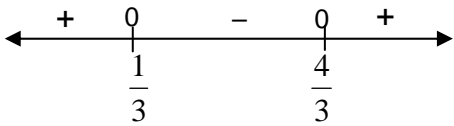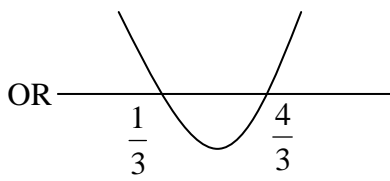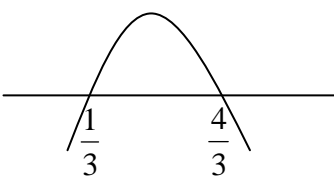NATIONAL
SENIOR CERTIFICATE

GRADE 12

Marks: 150

This memorandum consists of 25 pages.

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QUESTION 1

1.1.1	$x(x-1) = 30$ $x^2 - x = 30$ $x^2 - x - 30 = 0$ $(x-6)(x+5) = 0$ $x = 6$ or $x = -5$ OR $x(x-1) = 30$ $x^2 - x = 30$ $x^2 - x - 30 = 0$ $x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-30)}}{2(1)}$ $= \dfrac{1 \pm \sqrt{121}}{2}$ $= \dfrac{1 \pm 11}{2}$ $x = 6$ or $x = -5$	If implied as equation : No penalty If there is no equals sign or the equation is not = 0: No penalty If $x = 6$ is answer by inspection : 1 / 3 Both correct answers no calculation : 1 / 3	✓ simplification (multiplying out brackets) ✓ factors ✓ both answers (3) ✓ simplification (multiplying out brackets) ✓ substitution into formula ✓ both answers *(ca)* (3)
1.1.2	$3x^2 - 5x + 1 = 0$ $a = 3 \; b = -5 \; c = 1$ $x = \dfrac{-(-5) \pm \sqrt{25 - 4(3)(1)}}{2(3)}$ $= \dfrac{5 \pm \sqrt{13}}{6}$ $x = 1,4$ or $x = 0,2$ OR	**NOTE:** Penalty 1 for incorrect rounding off in either answer Using calculator incorrectly: Max: 2 / 4 Answers will be $x = 5,6$ or $4,4$ Incorrect formula: max 1 / 4 If $x = \dfrac{5 \pm \sqrt{37}}{6}$ then CA applies $x = 1,8$ and $-0,2$: Max 3 / 4 Correct answer only: 2 / 4 If factorising: 0 / 4 If $x = \dfrac{5 \pm \sqrt{13}}{6}$ only, then 2 / 4 If $x = 5 \pm \dfrac{\sqrt{13}}{6}$ only, then 1 / 4	✓ substitution into correct formula ✓ $\sqrt{13}$ ✓✓ values of x *(CA with formula)* (4)

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1.1.2 contd	$3x^2 - 5x + 1 = 0$ $x^2 - \dfrac{5}{3}x = -\dfrac{1}{3}$ $x^2 - \dfrac{5}{3}x + \dfrac{25}{36} = -\dfrac{1}{3} + \dfrac{25}{36}$ $\left(x - \dfrac{5}{6}\right)^2 = \dfrac{13}{36}$ $x - \dfrac{5}{6} = \dfrac{\pm\sqrt{13}}{6}$ $x = \dfrac{5 \pm \sqrt{13}}{6}$ $x = 1,4 \quad$ or $\quad x = 0,2$	✓ correct method of completing the square ✓ $\sqrt{13}$ ✓✓ values of x *(CA with formula)* (4)
1.1.3	$-9x^2 + 15x - 4 < 0$ $9x^2 - 15x + 4 > 0$ $(3x - 4)(3x - 1) > 0$ $x < \dfrac{1}{3}$ or $x > \dfrac{4}{3}$ Answer can be given as: $\quad x \in \left(-\infty\,;\dfrac{1}{3}\right) \cup \left(\dfrac{4}{3}\,;\infty\right)$ **OR** $-9x^2 + 15x - 4 < 0$ $(-3x + 4)(3x - 1) < 0$ $x < \dfrac{1}{3}$ or $x > \dfrac{4}{3}$ 	✓ factors ✓ correct inequality sign ✓ $\dfrac{1}{3}$; $\dfrac{4}{3}$ ✓ answer (4) ✓ factors ✓ correct inequality sign ✓ $\dfrac{1}{3}$; $\dfrac{4}{3}$ ✓ answer (4)
	NOTE: If stop at factorisation: 2 / 4 If incorrect factors: CA applies 3 / 4 If answer : $\dfrac{1}{3} < x < \dfrac{4}{3}$ then 3 / 4 If $x < \dfrac{1}{3}$ AND $x > \dfrac{4}{3}$ then 3 / 4	

Please turn over

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1.2	Substitute $x = y + 3$ in $x^2 - xy - 2y^2 - 7 = 0$	✓ substitution
	$(y+3)^2 - y(y+3) - 2y^2 - 7 = 0$	
	$y^2 + 6y + 9 - y^2 - 3y - 2y^2 - 7 = 0$	
	$2y^2 - 3y - 2 = 0$	✓ standard form
	$(2y+1)(y-2) = 0$	✓ factors
	$y = -\dfrac{1}{2}$ or $y = 2$	✓ both y-values
	$x = 2\dfrac{1}{2}$ or $x = 5$	✓ both x-values (5)

NOTE:
If the equation is changed to a linear equation, then max 2 / 5

There are no penalties for not putting = 0.

OR

	$y = x - 3$	✓ substitution
	$x^2 - x(x-3) - 2(x-3)^2 - 7 = 0$	
	$x^2 - x^2 + 3x - 2(x^2 - 6x + 9) - 7 = 0$	
	$0 = 2x^2 - 15x + 25$	✓ standard form
	$0 = (2x-5)(x-5)$	✓ factors
	$x = 2\dfrac{1}{2}$ or $x = 5$	✓ both x-values
	$y = -\dfrac{1}{2}$ or $y = 2$	✓ both y-values (5)

1.3	$\dfrac{10^{\frac{2009}{2}}}{10^{\frac{2011}{2}} - 10^{\frac{2007}{2}}}$	✓ convert to indices
	$= \dfrac{10^{\frac{2009}{2}}}{10^{\frac{2007}{2}}(100-1)}$	✓ common factor
	$= \dfrac{10}{99}$	✓ answer (3)

OR

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1.3 contd	$$\frac{10^{1004}\sqrt{10}}{10^{1005}\sqrt{10}-10^{1003}\sqrt{10}}$$	✓ convert to indices
	$$=\frac{10^{1004}\sqrt{10}}{\sqrt{10}(10^{1005}-10^{1003})}$$	✓ common factor
	$$=\frac{10^{1004}}{10^{1003}(100-1)}$$	✓ answer
	$$=\frac{10}{99}$$	(3)
	OR	
	$$\frac{\sqrt{10^{2009}}}{\sqrt{10^{2009}.10^2}-\sqrt{10^{2009}.10^{-2}}}$$	✓ convert to indices
	$$=\frac{\sqrt{10^{2009}}}{\sqrt{10^{2009}}(10-10^{-1})}$$	✓ common factor
	$$=\frac{1}{10-\dfrac{1}{10}}$$	✓ answer
		(3)
	$$=\frac{\dfrac{1}{99}}{10}$$	
	$$=\frac{10}{99}$$	
	OR	
	$$\frac{\sqrt{10^{2000}}\sqrt{10^9}}{\sqrt{10^{2000}.10^{11}}-\sqrt{10^{2000}.10^7}}$$	✓ convert to indices
	$$=\frac{\sqrt{10^{2000}}\sqrt{10^9}}{\sqrt{10^{2000}}\left(\sqrt{10^{11}}-\sqrt{10^7}\right)}$$	✓ common factor
	$$=\frac{\sqrt{10^9}}{\sqrt{10^{11}}-\sqrt{10^7}}$$	✓ answer
	$$=\frac{10\sqrt{10^7}}{100\sqrt{10^7}-\sqrt{10^7}}$$	(3)
	$$=\frac{10\sqrt{10^7}}{\sqrt{10^7}(100-1)}$$	
	$$=\frac{10}{99}$$	
	OR	

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	$\dfrac{\sqrt{10^{2007}}.\sqrt{10^2}}{\sqrt{10^{2007}.10^4} - \sqrt{10^{2007}}}$	✓ convert to indices
	$= \dfrac{10\sqrt{10^{2007}}}{\sqrt{10^{2007}}(\sqrt{10^4} - 1)}$	
	$= \dfrac{10}{100 - 1}$	✓ common factor
	$= \dfrac{10}{99}$	✓ answer (3)
	OR	
	Let $x = 2009$	
	$\dfrac{\sqrt{10^x}}{\sqrt{10^{x+2}} - \sqrt{10^{x-2}}}$	
	$= \dfrac{10^{\frac{x}{2}}}{10^{\frac{x}{2}}.10 - 10^{\frac{x}{2}}.10^{-1}}$	✓ convert to indices
	$= \dfrac{10^{\frac{x}{2}}}{10^{\frac{x}{2}}\left(10 - 10^{-1}\right)}$	✓ common factor
	$= \dfrac{1}{10 - \dfrac{1}{10}}$	✓ answer (3)
	$= \dfrac{1}{\dfrac{99}{10}}$	
	$= \dfrac{10}{99}$	
1.4	$\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$ $= 1 + 2\sqrt{2x^2} + 2x^2 - \sqrt{4}.\sqrt{2x^2}$ $= 1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2}$ $= 1 + 2x^2$	✓ expansion / multiplication $1 + 2\sqrt{2x^2} + 2x^2$ ✓ $\sqrt{8x^2} = 2\sqrt{2x^2}$ ✓ answer (3)
	OR $\left(1 + \sqrt{2x^2}\right)^2 - \sqrt{8x^2}$ $= 1 + \sqrt{8x^2} + 2x^2 - \sqrt{8x^2}$ $= 1 + 2\sqrt{2x^2} + 2x^2 - 2\sqrt{2x^2}$ $= 1 + 2x^2$	✓ expansion / multiplication $1 + \sqrt{8x^2} + 2x^2$ ✓ $\sqrt{8x^2} = 2\sqrt{2x^2}$ ✓ answer (3)

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| 1.4 contd | **OR** $\left(1+\sqrt{2x^2}\right)^2 - \sqrt{8x^2}$ $= 1 + 2\sqrt{2}x + 2x^2 - 2\sqrt{2}x \quad \text{or} \quad = 1 - 2\sqrt{2}x + 2x^2 + 2\sqrt{2}x$ $= 1 + 2x^2$

 Note: $\sqrt{x^2} = x$ if $x > 0$ and $-x$ if $x < 0$

 OR $\left(1+\sqrt{2x^2}\right)^2 - \sqrt{8x^2}$ $= \left(1 + (2x^2)^{\frac{1}{2}}\right)^2 - 8^{\frac{1}{2}}x$ $= 1 + 2.(2x^2)^{\frac{1}{2}} + 2x^2 - 8^{\frac{1}{2}}x$ $= 1 + 2.2^{\frac{1}{2}}x + 2x^2 - 8^{\frac{1}{2}}x$ $= 1 + 8^{\frac{1}{2}}x + 2x^2 - 8^{\frac{1}{2}}x$ $= 1 + 2x^2$

 Note: $\sqrt{x^2} = x$ if $x > 0$ and $-x$ if $x < 0$

 OR Let $2x^2 = y$ $\left(1+\sqrt{2x^2}\right)^2 - \sqrt{8x^2}$ $= (1 + \sqrt{y})^2 - \sqrt{4y}$ $= 1 + 2\sqrt{y} + y - 2\sqrt{y}$ $= 1 + y$ $= 1 + 2x^2$ | ✓expansion / multiplication
 ✓simplification
 ✓ answer

 (3)

 ✓expansion / multiplication
 $1 + 2.(2x^2)^{\frac{1}{2}} + 2x^2$
 ✓simplification
 ✓ answer

 (3)

 [22] |

Please turn over

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QUESTION 2

2.1.1	$T_n = 4n + 1$ **OR** $T_n = 5 + (n-1)(4)$ $\quad = 4n + 1$ **NOTE**: If $T_n = 5 + (n-1)(4)$ then full marks	✓✓✓ Answer only (3) ✓ $d = 4$ ✓ substitution ✓ answer (3)
2.1.2	$T_n = 5(25)^{n-1}$	✓ $r = 25$ ✓ answer (2)
2.2	The sequence is $1 ; 1 + d ; 1 + 2d ; 1 + 3d; \dots$ (AP) and $1 ; r ; r^2 ; r^3 ; \dots$ (GP) $\therefore 1 + d = r$ and $d = r - 1$ But $1 + 2d = r^2$ $\qquad\qquad r^2 = 1 + 2d$ $1 + 2(r-1) = r^2 \qquad (1+d)^2 = 1 + 2d$ $r^2 - 2r + 1 = 0 \quad$ OR $\quad 1 + 2d + d^2 = 1 + 2d$ $\quad (r-1)^2 = 0 \qquad\qquad d^2 = 0$ $\qquad r = 1 \qquad\qquad\quad d = 0$ $\qquad\qquad\qquad\qquad\qquad r = 1$ $\therefore d = 0$ $\therefore$ the one and only such sequence is $1 ; 1; 1 ; \dots$ Nomsa is correct. **OR** $T_1 = 1$ Let the sequence be $\quad 1 ; a ; b ; \dots$ Geometric: $\quad r = \dfrac{a}{1} = \dfrac{b}{a}$ $\qquad\qquad\qquad a^2 = b$ Arithmetic: $\quad d = a - 1 = b - a$ $\qquad\qquad\quad 2a - 1 = b$ $\qquad\qquad\quad 2a - 1 = a^2$ $\qquad\qquad\quad 0 = a^2 - 2a + 1$ $\qquad\qquad\quad 0 = (a-1)^2$ $\qquad\qquad\quad a = 1$ $\qquad\qquad\quad b = 1$ Sequence is $1 ; 1 ; 1 ; \dots$ Nomsa is correct If: Sequence is $1 ; 1 ; 1 ; 1 ; 1 ; 1 ; \dots$ Then $\quad d = 0$ $\qquad\qquad r = 1$ Therefore only one sequence $\qquad\qquad$ exists. Nomsa is correct Max 3 / 5 If the candidate only gives Sequence is $1 ; 1 ; 1 ; 1 ; 1 ; 1 ; \dots$ $\qquad\qquad$ then 2 / 5 If $ar^{n-1} = a + (n-1)d$ only $\quad$ then 1 / 5	✓ $1 + d = r$ ✓ $1 + 2d = r^2$ ✓ $r = 1$ ✓ $d = 0$ ✓ reason (5) ✓ Setting up sequence ✓ $a^2 = b$ ✓ $b = 2a - 1$ ✓ $a = 1$ ✓ $b = 1$ (5) **[10]**

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QUESTION 3

3.1	$-1 + 2 + 5 + ...$ **OR** $-1 \; ; 2 \; ; 5$	✓ all three terms (1)
3.2	$S_n = -1 + 2 + 5 + 8 + ...$ to 100 terms $S_n = \dfrac{n}{2}\big[2a + (n-1)d\big]$ $S_{100} = \dfrac{100}{2}\big[2(-1) + (100-1)(3)\big]$ Answer only: $4/4$ $\quad = 50[-2 + 297]$ $\quad = 14\,750$ **OR** $S_n = -1 + 2 + 5 + 8 + ...$ to 100 terms $T_{100} = 3(100) - 4$ $\quad = 296$ $S_n = \dfrac{n}{2}\big[T_1 + T_{100}\big]$ $S_{100} = \dfrac{100}{2}\big[-1 + 296\big]$ $\quad = 50[\,295\,]$ $\quad = 14\,750$ **NOTE**: If $S_n = -1 + 2 + 5 + 8 + ...$ to 99 terms $S_n = \dfrac{n}{2}\big[2a + (n-1)d\big]$ $S_{99} = \dfrac{99}{2}\big[2(-1) + (99-1)(3)\big]$ $\quad = \dfrac{99}{2}[-2 + 294]$ $\quad = 14454$ **Then 3 / 4** Apply consistent accuracy. This is the answer if series is $2 + 5 + 8 +$ $S_n = 2 + 5 + 8 + ...$ to 100 terms $S_n = \dfrac{n}{2}\big[2a + (n-1)d\big]$ $S_{100} = \dfrac{100}{2}\big[2(2) + (100-1)(3)\big]$ $\quad = 50[4 + 297]$ $\quad = 15050$ **Then 4 / 4**	✓ formula ✓ $n = 100$ ✓ substitution ✓ answer (4) **[5]**

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QUESTION 4

4.1	The first differences are 1; -1; -3; -5; ….	✓ pattern
	These form a linear pattern $T_n = 1 + (n-1)(-2)$ $= 3 - 2n$ **OR** $T_n = -2n + 3$ **ANSWER ONLY:** Full marks	✓ $d = -2$ ✓ answer (3)
4.2	Between the 35th and 36th terms of the quadratic sequence lies the 35th first difference 35th first difference $= 3 - 2(35)$ $= -67$ **OR** From the quadratic sequence: $P_{36} = -1158$ and $P_{35} = -1091$ 35th first difference $= -1158 - (-1091)$ $= -67$ ⬜ If substitute and get $T_{35} = -2(35) + 3 = -67$ and $T_{36} = -2(36) + 3 = -69$, leading to the answer -2 then **1 / 2**	✓ substitution of 35 into $T_n = -2n + 3$ ✓ answer (2) ✓ $P_{36} = -1158$ and $P_{35} = -1091$ ✓ answer (2)
4.3	Second difference of terms is -2. $P_n = an^2 + bn + c$ $a = -1$. $3a + b = 1$ $-3 + b = 1$ $b = 4$ $a + b + c = -3$ $-1 + 4 + c = -3$ $c = -6$ $P_n = -n^2 + 4n - 6$ ⬜ If the general term has been worked out correctly in 4.2 and not redone in 4.3 but answer just written down then **4 / 4** **OR**	✓ $a = -1$ ✓ substitution ✓ $b = 4$ ✓ $c = -6$ (4)

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| 4.3 contd | Second difference of terms is -2. $$P_n = an^2 + bn + c$$ $$a = -1.$$ $$P_0 = -6 = c$$ $$P_n = -n^2 + bn - 6$$ $$-3 = -(1)^2 + (1)b - 6$$ $$b = 4$$ $$P_n = -n^2 + 4n - 6$$ **OR** $$P_n = \frac{n-1}{2}\big[2(\text{first first difference}) + (n-2)(\text{second difference})\big] + P_1$$ $$P_n = \frac{n-1}{2}[2(1) + (n-2)(-2)] - 3$$ $$P_n = n - 1 - (n-2)(n-1) - 3$$ $$P_n = n - 1 - n^2 + 3n - 2 - 3$$ $$P_n = -n^2 + 4n - 6$$ **OR** $$P_n = (n-1)P_2 - (n-2)P_1 + 2nd \ difference \frac{(n-1)(n-2)}{2}$$ $$P_n = (n-1)(-2) - (n-2)(-3) - 2\frac{(n-1)(n-2)}{2}$$ $$P_n = -2n + 2 + 3n - 6 - n^2 + 3n - 2$$ $$P_n = -n^2 + 4n - 6$$ **OR** $$P_n = \frac{(n-2)(n-3)T_1 - 2(n-1)(n-3)T_2 + (n-2)(n-1)T_3}{2}$$ $$P_n = \frac{(n^2 - 5n + 6)(-3) - 2(n^2 - 4n + 3)(-2) + (n^2 - 3n + 2)(-3)}{2}$$ $$P_n = \frac{-3n^2 + 15n - 18 + 4n^2 - 16n + 12 - 3n^2 + 9n - 6}{2}$$ $$P_n = -n^2 + 4n - 6$$ **OR** | |

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4.3 contd	$P_2 - P_1 = T_1$ $P_3 - P_2 = T_2$ $P_4 - P_3 = T_3$ $P_n - P_{n-1} = T_{n-1}$ $P_n - P_1 = T_1 + T_2 + ... + T_{n-1}$ $P_n - P_1 = \dfrac{n-1}{2}\big[2(1) + (n-2)(-2)\big]$ $P_n - (-3) = (n-1)(3-n)$ $P_n = -n^2 + 4n - 6$	
4.4	Maximum value of T_n is $\dfrac{4(-1)(-6) - 4^2}{4(-1)} = -2$ The maximum value is negative and hence the sequence can not have any positive terms as the function is maximum valued **OR** $-n^2 + 4n - 6$ $= -(n-2)^2 + 4 - 6$ $= -(n-2)^2 - 2$ The function has a maximum-value of -2 and therefore the pattern will never have positive values. **OR** $T_n = -n^2 + 4n - 6$ $\dfrac{d}{dn}(T_n) = -2n + 4$ $0 = -2n + 4$ $n = 2$ $T_2 = -(2)^2 + 4(2) - 6$ $= -2$ The function has a maximum-value of -2 and therefore the pattern will never have positive values. **OR** As the sequence decreases from the second term onwards and the second term is negative, the sequence will never have a positive term. **OR** $T_n = -n^2 + 4n - 6$ $\dfrac{d}{dn}(T_n) = -2n + 4$ $\dfrac{d}{dn}(T_n) < 0$ for $n > 2$ and $T_2 < 0$ so the sequence decreases and stays negative	✓ max value – 2 ✓ explanation (2) ✓ max value – 2 ✓ explanation (2) ✓ max value – 2 ✓ explanation (2) ✓✓ answer (2) ✓✓ answer (2) **[11]**

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QUESTION 5

5.1	First year: 150	
	Second year: $150 + 18 = 168$	
	Third year: $168 + \dfrac{8}{9}(18) = 184$	
	Growth $= 18\left(\dfrac{8}{9}\right)^{n-2}$ after n years	✓ general terms
	17th year growth is $18\left(\dfrac{8}{9}\right)^{17-2} = 3,08$ cm	✓ answer (2)

	Yr 1	**Yr 2**	**Yr 3**	**Yr 4**	**Yr 5**	**Yr 6**	**Yr 7**	**Yr 8**	**Yr 9**
Ht	150	168	184	198,2	210,84	222,07	232,06	240,94	248,83
Inc		18	16	14,2	12,64	11,23	9,99	8,88	7,89
	Yr 10	**Yr 11**	**Yr 12**	**Yr 13**	**Yr 14**	**Yr 15**	**Yr 16**	**Yr 17**	
Ht	255,84	262,08	267,62	272,55	276,93	280,82	284,28	287,36	
Inc	7,01	6,24	5,54	4,93	4,38	3,89	3,46	3,08	

5.2	Height after 10 years	
	$=150 + \dfrac{18\left(1-\left(\dfrac{8}{9}\right)^{9}\right)}{1-\dfrac{8}{9}}$	✓ $n = 9$ ✓ substitution into sum formula
	NOTE: By writing out 9 terms and adding to 150 and answer correct, full marks. Answer only: 2 / 3	
	$= 150 + 105,8768146 \ldots$	
	$= 255,88$ cm	
	OR	
	$=150 + \dfrac{18\left(\left(\dfrac{8}{9}\right)^{9}-1\right)}{\dfrac{8}{9}-1}$	✓ answer (3)
	$= 150 + 105,8768146 \ldots$	
	$= 255,88$ cm	

5.3	Max height $= 150 +$ sum to infinity	✓ statement
	$= 150 + \dfrac{18}{1-\dfrac{8}{9}}$	✓ substitution into the sum to infinity formula
	$= 150$ cm $+ 162$ cm	✓ max height
	$= 312$ cm	(3)
	The tree will never reach a height of more than 312 cm.	**[8]**

NOTE:

If a candidate answers in 5.1 that the growth is $18\left(\dfrac{8}{9}\right)^{n-1} = 18\left(\dfrac{8}{9}\right)^{16} = 2,73$ cm then 1 / 2

The answer for 5.2 as continued accuracy uses $n = 10$,
Height after 10 years

$=150 + \dfrac{18\left(1-\left(\dfrac{8}{9}\right)^{10}\right)}{1-\dfrac{8}{9}} = 150 + 112,11 \ldots = 262,11$ cm

This is awarded 3/3 as consistent accuracy

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QUESTION 6

6.1	$\dfrac{1}{2}x^2 = -\dfrac{1}{x+1}+1$ $x^2(x+1) = -2 + 2(x+1)$ $x^3 + x^2 = -2 + 2x + 2$ $x^3 + x^2 - 2x = 0$ $x(x^2 + x - 2) = 0$ $x(x+2)(x-1) = 0$ $x = 0 \quad or \quad x = -2 \quad or \quad x = 1$ $y = 0 \quad or \quad y = \dfrac{1}{2}(-2)^2 \quad or \quad y = \dfrac{1}{2}(1)^2$ $\qquad y = 2 \qquad or \quad y = \dfrac{1}{2}$ $P(-2\,;2)$ $Q\left(1\,;\dfrac{1}{2}\right)$ **OR** $\dfrac{1}{2}(-2)^2 = 2 \quad \therefore \ (-2\,;2)$ lies on $f(x) = \dfrac{1}{2}x^2$ $-\dfrac{1}{(-2)+1}+1 = 2 \quad \therefore \ (-2\,;2)$ lies on $g(x) = -\dfrac{1}{x+1}+1$ $\therefore \ (-2\,;2)$ is one of the points P, O or Q. From the graph it is P $\dfrac{1}{2}(1)^2 = \dfrac{1}{2} \quad \therefore \ (-2\,;2)$ lies on $f(x) = \dfrac{1}{2}x^2 \therefore \left(1\,;\dfrac{1}{2}\right)$ is one of the points P, O or Q. From the graph it is Q $-\dfrac{1}{(1)+1}+1 = \dfrac{1}{2} \quad \therefore \ Q$ lies on $g(x) = -\dfrac{1}{x+1}+1$ $\therefore \left(1\,;\dfrac{1}{2}\right)$ is one of the points P, O or Q. From the graph it is Q	✓ equating ✓ multiplication by LCD ✓ standard form ✓ common factor ✓ factorisation of quadratic ✓ *y*-answer answer P(– 2 ; 2) answer $Q\left(1\,;\dfrac{1}{2}\right)$ (6) ✓ substitution ✓ substitution ✓ P lies on *f* and *g* ✓ substitution ✓ substitution ✓ Q lies on *f* and *g* (6)
6.2	For $m > 0$, $m = 1$ the equation of the axis of symmetry is $y = x + c$. $1 = (-1) + c$ $c = 2$ Therefore the equation is $y = h(x) = x + 2$.	✓ gradient $m = 1$ ✓ $c = 2$ (2)

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

6.3	The equation of the inverse of h is $x = y + 2$ $\therefore y = x - 2$ Answer only: Full marks	✓interchange x and y ✓ answer (2)
6.4	$g(x) = -\dfrac{1}{x+1} + 1 = \dfrac{-1+x+1}{x+1} = \dfrac{x}{x+1}$ $LHS = \dfrac{x}{x+1} + \dfrac{\dfrac{1}{x}}{\dfrac{1}{x}+1}$ $RHS = \left(\dfrac{-x}{1-x}\right)\left(\dfrac{x-1}{(x-1)+1}\right)$ $= \dfrac{x}{x+1} + \dfrac{1}{x+1}$ $= \dfrac{(1-x)x}{(1-x)x}$ $= \dfrac{x+1}{x+1}$ $= 1$ $= 1$ LHS = RHS **NOTE**: If substitute a value of x and prove it, then 0 / 3 **OR** $LHS = g(x) + g\left(\dfrac{1}{x}\right)$ $RHS = g(-x).g(x-1)$ $= -\dfrac{1}{x+1} + 1 - \dfrac{1}{\dfrac{1}{x}+1} + 1$ $= \left(-\dfrac{1}{-x+1}+1\right)\left(-\dfrac{1}{x-1+1}+1\right)$ $= -\dfrac{1}{x+1} + 2 - \dfrac{x}{1+x}$ $= \left(\dfrac{-1+1-x}{1-x}\right)\left(\dfrac{-1+x}{x}\right)$ $= -\dfrac{1+x}{1+x} + 2$ $= \left(\dfrac{-x}{1-x}\right)\left(\dfrac{x-1}{x}\right)$ $= -1 + 2$ $= \left(\dfrac{x}{x-1}\right)\left(\dfrac{x-1}{x}\right)$ $= 1$ $= 1$ LHS = RHS	✓ simplification of $g(x)$ ✓simplification of LHS ✓simplification of RHS (3) ✓ 2 substitutions correct. **NOTE**: not just rewriting $g(x)$ again ✓ simplification of LHS ✓simplification of RHS (3) **[13]**

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QUESTION 7

7.1	$y \in \left[-3\,;3\right]$ **OR** $-3 \le y \le 3$ **OR** y can be any value from -3 to 3 **NOTE**: Notation incorrect : 0 / 1	✓ answer (1)
7.2	x–value is 7,37° to the left of 90° B(82,63° ; 0,38) **NOTE**: Answer only : 3 / 3 x-value correct and y-value incorrect : 2 / 3 x-value incorrect and y-value correct : 1 / 3 If decimal part incorrect of x and y-value correct: 2 / 3	✓ method ✓ x-value ✓ y-value (3)
7.3	Period $= \dfrac{360°}{3}$ $\qquad\quad = 120°$ **NOTE**: Answer only : 2 / 2	✓ $\dfrac{360°}{3}$ ✓ answer (2)
7.4	$x = -180°$	✓ ✓ answer (2) **[8]**

QUESTION 8

8.1	$x > 0$ **OR** $x \in (0;\infty)$	✓ answer (1)
8.2	$y = 2^{-x}$ **OR** $y = \left(\dfrac{1}{2}\right)^{x}$	✓ answer (1)
8.3	$y = 0$	✓ answer (1)
8.4.1	Reflect the graph of f over the x-axis OR For each point the y-coordinate changes sign. **NOTE**: Reflect only : 0 / 1	✓ answer (1)
8.4.2	Reflect the graph of f over the line $y = x$. Then shift the graph down 5 units	✓✓ answer ✓ answer (3)

- Consistent Accuracy will apply as a general rule.
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8.4.2 contd	**OR** Sketch the graph of the inverse of f. Shift the graph of the inverse of f down by 5 units. **OR** Shift the graph 5 units LEFT. Reflect the graph over the line $y = x$.		
8.5	$\log_2 x < 3$ $-\log_2 x > -3$ For $-\log_2 x = -3$ $\quad\quad 2^3 = x$ $\quad\quad x = 8$ $f(x) > -3$ $0 < x < 8 \quad$ or $\quad x \in (0\,;\,8)$	**NOTE:** Notation incorrect: Answer $x < 8$: 2 / 3 Answer only correct: 3 / 3	✓ multiplication by -1 ✓ Notation ✓ critical values (3) **[10]**

QUESTION 9

Penalise ONCE in question 9 for early rounding off.

9.1	$A = P(1 - i)^n$ $15000 = 24000(1 - 0,18)^n$ $0,625 = (0,82)^n$ $n = \dfrac{\log 0,625}{\log 0,82}$ $= 2,37$ years	**NOTE:** If subs A and P incorrectly: Answer would be $n = -2,37$ years $\therefore n = 2,37$ years: 2 / 4 If subs A and P incorrectly: Answer would be $n = -2,37$ years : 1 / 4 Answer $n = 2,4$ years 4 / 4 Answer rounded to 3 years and all calculations shown and $n = 2,37$ shown: 4/4 Answer rounded to 3 years and $n = 2,37$ not shown: 3 / 4	✓ substitution ✓ simplification ✓ application of logs ✓ answer (4) Incorrect formula: 0/4
9.2.1	$130\,000\left(1 + \dfrac{0,18}{12}\right)^2$ $= 130000(1,015)^2$ $= R\ 133\ 929,25$	**NOTE:** -1 per error for incorrect substitution to a max of 2 marks	✓✓ substitution ✓ answer (3) Incorrect formula: 0/3

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9.2.2(a)	$133\,929,25 = \dfrac{x[1-(1,015)^{-54}]}{0,015}$ $2008,93875 = x[1-(1,015)^{-54}]$ $\qquad x = \text{R }3636,36$ **OR** $133\,929,25\left(1+\dfrac{0,18}{12}\right)^{54} = \dfrac{x\left[\left(1+\dfrac{0,18}{12}\right)^{54}-1\right]}{\dfrac{0,18}{12}}$ $299255,2087 = 82,29517136...x$ $\qquad x = \text{R }3636,36$ **OR** $130000\left(1+\dfrac{0,18}{12}\right)^{56} = \dfrac{x\left[\left(1+\dfrac{0,18}{12}\right)^{54}-1\right]}{\dfrac{0,18}{12}}$ $299255,2087 = 82,29517136...x$ $\qquad x = \text{R }3636,36$	✓ $n = 54$ ✓ substitution of 133 929,25 ✓ answer (3) ✓ $n = 54$ ✓ substitution of 133 929,25 ✓ answer (3) ✓ $n = 54$ ✓ $130000\left(1+\dfrac{0,18}{12}\right)^{56}$ ✓ answer (3)
9.2.2(b)	$\text{Total} = 3636,36 \times 54$ $\qquad = \text{R}196\,363,66$ **NOTE:** Accept answer = R 196	✓answer (1)
9.2.3	$130\,000 = \dfrac{x[1-(1,015)^{-54}]}{0,015}$ $\quad 1950 = x[1-(1,015)^{-54}]$ $\qquad x = \text{R }3529,68$ $\text{Total payments} = \text{R }3529,68 \times 54$ $\qquad\qquad\qquad = \text{R }190\,602,72$ **OR**	✓ 130 000 ✓ $i = 0,015$ ✓ answer 3529,68 ✓ answer R 190 602,72 (4)

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

| 9.2.3 contd | $130\,000\left(1+\dfrac{0,18}{12}\right)^{54} = \dfrac{x\left[\left(1+\dfrac{0,18}{12}\right)^{54}-1\right]}{\dfrac{0,18}{12}}$

 $290475,5842 = 82,29517136...x$
 $x = \text{R }3529,68$
 Total payments $= \text{R }3529,68 \times 54$
 $\qquad\qquad\qquad = \text{R }190\,602,72$

 OR

 $130\,000\left(1+\dfrac{0,18}{12}\right)^{55} = \dfrac{x\left(1+\dfrac{0,18}{12}\right)\left[\left(1+\dfrac{0,18}{12}\right)^{54}-1\right]}{\dfrac{0,18}{12}}$

 $290475,5842 = 82,29517136...x$
 $x = \text{R }3529,68$
 Total payments $= \text{R }3529,68 \times 54$
 $\qquad\qquad\qquad = \text{R }190\,602,72$ **NOTE:** Disregard the cents values if they are incorrect. | ✓ $130\,000\left(1+\dfrac{0,18}{12}\right)^{54}$
 ✓ $i = 0,015$
 ✓ answer $3529,68$

 ✓ answer
 R $190\,602,72$
 (4)

 ✓ $130\,000\left(1+\dfrac{0,18}{12}\right)^{55}$
 ✓ $i = 0,015$
 ✓ answer $3529,68$

 ✓ answer
 R $190\,602,72$
 (4) |
| 9.2.4 | R196 363,66 – R190 602,72
 =R5 760,96 | ✓ answer
 (1)
 [16] |

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 10

| 10.1 | $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ $$= \lim_{h \to 0} \frac{-2(x+h)^2 + 3 - (-2x^2 + 3)}{h}$$ $$= \lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 + 3 + 2x^2 - 3}{h}$$ $$= \lim_{h \to 0} \frac{h(-4x - 2h)}{h}$$ $$= \lim_{h \to 0}(-4x - 2h)$$ $$= -4x$$ **NOTE:** Penalty 1 mark only for incorrect notation (lim missing or = in incorrect place) Answer only : 0 / 5 Cannot give mark for answer if the answer is incorrect according to the working out, even if the answer is given as – 4x. | ✓ $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ ✓ $-2(x+h)^2 + 3$ ✓ simplification ✓ simplification ✓ answer (5) |
| 10.2 | $$y = x^2 - \frac{1}{2x^3}$$ $$y = x^2 - \frac{1}{2}x^{-3}$$ $$\frac{dy}{dx} = 2x + \frac{3}{2}x^{-4}$$ **OR** $$\frac{dy}{dx} = 2x + \frac{3}{2x^4}$$ **OR** $$\frac{dy}{dx} = 2x - (-3)\frac{1}{2}x^{-4}$$ | ✓ $2x$ ✓ $+\dfrac{3}{2}x^{-4}$ (2) **[7]** |

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 11

11.1	$0 = -x^3 + x^2 + 8x - 12$	✓ any one of factors
	$x^3 - x^2 - 8x + 12 = 0$	✓ quadratic factor
	$(x-2)(x^2 + x - 6) = 0$	✓ linear factors
	$(x-2)(x-2)(x+3) = 0$	✓✓ x-answers
	$x = 2$ or $x = -3$	(5)
	x-intercepts are $(2 ; 0)$ and $(-3 ; 0)$	
	OR	
	$0 = -x^3 + x^2 + 8x - 12$	
	$x^3 - x^2 - 8x + 12 = 0$	
	$(x+3)(x^2 - 4x + 4) = 0$	
	$(x+3)(x-2)(x-2) = 0$	
	$x = 2$ or $x = -3$	
	x-intercepts are $(2 ; 0)$ and $(-3 ; 0)$	
11.2	$f'(x) = -3x^2 + 2x + 8$	✓ $f'(x) = 0$
	$\quad 0 = 3x^2 - 2x - 8$	✓ $-3x^2 + 2x + 8 = 0$ or $3x^2 - 2x - 8 = 0$
	$\quad 0 = (x-2)(3x+4)$	✓ factors
	$x = 2$ or $x = -\dfrac{4}{3}$	✓ x-values
	turning points are $(2 ; 0)$ and $\left(-\dfrac{4}{3} ; -\dfrac{500}{27}\right)$	✓ y-values
	OR $\quad\quad\quad (2 ; 0)$ and $(-1,33 ; -18,52)$	(5)
	NOTE: If $= 0$ is omitted in 11.2: penalty 1 mark If not in coordinate form but coordinates implied: OK	

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

11.3		If candidate used function as $f(x) = x^3 - x^2 - 8x + 12$ then max 1 / 3 ✓ shape ✓ y-intercept ✓ turning pts (3)

The graph shows turning point $\left(-\dfrac{4}{3}; -\dfrac{500}{27}\right)$

11.4	$f''(x) = 0$ $\qquad\qquad$ $f''(x) = 0$ $6x - 2 = 0$ $\quad$ or $\quad$ $-6x + 2 = 0$ $\qquad x = \dfrac{1}{3}$ $\qquad\qquad\qquad x = \dfrac{1}{3}$ **OR** $x = \dfrac{2 - \dfrac{4}{3}}{2}$ $x = \dfrac{1}{3}$ **Note:** If write down $f''(x) = 6x - 2$ or $f''(x) = -6x + 2$ then 1 / 2	✓ method ✓ answer Answer only: Full marks (2)

11.5	$(2 \,;\, -3)$ and $\left(-\dfrac{4}{3} \,;\, -\dfrac{581}{27}\right)$ **OR** $(2 \,;\, -3)$ and $(-1{,}33 \,;\, -21{,}52)$	✓✓ each answer (2) **[17]**

- Consistent Accuracy will apply as a general rule.
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- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 12

12.1	$s(0) = 5(0)^3 - 65(0)^2 + 200(0) + 100$ $\qquad = 100$ metres **NOTE:** If subs $t = 8$, then answer $= 100$: $0/2$	✓ $t = 0$ ✓ answer $\qquad$ (2) Answer only: full marks
12.2	$s(t) = 5t^3 - 65t^2 + 200t + 100$ $s'(t) = 15t^2 - 130t + 200$ $s'(4) = 15(4)^2 - 130(4) + 200$ $\qquad = -80$ metres per minute **NOTE:** If used average rate of change between $t = 0$ and $t = 4$: $0/3$ If subs $t = 4$ into $s(t)$: $0/3$	✓ $s'(t) = 15t^2 - 130t + 200$ ✓ substitution $t = 4$ ✓ answer (-80) $\qquad$ (3)
12.3	The height of the car above sea level is decreasing at 80 metres per minute and the car is travelling downwards hence it is a negative rate of change. **OR** The **<u>vertical</u>** velocity of the car at $t = 4$ is 80 metres per minute. **NOTE:** Mark this CA even if answer to QUESTION 12.2 is completely inaccurate.	✓ speed 80 metres per minute ✓ downwards $\qquad$ (2)
12.4	$s'(t) = 15t^2 - 130t + 200$ $s''(t) = 30t - 130$ $\qquad 130 = 30t$ $\qquad\quad t = 4,3\dot{3}$ minutes **OR** $t = \dfrac{-(-130)}{2(15)}$ $t = 4,3\dot{3}$ minutes	✓ $s''(t) = 30t - 130$ ✓ $s''(t) = 0$ ✓ answer $\qquad$ (3) **[10]**

- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 13

13.1	$x + 3y \leq 18$ $x + y \leq 8$ $2x + y \leq 14$ $x, y \geq 0$ **OR** $6x + 18y \leq 108$ $8x + 8y \leq 64$ $14x + 7y \leq 98$ $x, y \geq 0$ **OR** $y \leq -\dfrac{1}{3}x + 6$ $y \leq -x + 8$ $y \leq -2x + 14$ $x, y \geq 0$ **NOTE**: If written as equations (inequality omitted): max 6 / 7 If inequalities sign the wrong way round: max 6 / 7 One should note that x and y should be counting numbers	✓✓ answer ✓✓ answer ✓✓ answer ✓ answer (7)
13.2	$P = 30x + 40y$ **NOTE**: If $P = 30A + 40B$ then 1 / 2	✓✓ answer (2)
13.3	$y = -\dfrac{3}{4}x + \dfrac{P}{40}$ **NOTE**: Please check diagram Maximum at (3 ; 5)	✓✓ answer (2) Answer only : Full marks
13.4	$-2 < m < -1$ **NOTE**: accept $1 < m < 2$: 2 / 2. If $\leq$ signs used then max 1 / 2	✓✓ answer (2) **[13]**

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