}} + x^{-2}$ $\frac{dy}{dx} = -3x^{-\frac{5}{2}} - 2x^{-3}$	✓ $2x^{-\frac{3}{2}}$ ✓ x^{-2} ✓ $-3x^{-\frac{5}{2}}$ ✓ $-2x^{-3}$ (4)

9.3	$f'(-1)=-7$ $f'(x)=2ax+b$ $-7=-2a+b$ $f(-1)=-7(-1)+3$ $=10$ $\therefore\ a-b+5=10$ $a-b=5............[1]$ $-2a+b=-7.........[2]$ $-a=-2.........[1]+[2]$ $a=2$ $b=-3$	✓ $f'(x)=2ax+b$ ✓substitusie van $x = -1$ ✓ $-7=-2a+b$ ✓$f(-1) = 10$ ✓$a = 2$ ✓$b = -3$ (6) **[15]**

VRAAG 10

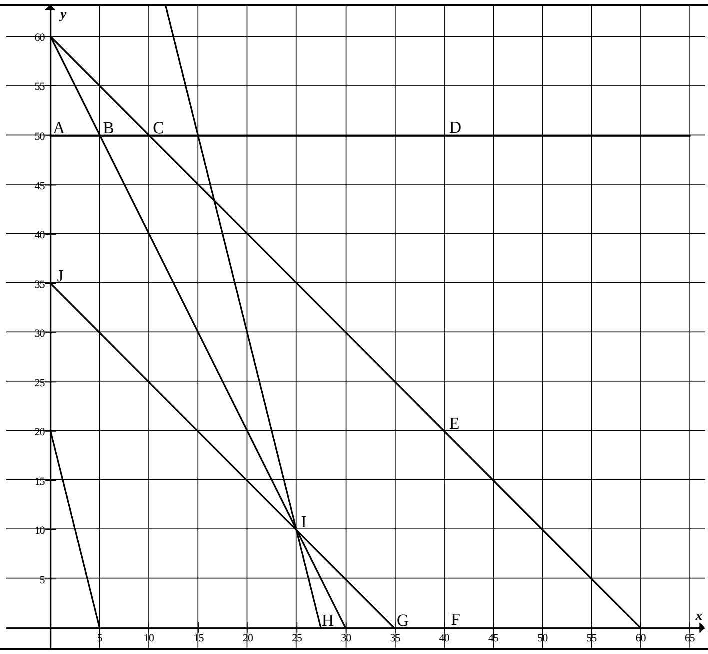
$f(x) = -x^3 - x^2 + x + 10$

10.1	$(0;10)$	✓ $(0;10)$ (1)
10.2	$0 = -x^3 - x^2 + x + 10$ $0 = -(x-2)(x^2 + 3x + 5)$ $x - 2 = 0$ of $x^2 + 3x + 5 = 0$ $x = 2$ $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2(1)}$ $= \frac{-3 \pm \sqrt{-11}}{2}$ wat geen oplossing het nie Dus is die enigste x-afsnit van $f(2;0)$	✓ $(x-2)$ ✓ $(x^2 + 3x + 5)$ ✓ $x = \frac{-3 \pm \sqrt{-11}}{2}$ ✓geen oplossing (4)
10.3	$f'(x) = -3x^2 - 2x + 1$ $0 = -3x^2 - 2x + 1$ $0 = (3x-1)(x+1)$ $x = \frac{1}{3}$ of $x = -1$ $y = -\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right) + 10$ of $y = -(-1)^3 - (-1)^2 + (-1) + 10$ $= \frac{275}{27}$ $= 9$ $\left(\frac{1}{3}; 10\frac{5}{27}\right)$ $(-1; 9)$	✓ $f'(x) = -3x^2 - 2x + 1$ ✓ $f'(x) = 0$ ✓faktore ✓x-waardes ✓ $\left(\frac{1}{3}; 10\frac{5}{27}\right)$ ✓ $(-1 ; 9)$ (6)
10.4		✓vorm ✓afsnitte ✓draaipunte (3) **[14]**

VRAAG 11

11.1	Lengte van die houer = $3x$ Volume $= l \times b \times h$ $9 = 3x \cdot x \cdot h$ $9 = 3x^2 h$ $h = \dfrac{3}{x^2}$	✓lengte van houer = $3x$ ✓$9 = 3x \cdot x \cdot h$ ✓$h = \dfrac{3}{x^2}$ (3)
11.2	$C = (2(3xh) + 2xh) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\dfrac{3}{x^2}\right) \times 50 + 600x^2$ $= \dfrac{1200}{x} + 600x^2$ **OF** $C = (h \times 8x) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\dfrac{3}{x^2}\right) \times 50 + 600x^2$ $= \dfrac{1200}{x} + 600x^2$	✓$(2(3xh) + 2xh) \times 50$ ✓$(2 \times 3x^2) \times 100$ ✓substitusie van $h = \dfrac{3}{x^2}$ (3) ✓$(h \times 8x) \times 50$ ✓$(2 \times 3x^2) \times 100$ ✓substitusie van $h = \dfrac{3}{x^2}$ (3)
11.3	$C = 1200x^{-1} + 600x^2$ $\dfrac{dC}{dx} = -1200x^{-2} + 1200x$ $0 = -1200x^{-2} + 1200x$ $1200x^3 = 1200$ $x^3 = 1$ $x = 1$ Dus is die breedte van die houer 1 meter.	✓$\dfrac{dC}{dx} = -1200x^{-2} + 1200x$ ✓$\dfrac{dC}{dx} = 0$ ✓$x^3 = 1$ ✓$x = 1$ (4) **[10]**

VRAAG 12

12.1		✓✓gebied ABIJ gearseer (2) **LET WEL:** Indien gebied BCEFGI gearseer is: gee EEN punt Indien enige ander gebied gearseer is: gee 0 punte
12.2	$x \leq 40$ $x + y \leq 60$ $y \geq 0$	✓ $x \leq 40$ ✓ $x + y \leq 60$ ✓ $y \geq 0$ (3)
12.3	$x = 25$	✓antwoord (1)
12.4	At I(25 ; 10), $P = 4(25) + 10 = 110$ Maksimum waarde van P is 110 as $x = 25$ en $y = 10$	✓ $x = 25$ ✓ $y = 10$ ✓substitusie ✓maksimum waarde van P is 110 (4)
12.5	$C = kx + y$ $y = -kx + C$ $-k < -1$ $k > 1$	✓ $y = -kx + C$ ✓ $k > 1$ (2) **LET WEL:** Slegs antwoord: gee TWEE punte **[12]**

TOTAAL: 150