basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2013

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages, 1 diagram sheet and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 11 questions.

2.   Answer ALL the questions.

3.   Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.

4.   Answers only will not necessarily be awarded full marks.

5.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

7.   ONE diagram sheet for QUESTION 11 is attached at the end of this question paper. Write your centre number and examination number on this diagram sheet in the spaces provided and insert the diagram sheet inside the back cover of your ANSWER BOOK.

8.   An information sheet with formulae is included at the end of this question paper.

9.   Number the answers correctly according to the numbering system used in this question paper.

10.   Write neatly and legibly.

QUESTION 1

1.1   Solve for x in each of the following:

        1.1.1 $x^2 - x - 12 = 0$ (3)

        1.1.2   (a) $2x^2 - 5x - 11 = 0$ (4)

                  (b) $2x^3 - 5x^2 - 11x = 0$ (2)

        1.1.3 $-3(x+7)(x-5) < 0$ (4)

1.2   Given: $y + 2 = x$ and $y = x^2 - x - 10$

        Solve for x and y simultaneously.   (6)

1.3   Simplify: $\dfrac{3^{2015} + 3^{2013}}{9^{1006}}$ (3)

                                                                                            [22]

QUESTION 2

2.1   Given the geometric sequence: $7 ; x ; 63 ; ...$

        Determine the possible values of x.   (3)

2.2   The first term of a geometric sequence is 15. If the second term is 10, calculate:

        2.2.1 T_{10} (3)

        2.2.2 S_9 (2)

2.3   Given: $0 ; -\dfrac{1}{2} ; 0 ; \dfrac{1}{2} ; 0 ; \dfrac{3}{2} ; 0 ; \dfrac{5}{2} ; 0 ; \dfrac{7}{2} ; 0 ; ...$

        Assume that this number pattern continues consistently.

        2.3.1   Write down the value of the 191st term of this sequence.   (1)

        2.3.2   Determine the sum of the first 500 terms of this sequence.   (4)

2.4   Given: $\sum\limits_{k=2}^{20}(4x-1)^k$

       2.4.1   Calculate the first term of the series $\sum\limits_{k=2}^{20}(4x-1)^k$ if $x=1$.   (2)

       2.4.2   For which values of x will $\sum\limits_{k=1}^{\infty}(4x-1)^k$ exist?   (3)
                                                                                                       [18]

QUESTION 3

3.1   Given the arithmetic sequence: $-3\,;\,1\,;\,5\,;\,...\,;\,393$

       3.1.1   Determine a formula for the nth term of the sequence.   (2)

       3.1.2   Write down the 4th, 5th, 6th and 7th terms of the sequence.   (2)

       3.1.3   Write down the remainders when each of the first seven terms of the
                  sequence is divided by 3.   (2)

       3.1.4   Calculate the sum of the terms in the arithmetic sequence that are
                  divisible by 3.   (5)
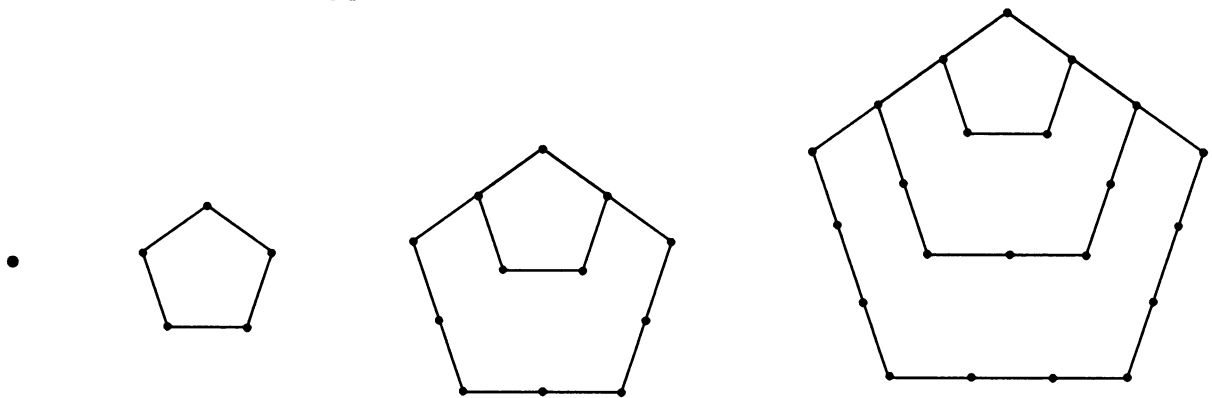
3.2   Consider the following pattern of dots:

       FIGURE 1   FIGURE 2   FIGURE 3   FIGURE 4

       If T_n represents the total number of dots in FIGURE n, then $T_1=1$ and $T_2=5$.
       If the pattern continues in the same manner, determine:

       3.2.1 T_5 (2)

       3.2.2 T_{50} (5)
   **[18]**

QUESTION 4

Given: $f(x) = -2x^2 - 5x + 3$

4.1   Write down the coordinates of the y-intercept of f.   (1)

4.2   Determine the coordinates of the x-intercepts of f.   (3)

4.3   Determine the coordinates of the turning point of f.   (3)

4.4   Sketch the graph of $y = f(x)$, clearly showing the coordinates of the turning points
       and the three intercepts with the axes.   (3)
                                                                                                        [10]

QUESTION 5

5.1   Sketched below are the graphs of $g(x) = k^x$, where $k > 0$ and $y = g^{-1}(x)$.
       (2 ; 36) is a point on g.
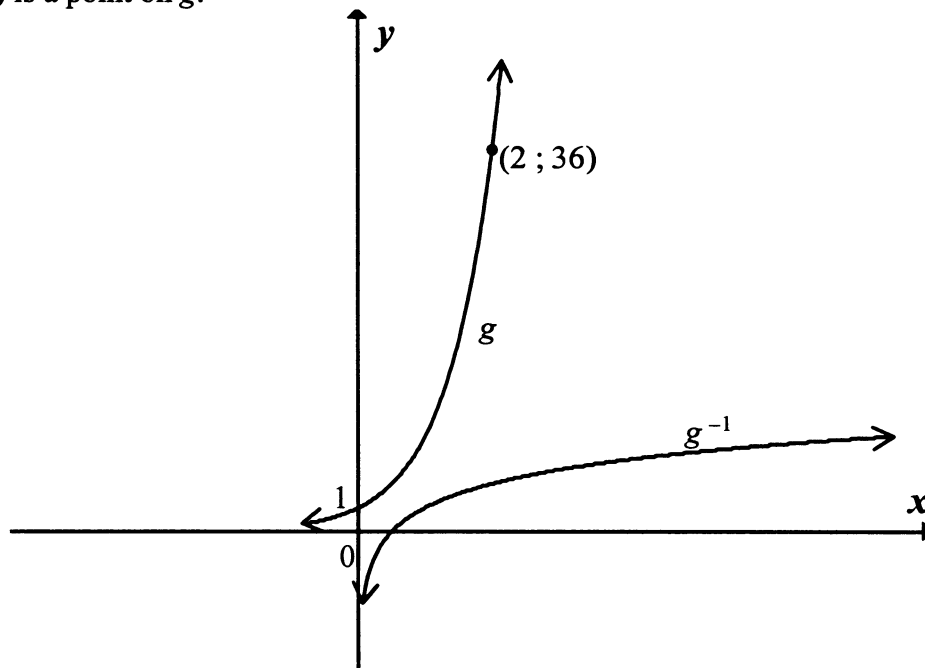
       5.1.1   Determine the value of k.   (2)

       5.1.2   Give the equation of g^{-1} in the form $y = ...$ (2)

       5.1.3   For which value(s) of x is $g^{-1}(x) \le 0$?   (2)

       5.1.4   Write down the domain of h if $h(x) = g^{-1}(x-3)$.   (1)

5.2   5.2.1   Sketch the graph of the inverse of $y = 1$.   (2)

       5.2.2   Is the inverse of $y = 1$ a function? Motivate your answer.   (2)
                                                                                                        [11]

WESTERN CAPE

QUESTION 6

A sketch of the hyperbola $f(x) = \dfrac{x-d}{x-p}$, where d and p are constants, is given below. The

dotted lines are the asymptotes. The asymptotes intersect at P and B(2 ; 0) is a point on f.
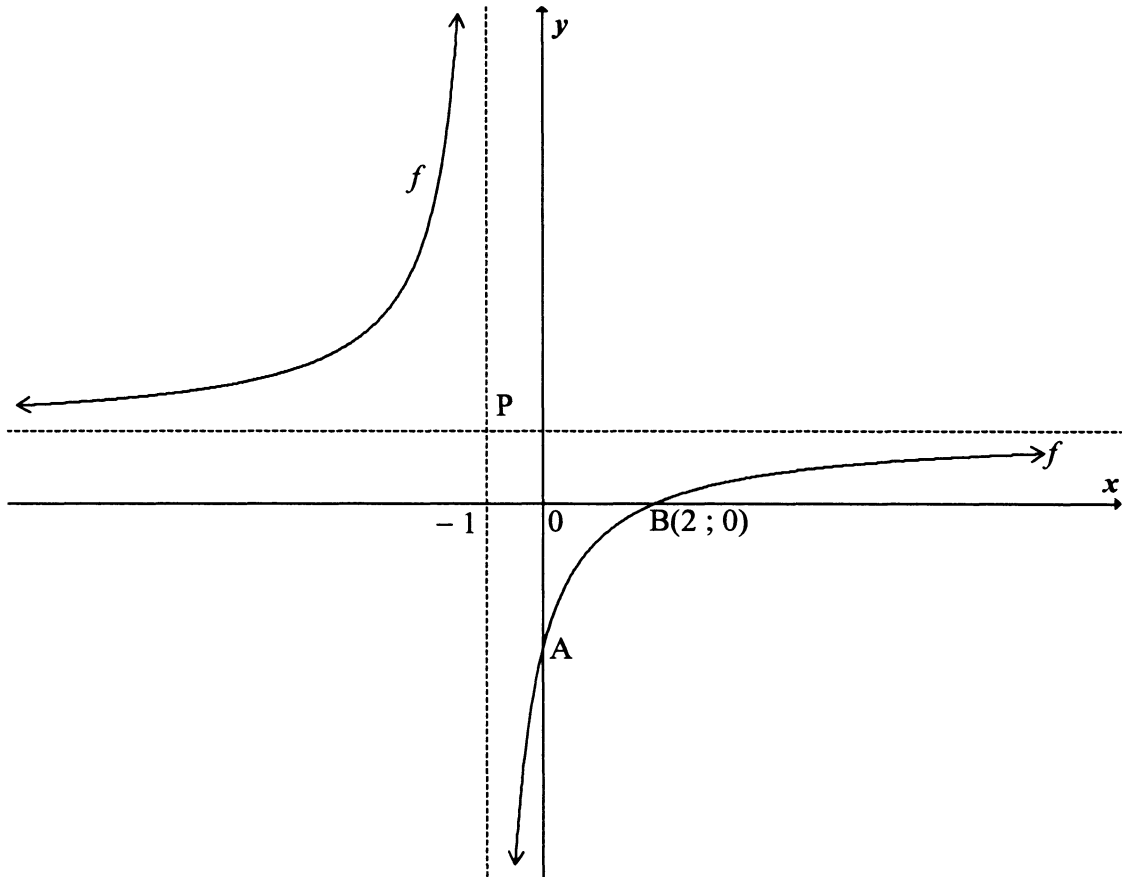
| 6.1.1 | Determine the values of d and p. | (2) |

| 6.1.2 | Show that the equation of f can be written as $y = \dfrac{-3}{x+1} + 1$. | (2) |

| 6.1.3 | Write down the coordinates of P. | (2) |

| 6.1.4 | Write down the coordinates of the image of $B(2 \; ; \; 0)$ if B is reflected about the axis of symmetry $y = x + 2$. | (2) |

6.2   The exponential function, $g(x) = p.2^x + q$ has a horizontal asymptote at $y = 1$ and
passes through $(0 \; ; -2)$. Determine the values of p and q.   (3)

                                                                                                  [11]

QUESTION 7

7.1   Mpho invests R12 500 for exactly k years. She earns interest at a rate of 9% per annum, compounded quarterly. At the end of k years, her investment is worth R30 440.

      7.1.1   Calculate the effective annual interest rate of Mpho's investment.   (2)

      7.1.2   Determine the value of k.   (5)

7.2   Darrel is planning to buy his first home. The bank will allow him to use a maximum of 30% of his monthly salary to repay the bond.

      7.2.1   Calculate the maximum amount that the bank will allow Darrel to spend each month on his bond repayments, if Darrel earns R18 480 per month.   (1)

      7.2.2   Suppose, at the end of each month, Darrel repays the maximum amount allowed by the bank. How much money does Darrel borrow if he takes 25 years to repay the loan at a rate of 8% p.a., compounded monthly? (The first repayment is made one month after the loan is granted.)   (4)

                                    [12]

QUESTION 8

8.1   Given: $f(x) = 3x^2 - 4$

      8.1.1   Determine $f'(x)$ from first principles.   (5)

      8.1.2   A(x ; 23), where $x > 0$, and B($-$ 2 ; y) are points on the graph of f. Calculate the numerical value of the average gradient of f between A and B.   (5)

8.2   Differentiate $y = \dfrac{x+5}{\sqrt{x}}$ with respect to x.   (3)

8.3   Determine the gradient of the tangent of the graph of $f(x) = -3x^3 - 4x + 5$ at $x = -1$.   (4)

                                      [17]

QUESTION 9

The function defined by $f(x) = x^3 + ax^2 + bx - 2$ is sketched below.
P(-1 ; -1) and R are the turning points of f.
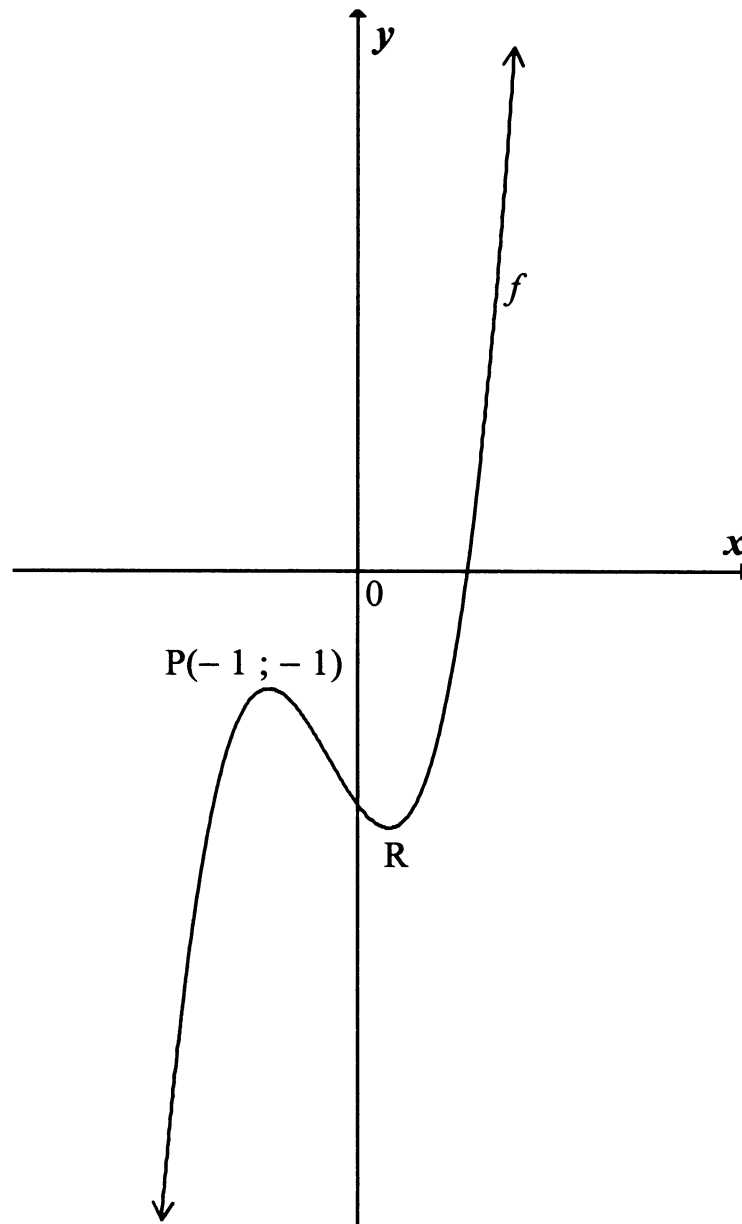
9.1   Show that $a = 1$ and $b = -1$.   (6)

9.2   Hence, or otherwise, determine the x-coordinate of R.   (3)

9.3   Write down the coordinates of a turning point of h if h is defined by $h(x) = 2f(x) - 4$.   (2)

[11]

QUESTION 10

An industrial process requires water to flow through its system as part of the cooling cycle. Water flows continuously through the system for a certain period of time.

The relationship between the time (t) from when the water starts flowing and the rate (r) at which the water is flowing through the system is given by the equation:

$$r = -0,2t^2 + 10t$$

where t is measured in seconds.

10.1	After how long will the water be flowing at the maximum rate?	(3)
10.2	After how many seconds does the water stop flowing?	(3)
		[6]

QUESTION 11

A company manufactures both short-sleeved shirts and long-sleeved shirts. The constraints below govern the production of the shirts per day.

- No more than 80 short-sleeved shirts can be produced per day.
- A minimum of 50 long-sleeved shirts must be produced per day.
- At most 5 long-sleeved shirts must be manufactured for every short-sleeved shirt.
- Each short-sleeved shirt has 5 buttons and each long-sleeved shirt has 4 buttons.
- At most 800 buttons are available for production per day.

Let the number of short-sleeved shirts be x and the number of long-sleeved shirts be y.

11.1	Write down the constraints which govern this system.	(4)
11.2	Sketch the system of constraints (inequalities) on the graph paper on DIAGRAM SHEET 1. Clearly indicate the feasible region.	(5)
11.3	A profit of R30 is made on each short-sleeved shirt and a profit of R20 is made on each long-sleeved shirt.	

11.3.1	Write down the profit function.	(1)
11.3.2	Determine the number of short-sleeved shirts and long-sleeved shirts that must be manufactured per day to provide the company with maximum profit.	(2)

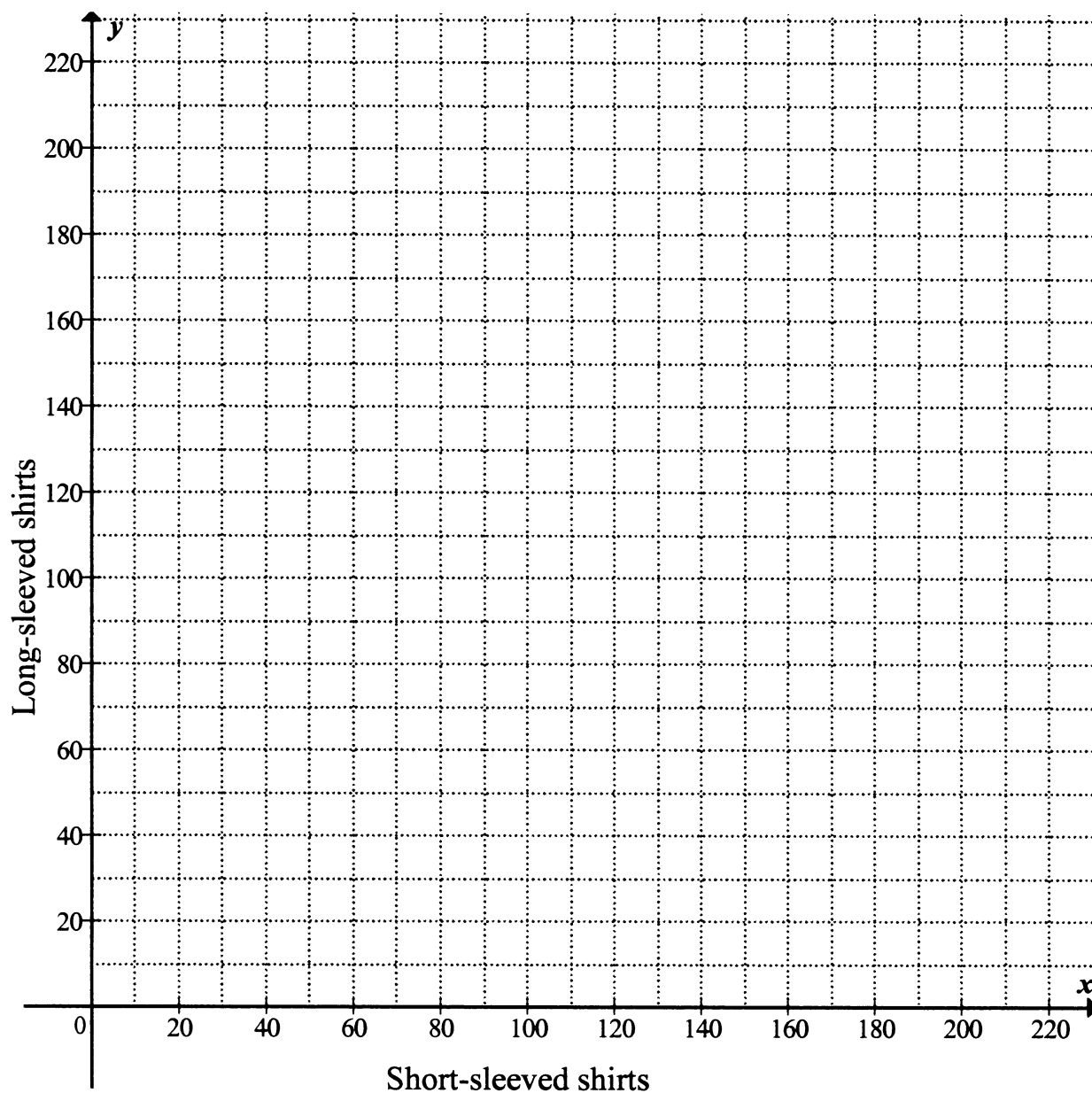| 11.4	If the objective profit function is given by $P = ax + by$, determine $\dfrac{a}{b}$ if P is maximised at each value of y between 100 and 160.	(2)
		[14]

TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 11.2

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \ ; \qquad r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \ ; \ -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x \ ; y) \to (x\cos\theta - y\sin\theta \ ; \ y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$