basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

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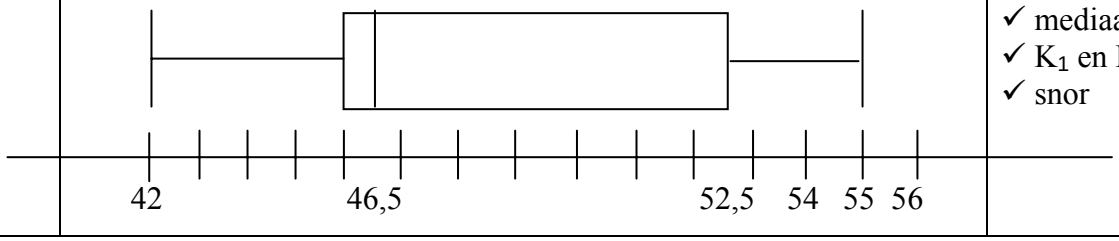
MEMORANDUM

PUNTE: 150

Hierdie memorandum bestaan uit 15 bladsye.

Kopiereg voorbehou

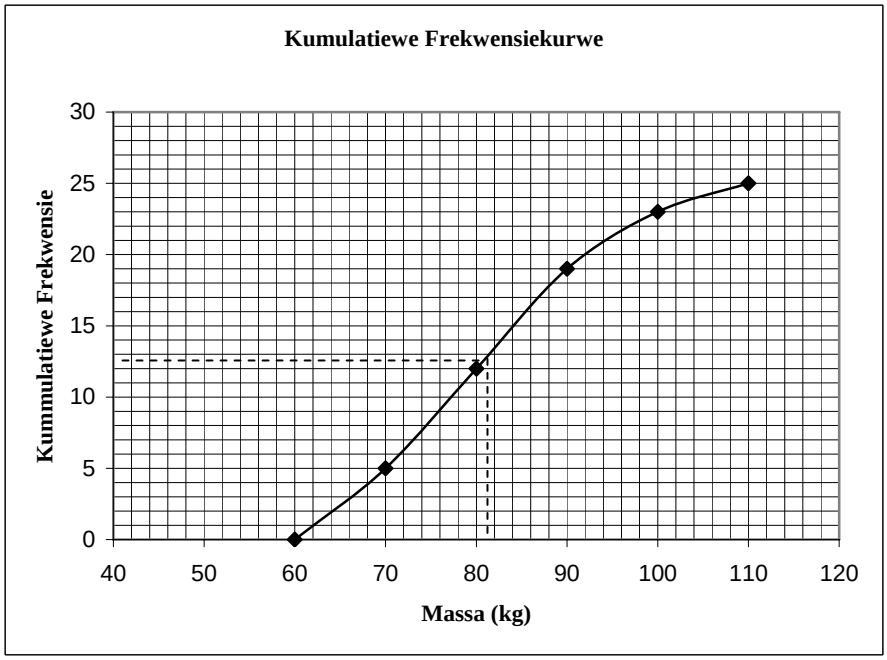
VRAAG 1

1.1	$\frac{55+55+50+47+42+3x}{8} = 48{,}375$ $\frac{249+3x}{8} = 48{,}375$ $3x = 138$ $x = 46$	✓ $\frac{249+3x}{8} = 48{,}375$ ✓ $3x = 138$ (2)
1.2		✓ maks en min ✓ mediaan ✓ K_1 en K_3 ✓ snor (4) **[6]**

VRAAG 2

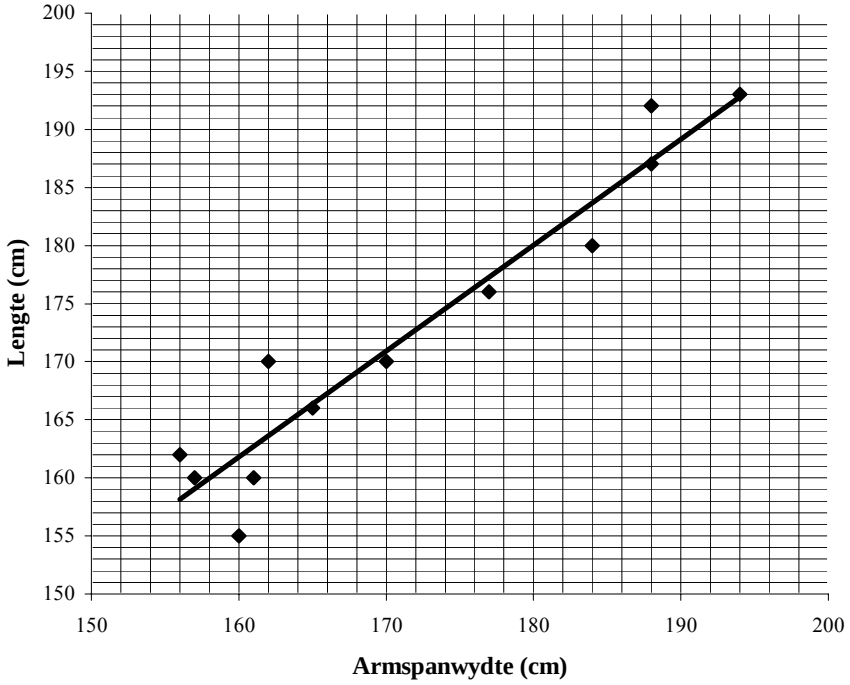
2.1		✓✓ Frekwensies ✓✓ Kumulatiewe Frekwensies (4)

Massa (kg)	Frekwensie	Kumulatiewe Frekwensie
$60 \le x < 70$	5	5
$70 \le x < 80$	7	12
$80 \le x < 90$	7	19
$90 \le x < 100$	4	23
$100 \le x < 110$	2	25

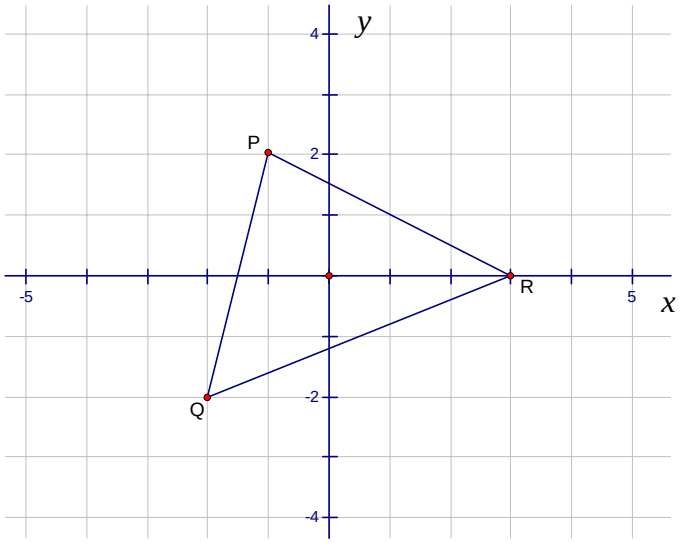
2.2		✓✓ alle punte akkuraat afgesteek 1 punt: 3 – 5 punte korrek 0 punte : 2 of minder punte korrek ✓ grafiek (3)
2.3	Gemiddelde = 79,28	✓✓ antwoord (2)

2.4	Standaardafwyking = 11,02 79,28 – 11,02 = 68,26 79,28+11,02 = 90,3 17 spelers lê in hierdie interval. $\frac{17}{25} = 68\%$	✓✓✓ st afw = 11,02 ✓ 17 spelers ✓ 68% (5) **[14]**

VRAAG 3

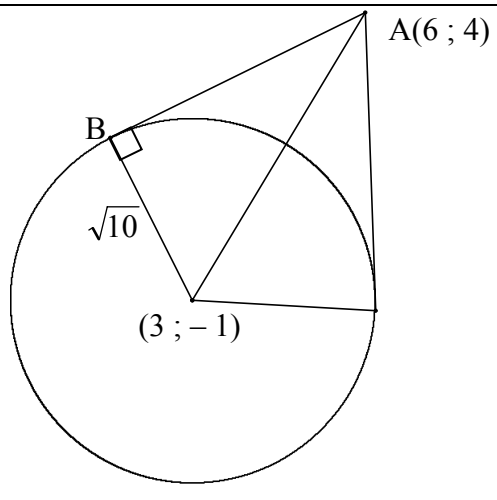
3.1 & 3.2		Vraag 3.1 4 punte: Alle punte akkuraat afgesteek 3 punte: 9 – 11 punte korrek 2 punte: 6 – 8 punte korrek 1 punts: 3 – 5 punte korrek 0 punte: as minder as 3 punte korrek (4) Vraag 3.2 ✓✓ lineêre beste paslyn (2)
3.3	Ja. Die verhouding tussen armspanwydte en lengte is 'n positiewe, lineêre een so ons kan verwag dat 'n persoon met 'n ondergemiddelde armspanwydte ook 'n ondergemiddelde lengte sal hê.	✓ Ja ✓ Rede (2) **[8]**

VRAAG 4



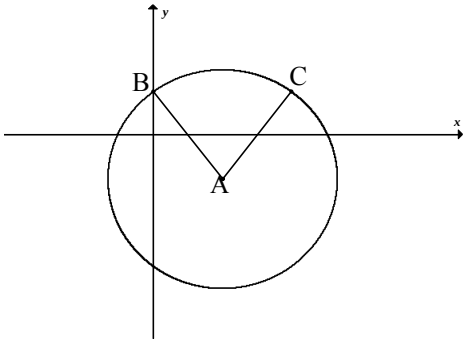
4.1	Laat β die inklinasiehoek van PQ wees. $\tan\beta = m_{PQ}$ $\tan\beta = \frac{2-(-2)}{-1-(-2)}$ $\tan\beta = 4$ $\beta = 75{,}96°$	✓ $\tan\beta = m_{PQ}$ ✓ $\tan\beta = 4$ ✓ antwoord (3)
4.2	$M\left(\frac{-1+3}{2};\frac{2+0}{2}\right)$ M (1 ; 1)	✓ *x*-waarde ✓ *y*-waarde (2)
4.3	$PQ = \sqrt{(-1+2)^2+(2+2)^2}$ $= \sqrt{17}$ $PR = \sqrt{(-1-3)^2+(2-0)^2}$ $= \sqrt{20}$ $QR = \sqrt{(0-(-2))^2+(3-(-2))^2}$ $= \sqrt{29}$ Omtrek = $\sqrt{29}+\sqrt{20}+\sqrt{17}$ = 13,98 eenhede = 14 tot die naaste heelgetal	✓ substitusie in korrekte formule ✓ antwoord ✓ antwoord ✓ som ✓ antwoord (5)
4.4	$y-1 = 4(x-1)$ $y = 4x-3$	✓ $m = 4$ ✓ substitusie van (1 ; 1) ✓ antwoord (3) **[13]**

VRAAG 5

5.1.1	$x^2 + y^2 - 8x + 6y$ $= (2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ Die punt lê vervolgens op die omtrek van die sirkel. **OF** $x^2 + y^2 - 8x + 6y = 15$ $(x-4)^2 + (y+3)^2 = 15 + 16 + 9$ $(x-4)^2 + (y+3)^2 = 40$ $(x-4)^2 + (y+3)^2$ $= (2-4)^2 + (-9+3)^2$ $= 2^2 + 6^2$ $= 40$ $\therefore$ Die punt lê op die omtrek van die sirkel.	✓ substitsie ✓ antwoord (2) ✓ substitusie ✓ antwoord (2)
5.1.2	$x^2 + y^2 - 8x + 6y = 15$ $(x-4)^2 + (y+3)^2 = 15 + 16 + 9$ $(x-4)^2 + (y+3)^2 = 40$ Middelpunt van sirkel $(4\ ;-3)$ $m_{rad} = \frac{-3-(-9)}{4-2}$ $m_{rad} = 3$ $m_{\tan} = -\frac{1}{3}$ $y + 9 = -\frac{1}{3}(x-2)$ $y = -\frac{1}{3}x - \frac{25}{3}$	✓✓ $(x-4)^2 + (y+3)^2 = 40$ ✓ middelpunt ✓ gradiënt van radius ✓ gradiënt van raaklyn ✓substitusie ✓ antwoord (7)
5.2		

	Radius AB $=\sqrt{10}$ Afstand vanaf A na middelpunt van sirkel is $=\sqrt{(6-3)^2+(4+1)^2}$ $=\sqrt{9+25}$ $=\sqrt{34}$ $AB^2 = 34-10$ $AB^2 = 24$ $AB = \sqrt{24}$ $AB = 2\sqrt{6}$ $AB = 4{,}90$ **OF** $r^2 = 10$ $r = \sqrt{10}$ radius $\perp$ raaklyn Vlgs Pythagoras $AB^2 = (6-3)^2 + (4+1)^2 - 10$ $= 24$ $AB = 4{,}90$	✓ radius = $\sqrt{10}$ ✓ substitusie in afstand formule ✓ $\sqrt{34}$ ✓ $AB^2 = 34-10$ ✓ antwoord (5) ✓ $r = \sqrt{10}$ ✓✓ $AB^2 = (6-3)^2 + (4+1)^2 - 10$ ✓ AB = 4,90 (5) **[14]**

VRAAG 6

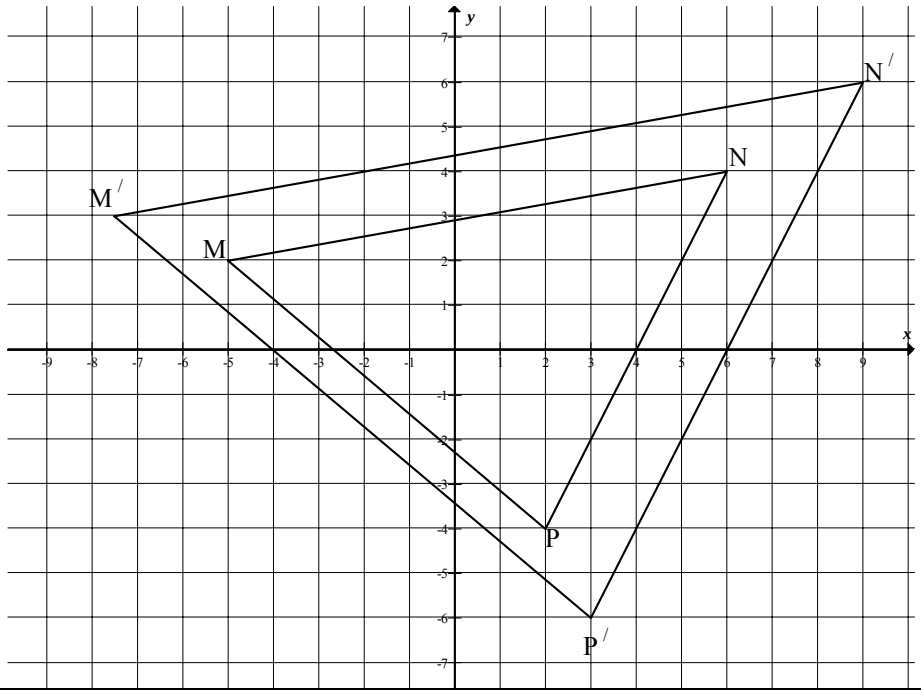


6.1	$9+(y+2)^2=25$ $(y+2)^2=16$ $y+2=\pm 4$ $y=2$ of $y=-6$ B(0 ; 2) **OF** $x=0$ $(0)^2-6(0)+y^2+4y=12$ $y^2+4y-12=0$ $(y+6)(y-2)=0$ $y=-6$ of $y=2$ B(0 ; 2)	✓ $x=0$ ✓ faktore ✓ antwoorde ✓ antwoord vir B (4) ✓ $x=0$ ✓ faktore ✓ antwoorde ✓ antwoord vir B (4)
6.2	C(6 ; 2)	✓✓ antwoord (2)
6.3	$\left(x-3\times\frac{3}{2}\right)^2+\left(y+2\times\frac{3}{2}\right)^2=\left(5\times\frac{3}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=\left(\frac{15}{2}\right)^2$ $\left(x-\frac{9}{2}\right)^2+(y+3)^2=56{,}25$	✓ elke deel $\times\frac{3}{2}$ ✓ antwoord (2)
6.4.1	$AB=\sqrt{(12-3)^2+(10-(-2))^2}$ $=\sqrt{9^2+12^2}$ $=15$	✓ substitusie ✓ antwoord (2)
6.4.2	Die radiusse is 5 en 10. $r_A+r_B=5+10$ $=15$ $=AB$ Die sirkels sal slegs in een punt sny.	✓ bymekaartel van radiusse ✓ antwoord (2) **[12]**

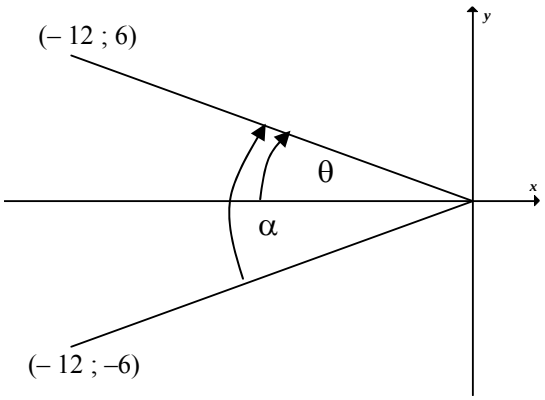
VRAAG 7

	$-3 = x\cos 150° - 2\sin 150°$	✓ uitbreiding
	$-3 = -x.\frac{\sqrt{3}}{2} - 2.\frac{1}{2}$	✓ substitusie
	$\frac{\sqrt{3}}{2}x = 2$	
	$x = \frac{4}{\sqrt{3}}$	✓ antwoord
	$y = x.\sin 150° + 2.\cos 150°$	✓ uitbreiding
	$y = \frac{4}{\sqrt{3}}.\frac{1}{2} + 2.\left(-\frac{\sqrt{3}}{2}\right)$	
	$= \frac{2}{3}.\sqrt{3} - \sqrt{3}$	
	$= -\frac{\sqrt{3}}{3}$	✓ antwoord
		[5]

VRAAG 8

8.1		✓✓✓ koördinate van nuwe punte (3)
8.2.1	$\frac{MN}{M'N'} = \frac{2}{3}$	✓✓ (2)
8.2.2	$\frac{\text{oppervlakte } \Delta MNP}{\text{oppervlakte } \Delta M'N'P'} = \frac{4}{9}$	✓✓ (2)
8.2.3	$\frac{\text{oppervlakte } \Delta MNP}{\text{oppervlakte } \Delta M''N''P''} = \left(\frac{4}{9}\right)^{n+1}$	✓✓ (2) **[9]**

VRAAG 9

9.1	$A^{/}(-12;-6)$	✓ antwoord (1)
9.2	$x^{/} = x\cos\alpha - y\sin\alpha$ $-12\cos\alpha - 6\sin\alpha = -12$ $-2\cos\alpha - \sin\alpha = -2$.....(1) $y^{/} = y\cos\alpha + x\sin\alpha$ $6\cos\alpha - 12\sin\alpha = -6$ $\cos\alpha = 2\sin\alpha - 1$ … (2) Vervang (2) in (1) $-2(2\sin\alpha - 1) - \sin\alpha = -2$ $-4\sin\alpha + 2 - \sin\alpha = -2$ $-5\sin\alpha = -4$ $\sin\alpha = \frac{4}{5}$ $\alpha = 53{,}13°$ **OF**  $\tan\theta = \frac{1}{2}$ $\theta = 26{,}565°$ $\alpha = 2(26{,}565°)$ $\alpha = 53{,}13°$	✓ substitusie ✓ substitusie ✓ vereenvoudiging ✓ substitusie ✓ vereenvoudiging ✓ antwoord (6) ✓✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26{,}565°$ ✓✓ $\alpha = 2(26{,}565°)$ ✓ antwoord (6) **[7]**

VRAAG 10

10.1.1	$\cos 28° = \sqrt{1-\sin^2 28°}$ $= \sqrt{1-a^2}$	✓ $\sqrt{1-\sin^2 28°}$ ✓antwoord (2)
10.1.2	$\cos 64°$ $= \cos 2(32°)$ $= 2\cos^2 32° - 1$ $= 2b^2 - 1$	✓ $\cos 2(32°)$ ✓ $2\cos^2 32° - 1$ ✓antwoord (3)
10.1.3	$\sin 4°$ $= \sin(32° - 28°)$ $= \sin 32° \cos 28° - \cos 32° \sin 28°$ $= \sqrt{1-b^2}.\sqrt{1-a^2} - ab$ **OF** $\sin 4°$ $= \sin(60° - 2\times 28°)$ $= \sin 60° \cos(2\times 28°) - \cos 60° \sin(2\times 28°)$ $= \frac{\sqrt{3}}{2}(1-2a^2) - \frac{1}{2}(2a)\sqrt{1-a^2}$ $= \frac{\sqrt{3}}{2} - \sqrt{3}a^2 - a\sqrt{1-a^2}$ **OF** $\sin 4°$ $= \sin(2\times 32° - 60°)$ $= \sin(2\times 32°)\cos 60° - \cos(2\times 32°).\sin 60°$ $= 2.b\sqrt{1-b^2}.\frac{1}{2} - \frac{\sqrt{3}}{2}(2b^2-1)$ $= b\sqrt{1-b^2} - \sqrt{3}b^2 + \frac{\sqrt{3}}{2}$ **OF** As gebruik sin(A+B) + sin(A – B) = 2.sinA.cosB met A = 28° en B = 32° $\sin 60° + \sin(-4°) = 2ab$ $\sin 4° = \frac{\sqrt{3}}{2} - 2ab$ **OF**	✓ $\sin(32° - 28°)$ ✓ uitbreiding ✓✓antwoord (4)

	As gebruik sin(A+B) + sin(A – B) = 2.sinA.cosB met A = 32° en B = 28° $\sin 60° + \sin(4°) = 2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin 4° = 2\sqrt{1-b^2}.\sqrt{1-a^2} - \frac{\sqrt{3}}{2}$ **OF** As gebruik $\sin 4° = 2\sin 2°.\cos 2°$ en $\sin 2° = \sin(30° - 28°) = \frac{1}{2}\left(\sqrt{1-a^2} - \sqrt{3}a\right)$ en $\sin 2° = \sin(32° - 30°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-b^2} - b\right)$ en $\cos 2° = \cos(30° - 28°) = \frac{1}{2}\left(\sqrt{3}\sqrt{1-a^2} + a\right)$ en $\cos 2° = \cos(32° - 30°) = \frac{1}{2}\left(\sqrt{3}b + \sqrt{1-b^2}\right)$ dan is $\sin 4° = \frac{1}{2}\left\{\sqrt{3}b\sqrt{1-a^2} - 3ab + \sqrt{1-a^2}.\sqrt{1-b^2} - \sqrt{3}a\sqrt{1-b^2}\right\}$ **OF** $\sin 4° = \frac{1}{2}\left\{3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab\right\}$	
10.2	$b\sqrt{1-a^2} - a\sqrt{1-b^2}$ $= \cos 32°.\sqrt{1-\sin^2 28°} - \sin 28°\sqrt{1-\cos^2 32°}$ $= \cos 32°.\cos 28° - \sin 28°.\sin 32°$ $= \cos(32° + 28°)$ $= \cos 60°$ $= \frac{1}{2}$	✓ substitusie ✓ cos 28° ✓ sin 32° ✓ saamgestelde hoek formule (4)
10.3.1	$\frac{\sin 130°.\tan 60°}{\cos 540°.\tan 230°.\sin 400°}$ $= \frac{\sin 50° \times \tan 60°}{\cos 180° \times \tan 50° \times \sin 40°}$ $= \frac{\sin 50° \times \sqrt{3}}{-1 \times \frac{\sin 50°}{\cos 50°} \times \cos 50°}$ $= -\frac{\sqrt{3}\cos 50°}{\cos 50°}$ $= -\sqrt{3}$	✓ $\sin 50°$ ✓ $\tan 50°$ ✓ $\sin 40^o$ ✓ $\cos 50^o$ ✓ $\frac{\sin 50°}{\cos 50°}$ ✓ -1 ✓ antwoord (7)

10.3.2	$(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=1-2\sin^2 75°$ $=\cos 150°$ $=\frac{-\sqrt{3}}{2}$ **OF** $\sin 75°$ $=\sin(45°+30°)$ $=\sin 45°.\cos 30°+\cos 45°.\sin 30°$ $=\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}.\frac{1}{2}$ $\sqrt{2}\sin 75°=\frac{\sqrt{3}}{2}+\frac{1}{2}=a$ $(1-\sqrt{2}\sin 75°)(1+\sqrt{2}\sin 75°)$ $=(1-a)(1+a)$ $=1-a^2$ $=1-\left(\frac{3}{4}+\frac{1}{4}+2.\frac{\sqrt{3}}{2}.\frac{1}{2}\right)$ $=-\frac{\sqrt{3}}{2}$	✓vereenvoudiging ✓$1-2\sin^2 75°$ ✓$\cos 150°$ ✓antwoord (4) ✓vereenvoudiging ✓$1-2\sin^2 75°$ ✓$\cos 150°$ ✓antwoord (4)
10.4	$\sin^2 x+\cos 2x-\cos x=0$ $\sin^2 x+(\cos^2 x-\sin^2 x)-\cos x=0$ $\cos^2 x-\cos x=0$ $\cos x(\cos x-1)=0$ $\cos x=0$ *or* $\cos x=1$ $x=\pm 90°+k.360°$ of $x=0°+k.360°$ $k\in Z$ $=k.360°$ i.e. $x=90°+k.180°$ of $x=k.360°\pm 90°, k\in Z$)	✓$(\cos^2 x-\sin^2 x)$ ✓$\cos^2 x-\cos x=0$ ✓faktore ✓$\cos x=0$ of $\cos x=1$ ✓$90°+k.360°$ ✓$k.360°$ ✓ $x=-90°+k.360°$ (7)
10.5.1	$x=0°;90°;180°$	✓✓✓ elke waarde (3)

10.5.2	$\frac{\cos 2x.\tan x}{\sin^2 x} = \frac{(\cos^2 x - \sin^2 x).\frac{\sin x}{\cos x}}{\sin^2 x}$ $= \frac{\cos^2 x - \sin^2 x}{\cos x.\sin x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x$	✓ $(\cos^2 x - \sin^2 x)$ ✓ $\frac{\sin x}{\cos x}$ ✓ antwoord ✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ ✓antwoord (5) **[39]**

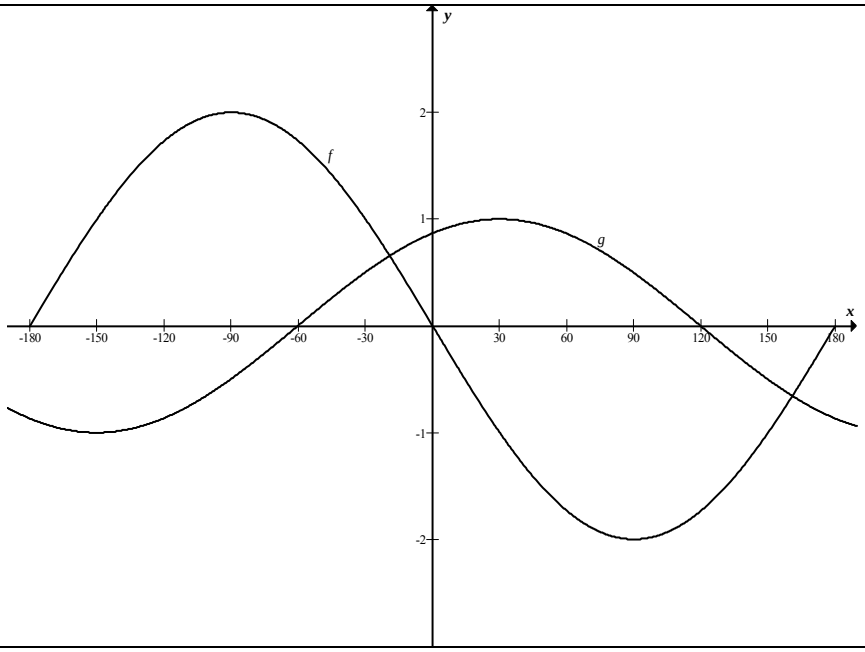
VRAAG 11

11.1	$EC^2 = DE^2 + DC^2 - 2DE.DC\cos\hat{C}$ $= (7,5)^2 + (9,4)^2 - 2.(7,5)(9,4)\cos 32°$ $= 25,03521844...$ $EC = 5,0$ meter	✓substitusie in cos reël ✓ 25,03521844... ✓antwoord (3)
11.2	$\frac{\sin D\hat{C}E}{7,5} = \frac{\sin 32°}{5,0}$ $\sin D\hat{C}E = \frac{7,5.\sin 32°}{5,0}$ $= 0,7948788963$ $D\hat{C}E = 52,6°$	✓ sin reël ✓ 0,7948788963 ✓antwoord (3)
11.3	Area van ΔDEC $= \frac{1}{2}DE.DC\sin\hat{D}$ $= \frac{1}{2}(7,5)(9,4)\sin 32°$ $= 18,7m^2$ OF Area van ΔDEC $= \frac{1}{2}CE.DC\sin 52,6°$ $= \frac{1}{2}(5,0)(9,4)\sin 52,6°$ $= 18,7m^2$	✓ substitusie ✓antwoord (2)

11.4	$\sin 32° = \frac{EG}{7,5}$ $EG = 7,5.\sin 32°$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5$ meter OF $EG = EC.\sin 52,6°$ $= (5,0).\sin 52,6°$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ OF	✓verhouding ✓substitusie ✓antwoord (3) **[11]**

	$\frac{1}{2}.DC.EG = \text{area } \Delta DEC$ $\frac{1}{2}(9,4)EG = 18,7$ $\therefore \ EG = \frac{18,7 \times 2}{9,4}$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$	

VRAAG 12

12.1	Periode = 360°	✓antwoord (1)
12.2	Amplitude = $\frac{1}{2}$	✓✓antwoord (2)
12.3		✓vorm ✓*x*-afsnitte ✓ amplitude (3)
12.4	2 oplossings	✓antwoord (1)
12.5	$-60° \le x \le 120°$ of $x \in [-60°; 120°]$	✓ $-60°; 120°$ ✓ notasie (2)
12.6	$-90° < x < 30°$ of $x \in (-90°; 30°)$	✓✓ $-90°; 30°$ ✓ notasie (3) **[12]**

TOTAAL: 150

Kopiereg voorbehou