![basic education Department: Basic Education REPUBLIC OF SOUTH AFRICA]

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

NOVEMBER 2012

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages, 1 diagram sheet and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1.   This question paper consists of 13 questions.

2.   Answer ALL the questions.

3.   Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.

4.   Answers only will not necessarily be awarded full marks.

5.   You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6.   If necessary, round off answers to TWO decimal places, unless stated otherwise.

7.   Diagrams are NOT necessarily drawn to scale.

8.   ONE diagram sheet for QUESTION 3.2 and QUESTION 7.3 is attached at the end of this question paper. Write your centre number and examination number on this sheet in the spaces provided and insert the sheet inside the back cover of your ANSWER BOOK.

9.   An information sheet with formulae is included at the end of this question paper.

10.   Number the answers correctly according to the numbering system used in this question paper.

11.   Write neatly and legibly.

QUESTION 1

The scatter plot below shows the age (in years) and the average height (in centimetres) of boys between 2 and 15 years.
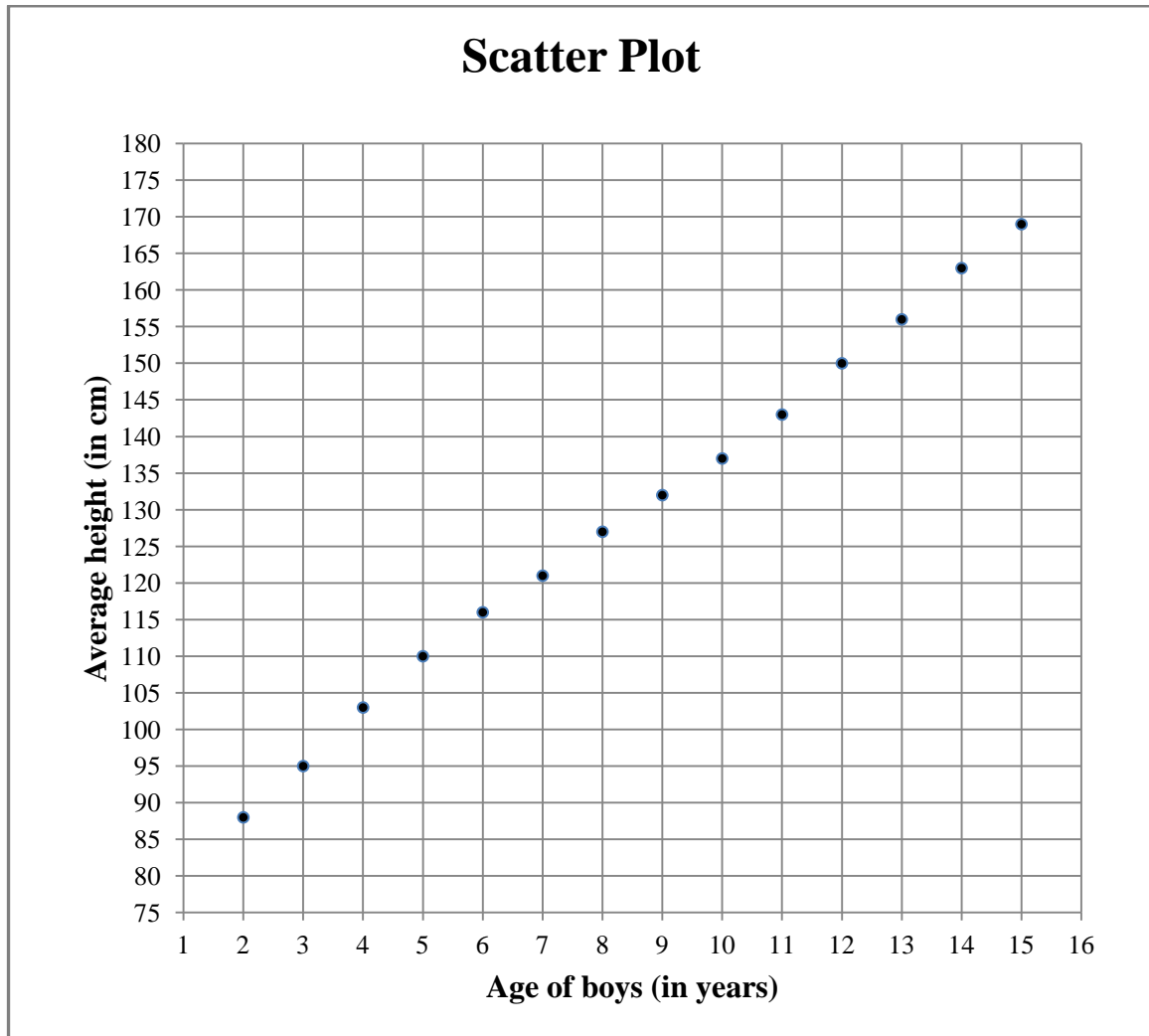
Scatter Plot

1.1   Use the scatter plot to determine the average height of a 7-year-old boy.   (1)

1.2   Describe the trend in the scatter plot.   (1)

1.3   What is the approximate increase in the average height per annum between the ages of 2 and 15 years?   (3)

1.4   Explain why the observed trend CANNOT continue indefinitely.   (1)

                                                                                                     [6]

QUESTION 2

Abe plays for his school's cricket team. The number of runs scored by Abe in the eight games that he batted in, is shown below. (Abe was given out in all of the games.)

$$21 \quad 8 \quad 19 \quad 7 \quad 15 \quad 32 \quad 14 \quad 12$$

2.1   Determine the average runs scored by Abe in the eight games.   (2)

2.2   Determine the standard deviation of the data set.   (2)

2.3   Abe's scores for the first three of the next eight games were 22, 35 and 2 respectively. Describe the effect of his performance on the standard deviation of this larger set having 11 data points.   (2)

2.4   Abe hopes to score an average of 20 runs in the first 16 games. What should his average in the last five games be so that he may reach his goal?   (3)
                                                                                                                                      [9]

QUESTION 3

In a certain school 60 learners wrote examinations in Mathematics and Physical Sciences. The box-and-whisker diagram below shows the marks (out of 100) that these learners scored in the Physical Sciences examination.
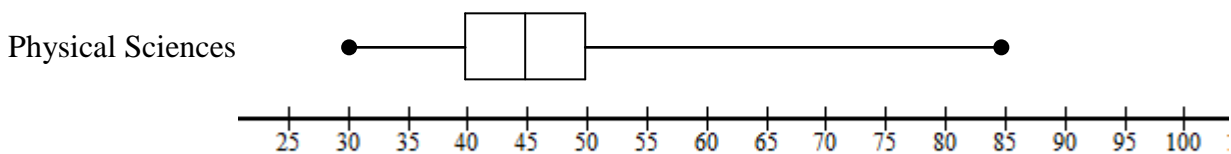

3.1   Write down the range of the marks scored in the Physical Sciences examination.   (1)

3.2   Use the information below to draw the box-and-whisker diagram for the Mathematics results on DIAGRAM SHEET 1.

         Minimum mark = 30
         Range = 55
         Upper quartile = 70
         Interquartile range = 30
         Median = 55   (4)

3.3   How many learners scored less than 70% in the Mathematics examination?   (2)

3.4   Joe claims that the number of learners who scored between 30 and 45 in Physical Sciences is smaller than the number of learners who scored between 30 and 55 in Mathematics. Is Joe's claim valid? Justify your answer.   (2)
                                                                                                                                      [9]
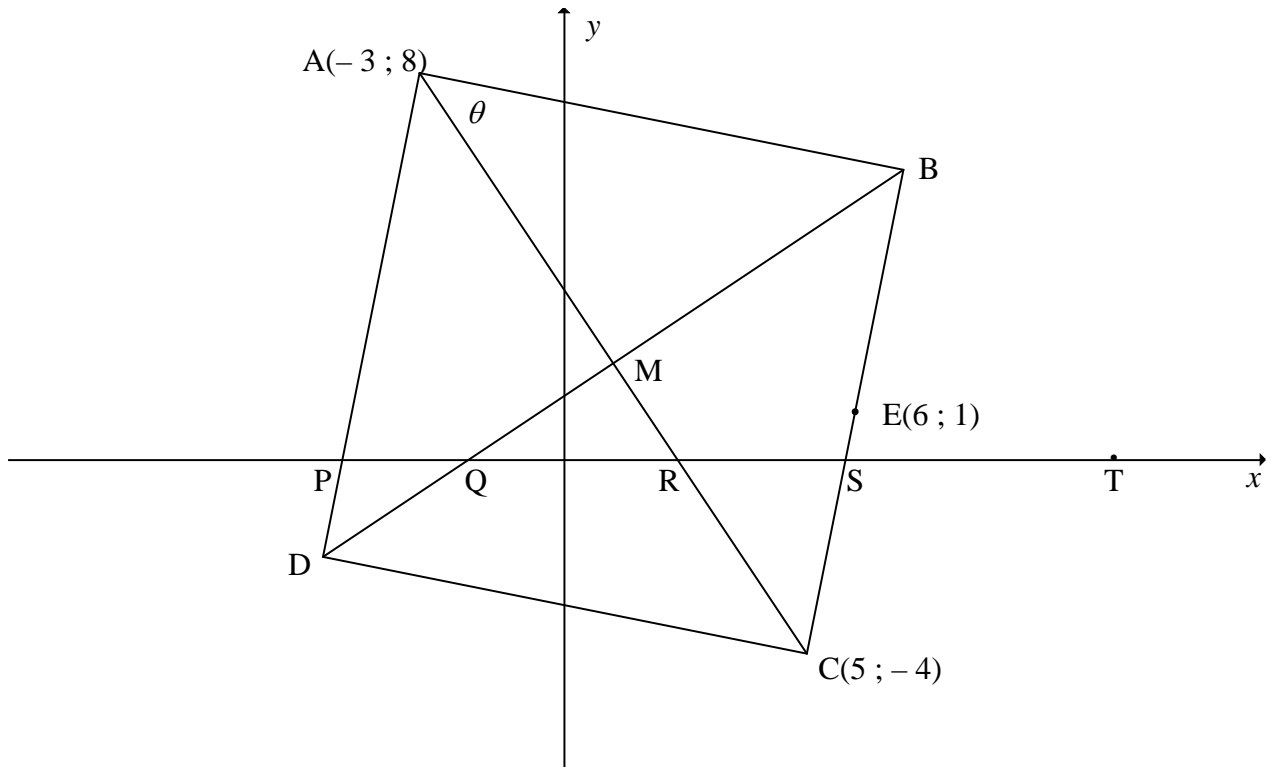
QUESTION 4

As part of an environmental awareness initiative, learners of Greenside High School were requested to collect newspapers for recycling. The cumulative frequency graph (ogive) below shows the total weight of the newspapers (in kilograms) collected over a period of 6 months by 30 learners.

CUMULATIVE FREQUENCY GRAPH
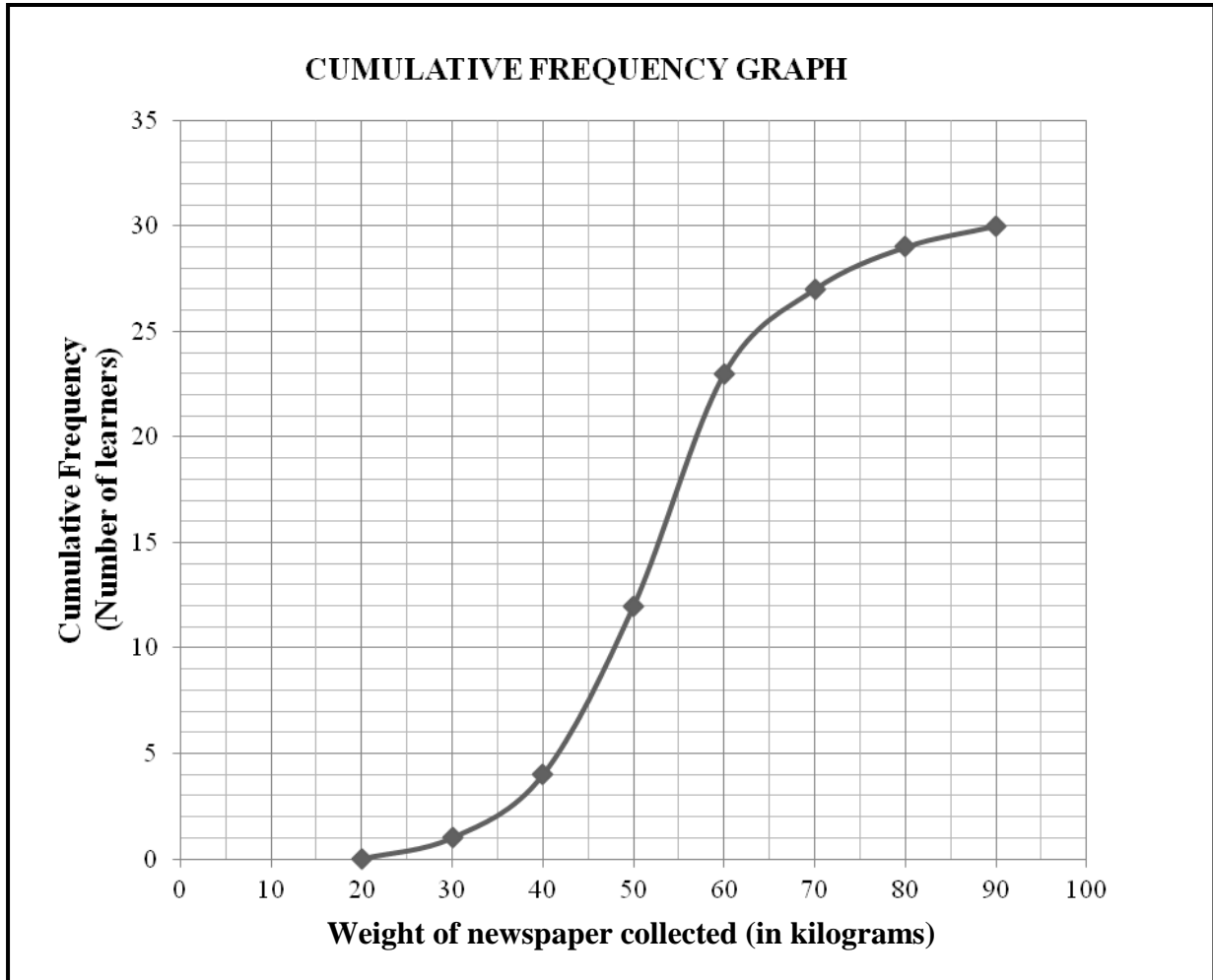
4.1   Determine the modal class of the weight of the newspapers collected.   (1)

4.2   Determine the median weight of the newspapers collected by this group of learners.   (1)

4.3   How many learners collected more than 60 kilograms of newspaper?   (2)

[4]

QUESTION 5

ABCD is a rhombus with A(– 3 ; 8) and C(5 ; – 4). The diagonals of ABCD bisect each other at M. The point E(6 ; 1) lies on BC.
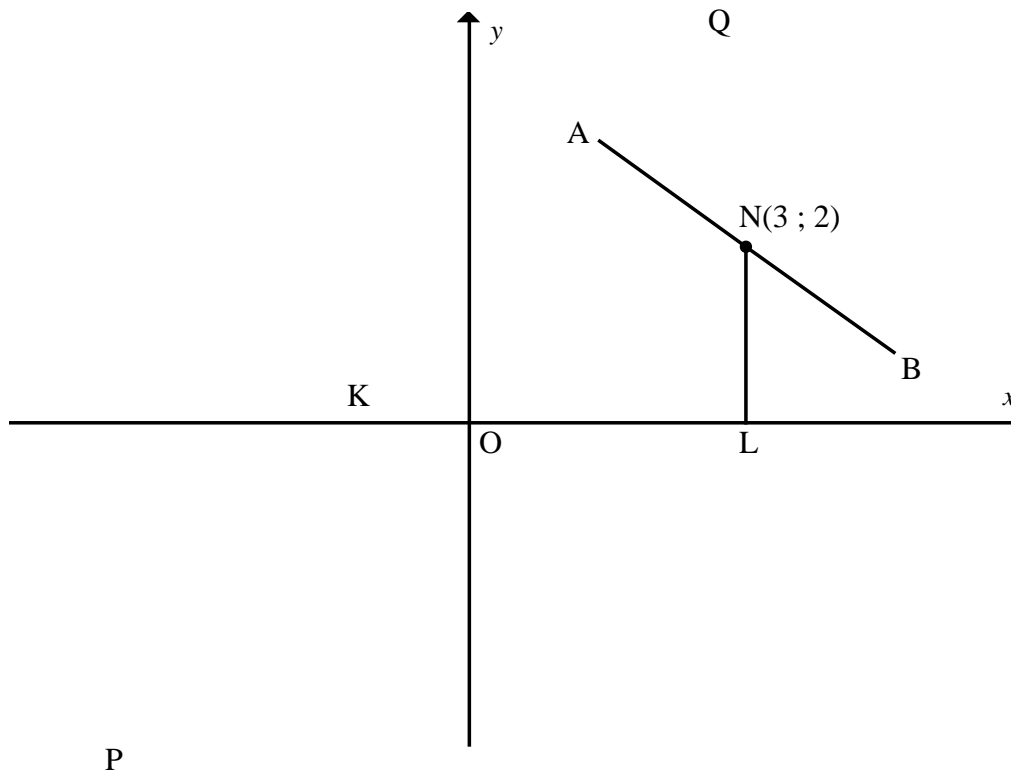

5.1	Calculate the coordinates of M.	(2)
5.2	Calculate the gradient of BC.	(2)
5.3	Determine the equation of the line AD in the form $y = mx + c$.	(3)
5.4	Determine the size of θ, that is $\hat{BAC}$. Show ALL calculations.	(6)

[13]

QUESTION 6

A circle centred at N(3 ; 2) touches the *x*-axis at point L. The line PQ, defined by the equation $y = \dfrac{4}{3}x + \dfrac{4}{3}$, is a tangent to the same circle at point A.


6.1   Why is NL perpendicular to OL?   (1)

6.2   Determine the coordinates of L.   (1)

6.3   Determine the equation of the circle with centre N in the form $(x - a)^2 + (y - b)^2 = r^2$ (3)

6.4   Calculate the length of KL.   (3)

6.5   Determine the equation of the diameter AB in the form $y = mx + c$.   (4)

6.6   Show that the coordinates of A are $\left(\dfrac{7}{5} \; ; \dfrac{16}{5} \right)$.   (3)

6.7   Calculate the length of KA.   (3)

6.8   Why is KLNA a kite?   (2)

6.9   Show that $A\hat{B}K = 45°$.   (3)

6.10   If the given circle is reflected about the *x*-axis, give the coordinates of the centre of the new circle.   (1)

                                                                                   [24]

QUESTION 7

Consider the diagram below where A(– 5 ; 2), B(– 4 ; 1) and C(– 3 ; 3) are the vertices of ∆ABC.
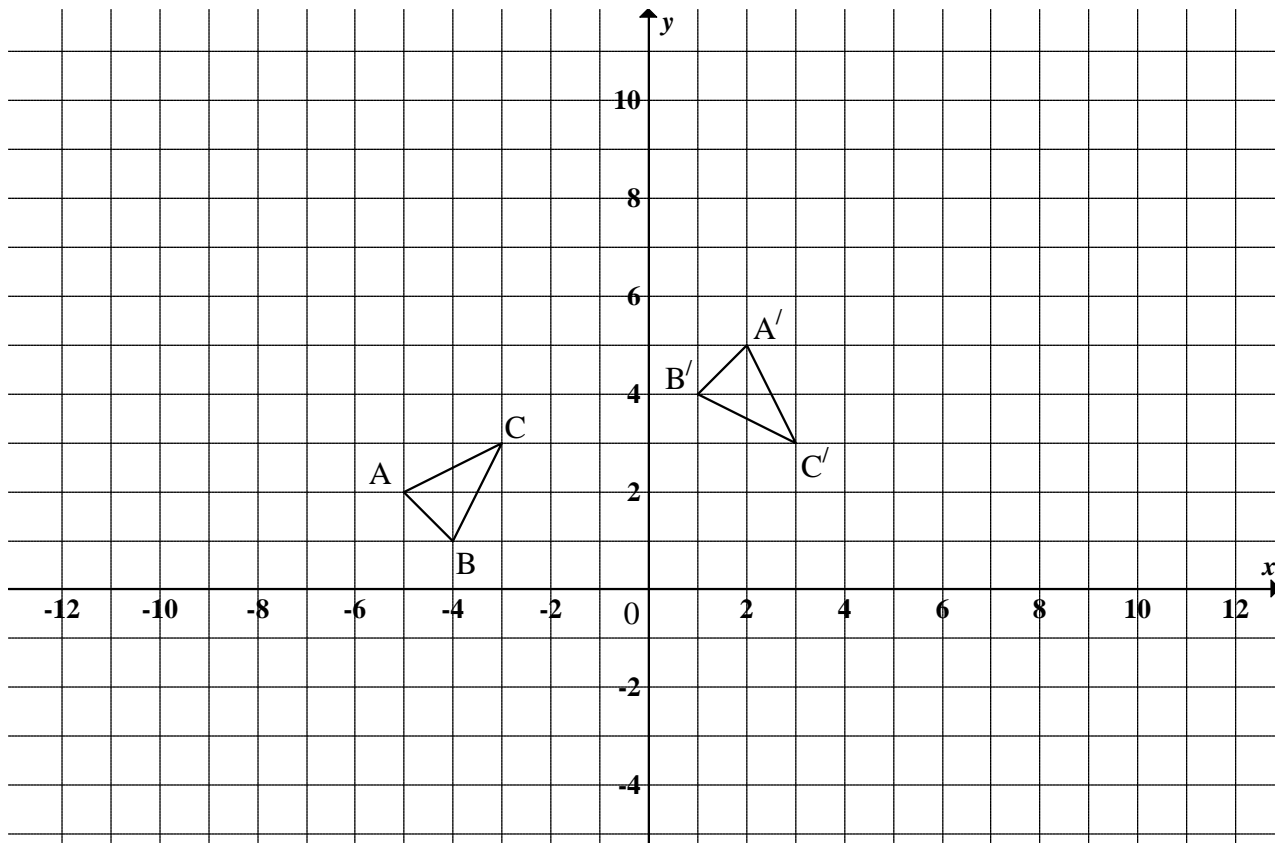
7.1   Describe the single transformation of ∆ABC to ∆A$^{/}$B$^{/}$C$^{/}$.   (2)

7.2   Write down the general rule of the transformation in QUESTION 7.1.   (2)

7.3   ∆ A$^{/}$B$^{/}$C$^{/}$ is enlarged by a scale factor of 2 to form ∆ A$^{//}$B$^{//}$C$^{//}$. Draw the enlargement on DIAGRAM SHEET 1.   (2)

7.4   Write down the general rule of the transformation in QUESTION 7.3.   (1)

7.5   ∆ABC is reflected about the *x*-axis and then it is reflected about the *y*-axis to form ∆DEF.

        7.5.1   Write down the coordinates of D, where D is the image of A after the transformation described above.   (2)

        7.5.2   Write down the general rule of this transformation in the form:
                  $(x ; y) \rightarrow (\ldots ; \ldots) \rightarrow (\ldots ; \ldots)$.   (2)

        7.5.3   Describe a single transformation that ∆ABC undergoes to form ∆DEF.   (2)
                                                                                              [13]
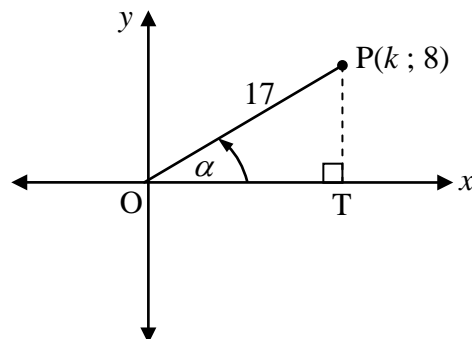
QUESTION 8

Answer this question WITHOUT using a calculator.

8.1   The point P(k ; 8) lies in the first quadrant
        such that OP = 17 units and TÔP = α
        as shown in the diagram alongside.


        8.1.1   Determine the value of k.   (2)

        8.1.2   Write down the value of $\cos\alpha$.   (1)

        8.1.3   If it is further given that $\alpha + \beta = 180°$, determine $\cos\beta$.   (2)

        8.1.4   Hence, determine the value of $\sin(\beta - \alpha)$.   (4)

8.2   Consider the expression: $\dfrac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$

        8.2.1   Prove that: $\dfrac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ (4)

        8.2.2   The above expression is undefined if $\sin 2x - \cos x = 0$. Solve this
                   equation in the interval $0° \le x \le 360°$.   (4)

                                                                                                 [17]

QUESTION 9

9.1   Simplify as far as possible: $\dfrac{\sin^2 \theta}{\sin(180° - \theta).\cos(90° + \theta) + \tan 45°}$ (5)

9.2   Simplify without the use of a calculator: $\dfrac{\sin 104°(2\cos^2 15° - 1)}{\tan 38°.\sin^2 412°}$ (8)

   **[13]**

QUESTION 10

The graphs of $f(x) = \sin(x + 30°)$ and $g(x) = -2\cos x$ for $-90° \leq x \leq 180°$ are given below. The graphs intersect at point P and point Q.
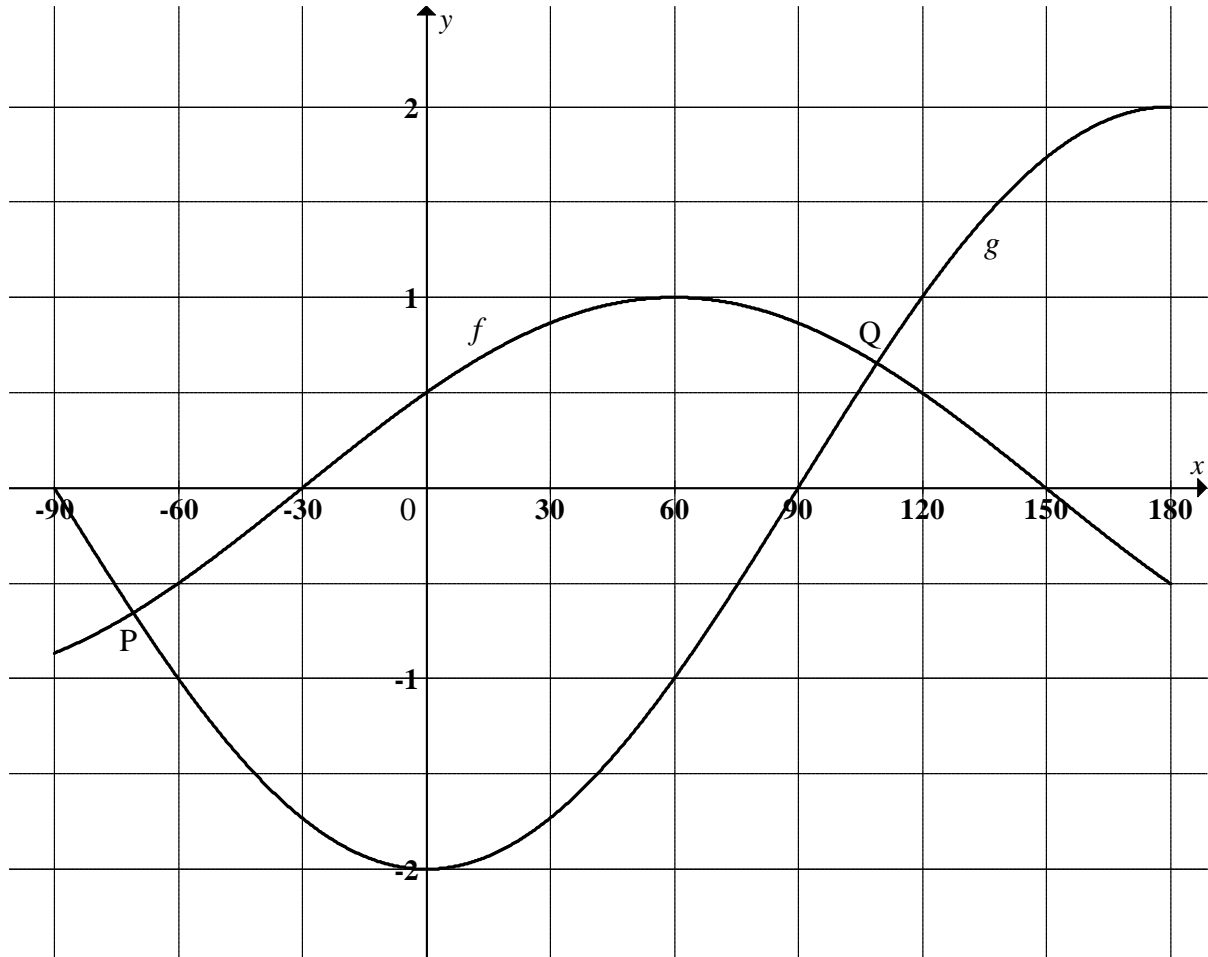
10.1   Calculate $f(0) - g(0)$.   (1)

10.2   Calculate the *x*-coordinates of point P and point Q.   (7)

10.3   For which values of *x* will $f(x) \geq g(x)$?   (2)

10.4   Graph *h* is obtained by the following transformation of *f*: $h(x) = 2f(x + 60°)$.
        Describe the relationship between *g* and *h*.   (2)

                                                                                                        [12]
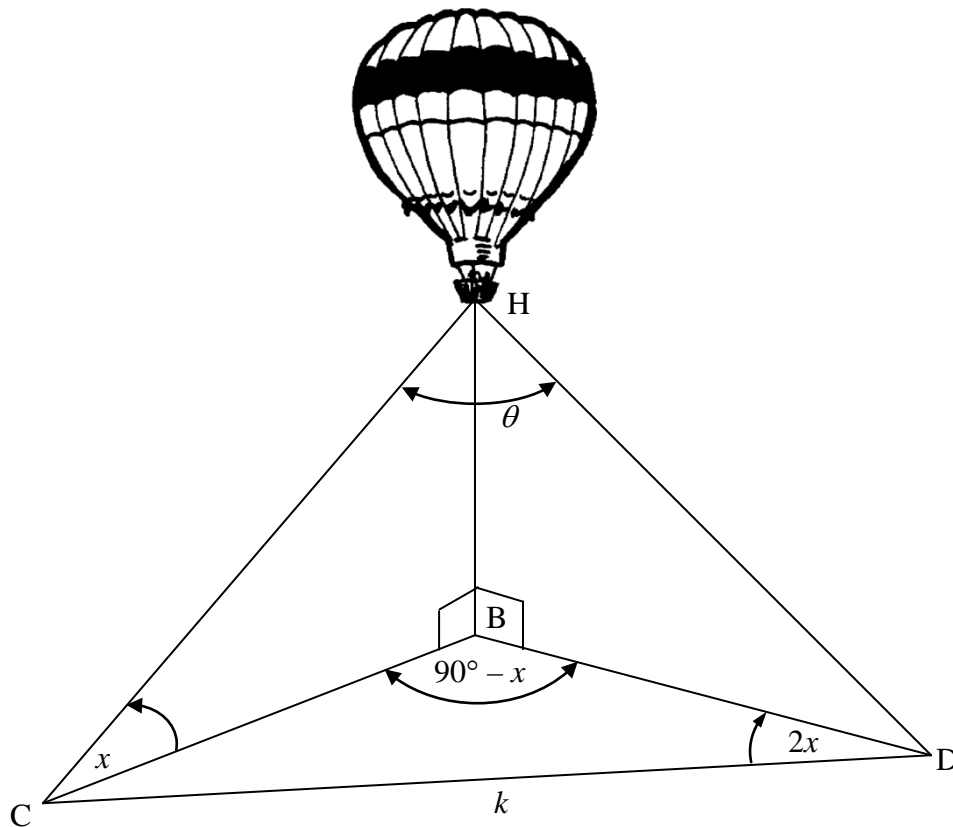
QUESTION 11

ABCD is a parallelogram with AB = 3 units, BC = 2 units and $A\hat{B}C = \theta$ for $0° < \theta \le 90°$.
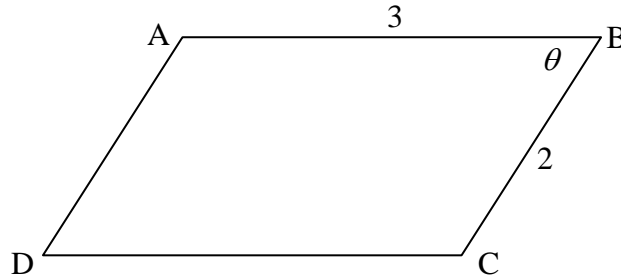
11.1   Prove that the area of parallelogram ABCD is $6\sin\theta$.   (3)

11.2   Calculate the value of θ for which the area of the parallelogram is $3\sqrt{3}$ square units.   (3)

11.3   Determine the value of θ for which the parallelogram has the maximum area.   (2)

[8]

QUESTION 12

A hot-air balloon H is directly above point B on the ground. Two ropes are used to keep the hot-air balloon in position. The ropes are held by two people on the ground at point C and point D. B, C and D are in the same horizontal plane. The angle of elevation from C to H is x. $\hat{CDB} = 2x$ and $\hat{CBD} = 90° - x$. The distance between C and D is k metres.

12.1   Show that CB = $2k \sin x$.   (5)

12.2   Hence, show that the length of rope HC is $2k \tan x$.   (3)

12.3   If $k = 40$ m, $x = 23°$ and HD = 31,8 m, calculate θ, the angle between the two ropes.   (4)

[12]

QUESTION 13

The face of a standard clock is positioned such that the centre is at the origin. At a certain time, the end of the minute hand is at the point P(2 ; 4). 37 minutes later, the end of the minute hand is at the point P$^{/}$ (*a ; b*).
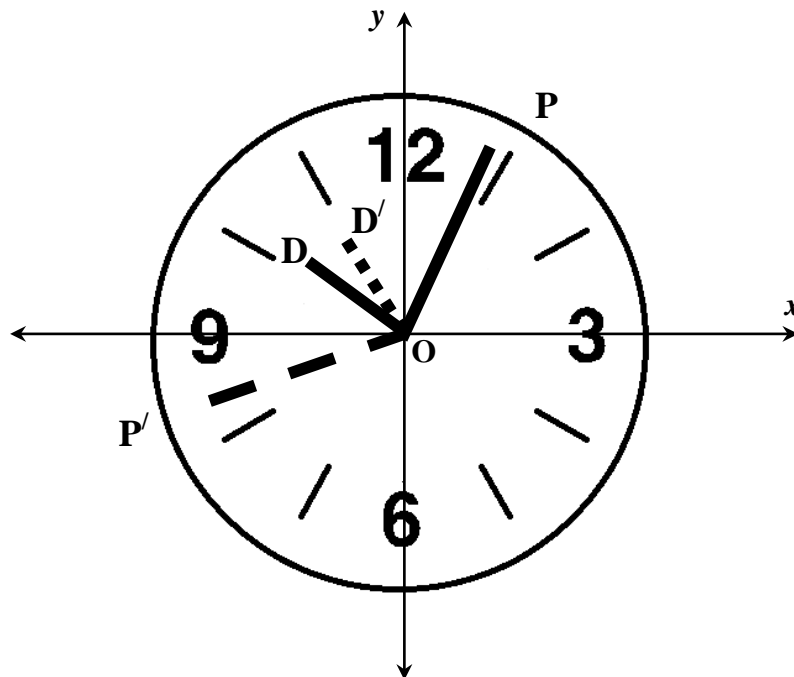
13.1   Determine the value of *a* and *b*.   (6)

13.2   OD is the position of the hour hand when the minute hand is at P and OD$^{/}$ is the position of the hour hand when the minute hand is at P$^{/}$. Calculate the angle between OD and OD$^{/}$.   (4)
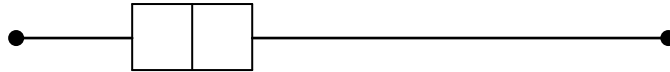
[10]

TOTAL: 150

CENTRE NUMBER:

EXAMINATION NUMBER:

DIAGRAM SHEET 1

QUESTION 3.2

Physical Science

Mathematics

25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
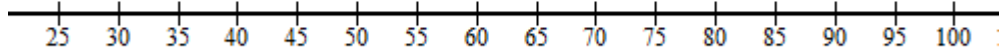
QUESTION 7.3
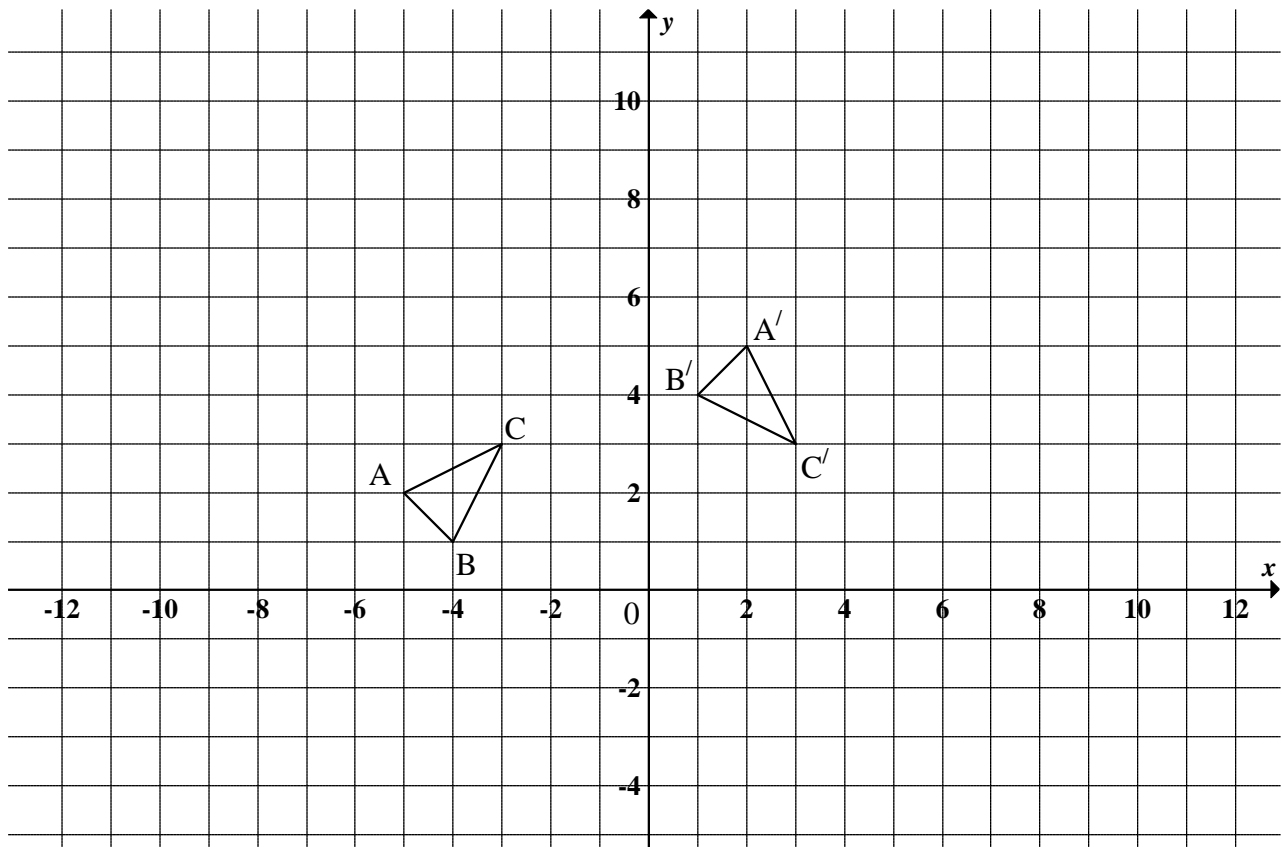
INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r - 1} \; ; \qquad r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1 + i)^n - 1\right]}{i} \qquad P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$In \; \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \; \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x \; ; \; y) \to (x\cos\theta - y\sin\theta \; ; \; y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \; or \; B) = P(A) + P(B) - P(A \; and \; B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$