NATIONAL CERTIFICATES (VOCATIONAL)

SUBJECT GUIDELINES

MATHEMATICS
NQF LEVEL 2

September 2007
INTRODUCTION

A. What is Mathematics?

*Reader’s Digest Oxford Complete Wordfinder* defines Mathematics as “the abstract science of number, quantity and space studied in its own right”.

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. Through mathematical problem solving, students can understand the world and use that understanding in their daily lives.

Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. The Mathematics Subject Outcomes and Assessment Standards are designed to allow all Further Education and Training (FET) (Vocational) students to develop into citizens who confidently deal with the Mathematics as and when it impinges on their daily lives, their community and the world in general. Mathematics in the Further Education and Training band also provides a platform to link with Higher Education.

B. Why is Mathematics important as a Fundamental?

The Mathematics programme (NQF Level 2 – 4) empowers students to:

- Communicate appropriately using descriptions in words, graphs, symbols, tables and diagrams.
- Use mathematical process skills to identify, pose and solve problems creatively and critically.
- Organise, interpret and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal concerns.
- Work collaboratively in teams and groups to enhance mathematical understanding.
- Collect, analyse and organise quantitative data to evaluate and comment on conclusions.
- Engage responsibly with quantitative arguments relating to local, national and global issues.

C. The link between Mathematics Learning Outcomes and the Critical and Developmental Outcomes

The Mathematics Learning Outcomes provide a platform for students to achieve the seven Critical Outcomes and five Developmental Outcomes.

The Critical Outcomes require students to be able to:

- Identify and solve problems and make decisions using critical and creative thinking.
- Work effectively with others as members of a team, group, organisation and community.
- Organise and manage themselves and their activities responsibly and effectively.
- Collect, analyse, organise and critically evaluate information.
- Communicate effectively using visual, symbolic and/or language skills in various modes.
- Use science and technology effectively and critically showing responsibility towards the environment and the health of others.
- Demonstrate an understanding of the world as a set of related systems by recognising that problem-solving contexts do not exist in isolation.

The Developmental Outcomes require students to be able to:

- Reflect on and explore a variety of strategies to learn more effectively.
- Participate as responsible citizens in the life of local, national and global communities.
- Be culturally and aesthetically sensitive across a range of societal contexts.
- Develop entrepreneurial opportunities.

D. Factors that contribute to achieving Mathematics Learning Outcomes

Students in the Further Education and Training band who are interested in the subject or who intend to follow a career path requiring Mathematics will, while ensuring that they are mathematically literate, work towards being able to:

- Completely use mathematical process skills, such as making conjectures, proving assertions and modelling situations.
• Calculate confidently and competently with and without calculators and use rational and irrational numbers with understanding.
• Competently produce useful equivalents for algebraic expressions and use such equivalents appropriately and with confidence.
• Use Mathematics to critically investigate and monitor the financial aspects of personal and community life and political decisions.
• Work with a wide range of patterns and transformations (translations, rotations, reflections) of functions and solve related problems.
• Describe, represent and analyse shape and space in two and three dimensions using various approaches in geometry (synthetic, analytic, transformational) and trigonometry in an interrelated or connected manner.
• Collect and use data to establish basic statistical and probability models, solve related problems and critically consider representations provided or conclusions reached.
• Use and understand the principles of differential calculus to determine the rate of change of a range of simple, non-linear functions and solve simple optimisation problems.
• Solve problems involving sequences and series in real-life and mathematical situations.
• Use and understand basic principles of integration to calculate volume and area.
• Use and understand other sets of numbers, such as complex numbers, to solve non-real equations particularly with reference to electrical problems.
• Solve non-routine, unseen problems using mathematical principles and processes.
• Investigate historical aspects of the development and use of mathematics in various cultures.
• Use available technology (the minimum being a scientific calculator) in calculations and in the development of models.

These mathematical skills and process abilities will, where possible, be embedded in contexts that relate to HIV/AIDS, human rights, indigenous knowledge systems and political, economic, environmental and inclusivity issues.
MATHEMATICS – LEVEL 2

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1. DURATION AND TUITION TIME
This is a one year instructional programme comprising 200 teaching and learning hours. The subject may be offered on a part-time basis provided all the assessment requirements are adhered to.

Students with special education needs (LSEN) must be catered for in a way that eliminates barriers to learning.

2. SUBJECT LEVEL FOCUS
Students will be able to:

- Recognise and work with numbers and their relationships and estimate, calculate and check solutions.
- Investigate and represent a wide range of algebraic expressions and functions and solve related problems.
- Describe, represent, analyse and explain properties of shape in two- and three-dimensional space with justification.
- Analyse data to establish statistical models to solve related problems.
- Plan personal finances in the context of income and expenditure, basic budgets and the impact of interest rates.

3. ASSESSMENT REQUIREMENTS

3.1. Internal assessment (25 percent)
All internal assessments must be finalised by an assessor with at least a certificate of competence.

3.1.1. Theoretical component

<table>
<thead>
<tr>
<th>Form:</th>
<th>Tests and assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and frequency:</td>
<td>One test and one assignment per term</td>
</tr>
<tr>
<td>Proportion of internal mark allocated to this component:</td>
<td>25%</td>
</tr>
</tbody>
</table>

3.1.2. Practical component
The practical component in Mathematics includes all exercises and applications completed by the student.

<table>
<thead>
<tr>
<th>Form:</th>
<th>Tests and assignments in a Portfolio of Evidence (PoE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and frequency:</td>
<td>One test and one assignment per term</td>
</tr>
<tr>
<td>Proportion of internal mark allocated to this component:</td>
<td>75%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Term 3</th>
<th>Term 4</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>8</td>
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</tbody>
</table>

*One of these must be an examination

- Some examples of practical assessments include, but are not limited to:
  A. Practical exercise work and applications to contextual problems
  B. Presentations (lectures, demonstrations, group discussions and activities, practical work, observation, role-play, self activity, judging and evaluation)
  C. Use of aids
  D. Exhibitions
  E. Visits
• Evidence in practical assessments
All evidence pertaining to evaluation of practical work must be reflected in the students’ Portfolios of Evidence (PoE). The tools and instruments constructed and used to conduct these assessments must be clear from evidence contained in the Portfolio of Evidence (PoE).

3.1.3. Processing of Internal assessment mark for the year
A year mark out of 100 is calculated by adding the marks of the theoretical component (25%) and the practical component (75%) of the internal continuous assessment (ICASS).

3.1.4. Moderation of internal assessment mark
Internal assessment is subjected to both internal and external moderation procedures as set out in the National Examination Policy for FET College Programmes.

3.2. External assessment (75 percent)
A National Examination is conducted annually in October or November by means of a paper set and moderated externally. At Level 2, students write one three-hour examination paper.

External assessment details and procedures are set out in Assessment Guidelines: Mathematics (Level 2).

4. WEIGHTED VALUES OF TOPICS

<table>
<thead>
<tr>
<th>TOPICS</th>
<th>WEIGHTED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Numbers</td>
<td>20</td>
</tr>
<tr>
<td>2. Functions</td>
<td>20</td>
</tr>
<tr>
<td>3. Space, Shape and Orientation</td>
<td>20</td>
</tr>
<tr>
<td>4. Statistical and Probability Models</td>
<td>20</td>
</tr>
<tr>
<td>5. Financial Mathematics</td>
<td>20</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

5. CALCULATION OF FINAL MARK
Continuous assessment: \( \frac{X}{100} \times \frac{25}{1} = \) a mark out of 25 (a)
Examination mark: \( \frac{X}{100} \times \frac{75}{1} = \) a mark out of 75 (b)
Final mark: \( (a) + (b) = \) a mark out of 100

All marks are systematically processed and accurately recorded to be available as hard copy evidence for, amongst others, moderation and verification purposes.

6. PASS REQUIREMENTS
The student must obtain a minimum of 30 percent in Mathematics. A pass will be condoned at 25 percent if it is the only subject stopping the student from progressing to Level 3.

7. SUBJECT AND LEARNING OUTCOMES
On completion of Mathematics Level 2, the student should have covered the following topics:

Topic 1: Numbers
Topic 2: Functions
Topic 3: Space, Shape and Orientation
Topic 4: Statistical and Probability Models
Topic 5: Financial Mathematics

7.1. Topic 1: Numbers

Subject Outcome 1: Use computational tools and strategies and make estimates and approximations.
Learning Outcomes
Students are able to:
• Use a scientific calculator competently and efficiently.
• Execute algorithms appropriately in calculations.
• Take and record measurements to the degree of accuracy of the instrument.

Subject Outcome 2: Demonstrate understanding of numbers and relationships among numbers and number systems and represent numbers in different ways.

Learning Outcomes
Students are able to:
• Identify rational numbers and convert between terminating or recurring decimals like $\frac{a}{b}; a, b \in \mathbb{Z}; b \neq 0$.
• Know, understand and apply the laws of exponents.
• Convert surds into rational forms.
• Identify and work with arithmetic progressions, sequences and series.
• Manipulate and apply simple and compound interest formula.

7.2. Topic 2: Functions

Subject Outcome 1: Work with a range of functions and patterns and solve problems.

Learning Outcomes
Students are able to:
• Sketch and interpret linear, quadratic functions centred on the $y$-axis, inverse, exponential and the basic trigonometric functions of sine, cosine and tangent functions. The graphs referred to are:
  
  $y = ax + q; y = ax^2 + q; y = \frac{a}{x} + q; y = ab^x + q, b > 0; y = a \sin x + q; y = a \cos x + q,

  
  $y = a \tan x + q$
• Convert between numerical, verbal, symbolic and graphical representations of information.
• Investigate and generalise the impact of $a$ and $q$ in the functions above.

Subject Outcome 2: Manipulate algebraic expressions.

Learning Outcomes
Students are able to:
• Find products of binomials with trinomials.
• Factorise by grouping and factorise trinomials.
• Simplify fractions with monomial denominators.

Subject Outcome 3: Solve algebraic equations.

Learning Outcomes
Students are able to:
• Solve linear equations.
• Solve quadratic equations by factorisation.
• Solve exponential equations of the form $ka^{x+y} = m$ by trial and error.
• Solve inequalities in one variable graphically.
• Solve equations simultaneously by numerical, algebraic and graphical methods.

Subject Outcome 4: Investigate the average rate of change of a function between two values of the independent variable.

Learning Outcomes
Students are able to:
• Investigate the gradient between two points on a curve with specific reference to linear functions.
• Investigate the gradient of a curve at a specific point.
7.3. **Topic 3: Space, Shape and Orientation**

**Subject Outcome 1**: Describe, represent, analyse and explain properties of shape in two- and three-dimensional space with justification.

**Learning Outcomes**

Students are able to:
- Investigate the effect on the volume and surface area of right prisms and cylinders, where one or more dimensions are multiplied by a constant factor $k$.
- Investigate and generalise formula for calculating the volume and surface area of right prisms and cylinders.

**Subject Outcome 2**: Describe and represent the properties of geometric shapes.

**Learning Outcomes**

Students are able to:
- Investigate polygons through Euclidean, co-ordinate and/or transformation geometry.
- Generalise the properties of different polygons.
- Expose alternate definitions of polygons.

**Subject Outcome 3**: Represent geometric figures on a Cartesian co-ordinate system.

**Learning Outcomes**

Students are able to:
- Plot points, lines and polygons on a Cartesian plane.
- Calculate the distance between two points.
- Calculate the gradient of a line segment joining two points.
- Calculate the midpoint of a line segment joining two points on a graph.
- Estimate distance, slope and midpoint from a graphical representation of a straight line.

**Subject Outcome 4**: Solve problems by constructing and interpreting geometrical and trigonometric models.

**Learning Outcomes**

Students are able to:
- Use transformation geometry to translate $p$ units horizontally and $q$ units vertically.
- Use transformation geometry to reflect on graphs about the x-axis, the y-axis and about the line $y = x$.
- Introduce the similarity of triangles and proportion of sides of triangles.
- Be familiar with scale drawings and building plans and read what is indicated by them from a geometric and trigonometric perspective.
- Know and use the definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$.
- Sketch the sin, cos and tan functions and discuss their periodicity.
- Research the history of the development of geometry and trigonometry in various cultures.

7.4. **Topic 4: Statistical and Probability Models**

**Subject Outcome 1**: Collect, organise and interpret univariate numerical data.

**Learning Outcomes**

Students are able to:
- Calculate measures of central tendency, namely mean, median and mode, of grouped and ungrouped data.
- Calculate measures of dispersion including range, percentiles, quartiles, interquartile and semi-interquartile range.
- Calculate errors in measurement of data.
- Detect and report on sources of bias in the data and data collection process.

**Subject Outcome 2**: Represent data effectively and appropriately.

**Learning Outcomes**

Students are able to:
- Create and interpret bar and compound bar graphs.
- Create and interpret histograms of grouped data.
• Create and interpret frequency polygons.
• Create and interpret pie charts.
• Create and interpret line and broken line graphs.

7.5. **Topic 5: Financial Mathematics**

**Subject Outcome 1:** Use Mathematics to plan and control personal and household budgets, income and expenditure.

**Learning Outcomes**

- Draw up an income/expenditure sheet describing plans for projected income and expenditure.
- Do calculations using computational tools efficiently and correctly and verify solutions in terms of the context.
- Draw up a personal and/or household budget.
- Record actual income and expenditure over a period and compare to the projected budget.
- Identify, discuss and explain variances and methods for financial control.

**Subject Outcome 2:** Use simple and compound interest to explain and define a variety of situations.

**Learning Outcomes**

- Differentiate between simple and compound interest and extrapolate the advantages and disadvantages of each in specific situations.
- Calculate simple and compound interest over different periods at specific rates.
- Do calculations using computational tools efficiently and correctly and verify solutions in terms of the context.
- Use solutions to calculations effectively to define the changes that occur over a period.

8. **RESOURCE NEEDS FOR THE TEACHING OF MATHEMATICS – LEVEL 2**

- **Physical resources**
  - Softcover spring file for Portfolio of Evidence (PoE)
  - Scientific calculators
  - Graph paper
  - Textbook or workbook
  - Computer memory stick or flash disk
  - Computer and printing facilities
  - Geometric sets
  - Chalk and chalkboards
  - Paper
  - Overhead projectors
  - Current newspapers and information about financial packages from banks and investment companies
  - Internet access or access to a good library or resource centre
  - Models

- **Human resources**

A lecturer must have NQF Level 4 Mathematics or equivalent with a teaching qualification to teach Level 2 Mathematics.