



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2013

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages, 3 diagram sheets and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Diagram sheets for QUESTION 1.1, QUESTION 3.1 and QUESTION 10.3 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert the sheets inside the back cover of your ANSWER BOOK.
9. An information sheet, with formulae, is included at the end of this question paper.
10. Number the answers correctly according to the numbering system used in this question paper.
11. Write neatly and legibly.

QUESTION 1

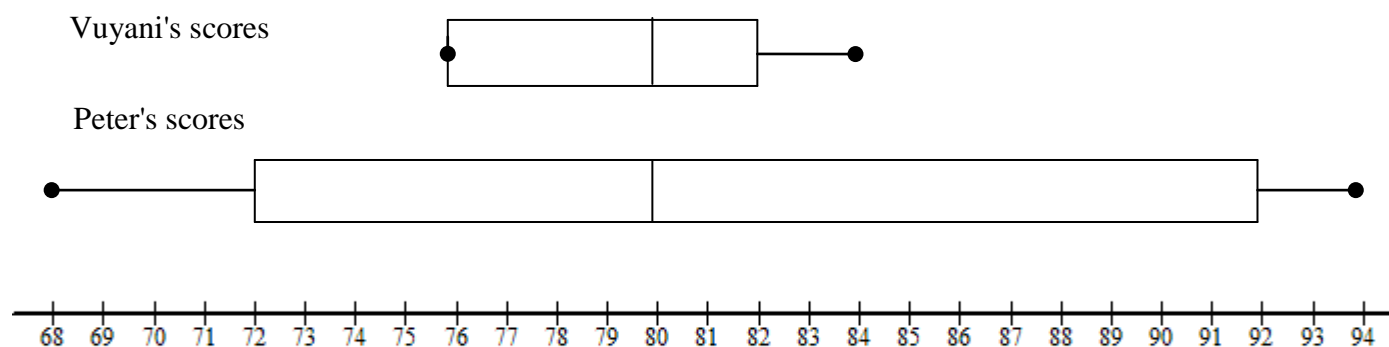
The table below gives the average rand/dollar exchange rate and the average monthly oil price for the year 2010.

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Exchange rate in R/\$	7,5	7,7	7,2	7,4	7,7	7,7	7,6	7,3	7,1	7,0	6,9	6,8
Oil price in \$	69,9	68,0	72,9	70,3	66,3	67,1	67,9	68,3	71,3	73,6	76,0	81,0

- 1.1 Draw a scatter plot on DIAGRAM SHEET 1 to represent the exchange rate (in R/\$) versus the oil price (in \$). (3)
 - 1.2 Describe the relationship between the exchange rate (in R/\$) and the oil price (in \$). (2)
 - 1.3 Determine the mean oil price. (2)
 - 1.4 Determine the standard deviation of the oil price. (2)
 - 1.5 Generally there is concern from the public when the oil price is higher than two standard deviations from the mean. In which month(s) would there have been concerns from the public? (2)
- [11]**

QUESTION 2

The box-and-whisker diagrams below represent Vuyani and Peter's scores for their School-based Assessment Tasks in a certain subject throughout the year.



- 2.1 Give the range of Peter's scores. (2)
 - 2.2 Give the minimum of Vuyani's scores. (1)
 - 2.3 Comment on who you think had a more consistent performance throughout the year. Motivate your answer by referring to values in the box-and-whisker diagrams. (2)
- [5]**

QUESTION 3

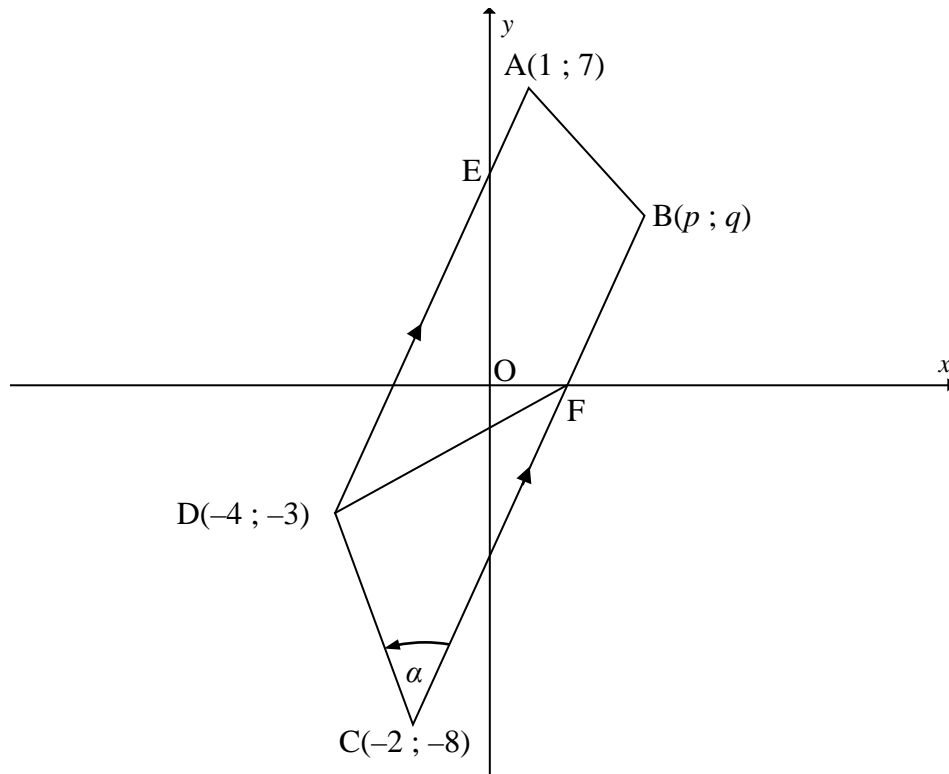
The average percentage of 150 learners for all their subjects is summarised in the cumulative frequency table below.

PERCENTAGE INTERVAL	CUMULATIVE FREQUENCY
$x \leq 10$	5
$x \leq 20$	21
$x \leq 30$	50
$x \leq 40$	70
$x \leq 50$	88
$x \leq 60$	110
$x \leq 70$	135
$x \leq 80$	142
$x \leq 90$	147
$x \leq 100$	150

- 3.1 Draw the ogive (cumulative frequency graph) representing the above data on DIAGRAM SHEET 2. (4)
- 3.2 Use the ogive to approximate the following:
- 3.2.1 The number of learners who scored less than 85% (2)
- 3.2.2 The interquartile range (Show ALL calculations.) (3)
- [9]**

QUESTION 4

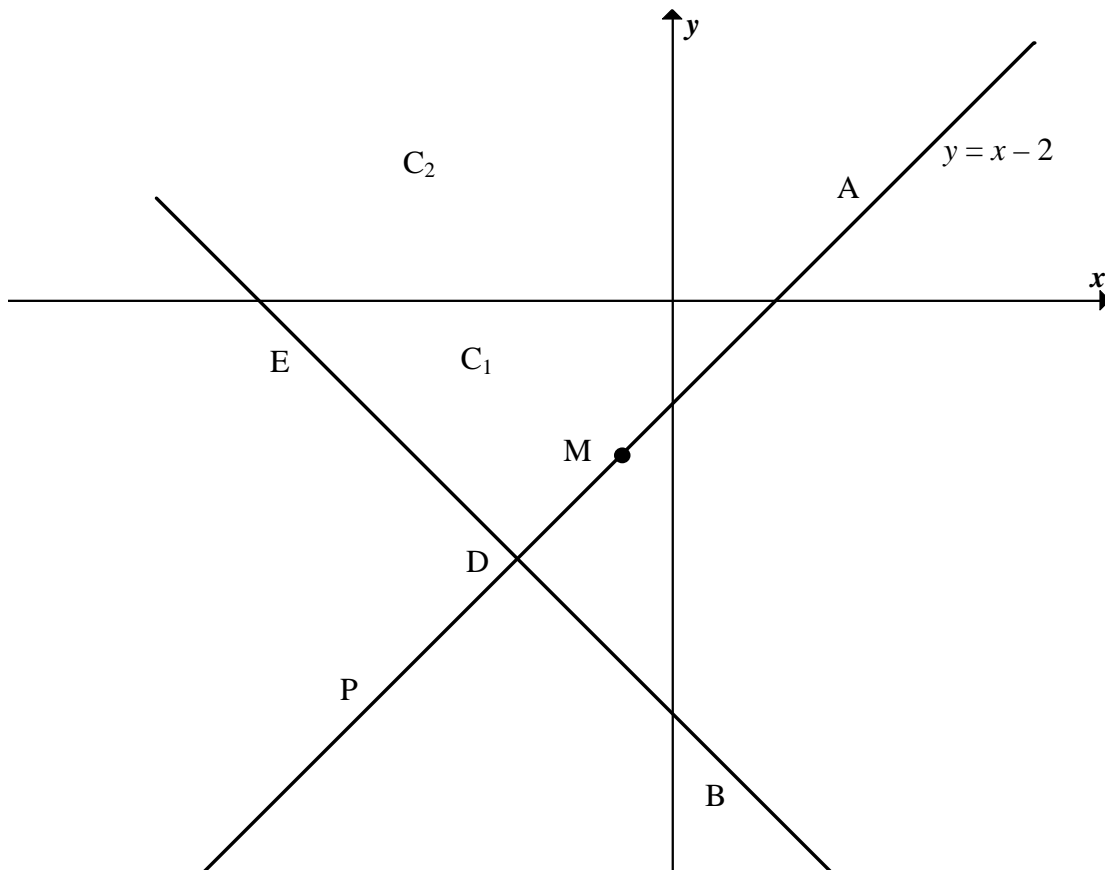
In the diagram below, trapezium ABCD with $AD \parallel BC$ is drawn. The coordinates of the vertices are $A(1 ; 7)$; $B(p ; q)$; $C(-2 ; -8)$ and $D(-4 ; -3)$. BC intersects the x -axis at F. $\widehat{DCB} = \alpha$.



- 4.1 Calculate the gradient of AD. (2)
- 4.2 Determine the equation of BC in the form $y = mx + c$. (3)
- 4.3 Determine the coordinates of point F. (2)
- 4.4 $AB'CD$ is a parallelogram with B' on BC. Determine the coordinates of B' , using a transformation $(x ; y) \rightarrow (x + a ; y + b)$ that sends A to B' . (2)
- 4.5 Show that $\alpha = 48,37^\circ$. (4)
- 4.6 Calculate the area of $\triangle DCF$. (6)
- [19]**

QUESTION 5

Circles C_1 and C_2 in the figure below have the same centre M . P is a point on C_2 . PM intersects C_1 at D . The tangent DB to C_1 intersects C_2 at B . The equation of circle C_1 is given by $x^2 + 2x + y^2 + 6y + 2 = 0$ and the equation of line PM is $y = x - 2$.



- 5.1 Determine the following:
 - 5.1.1 The coordinates of centre M (3)
 - 5.1.2 The radius of circle C_1 (1)
- 5.2 Determine the coordinates of D , the point where line PM and circle C_1 intersect. (5)
- 5.3 If it is given that $DB = 4\sqrt{2}$, determine MB , the radius of circle C_2 . (3)
- 5.4 Write down the equation of C_2 in the form $(x - a)^2 + (y - b)^2 = r^2$. (2)
- 5.5 Is the point $F(2\sqrt{5}; 0)$ inside circle C_2 ? Support your answer with calculations. (4)

[18]

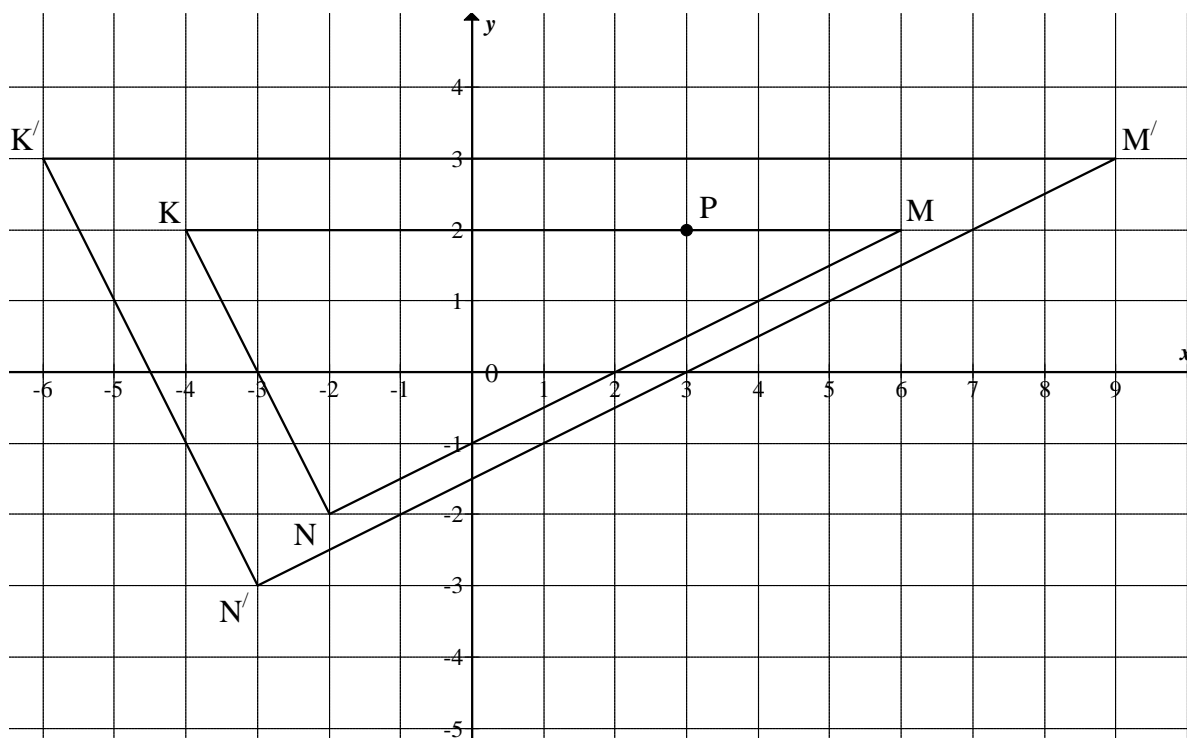
QUESTION 6

6.1 Write down the coordinates of the image of point A(−5 ; 3) after it has undergone the following transformations:

6.1.1 Translation of 3 units downwards and 4 units to the right (2)

6.1.2 Reflection about the x -axis (2)

6.2 Consider the following diagram:



6.2.1 In the above diagram, triangle KMN is enlarged by a certain factor to form triangle $K'M'N'$. Determine the factor of enlargement. (2)

6.2.2 Give the general rule for the transformation in QUESTION 6.2.1. (2)

6.2.3 Use the answer to QUESTION 6.2.2 to determine the image P' of $P(3 ; 2)$. (2)

6.2.4 M is the reflection of K about the line with equation $x = a$. Determine the value of the constant a . (2)

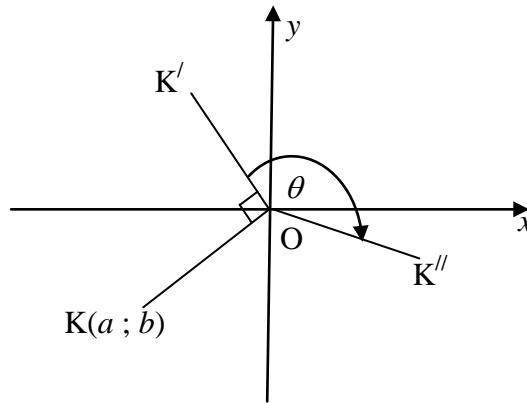
6.2.5 ΔKMN is rotated 180° about the origin to form $\Delta K''M''N''$. Give the coordinates of K'' . (2)

6.2.6 ΔKMN is translated 3 units to the right and 1 unit upwards to obtain $\Delta K'''M'''N'''$. Write down the ratio of $\frac{K'K'''}{K'M'''}$ after the translation. (3)

[17]

QUESTION 7

In the diagram below, point $K(a ; b)$ is rotated clockwise through an angle of 90° about the origin to K' and then rotated clockwise through an angle θ to K'' .



- 7.1 Write down the coordinates of point K' in terms of a and b . (2)
- 7.2 Write down the coordinates of K'' in terms of a , b , $\sin\theta$ and $\cos\theta$. Simplify if necessary. (2)
- 7.3 $T(-4 ; -2)$ is rotated clockwise through an angle of $(90^\circ + \theta)$ about the origin to obtain image T' . Determine, in the simplest form, the coordinates of T' in terms of θ . (2)
- 7.4 Hence, or otherwise, calculate the size of θ if it is given that $T'(2\sqrt{3} + 1 ; \sqrt{3} - 2)$ and $90^\circ < \theta < 180^\circ$. (5)
- [11]**

QUESTION 8

8.1 Simplify as far as possible: $1 - \sin^2 \theta + 3 - \cos^2 \theta$ (2)

8.2 Simplify WITHOUT the use of a calculator: $\sqrt{4^{\sin 150^\circ} \times 2^{3 \tan 225^\circ}}$ (4)

8.3 Prove that $\frac{\cos^2 x \sin^2 x + \cos^4 x}{1 - \sin x} = 1 + \sin x$ (4)

8.4 Prove that for any angle θ , $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$.
(Hint: $3\theta = \theta + 2\theta$) (4)

8.5 If $x = \cos 20^\circ$, use QUESTION 8.4 to show that $8x^3 - 6x - 1 = 0$. (2)
[16]

QUESTION 9

9.1 Simplify to ONE trigonometric function WITHOUT using a calculator:

$$\frac{\cos 160^\circ \tan 200^\circ}{2 \sin(-10^\circ)}$$
 (6)

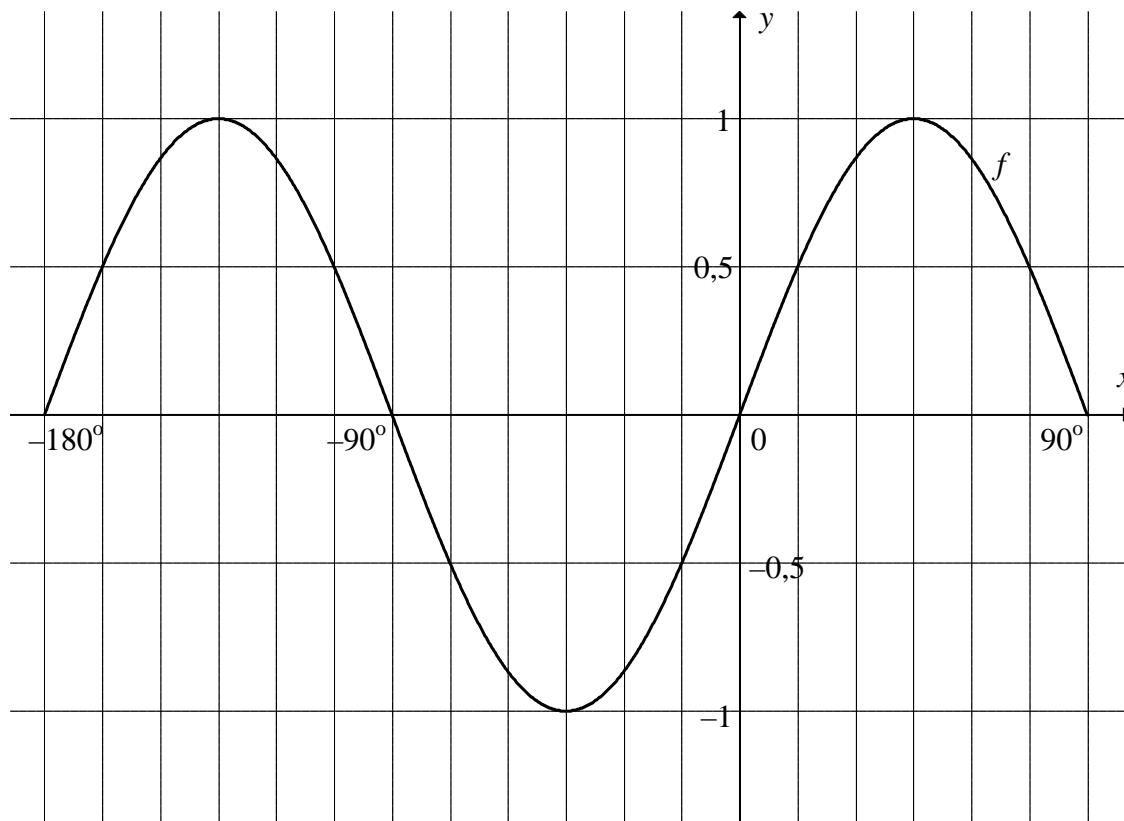
9.2 Consider $\cos(x + 45^\circ) \cos(x - 45^\circ)$.

9.2.1 Show that $\cos(x + 45^\circ) \cos(x - 45^\circ) = \frac{1}{2} \cos 2x$. (4)

9.2.2 Hence, determine a value of x in the interval $0^\circ \leq x \leq 180^\circ$ for which $\cos(x + 45^\circ) \cos(x - 45^\circ)$ is a minimum. (3)
[13]

QUESTION 10

The graph of $f(x) = \sin 2x$ for $-180^\circ \leq x \leq 90^\circ$ is shown in the sketch below.

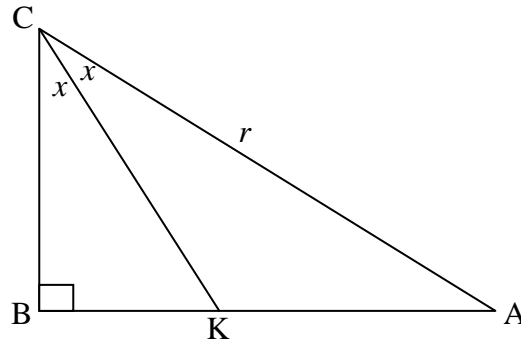


- 10.1 Write down the range of f . (2)
- 10.2 Determine the period of $f\left(\frac{3}{2}x\right)$. (2)
- 10.3 Draw the graph of $g(x) = \cos(x - 30^\circ)$ for $-180^\circ \leq x \leq 90^\circ$ on the system of axes on DIAGRAM SHEET 3. Clearly label ALL x -intercepts and turning points. (4)
- 10.4 Hence, or otherwise, determine the values of x in the interval $-180^\circ \leq x \leq 90^\circ$ for which $f(x) \cdot g(x) < 0$. (4)
- 10.5 Describe the transformation that graph f has to undergo to form $y = \sin(2x + 60^\circ)$. (2)
- 10.6 Determine the general solution of $\sin 2x = \cos(x - 30^\circ)$. (6)

[20]

QUESTION 11

In the diagram below, ABC is a right-angled triangle. KC is the bisector of \hat{ACB} .
 $AC = r$ units and $\hat{BCK} = x$.



- 11.1 Write down AB in terms of x and r . (2)
- 11.2 Give the size of \hat{AKC} in terms of x . (1)
- 11.3 If it is given that $\frac{AK}{AB} = \frac{2}{3}$, calculate the value of x . (8)

[11]**TOTAL: 150**

CENTRE NUMBER:

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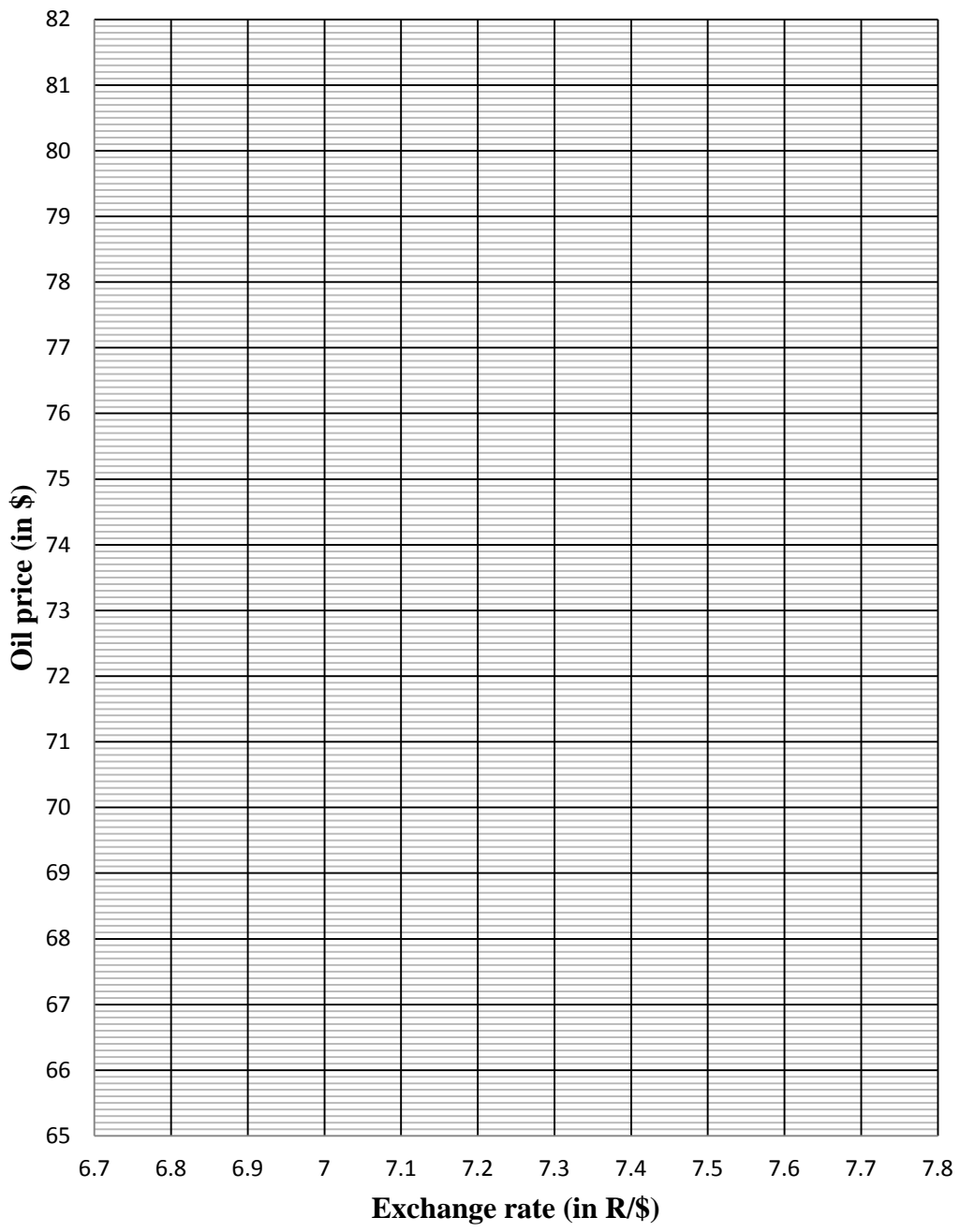
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DIAGRAM SHEET 1

QUESTION 1.1

Scatter plot of exchange rate versus oil price



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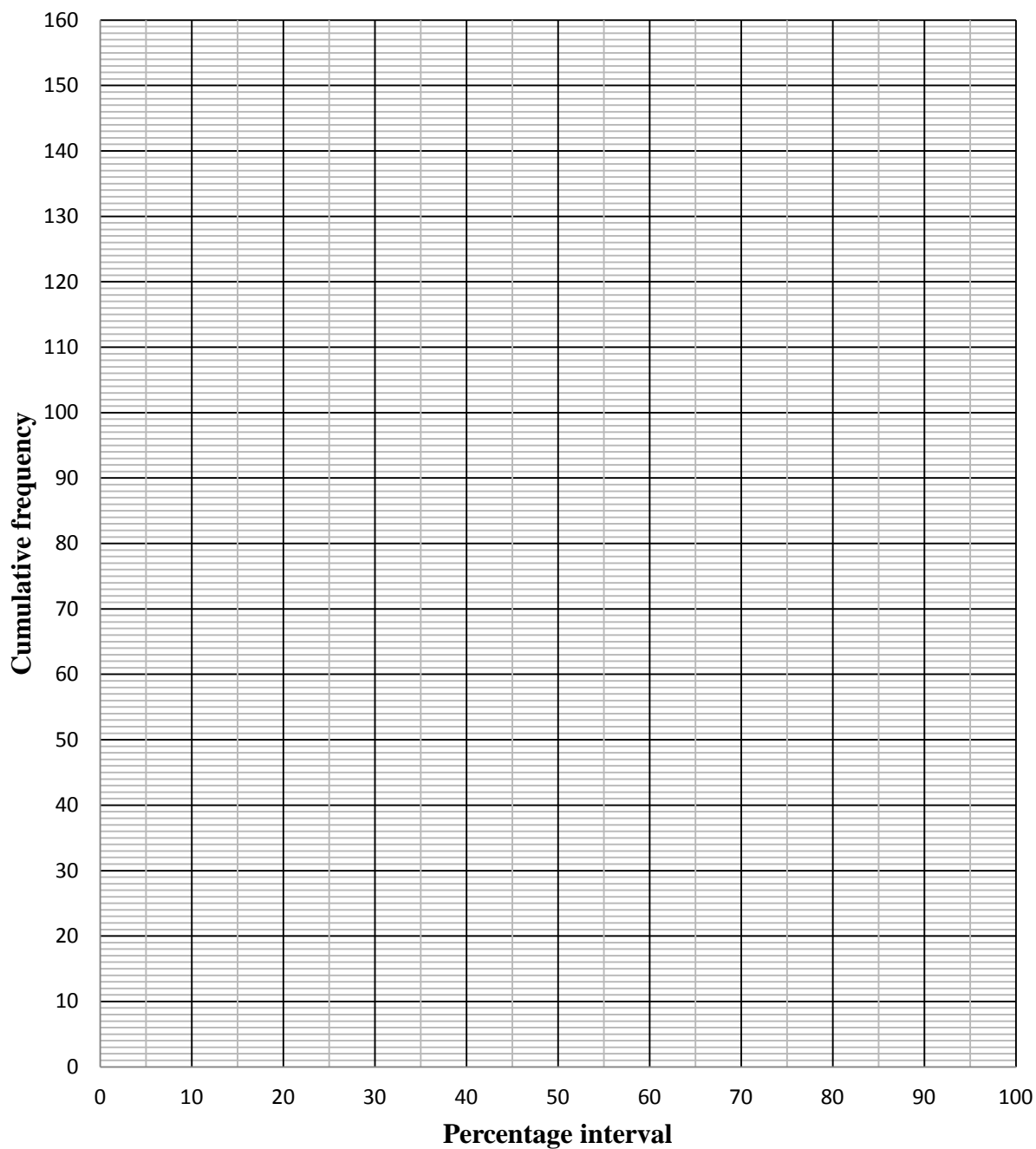
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DIAGRAM SHEET 2

QUESTION 3.1

Cumulative Frequency Graph



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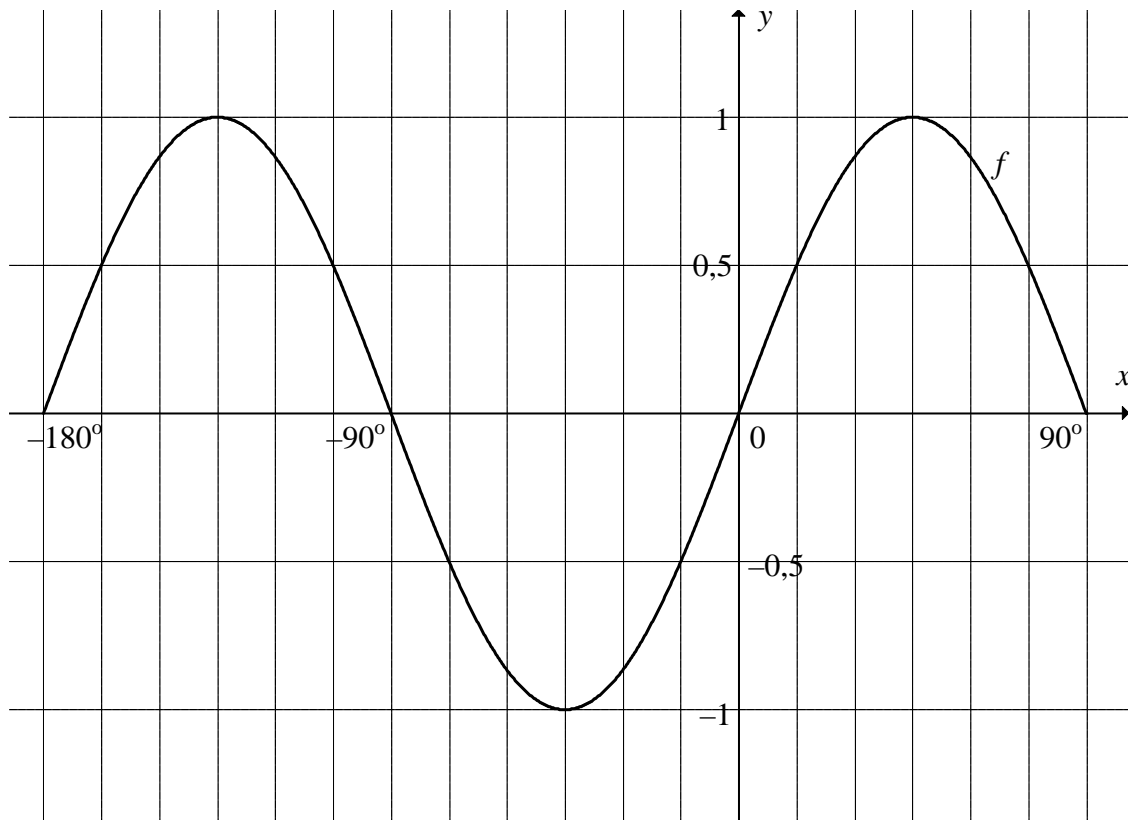
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DIAGRAM SHEET 3

QUESTION 10.3



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$\sum_{i=1}^n 1 = n$$

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$(x; y) \rightarrow (x \cos \theta - y \sin \theta; y \cos \theta + x \sin \theta)$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$