



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2


FEBRUARY/MARCH 2011

MEMORANDUM

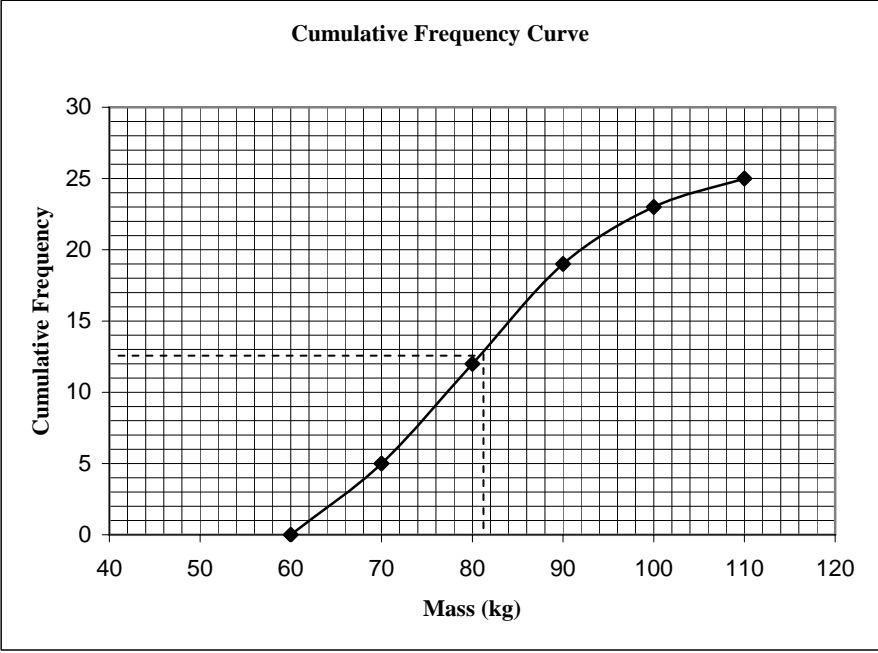
MARKS: 150

This memorandum consists of 15 pages.

QUESTION 1

1.1	$\frac{55 + 55 + 50 + 47 + 42 + 3x}{8} = 48,375$ $\frac{249 + 3x}{8} = 48,375$ $3x = 138$ $x = 46$	$\checkmark \frac{249 + 3x}{8} = 48,375$ $\checkmark 3x = 138$ <p style="text-align: right;">(2)</p>
1.2		<ul style="list-style-type: none"> ✓ max and min ✓ median ✓ Q₁ and Q₃ ✓ whiskers <p style="text-align: right;">(4) [6]</p>

QUESTION 2

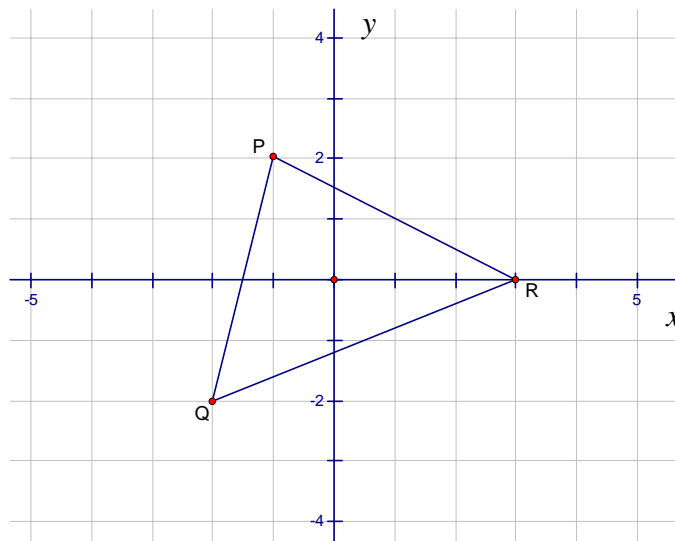
2.1	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="padding: 5px;">Mass (kg)</th> <th style="padding: 5px;">Frequency</th> <th style="padding: 5px;">Cumulative Frequency</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">$60 \leq x < 70$</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">5</td> </tr> <tr> <td style="padding: 5px;">$70 \leq x < 80$</td> <td style="padding: 5px;">7</td> <td style="padding: 5px;">12</td> </tr> <tr> <td style="padding: 5px;">$80 \leq x < 90$</td> <td style="padding: 5px;">7</td> <td style="padding: 5px;">19</td> </tr> <tr> <td style="padding: 5px;">$90 \leq x < 100$</td> <td style="padding: 5px;">4</td> <td style="padding: 5px;">23</td> </tr> <tr> <td style="padding: 5px;">$100 \leq x < 110$</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">25</td> </tr> </tbody> </table>	Mass (kg)	Frequency	Cumulative Frequency	$60 \leq x < 70$	5	5	$70 \leq x < 80$	7	12	$80 \leq x < 90$	7	19	$90 \leq x < 100$	4	23	$100 \leq x < 110$	2	25	<ul style="list-style-type: none"> ✓✓ Frequencies ✓✓ Cumulative Frequencies <p style="text-align: right;">(4)</p>
Mass (kg)	Frequency	Cumulative Frequency																		
$60 \leq x < 70$	5	5																		
$70 \leq x < 80$	7	12																		
$80 \leq x < 90$	7	19																		
$90 \leq x < 100$	4	23																		
$100 \leq x < 110$	2	25																		
2.2	<p style="text-align: center;">Cumulative Frequency Curve</p> 	<ul style="list-style-type: none"> ✓✓ plotting points 1 mark: 3 – 5 points correctly 0 marks : 2 or less points correctly plotted ✓ graph <p style="text-align: right;">(3)</p>																		
2.3	Mean = 79,28	<ul style="list-style-type: none"> ✓✓ answer <p style="text-align: right;">(2)</p>																		

2.4	Standard Deviation = 11,02 $79,28 - 11,02 = 68,26$ $79,28 + 11,02 = 90,3$ 17 players lie in this interval. $\frac{17}{25} = 68\%$	✓✓✓ sd = 11,02 ✓ 17 players ✓ 68% (5) [14]
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QUESTION 3

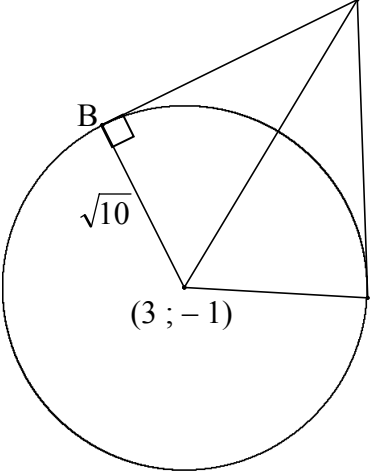
3.1 & 3.2	<p>Scatter Plot showing Arm Span vs Height</p>	Question 3.1 4 marks: All points plotted correctly. 3 marks: 9 – 11 points correct 2 marks: 6 – 8 points correct 1 marks: 3 – 5 points correct 0 marks if less than 3 points plotted correctly. (4) Question 3.2 ✓✓ linear best fit Line (2)
3.3	Yes. The relationship between arm span and height is a positive, linear one so we can expect a person with below average arm span to have below average in height.	✓ Yes ✓ Reason (2) [8]

QUESTION 4



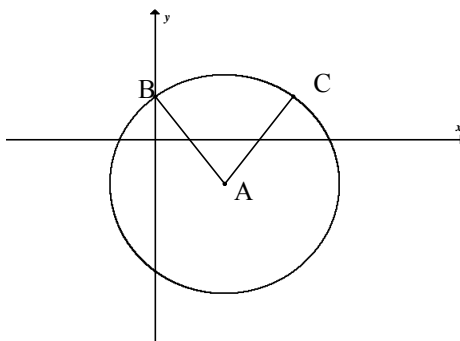
<p>4.1</p>	<p>Let β be the angle of inclination of PQ. $\tan \beta = m_{PQ}$ $\tan \beta = \frac{2 - (-2)}{-1 - (-2)}$ $\tan \beta = 4$ $\beta = 75,96^\circ$</p>	<p>✓ $\tan \beta = m_{PQ}$ ✓ $\tan \beta = 4$ ✓ answer (3)</p>
<p>4.2</p>	<p>$M\left(\frac{-1+3}{2}; \frac{2+0}{2}\right)$ $M(1; 1)$</p>	<p>✓ x-value ✓ y-value (2)</p>
<p>4.3</p>	<p>$PQ = \sqrt{(-1+2)^2 + (2+2)^2}$ $= \sqrt{17}$ $PR = \sqrt{(-1-3)^2 + (2-0)^2}$ $= \sqrt{20}$ $QR = \sqrt{(0-(-2))^2 + (3-(-2))^2}$ $= \sqrt{29}$ Perimeter = $\sqrt{29} + \sqrt{20} + \sqrt{17}$ $= 13,98$ units $= 14$ to the nearest whole number</p>	<p>✓ substitution into correct formula ✓ answer ✓ answer ✓ sum ✓ answer (5)</p>
<p>4.4</p>	<p>$y - 1 = 4(x - 1)$ $y = 4x - 3$</p>	<p>✓ $m = 4$ ✓ substitution of (1 ; 1) ✓ answer (3) [13]</p>

QUESTION 5

<p>5.1.1</p>	$x^2 + y^2 - 8x + 6y$ $= (2)^2 + (-9)^2 - 8(2) + 6(-9)$ $= 4 + 81 - 16 - 54$ $= 15$ <p>Hence, the point lies on the circumference of the circle.</p> <p>OR</p> $x^2 + y^2 - 8x + 6y = 15$ $(x - 4)^2 + (y + 3)^2 = 15 + 16 + 9$ $(x - 4)^2 + (y + 3)^2 = 40$ $(x - 4)^2 + (y + 3)^2$ $= (2 - 4)^2 + (-9 + 3)^2$ $= 2^2 + 6^2$ $= 40$ <p>∴ The point lies on the circumference of the circle.</p>	<p>✓ substitution ✓ answer (2)</p> <p>✓ substitution ✓ answer (2)</p>
<p>5.1.2</p>	$x^2 + y^2 - 8x + 6y = 15$ $(x - 4)^2 + (y + 3)^2 = 15 + 16 + 9$ $(x - 4)^2 + (y + 3)^2 = 40$ <p>Circle centre (4 ; -3)</p> $m_{rad} = \frac{-3 - (-9)}{4 - 2}$ $m_{rad} = 3$ $m_{tan} = -\frac{1}{3}$ $y + 9 = -\frac{1}{3}(x - 2)$ $y = -\frac{1}{3}x - \frac{25}{3}$	<p>✓✓ $(x - 4)^2 + (y + 3)^2 = 40$ ✓ centre</p> <p>✓ gradient of radius</p> <p>✓ gradient of tangent</p> <p>✓ substitution ✓ answer (7)</p>
<p>5.2</p>		<p>✓ radius = $\sqrt{10}$</p>

<p>Radius $AB = \sqrt{10}$ Distance from A to centre of circle is $= \sqrt{(6-3)^2 + (4+1)^2}$ $= \sqrt{9+25}$ $= \sqrt{34}$ $AB^2 = 34 - 10$ $AB^2 = 24$ $AB = \sqrt{24}$ $AB = 2\sqrt{6}$ $AB = 4,90$</p> <p>OR $r^2 = 10$ $r = \sqrt{10}$ Radius \perp tangent By Pythagoras $AB^2 = (6-3)^2 + (4+1)^2 - 10$ $= 24$ $AB = 4,90$</p>	<p>✓ subs into distance formula</p> <p>✓ $\sqrt{34}$</p> <p>✓ $AB^2 = 34 - 10$</p> <p>✓ answer (5)</p> <p>✓ $r = \sqrt{10}$</p> <p>✓✓</p> <p>$AB^2 = (6-3)^2 + (4+1)^2 - 10$</p> <p>✓ $AB = 4,90$ (5)</p> <p>[14]</p>
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QUESTION 6



<p>6.1</p>	$9 + (y + 2)^2 = 25$ $(y + 2)^2 = 16$ $y + 2 = \pm 4$ $y = 2 \text{ or } y = -6$ $B(0 ; 2)$ <p>OR</p> $x = 0$ $(0)^2 - 6(0) + y^2 + 4y = 12$ $y^2 + 4y - 12 = 0$ $(y + 6)(y - 2) = 0$ $y = -6 \text{ or } y = 2$ $B(0 ; 2)$	<p>✓ $x = 0$</p> <p>✓ factors ✓ answers ✓ answer for B (4)</p> <p>✓ $x = 0$</p> <p>✓ factors ✓ answers ✓ answer for B (4)</p>
<p>6.2</p>	<p>$C(6 ; 2)$</p>	<p>✓✓ answer (2)</p>
<p>6.3</p>	$\left(x - 3 \times \frac{3}{2}\right)^2 + \left(y + 2 \times \frac{3}{2}\right)^2 = \left(5 \times \frac{3}{2}\right)^2$ $\left(x - \frac{9}{2}\right)^2 + (y + 3)^2 = \left(\frac{15}{2}\right)^2$ $\left(x - \frac{9}{2}\right)^2 + (y + 3)^2 = 56,25$	<p>✓ each part $\times \frac{3}{2}$</p> <p>✓ answer (2)</p>
<p>6.4.1</p>	$AB = \sqrt{(12 - 3)^2 + (10 - (-2))^2}$ $= \sqrt{9^2 + 12^2}$ $= 15$	<p>✓ substitution</p> <p>✓ answer (2)</p>
<p>6.4.2</p>	<p>The radii are 5 and 10.</p> $r_A + r_B = 5 + 10$ $= 15$ $= AB$ <p>The circles will only intersect at one point.</p>	<p>✓ addition of radii</p> <p>✓ answer (2)</p> <p>[12]</p>

QUESTION 7

	$-3 = x \cos 150^\circ - 2 \sin 150^\circ$ $-3 = -x \cdot \frac{\sqrt{3}}{2} - 2 \cdot \frac{1}{2}$ $\frac{\sqrt{3}}{2} x = 2$ $x = \frac{4}{\sqrt{3}}$ $y = x \cdot \sin 150^\circ + 2 \cdot \cos 150^\circ$ $y = \frac{4}{\sqrt{3}} \cdot \frac{1}{2} + 2 \cdot \left(-\frac{\sqrt{3}}{2} \right)$ $= \frac{2}{\sqrt{3}} \cdot \sqrt{3} - \sqrt{3}$ $= -\frac{\sqrt{3}}{3}$	✓ expansion ✓ substitution ✓ answer ✓ expansion ✓ answer
		[5]

QUESTION 8

8.1		✓✓✓ coordinates of new points (3)
8.2.1	$\frac{MN}{M'N'} = \frac{2}{3}$	✓✓ (2)
8.2.2	$\frac{\text{area } \Delta MNP}{\text{area } \Delta M'N'P'} = \frac{4}{9}$	✓✓ (2)
8.2.3	$\frac{\text{area } \Delta MNP}{\text{area } \Delta M''N''P''} = \left(\frac{4}{9} \right)^{n+1}$	✓✓ (2) [9]

QUESTION 9

9.1	$A'(-12; -6)$	✓ answer (1)
9.2	$x' = x \cos \alpha - y \sin \alpha$ $-12 \cos \alpha - 6 \sin \alpha = -12$ $-2 \cos \alpha - \sin \alpha = -2 \dots \dots (1)$ $y' = y \cos \alpha + x \sin \alpha$ $6 \cos \alpha - 12 \sin \alpha = -6$ $\cos \alpha = 2 \sin \alpha - 1 \dots (2)$ <p>Substitute (2) into (1)</p> $-2(2 \sin \alpha - 1) - \sin \alpha = -2$ $-4 \sin \alpha + 2 - \sin \alpha = -2$ $-5 \sin \alpha = -4$ $\sin \alpha = \frac{4}{5}$ $\alpha = 53,13^\circ$ <p>OR</p> <p>$\tan \theta = \frac{1}{2}$</p> $\theta = 26,565^\circ$ $\alpha = 2(26,565^\circ)$ $\alpha = 53,13^\circ$	✓ substitution ✓ substitution ✓ simplification ✓ substitution ✓ simplification ✓ answer (6)
		✓✓ $\tan \theta = \frac{1}{2}$ ✓ $\theta = 26,565^\circ$ ✓✓ $\alpha = 2(26,565^\circ)$ ✓ answer (6) [7]

QUESTION 10

10.1.1	$\cos 28^\circ = \sqrt{1 - \sin^2 28^\circ}$ $= \sqrt{1 - a^2}$	$\checkmark \sqrt{1 - \sin^2 28^\circ}$ \checkmark answer (2)
10.1.2	$\cos 64^\circ$ $= \cos 2(32^\circ)$ $= 2 \cos^2 32^\circ - 1$ $= 2b^2 - 1$	$\checkmark \cos 2(32^\circ)$ $\checkmark 2 \cos^2 32^\circ - 1$ \checkmark answer (3)
10.1.3	$\sin 4^\circ$ $= \sin(32^\circ - 28^\circ)$ $= \sin 32^\circ \cos 28^\circ - \cos 32^\circ \sin 28^\circ$ $= \sqrt{1 - b^2} \cdot \sqrt{1 - a^2} - ab$ <p>OR</p> $\sin 4^\circ$ $= \sin(60^\circ - 2 \times 28^\circ)$ $= \sin 60^\circ \cos(2 \times 28^\circ) - \cos 60^\circ \sin(2 \times 28^\circ)$ $= \frac{\sqrt{3}}{2} (1 - 2a^2) - \frac{1}{2} (2a) \sqrt{1 - a^2}$ $= \frac{\sqrt{3}}{2} - \sqrt{3}a^2 - a\sqrt{1 - a^2}$ <p>OR</p> $\sin 4^\circ$ $= \sin(2 \times 32^\circ - 60^\circ)$ $= \sin(2 \times 32^\circ) \cos 60^\circ - \cos(2 \times 32^\circ) \cdot \sin 60^\circ$ $= 2b\sqrt{1 - b^2} \cdot \frac{1}{2} - \frac{\sqrt{3}}{2} (2b^2 - 1)$ $= b\sqrt{1 - b^2} - \sqrt{3}b^2 + \frac{\sqrt{3}}{2}$ <p>OR</p> <p>Using $\sin(A+B) + \sin(A - B) = 2 \cdot \sin A \cdot \cos B$ With $A = 28^\circ$ and $B = 32^\circ$ $\sin 60^\circ + \sin(-4^\circ) = 2ab$</p> $\sin 4^\circ = \frac{\sqrt{3}}{2} - 2ab$ <p>OR</p>	$\checkmark \sin(32^\circ - 28^\circ)$ \checkmark expansion $\checkmark \checkmark$ answer (4)

	<p>Using $\sin(A+B) + \sin(A - B) = 2.\sin A.\cos B$ With $A = 32^\circ$ and $B = 28^\circ$ $\sin 60^\circ + \sin(4^\circ) = 2\sqrt{1-b^2}.\sqrt{1-a^2}$ $\sin 4^\circ = 2\sqrt{1-b^2}.\sqrt{1-a^2} - \frac{\sqrt{3}}{2}$</p> <p>OR Using $\sin 4^\circ = 2 \sin 2^\circ.\cos 2^\circ$ and $\sin 2^\circ = \sin(30^\circ - 28^\circ) = \frac{1}{2}(\sqrt{1-a^2} - \sqrt{3}a)$ and $\sin 2^\circ = \sin(32^\circ - 30^\circ) = \frac{1}{2}(\sqrt{3}\sqrt{1-b^2} - b)$ and $\cos 2^\circ = \cos(30^\circ - 28^\circ) = \frac{1}{2}(\sqrt{3}\sqrt{1-a^2} + a)$ and $\cos 2^\circ = \cos(32^\circ - 30^\circ) = \frac{1}{2}(\sqrt{3}b + \sqrt{1-b^2})$ then $\sin 4^\circ = \frac{1}{2} \{ \sqrt{3}b\sqrt{1-a^2} - 3ab + \sqrt{1-a^2}.\sqrt{1-b^2} - \sqrt{3}a\sqrt{1-b^2} \}$</p> <p>OR $\sin 4^\circ = \frac{1}{2} \{ 3\sqrt{1-b^2}\sqrt{1-a^2} + \sqrt{3}a\sqrt{1-b^2} - \sqrt{3}b\sqrt{1-a^2} - ab \}$</p>	
<p>10.2</p>	$b\sqrt{1-a^2} - a\sqrt{1-b^2}$ $= \cos 32^\circ.\sqrt{1-\sin^2 28^\circ} - \sin 28^\circ\sqrt{1-\cos^2 32^\circ}$ $= \cos 32^\circ.\cos 28^\circ - \sin 28^\circ.\sin 32^\circ$ $= \cos(32^\circ + 28^\circ)$ $= \cos 60^\circ$ $= \frac{1}{2}$	<p>✓ substitution ✓ $\cos 28^\circ$ ✓ $\sin 32^\circ$ ✓ compound angle formula</p> <p>(4)</p>
<p>10.3.1</p>	$\frac{\sin 130^\circ.\tan 60^\circ}{\cos 540^\circ.\tan 230^\circ.\sin 400^\circ}$ $= \frac{\sin 50^\circ \times \tan 60^\circ}{\cos 180^\circ \times \tan 50^\circ \times \sin 40^\circ}$ $= \frac{\sin 50^\circ \times \sqrt{3}}{-1 \times \frac{\sin 50^\circ}{\cos 50^\circ} \times \cos 50^\circ}$ $= -\frac{\sqrt{3} \cos 50^\circ}{\cos 50^\circ}$ $= -\sqrt{3}$	<p>✓ $\sin 50^\circ$ ✓ $\tan 50^\circ$ ✓ $\sin 40^\circ$ ✓ $\cos 50^\circ$ ✓ $\frac{\sin 50^\circ}{\cos 50^\circ}$ ✓ -1 ✓ answer</p> <p>(7)</p>

10.5.2	$\frac{\cos 2x \cdot \tan x}{\sin^2 x} = \frac{(\cos^2 x - \sin^2 x) \cdot \frac{\sin x}{\cos x}}{\sin^2 x}$ $= \frac{\cos^2 x - \sin^2 x}{\cos x \cdot \sin x}$ $= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$ $= \frac{\cos x}{\sin x} - \tan x$	<p>✓ $(\cos^2 x - \sin^2 x)$</p> <p>✓ $\frac{\sin x}{\cos x}$</p> <p>✓ answer</p> <p>✓ $\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}$</p> <p>✓ answer</p> <p>(5) [39]</p>
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QUESTION 11

11.1	$EC^2 = DE^2 + DC^2 - 2DE \cdot DC \cos \hat{C}$ $= (7,5)^2 + (9,4)^2 - 2 \cdot (7,5)(9,4) \cos 32^\circ$ $= 25,03521844\dots$ $EC = 5,0 \text{ metres}$	✓ substitution into cosine rule ✓ 25,03521844... ✓ answer (3)
11.2	$\frac{\sin \hat{DCE}}{7,5} = \frac{\sin 32^\circ}{5,0}$ $\sin \hat{DCE} = \frac{7,5 \cdot \sin 32^\circ}{5,0}$ $= 0,7948788963$ $\hat{DCE} = 52,6^\circ$	✓ sin rule ✓ 0,7948788963 ✓ answer (3)
11.3	Area of $\triangle DEC$ $= \frac{1}{2} DE \cdot DC \sin \hat{D}$ $= \frac{1}{2} (7,5)(9,4) \sin 32^\circ$ $= 18,7m^2$ <p>OR</p> Area of $\triangle DEC$ $= \frac{1}{2} CE \cdot DC \sin 52,6^\circ$ $= \frac{1}{2} (5,0)(9,4) \sin 52,6^\circ$ $= 18,7m^2$	✓ substitution ✓ answer (2)
11.4	$\sin 32^\circ = \frac{EG}{7,5}$ $EG = 7,5 \cdot \sin 32^\circ$ $= 4,0$ $EF = (4 + 3,5)$ $= 7,5 \text{ metres}$ <p>OR</p> $EG = EC \cdot \sin 52,6^\circ$ $= (5,0) \cdot \sin 52,6^\circ$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$ <p>OR</p>	✓ ratio ✓ substitution ✓ answer (3) [11]

$\frac{1}{2} \cdot DC \cdot EG = \text{area } \triangle DEC$ $\frac{1}{2} (9,4) EG = 18,7$ $\therefore EG = \frac{18,7 \times 2}{9,4}$ $= 4,0$ $EF = 4,0 + 3,5$ $= 7,5$	
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QUESTION 12

12.1	Period = 360°	✓ answer (1)
12.2	Amplitude = $\frac{1}{2}$	✓✓ answer (2)
12.3		✓ shape ✓ x intercepts ✓ amplitude (3)
12.4	2 solutions	✓ answer (1)
12.5	$-60^\circ \leq x \leq 120^\circ$ or $x \in [-60^\circ; 120^\circ]$	✓ $-60^\circ; 120^\circ$ ✓ notation (2)
12.6	$-90^\circ < x < 30^\circ$ or $x \in (-90^\circ; 30^\circ)$	✓✓ $-90^\circ; 30^\circ$ ✓ notation (3) [12]

TOTAL: 150