basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

PHYSICAL SCIENCES

CURRICULUM SUPPORT DOCUMENT

2010

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PHYSICS

APRIL 2010

Purpose of this Document

This document is intended to serve as a resource for teachers and learners. It provides notes, examples, problem-solving exercises with solutions and examples of practical activities.

How to obtain maximum benefit from this resource

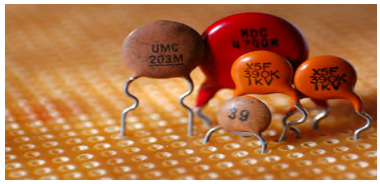
This resource contains many problem-solving exercises, quantitative-type questions and qualitative-type questions. The reason for this is that learners can improve their understanding of concepts if given the opportunity to answer thought provoking questions and grapple with problem-solving exercises both in class, as classwork activities and outside the classroom as homework activities.

PHYSICS CONTENT

Electricity and Magnetism

Electrostatics

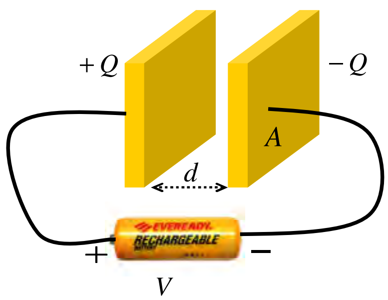
Capacitance and Capacitive Circuits



Capacitance

Basic Concepts

- *Capacitor - charge and energy storing device*
- *Parallel –plate Capacitor*



$$C = \frac{Q}{V}$$

(C) (V)

$1F \equiv 1C/V$

C is always positive

Example 1:

A 10 μF capacitor is connected to a 24 V battery. What is the charge on each plate?

$$C = \frac{Q}{V}$$

$$\because Q = CV = (10\times10^{-6}F)(24V) = \underline{240\mu C}$$

$$C = \frac{\varepsilon_o A}{d}$$

$A \equiv$ Area of plates (m^2)
$d \equiv$ Plate separation distance (m)
$\varepsilon_o \equiv$ Permittivity of free space
$\varepsilon_o = 8.85\times10^{-12}\,C^2.N^{-1}.m^{-2}$

$$E = \frac{V}{d}$$

$E \equiv$ Electric field stength ($N.m^{-1}$)
$d \equiv$ Plate separation distance (m)
$V \equiv$ Potential difference (V)

Example 2:

A parallel plate capacitor is constructed with plates having dimensions (6 cm by 5 cm) and being separated by a distance of 0.5 mm. If a potential of 18 V is applied across the capacitor, determine the charge on each plate.

Reasoning Strategy

$$C = \frac{\varepsilon_o A}{d} = \frac{\varepsilon_o (l\times b)}{d} \qquad C = \frac{Q}{V} \Rightarrow Q = CV$$

$$C = \frac{\varepsilon_o A}{d}$$
$$= \frac{\varepsilon_o (l\times b)}{d}$$
$$= \frac{(8.85\times10^{-12}\,C^2.N^{-1}.m^{-2})(6\times10^{-2}\,m)(5\times10^{-2}\,m)}{0.5\times10^{-3}\,m}$$
$$= 5.31\times10^{-11}\,F$$

$$C = \frac{Q}{V} \Rightarrow$$
$$Q = CV = (5.31\times10^{-11}\,F)(18V) = 9.56\times10^{-10}\,C$$

Activity 1

1.1 Using the appropriate equations and the definition of the farad, show that
$1F = 1C^2.N^{-1}.m^{-1}$

1.2 In example 2 , what separation distance, d, is necessary to give each plate a charge of 3 μC ? Assume that all other quantities remain unchanged.

- ***The Dielectric*** *– A material inserted between the plates of a capacitor to increase its capacitance*

- *See Appendix 1 for details*

$$\vec{E} = \frac{\vec{E_o}}{\kappa}$$

$E \equiv$ Reduced field ($N.m^{-1}$)
$E_o \equiv$ Original field ($N.m^{-1}$)
$\kappa \equiv$ dielectric constant (dimensionless)

$$C = \kappa \frac{\varepsilon_o A}{d}$$

$C \equiv$ Capacitance with the dielectric

Material	*Dielectric Constant, κ*	*Dielectric Strength ($V.m^{-1}$)*
		E_{max}
Vacuum	**1.000 00**	**-**
Air	**1.000 59**	**3×10^6**
Pyrex Glass	**5.6**	**14×10^6**
Polystyrene	**2.56**	**24×10^6**
Paper	**3.7**	**16×10^6**
Water	**80**	**-**
Neoprene Rubber	**6.7**	**12×10^6**
Teflon	**2.1**	**60×10^6**

Activity 2

2.1 *If the electric field, potential difference and capacitance of a parallel plate capacitor before the introduction of a dielectric are respectively E_o, V_o and C_o, show that the potential difference and capacitance (with the dielectric) is given by the equations below.*

$$V = \frac{V_o}{\kappa}$$

$$C = \kappa C_o$$

2.2

A parallel-plate capacitor has plates with an area of 0.012 m^2 and a separation of 0.88 mm. The space between the plates is filled with polystyrene.

(a) What is the potential difference between the plates when the charge on the capacitor plates is 4.7μC?
(b) What is the potential difference between the plates when the polystyrene is removed and the gap between the plates is filled with Air?

Dielectric Breakdown

If the electric field across a dielectric is large enough, it can literally tear the atoms apart thereby allowing the dielectric to conduct electricity. The maximum electric field a dielectric can withstand is called the ***Dielectric strength*** *($V.m^{-1}$)*

> **For example,** if the Dielectric Strength of air exceeds 3 million volts per meter, dielectric breakdown will occur leading to a tiny spark on a small scale or a bolt of lightning on a larger scale.

Activity 3

A parallel plate capacitor is constructed with a plate of area 0.028 m^2, and a separation distance of 0.550 mm. the space between the plates is filled with a dielectric material of dielectric constant,κ. When the capacitor is connected to a 12 V battery, each plate has a charge of 3.62×10^{-8} C.

(i) What is the value of the dielectric constant?
(ii) What material is the dielectric made from?
(iii) If the separation distance is held constant, calculate the potential difference that would lead to dielectric breakdown.

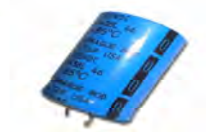
Activity 4: Conceptual Question

If you were asked to design a capacitor where small size and large capacitance were required, what factors would be important in your design?

Different types of capacitors

1. Electrolytic Capacitors (Electrochemical type capacitors)

The most important characteristic of electrolytic capacitors is that they have polarity. They have a positive and a negative electrode. [Polarised] This means that it is very important which way round they are connected. If the capacitor is subjected to voltage exceeding its working voltage, or if it is connected with incorrect polarity, it may burst.



2. Tantalum Capacitors

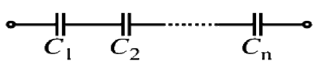
Tantalum Capacitors are electrolytic Capacitors that use a material called tantalum for the electrodes. Tantalum capacitors are superior to Aluminium electrolytic capacitors in temperature and frequency characteristics. These capacitors have polarity as well. Capacitance can change with temperature as well as frequency, and these types are very stable.

3. Ceramic Capacitors

Ceramic capacitors are constructed with materials such as titanium acid barium used as the dielectric. Internally, these capacitors are not constructed as a coil, so they can be used in high frequency applications. Typically, they are used in circuits which bypass high frequency signals to ground. These capacitors have the shape of a disk. Their capacitance is comparatively small.

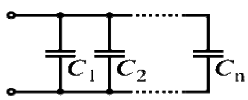
Capacitive circuits

Capacitors in Series:



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots.. + \frac{1}{C_n}$$

Capacitors in Parallel:



$$C_p = C_1 + C_2 + \ldots. + C_n$$

Capacitance & Capacitive Circuits: Everyday Applications

Electronic Flash Units

An electronic flash unit contains a capacitor that can store a large amount of charge. When the charge is released, the resulting flash can be a short as a millisecond. This allows photographers to "freeze" motion.

Defibrillator

When a person's heart undergoes ventricular fibrillation – the rapid, uncontrolled twitching of the heart muscles, a powerful jolt of electrical energy is required to restore the heart's regular beating. The device that is used to deliver the energy is called a defibrillator and it uses a capacitor to store the energy required.

Energy storage

A capacitor can store electric energy when disconnected from its charging circuit, so it can be used like a temporary battery. Capacitors are commonly used in electronic devices to maintain power supply while batteries are being changed.

Measuring Humidity in Air

Changing the dielectric: The effects of varying the physical and/or electrical characteristics of the **dielectric** can also be of use. Capacitors with an exposed and porous dielectric can be used to measure humidity in air.

Measuring Fuel level

Changing the distance between the plates: Capacitors are used to accurately measure the fuel level in airplanes

Tuned Circuits

Capacitors and inductors are applied together in tuned circuits to select information in particular frequency bands. For example, radio receivers rely on variable capacitors to tune the station frequency.

Signal Coupling

Because capacitors pass AC but block DC signals (when charged up to the applied dc voltage), they are often used to separate the AC and DC components of a signal. This method is known as *AC coupling* or "capacitive coupling".

Power conditioning

Resevoir are used in power supplies where they smooth the output of a full or half wave rectifer. Audio equipment, for example, uses several capacitors to shunt away power line hum before it gets into the signal circuitry.

APPENDIX 1: Dielectric

If the molecules in dielectric have a permanent dipole moments, they will align with the electric field as shown in the diagram. This results in a negative charge on the surface of the slab near the positive plate and a positive charge on the surface of the slab near the negative plate. Since electric field line start on positive charges and terminate on negative charge, it is clear that fewer electric field lines exist between the plates and there is a reduced field, E *, in the dielectric which is characterized with a dimensionless constant called the dielectric constant,* κ

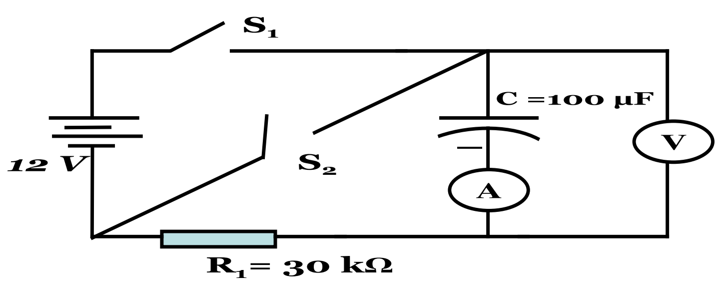
Experiment

Charging and Discharging of a Capacitor

Materials and Equipment Required

1. Battery (6v or 1.5v x 4)
2. 2 x SPST switches
3. 3 x 100μF capacitor and 3x 10 μ F capacitor
4. 3 x 10kΩ and 3 x 1kΩ resistors
5. 2 x digital meters (one set to measure current and the other voltage)
7. Conducting leads
8. Stopwatch

Connect the circuit shown below.



Note: for charging, S_1 is closed and S_2 is opened
And for discharging S_1 is opened and S_2 is closed

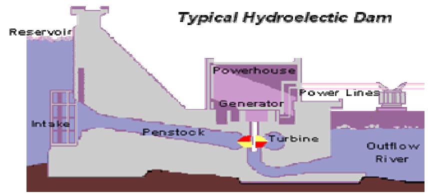
Observations

- *Close S_1* ***(charging)***
- *Observe the readings on the voltmeter and ammeter – conclusion*
- *After a few minutes how does the voltmeter reading compare with the source voltage.*
- *After a few minutes open S_1, and close S_2* ***(discharging)***
- *Observe the readings on the voltmeter and ammeter – conclusion*
- *After a few minutes short out the cap to completely drain it (re-setting)*

Electrodynamics

Generators

Convert mechanical energy into electrical energy
Used in hydroelectric power generation



Hydroelectric and coal-fired power plants produce electricity in virtually the same way. In both cases a moving fluid is used to rotate the turbine blades which then turns a metal shaft positioned in the generator (which produces electricity).

Turbines

In a coal-fired power plant steam is used to turn the turbine blades; while a hydroelectric plant harnesses the energy of falling water to turn the turbine blades.

Power lines connected to the generator help carry the power to our homes. In South Africa about 95% of our electricity is obtained from coal-fired power generators.

We all know how important electricity is to our everyday living. So you see, Science has a huge impact on human development. Shortly we are going to be studying the Physics involved in electrical power generation.

Now lets have a look a quotation for a recent article in a South African Journal.

"Clean coal technologies, coal cost and quality, environmental considerations, sustainable development, the growth of the South African economy and Government's regulation of the electricity industry are the main challenges to the continued use of coal as South Africa's primary energy source."

JEFFREY, L.S. 2005. The Journal of The South African Institute of Mining and Metallurgy. Issue No.2: 95-102, February

What are your views on this quotation?

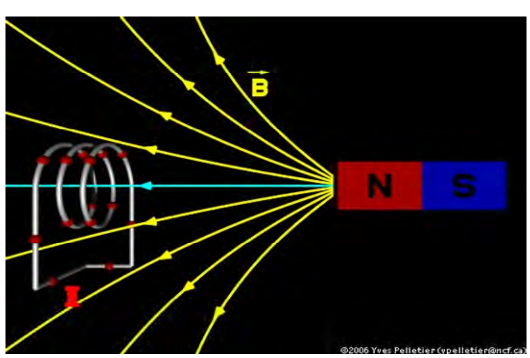
Motors

Converts electrical energy into mechanical energy

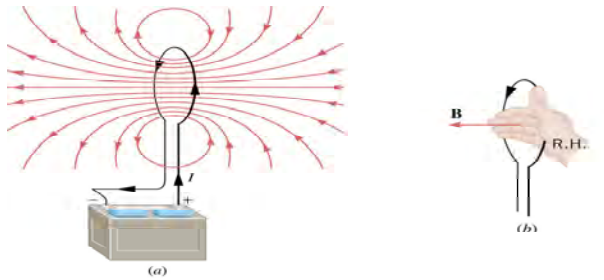
Uses

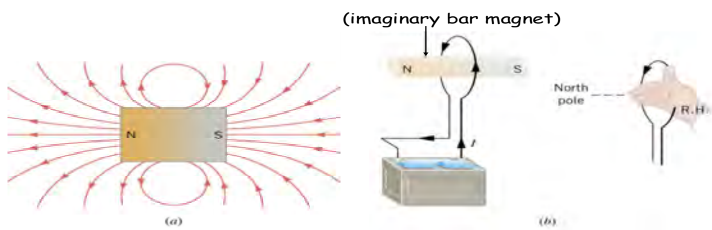
- **Electric lifts** - An electric motor moves the lift up and down. Another operates the doors.
- **Cars** - Cars have several electric motors. The starter motor turns the engine to get it going. Motors are used to work the windscreen wipers, electric windows, electric side mirror etc.
- Can you list other uses of motors?

Introductory Concepts



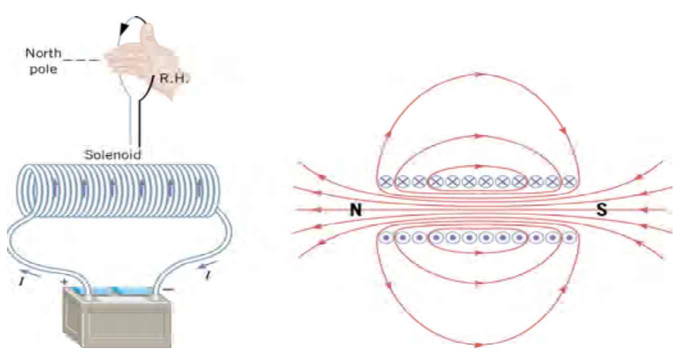
1. Magnetic field pattern around a current-carrying loop.





Slide 9

2. Magnetic field around a solenoid – (coil of wire)



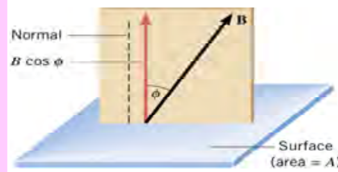
⊙ Current out of page
⊗ Current into page

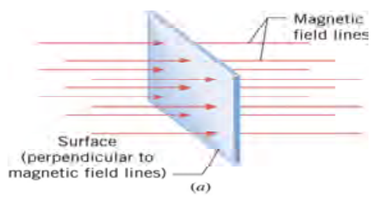
3. Magnetic Flux

Given a loop of wire of area, A, in the presence of a magnetic field **B**. The magnetic flux, Φ ,through the loop is proportional to the total number of field lines passing through the surface and is given by:

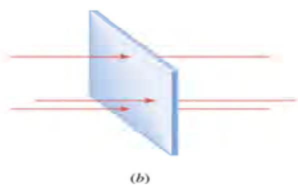
$$\Phi = BA\cos\phi$$

$\Phi \equiv$ Magnetic flux (Wb)
$\{1\ \text{Weber (Wb)} = 1\ \text{telsa} \cdot \text{meter}^2\}$
$B \equiv$ magnetic field, tesla (T)
$A \equiv$ area (m^2)
$\phi \equiv$ angle between B and A



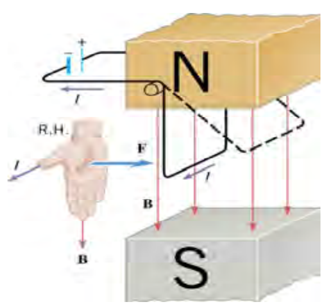


High Magnetic Flux



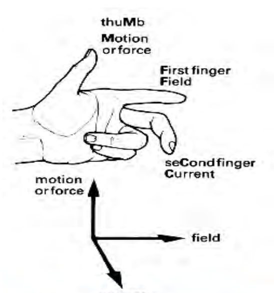
Low Magnetic Flux

4. Force acting on a current – carrying wire



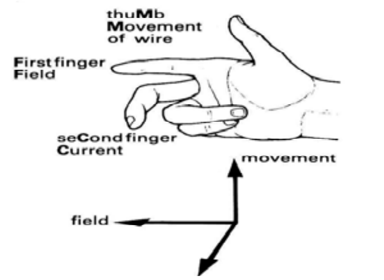
$$F = ILB\sin\theta$$

Note: θ Is the angle between **I** (conventional current) and **B**



Fleming's Left Hand Rule for Motors

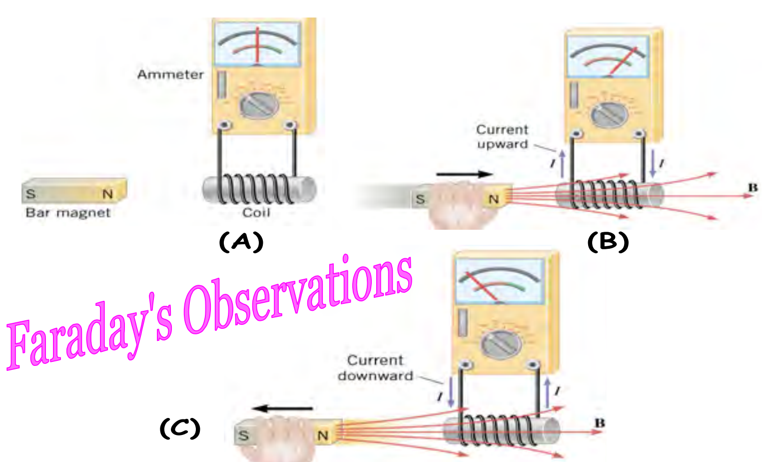
TRY IT!!!



Fleming's Right Hand Rule
for Generators

Faraday's Law

"The induced electromotive force or EMF in any closed circuit is equal to the rate of change of the magnetic flux through the circuit."



Lenz's law

"Lenz's law states that the induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop."

Faraday's Law stated mathematically

$$\varepsilon = -N\left(\frac{\Phi - \Phi_0}{t - t_0}\right) = -N\frac{\Delta\Phi}{\Delta t}$$

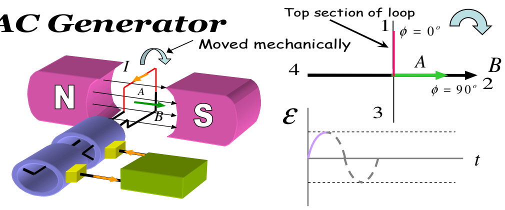
$N \equiv$ no. of turns or loops

$\Delta\Phi \equiv$ change in flux (tesla.meter^2 $\equiv 1 weber(Wb)$)

$\Delta t \equiv$ change in time (s)

Application of Faraday's law of Electromagnetic Induction

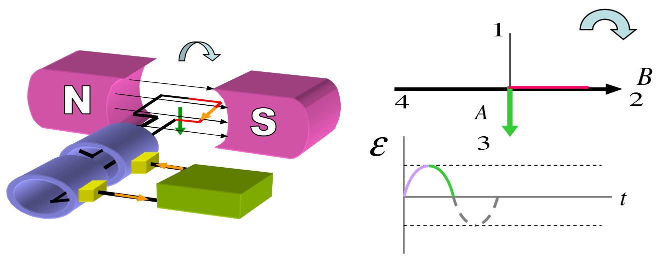
AC Generator



ϕ is the $\angle$ between A (Green Vector) and B

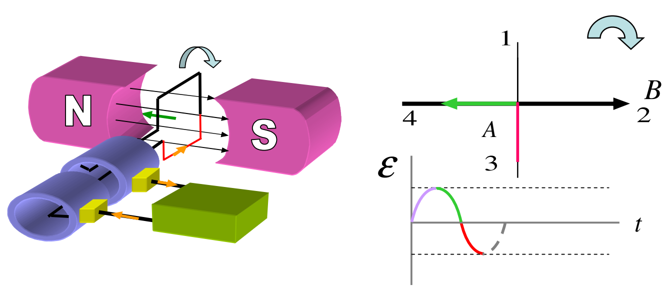
As the ***loop*** is rotated from the position 1 ($\phi = 0^o$) to 2 ($\phi = 90^o$) the flux (involving vectors A and B) is positive and decreasing.

$\Delta\Phi \prec 0$ and $\varepsilon \succ 0$



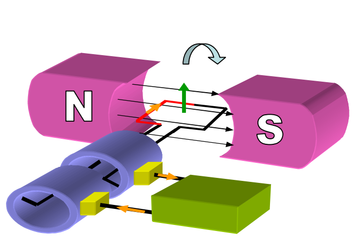
As the ***loop*** is rotated from the position 2 ($\phi = 90^o$) to 3 ($\phi = 180^o$) the flux is negative and increasing.

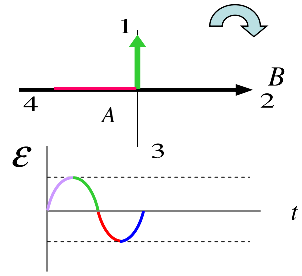
$\Delta\Phi \prec 0$ and $\varepsilon \succ 0$



As the ***loop*** is rotated from the position 3 ($\phi = 180^o$) to 4 ($\phi = 270^o$) the flux is negative and decreasing.

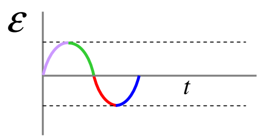
$\Delta\Phi \succ 0$ and $\varepsilon \prec 0$





As the ***loop*** is rotated from the position 4 ($\phi = 270^o$) to 1 ($\phi = 360^o$) the flux is positive and increasing.

$$\Delta\Phi \succ 0 \text{ and } \varepsilon \prec 0$$



$\phi = \omega t$ (From circular motion)

$$\begin{aligned} \varepsilon &= -N\frac{d\Phi}{dt} \\ &= -N\frac{d(BA\cos\phi)}{dt} \\ &= -N\frac{d(BA\cos\omega t)}{dt} \\ &= NBA\omega\sin\omega t \\ &= \varepsilon_o\sin\omega t \end{aligned}$$

$\varepsilon = \varepsilon_o \sin\omega t$

or

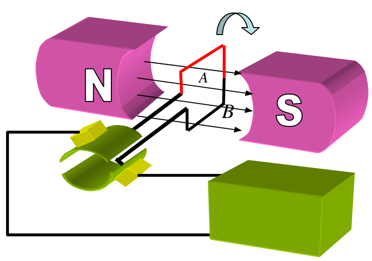
$V = V_{max} \sin\omega t$

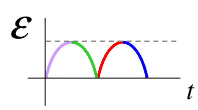
Note:

$\omega = 2\pi f$

1 rev = 2π rad

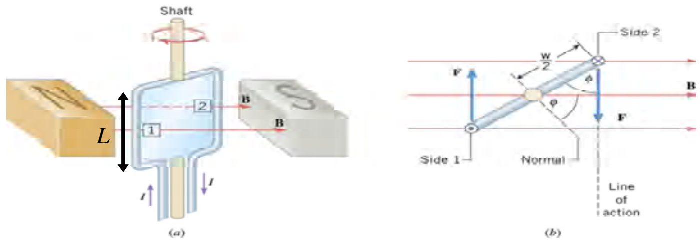
DC Generator





Similar to AC generator except the contacts to the rotating loop are made by a **split ring** or commutator. Here the output voltage always has the same polarity and the current is a pulsating DC current. The contacts to the split rings reverse their role every half-cycle. At the same time the polarity of the induced emf reverses and hence the polarity of the split ring (which is the same as the output voltage) remains the same.

Torque on a current –carrying coil



$$F = ILB\sin\theta$$

$$\tau = rF\sin\phi$$

For $\theta = 90^0$ i.e. $I \perp B$,

$F = ILB$

$$\begin{aligned}\tau &= rF\sin\phi \\ &= rILB\sin\phi \\ &= 2\times\left(\tfrac{w}{2}\right)(ILB)\sin\phi \\ &= wLIB\sin\phi \\ &= AIB\sin\phi\end{aligned}$$

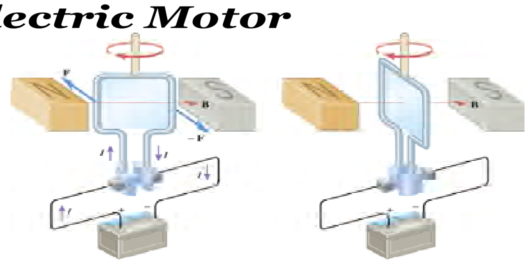
For N turns:

$$\tau = NAIB\sin\phi$$

Maximum torque:

$$\tau_{max} = NAIB$$

Electric Motor



In (a), the loop experiences a torque and rotates clock-wise. Fig (b) shows that at some point in the rotation the brushes momentarily loose contact with the split rings and no current flows in the coil. But the Inertia of the coil causes it to continue rotating. The brushes eventually make contact again with the split rings and the process continues. Split rings ensure a unidirectional current.

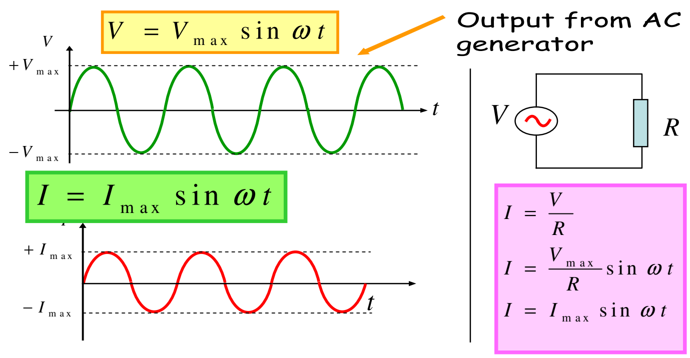
Uses of AC Generators

The main generators in nearly all electric power plants are AC generators. This is because a simple electromagnetic device called a transformer makes it easy to increase or decrease the voltage of alternating current. Almost all household appliances utilize AC.

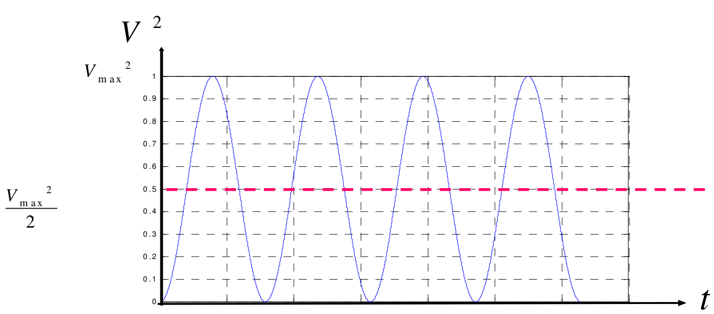
Uses of DC Generators

Factories that do electroplating and those that produce aluminium, chlorine, and some other industrial materials need large amounts of direct current and use DC generators. So do locomotives and ships driven by diesel-electric motors. Because commutators are complex and costly, many DC generators are being replaced by AC generators combined with electronic rectifiers.

Alternating Current



Note that V_{av} and I_{av} are both zero so they convey little information about the actual behaviour of V and I. A more useful and appropriate type of average called the rms (root mean squared) is used.



$$V^2 = V_{max}^2 \sin^2 \omega t$$

$$V_{av}^2 = \frac{V_{max}^2}{2}$$

$$V_{rms} = \sqrt{V_{av}^2} = \sqrt{\frac{V_{max}^2}{2}} = \frac{1}{\sqrt{2}} V_{max}$$

Similarly:

$$I_{rms} = \frac{1}{\sqrt{2}} I_{max}$$

$$I_{rms} = \frac{1}{\sqrt{2}} I_{max}$$

$$V_{rms} = \frac{1}{\sqrt{2}} V_{max}$$

$$P_{rms} = I_{rms} V_{rms}$$

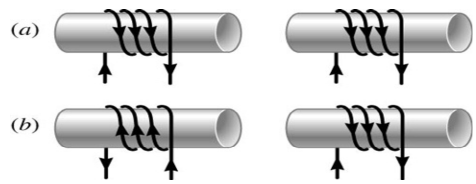
In SA our mains supply is 220V (rms) AC (50 Hz).
What is the peak or maximum voltage?

$$\begin{aligned} V_{max} &= \sqrt{2} \times V_{rms} \\ &= \sqrt{2} \times 220V \\ &= \underline{311.13V} \end{aligned}$$

Exercises

Problem 1

For each electromagnet at the left of the drawing, explain whether it will be attracted to or repelled from the adjacent electromagnet at the right.

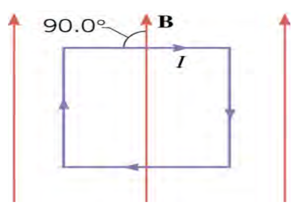


Problem 2

The 1200 turn coil in a dc motor has an area per turn of 1.1×10^{-2} m^2. The design for the motor specifies that the magnitude of the maximum torque is 5.8 N · m when the coil is placed in a 0.20 T magnetic field. What is the current in the coil?

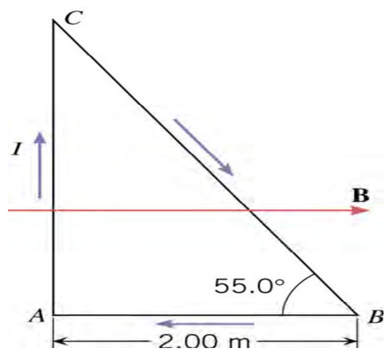
Problem 3

A square coil of wire containing a single turn is placed in a uniform 0.25 T magnetic field, as the drawing shows. Each side has a length of 0.32 m, and the current in the coil is 12 A. Determine the magnitude of the magnetic force on each of the four sides.



Problem 4

The triangular loop of wire shown in the drawing carries a current of $I = 4.70$ A. A uniform magnetic field is directed parallel to side *AB* of the triangle and has a magnitude of 1.80 T. (a) Find the magnitude and direction of the magnetic force exerted on each side of the triangle.
(b) Determine the magnitude of the net force exerted on the triangle.



Problem 5

Two pieces of the same wire have the same length. From one piece, a square coil containing a single loop is made. From the other, a circular coil containing a single loop is made. The coils carry different currents. When placed in the same magnetic field with the same orientation, they experience the same torque. What is the ratio I_{square}/I_{circle} of the current in the square coil to that in the circular coil?

Problem 6

A generator has a square coil consisting of 248 turns. The coil rotates at 79.1 rad/s in a 0.170 T magnetic field. The peak out put of the generator is 75.0 V. What is the length of one side of the coil?

Problem 7

The maximum strength of the earth's magnetic field is about 6.9×10^{-5} T near the south magnetic pole. In principle, this field could be used with a rotating coil to generate 60.0 Hz ac electricity. What is the minimum number of turns (area per turn = 0.022 m^2) that the coil must have to produce an rms voltage of 120 V?

Problem 8

The coil within an ac generator has an area per turn of 1.2×10^{-2} m^2 and consists of 500 turns. The coil is situated in a 0.13 T magnetic field and is rotating at an angular speed of 34 rad/s. What is the emf induced in the coil at the instant when the normal to the loop makes an angle of 27° with respect to the direction of the magnetic field?

Problem 9

An emf is induced in a conducting loop of wire 1.12 m long as its shape is changed from square to circular. Find the average magnitude of the induced emf if the change in shape occurs in 4.25 s and the local 0.105-T magnetic field is perpendicular to the plane of the loop.

Electric Circuits

Overview of concepts

- ***Current*** – *rate of flow of electric charge, I (A)*

$$I = \frac{\Delta Q}{\Delta t}$$

$$1\,A \equiv 1\,C \cdot s^{-1}$$

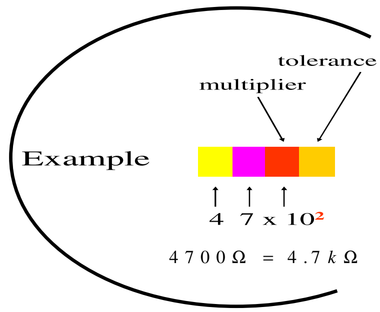
- ***Resistance*** – *opposition to current flow, R (Ω)*
  - *Temperature dependent.*
- ***EMF*** – *voltage measured when battery is not supplying current to an external circuit.*
- ***PD*** – *voltage measured when battery is supplying current to an external circuit.*

Ohm's law

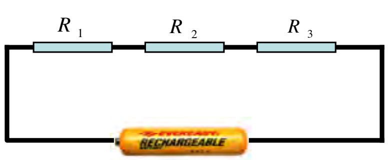
$$I = \frac{V}{R}$$

Resistor Colour Code

Black	0	
Brown	1	
Red	2	
Orange	3	
Yellow	4	
Green	5	
Blue	6	
Violet	7	
Gray	8	
White	9	
Gold		5%
Silver		10%

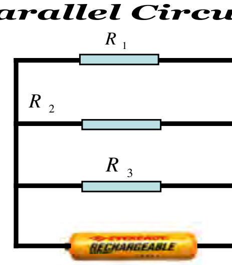


Series Circuit



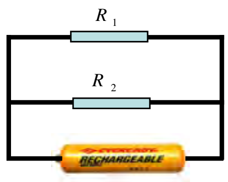
$$R_s = R_1 + R_2 + R_3$$

Parallel Circuit



$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Resistors in parallel (special case)



$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_p = \frac{R_1 \times R_2}{R_1 + R_2}$$

Product (numerator)

Sum (denominator)

Problems: Equivalent Resistance and Circuit Analysis

Equivalent Resistance Problem 1

Given three resistors, $R_1 = 100\ \Omega$, $R_2 = 30\ \Omega$ and $R_3 = 15\ \Omega$. Find the equivalent resistance when:

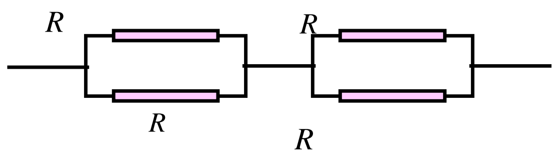
(i) they are all connected in series

(ii) they are all connected in parallel

DRAW DIAGRAMS FOR EACH

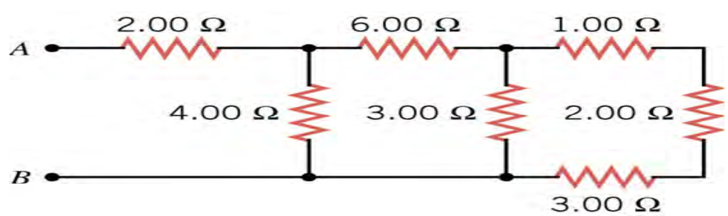
Equivalent Resistance Problem 2

You have four identical resistors, each with a resistance of R. You are asked to connect these four together so that the equivalent resistance of the resulting combination is R. How many ways can you do it? There is more than one way. Justify your answers.



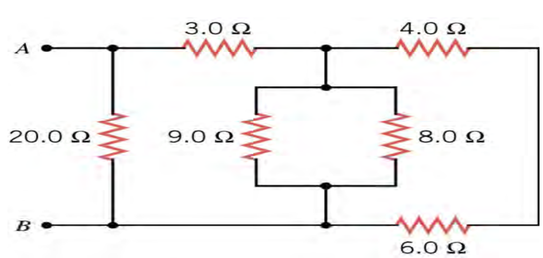
Equivalent Resistance Problem 3

Find the equivalent resistance between points A and B in the drawing.



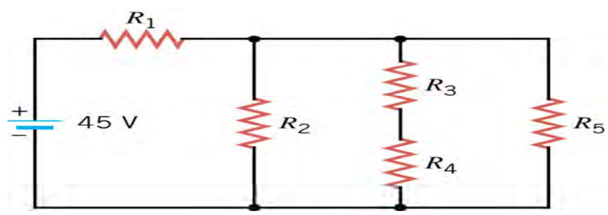
Equivalent Resistance Problem 4

Determine the equivalent resistance between the points A and B for the group of resistors in the drawing.



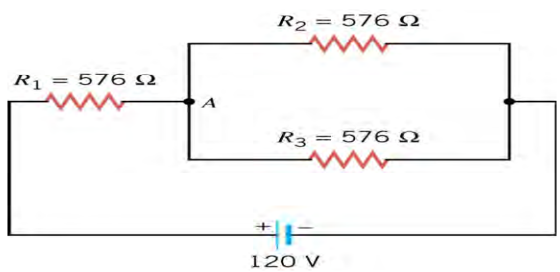
Circuit Analysis Problem 1

The circuit in the drawing contains five identical resistors. The 45-V battery delivers 58 W of power to the circuit. What is the resistance R of each resistor?



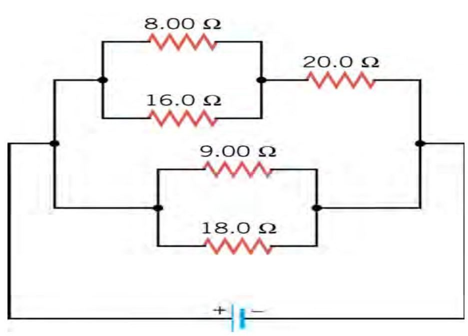
Circuit Analysis Problem 2

Determine the power supplied to each of the resistors in the drawing.



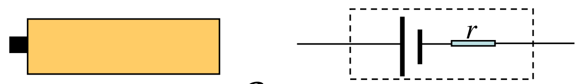
Circuit Analysis Problem 3

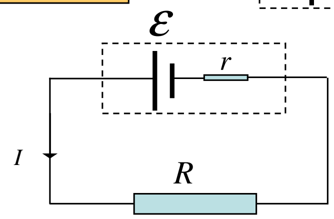
The current in the 8.00 W resistor in the drawing is 0.500 A. Find the current in (a) the 20.0 W resistor and in (b) the 9.00 W resistor.



Internal Resistance

A real battery has internal resistance.





$$\varepsilon = pd + Ir$$
$$= IR + Ir$$
$$= I(R + r)$$

$$\boxed{\varepsilon = I(R + r)}$$

If $R \succ\succ r$, the effect of internal resistance is negligible. $PD \cong \varepsilon$

Problems: Internal Resistance

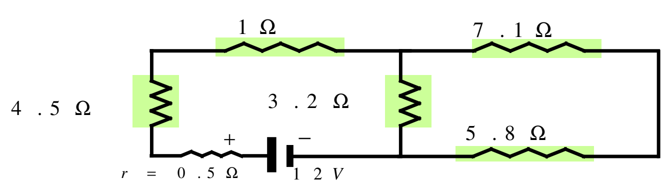
Internal Resistance Problem 1

Given the circuit below. The battery has an EMF of 12 V and an internal resistance of 0.5Ω. Determine:

(i) The current flowing through the 7.1Ω and 3.2Ω resistors.

(ii) The current flowing through the battery

(iii) The PD between the terminals of the battery



Internal Resistance Problem 2

A battery delivering a current of 55.0 A to a circuit has a terminal voltage of 23.4 V. The electric power being dissipated by the internal resistance of the battery is 34.0 W. Find the EMF of the battery.

Internal Resistance Problem 3

A 75.0 Ω and a 45.0 Ω resistor are connected in parallel. When this combination is connected across a battery, the current delivered by the battery is 0.294 A. When the 45.0 Ω resistor is disconnected, the current from the battery drops to 0.116 A. Determine

(a) the EMF and

(b) the internal resistance of the battery.

Practical Investigation

A how to guide .... 9 STEPS

1. **Observation**
2. **Question**
3. **Hypothesis (Prediction)**
4. **Variables**
5. **Procedure**
6. **Materials**
7. **Data Tables**
8. **Conduct Investigation**
9. **Conclusion**

STEP 1: *Observation*

- ***A list of facts that describe an object.***
- ***It involves the five senses.***
- ***A good observation is detailed, accurate, unbiased, informative, helpful.***

STEP 2: *Question*

- ***A good scientific question will always***

  inform or enlighten the investigation

  i.e. the way forward becomes clear.

STEP 3: *Hypothesis*

- ***A predicted answer to you research question.***
- ***It is always written in the IF…THEN… BECAUSE…format***

STEP 4: *Variables*

- *Controlled Variables* – ***the ones that remain the same throughout the investigation.***
- *Independent variable* – ***the one we can control (i.e. change).***
- *Dependent variable* – ***respond to changes made to the independent variable.***

STEP 5: *Procedure*

- ***A scientific procedure (recipe) containing a comprehensive set of steps that ensure the reproducibility of the desired results of an investigation.***

STEP 6: *Materials*

- ***An "all you need list" of items for the accomplishment of the investigation.***

STEP 7: *Data tables*

- ***A data table is used to record of all measurements made during the investigation.***
- ***A data table should include the dependent and independent variable.***

STEP 8: *Conduct Investigation*

- ***Do exactly what you set out to do in the STEP 4 of the process.***

STEP 9: *Conclusion*

- ***A good conclusion will answer the research question set out in STEP 1.***
- ***Average data obtained in the investigation will be quoted and compared.***

Putting the 9-Step Procedure (9SP) into practice.

Practical Investigation

Capacitance of a parallel Plate Capacitor

Instructions to learner.

We know that a parallel plate capacitor is made up of two identical plates that are parallel to each other and some distance apart from each other.
Design a practical investigation on the capacitance of the parallel plate capacitor using the 9–Step method above.

Electric Circuits

Practical 1: *Internal Resistance*

Task:

(i) Design an experiment using appropriate materials to show a distinct difference between EMF and PD.

(ii) Use the circuit to determine the internal resistance of a single cell.

Note: One could use more than one cell.

Practical 2: *Resistor Networks*

Materials List

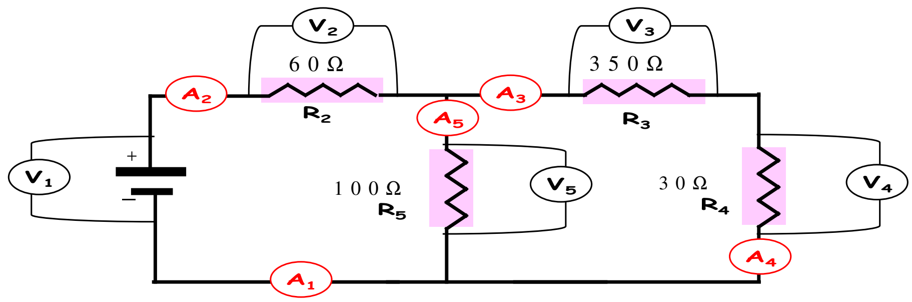
4 x (1.5 V) cells
5 Voltmeters
5 Ammeters
4 x 1k Ω resistors
4 x 100 Ω resistors
4 x 10 Ω resistors

Task

(i) Design the series-parallel network shown below using your knowledge of equivalent resistance and the given materials.

(ii) Connect the ammeters and voltmeters as shown in the circuit below.

(iii) Tabulate all the voltmeter (V) and ammeter readings (A)



	1	2	3	4	5
Ⓥ					
Ⓐ					

Questions:

1. Compare $\mathbf{A_1}$ and $\mathbf{A_5}$. What can you conclude?
2. Compare $\mathbf{A_1}$ and $\mathbf{A_2}$ and $\mathbf{A_3}$. What can you conclude?
3. Compare $\mathbf{A_2}$ and $\mathbf{A_4}$. What can you conclude?
4. Compare $\mathbf{A_3}$ and $\mathbf{A_4}$and $\mathbf{A_5}$. What can you conclude?
5. Compare $\mathbf{V_1}$, $\mathbf{V_2}$ and $\mathbf{V_3}$. What can you conclude?
6. Compare $\mathbf{V_3}$, $\mathbf{V_4}$ and $\mathbf{V_5}$. What can you conclude?
7. Which voltmeter and ammeter reading would you use determine the total resistance in the circuit.

Physics Test

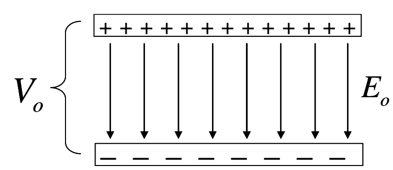
Capacitors, Electric Circuits and Electrodynamics

QUESTION 1: CAPACITORS (12 MARKS)

1.1 The plates of a particular parallel plate capacitor are uncharged. Is the capacitance of the capacitor zero? Explain.

[2]

1.2 Given the parallel-plate capacitor below having a capacitance, C_o.



A dielectric material, having a dielectric constant κ, is inserted between the plates. Using appropriate equations show that the capacitance of the capacitor increases.

[5]

1.3 As a crude model for lightning, consider the ground to be one plate of a parallel-plate capacitor and a cloud at an altitude of 550 m to be the other plate. Assume the surface area of the cloud to be the same as the area of a square that is 0.50 km on a side.

1.3.1 What is the capacitance of this capacitor?

[2]

1.3.2 How much charge can the cloud hold before the dielectric strength of the air is exceeded and a spark (lightning) results?

[3]

QUESTION 2: ELECTRIC CIRCUITS (11 MARKS)

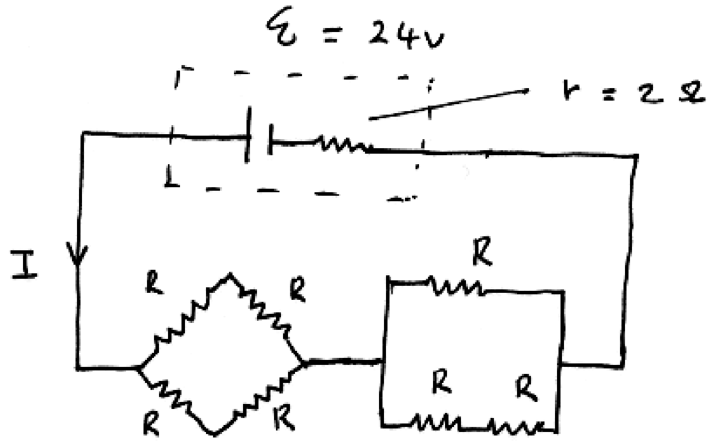
2.1 A number of light bulbs are to be connected to a single electrical outlet. Will the bulbs provide more brightness if they are connected in series or in parallel? Why?

[3]

2.2 You have four identical resistors, each with a resistance of *R*. You are asked to connect these four together so that the equivalent resistance of the resulting combination is $4R/3$.

[2]

2.3 Given a battery of EMF 24 volts having an internal resistance of 2Ω. When the battery is connected across a network (shown below) of identical resistors, the current in the circuit is 2A. Determine the resistance of a single resistor.



QUESTION 3: ELECTRODYNAMICS (12 MARKS)

3.1 Two coils have the same number of circular turns and carry the same current. Each rotates in a magnetic field. Coil 1 has a radius of 5.0 cm and rotates in a 0.18-T field. Coil 2 rotates in a 0.42-T field. Each coil experiences the same maximum torque. What is the radius (in cm) of coil 2?

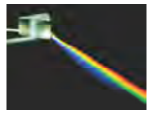
[4]

3.2 A magnetic field increases from 0 to 0.2T in 1.5s. How many turns of wire are needed in a circular coil of 12 cm diameter to produce an induced EMF of 6.0V?

[4]

3.3 A rectangular coil 25 cm by 35 cm has 120 turns. This coil produces an RMS voltage of 65 V when it rotates with an angular speed of 190 rad/s in a magnetic field of strength *B.* Find the value of *B.*

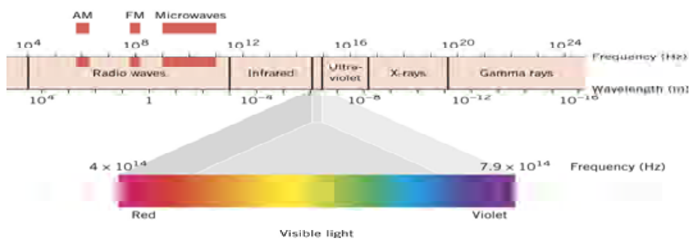
[4]



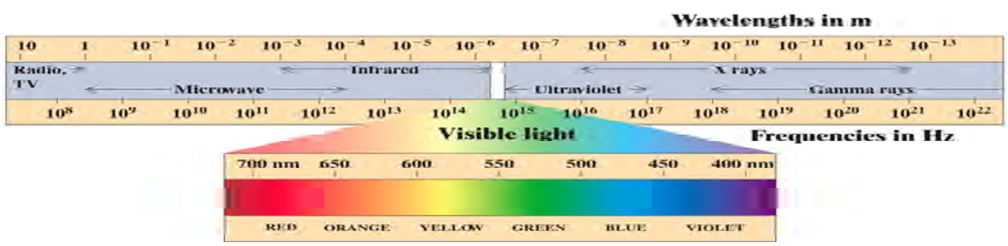
COLOUR
&
COLOUR MIXING

1. Introduction

Visible light is a small part of the complete **electromagnetic spectrum**



Red light has the largest λ, while **violet light** has the smallest λ



Each colour runs smoothly into the next, but one can assign approximate *wavelength ranges to each colour*

White light contains all colours

The wavelength and frequency of light are related by

$$c = f\lambda$$

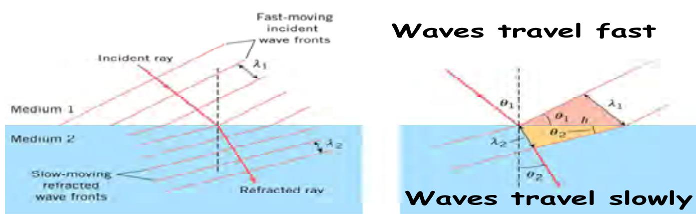
(c = 3.00 × 10⁸ m/s) in vacuum

<u>Example 1</u> Calculate the wavelength of light with a frequency of 6×10^{14} Hz

<u>Example 2</u> Calculate the frequency of light with a wavelength of 650 nm

2. Refraction and Dispersion of Light

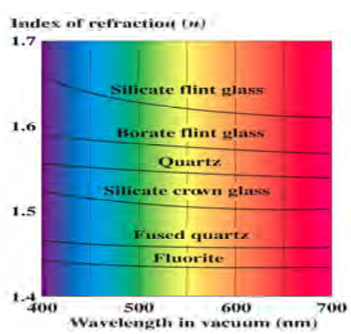
When a wave passes from one medium to another in which its speed is different, the wave is refracted (bent)



The amount the wave is bent depends on the change in speed of the wave between the two media

The speed of light in a medium (except the vacuum) depends on its wavelength (i.e. colour)

E.g., in glass, red light travels faster than violet light

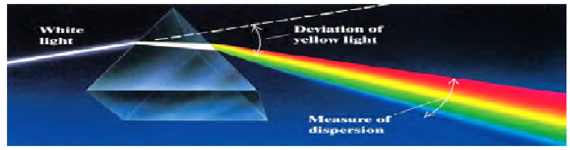


Thus, the amount light is bent or refracted by a glass prism depends on its colour

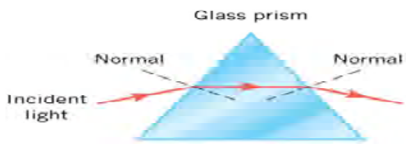
Light is said to be **DISPERSIVE**

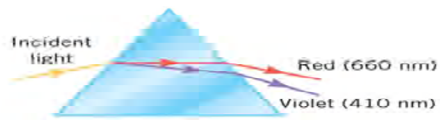
Refraction of White Light by a Prism

When **white light** is incident on a prism, the component colours are refracted by different amounts



A rainbow is seen on exiting the prism

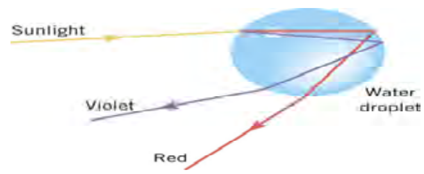


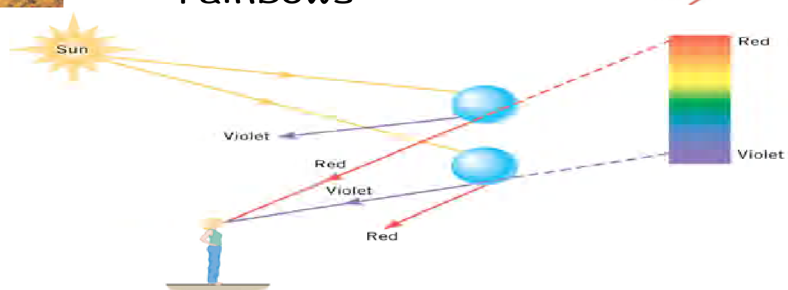


Violet deviates most
Red deviates the least

Rainbows

The dispersion of sunlight by water drops creates rainbows





3. Addition and Subtraction of Light

Spectrum colours vary smoothly from **violet** to **red**

However, we can approximate the spectrum using only three separate bands of colour called the **ADDITIVE PRIMARY COLOURS** representing equal wavelength intervals

Additive Primary Colours of Light
RED, **GREEN**, & **BLUE**
Red Light + **Green Light** + **Blue Light** = White Light

Colour Mixing by Addition of Light

Combining the additive primaries of light in various ways makes it possible to create all colours of light

E.g., two additive primaries in equal quantities:

Blue + Green = Cyan

Red + Green = Yellow

Red + Blue = Magenta

Adding all three additive primaries in equal quantities results in white light

Any pair of colours of light that combine to give white light are said to be
COMPLIMENTARY COLOURS

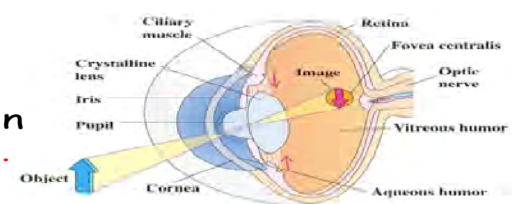
Cyan is complimentary to **Red**

Yellow is complimentary to **Blue**

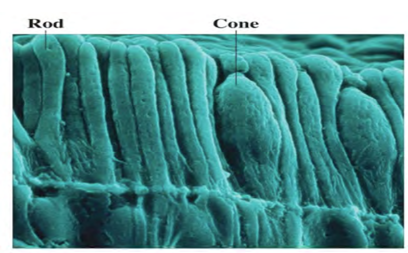
Magenta is complimentary to **Green**

The Eye

There are two types of photosensitive receptors in the retina: rods and cones.

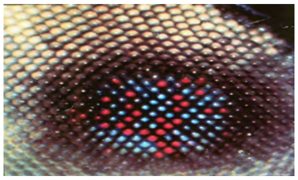


Rods convey no colour information, but are very sensitive to light. They predominate closer to the periphery of the retina.

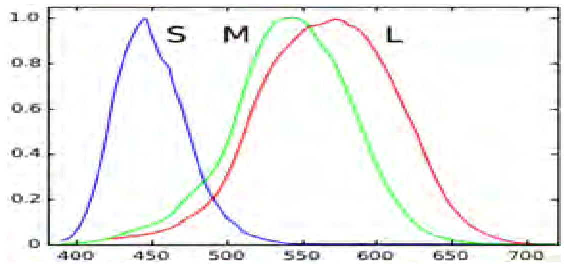


(b) Scanning electron micrograph showing retinal rods and cones in cross section

The less abundant cones are responsible for colour vision and are very dense in the fovea.

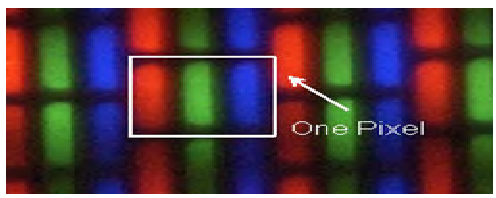


Because humans usually have three kinds of cones, with different photopsins, which respond to variation in colour in different ways, they have trichromatic vision



TV's use Colour Addition!

Each pixel contains three dots: red, green and blue



This allows a TV to reproduce a wide range of colours through colour addition when viewed from a distance

Color Mixing by Subtraction of Light

(i.e. make colours by passing white light through filters which selectively transmit)

Filters transmit only certain colours and absorb the rest

E.g. **red** filters transmit only **red** light, while they absorb **blue** and **green** light

Obviously then **red**, **green** and **blue** filters are not appropriate for making colours of light by subtraction from white light

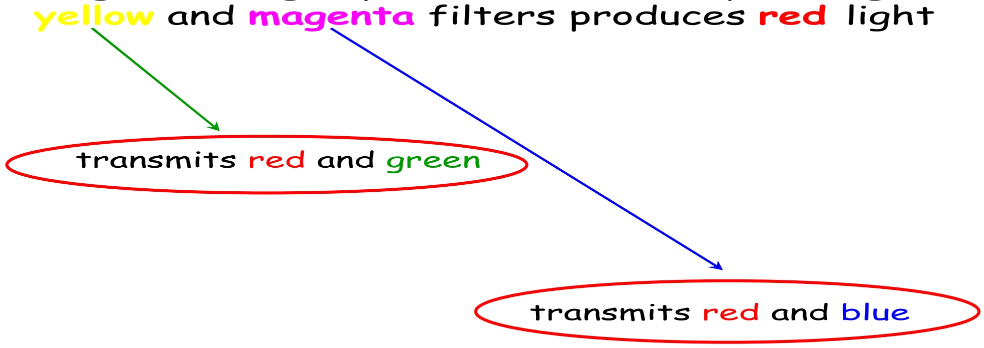
(adding these filters in succession eliminates all colours from white light!)

Filters of the so-called subtractive primary colours can, however, be used successively to make all other colours of light by subtraction

SUBTRACTIVE PRIMARY COLOURS
CYAN, MAGENTA and YELLOW

Note the subtractive primary colours are the complimentary colours of the additive primary colours

E.g. white light passed successively through yellow and magenta filters produces **red** light



4. The Colour of Objects

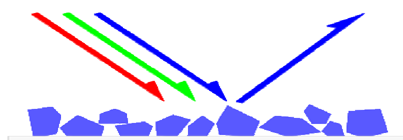
Objects appear a certain colour because of a number of factors

Some objects are sources of light (e.g. the sun, fires, light bulbs etc.) The colour they appear is affected by the physical processes occurring inside these materials

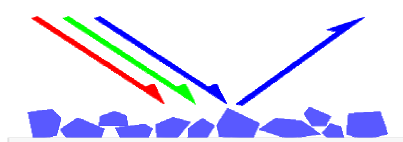
All objects absorb, reflect, or transmit light that is incident on them to varying degrees affecting the colour they appear

Selective Reflection
(the colour of opaque objects)

Opaque objects do not transmit light, but their surface may selectively reflect certain colours due to pigments on the object's surface



For example, an opaque object that appears **blue** in white light appears so because its **blue** pigments reflect only the **blue** component of the light, while absorbing the rest (**red** and **green**)



What about a **red** apple in **blue** light?

Selective Transmission
(the colour of transparent objects)

Light may take on a particular colour as it passes through a transparent object that selectively absorbs some wavelengths and transmits the rest

The object then appears the same colour as the light it is able to transmit

Example:

Red glass absorbs all colours of white light except **red**. In other words, **red** light passes through a **red glass** filter unaffected.

Similarly, a **blue** glass filter allows only **blue** light through.

Paint

Paints appear the colour that they do due to pigments that selectively absorb/reflect light

E.g. **red** paint contains **red pigment** that absorbs **green** and **blue** light and reflects only **red** light

By using different quantities of paints of the subtractive primary colours, one can make any colour of paint
(that is, the subtractive primaries are the primary colours of paint!)

Printing

The printing industry uses
colour subtraction too!

Ink selectively reflects

Photography

Film is made from three layers of photosensitive material, each of which responds to one of the additive primary colours

When developed, dye images in one of the subtractive primaries form in each layer

The varying densities of these filters control the light that passes through

Colour Exercises

1. Why will the leaves of a rose be heated more than the petals when illuminated with red light?

2. What are the complements of
   a) cyan,
   b) yellow,
   c) red?

3.
   a) red light + blue light =
   b) white light – red light =
   c) white light – blue light =

4. Why do people in hot desert countries wear white clothes?

5. If sunlight were green instead of white, what colour garment would be most advisable on an uncomfortably hot day? On a very cold day?

6. What colour would red cloth appear if it were illuminated by sunlight? By cyan light?

7. Why does a white piece of paper appear white in white light, red in red light, blue in blue light, and so on for every colour?

8. How could you use spotlights at a play to make yellow clothes of the performers suddenly change to black?

9. White light passes through a green filter and is observed on a screen. Describe how the screen will look if a second green filter is placed between the first filter and the screen. Describe how the screen will look if a red filter is placed between the green filter and the screen.

10. White light passes through a cyan filter, which is, in turn, followed by a second filter. What colour emerges if the second filter is

    a) yellow,
    b) magenta,
    c) blue,

d) green?

11. What colour results from the addition of equal intensities of

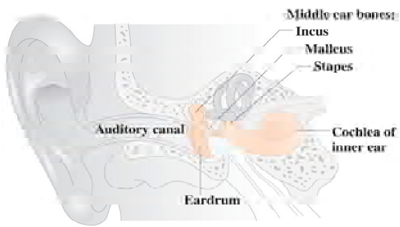
    a) magenta and green light, and
    b) blue and yellow light?

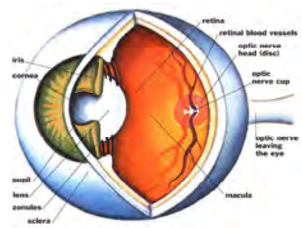
12. White light passes through a yellow filter, which is, in turn followed by a second filter. What colour emerges if the second filter is

    a) green,
    b) cyan,
    c) magenta, or
    d) blue?

13. Why is the sky blue except at sunset when it turns reddish?

Doppler Effect





What is the Doppler Effect?

Sound:

"A change in frequency heard by a listener due to relative motion between the sound source and the listener"

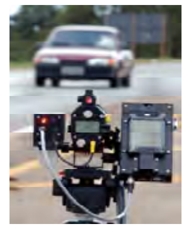
Light:

"A change in colour seen by an observer due to relative motion between the light source and the observer"

Some Everyday Examples:

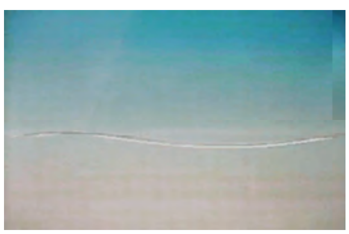
Sound waves: The pitch of an ambulance siren changes as the ambulance passes you

Water waves: The bow wave of a ship is an example of the Doppler Effect



Light waves: The radar guns used by traffic cops utilise the Doppler Effect

1. Wave Basics

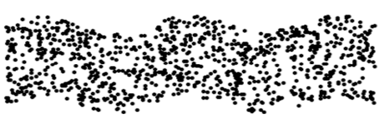


A wave is a travelling disturbance carrying energy from one place to another

NB: Disturbance and energy move
No bulk movement of material

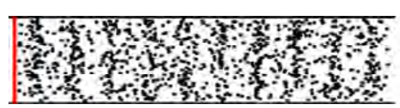
Transverse waves

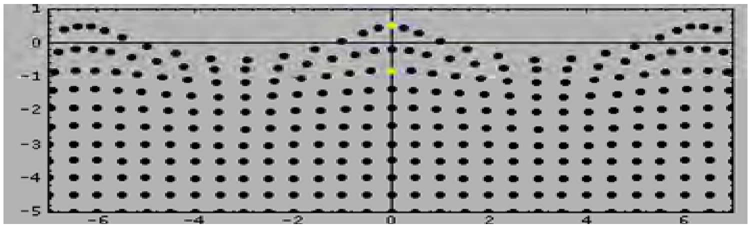
Disturbance occurs perpendicular to the direction of travel of the wave



Longitudinal waves

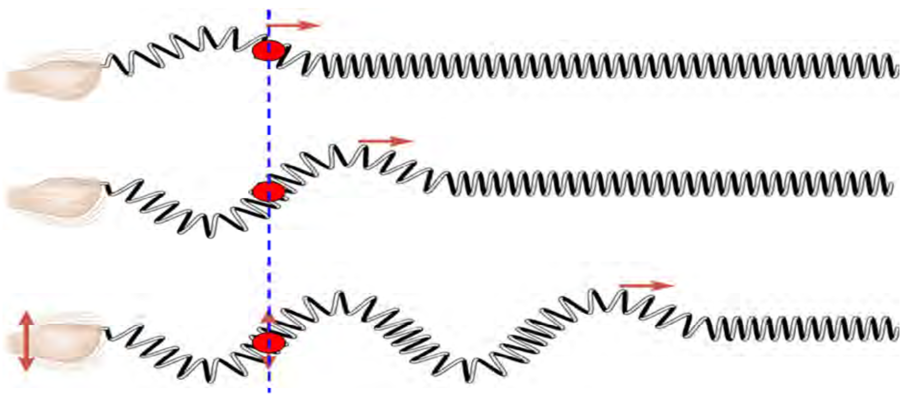
Disturbance occurs parallel to the direction of travel of the wave





Water waves are neither longitudinal nor transverse, but rather a combination of the two

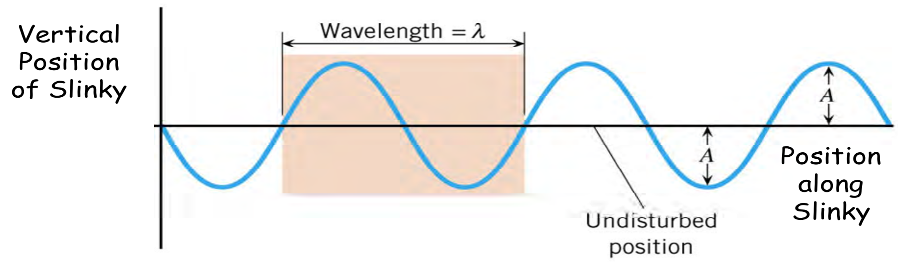
Example: Transverse Wave in a Slinky



Position of slinky depends on two things:

- when you look
- where you look

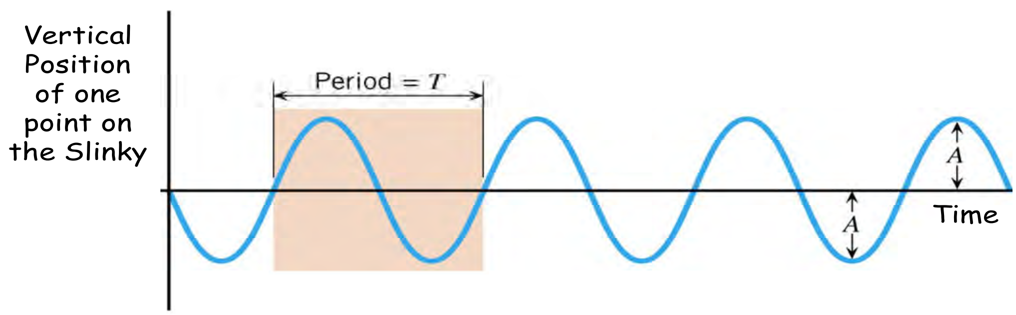
At particular time (i.e. a photo taken of the slinky):



Amplitude A: the maximum excursion of a particle of the medium from the particle's undisturbed position

Wavelength λ: the horizontal length of one cycle of the wave

At particular point on slinky:



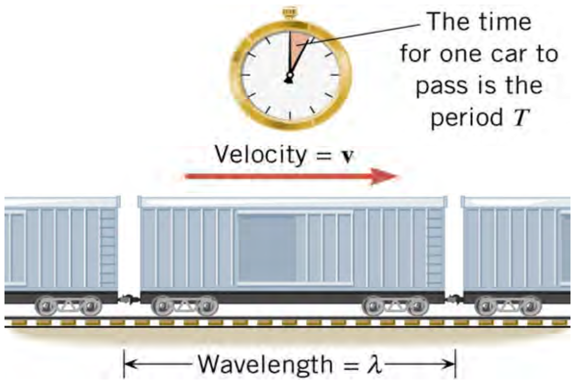
Period T: the time required for a single point on the wave to complete one up-down cycle

OR

the time it takes the wave to travel a distance of one wavelength

Frequency, Period and Wavelength relations

$$f = \frac{1}{T}$$



$$v = \lambda / T$$

$$v = f\lambda$$

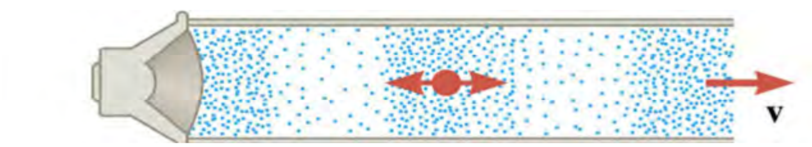
Graphs of Wave Motion

2. Sound

Sound waves are longitudinal waves (created by a vibrating object) with particles of the medium vibrating in the direction parallel to the wave's propagation

speaker diaphragm vibrates back and forth

No mass movement of air like wind!

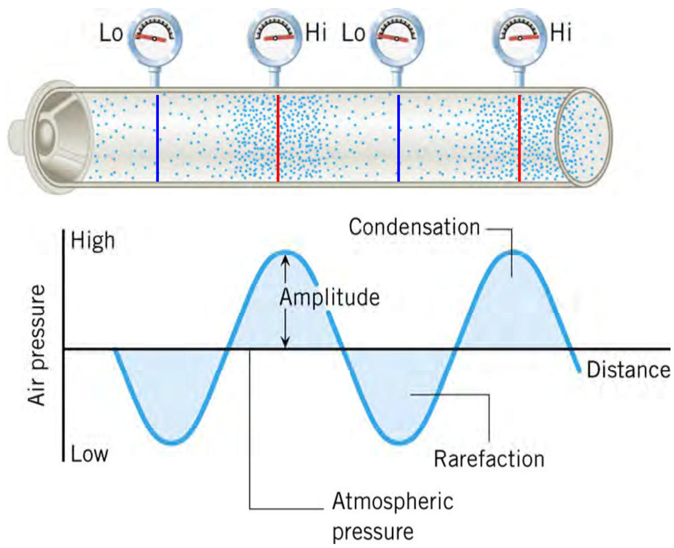


Each particle oscillates about a fixed position and collides with its neighbours passing the disturbance along

Sound waves consist of a pattern of high and low pressures propagating through space

At a particular time:

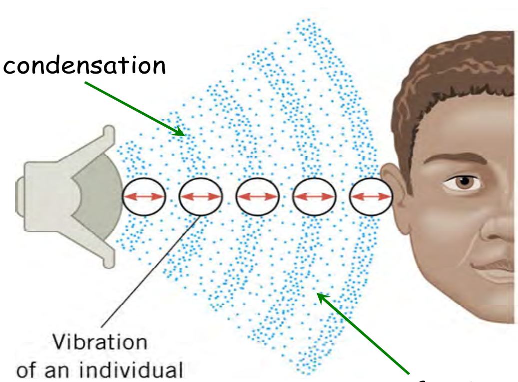
Wave front: imaginary line connecting neighbouring points 'in phase'



Condensations and rarefactions arriving at the ear cause it to vibrate at the same frequency as the speaker diaphragm

The brain interprets this as sound

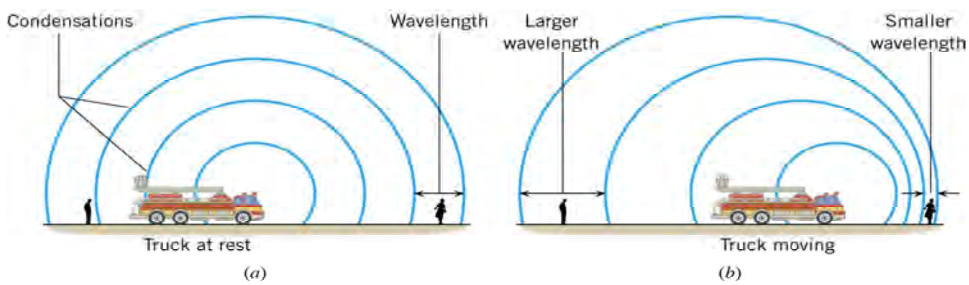
speaker diaphragm vibrates back and forth



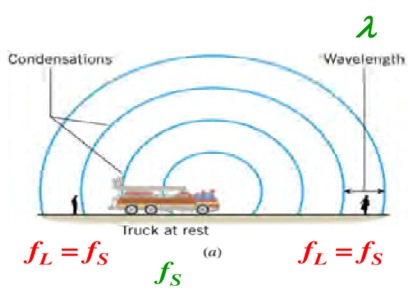
3. The Doppler Effect (Sound)

When either the listener or the sound source move, the frequency heard by the listener is different to that when both are stationary

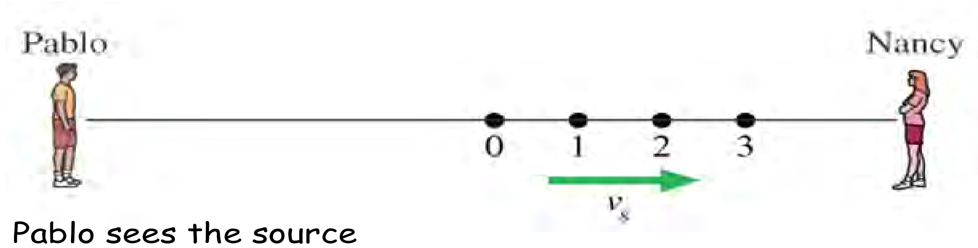
pitch changes!



3.1 Case 1: Moving Source Stationary Listener



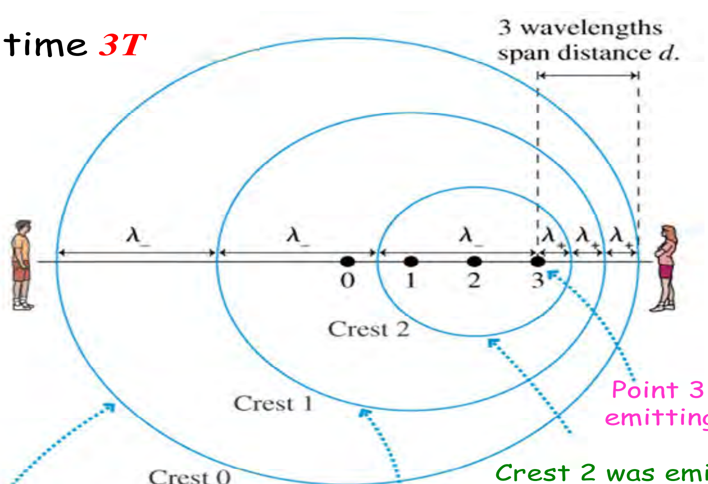
The dots are the positions of the source at $t = 0, T, 2T$ and $3T$



Pablo sees the source receding at speed v_S

Nancy sees the source approaching at speed v_S

Snapshot at time $3T$



Point 3 just emitting now

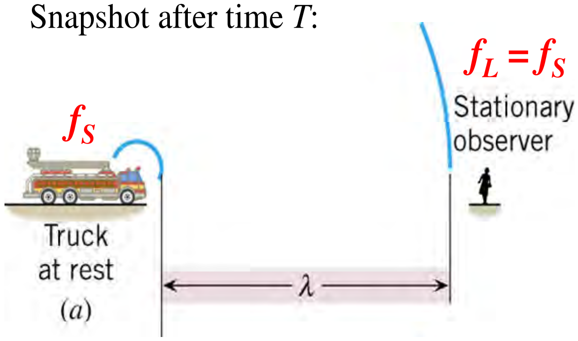
Crest 2 was emitted at $t = 2T$ (wavefront is circle of radius λ centre 2)

Crest 1 was emitted at $t = T$ (wavefront is circle of radius 2λ centre 1)

Crest 0 was emitted at $t = 0$ (wavefront is circle of radius 3λ centre 0)

Consider source moving towards stationary listener:

Snapshot after time T:



source speed

$$\lambda' = \lambda - v_S T$$

sound speed

$$v = \frac{\lambda'}{T_L}$$

$$f_L = \frac{v}{\lambda'}$$

$$f_L = \frac{v}{\lambda'} \qquad \left(\lambda' = \lambda - v_S T\right)$$

Moving Source:

$$f_L = f_S \frac{v}{v \pm v_S}$$

+ : source away
− : source towards

Example 1

A whistle of frequency 540 Hz moves in a circle at a constant speed of 24.0 m/s. What are (i) the lowest and (ii) the highest frequencies heard by a listener a long distance away at rest with respect to the centre of the circle?

Example 2

You are standing at x = 0 m, listening to a sound that is emitted at frequency f_S . At t = 0 s, the sound source is at x = 20 m and moving toward you at a steady 10 m/s.

Draw a graph showing the frequency you hear from t = 0 s to t = 4 s.

Important:

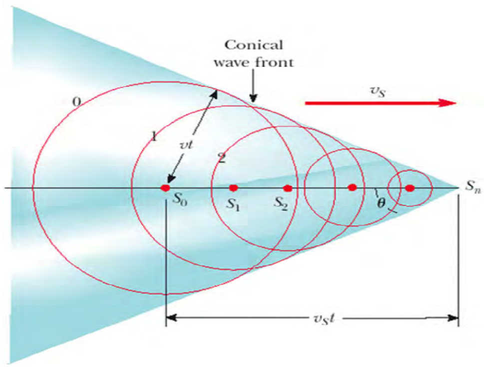
With the source approaching the listener, the pitch heard by the listener is higher than when the source is stationary.

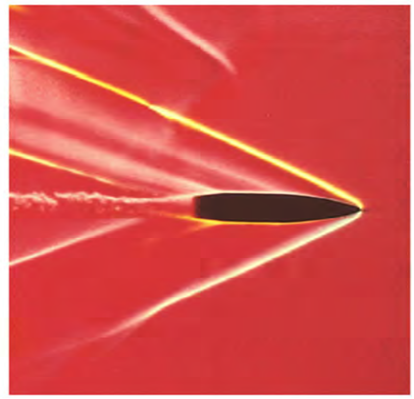
However, as the source gets closer, the pitch does not increase further; only the loudness increases!

As the source passes and begins to recede from the listener, the pitch heard by the listener drops to a value that is lower than when the source is stationary.

However, as the source recedes, the pitch does not decrease further; only the loudness drops!

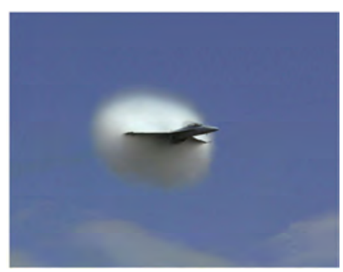
Extreme Case: Source moving at speed of sound or faster





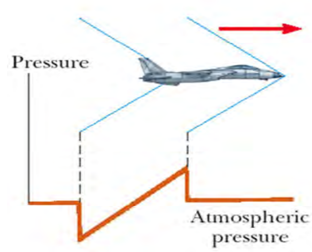
Source Faster Than Speed of Sound

Other Doppler examples with moving sources:

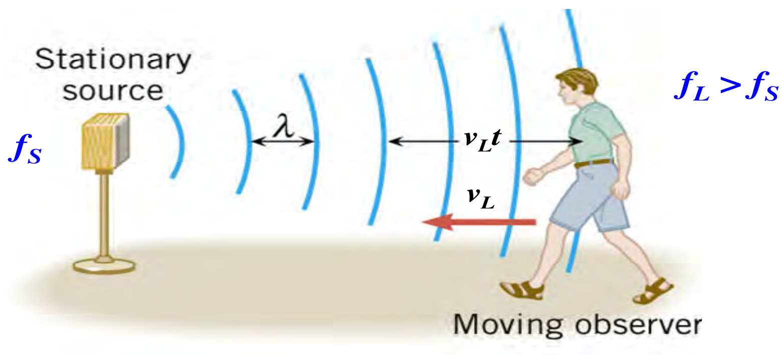


Sonic boom

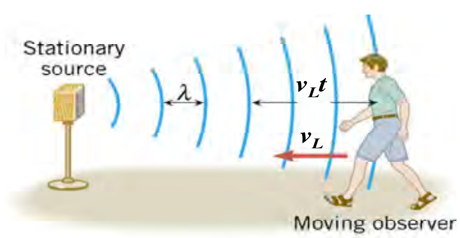
Bow wave
(water waves)



3.2 Case 2: Stationary Source Moving Listener



Consider listener moving towards stationary source:



$$f_L = f_S + \textbf{?}$$

Unlike in Case 1, the waves are not squashed or stretched

Moving Listener:

$$f_L = f_S\left(\frac{v \pm v_L}{v}\right)$$

+ : listener towards
– : listener away

Case 1
moving source

$$f_L = f_S \frac{v}{v \pm v_S}$$

Case 2
moving listener

$$f_L = f_S \left(\frac{v \pm v_L}{v}\right)$$

$$f_L = f_S \frac{v \pm v_L}{v \pm v_S}$$

Numerator $\pm$:
+ listener towards
– listener away

Denominator $\pm$:
+ source away
– source towards

NB: applies only in frame where medium is at rest!

Example 3

A French submarine and a U.S. submarine move head-on during manoeuvres in motionless water in the North Atlantic. The French sub moves at 50.0 km/h, and the U.S. sub at 70.0 km/h. The French sub sends out a sonar signal (sound wave in water) at 1000 Hz. Sonar waves travel at 5470 km/h.

a) What is the signal's frequency as detected by the U.S. sub?

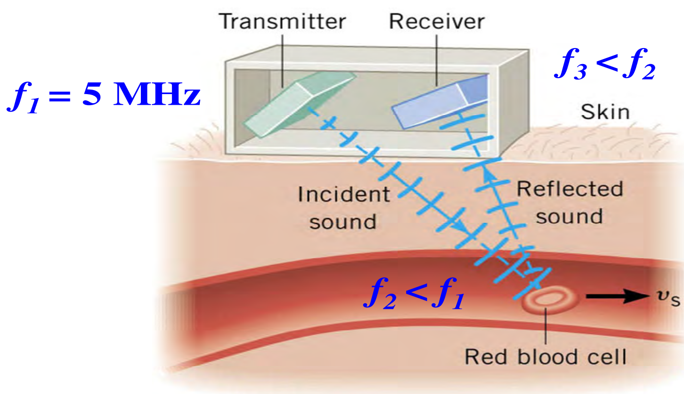
b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?

Important Fact:

When a sound wave reflects off a surface, the surface acts like a source of sound emitting a wave of the same frequency as that heard by a listener travelling with the surface

4. Applications of Doppler in Medicine

Doppler Flow Meter



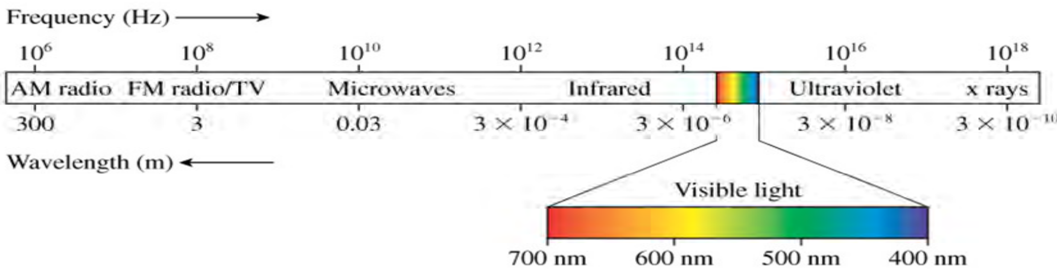
V_S ~ 0.1 m/s leads to $f_1 - f_3$ ~ 600 Hz

Used to locate regions where blood vessels have narrowed

5. The Doppler Effect (Light)

The Doppler effect applies to all waves

For example, the Doppler effect applies also to light (an electromagnetic wave)

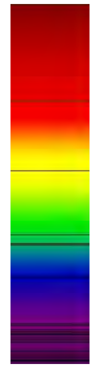


When a light source moves away from an observer, the frequency of the light observed is less than that emitted (equivalently the wavelength of the light observed is greater)

Since a shift to lower frequencies is towards the red part of the spectrum, this is called a **redshift**

Redshift

The Doppler effect for light is used in astronomy to measure the velocity of receding astronomical bodies



It is also used to measure car speeds using radio waves emitted from radar guns

Doppler Effect Exercises

Unless otherwise stated take the speed of sound in air to be 340 m/s.

1. An opera singer in a convertible sings a note at 600 Hz while cruising down the highway at 90 km/hr. What is the frequency heard by,

    a) a person standing beside the road in front of the car?
    b) a person on the ground behind the car?

2. A mother hawk screeches as she dives at you. You recall from biology that female hawks screech at 800Hz, but you hear the screech at 900 Hz. How fast is the hawk approaching?

3. A whistle you use to call your hunting dog has a frequency of 21 kHz, but your dog is ignoring it. You suspect the whistle may not be working, but you can't hear sounds above 20 kHz. To test it, you ask a friend to blow the whistle, then you hop on your bicycle. In which

direction should you ride (toward or away from your friend) and at what minimum speed to know if the whistle is working?

4. A friend of yours is loudly singing a single note at 400 Hz while racing toward you at 25.0 m/s.

   a) What frequency do you hear?
   b) What frequency does your friend hear if you suddenly start singing at 400 Hz?

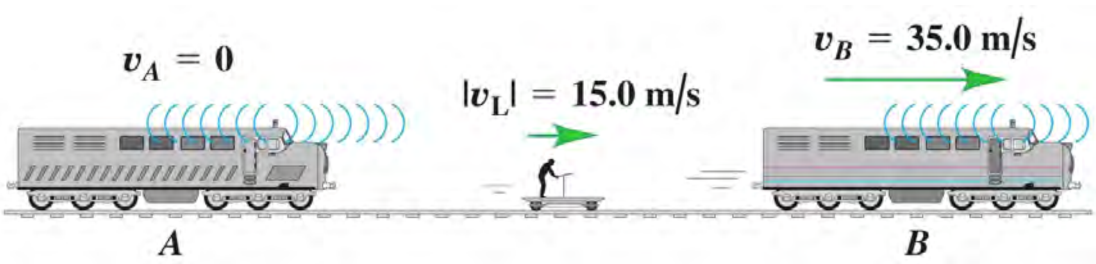
5. When a car is at rest, its horn emits a frequency of 600 Hz. A person standing in the middle of the street hears the horn with a frequency of 580 Hz. Should the person jump out of the way? Account for your answer.

6. The security alarm on a parked car goes off and produces a frequency of 960 Hz. As you drive toward this parked car, pass it, and drive away, you observe the frequency to change by 95 Hz. At what speed are you driving?

7. Suppose you are stopped for a traffic light, and an ambulance approaches you from behind with a speed of 18 m/s. The siren on the ambulance produces sound with a frequency of 955 Hz. What is the wavelength of the sound reaching your ears?

8. A speeder looks in his rear-view mirror. He notices that a police car has pulled up behind him and is matching his speed of 38 m/s. The siren on the police car has a frequency of 860 Hz when the police car and the listener are stationary. What frequency does the speeder hear when the siren is turned on in the moving police car?

9. Two train whistles, *A* and *B*, each have a frequency of 444 Hz. *A* is stationary and *B* is moving toward the right (away from *A*) at a speed of 35.0 m/s. A listener is between the two whistles and is moving toward the right with a speed of 15.0 m/s.



a) What is the frequency from *A* as heard by the listener?

b) What is the frequency from *B* as heard by the listener?

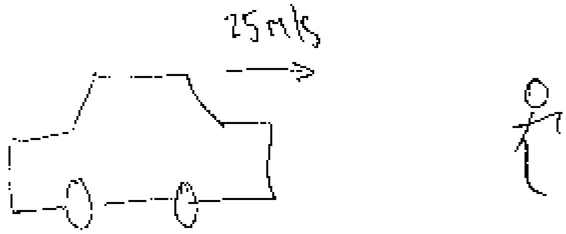
10. The siren of a fire engine that is driving northward at 30.0 m/s emits a sound of frequency 2000 Hz. A truck in front of this fire engine is moving northward at 20.0 m/s.

    a) What is the frequency of the siren's sound that the fire engine's driver hears reflected from the back of the truck?

Doppler Effect Exercises Solutions

1. 90 km/hr = $90\,000/60^2$ m/s = 25 m/s

a)



$f_s = 600$ Hz $\qquad f_L$

$$f_L = f_s \frac{v}{v - v_s} \quad \text{moving source}$$

$$= (600)\frac{(340)}{340 - 25} = 648 \text{ Hz}$$

(i.e. > 600 Hz)

b) $$f_L = f_s \frac{v}{v + v_s} \quad \text{moving source}$$

$$= (600)\frac{(340)}{340 + 25} = 559 \text{ Hz}$$

(i.e < 600 Hz)

2. $f_L = f_s \dfrac{V}{V - V_s}$ moving source 2

↳ approaching!

$$900 = 800 \frac{340}{340 - V_s}$$

$$900(340) - 900V_s = 800(340)$$

$$V_s = \frac{(900-800)(340)}{900}$$

$$= 38 \text{ m/s}$$

3. Need to ride away from friend so that the sound heard is reduced in frequency

$$f_L = f_s \left(\frac{v - v_L}{v}\right)$$ moving listener 3

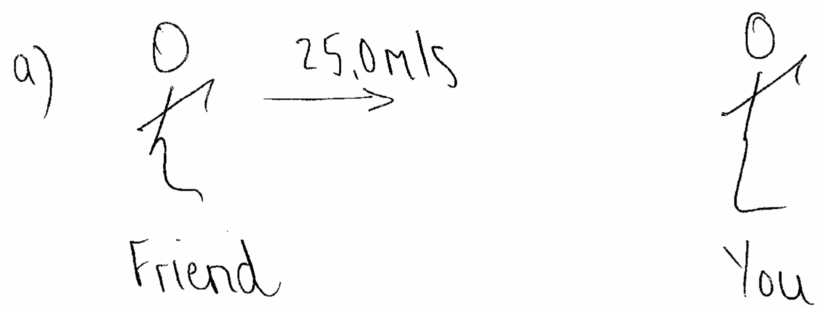
$$20 \times 10^3 = 21 \times 10^3 \left(\frac{340 - v_L}{340}\right)$$

$$v_L = 340 - \frac{(20 \times 10^3)(340)}{21 \times 10^3}$$

$$= \underline{16 \text{ m/s}}$$

4.

a)



$$f_L = f_s \frac{v}{v - v_s}$$ moving source

$$= (400) \frac{(340)}{340 - 25{,}0}$$

$$= \underline{432 \text{ Hz}}$$ (i.e. > 400 Hz)

b) $f_L = f_s \left(\frac{v + v_L}{v} \right)$ moving listener 4

$= (400) \left(\frac{340 + 25}{340} \right)$

$= 429 \text{ Hz}$

$\longrightarrow$

5. No need to jump; since the frequency heard is less than that when the car is at rest, it must be moving away from the person.

6. Approaching moving listener stationary source

$f_{approach} = f_s \left(\frac{v + v_L}{v} \right)$

$f_{away} = f_s \left(\frac{v - v_L}{v} \right)$

Now,

$$f_{approach} - f_{away} = f_s\left(\frac{2V_L}{V}\right)$$

$$95 = 960\left(\frac{2V_L}{340}\right)$$

$$V_L = 16.8 \text{ m/s}$$

7. $$f_L = f_s \frac{V}{V - V_s}$$ moving source stationary listener

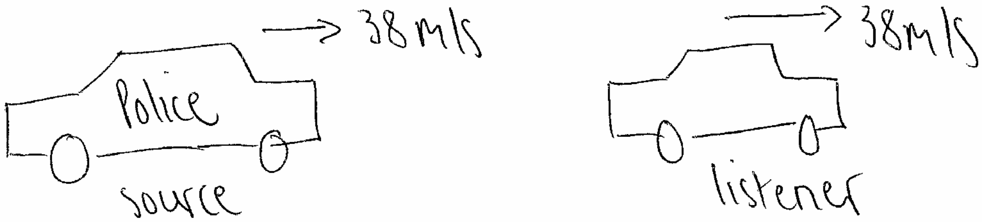
$$= (955)\frac{340}{340 - 18}$$

$$= 1008 \text{ Hz}$$

$V = f\lambda \quad \therefore \lambda_L = \frac{340}{1008} = 34 \text{ cm}$

6

8.



$$f_L = f_s \frac{V - V_L}{V - V_S}$$

both listener & source moving

$$= (860) \frac{340 - 38}{340 - 38}$$

$$= 860 \text{ Hz}$$

9. a) $$f_L = f_s \left(\frac{V - V_L}{V} \right)$$

moving listener stationary source

$$= 444 \left(\frac{340 - 15{,}0}{340} \right)$$

$$= 424 \text{ Hz}$$

b) $f_L = f_S\left(\frac{V + V_L}{V + V_S}\right)$ moving source & listener 7

$= (444)\left(\frac{340 + 15{,}0}{340 + 35{,}0}\right)$

$= 420\ \text{Hz}$

$\longrightarrow$

10. First find the frequency the truck hears:

$f_L = f_S\left(\frac{V - V_L}{V - V_S}\right)$ moving source & listener

$= 2000\left(\frac{340 - 20{,}0}{340 - 30{,}0}\right)$

$= 2064\ \text{Hz}$

The truck now reflects this back

$f_L = f_S\left(\frac{V + V_L}{V + V_S}\right)$ moving source & listener

8

$$\therefore f_L = (2064)\left(\frac{340 + 30.0}{340 + 20.0}\right)$$

$$= \underline{2121\ Hz}$$

$\longrightarrow$

1. FORCE

1.1 Contact and non-contact forces

1.1.1 Contact forces: There is physical contact between the interacting objects. Examples of these are: kicking a ball; friction between two surfaces; tension in cables; and pushing a cart.

1.1.2 Non-Contact forces: (No physical contact between interacting objects) e.g. force between two masses (gravitational force); force between two charges (electrostatic force); force between two magnetic poles (magnetic force)

1.2 Free-body diagrams and Force diagrams

1.2.1 Force diagram

A force diagram shows the object of interest with all forces acting on it. The forces are illustrated using arrows of appropriate length.

ACTIVITY 1

Draw a force diagram for an object accelerating down a rough inclined plane.

1.2.2 Free-body diagram

In a free-body diagram, the object of interest is drawn as a dot and all the forces acting on it are drawn as arrows pointing away from the dot.

ACTIVITY 2

Draw a free-body diagram for an object accelerating down a rough inclined plane.

1.3 Newton's Third Law (N3)

When object A exerts a force on object B, then object B simultaneously exerts an oppositely directed force of equal magnitude on object A.

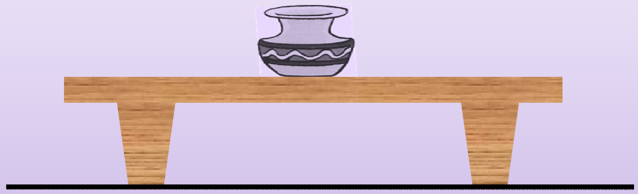
ACTIVITY 3

List the properties of Newton 3 pairs (action-reaction).

1.4 Application of Newton's Third Law

ACTIVITY 4

You are given a vase resting on a table, as shown below.



(a)Identify one contact force.

(b) Identify one non-contact force.

ACTIVITY 5

Given a vase resting on a table used in activity 2

(a) Identify all the action—reaction forces for the vase.

(b) Identify all the action—reaction forces for the table.

ACTIVITY 6

A horse is pulling a cart along a road (as shown above). We know from Newton's third law that the force exerted by the horse on the cart is equal and opposite to the force exerted by the cart on the horse. How then is it possible for motion to occur?

ACTIVITY 7

A ball is held in a person's hand (outstretched). (a) Identity all the N3 pairs for the ball. (b) If the ball is dropped, identity all the N3 pairs for the ball while it is falling? Neglect air resistance.

2. Momentum & Impulse

2.1 Definition of momentum:

Momentum is the product of the mass and velocity of an object, and is in the same direction as the object's velocity. Momentum is *vector* quantity so direction is very important in calculations.

2.2 Vector nature of momentum

EXAMPLE 1

A 2 kg cannon ball is fired vertically upward with an initial velocity of 25m.s^{-1}. Calculate the momentum of the ball at t = 2 s and t = 3 s.

2.3 Newton's second law & momentum

The net (or resultant) force acting on an object is equal to the rate of change of (linear) momentum.

ACTIVITY 1

Show that Newton's second law can be expressed as:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

2.4 Relationship between net force and change in momentum.

If a net force is applied to an object, then the object will experience a change in momentum.

The converse is also true. If an object undergoes a change in momentum, then there has to be a net force being applied on the object.

In other words, the net force & change in momentum are not mutually exclusive.

Also note that the net force and the change in momentum always act in the same direction.

ACTIVITY 2

Is there any relationship between Newton's First law and Newton's second law (in momentum form)?

2.5 Calculating change in momentum for various scenarios:

EXAMPLE 2: A net force is applied and the object's velocity increases in the direction of motion.

The firing of a rocket ($m = 1\times10^6$ kg) results in a net force of 4×10^7 N being exerted on a rocket. If this force is applied for $\Delta t = 30$ s, calculate the change in momentum. Take up as (+).

EXAMPLE 3: A net force is applied the object's velocity decreases in the direction of motion.

A 25 kg box travelling East on a rough surface experiences a net force of 50 N, West. If the net force acts for 3 s, calculate the change in momentum. Take East as (+).

EXAMPLE 4: A net force is applied and the object's velocity is reversed.

A 0.06 kg tennis ball travelling horizontally strikes a racquet with a speed of 60 m/s (East). The ball is returned with speed of 50 m/s in the opposite direction. Taking east as positive , calculate the change in momentum.

ACTIVITY 3

Draw vector diagrams to illustrate the relationship between the initial momentum, the final momentum and the change in momentum in each of the above cases.

2.6 Momentum Conservation

Clarifying the meaning of a few terms

Closed System We define a system as a small portion of the universe that we are interested in and we ignore the rest of the universe outside of the defined system. A system could be a single particle or object or it could be a collection of objects e.g. two cars colliding.

Internal forces. These are forces between particles or objects that constitute the system. E.g. when two cars collide, the forces they exert during the collision are internal to the system.

External forces. These are forces outside the defined system that are exerted on the system.

If the net external force acting on an isolated system of two or more particles is zero, then the linear momentum of that system is conserved

$$F_{net} = \frac{\Delta p}{\Delta t}$$
$$0 \times \Delta t = \Delta p$$
$$\therefore \Delta p = (p_f - p_i) = 0$$
$$\Rightarrow \boxed{p_f = p_i}$$
$$\boxed{\text{Momentum Conservation}}$$

ACTIVITY 4

A ball dropped from a building has a momentum that is increasing with time. Does this mean that momentum conservation has been violated? Assume no air resistance.

2.7 Application of conservation of momentum

Perfectly Elastic Collision

Consider a system of two objects m_1 and m_2 initially moving at u_1 and u_2 respectively. Masses m_1 and m_2 collide and thereafter move with final velocities v_1 and v_2 respectively.

$$F_{12} = -F_{21}$$
$$\Rightarrow \frac{\Delta p_2}{\Delta t} = -\frac{\Delta p_1}{\Delta t}$$
$$\Rightarrow \Delta p_2 = -\Delta p_1$$
$$m_2(v_2 - u_2) = -m_1(v_1 - u_1)$$
$$m_2 v_2 - m_2 u_2 = -m_1 v_1 + m_1 u_1$$
$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$
$$p_{Total\ After} = p_{Total\ Before}$$

Properties of perfectly elastic collisions

- Momentum is conserved (as seen above)
- Kinetic energy is conserved i.e. total kinetic energy before collision is equal to the total kinetic energy after collision.

ACTIVITY 5

A 3.0 kg cart moving East with a speed of 1.0 m/s collides head-on with a 5.0 kg cart that is initially moving West with a speed of 2.0 m/s. After the collision, the 3.0 kg cart is moving to the left with a speed of 1.0 m/s. Ignore friction.

(a) What is the final velocity of the 5.0kg cart?

(b) What impact would friction have, if considered?

(c) Calculate the change in momentum for each mass. Are these values consistent with theory?

ACTIVITY 6

"People generally say that during a head-on collision it is better to be in the more massive vehicle." Do you think it really makes any difference at all?

Perfectly Inelastic Collision

Consider a system of two objects m_1 and m_2 initially moving at u_1 and u_2 respectively. Masses m_1 and m_2 collide, couple, and thereafter move with a common final velocity v.

$$
\begin{aligned}
F_{12} &= -F_{21} \\
\Rightarrow \frac{\Delta p_2}{\Delta t} &= -\frac{\Delta p_1}{\Delta t} \\
\Rightarrow \Delta p_2 &= -\Delta p_1 \\
m_2(v - u_2) &= -m_1(v - u_1) \\
m_2 v - m_2 u_2 &= -m_1 v + m_1 u_1 \\
m_1 v + m_2 v &= m_1 u_1 + m_2 u_2 \\
(m_1 + m_2)v &= m_1 u_1 + m_2 u_2 \\
p_{Total\ After} &= p_{Total\ Before}
\end{aligned}
$$

Properties of perfectly inelastic collisions

• Momentum is conserved (as seen above)

• Kinetic energy is **NOT** conserved i.e. total kinetic energy before collision is not equal to the total kinetic energy after collision.

ACTIVITY 7

The energy released by the exploding gunpowder in a cannon propels the cannonball forward. Simultaneously the cannon recoils. Which the greater kinetic energy, the launched cannonball or the recoiling cannon? Explain.

ACTIVITY 8

A 35 kg girl is standing near and to the left of a 43 kg boy on the frictionless surface of a frozen pond. The boy throws a 0.75 kg ice ball to the girl with a horizontal speed of 6.2 m/s. What are the velocities of the boy and the girl immediately after the girl catches the ice ball?

2.8 Definition of Impulse

Impulse is defined as the *product of net force,* F_{net}, *and the contact time*, Δt. Unit: N.s

OR

Impulse is defined as the *change in momentum*, Δp. Unit: $kg.m.s^{-1}$

Note: 1N.s =1 $kg.m.s^{-1}$

Impulse is a vector quantity and points in the same direction as the net force and change in momentum.

Impulse-Momentum Theorem

$$F_{net}\Delta t = \Delta p$$

For the same impulse, a longer contact time has a smaller associated net (or contact) force while a shorter contact time has a larger associated net (or contact). This will be discussed further when we look at real-world applications of impulse.

ACTIVITY 9

Does a large force always produce a larger impulse on an object than a smaller force? Explain your answer.

2.9 Calculation involving impulse

EXAMPLE 5

A 1000 kg car is travelling due west on the M7 at 30m.s^{-1}. The driver of the car is busy talking on his cell phone and is not aware of a stationary horse and trailer (fully loaded with steel blocks) directly in front of him. The car collides with the truck and comes to rest in a time of 2ms.

(a) Calculate the **impulse** for the car.

(b) Calculate the net force exerted on the car during the collision.

(c) How long would the car have to have been in contact with a huge sand pile in the middle of the road if the force exerted on the car is 5% of that experienced by the collision with the horse and trailer?

ACTIVITY 10

A rubber ball of mass 125 g is dropped from a 1.30 m high table. The ball rebounds after striking the floor and reaches a height of 0.85 m.

(a) Calculate the impulse delivered to the ball.

(b) Illustrate the change in momentum using vectors.

(c) Determine (and illustrate) the net force acting on the ball during impact with the floor if the time of contact is 1.5ms.

2.10 Real-World applications of Impulse

Impulse and sport

Boxing. How is the impulse – momentum theorem applied in boxing?

Cricket. A batsman would generally be told to follow -through when playing a shot. Why?

Impulse and road safety

Airbags. In what way ("physics") do airbags help minimize injury during severe collisions (which require the deployment of airbags)?

Crumple zones. What are crumple zones? What purpose do they serve as far as safety is concerned?

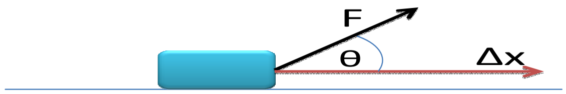
Arrestor Beds. What are arrestor beds? Where are they found? What purpose do they serve as far as safety is concerned?

3.Work, Energy & Power

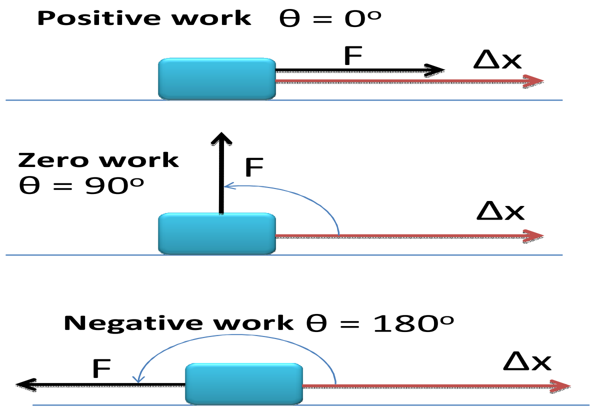
3.1 Definition of Work

The product of the magnitude of the displacement and the component of the force acting in the direction of the displacement.

Work is a scalar and is measure in joules (J)



$$W = F \Delta x \cos\theta$$



ACTIVITY 1

Discuss whether any work is being done by each of the following agents. If so, state whether the work done is positive or negative:

(a)A chicken scratching the ground looking for worms,

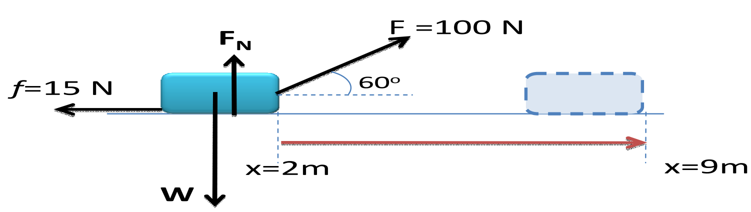
(b) A boy sitting at the table and studying for his Physics test,

(c) A 2010 stadium construction crane lifting a bucket of concrete and

(d) The gravitational force on the bucket in part (c).

ACTIVITY 2

Calculate the work done on the box by each of the forces shown below. Hence calculate the net work done on the box.



TECHNIQUE FOR CALCULATING NET WORK DONE ON A DISPLACED (1-D) OBJECT ACTED UPON BY SEVERAL FORCES

1. Draw a force diagram showing only components that act along the plane of motion
2. Ignore all forces and components that are perpendicular to the plane of motion
3. Calculate the resultant force
4. Multiply the displacement by this resultant force to obtain the net work done.

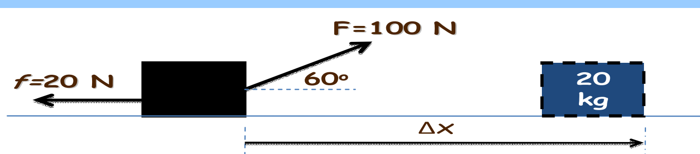
$$W_{net} = F_{net}\Delta x$$

3.2 Work - Energy Theorem

The net work done on a system is equal to the change in the kinetic energy of the system i.e. $W_{net} = \Delta K = K_f - K_i$

EXAMPLE 1: Work-Energy Theorem - Horizontal planes

A 20 kg box is pulled, as shown, across a rough floor. If the box was initially at rest, find the magnitude of the momentum after the box has been displaced 5m using energy methods.



ACTIVITY 3: Work-Energy Theorem - Vertical Planes

A 3 kg steel ball is fired straight up from the ground at a speed of 15 m/s. Use the work-energy theorem to calculate the speed of the ball when it has been displaced 5m. Ignore air resistance.

EXAMPLE 2: Work-Energy Theorem-Inclined Planes

A 10 kg crate of tomatoes is pulled up a rough plane, inclined at 20.0° to the horizontal, by a pulling force of 120 N that acts parallel to the incline. The frictional force between the plane and the crate is 92.09 N, and the crate is displaced 8.0 m. (a) How much work is done by the gravitational force on the crate? (b) Determine the increase in internal energy of the crate-incline system due to friction. (c) How much work is done by the pulling force on the crate? (d) What is the change in kinetic energy of the crate? (e) What is the speed of the crate after being pulled 8.0 m, if it has an initial speed of 2.50 m/ s.?

ACTIVITY 4: Work-Energy Theorem (Conceptual)

Using the work-energy theorem explain why a box set in motion across a rough floor eventually comes to rest?

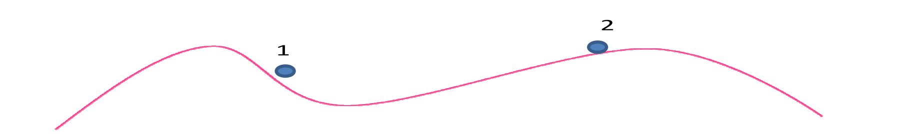
3.3 Terms and concepts involving energy.

- System – For most applications we define the object and earth as a system.
- Isolated or closed system – One that has no external forces acting on it.
- Internal forces (conservative forces) – e.g. gravitational force
- External forces (non-conservative forces) – e.g. tension, friction, air resistance
- Kinetic energy – the energy an object possess due to its motion. $K = \frac{1}{2}mv^2$
- Potential energy – the energy an object possess due to its height relative to some reference level. $U = mgh$
- Mechanical energy – sum of the kinetic and potential (gravitational) energy of an object.

3.4 Conservation of Energy

If we consider only the conservative gravitational force, the mechanical energy of an isolated system is constant i.e. mechanical energy is conserved.

$$ME = \text{constant} \Rightarrow K_1 + U_1 = K_2 + U_2$$



EXAMPLE 3: Conservation of Mechanical Energy

A ball is driven with a golf club from ground level with an initial speed of 50 m/s causing it to rise to a height of 30.3 m. Ignore air resistance.

(a) Determine the speed of the ball at its highest point.

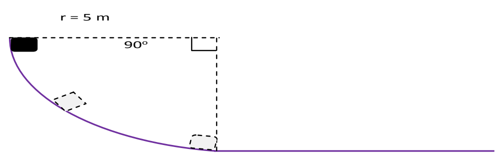
(b) If the magnitude of ball's momentum is 2.28 kg.m.s^{-1} at a point 8.9 m below the highest point, determine (in grams) the mass of the ball.

ACTIVITY 5 : Conservation of Mechanical Energy

A block starts from rest and slides down a frictionless circular section as shown below. Calculate the speed of the block

(a) at the bottom of the circular section

(b) when it has travelled halfway along the circular section.



If we consider the conservative gravitational force as well as other non-conservative forces such as friction, air resistance and tension then the mechanical energy of the system is not constant i.e. mechanical energy is not conserved. But, note, that <u>energy conservation still holds true</u>.
The following equations are used whenever external forces are present.

$$\boxed{W_{other} + K_1 + U_1 = K_2 + U_2}$$

OR

$$\boxed{W_{other} = \Delta K + \Delta U}$$

W_{other} represents the work done by friction, tension and air resistance.

EXAMPLE 4: Application of Conservation of Energy

A 9 kg block is released from rest and slides down a 10 m plane inclined at 45° to the horizontal. Calculate the magnitude of the frictional force if the block has a speed of 4.985 $m.s^{-1}$ 5m down the plane (measured from the top).

ACTIVITY 6: Conservation Energy

A 55 kg skier starts from rest and coasts down a mountain slope inclined at 25° with respect to the horizontal. The kinetic friction between her skis and the snow is 97.7N. She travels 12m down the slope before coming to the edge of a cliff. Without slowing down, she skis off the cliff and lands downhill at a point whose vertical distance is 4m below the edge. Using energy methods, determine her speed just before she lands? Ignore air resistance

ACTIVITY 7 : Conservation Energy

A 5 kg block is travelling at 9 m/s when it approaches the bottom of a ramp inclined at 30° to the horizontal. How far up the plane does the block comes to rest if the frictional force is 28.81N?

3.5 Power

The rate at which work is done or energy is expended.

$P = \frac{W}{t}$ (w) $1w = J.s^{-1}$

EXAMPLES 5

A 700 N police officer in training, climbs a 15 m vertical rope in a time of 9 s. Calculate the power output of the officer.

ACTIVITY 8

A rock climber and a hiker (having equal masses) both start off simultaneously at the foot of a mountain. The hiker takes a longer but easier route spiraling up around the mountain and is the first to arrive at the top. Later the climber arrives at the top.

(a)Which one (climber or hiker) does more work in getting to the top of the mountain?

(b) Which one expends more power in getting to the top of the mountain?

If a force F causes the object to move at a constant velocity, then the average power is given by: $P = F v$

EXAMPLE 6

A car is travelling on a horizontal road at a speed of 25 m/s. Calculate the power (in kW) delivered to the wheels of the car if the friction between the road and wheels is 1900 N and the air resistance experienced is 1400 N.

ACTIVITY 9

A 800 kg car is stuck at the bottom of a hill inclined at 30° with respect to the horizontal. While being pulled up along the incline, the car experiences a frictional force of 2700 N. What should the power rating of a motor be if it is to pull the car up the incline at a constant speed of 3 m/s?

ACTIVITY 10

A motorcycle (mass of cycle plus rider is 270 kg) is traveling at a steady speed of 30 m/s. The force of air resistance acting on the cycle and rider is 240 N. Find the power necessary to sustain this speed if (a) the road is level and (b) the road is sloped upward at 37.0° with respect to the horizontal.

ACTIVITY 11: Lift problem

An empty lift car of mass 1700 kg stops at some level of a shopping mall and three 75 kg men and two 50 kg woman get in. Calculate the power (in kW) delivered by motor if 5000 N of friction is experienced and the lift car is (a) going up at a constant speed of 4 m/s and (b) going down at a constant speed of 4 m/s.

EXAMPLE 7 : Pumping water from a borehole.

A pump is needed to lift water through a distance of 25 m at a steady rate of 180kg/min. What is the minimum power (kW) motor that could operate the pump if (a) the velocity of the water is negligible at both the intake and outlet? (b) The velocity at the intake is negligible but at the outlet the water is moving with a speed of 9m/s.

ACTIVITY 12: Borehole problem

A pump is rated 9 kW. An engineer claims that based on his past experience using the pump, it is only 80% efficient. Is the pump suitable to lift 950 kg (approximately 238 gallons) of water per min from a 40 m deep borehole and eject the water at a speed of 15 m/s?

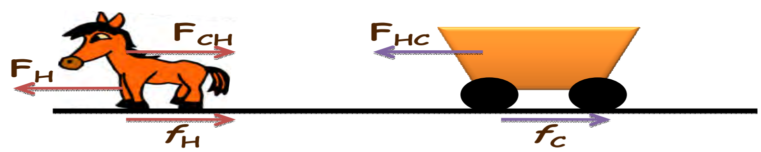
Newton's Third Law

Exercises and Answers

NTL-1

A horse is pulling a cart along a road. We know from Newton's third law that the force exerted by the horse on the cart is equal and opposite to the force exerted by the cart on the horse. How then is it even possible for motion to occur?

Free-body diagram for horse and cart.

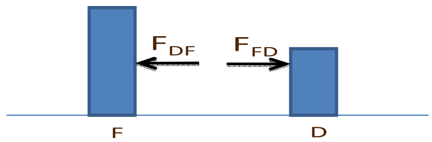


- Note that the Newton 3 pairs (action-reaction) act on different objects and thus do not cancel out
- The motion of the horse and cart depends on the forces acting on them.
- For horse: $F_H > F_{CH} + f_H$
- For cart: $F_{HC} > f_C$

NTL-2

A father and his Grade R daughter, both wearing ice skates, are standing on ice and facing each other. Using their hands, they push off against one another. (i) Compare the magnitudes of the pushing forces they each experience. (ii) Compare the magnitudes of their accelerations? Give reasons for your answers.

• Force exerted by father on daughter is F_{FD}
•Force exerted by daughter on father is F_{DF}



(a)The magnitudes of these forces are equal
(b) $F_R = m\, a$

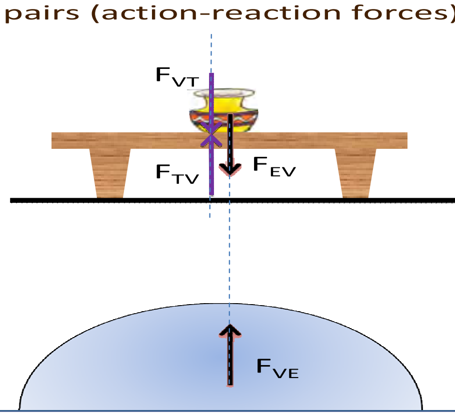
The magnitude of the resultant force on daughter and father is equal. Thus $a \propto \dfrac{1}{m}$.

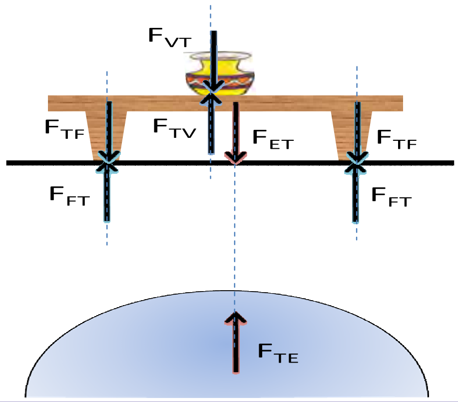
Since $m_F \succ m_D \Rightarrow a_F \prec a_D$. Thus the daughter experiences a greater acceleration

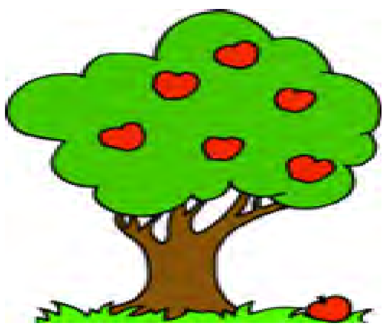
NTL-3

Given a vase resting on a table as shown below.
(i)Identify all the Newton 3 pairs (action-reaction forces) for the vase.
(ii) Identify all the Newton 3 pairs (action-reaction forces for the table.

(i) Newton 3 pairs (action-reaction forces) for the vase.







Newton's Third Law

Exercises and Answers

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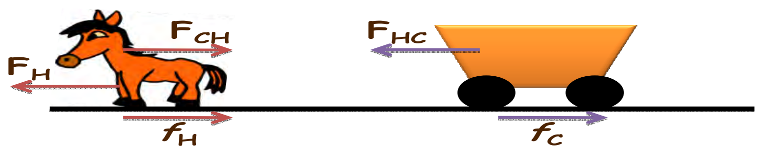

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NTL-2

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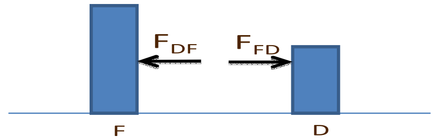

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NTL-3

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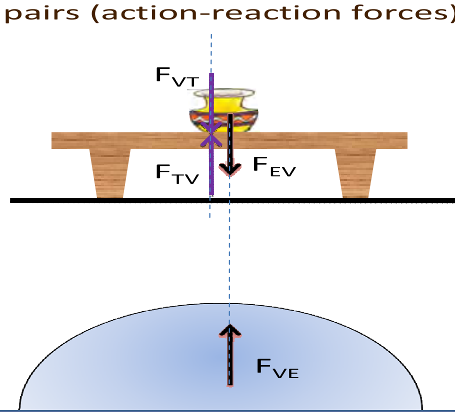
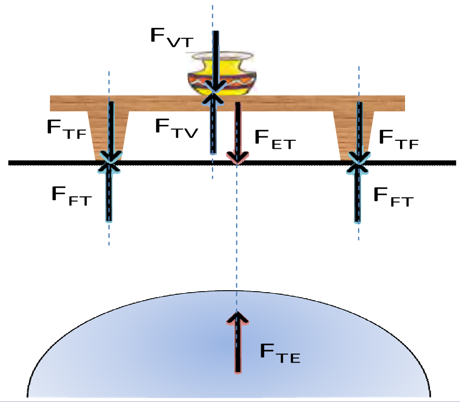




Impulse & Momentum

Key: “I & M" refer to “Impulse and Momentum.

I & M-1

A 2kg ball is thrown vertically upward with an initial velocity of 25 m.s^{-1}. Calculate the momentum of the ball at t=2s and t=3s.

Take up as +ve ($a = -g = -9.8\,m/s^2$)

$t = 2s$:

$v_f = V_i + at = 25 m.s^{-1} + (-9.8 m/s^2)(2s) = 5.40\ m.s^{-1}$ up.

$p_f = m v_f = (2kg)(5.40 m.s^{-1}) = \underline{10.8\ kg.m.s^{-1}}, up$

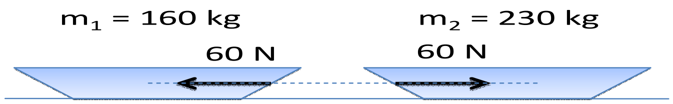
$t = 3s$:

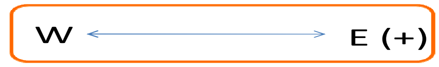
$v_f = V_i + at = 25 m.s^{-1} + (-9.8 m/s^2)(3s) = -4.40\ m.s^{-1}$ down.

$p_f = m v_f = (2kg)(-4.40 m.s^{-1}) = -8.80\ kg.m.s^{-1} = \underline{8.80\ kg.m.s^{-1}}$, down

I & M-2

Two groups of tourists meet while canoeing in in a dam. Both canoes are stationary and lie in a straight line in close proximity of each other. A person from the first canoe pushes on the other canoe with a force of 60N. Find the momentum of each canoe after 1.3 s of pushing if the total masses of canoes 1 and 2 are 160 and 230 kg respectively.





$$a_1 = \frac{F}{m_1} = \frac{-60\,N}{160\,kg} = -0.38\ m.s^{-2}$$

$$a_2 = \frac{F}{m_2} = \frac{-60\,N}{230\,kg} = +0.26\ m.s^{-2}$$

$$v_{f1} = v_{i1} + a_1 t = 0 + (-0.38\ m.s^{-2})(1.3\,s) = -0.49\,m.s^{-1}$$

$$v_{f2} = v_{i2} + a_2 t = 0 + (0.26\ m.s^{-2})(1.3\,s) = +0.34\,m.s^{-1}$$

$$P_{f1} = m_1 v_{f1} = (160)(-0.49) = -78\,kg.m.s^{-1} = 78\,kg.m.s^{-1}, West$$

$$P_{f1} = m_1 v_{f1} = (230)(0.34) = +78\,kg.m.s^{-1}, East$$

I & M-3

The energy released by the exploding gunpowder in a cannon propels the cannonball forward. Simultaneously the cannon recoils. Which has the greater kinetic energy, the launched cannonball or the recoiling cannon? Explain, assuming that momentum conservation applies.

This is an inelastic collision.

$$P_{before} = P_{after}$$

$$0 = P_m + P_M \Rightarrow P_m = -P_M \Rightarrow |P_m| = |P_M| \quad \text{(magnitudes are equal)}$$

$$\boxed{E_K = \frac{P^2}{2m}} \quad \text{(Try to prove this!!)}$$

Since the magnitudes of the momenta are equal, it means that:

$$E_K \propto \frac{1}{m} \Rightarrow E_{K.m} \succ E_{K,M}$$

I & M-4

An ice boat is coasting on a frozen lake at a constant velocity. From a bridge stunt man jumps straight down into the boat. Ignore friction and air resistance. (a) Does the total horizontal momentum of the boat plus the jumper change? (b) Does the speed of the boat itself change? Explain your answers.

$$P_{before} = m_b v_b + m_s(0) = m_b v_b$$
$$P_{after} = (m_b + m_s)V$$
$$P_{before} = P_{after}$$

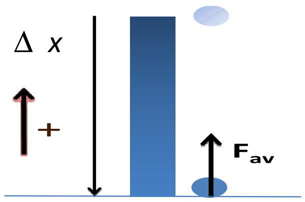
since the mass has increases, the velocity after collision must decrease for momentum conservation to hold true.

(a) Total horizontal momentum for boat plus stuntman does not change.

(b) The speed of the boat decreases.

I & M-5

A 60 kg student falls off a wall, strikes the ground and comes to rest in a time of 10ms. The average force exerted on him by the ground is + 21000 N where the upward direction is taken to be the positive direction. Calculate height of the wall assuming that the only force acting on him during the collision is that due to the ground.



First consider the collision with the floor. Just before striking the floor the learner has an initial velocity. After striking the floor his velocity is zero.

$$F_{av}\Delta t = m(v_f - v_i) = m(0 - v_i) = -mv_i$$

$$v_i = \frac{F_{av}\Delta t}{-m} = -\frac{(21000N)(10\times10^{-3}s)}{60kg} = -3.50m.s^{-1}$$

$$\Rightarrow v_i = 3.50m.s^{-1}\downarrow$$

Now consider the height through which the learner falls. The initial velocity is zero and the final velocity is 3.5m.s^{-1} (down).

$$v_i = 0, v_f = -3.5m.s^{-1}, a = -g = -9.8m.s^{-2}$$

$$\boxed{v_f^2 = v_i^2 + 2a\Delta x}$$

$$-\Delta x = \frac{v_f^2 - v_i^2}{2(-g)} = \frac{(-3.5)^2 - (0)^2}{2(-9.8)} = \frac{12.25}{-19.6} = -0.63m$$

$$\therefore \Delta x = \underline{0.63m}$$

I & M-6

You are standing still and then take a step forward. We know that your initial momentum is zero while your final momentum is not zero. Does this mean that momentum is not conserved.

You and the earth form an isolated system.

$$P_{before} = P_{after}$$

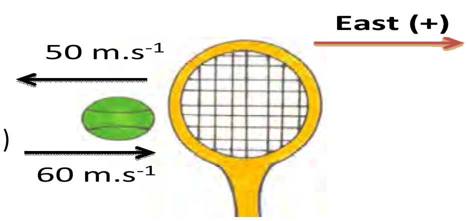
$$0 = m_{you}v_{you} + M_{earth}V_{earth}$$

$$M_{earth} \gg m_{you} \Rightarrow V_{earth} \ll v_{you}$$

So the earth does move when you take a step forward, but It is not visible because it is very small.

I & M-7

A 0.06 kg tennis ball travelling horizontally strikes a racquet with a speed of 60 m/s. The ball is returned with speed of 50 m/s in the opposite direction. (i) Determine the impulse delivered to the ball by the racquet. (ii) Determine the force exerted on the ball by the racquet is the contact time is 2 ms.



(a)

$\Delta P = m(v_f - v_i)$

$= (0.06\,kg)(-50\,m.s^{-1} - 60\,m.s^{-1})$

$= -6.60\ kg.m.s^{-1}$

$= 6.60\ kg.m.s^{-1}, West.$

(b)

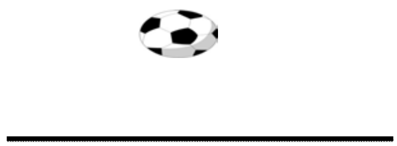
$\Delta P = F_{av}\Delta t$

$$F_{av} = \frac{\Delta P}{\Delta t} = \frac{-6.60\ kg.m.s^{-1}}{1\times10^{-3}\,s} = -6600N = 6600\ N, West$$

I & M-8

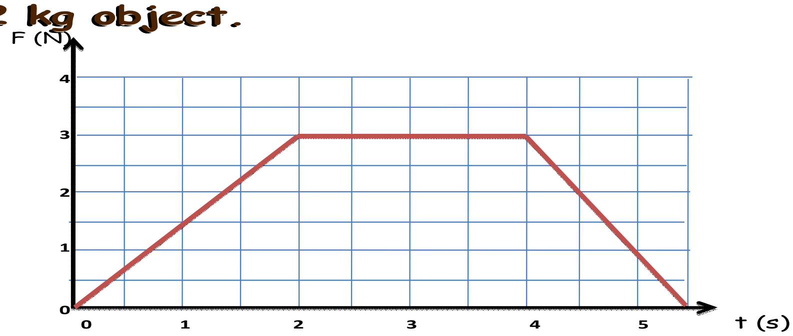
A ball dropped from a building has a momentum that is increasing with time. Does this mean that momentum conservation has been violated.

Momentum conservation does not apply in this situation here.
The ball is accelerating, so there is a net force (external force) that is acting on the ball. Momentum is only conserved for an isolated system i.e. a system that has no net force is acting.



I & M-9

The forces in the force-time graph below act on a 2 kg object.



(i) Find the impulse of the force
(ii) Calculate the final velocity of the object if it was initially at rest.
(iii) Calculate the final velocity of the object if it was initially moving at -3 m/s .

(i) To find the impulse, determine the area under the F-t graph.

$$F\Delta t = \tfrac{1}{2}(2)(3) + (2)(3) + \tfrac{1}{2}(1.5)(3) = 11.25\ N.s = 11.25\ kg.m\,s^{-1}$$

(ii)

$$\Delta P = m(v_f - v_i) = F\Delta t = 11.25\ kg.m\,s^{-1}$$

$$v_f = \frac{F\Delta t}{m} + v_i = \frac{11.25\ kg.m\,s^{-1}}{2\,kg} + 0 = \underline{5.63\,m.s^{-1}}$$

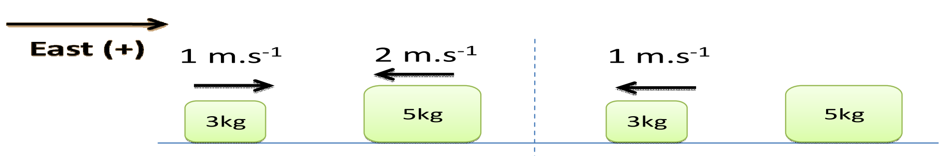
(iii)

$$\Delta P = m(v_f - v_i) = F\Delta t = 11.25\ kg.m\,s^{-1}$$

$$v_f = \frac{F\Delta t}{m} + v_i = \frac{11.25\ kg.m\,s^{-1}}{2\,kg} - 3\,m\,s^{-1} = \underline{2.63\,m.s^{-1}}$$

I & M-10

A 3.0 kg cart moving to the right with a speed of 1.0 m/s has a head-on collision with a 5.0 kg cart that is initially moving to the left with a speed of 2.0 m/s. After the collision, the 3.0 kg cart is moving to the left with a speed of 1.0 m/s. What is the final velocity of the 5.0kg cart?



I&M -10 SOLUTION

$$P_{before} = P_{after}$$

$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

$$(3)(1) + (5)(-2) = 3(-1) + 5v_{f2}$$

$$5v_{f2} = -4$$

$$v_{f2} = -0.8\ m.s^{-1} = \underline{0.8\,m.s^{-1}, West}$$

I & M-11

A 35 kg girl is standing near and to the left of a 43 kg boy on the frictionless surface of a frozen pond. The boy throws a 0.75 kg ice ball to the girl with a horizontal speed of 6.2 m/s. What are the velocities of the boy and the girl immediately after the girl catches the ice ball?

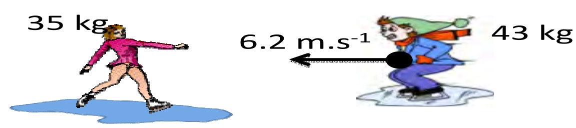
For Boy

$P_{before} = P_{after}$

$(m_b + m_B)(0) = m_b v_{fb} + m_B v_{fB}$

$0 = (0.75)(6.2) + (43)v_{fB}$

$v_{fB} = -0.11 m.s^{-1} = 0.11 m.s^{-1}, East$



For Girl

$P_{before} = P_{after}$

$(m_G)(0) + m_b v_{ib} = (m_b + m_G)v_G$

$0 + (0.75)(6.2) = (35.75)v_G$

$v_{fB} = 0.13 m.s^{-1}, West$

I&M -11 SOLUTION

I & M-12

Explain the "Physics" of airbags, seatbelts and arrestor beds.

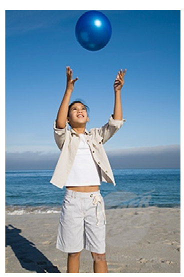
(i) The physics is simply that a greater contact time with a device like an airbag results in the occupants experiencing a smaller average force (impulsive) ,thereby minimizing injury, during collisions. During severe head-on collisions air bags will deploy. The seat belt provides an unbalanced force mainly to the middle section of the body, but not the upper areas like neck and head and lower areas like knees (which the air bags will take care of). So a combination seatbelts and airbags is ensures maximum safety.

I&M -12 SOLUTION

MECHANICS: VERTICAL PROJECTILE MOTION

PROBLEM-SOLVING EXERCISES AND SOLUTIONS

Key: "VPR" on the pages below is shorthand for Vertical Projectile motion.



Vertical Projectile Motion

VPM-1

Two objects are thrown vertically upward, first one, and then, a bit later, the other. Is it possible that both reach the same maximum height at the same instant? Account for your answer.

Since object 2 is thrown a bit later, it must be projected up with a smaller velocity for both balls to reach maximum height at the same instant.

Taking up as (+)

An object thrown up at v_i rises to a maximum height Δx given by:

$$\Delta x = \frac{v_f^2 - v_i^2}{2(-g)} = \frac{(0)^2 - v_i^2}{2(-g)} = \frac{v_i^2}{2g}$$

$$\boxed{\Delta x = \frac{v_i^2}{2g}}$$

since $v_{i,object2} < v_{i,object1}$

$\Delta x_{\max,object1} > \Delta x_{\max,object2}$

VPM-1 SOLUTION

VPM-2

Two students, Anne and Joan, are bouncing straight up and down on a trampoline. Anne bounces twice as high as Joan does. Assuming both are in free-fall, find the ratio of the time Anne spends between bounces to the time Joan spends.

Take ↑ (+)

Consider an object thrown up with initial velocity, v_i, that reaches maximum height, Δx, in a time t .

VPM-2 SOLUTION

$$0 = v_i - gt \Rightarrow v_i = gt$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0 = (gt)^2 - 2g\Delta x$$

$$t = \sqrt{\frac{2\Delta x}{g}}$$

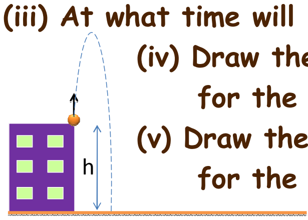
$$t_J = \sqrt{\frac{2\Delta x}{g}}$$

$$t_A = \sqrt{\frac{2(2\Delta x)}{g}} = \left(\sqrt{\frac{2\Delta x}{g}}\right)\sqrt{2}$$

$$\frac{t_A}{t_B} = \sqrt{2}$$

VPM-3

A ball is projected vertically upward with a velocity of 30 m/s. It strikes the ground after 8s. (i) Determine the height of the building.
(ii) Find the height of the ball relative to the ground as well as its velocity at t=2s and t=7s.
(iii) At what time will its velocity be 25 m/s↓.



(iv) Draw the position vs. time graph for the motion.

(v) Draw the velocity vs. time graph for the motion.

(i) Take up as positive.

$v_i = 30 m.s^{-1}, g = -9.8 m.s^{-2}$. After 8s:

$\Delta x = -h = v_i t + \frac{1}{2}at^2$

$-h = (30)(8) + \frac{1}{2}(-9.8)(8)^2 = -73.60 m$

$h = \underline{73.60 m}$

VPM-3
SOLUTION

$(ii)\ v_i = 30 m.s^{-1}, g = -9.8 m.s^{-2}$

At t=0, balls position with respect the ground is 73.60m. The position of the ball at any time t is:

$x(t) = x(0) + v_i t + \frac{1}{2}at^2 = 73.60 + 30t - 4.9t^2$

$\Rightarrow \qquad \boxed{x(t) = 73.60 + 30t - 4.9t^2}$

And, the velocity of the ball at any time t is:

$$\boxed{v(t) = 30 - 9.8t}$$

$x(3) = 73.60 + 30(2) - 4.9(2)^2 = 114m$ (above ground)

$v(3) = 30 - 9.8(3) = 10.4 m.s^{-1} \uparrow$

$x(5) = 73.60 + 30(7) - 4.9(7)^2 = 43.50m$ (above ground)

$v(5) = 30 - 9.8(7) = -38.60 m.s^{-1} = 38.60 m.s^{-1} \downarrow$

VPM-3
SOLUTION

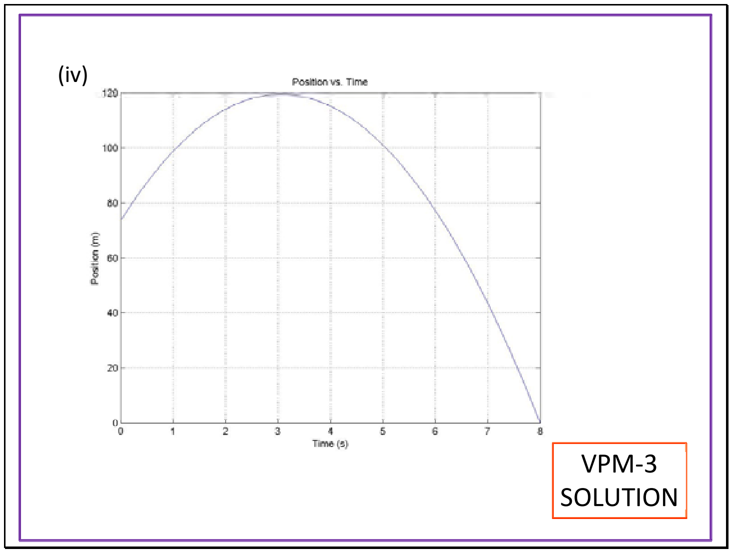
(iii)

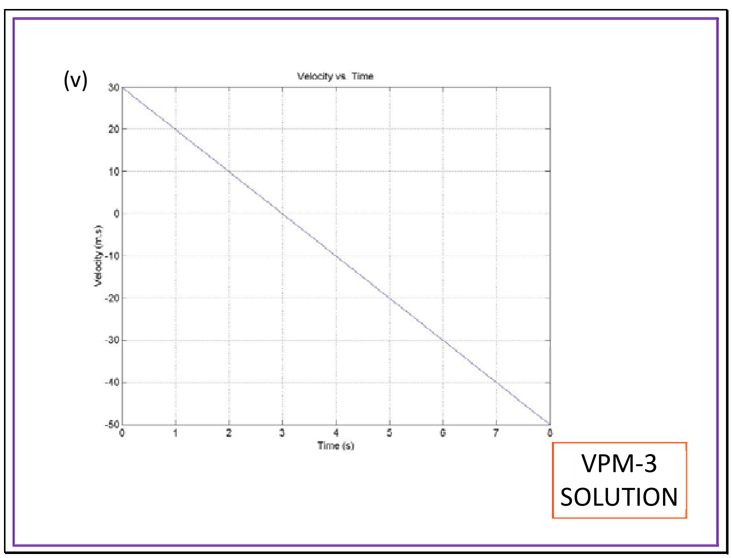
$v_f = v_f + at$

$-25 = 30 - 9.8t$

$t = \frac{-55 m.s^{-1}}{-9.8 m.s^{-2}} = 5.61s$

VPM-3
SOLUTION


(iv)
Position vs. Time
Position (m)
Time (s)
VPM-3
SOLUTION


(v)
Velocity vs. Time
Time (s)
VPM-3
SOLUTION

VPM-4

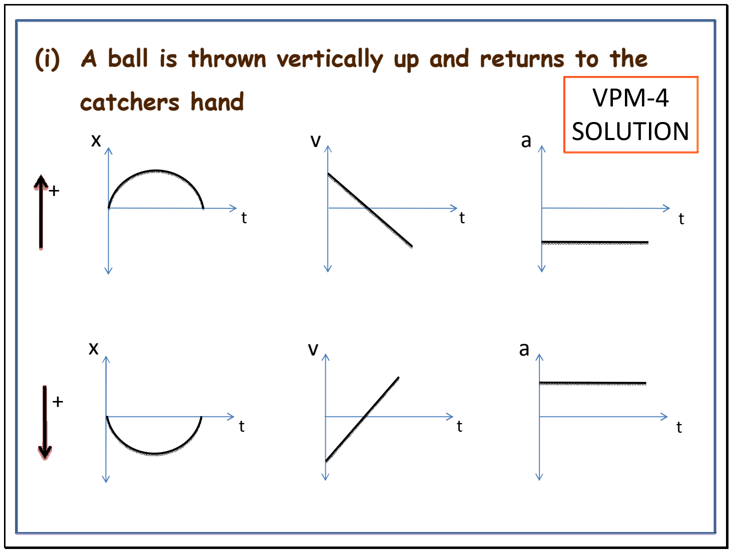
Sketch a-t, v-t and x-t graphs for the following:

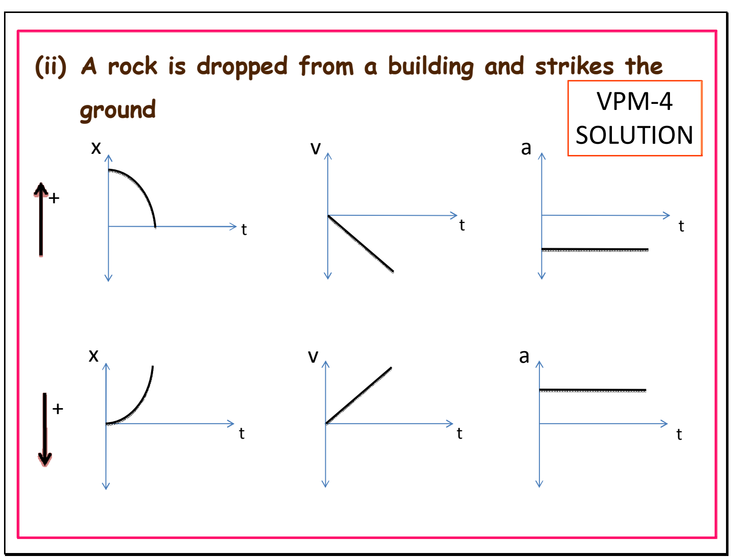
(i)A ball is thrown vertically up and returns to the catchers hand

(ii)A rock is dropped from a building and strikes the ground

(iii) A golf ball is thrown down, bounces off the floor, and caught at its maximum height.

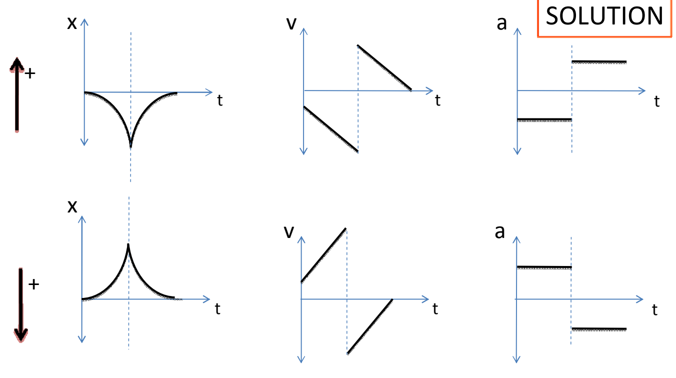
First take up as positive, then repeat for down as positive


(i) A ball is thrown vertically up and returns to the catchers hand
VPM-4 SOLUTION
x
t
v
t
a
t
+
x
t
v
t
a
t
+


(ii) A rock is dropped from a building and strikes the ground
VPM-4 SOLUTION
x
t
v
t
a
t
+
x
t
v
t
a
t
+

(iii) A golf ball is thrown down, bounces off the floor, and caught at its maximum height.

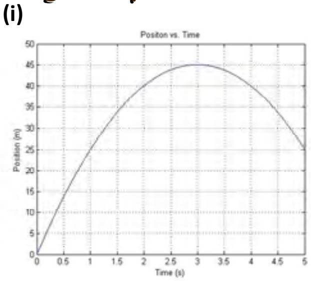
VPM-4 SOLUTION



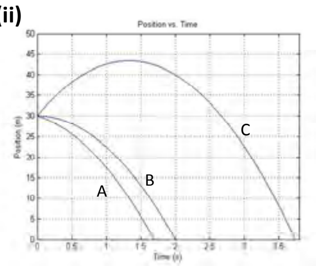
VPM-5

Study the following motion graphs and try to determine the physical situations that these might represent.

(i)



(ii)



(i) This could represent an object that was thrown upward from the ground and caught before it striking the ground.

(ii) **A** represents an object that was thrown downward

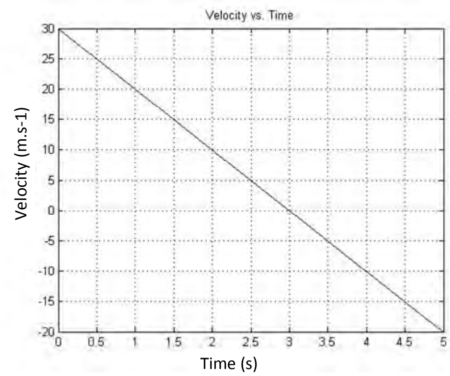
B represents an object that was dropped

C represents an object that was thrown upward, reaches maximum height before falling to the ground.

VPM-5 SOLUTION

VPM-6

Given the following velocity -time



(i) What is the initial velocity of the object?

(ii) What is the instantaneous velocity of the object at (a) t=1s, (b) t=4.5s?

(iii) How does the object take to reach maximum height?

(iv) Determine the position of object with respect to the ground at (a) t=2s, (a) t=4s

(v) Draw the position vs. time graph for the first 4s of the motion.

VPM-6 SOLUTION

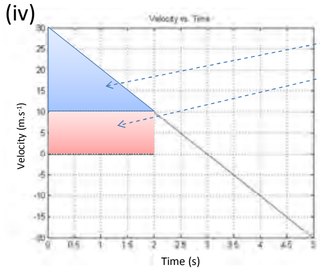
(i) $v_i = 30\ m.s^{-1}$

(ii) $v(1)=20\ m.s^{-1}$, $v(4.5)= -15\ m.s^{-1}= 15\ m.s^{-1}$,downward

(iii) Reaches max height when velocity is $0\ m.s^{-1}$.

This happens at t=3s

(iv)



$$A_1 = \tfrac{1}{2}bh = \tfrac{1}{2}(2)(30-10) = 20m$$

$$A_2 = bh = (2)(10-0) = 20m$$

$$A_{Total} = A_1 + A_2 = 40m$$

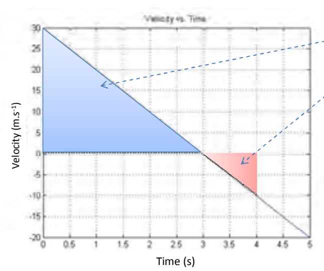
$$\Delta x = 40m.$$

Since $x(0)=0$

$$x(2) - x(0) = 40m$$

$$x(2) = 40m$$

Object is 40 m above ground.



$$A_1 = \tfrac{1}{2}bh = \tfrac{1}{2}(3)(30-0) = 45m$$

$$A_2 = \tfrac{1}{2}bh = \tfrac{1}{2}(1)(-10-0) = -5m$$

$$A_{Total} = A_1 + A_2 = 40m$$

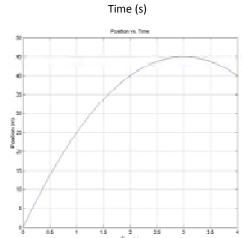
$$\Delta x = 40m.$$

Since $x(0)=0$

$$x(4) - x(0) = 40m$$

$$x(4) = 40m$$

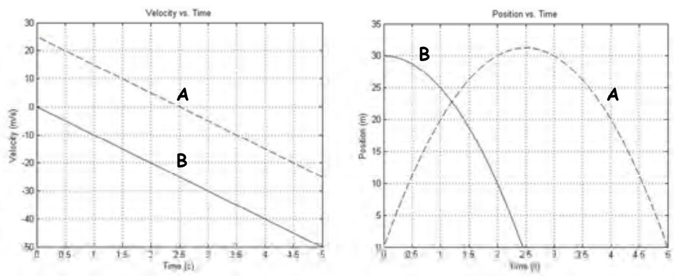
(v)



VPM-6 SOLUTION

VPM-7

Given the v-t and x-t graphs for two different objects



(i) Write the equations of motion [x(t) and v(t)] for both objects.

(ii) What is the distance between the objects at t=1s and t=2s.

(iii) At what time (s) is the speed of object A 10 m/s.

(iv) Calculate the area under the v-t graph for object B for t=0s to t=2s, and confirm using its x-t graph.

(i)

$A:$

$v(t) = 25 - 10t$

$x(t) = 25t - 5t^2$

$B:$

$v(t) = -10t$

$x(t) = 30 - 5t^2$

(ii)

$x_A(1)$=25m

$x_B(1)$=20m

$x_A(1)$-$x_B(1)$=5m

(iii) $t = 1.5s$ and $3.5s$

VPM-7 SOLUTION

(iv) $A = \frac{1}{2}(2)(-20) = -20m$

$x(2) - x(0) = -20m \Rightarrow x(2) - 30m = -20m$

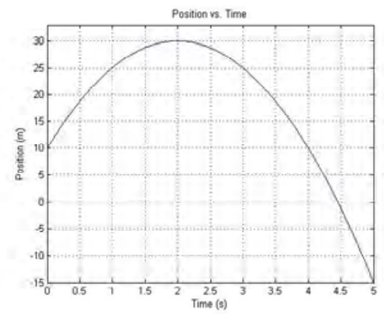
$x(2) = 10m$. Object B is 10m above ground.

Look at x-t for B: x(2)=10m.

This confirms the answer obtained from the area under the v-t graph between 0s and 2 s

VPM-8

Given the following x-t graph.



(i) Calculate the time taken to reach maximum height.

(ii) How long after launch does the object pass its launch point.

(iii) Calculate the initial velocity of the object.

(iv) Determine v(t) for the object.

(v) Draw the v-t graph for t=0 to t=4s.

(vi) Calculate the position of the object (with respect to ground) at t=3s.

(i) t=2s

(ii) After 4s

(iii) Taking upward as positive and $g=10m.s^{-2}$:

Between t=0s and t=2s

$v_i = ?, v_f = 0, t = 2s, a = -10m.s^{-2}$

$v_f = v_i + at$

$v_i = v_f - at = 0 - (-10)(2) = 20m.s^{-1}, up$

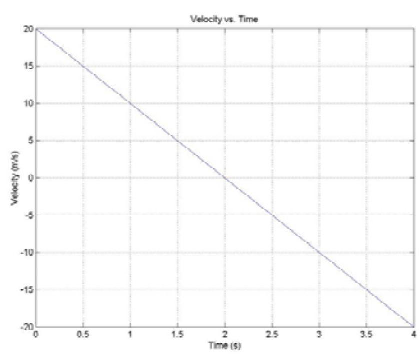
(iv) v(t)=10-10t

(v) Next slide

(vi) X =30m

VPM-8 SOLUTION

(v)



VPM-8 SOLUTION

VPM-9

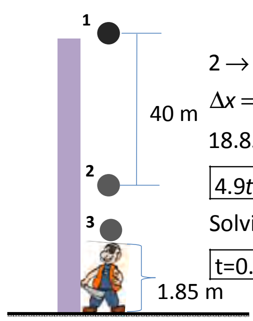
A rock accidentally falls from rest from the side of a 60 m high building. When the rock is 20 m above the ground, a 1.85 m tall man looks up and sees the rock directly above him. Calculate the maximum amount of time the man has to get out of the way and avoid the impending danger?

$1 \rightarrow 2: v_1 = 0 m.s^{-1}, v_2 = ? g = 9.8 m.s^{-2}, \Delta x = 40 m$

$v_2^2 = v_1^2 + 2g\Delta x$

$v_2^2 = 0 + 2(9.8)(40) = 784$

$v_2 = 28 m.s^{-1}, down$



$2 \rightarrow 3: v_2 = 28 m.s^{-1}, g = 9.8 m.s^{-2},$

$\Delta x = 20 - 1.85 = 18.85 m$

$18.85 = 28t + 4.9t^2$

$\boxed{4.9t^2 + 28t - 18.85 = 0}$

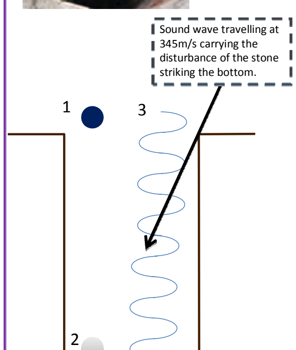
Solving the quadratic:

$\boxed{t=0.59 \text{ s}}$

VPM-9
SOLUTION

VPM-10

A cave explorer drops a stone from rest into a hole. The speed of sound in air on that day is 345 m/s, and the sound of the stone hitting the bottom of the hole is heard 3.50 s after the stone is dropped. What is the dept of the hole?



Let height of hole = h, $t_{1\to2} = t_x$ and $t_{2\to3} = t_y$

$$\boxed{t_x + t_y = 3.5s}$$

$1 \to 2$ (taking down as postitive)

$v_1 = 0, g = 9.8m.s^{-2}$

$h = \Delta x = \frac{1}{2}(9.8)t_x^{\ 2} = 4.9t_x^{\ 2}$

$\boxed{h = 4.9t_x^{\ 2}}$

$2 \to 3$ (sound wave)

$345 = \frac{h}{t_y} \Rightarrow \boxed{h = 345t_y}$

$\therefore 4.9t_x^{\ 2} = 345t_y$

$but\ \ t_y = 3.5 - t_x$

$4.9t_x^{\ 2} = 345(3.5 - t_x)$

$\boxed{4.9t_x^{\ 2} + 345t_x - 1207.50 = 0}$

sloving :

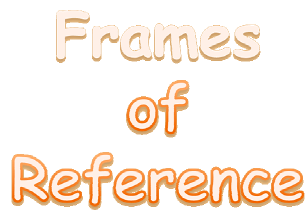
$t_x = 3.34s \Rightarrow \boxed{h = 54.71m}$

VPM-10 SOLUTION

MECHANICS: Frames of Reference:

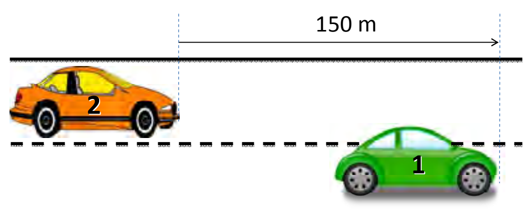
Problem Solving Exercises and Solutions

KEY: "FOR" on the pages below is shorthand for Frames of Reference.



FOR-1

Two cars are travelling on separate lanes on a two lane freeway. How long does it take a car 2 travelling at 100 km/h to overtake a car 1 travelling at 80 km/h if the distance between the their front bumpers is 150 m.



$v_{1g} = 100\ km.h^{-1}, v_{2g} = 80\ km.h^{-1}$

$\boxed{v_{AB} = -v_{BA}}$

$v_{12} = v_{1g} + (-v_{g2}) = 100 - 80 = 20\ km.h^{-1}$

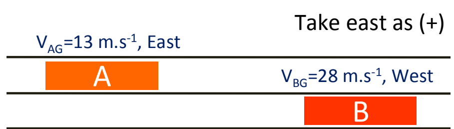
$$t = \frac{d}{v_{12}} = \frac{150 \times 10^{-3}\,km}{20 km.h^{-1}} = 7.50 \times 10^{-3}\,h = 27s.$$

Exercise: confirm using kinematic equations

FOR-1 SOLUTION

FOR-2

Two passenger trains are passing each other on adjacent tracks. Train A is moving east with a speed of 13 m/s, and train B is traveling west with a speed of 28 m/s. (a) What is the velocity (magnitude and direction) of train A as seen by the passengers in train B? (b) What is the velocity (magnitude and direction) of train B as seen by the passengers in train A?



$$(a)\ v_{AB} = v_{AG} + (-v_{GB}) = 13 + 28 = 41 m.s^{-1}, East$$

$$(b)\ v_{BA} = v_{BG} + (-v_{GA}) = -28 - 13 = -41 m.s^{-1} = 41 m.s^{-1}, West$$

FOR-2 SOLUTION

FOR-3

On a pleasure cruise a boat is traveling relative to the water at a speed of 5.0 m/s due south. Relative to the boat, a passenger walks toward the back of the boat at a speed of 1.5 m/s. (a) What is the magnitude and direction of the passenger's velocity relative to the water? (b) How long does it take for the passenger to walk a distance of 27 m on the boat?

(c) How long does it take for the passenger to cover a distance of 27 m on the water?

$$(a)\ v_{PW} = v_{PB} + v_{BW} = -1.5 + 5 = 3.5 m.s^{-1}, South$$

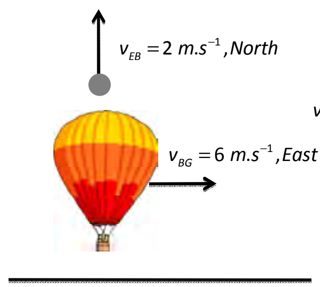
$$(b)\ t = \frac{d}{v} = \frac{27m}{v_{PB}} = \frac{27m}{1.5 m.s^{-1}} = 18s$$

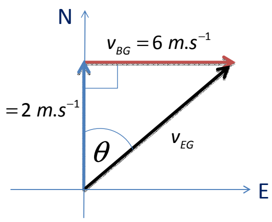
$$(c)\ t = \frac{d}{v} = \frac{27m}{v_{PW}} = \frac{27m}{3.5 m.s^{-1}} = 7.71s$$

FOR-3
SOLUTION

FOR-4

You are in a hot-air balloon that, relative to the ground, has a velocity of 6.0 m/s in a direction due east. You see a hawk moving directly away from the balloon in a direction due north. The speed of the hawk relative to you is 2.0 m/s. What are the magnitude and direction of the hawk's velocity relative to the ground? Express the directional angle relative to due east.





$$\vec{v}_{EG} = \vec{v}_{EB} + \vec{v}_{BG}$$

FOR-4 SOLUTION

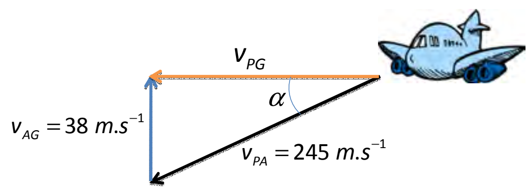
$$v_{EG} = \sqrt{(v_{EB})^2 + (v_{BG})^2} = \sqrt{(2)^2 + (6)^2} = \underline{6.32\ m.s^{-1}}$$

$$\tan\theta = \frac{v_{BG}}{v_{EG}} = 3 \Rightarrow$$

$\theta = \tan^{-1}(3) = 71.57^o$, East of North.

FOR-5

The captain of a plane wishes to proceed due west. The cruising speed of the plane is 245 m/s relative to the air. A weather report indicates that a 38.0 m/s wind is blowing from the south to the north. In what direction, measured with respect to due west, should the pilot head the plane relative to the air?



$$\sin\alpha = \frac{v_{AG}}{v_{PA}} = \frac{38}{245} = 0.16$$

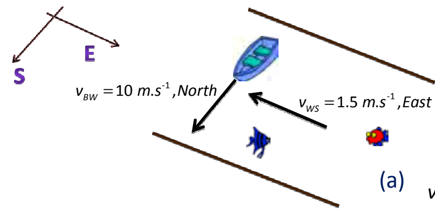
$$\alpha = \sin^{-1}(0.16) = 8.92^o,\ South\ of\ West$$

FOR-5 SOLUTION

FOR-6

A river flows east at 1.5 m/s. A boat crosses the river from south shore to north shore by maintaining a constant velocity of 10 m/s due north relative to the water. (i) What is the velocity of the boat relative to the shore, (ii) If the river is 300 m wide, how far downstream has the boat moved by the time it reaches the north shore.

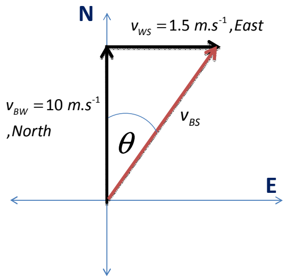
$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$



(a) $v_{BS} = \sqrt{(v_{BW})^2 + (v_{WS})^2}$

$= \sqrt{(10)^2 + (1.5)^2}$

$= \underline{10.11\ m.s^{-1}}$



$\tan\theta = \frac{v_{WS}}{v_{BW}} = 0.15 \Rightarrow$

$\theta = \tan^{-1}(0.15) = 8.53^o$, East of North.

(b) $t = \frac{d}{v_{BS}} = \frac{300m}{10.11\ m.s^{-1}} = 29.67\ s$

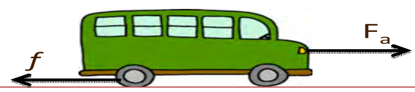
FOR-6 SOLUTION

Work, Energy & Power

Key: "WEP" refers to "Work Energy Power" on the following pages.

WEP-1

A mini-bus driver, travelling on a straight horizontal road, wonders why the speed of his vehicle is constant even though he has his foot on the accelerator - applying a constant value of "acceleration". Supply the driver with a reason for his observation.



The force applied (provided by the engine) to move the mini-bus forward is controlled by the accelerator. At the time of observation, this applied force is equal to the opposing frictional force.

$$W_{net} = F_{net}\Delta x = (F_a - f)\Delta x = 0\ J$$
$$W_{net} = \Delta K = 0\ \ (Work - Energy\ theorem)$$
$$\therefore K_f = K_i \Rightarrow v_f = v_i \Rightarrow v \text{ is constant}$$

WEP-2

In the following two scenarios ignore friction and air resistance. Car X approaches a hill. The driver turns off the engine at the bottom of the hill, and the car freewheels up the hill. Car Y, with its engine running, is driven up the hill at a constant speed. In which scenario is the principle of conservation of mechanical energy observed? Explain your answer.

Mechanical energy is conserved for car X. As the car goes up the hill the kinetic energy decreases (car slows down) while the gravitational potential energy increases.

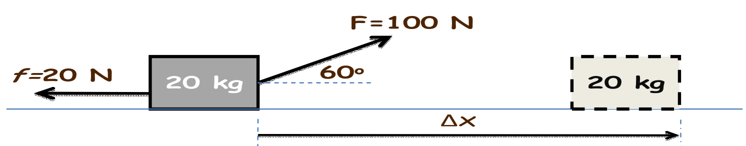
$$ME = \text{constant (conserved)}$$
$$\Delta K + \Delta U = 0$$

Mechanical energy is not conserved for car Y. As the car goes up the hill the kinetic energy remains constant while the gravitational potential energy increases. As car Y proceeds up the hill, its mechanical energy increases

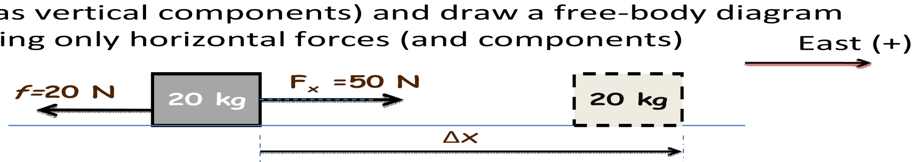
$$ME \neq \text{constant (conserved)}$$
$$\Delta K + \Delta U \succ 0$$

WEP-3

A 20 kg box is pulled, as shown, across a rough floor. If the box was initially at rest, find the magnitude of the momentum after the box has been displaced 5m using energy methods.



Because the motion is horizontal, we ignore all vertical forces (as Well as vertical components) and draw a free-body diagram Showing only horizontal forces (and components)



$$W_{net} = F_R \Delta x = (F_x - F)\Delta x = (50N - 20N)(5m) = 150J$$
$$W_{net} = \Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2) = \tfrac{1}{2}(20)(v_f^2 - 0^2) = 10v_f^2$$

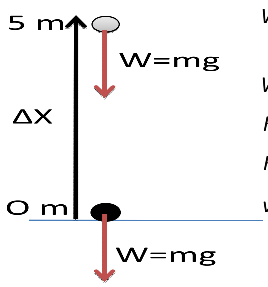
$$10v_f^2 = 150$$
$$v_f = 3.87m.s^{-1}, East$$

$$P_f = mv_f = (20kg)(3.87m.s^{-1}) = 77.46kg.m.s^{-1}, East$$

WEP-4

A 3 kg steel ball is fired straight up from the ground at a speed of 15 m/s. Use the work-energy theorem to calculate the speed of the ball when it has been displaced 5m.

Exercise: Confirm using kinematic equations.
Exercise: Confirm using conservation of energy.



$$W_{ext} = w\Delta x = -mg(x_2 - x_1) = -(3)(9.8)(5) = -147\,J$$

$$W_{ext} = \Delta K = K_f - K_i$$

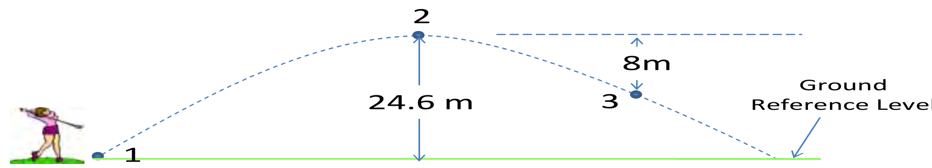
$$K_f = W_{ext} + K_1 = -147 + \tfrac{1}{2}(3)(15^2) = 190.5\,J$$

$$K_f = \tfrac{1}{2}(3)(v_f^2) = 190.5$$

$$v_f = \sqrt{\tfrac{2}{3}(190.5)} = 11.27\,m.s^{-1}$$

WEP-5

A 47.0 g golf ball is driven from the tee with an initial speed of 52.0 m/s and rises to a height of 24.6 m. (i) Neglect air resistance and determine the kinetic energy of the ball at its highest point. (ii)What is its speed when it is 8.0 m below its highest point?



(i) $K_1 + U_1 = K_2 + U_2$ (conservation of M E)

$K_2 = K_1 + U_1 - U_2 = \frac{1}{2}(47\times10^{-3})(52)^2 + (47\times10^{-3})(9.8)(24.6) - 0$

$K_2 = 74.87\,J$

(ii) $K_2 + U_2 = K_3 + U_3$ (conservation of M E)

$K_3 = K_2 + U_2 - U_3 = 74.87 + (47\times10^{-3})(9.8)(24.6) - (47\times10^{-3})(9.8)(24.6-8)$

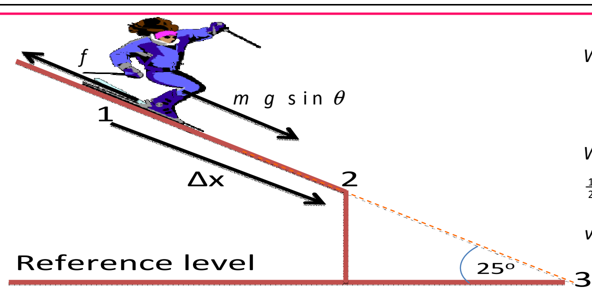
$K_3 = 78.56\,J$

$\frac{1}{2}mv_3^2 = 78.56$

$v_3 = \sqrt{(2)(78.56)/(47\times10^{-3})} = 57.82\,m.s^{-1}$

WEP-6

A 55 kg skier starts from rest and coasts down a mountain slope inclined at 25° to the horizontal. The kinetic friction between her skis and the snow is 97.7N. She travels 12m down the slope before coming to the edge of a cliff. Without slowing down, she skis off the cliff and lands downhill at a point whose vertical distance is 4m below the edge. Using energy methods, determine her speed just before she lands?



$W_{net} = F_{net}\Delta x = (mg\sin\theta - f)(\Delta x)$

$= \left((55)(9.8)\sin 25^0 - 97.7\right)(12)$

$= 1561.09\,J$

$W_{net} = K_2 - K_1 = K_2$

$\frac{1}{2}mv_2^2 = 1561.09$

$v_2 = \sqrt{\frac{2(1561.09)}{55}} = 7.53\,m.s^{-1}$

Assuming air resistance is negligible:

$K_2 + U_2 = K_3 + U_3$ (Cons. of M E)

$1561.09 + mgh = K_3 + 0$

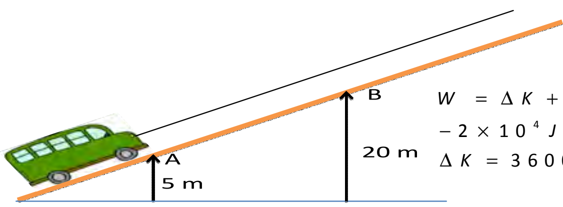
$K_3 = 1561.09 + mgh = 561.09 + (55)(9.8)(4) = 3717.09$

$\frac{1}{2}mv_3^2 = 3717.09$

$v_3 = \sqrt{\frac{2(3717.09)}{55}} = \underline{11.63\,m.s^{-1}}$

WEP-7

A 1200 kg mini-bus (stuck – engine off) is being pulled up from a from a point *A* ,5 m above the ground, to a point *B* ,20 m above the ground. Work done by friction is -2×10^{4} J and work done by chain mechanism (to help the car up the bank) is $+2\times10^{5}$ J . What is the change in the car's kinetic energy?



$$W = \Delta K + \Delta U$$

$$-2\times10^{4} J + 2\times10^{5} J = \Delta K + (1200)(9.8)(20-15)$$

$$\Delta K = 3600 J$$

OR

$$W_{ext} = \Delta K \quad \text{(Work-Energy Theorem)}$$

$$w_f + w_c + w_g = \Delta K$$

$$-2\times10^{4} J + 2\times10^{5} J + \left[-(1200)(9.8)(20-15)\right] = \Delta K$$

$$\Delta K = 3600 J$$

WEP-8

A 80 kg truck driver accelerate his 2000 kg truck from rest at rate of 8m.s^{-2}. If the trucks displacement is 300 m, calculate the power (in kW) expended to accomplish this on a frictionless road.

$$W_{net} = F_R \Delta x = m a \Delta x = (2000 + 80)(8)(300) = 4.99 \times 10^6 \, J$$

$$\Delta x = v_i t + \tfrac{1}{2} a t^2$$

$$300 = 0 + \tfrac{1}{2}(8) t^2 \Rightarrow t = 8.66 \, s$$

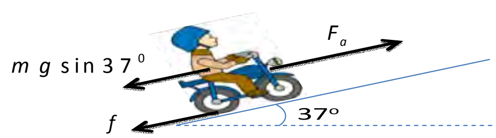
$$P = \frac{W}{t} = \frac{4.99 \times 10^6 \, J}{8.66 \, s} = 5.76 \times 10^5 \, \text{W} = \underline{576 \text{ kW}}$$

WEP-9

A motorcycle (mass of cycle plus rider is 270 kg) is traveling at a steady speed of 30 m/s. The force of air resistance acting on the cycle and rider is 240 N. Find the power necessary to sustain this speed if (a) the road is level and (b) the road is sloped upward at 37.0° with respect to the horizontal

(a) The power developed by the engine is:

$$P = Fv = (240 \, N)\left(30 \, m.s^{-1}\right) = 7200 \, \text{W}$$



(b) The power developed by the engine is:

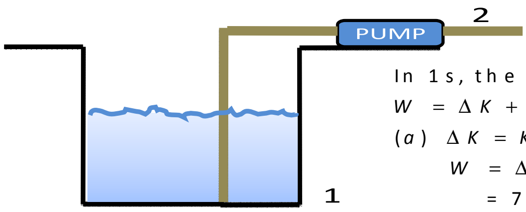
$$P = Fv = (240 \, N + mg \sin\theta)\left(30 \, m.s^{-1}\right)$$

$$= [240 \, N + (270)(9.8)\sin 37^0)](30)$$

$$= 5.50 \times 10^4 W$$

WEP-10

A pump is needed to lift water through a distance of 25m at a steady rate of 180kg/min. What is the minimum power motor that could operate the pump if (a) the velocity of the water is negligible at both the intake and outlet? (b) The velocity at the intake is negligible but at the outlet the water is moving with a speed of 9m/s.



In 1s, the mass lifted = (180kg/60s) × (1s) = 3kg

$W = \Delta K + \Delta U$

(a) $\Delta K = K_2 - K_1 = 0$

$W = \Delta U = mg(h_2 - h_1) = (3)(9.8)(25 - 0)$

$= 735\ J$

$P = \frac{W}{t} = \frac{735\ J}{1\ s} = \underline{735\ W}$

In 1s, the mass lifted = (180kg/60s) × (1s) = 3kg

(b) $W = \Delta K + \Delta U = \frac{1}{2}m(v_2^2 - v_1^2) + mg(h_2 - h_1)$

$= \frac{1}{2}(3)(9^2 - 0) + (3)(9.8)(25 - 0)$

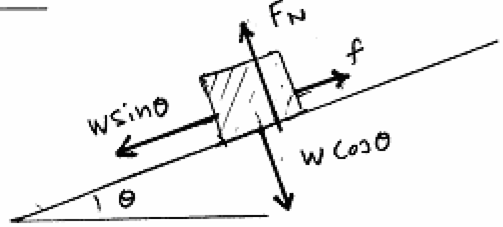
$= 856.5\ J$

$P = \frac{W}{t} = \frac{856.5\ J}{1\ s} = \underline{856.5\ W}$

Answers to activities and exercises that appear on pages 82 to 99.

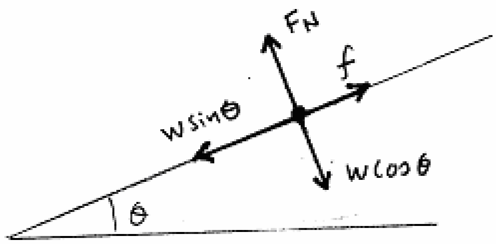
FORCE

ACTIVITY 1



(Force Diagram)

ACTIVITY 2



(Free Body Diagram)

ACTIVITY 3.

(i) Equal in magnitude

(ii) oppositely directed

(iii) Act along the same line of action (co-linear)

(iv) Act on different objects

(v) Act simultaneously

ACTIVITY 4

(a) The force exerted by vase on table and the force exerted by table on vase.

(b) The force exerted by the earth on the vase and the force exerted by the vase on the earth.

ACTIVITY 5

(a) i) The force exerted by table on vase and the force exerted by the vase on the table.

ii) The force exerted by the earth on the vase and the force exerted by the vase on the earth.

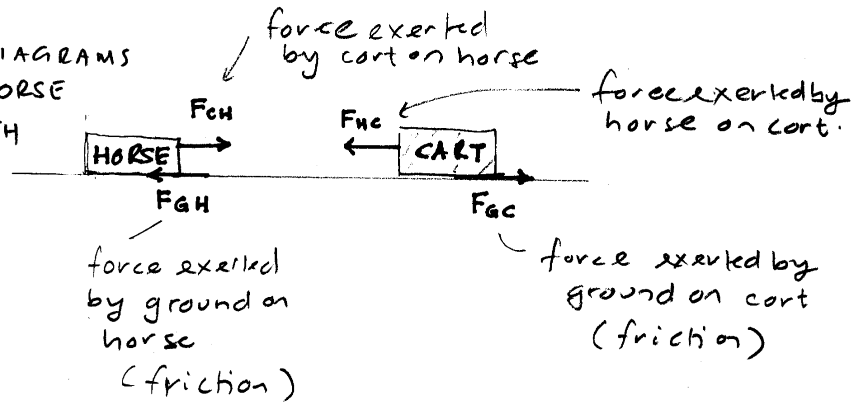
(b) i) The force exerted by the vase on the table and the force exerted by the table on the vase.

ii) Force exerted by the table legs on the floor and the force exerted by the floor on the table legs.

iii) Force exerted by the Earth on the table and the force exerted by the table on the earth.

ACTIVITY 6 :

FORCE DIAGRAMS FOR HORSE & CARTH



The horse will move when: $F_{GH} > F_{CH}$

The cart will move when: $F_{HC} > F_{GC}$

Recall: N3 pairs such as F_{CH} and F_{HC} act on different objects.

ACTIVITY 7 :

(a) i) The force exerted by the ball on the person's hand and the force exerted by the person's hand on the ball.

ii) The force exerted by the earth on the ball and the force exerted by the ball on the earth.

(b) While the ball is falling only (ii) above is present.

2. MOMENTUM & IMPULSE

Example 1:

$\uparrow +ve \quad g = -9.8\ m/s^2$

$V_f = V_i + at$

$V_f = 25 - 9.8t$

$$\boxed{V_f(t) = 25 - 9.8t}$$

$V_f(2) = 25 - 9.8(2)$

$= 5.40\ m.s^{-1} \uparrow$

$P_f(2) = mV_f(2)$

$= (2\ kg)(5.40\ m.s^{-1})$

$= \underline{10.8\ kg.m.s^{-1}} \uparrow$

$V_f(3) = 25 - 9.8(3) = -4.40\ m.s^{-1} = 4.40\ m.s^{-1} \downarrow$

$P_f(3) = mV_f(3) = (2\ kg)(-4.40\ m.s^{-1}) = -8.80\ kg.m.s^{-1}$

$\therefore \quad P_f(3) = \underline{8.80\ kg.m.s^{-1}} \downarrow$

ACTIVITY 1

$F_{net} = ma = m\frac{\Delta V}{\Delta t} = m\frac{(V_f - V_i)}{\Delta t}$

$\therefore \quad F_{net} = \frac{mV_f - mV_i}{\Delta t}$

$= \frac{p_f - p_i}{\Delta t}$

$$\therefore \quad F_{net} = \frac{\Delta p}{\Delta t}$$

ACTIVITY 2

According to N2, $F_{net} = \frac{\Delta p}{\Delta t}$ $\Rightarrow$

an objects momentum only changes if a net force is applied to it. A momentum change means a change in the state of motion. Newton's first low states that an object will maintain its state of motion, unless acted upon by a net force. So a net force is required to change an objects state of motion. Clearly, Newton 1 and Newton 2 (in momentum form) are related.

EXAMPLE 2

$$a = \frac{F_{net}}{m} = \frac{4 \times 10^7 \text{ N}}{1 \times 10^6 \text{ kg}} = 40 \text{ m}\cdot\text{s}^{-2} \uparrow$$

$$\Delta v = at = (40 \text{ m}\cdot\text{s}^{-2})(30 \text{ s}) = 1200 \text{ m}\cdot\text{s}^{-1} \uparrow$$

$$\Delta P = m \Delta v = 1.20 \times 10^9 \text{ kg}\cdot\text{m}\cdot\text{s}^{-1} \uparrow$$

EXAMPLE 3

$$a = \frac{F_{net}}{m} = \frac{-50\,N}{25\,kg} = -2\,m\cdot s^{-2} = 2\,m\cdot s^{-2}, \text{WEST}$$

$$\Delta v = at = (-2\,m\cdot s^{-2})(3\,s) = -6\,m\cdot s^{-1} = 6\,m\cdot s^{-1}, \text{WEST}$$

$$\Delta P = m\Delta v = (25\,kg)(-6\,m\cdot s^{-1}) = -150\,kg\cdot m\cdot s^{-1}$$

$$= 150\,kg\cdot m\cdot s^{-1}, \text{WEST}$$

EXAMPLE 4

$$\Delta P = m\Delta v$$

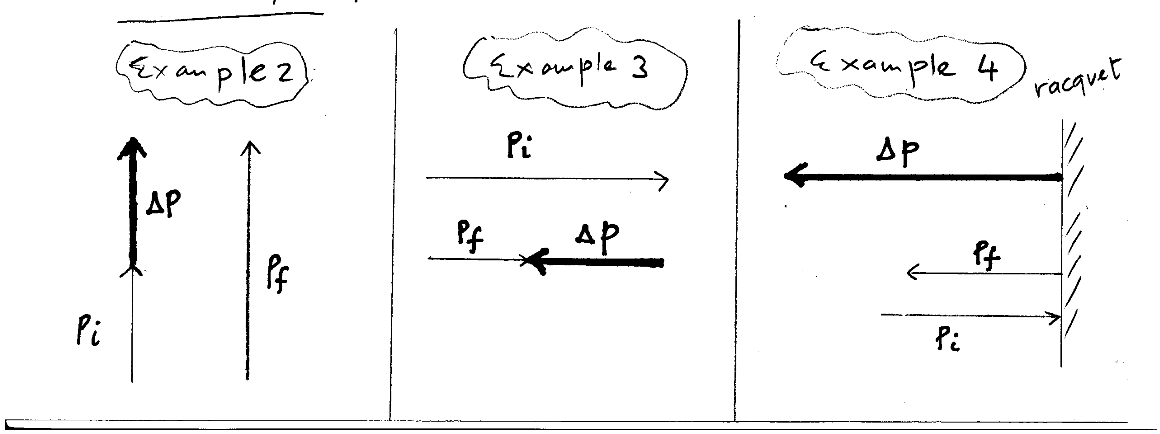
$$= m(v_f - v_i)$$

$$= (0.06\,kg)(-50 - 60)\,m\cdot s^{-1}$$

$$= -6.60\,kg\cdot m\cdot s^{-1}$$

$$= 6.60\,kg\cdot m\cdot s^{-1}, \text{west}$$

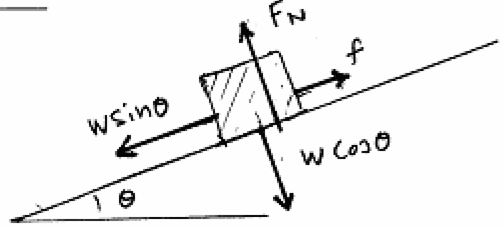
ACTIVITY 3:



Answers to activities and exercises that appear on pages 82 to 99.

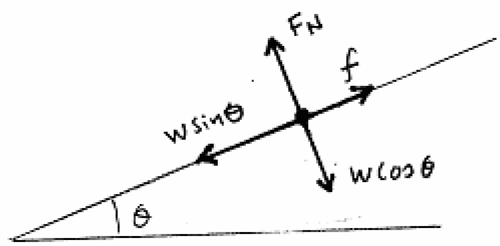
FORCE

ACTIVITY 1



(Force Diagram)

ACTIVITY 2



(Free Body Diagram)

ACTIVITY 3.

(i) Equal in magnitude

(ii) oppositely directed

(iii) Act along the same line of action (co-linear)

(iv) Act on different objects

(v) Act simultaneously

ACTIVITY 4

(a) The force exerted by vase on table and the force exerted by table on vase.

(b) The force exerted by the earth on the vase and the force exerted by the vase on the earth.

ACTIVITY 5

(a) i) The force exerted by table on vase and the force exerted by the vase on the table.

ii) The force exerted by the earth on the vase and the force exerted by the vase on the earth.

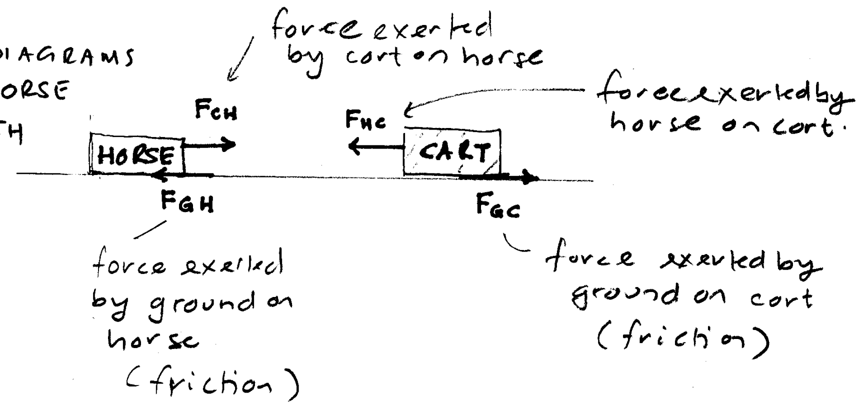
(b) i) The force exerted by the vase on the table and the force exerted by the table on the vase.

ii) Force exerted by the table legs on the floor and the force exerted by the floor on the table legs.

iii) Force exerted by the Earth on the table and the force exerted by the table on the earth.

ACTIVITY 6:

FORCE DIAGRAMS FOR HORSE & CARTH



The horse will move when: $F_{GH} > F_{CH}$

The cart will move when: $F_{HC} > F_{GC}$

Recall: N3 pairs such as F_{CH} and F_{HC} act on different objects.

ACTIVITY 7:

(a) i) The force exerted by the ball on the person's hand and the force exerted by the person's hand on the ball.

ii) The force exerted by the earth on the ball and the force exerted by the ball on the earth.

(b) While the ball is falling only (ii) above is present.

2. MOMENTUM & IMPULSE

Example 1:

↑ +ve $g = -9.8\ m/s^2$

$V_f = V_i + at$

$V_f = 25 - 9.8t$

$\boxed{V_f(t) = 25 - 9.8t}$

$V_f(2) = 25 - 9.8(2)$

$= 5.40\ m.s^{-1}$ ↑

$P_f(2) = mV_f(2)$

$= (2\ kg)(5.40\ m.s^{-1})$

$= 10.8\ kg.m.s^{-1}$ ↑

$V_f(3) = 25 - 9.8(3) = -4.40\ m.s^{-1} = 4.40\ m.s^{-1}$ ↓

$P_f(3) = mV_f(3) = (2\ kg)(-4.40\ m.s^{-1}) = -8.80\ kg.m.s^{-1}$

$\therefore P_f(3) = 8.80\ kg.m.s^{-1}$ ↓

ACTIVITY 1

$F_{net} = ma = m\frac{\Delta V}{\Delta t} = m\frac{(V_f - V_i)}{\Delta t}$

$\therefore F_{net} = \frac{mV_f - mV_i}{\Delta t}$

$= \frac{p_f - p_i}{\Delta t}$

$\therefore F_{net} = \frac{\Delta p}{\Delta t}$

ACTIVITY 2

According to N2, $F_{net} = \frac{\Delta p}{\Delta t} \Rightarrow$

an objects momentum only changes if a net force is applied to it. A momentum change means a change in the state of motion. Newton's first low states that on object will maintain its state of motion, unless acted upon by a net force. So a net force is required to change an objects state of motion. Clearly, Newton 1 and Newton 2 (in momentum form) are related.

EXAMPLE 2

$$a = \frac{F_{net}}{m} = \frac{4 \times 10^7 \text{ N}}{1 \times 10^6 \text{ kg}} = 40 \text{ m} \cdot \text{s}^{-2} \uparrow$$

$$\Delta v = at = (40 \text{ m} \cdot \text{s}^{-2})(30 \text{ s}) = 1200 \text{ m} \cdot \text{s}^{-1} \uparrow$$

$$\Delta P = m \Delta v = 1.20 \times 10^9 \text{ kg} \cdot \text{m} \cdot \text{s}^{-1} \uparrow$$

EXAMPLE 3

$$a = \frac{F_{net}}{m} = -\frac{50\,N}{25\,kg} = -2\,m\cdot s^{-2} = 2\,m\cdot s^{-2},\ \text{WEST}$$

$$\Delta v = at = (-2\,m\cdot s^{-2})(3\,s) = -6\,m\cdot s^{-1} = 6\,m\cdot s^{-1},\ \text{WEST}$$

$$\Delta P = m\,\Delta v = (25\,kg)(-6\,m\cdot s^{-1}) = -150\,kg\cdot m\cdot s^{-1}$$

$$= 150\,kg\cdot m\cdot s^{-1},\ \text{WEST}$$

EXAMPLE 4

$$\Delta P = m\,\Delta v$$

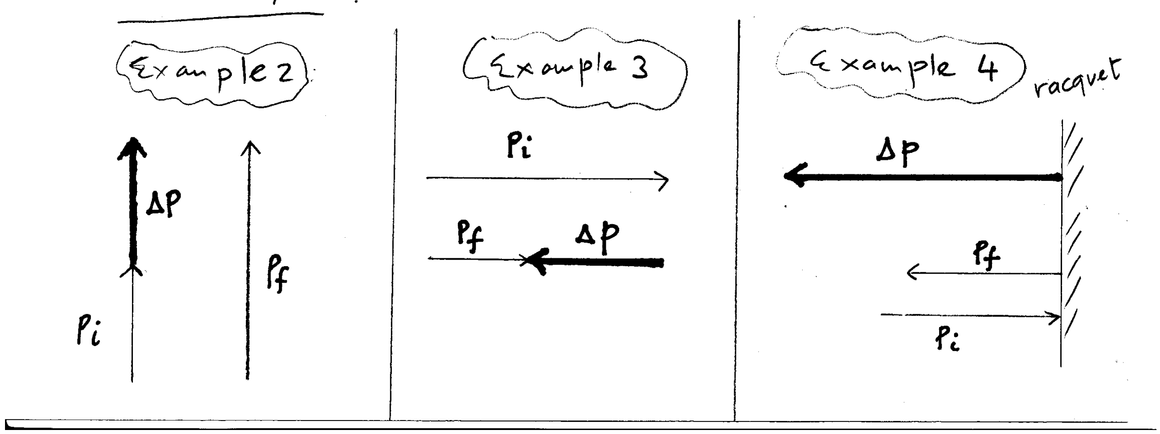
$$= m(v_f - v_i)$$

$$= (0.06\,kg)(-50 - 60)\,m\cdot s^{-1}$$

$$= -6.60\,kg\cdot m\cdot s^{-1}$$

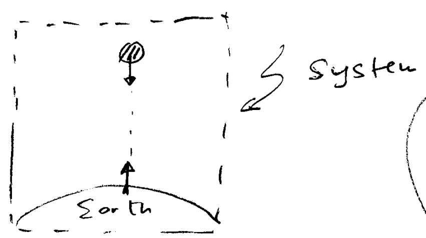
$$= 6.60\,kg\cdot m\cdot s^{-1},\ \text{west}$$

ACTIVITY 3 :



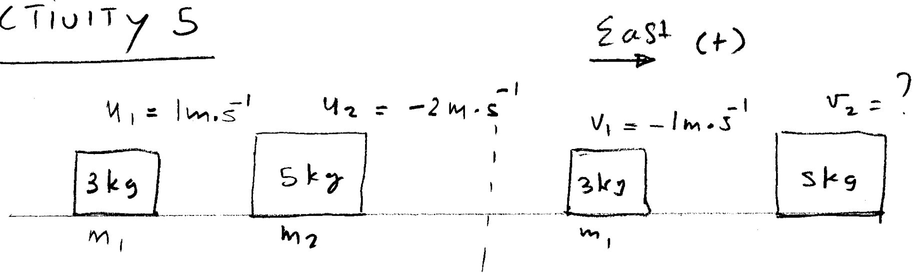
ACTIVITY 4

No. If we consider the earth and the ball as a system, then there will be no external force acting on the ball.



Note:
The momentum of the ball on its own is not conserved, but the momentum of the Earth-Ball System is conserved.

ACTIVITY 5



(a) $P_{before} = P_{after}$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(3)(1) + (5)(-2) = (3)(-1) + 5 v_2$$

$$-7 + 3 = 5 v_2$$

$$-4 = 5 v_2$$

$$v_2 = -0.8 \text{ m.s}^{-1}$$

$$= 0.8 \text{ m.s}^{-1}, \text{ West}$$

(b) If friction is present, then the net force on the system containing the 3 kg and 5 kg would not be an isolated system and hence momentum will not be conserved

(c) For the 3 kg: $\Delta P_1 = m(v_1 - u_1)$

$= 3(-1-1)$

$= -6\ kg.m.s^{-1}$

For the 5 kg: $\Delta p_2 = m(v_2 - u_2)$

$= 5(-0.8+2)$

$= 5(1.2)$

$= +6\ kg.m.s^{-1}$

This does agree with theory:

$$\Delta P_1 = -\Delta P_2$$

ACTIVITY 6

It does make a difference. During the collision, the magnitude of the force each object experiences is equal. Since the time is the same ⇒

$$\Delta P_1 = -\Delta P_2$$

$$\Rightarrow |\Delta P_1| = |\Delta P_2|$$

$$\Rightarrow m\Delta v = const$$

⇒ the smaller mass has a larger change in velocity. This implies that it is safer to be in the larger vehicle.

ACTIVITY 7

$$P_{before} = P_{after}.$$

$$0 = P_b + P_c$$

$$\therefore P_b = -P_c$$

$$|P_b| = |P_c| = P$$

$$E_k = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \times \frac{m v^2}{m}$$

$$= \frac{1}{2} \frac{m^2 v^2}{m}$$

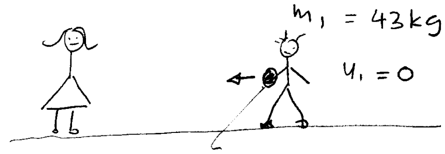
$$E_k = \frac{1}{2} \frac{P^2}{m}$$

$$\therefore E_k \propto \frac{1}{m}$$

$\Rightarrow$ The cannon ball carries the greater kinetic energy.

ACTIVITY 8

← WEST (+)



$m_1 = 43 kg$

$u_1 = 0$

$m_2 = 0.75 kg$

$u_2 = 0$, $v_2 = 6.2 m.s^{-1}$

For the Boy - Ball system.

$$P_{before} = P_{after}$$

$$0 = m_1 v_1 + m_2 v_2$$

$$= (43)(v_1) + (0.75)(6.2)$$

$$v_1 = -0.11 m.s^{-1} = 0.11 m.s^{-1} \text{ (EAST)}$$ ← Boy.

For the girl - ball system.

$m_2 = 0.75 kg$, $u_2 = 6.2\ m.s^{-1}$

$m_3 = 35 kg$; $u_3 = 0$

$P_{before} = P_{after}$.

$$m_2 u_2 + m_3 u_3 = (m_2 + m_3) v$$

$$(0.75)(6.2) + (35)(0) = (35.75) v$$

$$v = 0.13\ m.s^{-1} \text{ (WEST)}$$

ACTIVITY 9:

Consider a moving object. If we wanted to stop the object using a large force, this could be achieved quickly. If we wanted to stop the object using a smaller force, it would take longer. In both cases the impulse (change in momentum) is the same. So the larger force and smaller force produce the same impulse.

EXAMPLE 5

(a) $\Delta P = m(v - u) = (1000\ kg)(0 - 30\ m.s^{-1})$

$= -30\ 000\ kg.m.s^{-1}$

$= 30\ 000\ kg.m.s^{-1}$, EAST

(b) $F_{net} = \frac{\Delta P}{\Delta t} = \frac{-30\ 000\ kg.m.s^{-1}}{2 \times 10^{-3}\ s}$

$= \frac{-30\ 000\ N.s}{2 \times 10^{-3}\ s}$

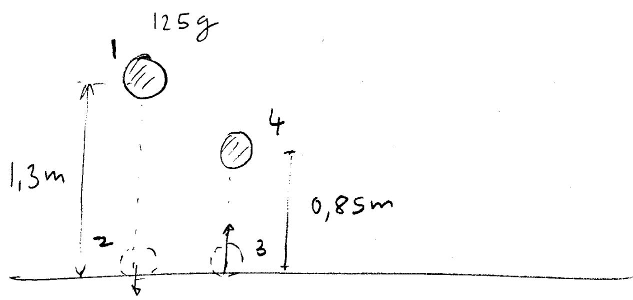
$= -1.50 \times 10^{7}\ N$

$= 1.50 \times 10^{7}\ N$, EAST

(c) $F_{net} = \frac{5}{100} \times (-1,50 \times 10^{7}) = -7.50 \times 10^{5}\ N$

$\Delta t = \frac{\Delta P}{F_{net}} = \frac{-30\ 000\ N.s}{-7.50 \times 10^{5}\ N} = 4 \times 10^{-2}\ s = 40\ ms$

ACTIVITY 10:



For the motion from 1→2, taking ↓+

$$\Delta x = 1.3 = \frac{V_2^2 - V_1^2}{2g} = \frac{V_2^2 - 0}{2(9.8)}$$

$$\therefore V_2^2 = (1.3)(2)(9.8) = 25.48$$

$$\therefore V_2 = 5.05 \text{ m.s}^{-1} \downarrow$$

For the motion from 3→4, taking ↑+.

$$\Delta x = 0.85 = \frac{V_4^2 - V_3^2}{2(-9.8)} = \frac{0 - V_3^2}{-19.6}$$

$$\therefore -V_3^2 = (-19.6)(0.85) = -16.66$$

$$\therefore V_3 = 4.08 \text{ m.s}^{-1} \uparrow$$

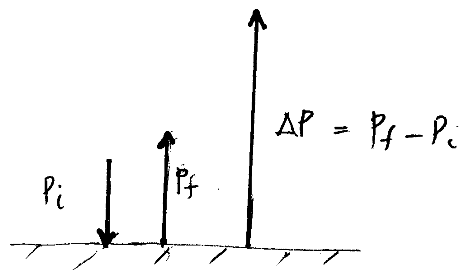
If we consider 2 and 3 only for the momentum "picture." ↓+

$$V_i = 5.05 \text{ m.s}^{-1}, \quad V_f = -4.08 \text{ m.s}^{-1}$$

$$\therefore (a) \quad \Delta P = m(V_f - V_i) = \left(\frac{125}{1000}\right)(-4.08 - 5.05)$$

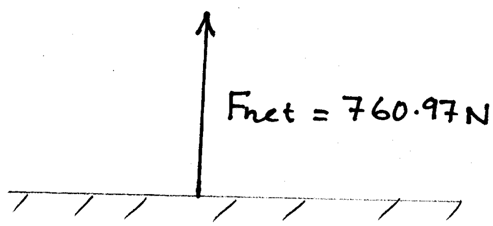
$$= -1.14 \text{ kg.m.s}^{-1} = 1.14 \text{ kg.m.s}^{-1} \uparrow$$

(b)



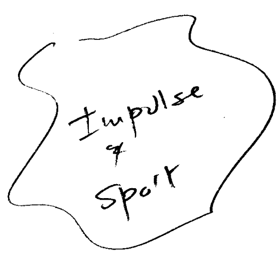
(c) $F_{net} = \frac{\Delta P}{\Delta t} = \frac{-1.14 \text{ N.s}}{1.5 \times 10^{-3} \text{ s}} = -760.97 \text{ N}$

$\Rightarrow F_{net} = 760.97 \text{ N}, \uparrow$



REAL-WORLD APPLICATION OF IMPULSE

(a) Boxing – gloves help increase contact time of punches, minimizing the impact of the force (punch).



– Also, a boxer being hit generally "rides" a punch. Move in the same direction as the punch. This increase the contact time and minimizes the impact of the force.

(b) Cricket – When a batsman follows through, the contact time is increased, and the change in velocity is greater.

Impulse & Road Safety

(a) Air bags deploy during serious collisions. Passangers' upper body (head, neck) moving foward makes contact with airbag. The large contact time ⇒ Impact of force is reduced, preventing serious injury

(b) Certain sections in vehicles have crumple zones. The crumple zones break, fold or squash very easily during collisions. This helps to prevent or limit the rebounding of the vehicles. Rebounding results in large contact forces.

(c) Arrestor Beds are run-off sections alongside roads. These sections contain soft sand. Any vehicle that has lost control can move into the arrestor bed. The large contact time (tyres and sand) gradually slow the vehicle to a stop. Change in momentum is gradual ⇒ safety of driver is ensured.

WORK, ENERGY & POWER

ACTIVITY 1

(a) Yes, positive work

(b) No work

(c) yes, positive work

(d) yes, Negative

ACTIVITY 2

USING THE DEFINITION

$F_N \perp \Delta x \Rightarrow W_{F_N} = 0\,J$

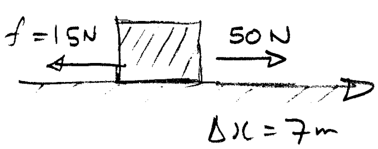
$W \perp \Delta x \Rightarrow W_W = 0\,J$

$W_F = F\,\Delta x \cos 60^\circ = (100N)(9-2)m \cos 60^\circ = 350J$

$W_f = f\,\Delta x \cos 180^\circ = (15N)(9-2)m \cos 180^\circ = -105J$

$W_{net} = 0J + 0J + 350J - 105J = \underline{245J}$

USING THE TECHNIQUE



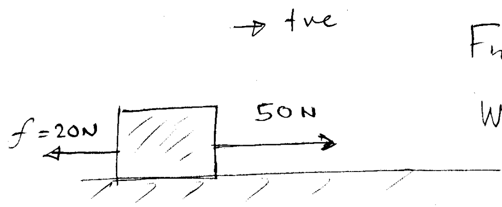
$F_{net} = 50 - 15 = 35N$

$W_{net} = F_{net}\,\Delta x$

$= 35N \times 7m$

$= 245J$

EXAMPLE 1



$F_{net} = 50N - 20N = 30N$

$W_{net} = F_{net}\,\Delta x = 30N \times 5m$

$= 150J$

$W_{net} = \Delta K = K_2 - K_1$

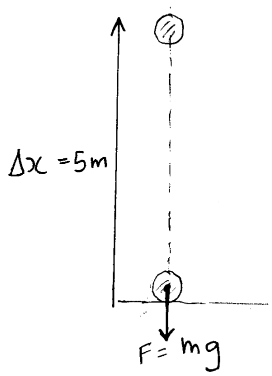
$\therefore \quad 150 = \frac{1}{2} m v_f^2 - 0$

$\therefore \quad v_f^2 = \frac{2(150)}{20} = 15$

$\therefore \quad v_f = 3.87\ m.s^{-1}$

$P_f = m v_f = \underline{77.46\ kg.m.s^{-1}} \rightarrow$

ACTIVITY 3



$W_{net} = F_{net}\,\Delta x$

$= -mg\,\Delta x$

$= -(3)(9.8)(5)$

$= -147J$

$W_{net} = \Delta K = K_2 - K_1$

$\therefore K_2 = W_{net} + K_1$

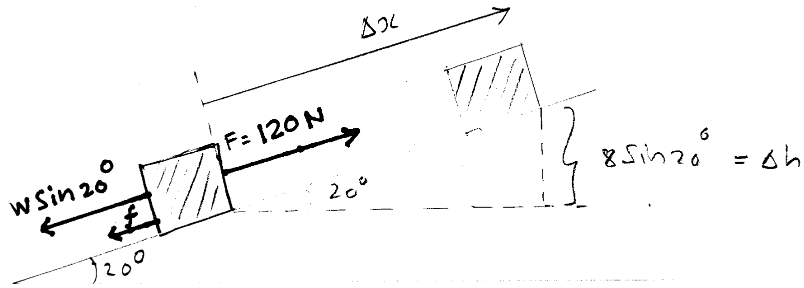
$= -147J + \frac{1}{2}(3)(15)^2$

$\frac{1}{2}mv^2 = 190.5J$

$v^2 = \frac{(190.5)(2)}{3} = 127$

$\Rightarrow \quad v = \underline{11.27\ m.s^{-1}}$

EXAMPLE 2



(a) $W_g = (W\sin 20^\circ)(8\,\text{m})\cos 180^\circ$

$= -(mg\sin 20^\circ)(8)$

$= -(10)(9.8)(\sin 20^\circ)(8)$

$= \underline{-268.14\,\text{J}}$

OR

$W_g = -mg\Delta h$

$= -(10)(9.8)(8\sin 20^\circ)$

$= \underline{-268.14\,\text{J}}$

(b) $W_f = f\Delta x\cos 180^\circ$

$= (15)(8)(-1)$

$= -120\,\text{J}$

(c) $W_F = (120)(8)\cos 0^\circ$

$= 960\,\text{J}$

(d) $W_{net} = W_g + W_f + W_F = 571.86\,\text{J}$

$W_{net} = \Delta K = 571.68\,\text{J}$

(e) $W_{net} = \frac{1}{2}m(v^2 - u^2)$

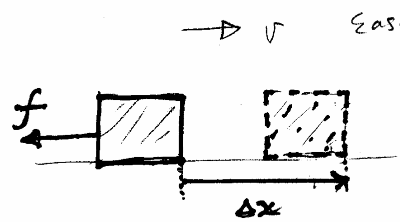
$v^2 = \frac{2W_{net}}{m} + u^2$

$= \frac{2(571.68)}{10} + (2.50)^2$

$= 120.62$

$v = \underline{10.98\,\text{m.s}^{-1}}$

ACTIVITY 4



$F_{net} = -f$

$W_{net} = -f\Delta x$

$W_{net} = \Delta K = -f\Delta x$

$\therefore \Delta K < 0$

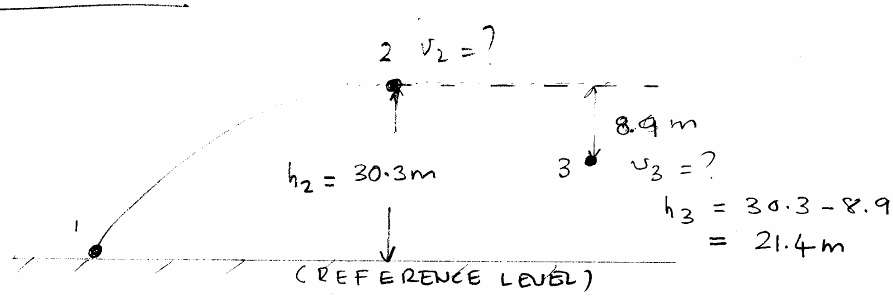
$\therefore K_2 - K_1 < 0$

$\therefore K_2 < K_1$

$\Rightarrow v_2 < v_1$

$\Rightarrow$ Box is slowing down.

EXAMPLE 3



$h_1 = 0\,m$

$v_1 = 50\,m.s^{-1}$

$K_1 + U_1 = K_2 + U_2$ (Cons. of Mech. Eng)

$\frac{1}{2} m v_1^2 + 0 = \frac{1}{2} m v_2^2 + m g h_2$

$\frac{1}{2} v_2^2 = \frac{1}{2} v_1^2 - g h_2$

$\therefore v_2^2 = v_1^2 - 2 g h_2$

$= (50)^2 - 2(9.8)(30.3)$

$= 1906.12$

$\therefore v_2 = 43.66\ m.s^{-1}$

$$K_1 + U_1 = K_3 + U_3$$

$$\frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mv_3^2 + mgh_3$$

$$v_1^2 = v_3^2 + 2gh_3$$

$$V_3^2 = V_1^2 - 2gh_3$$

$$= (50)^2 - 2(9.8)(21.4)$$

$$= 2080.56$$

$$\therefore V_3 = 45.61\ m.s^{-1}$$

$$P_3 = m v_3$$

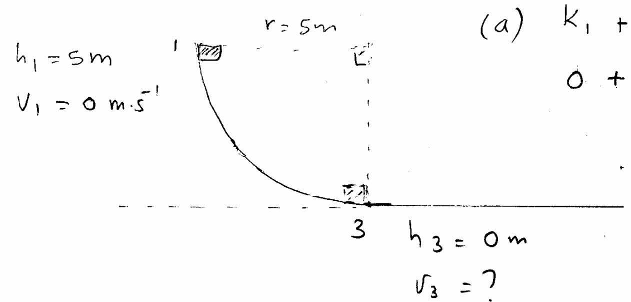
$$2.28\ kg.m.s^{-1} = m\ 45.61\ m.s^{-1}$$

$$\therefore m = \frac{2.28\ kg.m.s^{-1}}{45.61\ m.s^{-1}}$$

$$= 0.05\ kg$$

$$= \underline{50g}$$

ACTIVITY 5



(a) $K_1 + U_1 = K_3 + U_3$

$$0 + mgh_1 = \frac{1}{2}mv_3^2 + 0$$

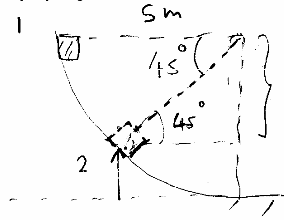
$$\therefore V_3^2 = 2gh_1$$

$$= 2(9.8)(5)$$

$$= 98$$

$$V_3 = \underline{9.90\ m.s^{-1}}$$

(b) $h_1 = 5\,m$
$v_1 = 0\,m\cdot s^{-1}$



$x = 5\sin 45^\circ = 3.54\,m$

$h_2 = 5 - 3.54$
$= 1.46\,m$

$v_2 = ?$

$$K_1 + U_1 = K_2 + U_2$$

$$0 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2$$

$$\therefore\ 2gh_1 = v_2^2 + 2gh_2$$

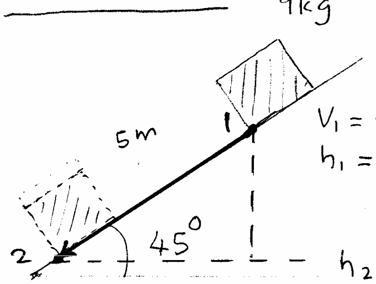
$$v_2^2 = 2g(h_1 - h_2)$$

$$= 2(9.8)(5 - 1.46)$$

$$= 69.30$$

$$v_2 = \underline{8.32\,m\cdot s^{-1}}$$

EXAMPLE 4:



$V_1 = 0$
$h_1 = 5\sin 45^\circ = 3.54\,m$

$h_2 = 0$
$v_2 = 4.985\,m\cdot s^{-1}$

$$K_1 + U_1 + W_f = K_2 + U_2$$

$$0 + mgh_1 + W_f = \frac{1}{2}mv_2^2 + 0$$

$$W_f = \frac{1}{2}mv_2^2 - mgh_1$$

$$= \frac{1}{2}(9)(4.985)^2 - 9(9.8)(3.54)$$

$W_f = -200.01\ J$

$W_f = f\Delta x \cos 180^\circ$

$= -f\Delta x$

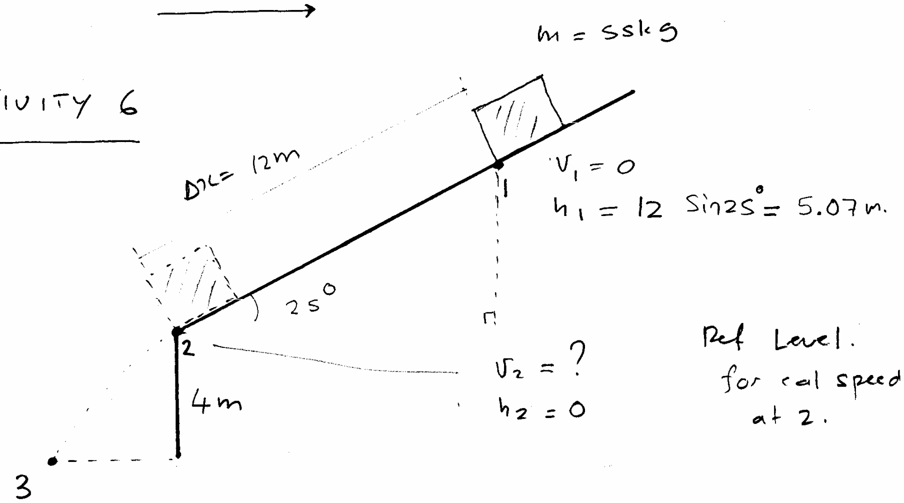
$= -f(5)$

$= -5f$

$\therefore \quad -5f = -200.01$

$f = 40\ N$

ACTIVITY 6



$\therefore$ Apply cons of Energy for 1 and 2

$K_1 + U_1 + W_f = K_2 + U_2$

$0 + mgh_1 + (-f\Delta x) = \frac{1}{2}mv_2^2 + 0$

$(55)(9.8)(5.07) + (-97.7)(12) = \frac{1}{2}(55)v_2^2$

$\therefore \quad v_2 = 7.53\ m.s^{-1}$

→ speed at the bottom of the slope.

Apply conservation of Mech. Energy for points 2 and 3.

$$K_2 + U_2 = K_3 + U_3.$$

Using landing point as ref. Level.

$v_2 = 7.53\ m.s^{-1}$, $h_2 = 4m$

$v_3 = ?$, $h_3 = 0m$

$$k_2 + U_2 = k_3 + U_3$$

$$\frac{1}{2} m v_2^2 + m g h_2 = \frac{1}{2} m v_3^2 + 0$$

$$\therefore \quad v_3^2 = v_2^2 + 2 g h_2$$

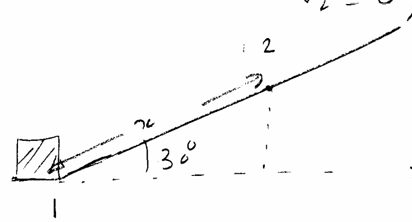
$$= (7.53)^2 + 2(9.8)(4)$$

$$= 135.17$$

$$\therefore \quad v_3 = \underline{11.63\ m.s^{-1}}$$

ACTIVITY 7

$V_2 = 0$, $h_2 = x \sin 30^\circ = \frac{x}{2}$



$h_1 = 0$

$v_1 = 9\ m.s^{-1}$

$$K_1 + U_1 + W_f = K_2 + U_2$$

$$\frac{1}{2}(5)(9)^2 + 0 + (-28.81)(x) = (5)(9.8)\frac{x}{2}$$

$$202.50 - 28.81x = 24.5x$$

$$46.31x = 202.50$$

$$x = \underline{4.37m}$$

EXAMPLE 5

$$W = \underset{w}{mg}\,\Delta x = (700\text{ N})(15\text{ m}) = 10500\text{ J}$$

$$P = \frac{W}{t} = \frac{10500\text{ J}}{9\text{ s}} = 1166.67\text{ W}$$

ACTIVITY 8:

(a) $W = mgh$. Their masses are equal and so is the vertical height between starting and finishing positions.

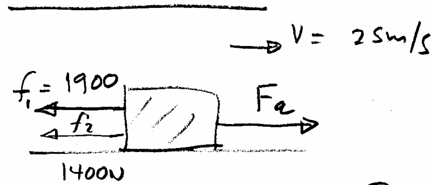
$\therefore W_c = W_H$.

(b) $t_c < t_H$, $P = \frac{W}{t}$

$\therefore P_c > P_H$

The climber expends more power in getting to the top.

EXAMPLE 6



$\boxed{a = 0} \Rightarrow$

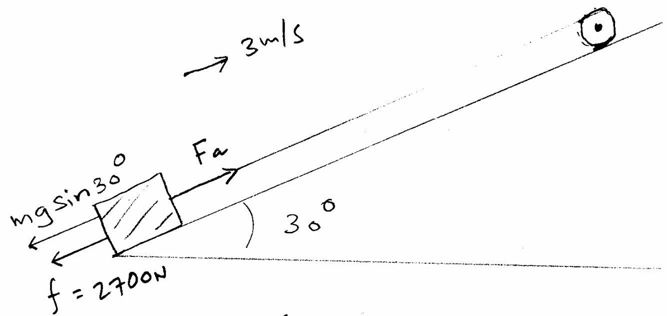
$F_a = f_1 + f_2$

$= 3300\text{ N}$

$P = Fv \Rightarrow P = (3300\text{ N})(25\text{ m}\cdot\text{s}^{-1})$

$= \underline{82.5\text{ kW}}$

ACTIVITY 9 :



$$a = 0$$

$$\therefore \quad F_{net} = 0$$

$$\Rightarrow \quad F_a = mg \sin 30^\circ + 2700$$

$$= (800)(9.8)(\tfrac{1}{2}) + 2700$$

$$= 6620 \text{ N}$$

$$P = Fv \quad \Rightarrow \quad P = (6620)(3) = 19860 \text{ W}$$

$$P = \underline{19.86 \text{ kW}}$$

ACTIVITY 10.

(a) $P = Fv = (240\text{ N})(30\text{ m.s}^{-1}) = 7200\text{ W}$

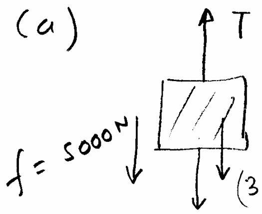
(b) $P = Fv = \left[(270)(9.8)\sin 37^\circ + 240\right](30)$

$$= 54972.08 \text{ W}$$

$$= \underline{5.50 \times 10^4 \text{ W}}$$

ACTIVITY 11 :-

(a)



$a = 0$

$w_2 = (3 \times 75 + 2 \times 50) \times 9.8 = 3185\ N$

$w_1 = 1700\,kg \times 9.8 = 16660\ N$

$\uparrow \quad T = f + w_1 + w_2$

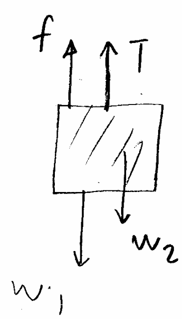
$= 24845\ N$

$P = Fv = Tv = (24845\ N)(4\ m.s^{-1})$

$= 99380\ W$

$= 99.38\ kW$

(b)



$a = 0$

$\Rightarrow$

$T = w_1 + w_2 - f$

$= 16660\ N + 3185\ N - 5000\ N$

$= 14845\ N$

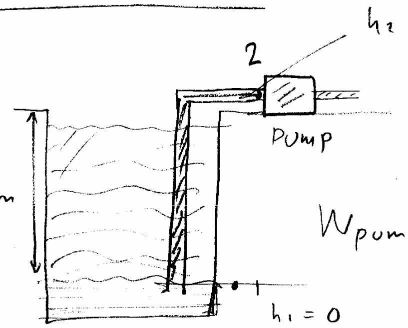
$P = Tv = (14845\ N)(4\ m.s^{-1})$

$= 59380\ W$

$= 59.38\ kW$

EXAMPLE 7:

(a)



For 1 min:

$$W_{pump} = \Delta K + \Delta U$$

$$= \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

$$v_1 = v_2 = 0$$

$$\Rightarrow W_{pump} = \Delta U = mg(h_2 - h_1)$$

$$= (180)(9.8)(25 - 0)$$

$$= 44100 \text{ J}$$

$$P = \frac{W_{pump}}{t}$$

$$= \frac{44100 \text{J}}{60 \text{s}} = 735 \text{ W}$$

(b)

$$W_{pump} = \Delta K + \Delta U$$

$$= \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

$$= \frac{1}{2}(180)(9^2 - 0^2) + 180(9.8)(25)$$

$$= 51390 \text{ J}$$

$$P = \frac{W_{pump}}{t} = \frac{51390 \text{J}}{60 \text{s}} = 856.50 \text{W}$$

ACTIVITY 12:

For 1 min:

$W_{ork} = \Delta K + \Delta U$

$= \frac{1}{2} m (v_2^2 - v_1^2) + mg(h_2 - h_1)$

$= \frac{1}{2}(950)(15^2 - 0^2) + 950(9.8)(40-0)$

$= 4.79 \times 10^5 \text{ J}$

$P_{required} = \frac{Work}{t} = \frac{4.79 \times 10^5 \text{ J}}{60 \text{ s}} = 7.99 \text{ kW}$

$P_{actual} = \frac{80}{100} \times 9000 \text{ W} = 7.20 \text{ kW}$

$P_{actual} < P_{required}$

$\Rightarrow$ Pump is <u>not</u> suitable <u>for the job</u>.

Some examples of practical activities

Physical Sciences: Physics	**Grade 11**
Friction: To determine the factors that affect the size of the frictional force between two surfaces	

Broad Knowledge Area:

Mechanics

Theme:

Force, momentum and impulse

Lesson Outcomes

Attainment is evident when the learner is able to:

1. Recognize that weight and surface type affect friction.
2. Recognize that surface area does NOT affect the friction.
3. Identify the dependent variable and independent variables
4. Identify control variables
5. Control variables
6. Recognize that some things are hard to measure like friction because the spring scale needle vibrates.

Apparatus:

4 small wood blocks for each group

Small screw hooks that can be screwed into the blocks to hook the blocks together.

1 spring scale for each group (if spring scales are not available you may

substitute a rubber band and note the amount the rubber band stretches).

Different surfaces like a table, carpet, glass, sandpaper, tiles, oil, water, etc.

Procedure

Learners should plan and conduct an investigation to determine the factors that affect the frictional force between two surfaces.

It is left to the discretion of the teacher, whether learners do this practical activity in groups or individually.

Hints to teacher:

- Tell the learners, a few day/s before the practical activity, to list the factors they think affects the size of the frictional force.
- Allow the learners select the equipment and let them try various combinations.
- At this point of the investigation do not tell the learners what combinations to try out. Allow them to explore combinations such as a different sides, different surfaces, a train (one hooked after the other), stacking on top, or combinations thereof.
- Regroup the learners together as a whole class after approximately 15 minutes of experimentation to discuss preliminary results. At this point you could remind Learners to control variables, remind them that they should not pull the spring scale at an angle and that the different sides of the block might have a different grain which can affect results.
- Let the learners go back into their groups so that they can fine tune their results. Have one representative from each group make a brief, final presentation of their results.

Further Questions

1. What happens if I double the weight by stacking one block on top of the

other?

2. What happens if I keep the weight the same but double the surface area?

3. What happens if I double the surface area and double the weight?

4. How does the surface type affect the frictional force? Answer: The answers will vary. Typically the smoother the surface is the less friction. However, sometimes glass which is very smooth will produce a large frictional force, specifically if it is very clean. FYI: There is a weak vacuum that is formed that pulls the blocks together when there is little or no air between the surfaces.

Conclusions:

Get learners to write conclusions that answer the questions that they investigated.

Physical Sciences: Physics	Grade 11
Torque: To show the moments of Force and to investigate the factors that cause the turning of a balanced object	

Broad Knowledge Area:

Mechanics

Theme:

Force, momentum and impulse

Lesson Outcomes

Attainment is evident when the learner is able to:

1. To show the moment of Force on a beam.
2. To determine the relationship between the distance from the fulcrum and the force on the object.
3. To predict the position of a single load on a beam in order to balance the beam.

Apparatus:

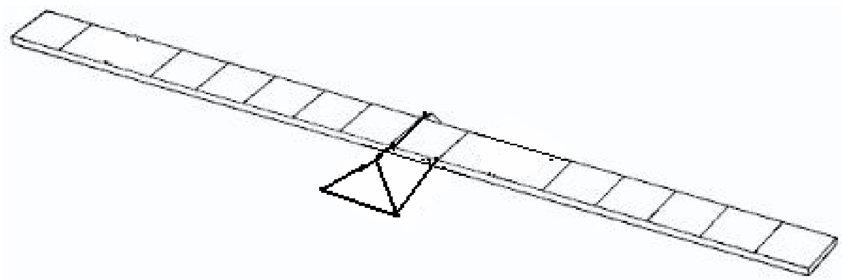
Simple beam with markings at regular intervals or a pivoted meter stick with sliding weights or a torque bar

Several mass pieces. E.g. 50g, 20g, 100g, etc

Triangular block

Procedure:

1. Balance the beam (or meter stick or torque bar) on the triangular block.



2. Place a mass so that the beam rotates.
3. Balance the beam without removing or changing the position of the mass that you placed in step two above.
4. Repeat steps 2 and 3 by placing different masses at different positions. Try out any variations that you can think of. (adding masses on top of other masses, etc)
5. Record your results appropriately.
6. Place two masses (same and/or different) at two different positions on the same side of the fulcrum and then try to balance the beam using only one other mass piece.

Hints to teacher:

1. The pattern in the results can be described in several ways.

   A learner who says words to the effect that, "doubling the load on one side requires the distance on the other side to be doubled" has spotted the pattern. One who says that, "the product of load and distance is the same on both sides of the beam when it is balanced" has provided a more general description that can be used to make predictions. In other words, the beam balances when the anti-clockwise moment equals the clockwise moment.

   Different learners will require different amounts of support in this. The most able will not only identify a pattern but will see for themselves that they can use it to make predictions of load position in order to achieve balance. Others will not see a pattern at all unless it is directly pointed out to them. It is worth explaining that the pattern is important because of its predictive power, which can be applied in many practical situations.

2. Learners' application of the predictive power of their new learning can be tested by moving the multiple loads to two marks from the pivot, and asking them to say where the single load must be placed for balance.

   The number of loads here provides a 'measurement' of weight, or force.

3. The product of the force and its distance from the pivot is a measure of its turning effect, and is called the **moment** of the force.

   For balance, the sum of the 'clockwise' moments is the same as the sum of the 'anticlockwise' moments. Large forces on one side of the fulcrum can be balanced by smaller forces on the other, provided that the smaller force is further from the fulcrum.

4. To illustrate the turning effect of a force, demonstrate with the classroom door. Try pushing it at the edge, then close to the hinge, then at intermediate positions. Compare the effects. You could try pushing near the hinge while a pupil pushes (from the other side) farther out. If you do this then take care that fingers cannot be trapped if the door closes.

Physical Sciences: Physics	Grade 11
Motion: To plan and conduct an experiment to test the following idea: an object will always move in the direction of the net force that is exerted on it by other objects	

Broad Knowledge Area:

Mechanics

Theme:

Force, momentum and impulse

Lesson Outcomes

Attainment is evident when the learner is able to:

1. Make a hypothesis
2. Test a hypothesis
3. Plan an investigation
4. Conduct an investigation
5. Collect relevant data
6. Analyse data
7. Formulate a relationship between variables
8. Make predictions

Apparatus:

1. Dynamics cart
2. dynamics track
3. spring scale calibrated in Newtons
4. masking tape
5. pulleys
6. mass pieces to hang
7. ramp

8. a few books

Hints to the teacher

It could be an idea to get the learners to do the following when engaged with this practical task:

1. Write down the idea that they are going to test.
2. Brainstorm the task and make a list of possible experiments whose outcomes can be predicted with the help of the idea. Decide whether testing an idea requires that you design experiments to prove the idea or to disprove the idea.
3. Briefly describe your chosen design Include a labeled sketch.
4. Draw a free body diagram of the object while the forces are being exerted on it.
5. Use the idea under test to make a prediction about the outcome of the experiment.
6. Perform the experiment. Record your observations.
7. Did the outcome of the experiment support the prediction?
8. Based on your prediction and the outcome of your experiment, can you say that the idea is proved, disproved?
9. Describe additional assumptions that you used to make a prediction about the outcome of your testing experiment. How can the assumptions affect your judgment?

Physical Sciences: Physics	
Capacitance: Discharging a capacitor	Grade 11

Broad Knowledge Area:

Electricity and Magnetism

Theme:

Electrostatics, capacitor as a circuit device

A capacitor is a device used to store electric charge. The capacitance of a capacitor is a measure of the quantity of charge, Q, it can store for a given potential difference, V. Capacitance is defined by the following equation:

$$C = Q/V$$

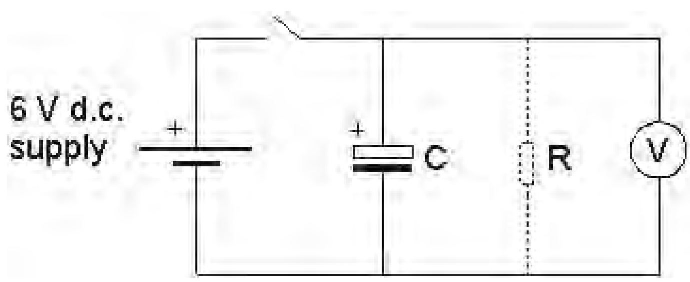
and so the units of capacitance are CV^{-1}. 1 CV^{-1} is called 1Farad (1F)

The capacitor is being studied here as it gives us another example of an *exponential* variation.

1. Preparation: a) Remind yourself how to measure the slope of a curved graph at a given point.

   b) See part 3 below.

2. The aim of the experiment is to plot a graph which shows how the voltage across a capacitor varies as it is discharging through a resistor.



R = 75 kΩ. If the voltmeter is an "analogue" type. Use the 7·5v calibration (on this calibration, it has a resistance of 75 kΩ).

Do the experiment first *without* the resistor R in the circuit.

When the switch is closed, the capacitor charges (almost immediately) to the same voltage as the supply. As soon as the switch is opened, the capacitor starts to discharge through the voltmeter. (When using the 7·5v calibration of the voltmeter, its resistance is 75 kΩ.)

- charge the capacitor, read the voltmeter **with the switch closed;** this is the voltage at t = zero

- open the switch and start a watch simultaneously

- measure the time taken for the voltage to fall to, for example, 5 volts

- recharge C and measure the time taken for the voltage to fall to some lower value, for example, 4·5 volts

- repeat for other voltages.

Repeat one or two of the readings with the 75 kΩ resistor connected in parallel with the voltmeter, as shown.

Plot a graph of voltage against time.

3. If the graph is *exponential*, it will be found that the rate of fall of voltage is *directly proportional* to voltage.

   Or, fall in voltage per second = (a constant) × voltage

   but fall in voltage per second is the *slope* of the graph

   so, if we measure the slope at various voltages v we should find that

$$\text{gradient} / v = \text{a constant}$$

4. a) Prove that your graphs are exponential. To do this, measure the slope at three points on the curve, for example, at v = 5 V, v = 3·5 V and v = 1·5 V.

   b) Another way to prove that the results show an exponential fall in voltage is to find how long it takes for the voltage to fall to half of its starting value. This "halving time" should be constant no matter what time you consider as the start. (You could, of course, consider the time taken for the voltage to fall to some other fraction of its initial value.)

   c) In theory, how long would it take to completely discharge a capacitor? In practice, how long (approximately) did it take? Why is there this difference between theory and practice?

Some suggestions when studying Conservation of Momentum.

Objectives:

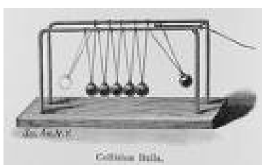
The learners will apply two of Newton's Laws of Motion discovering that Momentum is conserved.

Materials:

Newton's Cradle
Carts
Planks with skates screwed to the bottom
"Crash Dummy Motorcycle"

Strategy:

NEWTON'S CRADLE—Collision



Pull one ball out. Ask "What will happen when I let go?" Let everyone contribute. Then let go. See what actually happens. Do not get into a big discussion at this point! Come back to this at the end.

TWO CART COLLISION--
Define Momentum: Mass x Velocity. Have two carts of equal mass collide with each other from opposite directions. Ask "What happened?" Let everyone contribute. (Newton III; Momentum is Conserved)
Then have the two carts collide when one of the carts is the same mass as previously and the other has a third cart stacked on top-a larger mass. Ask "What happens?" Let everyone contribute. (Discussions will include Newton III, Newton II and Conservation of Momentum.)

PLANK WITH ROLLER SKATES ATTACHED--
Have a student walk the plank. Ask "What happened?" (Plank goes the other way.) Let every student contribute. Have students of different weights take turns. Observe any difference this makes. (Newton III, Momentum is Conserved)

CRASH DUMMY MOTORCYCLE--
Construct a "Wall" at the end of an inclined plane. Have the toy motorcycle with a dummy rider crash into the wall. Ask "What happened?" (Newton III, also Newton I, Momentum is Conserved)

NEWTON'S CRADLE REVISITED--
Go back to the Newton's Cradle. Again pull out one ball. Let go. Ask "What

happened?" and "Why?" Students should be able to discuss the results in terms of Newton's Laws for each ball's collision with the next ball. They should also recognize that momentum is conserved in each collision. Now try this with two, three, or even four balls. They should be able to extend their conclusions to these unequal mass collisions.

<u>Further Questions on Doppler Effect</u>

At start tests idea of relative velocity and then checks qualitative understanding:

The table below shows several situations in which the Doppler effect may arise. The first two columns indicate the velocities of the sound source and the observer, where the length of each arrow is proportional to the speed. For each situation, fill in the empty columns by deciding first whether the Doppler Effect occurs and then, if it does, whether the wavelength of the sound and the frequency heard by the observer increase, decrease, or remain the same compared to the case when there is no Doppler effect. Provide a reason for your answer.

	Velocity of source	Velocity of observer	Doppler effect occurs?	Wavelength	Frequency heard by observer
A	●	●			
B	→	●			
C	←	●			
D	→	→			
E	←	←			
F	→	←			
G	←	→			

A: Moving Source Only

Qualitative question including frequency change with time:

A music fan at a swimming pool is listening to a radio on a diving platform. The radio is playing a constant frequency tone when this fellow, clutching his radio, jumps off. Describe the Doppler effect heard by a) a person left behind on the platform, and b) a person down below floating on a rubber raft. In each case, specify 1) whether the observed frequency is constant, and 2) how the observed frequency changes during the fall, if it does change. Give your reasoning.

Simple "plug and chug":

…

Solve for speed:

A bird is flying directly toward a stationary bird-watcher and emits a frequency of 1250 Hz. The bird-watcher, however, hears a frequency of 1290 Hz. What is the speed of the bird?

Solve for speed and direction:

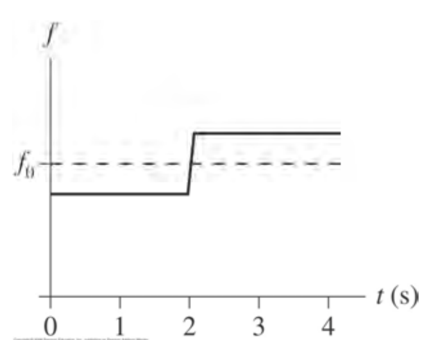
A bat locates insects by emitting ultrasonic "chirps" and then listening for echoes from the bugs. Suppose a bat chirp has a frequency of 25 kHz. How fast would the bat have to fly, and in what direction, for you to just barely be able to hear the chirp at 20 kHz?

Two simultaneous equations (f_s and v_s unknown):

Standing on a pavement, you hear a frequency of 560 Hz from the siren of an approaching ambulance. After the ambulance passes, the observed frequency of the siren is 480 Hz. Determine the ambulance's speed from these observations.

Interpretation of graph:

You are standing at $x = 0$ m, listening to a sound that is emitted at frequency f_0. The graph alongside shows the frequency you hear during a 4-second interval. Which of the following describes the sound source? Explain your choice.



a) It moves from left to right and passes you at $t = 2$s.
b) It moves from right to left and passes you at $t = 2$s.
c) It moves toward you but doesn't reach you. It then reverses direction at $t = 2$ s.
d) It moves away from you until $t = 2$ s. It then reverses direction and moves toward you but doesn't reach you.

B: Moving Listener Only

Simple "plug and chug":

The frequency of a certain police car's siren is 1550 Hz when at rest. What frequency do you detect if you move with a speed of 30.0 m/s a) toward the car, and b) away from the car?

Simple qualitative:

A large church has part of the organ in the front of the church and part in the back. A person walking rapidly down the aisle while both segments are playing at once reports that the two segments sound out of tune. Why?

Ties together previous concepts of waves:

A source *S* generates circular waves on the surface of a lake; the pattern of wave crests is shown in the figure below. The speed of the waves is 5.5 m/s, and the crest-to-crest separation is 2.3 m. You are in a small boat heading directly toward *S* at a constant speed of 3.3 m/s with respect to the shore. What frequency of waves do you observe? (need to include picture still)

C: Reflections Involving Two-step Application of Equations with Only One of Source or Listener Moving at a Time:

Simple (led through steps):

A toy rocket moves at a speed of 242 m/s directly toward a stationary pole (through stationary air) while emitting sound waves at frequency f = 1250 Hz.

a) What frequency f' is sensed by a detector that is attached to the pole?
b) Some of the sound reaching the pole reflects back to the rocket, which has an onboard detector. What frequency f'' does it detect?

Harder (not led in steps):

A stationary motion detector sends sound waves of 0.150 MHz toward a truck approaching at a speed of 45.0 m/s. What is the frequency of the waves reflected back to the detector?

+ unit conversion:

A Doppler flow meter uses ultrasound waves to measure blood-flow speeds. Suppose the device emits sound at 3.5 MHz, and the speed of sound in human tissue is taken to be 1540 m/s. What frequency is detected back by the meter if blood is flowing normally in the large leg arteries at 20 cm/s directly away from the sound source?

2 simultaneous equations:

A 2.00 MHz sound wave travels through a pregnant woman's abdomen and is reflected from the fetal heart wall of her unborn baby. The heart wall is moving toward the sound receiver as the heart beats. The reflected sound is then detected by the detector and has a frequency that differs from that emitted by 85 Hz. The speed of sound in body tissue is 1500 m/s. Calculate the speed of the fetal heart wall at the instant this measurement is made?

D: Both Source and Listener Moving

Simple plug and chug:

A railroad train is travelling at 30.0 m/s in still air. The frequency of the note emitted by the train whistle is 262 Hz. What frequency is heard by a passenger on a train moving in the opposite direction to the first at 18.0 m/s and a) approaching the first? b) receeding from the first?

Solve for v (re-arrange equation):

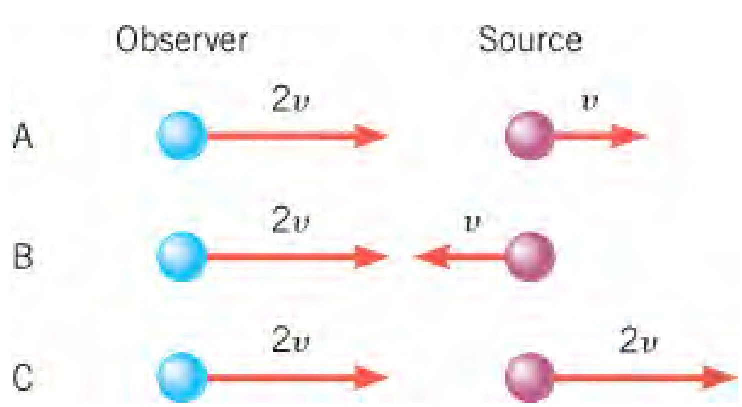
An ambulance with a siren emitting a whine at 1600 Hz overtakes and passes a cyclist pedalling a bike at 2 m/s. After being passed, the cyclist hears a frequency of 1590 Hz. How fast is the ambulance moving?

Two trucks travel at the same speed. They are far apart on adjacent lanes and approach each other essentially head-on. One driver hears the horn of the other truck at a frequency that is 1.14 times the frequency he hears when the trucks are stationary. The speed of sound is 343 m/s. At what speed is each truck moving?

E: Doppler for light:

An astronomer measures the Doppler change in frequency for the light reaching the earth from a distant star. From this measurement, explain how the astronomer can deduce that the star is receeding from the earth.

The drawing shows three situations A, B and C in which an observer and a source of electromagnetic waves are moving along the same line. In each case the source emits a wave of the same frequency. The arrows in each situation denote velocity vectors relative to the ground and have the indicated magnitudes, either v or $2v$. Rank the frequencies of the observed waves in descending order (largest first) according to magnitude. Explain your reasoning.



Answers to Further Questions on Doppler Effect
(These questions appear on pages 188 to 192)

So the concept of relative motion is crucial!
Two objects have relative velocity if the distance between them is changing!

Complete the table below:

Doppler Solutions

	Velocity of source	Velocity of observer	Doppler effect occurs?
A	●	●	✗
B	→	●	✓
C	←	●	✓
D	→	→	✗
E	←	←	✗
F	→	←	✓
G	←	→	✓

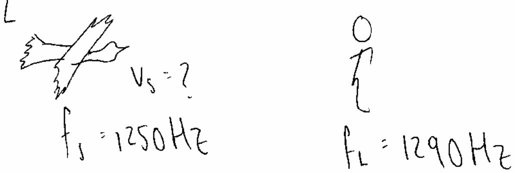
A1

a) A person left on the platform sees the fan and radio moving away.
He hears a sound of a lower frequency (lower pitch)

b) The person on the raft sees the fan and radio approaching.
He hears a sound of higher frequency (higher pitch)

In both cases, the frequency heard does not vary with time. Doppler effect depends only on the relative velocities of the sound source & listener and not on the distance between them.

A2



$$f_L = f_s \frac{1}{1 - v_s/v}$$

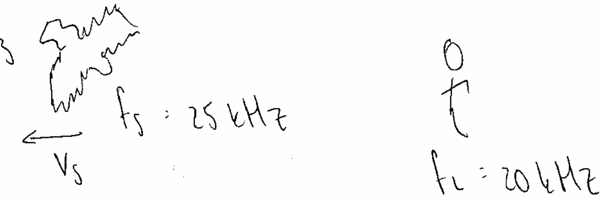
$$1290 = 1250 \frac{1}{1 - v_s/340}$$

$$\Rightarrow 1 - v_s/340 = \frac{1250}{1290}$$

$$\therefore v_s = 340\left(1 - \frac{1250}{1290}\right)$$

$$= 10{,}5\ m.s^{-1} \longrightarrow$$

A3



Since the frequency heard is less than that emitted, the bat must be flying away from the listener.

$$f_L = f_s \frac{v}{v + v_s}$$

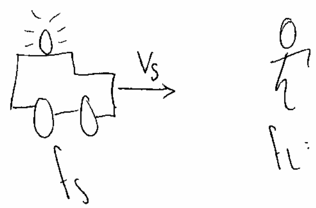
$$20 \times 10^3 = 25 \times 10^3 \cdot \frac{340}{340 + v_s}$$

$$\Rightarrow \frac{340 + v_s}{340} = 25/20$$

$$\therefore v_s = 340\left(25/20 - 1\right)$$

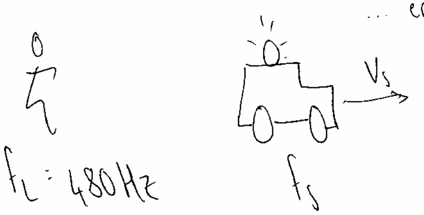
$$= 85\ m.s^{-1} \longrightarrow$$

A4



$f_L = 560\,Hz$

$$f_L = f_s \frac{V}{V - V_s} \Rightarrow 560 = f_s \frac{340}{340 - V_s} \quad \ldots eq\,1$$



$f_L = 480\,Hz$

$$f_L = f_s \frac{V}{V + V_s} \Rightarrow 480 = f_s \frac{340}{340 + V_s} \quad \ldots eq\,2$$

From eq 1: $f_s = \frac{560(340 - V_s)}{340} \quad \ldots eq\,3$

Subst in eq 2: $480 = \frac{560(340 - V_s)}{340} \times \frac{340}{(340 + V_s)}$

$\therefore 480(340 + V_s) = 560(340 - V_s)$

$\Rightarrow 1040\,V_s = 80 \times 340$

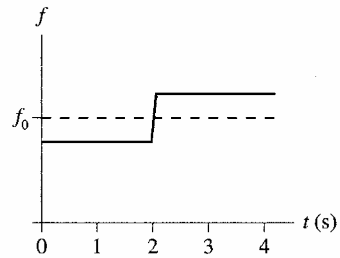
$\therefore V_s = 26.2\ m.s^{-1}$

Subst in eq 3:

$$f_s = \frac{560(340 - 26.2)}{340}$$

$= 517\,Hz$

Question A5
(Interpretation of graph)



You are standing at x = 0 m, listening to a sound that is emitted at frequency f_0. The graph above shows the frequency you hear during a 4-second interval. Which of the following describes the sound source? Explain your choice.

- ✗ It moves from left to right and passes you at t = 2s.
- ✗ It moves from right to left and passes you at t = 2s.
- ✗ It moves toward you but doesn't reach you. It then reverses direction at t =2 s.
- ✓ It moves away from you until t = 2 s. It then reverses direction and moves toward you but doesn't reach you.

B1

a) 30 m s⁻¹

$f_s = 1550$ Hz

$f_L = ?$

$f_L = f_s \frac{v + v_L}{v} = 1550 \frac{(340 + 30)}{340}$

$= 1687$ Hz

b) 30 m s⁻¹

$f_s = 1550$ Hz

$f_L = ?$

$f_L = f_s \frac{v - v_L}{v} = 1550 \frac{(340 - 30)}{340}$

$= 1413$ Hz

B2 As he walks, the sound from the organ at the back is heard at a lower pitch (compared to when stationary) while the organ in the front is heard at a higher pitch. They are thus heard at different frequencies & so appear out of tune.

B3

$\lambda = 2{,}3\ m$

$v = 5.5\ m.s^{-1}$

$v = f_s \lambda$

$5.5 = f_s (2{,}3)$

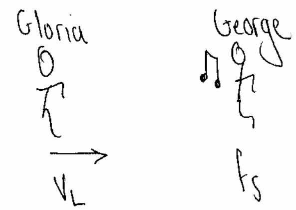
$\therefore f_s = \frac{5{,}5}{2{,}3} = 2{,}39\ Hz$

$f_L = f_s \frac{v + v_L}{v} = 2{,}39 \frac{5.5 + 3{,}3}{5{,}5}$

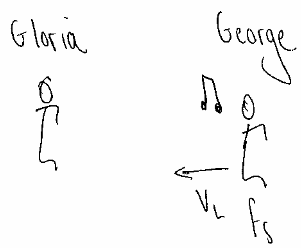
$= 3{,}82\ Hz$

B4

No!



$f_L^1 = f_s \frac{v + v_L}{v}$ Ⓐ



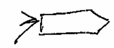
$f_L^2 = f_s \frac{v}{v - v_L}$ Ⓑ

Comparing Ⓐ & Ⓑ:

$f_L^1 \neq f_L^2$

∴ who moves matters!

C1



$v_s = 242 \text{ m.s}^{-1}$

$f_s = 1250 \text{ Hz}$

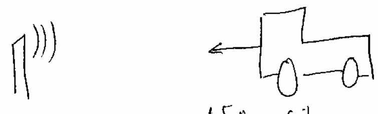
a) $f' = f_s \frac{v}{v - v_s} = 1250 \frac{340}{340 - 242}$

$= 4337 \text{ Hz}$

b) $f'' = f' \frac{v + v_L}{v} = 4337 \frac{(340 + 242)}{340}$

$= 7423 \text{ Hz}$

2



$f_s = 0{,}150$ MHz

First find frequency driver of truck hears:

$$f' = f_s \frac{v + v_L}{v} = 0{,}150 \times 10^6 \, \frac{340 + 45}{340}$$

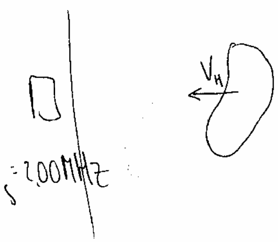
$$= 0{,}170 \times 10^6 \text{ Hz}$$

This is the frequency the truck acts as a source of. Now find frequency detected by the detector:

$$f'' = f' \frac{v}{v - v_s} = 0{,}170 \times 10^6 \, \frac{340}{340 - 45}$$

$$= 0{,}196 \times 10^6 \text{ Hz}$$

3



$f_s = 2{,}00$ MHz

First consider the frequency 'heard' by the heart wall:

$$f' = f_s \frac{v + v_H}{v} = 2{,}00 \times 10^6 \, \frac{1540 + v_H}{1540} \quad (1)$$

Now consider the frequency detected by the detector: subst (1)

$$f'' = f' \frac{v}{v - v_H} = 2{,}00 \times 10^6 \, \frac{1540 + v_H}{1540} \times \frac{1540}{1540 - v_H}$$

$$2{,}00 \times 10^6 + 85 = 2{,}00 \times 10^6 \times \frac{1540 + v_H}{1540 - v_H}$$

$$(\cancel{1540} - v_H) + \frac{85}{2{,}00 \times 10^6}(1540 - v_H) = \cancel{1540} + v_H$$

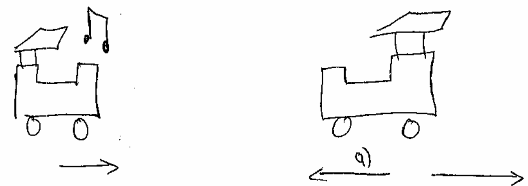
$$\therefore \frac{85}{2{,}00 \times 10^6} 1540 = \left(2 + \frac{85}{2{,}00 \times 10^6}\right) v_H$$

$$\therefore v_H = 0{,}033 \text{ m.s}^{-1}$$

$$= 3{,}3 \text{ cm.s}^{-1}$$

V1

a)



30,0 m/s

$f_s = 262$ Hz

18,0 m/s

$$f_L = f_s \frac{v + v_L}{v - v_s} = 262 \frac{340 + 18}{340 - 30}$$

$$= 302 \text{ Hz}$$

b)

18,0 m/s

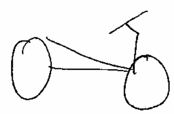
30,0 m/s

$f_s = 262$ Hz

$$f_L = f_s \frac{v - v_L}{v + v_s} = 262 \frac{340 - 18}{340 + 30}$$

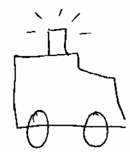
$$= \underline{228 \text{ Hz}}$$

Q2



2 m/s →

$f_L = 1590$ Hz



v_s →

$f_s = 1600$ Hz

$$f_L = f_s \frac{v + v_L}{v + v_s}$$

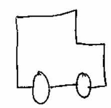
$$1590 = 1600 \frac{340 + 2}{340 + v_s}$$

$$\Rightarrow 340 + v_s = \frac{1600}{1590}(342)$$

$$\Rightarrow v_s = \frac{1600}{1590}(342) - 340$$

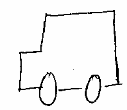
$$= \underline{4{,}2 \text{ m.s}^{-1}} \rightarrow$$

Q3



v →

f_s



← v

$f_L = 1{,}14 f_s$

$$f_L = f_s \frac{v + v_L}{v - v_s}$$

$$1{,}14 f_s = f_s \frac{340 + v}{340 - v}$$

$$1{,}14(340 - v) = 340 + v$$

$$340(1{,}14 - 1) = v(1 + 1{,}14)$$

$$\therefore v = 340 \frac{(1{,}14 - 1)}{(1{,}14 + 1)}$$

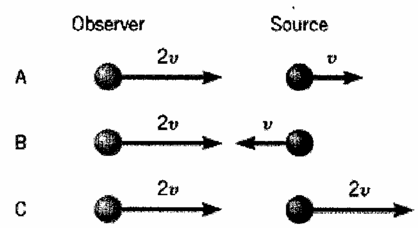
$$= \underline{22 \text{ m/s}} \rightarrow$$

E1

If the spectral lines are shifted towards lower frequencies (towards the red) then the astronomer can deduce that the star is moving away from the earth

Question E2
(Qualitative)

The drawing shows three situations A, B and C in which an observer and a source of electromagnetic waves are moving along the same line. In each case the source emits a wave of the same frequency. The arrows in each situation denote velocity vectors relative to the ground and have the indicated magnitudes, either v or $2v$. Rank the frequencies of the observed waves in descending order (largest first) according to magnitude. Explain your reasoning.



$f_B > f_A > f_C$

F1

A & C : nothing

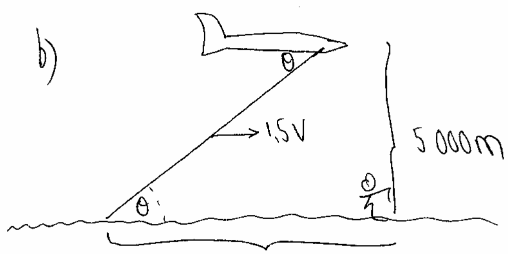
B : sonic boom

F2

a) $\sin\theta = \frac{v}{1.5v}$

$\therefore \theta = 41.8°$

b)



$\frac{5000}{\tan\theta} = 5592$ m

Now find time for shock wave to reach [stick figure]

$t = \frac{5592}{1.5(340)} = \underline{11s}$