FURTHER EDUCATION & TRAINING PHASE (FET)

MATHEMATICS

SBA EXEMPLAR BOOKLET

GRADES 10-12

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Read to Lead
A Reading Nation is a Leading Nation

2030 NDP

FOREWORD

The Department of Basic Education has pleasure in releasing a subject exemplar booklet for School Based Assessment (SBA) to assist and guide teachers with the setting and development of standardised SBA tasks and assessment tools. The SBA booklets have been written by teams of subject specialists to assist teachers to adapt teaching and learning methods to improve learner performance and the quality and management of SBA.

The primary purpose of this SBA exemplar booklet is to improve the quality of teaching and assessment (both formal and informal) as well as the learner's process of learning and understanding of the subject content. Assessment of and for learning is an ongoing process that develops from the interaction of teaching, learning and assessment. To improve learner performance, assessment needs to support and drive focused, effective teaching.

School Based Assessment forms an integral part of teaching and learning, its value as a yardstick of effective quality learning and teaching is firmly recognised. Through assessment, the needs of the learner are not only diagnosed for remediation, but it also assists to improve the quality of teaching and learning. The information provided through quality assessment is therefore valuable for teacher planning as part of improving learning outcomes.

Assessment tasks should be designed with care to cover the prescribed content and skills of the subject as well as include the correct range of cognitive demand and levels of difficulty. For fair assessment practice, the teacher must ensure that the learner understands the content and has been exposed to extensive informal assessment opportunities before doing a formal assessment activity.

The exemplar tasks contained in this booklet, developed to the best standard in the subject, is aimed to illustrate best practices in terms of setting formal and informal assessment. Teachers are encouraged to use the exemplar tasks as models to set their own formal and informal assessment activities.

MR HM MWELI

DIRECTOR-GENERAL

DATE: 13/09/2017

MATHEMATICS

GRADE 10

A. Investigating Parabolas

TIME: 50 minutes **TOTAL MARKS: 35**

NAME: Memo **CLASS: 10**

DEFINITIONS:
Translate – to translate a graph means to move the graph. A graph may be translated vertically up/down or horizontally left/right.
Reflect – to reflect a graph is to draw a mirror image of the graph on the opposite side of a straight line.

In this investigation you will be looking for changes in the graphs, i.e. have the graphs become steeper or shallower/wider, been translated and/or reflected and what causes the change(s).

NB : When a number is squared, the answer is always positive

INSTRUCTIONS FOR EACH QUESTION:
- Complete the tables.
- Plot the points from the tables on the Cartesian plane alongside the question and join the points to draw the graphs using different colours for each graph.
- Label the graphs b and c.
- Graphs must be drawn free hand. **DO NOT** use a ruler to draw the graphs.
- Answer the question based on your observations of the graphs.
- Marks will be awarded as follows:

Completing table	Graph	Answers to questions 1 – 5	Total
1 mark – accurate entries	1 mark - shape	1 mark per answer.	7 marks per question
	1 mark - accurate turning point		

1a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = x^2 + 1$

x	-2	-1	0	1	2
y					

c. $y = x^2 + 2$

x	-2	-1	0	1	2
y					

Compare the two graphs you have drawn to the original graph $y = x^2$.
Explain what happens to the graph when a number is added to x^2.

Draw the graphs here

2a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = x^2 - 5$

x	-3	-2	-1	0	1	2	3
y							

c. $y = x^2 - 3$

x	-3	-2	-1	0	1	2	3
y							

Compare the two graphs you have drawn to the original graph $y = x^2$.
Explain what happens to the graph when a number is subtracted from x^2.

Draw the graphs here

3a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = 2x^2$

x	-1	0	1
y			

c. $y = 3x^2$

x	-1	0	1
y			

Compare the two graphs you have drawn to the original graph $y = x$.
Explain what happens to the graph when $x^2$is multiplied by a number greater than 1.

Draw the graphs here

4a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
y					

c. $y = \frac{1}{4}x^2$

x	-4	-2	0	2	4
y					

Compare the two graphs you have drawn to the original graph $y = x^2$.

Explain what happens to the graph when x^2 is multiplied by a number between 0 and 1.

Draw the graphs here

5a. $y = x^2$	Draw the graphs here

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = -x^2$

x	-2	-1	0	1	2
y					

c. $y = -2x^2$

x	$-1\frac{1}{2}$	-1	0	1	$1\frac{1}{2}$
y					

Compare the two graphs you have drawn to the original graph $y = x^2$.
Explain what happens to the graph when the coefficient of x^2 is negative.

1a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = x^2 + 1$

x	-2	-1	0	1	2
y	**5**	**2**	**1**	**2**	**5**

√

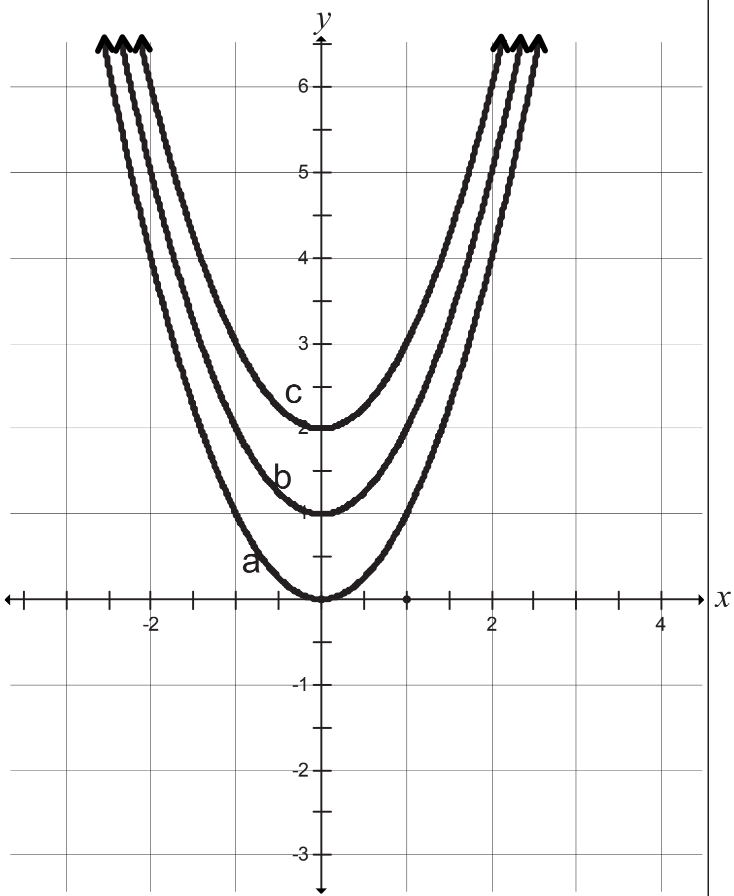
c. $y = x^2 + 2$

x	-2	-1	0	1	2
y	**6**	**3**	**2**	**3**	**6**

√

Compare the two graphs you have drawn to the original graph $y = x^2$.
Explain what happens to the graph when a number is added to x^2.

The graph is shifted vertically upwards by the number of units that is added to x^2 √



For graph b and c:
Turning point √√
Shape √√

2a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = x^2 - 5$

x	-3	-2	-1	0	1	2	3
y	**4**	**-1**	**-4**	**-5**	**-4**	**-1**	**4**

√

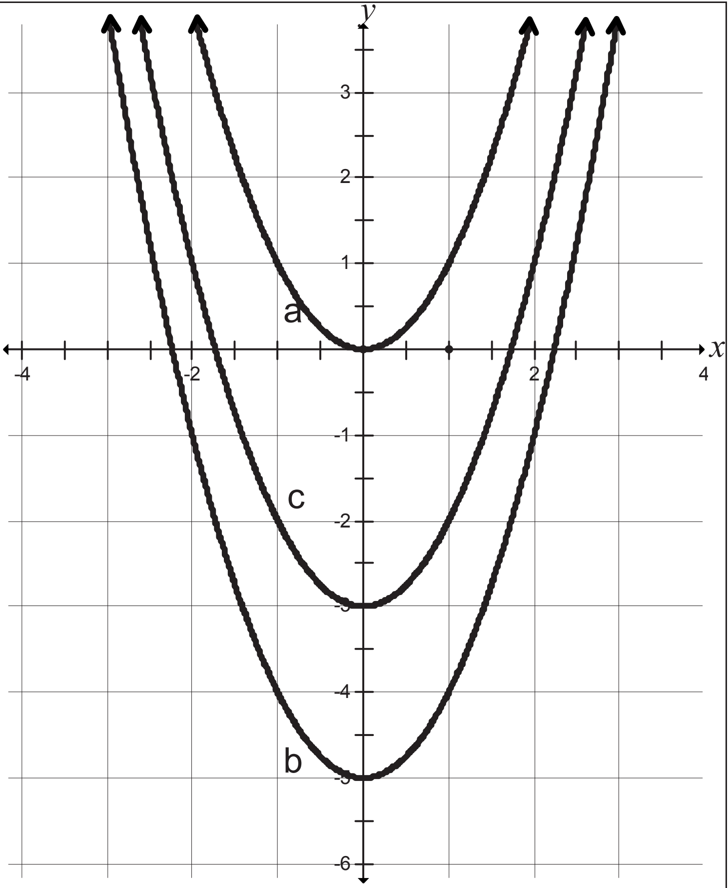
c. $y = x^2 - 3$

x	-3	-2	-1	0	1	2	3
y	**6**	**1**	**-2**	**-3**	**-2**	**1**	**6**

√

Compare the two graphs you have drawn to the original graph $y = x^2$.
Explain what happens to the graph when a number is subtracted from x^2.

The graph is shifted vertically down by the number of units subtracted from x^2 √



For graph b and c:
Turning point √√
Shape √√

3a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = 2x^2$

x	-1	0	1
y	**2**	**0**	**2**

√

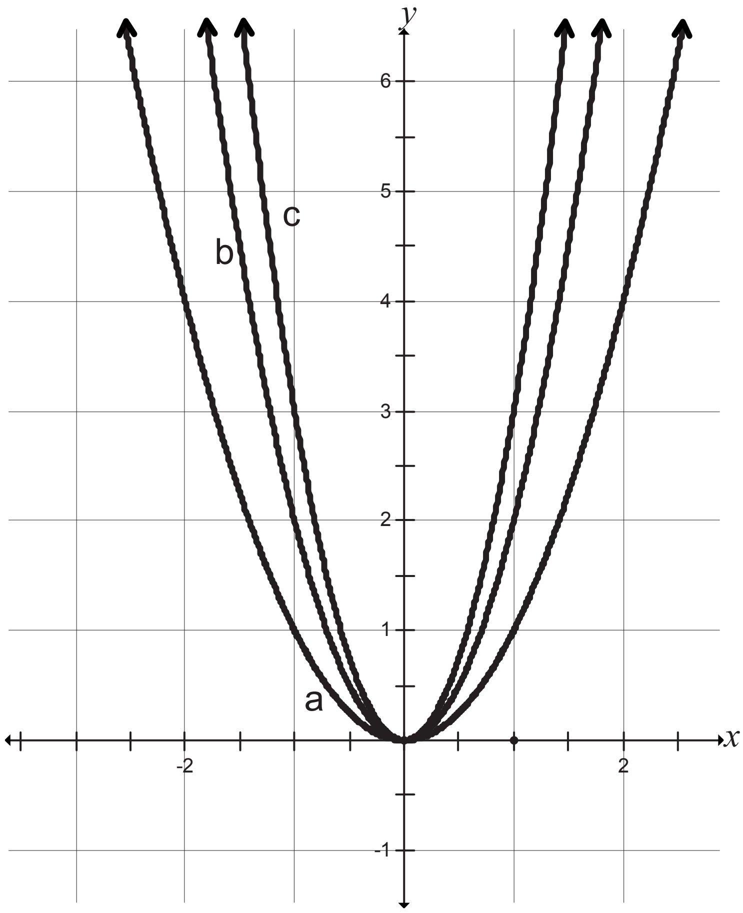
c. $y = 3x^2$

x	-1	0	1
y	**3**	**0**	**3**

√

Compare the two graphs you have drawn to the original graph $y = x^2$. Explain what happens to the graph when x^2 is multiplied by a number greater than 1.

As the coefficient increases, the arms of the graph get closer to the y-axis, i.e. the graph gets steeper/width decreases/narrower/thinner √



For graph b and c:
Turning point √√
Shape √√

4a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
y	**2**	$\frac{1}{2}$	**0**	$\frac{1}{2}$	**2**

√

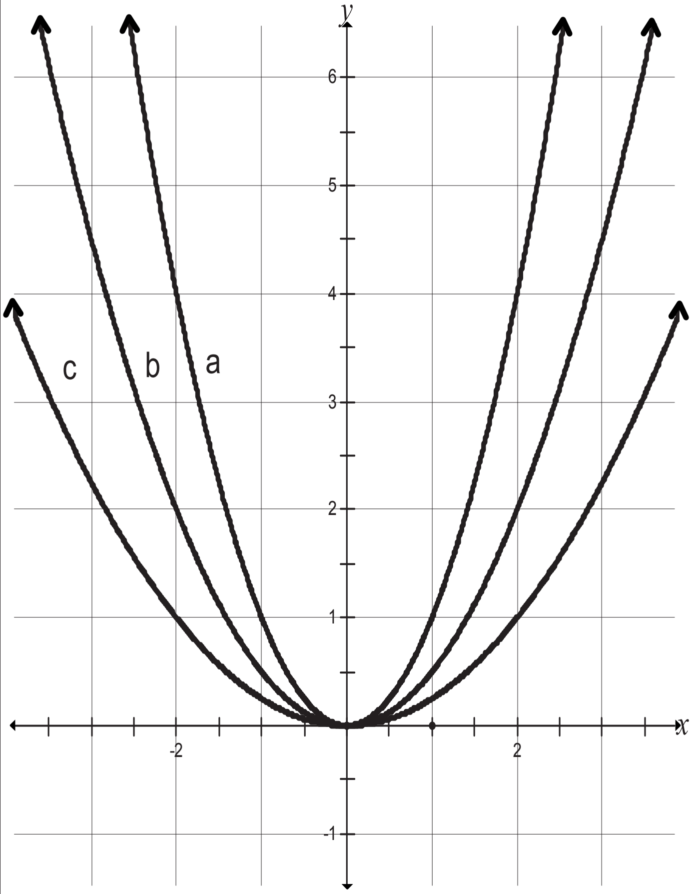
c. $y = \frac{1}{4}x^2$

x	-4	-2	0	2	4
y	**4**	**1**	**0**	**1**	**4**

√

Compare the two graphs you have drawn to the original graph $y = x^2$.
Explain what happens to the graph when x^2 is multiplied by a number between 0 and 1.

The smaller the coefficient of x^2, the wider the graph becomes √



For graph b and c:
Turning point √√
Shape √√

5a. $y = x^2$

x	-2	-1	0	1	2
y	4	1	0	1	4

b. $y = -x^2$

x	-2	-1	0	1	2
y	**-4**	**-1**	**0**	**-1**	**-4**

√

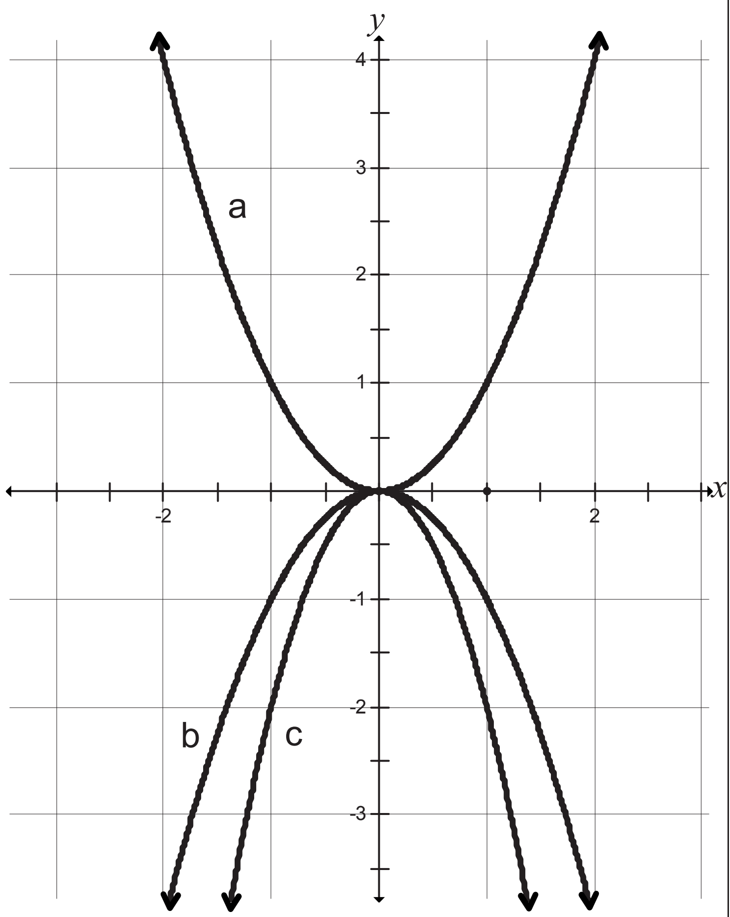
c. $y = -2x^2$

x	$-1\frac{1}{2}$	-1	0	1	$1\frac{1}{2}$
y	$-4\frac{1}{2}$	**-2**	**0**	**-2**	$-4\frac{1}{2}$

√

Compare the two graphs you have drawn to the original graph $y = x^2$.
Explain what happens to the graph when the coefficient of x^2 is negative.

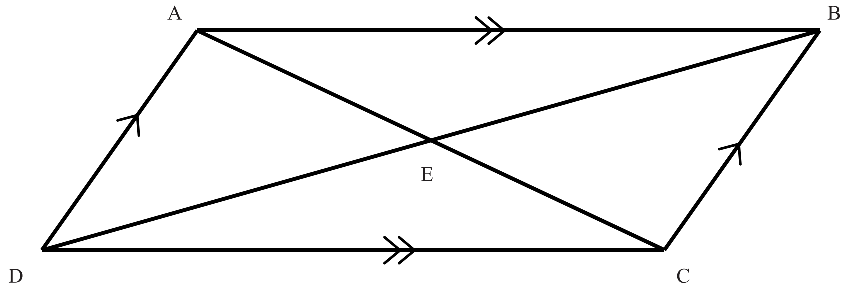
The graph is reflected about the x-axis√



For graph b and c:
Turning point √√
Shape √√

B. An investigation into the relation between the diagonals of a parallelogram **Marks :45**

1. The diagram below shows a parallelogram ABCD.



2. Use a ruler and a compass to measure the sections of the diagonals of the parallelogram and complete the first row of the table below.

Sections of the diagonals of a parallelogram				
	AE	CE	BE	DE
1				
2				
3				
4				
5				
6				

3. In your own words, make a conjecture about the intersection of the diagonals of a parallelogram:

____

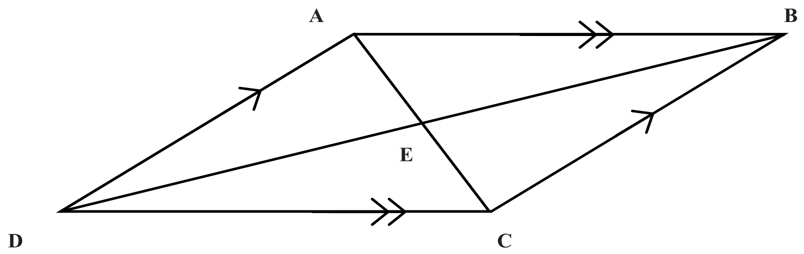
____

4. Use the diagrams on the back of the page to complete the rest of the table.

5. Now complete the following statement:

"The diagonals of a parallelogram __________________________."

2



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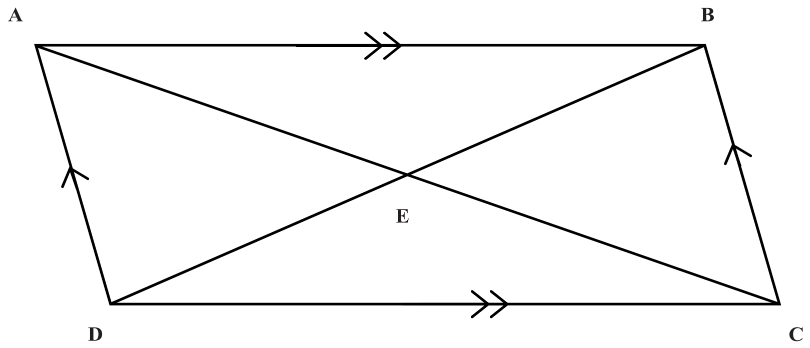

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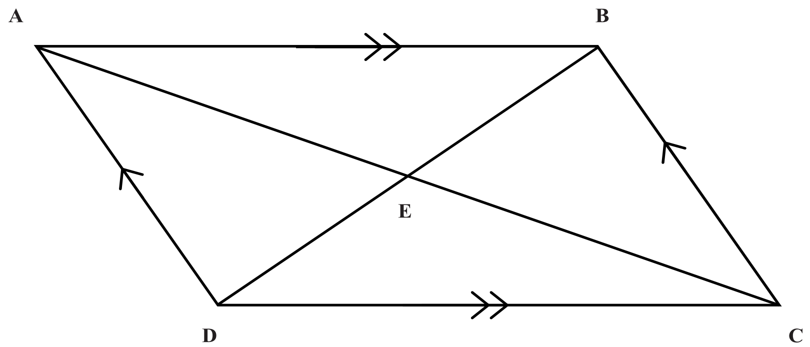

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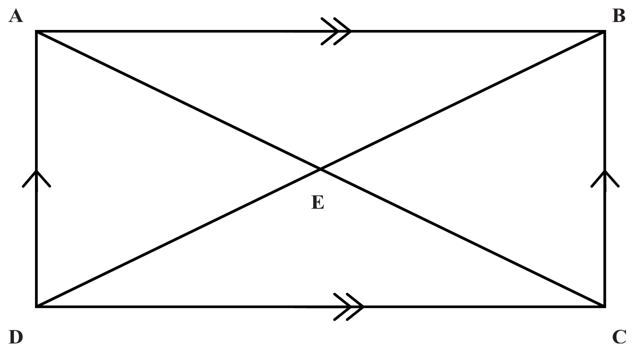

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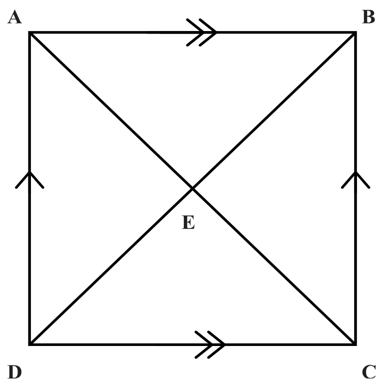


Assessment Tool

Learner's name: ....

	0	1	2	3	Weight	Mark
Measurements	Did not make any measurements	Measured the segments of the first diagram only	Only measured the segments of some of the diagrams, but not all	Measured the segments of all diagrams	**3**	/**9**
Accuracy	No measurements were correct	Less than half the measurements were correct	Majority of the measurements were correct	All measurements were correct	**4**	/**12**
Conjecture in own words	No attempt was made to make a conjecture	A conjecture was made, but was irrelevant to the question under investigation	A relevant conjecture was made, but did not describe the theorem properly	The conjecture did describe the theorem in an understandable way	**3**	/**9**
Theorem	No attempt was made to complete the theorem	The theorem was completed, but was irrelevant to the question under investigation	The theorem was completed, but not mathematically correct	The theorem was completed correctly	**5**	/**15**
					TOTAL	/**45**

C. **The influence of the value of *a* on the graphs of the different functions Marks: 45**

Outcomes:

At the end of this activity you should be able to tell the effect that the value of *a* has on the graphs of the different algebraic functions.

1. The graph of $y = ax$

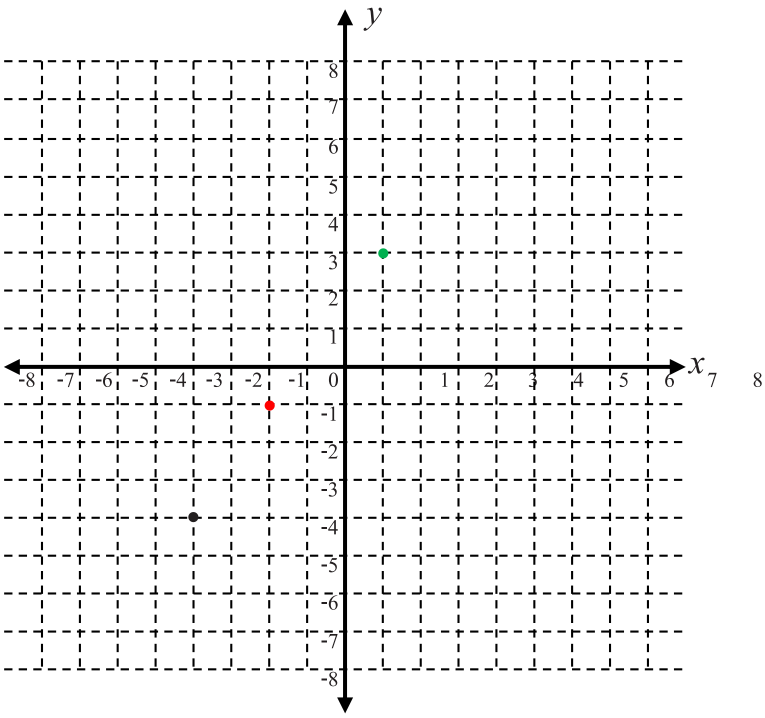
1.1 Use the table below and calculate the values of the different y's for the given values of x:

	x	-4	-2	0	1	3
Value of y_1	$y_1 = x$	-4				
Value of y_2	$y_2 = \frac{1}{2}x$		-1			
Value of y_3	$y_3 = 3x$				3	

(3)

1.2 Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.

(3)



1.3 In your own words, write down your conclusion about the effect of the value of *a* on the graph of $y=ax$: (2)

1.4 When *a* changes, the .... of the graph changes accordingly. (1)
Therefore the value of *a* indicates the ....of the straight line graph. (1)

2. The graph of $y = ax^2$

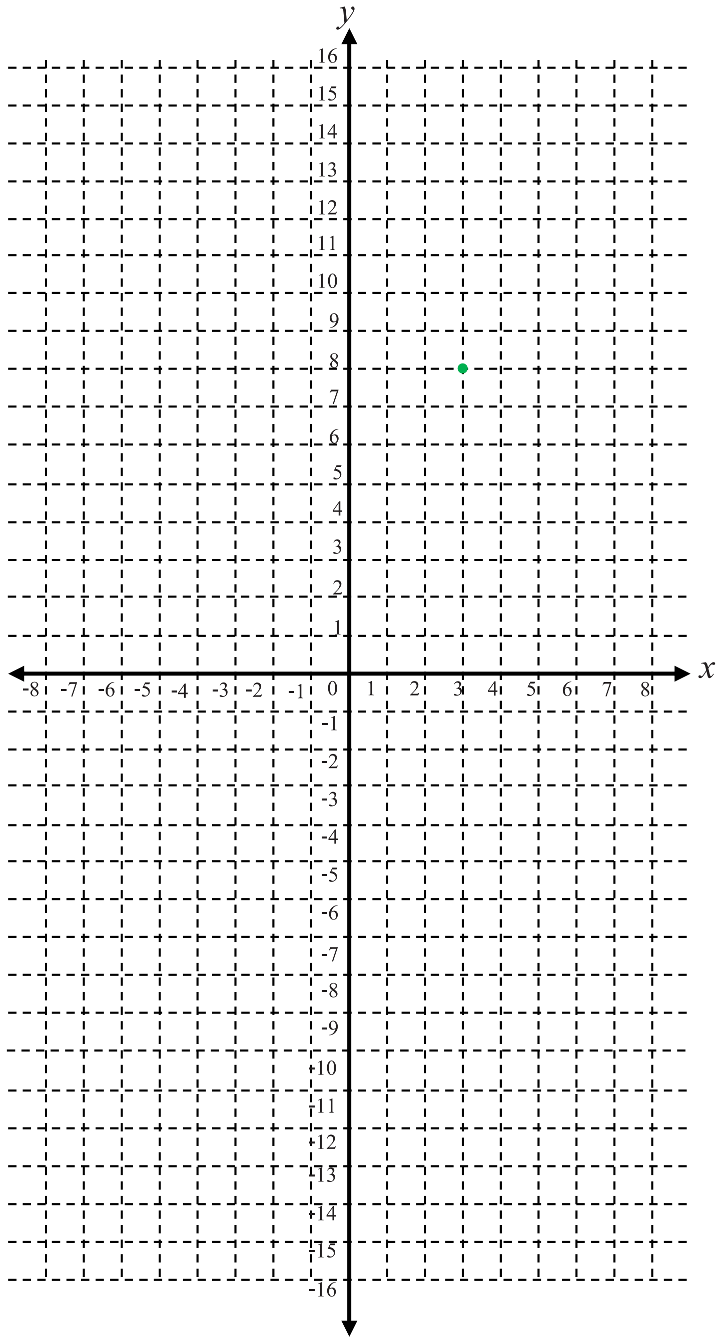
2.1 Use the table below and calculate the values of the different y's for the given values of x:

	x	-3	-2	0	2	3
Value of y_1	$y_1 = x^2$	9				
Value of y_2	$y_2 = \frac{1}{2}x^2$		2			
Value of y_3	$y_3 = 2x^2$				8	
Value of y_4	$y_4 = -x^2$					-9

(3)

2.2 Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.

(4)



2.3 In your own words, write down your conclusion about the effect of the value of *a* on the graph of $y = ax^2$:

____ (1)

2.4 When *a* changes, the .... of the graph changes accordingly. (1)
Therefore the value of *a* indicates the .... of the parabolic graph. (1)

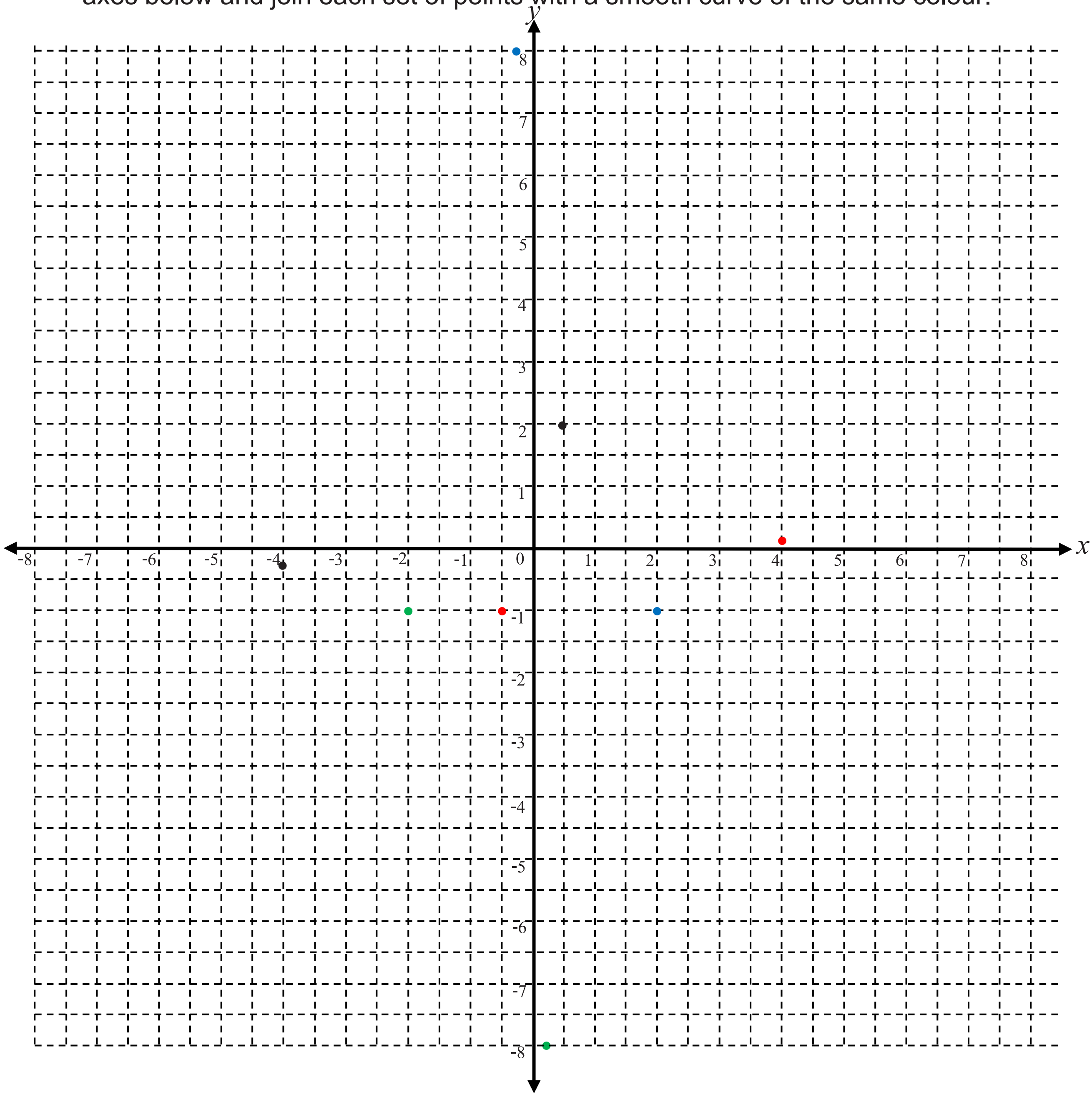
3. The graph of $y = \frac{a}{x} = a.\frac{1}{x}$

3.1 Use the table below and calculate the values of the different y's for the given values of x:

	x	- 4	- 2	- 1	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
Value of y_1	$y_1 = \frac{1}{x}$	$-\frac{1}{4}$							2			
Value of y_2	$y_2 = \frac{1}{2}\cdot\frac{1}{x}$				- 1							$\frac{1}{8}$
Value of y_3	$y_3 = 2.\frac{1}{x}$		- 1					8				
Value of y_4	$y_4 = -2.\frac{1}{x}$					8					- 1	

(4)

3.2 Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.



(4)

3.3 In your own words, write down your conclusion about the effect of the value of *a* on the graph of $y = \frac{a}{x}$:

____ (2)

3.4 When *a* changes, the .... of the graph changes accordingly. (1)
Therefore the value of *a* indicates the .... of the hyperbolic graph. (1)

D. The graph of $y = ab^x$; $b > 0$

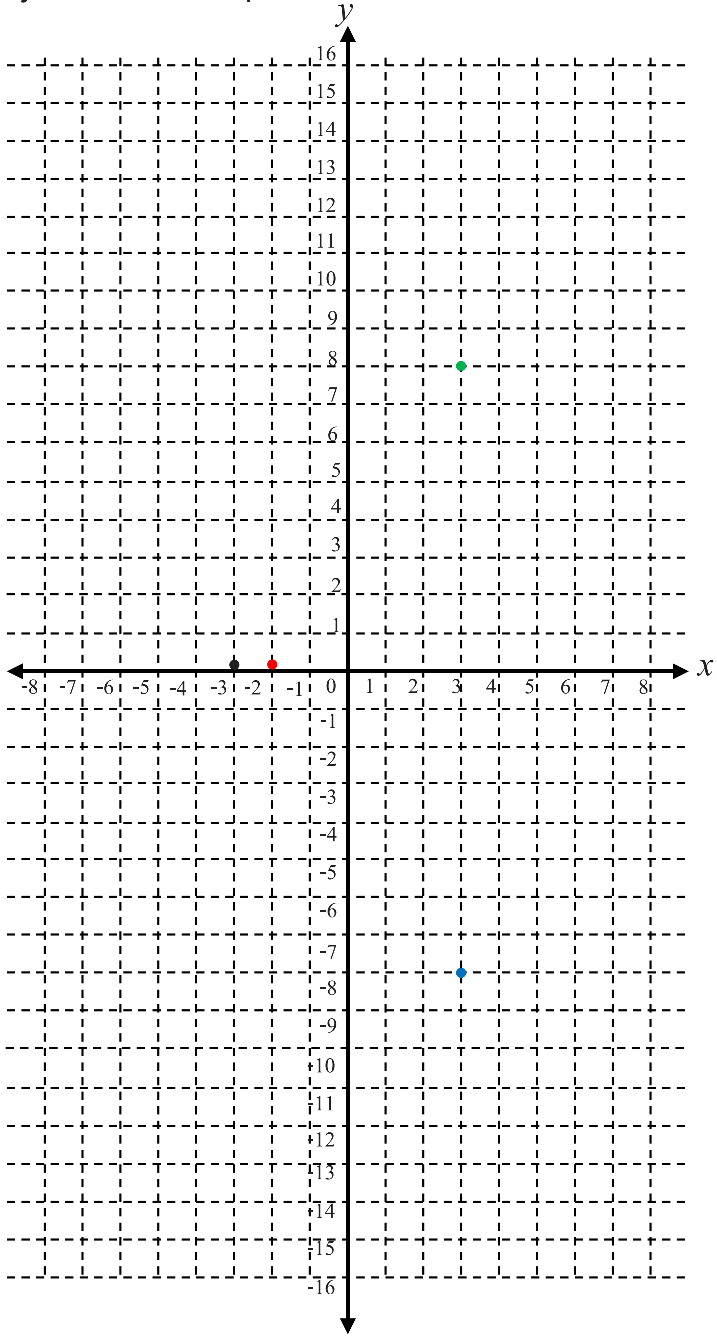
4.1 Use the table below and calculate the values of the different y's for the given values of x:

	x	-3	-2	0	2	3
Value of y_1	$y_1 = 2^x$	$\frac{1}{8}$				
Value of y_2	$y_2 = \frac{1}{2}.2^x$		$\frac{1}{8}$			
Value of y_3	$y_3 = 2.2^x$				8	
Value of y_4	$y_4 = -2^x$					-8

(4)

4.2 Use the table to plot the different sets of points in different colours on the set of axes below and join each set of points with a smooth curve of the same colour.

(4)



4.3 In your own words, write down your conclusion about the effect of the value of a on the graph of $y = ab^x$:

____ (2)

4.4 When a changes, the .... of the graph changes accordingly. (1)
Therefore value of a indicates the .... of the exponential graph. (1)

E. Transform the graph of $y = \sin x$ to the graph of $y = a \sin x$, $a \neq 0$

Mark: 40

1. **Outcomes:**
   After this activity you should be able to:
   1.1 state what effect a change in the value of constant a will have on the function $y = a \sin x$
   1.2 draw the graphs of $y = a \sin x$ using the properties of the function

2. Complete the following table with assistance of your calculator: (3)

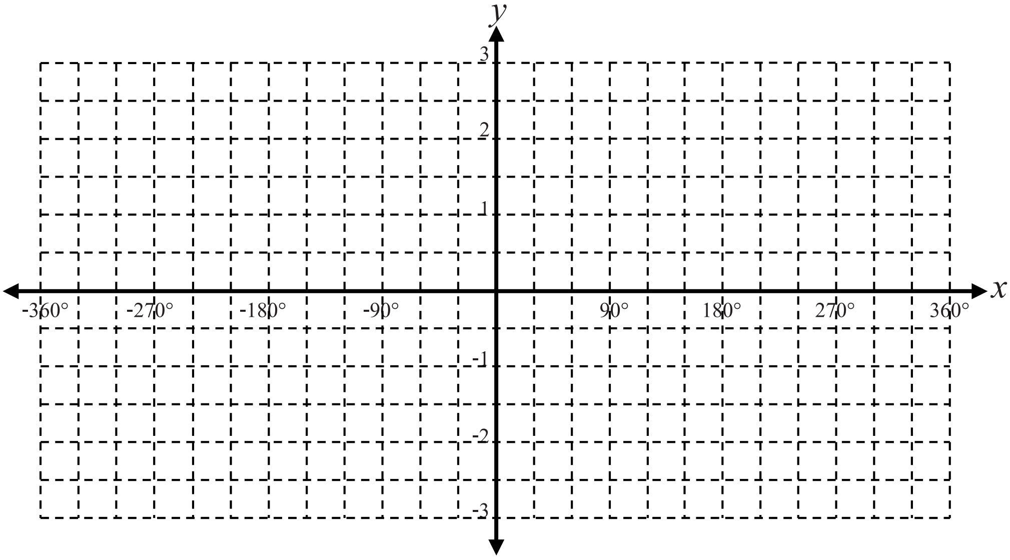
x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°	-90°	-60°	-30°	0°
$\sin x$													
$2\sin x$													
$\frac{1}{2}\sin x$													

(3)

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin x$													
$2\sin x$													
$\frac{1}{2}\sin x$													

3. 3.1 Use the set of axes below to draw the graph of $y = \sin x$ (2)

3.2 Draw the graphs of $y = 2 \sin x$ and $y = \frac{1}{2}\sin x$ on the same set of axes. (4)



4. Use your graphs to complete the table below.

	a	x-intersects					Turning points				Amplitude	Period
sin x												
$\frac{1}{2}\sin x$												
2 sin x												

(4x3) = 12

5. 5.1 How did a change in the value of *a* affect the x-intercepts of the sine function?

____ (2)

5.2 How did a change in the value of *a* affect the turning points of the sine function?

____ (2)

5.3 How did a change in the value of *a* affect the amplitude of the sine function?

____ (2)

5.4 How did a change in the value of *a* affect the period of the sine function?

____ (2)

6. Make a general conclusion on the effect of the value of *a* on the graph of the sine function.

____ (2)

7. Make a prediction on the effect of the value of *a* on the graphs of the cosine and tangent functions. Investigate your predictions.

____ (2)

____

F. **Transform the graph of y = cos x to the graph of y = *a* cos x, *a* ≠ 0**
Mark: 40

1. **Outcomes:**
   After this activity you should be able to:
   1.1 state what effect a change in the value of constant *a* will have on the function y = *a* cos x
   1.2 draw the graphs of y = *a* cos x using the properties of the function

2. Complete the following table with assistance of your calculator:

(3)

x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°	-90°	-60°	-30°	0°
$\cos x$													
$2\cos x$													
$\frac{1}{2}\cos x$													

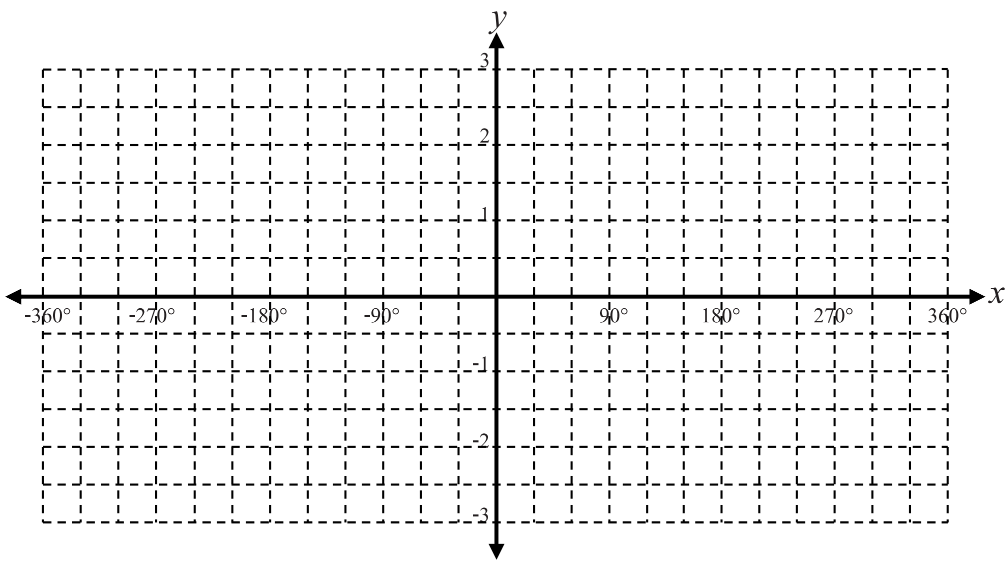
(3)

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\cos x$													
$2\cos x$													
$\frac{1}{2}\cos x$													

3. 3.1 Use the set of axes below to draw the graph of $y = \cos x$ (2)

   3.2 Draw the graphs of y = 2 cos x and y = $\frac{1}{2}\cos x$ on the same set of axes.

(4)



4. Use your graphs to fill out the table below:

	a	x-intersects					Turning points				Amplitude	Period
$\cos x$												
$\frac{1}{2}\cos x$												
$2\cos x$												

(4x3)=12

5. 5.1 How did a change in the value of *a* affect the x-intercepts of the cosine function?

____ (2)

5.2 How did a change in the value of *a* affect the turning points of the cosine function?

____ (2)

5.3 How did a change in the value of *a* affect the amplitude of the cosine function?

____ (2)

5.4 How did a change in the value of *a* affect the period of the cosine function?

____ (2)

6. Make a general conclusion on the effect of the value of *a* on the graph of the cosine function.

____

____ (2)

7. Was your prediction regarding the effect of the value of *a* on the graph of the cos function correct?
Make a prediction on the effect of the value of *a* on the graph of the tangent function.
Investigate your predictions.

____

____ (2)

PROJECT GRADE 10

MARKS: 50

INSTRUCTIONS

1. This project must be done at home.
2. Learners must collect data on the days' prescribed data, using the following sources:

   (a) Daily newspapers
   (b) TV services
   (c) Radio services or any other source.

3. Show calculations in determining your answers.
4. Submit your work on the specified date.
5. Five (5) marks will be awarded to learners who submitted on time.

ACTIVITY

You are required to collect data of minimum and maximum temperatures from Monday (date) to Friday (date) of Johannesburg and Cape Town on a daily basis.
Use the following table to collect data for these two cities.

(N.B. A teacher can decide the period for which learners can collect data. The period given above is an exemplar.)

JOHANNESBURG TEMPERATURES

	DAYS					
		Monday	Tuesday	Wednesday	Thursday	Friday
TEMPERATURE	Minimum					
	Maximum					

CAPE TOWN TEMPERATURES

	DAYS					
		Monday	Tuesday	Wednesday	Thursday	Friday
TEMPERATURE	Minimum					
	Maximum					

(10)

QUESTIONS

1. Determine the

1.1. Mean of the minimum temperature of Cape Town. (2)
1.2. Median of the maximum temperatures of Cape Town. (2)
1.3. Mode of the minimum temperatures of Johannesburg. (2)
1.4. Range of maximum temperatures of both cities. (4)

2. If you were to travel to Cape Town on the 13th of May, what type of clothes would you take along, considering the temperature of that day? (2)

3. Determine the five number summary of the maximum temperatures of both Johannesburg and Cape Town starting from Monday to Friday. (10)

4. Now draw the box and whisker diagram for two cities based on question 3 above. (10)

5. Between the two cities, which is cooler than the other? Motivate your answer and base your argument on statistical calculation. (3)

6. Submission on time. (5)

[TOTAL: 50]

MARKING GUIDELINE

JOHANNESBURG TEMPERATURES

	DAYS					
		Monday	Tuesday	Wednesday	Thursday	Friday
TEMPERATURE	Minimum	7	7	8	8	8
	Maximum	21	21	22	24	23

[5]

CAPE TOWN TEMPERATURES

	DAYS					
		Monday	Tuesday	Wednesday	Thursday	Friday
TEMPERATURE	Minimum	14	14	15	14	13
	Maximum m	24	22	24	23	21

[5]

1.
   1.1. $\frac{14+14+15+14+13}{5} = \frac{70}{5} = 14$ (2)

   1.2. 23 (2)

   1.3. 7 and 8 (2)

   1.4.
      1.4.1. Range of max (Johannesburg) = 24 – 21 = 3 (2)
      1.4.2. Range of max (Cape Town) = 24 – 21 = 3 (2)

2. Consider warm clothes. (2)

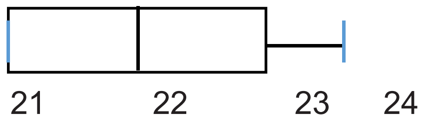
3. Five number summary (maximum)

			A Johannesburg	B Cape Town
3.1.	Minimum	=	21	21
3.2.	Lower quartile	=	21	22
3.3.	Median	=	22	23
3.4.	Upper quartile	=	23	24
3.5.	Maximum	=	24	24

(10)

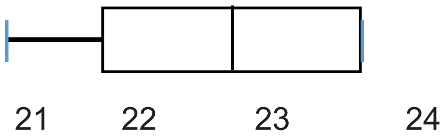
4. Box and whisker plot

A.



(5)

B.



(5)

5. Johannesburg, since the median or average temperature is 8 and that of Cape Town is 14. (3)

6. Submission on time. (5)

Grade 11

A. Practical Investigation: Ratios

INSTRUCTIONS

1. **Answer ALL the questions**
2. **Show all calculations (steps), diagrams, etc. you used in determining answers.**
3. **Marks will be awarded for accuracy and correct conclusions.**

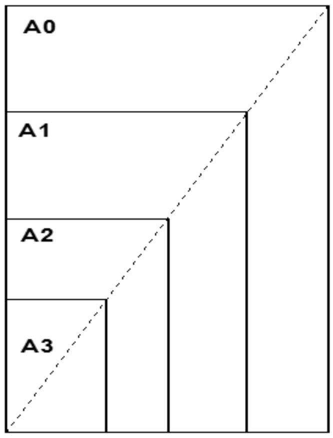
Marks: 50

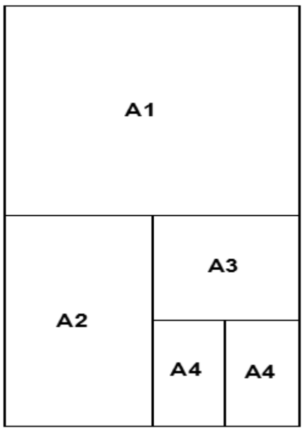
1 Solve the following equation for x in terms of y: $x^2 - 3xy + 2y^2 = 0$ and hence show that the ratio $x : y$ is 1 : 1 or 2 : 1. (4)

2 Most paper is cut to internationally agreed sizes: A0, A1, A2, …A7 with the property that the A1 sheet is half the size of the A0 sheet and has the same shape as the A0 sheet, the A2 sheet is half the size of the A1 sheet and has the same shape and so on.

2.1 Explain what it means that all sheets have the same shape. (2)

2.2 Consider the diagrams below:
All sheets have the same shape An A0 sheet folds into two A1 sheets

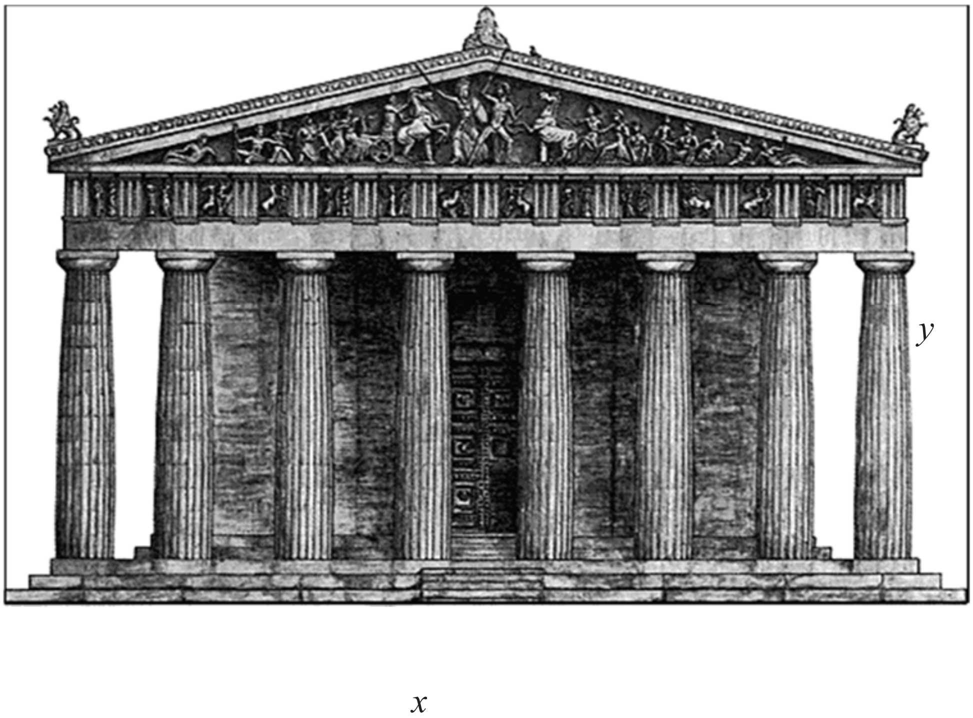




2.2.1 Find the ratio of the length to the breadth of each rectangular piece of paper. (5)

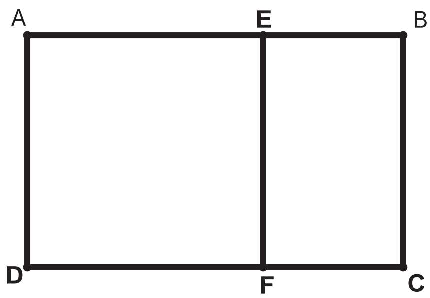
2.2.2 Why is $-\sqrt{2}$ not accepted as one of the possible answers in 2.2.1? (2)

3 The golden rectangle has been recognised through the ages as being aesthetically
pleasing. It can be seen in the architecture of the Greeks, in sculptures and in Renaissance paintings.



3.1 Measure x and y and hence estimate the golden ratio $x : y$ (2)

3.2 The golden rectangle has the property that: when a square is cut out from the rectangle with sides equal to the shortest side of the rectangle, then a smaller rectangle with the same shape is left.

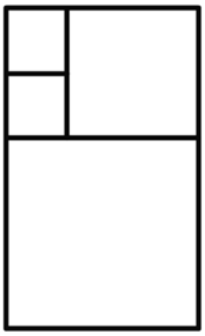


$$\frac{AB}{BC} = \frac{BC}{EB}$$

The process can be continued indefinitely, producing smaller and smaller rectangles. Using this information, calculate the ratio $x : y$ in surd form. (7)

3.3 Write the ratio obtained in 3.2 above (often called phi: ϕ) correct; you answer to 3 decimal places. (2)

4 A sequence of rectangles can be built up in the following way : start with a rectangle made by two identical squares placed next to each other. The next rectangle is formed by adding a square onto the longer side of this rectangle. The process can be continued indefinitely. Investigate the ratio of the length to the breadth of these rectangles up to at least 10 rectangles.
Let the first rectangle have dimensions of 2 units by 1 unit.



(11)

5 A line segment, AB can be divided in the golden ratio at C in the following way:

5.1 At B, draw a perpendicular to AB and mark off BD equal to half AB. (2)

5.2 Join AD, on DA mark off DE = DB, furthermore on AB mark off AC = AE (2)

5.3 Now prove that this construction divides AB so that AC:CB=ϕ (10)

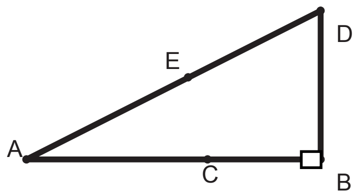
TOTAL: 50

MARKING MEMORANDUM

Grade 11 Investigation: Ratios **Marks: 50**

1	If $x^2-3xy+2y^2=0$, then $(x-y)(x-2y)=0$ and hence $x=y$ or $x=2y$ and dividing both sides by y, $\frac{x}{y}=1$ or $\frac{x}{y}=2$. This can be written as $x:y=1:1$ or $x:y$ = 2 : 1	✓factors ✓values of x ✓dividing by y ✓$x:y=1:1$ ✓$x:y$= 2 : 1 (5)
2.1	It means that the shape of the sheet will always be a rectangle.	✓✓answer (2)
2.2.1	Let length of larger sheet be x and breadth be y. Then the length and breadth of the sheet one size smaller will be y and $\frac{x}{2}$. Since these sheets have the same shape, the ratio of the lengths and breadths is the same: $\frac{x}{y}=\frac{y}{\frac{x}{2}}$ $\therefore \frac{x^2}{2}=y^2$ $\therefore x^2=2y^2$ $\therefore \frac{x^2}{y^2}=2$ Taking the square root of both sides, we get: $\frac{x}{y}=\sqrt{2}$	✓$\frac{x}{y}=\frac{y}{\frac{x}{2}}$ ✓$\frac{x^2}{2}=y^2$ ✓$\frac{x^2}{y^2}=2$ ✓square roots ✓correct answer (5)
2.2.2	$-\sqrt{2}$ has no practical meaning, since x and y are the dimensions of a quadrilateral and have to be positive.	✓ x and y are the dimensions ✓positive (2)
3.1	The ratio $\frac{x}{y}\approx 1{,}6$	✓measuring ✓ratio

		NB: measurement may vary but ratio will be the same. (2)
3.2	Letting the length be x and the breadth y, the square that will be removed will be y by y and the dimensions of the new rectangle will be y by $(x - y)$. Since the larger and the smaller rectangles have the same shape: $\frac{x}{y} = \frac{y}{x-y}$ $\therefore x^2 - xy = y^2$ $\therefore x^2 - xy - y^2 = 0$ Using the quadratic formula to solve for x in terms of y, we get: $x = \frac{-(-y) \pm \sqrt{(-y)^2 - 4(1)(-y^2)}}{2(1)}$ $= \frac{1 \pm \sqrt{5}}{2} y$ Again, $\frac{1-\sqrt{5}}{2}$, which is a negative number has no practical meaning in this case, so $\frac{x}{y} = \frac{1+\sqrt{5}}{2}$ or $x : y = 1 + \sqrt{5} : 2$.	✓ y by y ✓ y by $(x - y)$ ✓ratio ✓equation in standard form ✓Subst into quadratic formula ✓ $\frac{1 \pm \sqrt{5}}{2} y$ ✓ $x : y = 1 + \sqrt{5} : 2$ (7)
3.3	This ratio: $\frac{1+\sqrt{5}}{2} \approx 1{,}6180\ldots$ is called the golden ratio and a rectangle in which the ratio of the length to the breadth is $\phi = \frac{1+\sqrt{5}}{2}$, is called a golden rectangle.	✓ $\frac{1+\sqrt{5}}{2} \approx 1{,}618033989$ ✓1,618 (2)
4	The sequence is as follows: 2:1 = 2 3:2 = 1,5 5:3 = $1{,}\dot{6}$	✓2:1 ✓3:2 = 1,5

	8:5 = 1,6 13:8 = 1,625 21:13 = 1,615384 … 34:21 = 1,619047… 55:34 = 1,617647… 89:55 = 1,618181… 144:89 = 1,61787… Successive ratios seem to be approaching the golden ratio, phi (ϕ)	✓5:3 = $1,\dot{6}$ ✓8:5 = 1,6 ✓13:8 = 1,625 ✓21:13 = 1,615384 … ✓34:21 = 1,619047… ✓55:34 = 1,617647… ✓89:55 = 1,618181… ✓144:89 = 1,61787… ✓conclusion (11)
5.1	Let AB = x, then $DB = \frac{x}{2}$ 	✓BD $\perp$ AB ✓$DB = \frac{x}{2}$ (2)
5.2	See diagram in 5.1	✓✓correct construction (2)
5.3	By the Theorem of Pythagoras, $AD^2 = x^2 + \left(\frac{x}{2}\right)^2$ $= \frac{5x^2}{4}$ $\therefore AD = \frac{\sqrt{5}}{2}x$ Hence AE = AD – ED = $\frac{\sqrt{5}}{2}x - \frac{1}{2}x$ $= \frac{\sqrt{5}-1}{2}x$ AC=AE$= \frac{\sqrt{5}-1}{2}x$	✓using Pythagoras Theorem ✓$\frac{5x^2}{4}$ ✓AD ✓ AC = AE ✓$\frac{\sqrt{5}}{2}x - \frac{1}{2}x$ ✓$\frac{\sqrt{5}-1}{2}x$ ✓AC ✓CB = $\frac{3-\sqrt{5}}{2}x$ ✓$\frac{AC}{CB} = \frac{\sqrt{5}-1}{2}x \div \frac{3-\sqrt{5}}{2}x$

	And CB = $x-\left(\frac{\sqrt{5}-1}{2}\right)x=\frac{3-\sqrt{5}}{2}x$ The ratio $\frac{AC}{CB}=\frac{\sqrt{5}-1}{2}x\div\frac{3-\sqrt{5}}{2}x=\frac{\sqrt{5}-1}{2}\times\frac{2}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$ $=\frac{1+\sqrt{5}}{2}$ the golden ratio! The rationalising of denominators is not explicitly mentioned in the curriculum, so the use of a calculator to show that $\frac{\sqrt{5}-1}{3-\sqrt{5}}=1{,}618\ldots$would be fine.	✓ $=\frac{1+\sqrt{5}}{2}$ = 1,618 (10)
GRAND TOTAL		**50 MARKS**

Grade 11

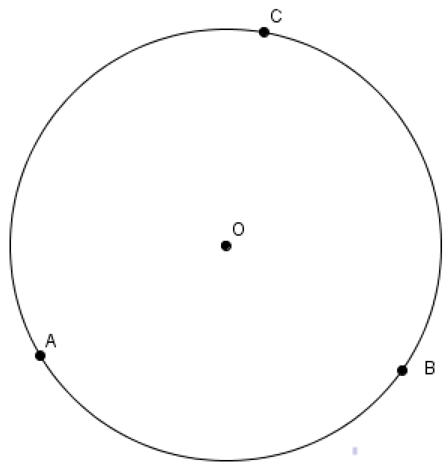
B. **Mathematics Investigation on Euclidean GeometryMARKS: 50**

Topic: *Investigate the relationship between the angle at the circumference and the angle at the centre of a circle.*

Instructions to teachers

First do the definitions of the properties of a circle.

Teach them how to measure an angle.



Instructions to learners

Is there a relationship between the angle at the circumference and the angle at the centre?

1. Draw any circle
2. Choose a point on the circumference of a circle, now choose two other points on the alternate segment of the circumference of the circle, as in the diagram.
3. Join chord AC and BC.
4. Join radii AO and BO
5. Measure $A\hat{C}B$ and $A\hat{O}B$
6. Complete the table

Angle at circumference	Angle at centre

7. Write down your conjecture.
8. Repeat this process for two or three other circles to see if it agrees with your conjecture.

Does this relationship only hold for one angle at the circumference?

1. Draw a different circle, with three points on the one segment of the circle and two other points on the alternate segment.
2. Join similarly as in phase 1.
3. Measure the angles.
4. Complete the table

Point 1 Angle at circumference	
Point 2 Angle at circumference	
Point 3 Angle at circumference	
Angle at centre	

5. Write down your conjectures.
6. Repeat this process for two or three other circles to see if it agrees with your conjecture.

What if the point is not on the circumference of the circle?

1. Draw a different circle with point D **outside** the circle and point G **inside** the circle and any two points E and F on the circumference of the circle.
2. Join line DE, FD, EG and FG
3. Join radii EO and FO.
4. Measure the angles.

Angle outside	
Angle inside	
Angle at centre	

5. Write down your findings.
6. Does this correspond to your conjecture in Phase 1 & 2? What does this tell you?

What if the points are in the same segment?

1. Draw a different circle, with points P, Q and R in the **same** segment.
2. Join the radii.
3. Join the line segments.
4. Measure the angles.
5. What do you notice?
6. Measure the reflex angle at the centre.
7. Complete the table.

Angle at circumference	
Angle at centre	
Reflex angle at centre	

8. Write down your findings.
9. How does this influence your conjecture?

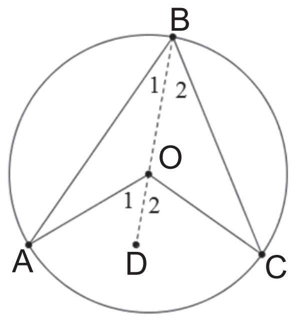
Summarise your findings/conjectures

Are there any restrictions?

Illustrate your understanding of the investigation by providing appropriate examples.

Can we prove our conjecture?

Use the diagram below in completing the statements and reason in attempting to prove one of the above conjectures.



STATEMENT	REASON
In ΔAOB, $OA = OB$	…………………………………
$\hat{O}_1 = \hat{B}_1 + \hat{A}$	…………………………………
But $\hat{A} = \hat{B}_1$	…………………………………
Therefore $\hat{O}_1 =$ …………………	
In ΔCOB $CO = BO$	……………………………
$\hat{O}_2 = \hat{B}_2 + \hat{C}$	…………………………………
But $\hat{C} = \hat{B}_2$	…………………………………………
Therefore $\hat{O}_2 =$ …………………	
Therefore $\hat{O}_1 + \hat{O}_2 =$ …………………	
$\hat{O}_1 + \hat{O}_2 = 2($…………………$)$	
$\Rightarrow A\hat{O}C =$ …………	

Rubric

ITEM	CRITERIA			
Presentation	Impression of a last-minute effort 0	Task is satisfactory 2	Task is neat, easy and a pleasure to follow; effort was made 5	
Construction	No constructions done 0	Most of the constructions neatly and correctly done 2	All constructions correctly done 3	Completed all constructions neatly and correctly done 5
Additional constructions	None done 0	At least two more circles were drawn, but incomplete 3	At least two more circles were drawn and completed 5	
Reading/ Measurements	No readings 0	Most readings done 2	All readings/ measurements correctly done 5	
Conclusion 1	No idea 0	Misinterpretation 2	Correctly completed 5	
Conclusion 2	No idea 0	Misinterpretation 2	Correctly completed 5	
Restrictions	No idea 0	Did not notice all the restrictions 2	All the restrictions were mentioned 5	
Examples	No examples 0	Inappropriate examples 2	Ample examples illustrating the conjectures 5	
Proving the conjecture	Did not attempt 0	Incomplete attempt 3	Completely correct 10	

TOTAL: 50

MARKING TOOL

Grade 11 Mathematics Investigation on Euclidean Geometry

MARKS: 50 **DUE DATE: ……………………**

ITEM	CRITERIA			
Presentation	Impression of a last-minute effort 0	Task is satisfactory 2	Task is neat, easy and a pleasure to follow; effort was made 5	
Construction	No constructions done 0	Most of the constructions neatly and correctly done 2	All constructions correctly done 3	Completed all constructions neatly and correctly done 5
Additional constructions	None done 0	At least two more circles weredrawn,but incomplete 3	At least two more circles were drawn and completed 5	
Reading/ Measurements	No readings 0	Most readings done 2	All readings/ measurements correctly done 5	
Conclusion 1	No idea 0	Misinterpretation 2	Correctly completed 5	
Conclusion 2	No idea 0	Misinterpretation 2	Correctly completed 5	
Restrictions	No idea 0	Did not notice all the restrictions 2	All the restrictions were mentioned 5	
Examples	No examples 0	Inappropriate examples 2	Ample examples illustrating the conjectures 5	
Proving the conjecture	Did not attempt 0	Incomplete attempt 3	Completely correct 10	

TOTAL: 50

PROJECT THEME: BEST FUNDING OPTIONS FOR SCHOOL PROJECTS

Key investigative question: What is the best option for your school to fund its computerisation project?

INSTRUCTIONS

This project is designed to be done by groups of at most three and at least two. Where the number in the class is not divisible by three, one or two groups must have two members and approach two financial institutions when tackling the second task described below. It is suggested that each group member approaches a different financial institution. However, it does **NOT** need to be different from the institutions approached by others in the group or by other groups. It is suggested that time be allocated to choosing financial institutions so that enough different companies are identified. Your teacher will provide you with a letter that you can use to make contact with the institutions.

TASK

Visit any two or three financial institutions such as banks and lending institutions and find out the following information:

- The financial institution's lending and investment rates for different amounts of up to R100 000.00
- The key terms and conditions of their loans.
- The methods they use to calculate their interest. (Please consider both compound interest and simple interest rates.)
- Factors affecting the changes in interest rates.

HINT: Design a small questionnaire that you will use to gather the information. Collect as many pamphlets as possible that deal with the financial institution's lending rates. Your questionnaire should have different sections as follows and should contain no more than 15 questions and no less than 10 questions:

SECTION A: GENERAL INFORMATION – In this section you aim at collecting general information about the institution and the respondents, hence fewer than 5 questions are still acceptable. You may also include multiple choice questions where the respondent may choose the answers. For example:

What type of a financial institution is this?

A. Lending B. Investment C. Both

SECTION B: LENDING AND BORROWING – In this section you will be aiming to collect the information with regard to the lending and investments rates of the company, the terms and conditions of loans and investments, and methods of calculating interest on loans and investments.

QUESTION 1

1.1 Design rates tables that show the difference between the lending and investment rates of the two financial institutions. You need to include values of money from R10 000.00 up to R100 000. 00 **(10)**

1.2 Draw a compound line graph using the ANNEXURE at the end of this question paper that illustrates the differences in lending rates for the two financial institutions. **(5)**

1.3 Draw up a suitable conclusion from the information **(5)**

[20]

QUESTION 2

Your school decides to buy twenty new sets of computers for the computer laboratory. The price of each computer set is R4 999.00. The school decides to pay 80% of the total cost of the computers in five years. The remaining 20% will be funded by the scrap value of the current set of computers.

2.1 Calculate the current scrap value of the computers (2)

2.2 The school needs advice on where to borrow the money. Advise, using calculations from the information collected in QUESTION 1, the best option for the school to fund their project and repay the money within five years. Generalise your solution such that the school needs to just enter an amount and calculate their best option. Use this formula to calculate the amount the school will repay after borrowing

$$P = \frac{r \times A}{1-(1+r)^{-n}}$$

Where, P = Payment Amount, A = Loan Amount r = Rate of Interest (compounded) n = q

Number of Payments (12)

2.3 The school decides to invest for the future computer sales after five years. The depreciation of the computers is at the rate of 12% per annum on a reducing balance method. The value of the new computers appreciates at a rate of 8% per annum on a straight line method.

2.3.1 Calculate the value of one computer at the end of five years (scrap value) (3)

2.3.2 Calculate the value of one **NEW** computer at the end of five years. (3)

2.3.3 If the school decides to set up a sinking fund for the twenty computers, calculate the value of the sinking fund. ***(Sinking Fund = Price of New Computers after 5 Years – Price of Computers after 5 Year*** (4)

2.3.4 With relevant calculations, advise the school on how to set up the sinking fund with one of the financial institutions. (8)

2.4 Draw up conclusions based on your findings. (3)

[35]

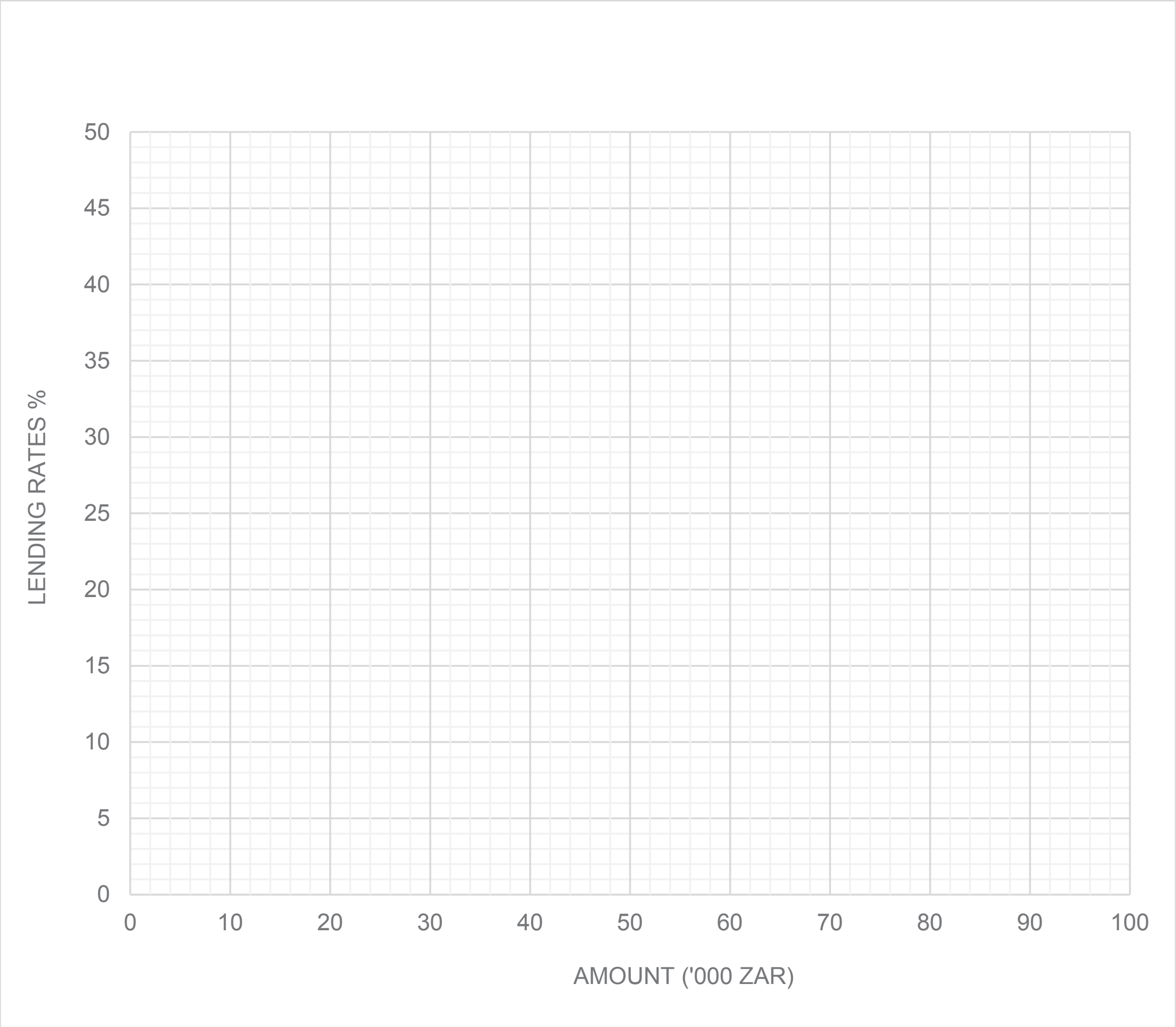
2.5 ***You are required to present your results to the class and show evidence that the data you collected were authentic. All results must be summarised on a chart .You shall be assessed using a rubric (See rubric below)***

Use the following as your marking guidelines for authenticity of data and data presentation (Question 2.5)

	[5]	**[3]**	**[2]**	**[0]**
Evidence of data collection	Evidence clear and in line with results	Evidence clear but not in line with results	Not very clear evidence	No evidence
Data authenticity	Data is clearly authentic	Data is partly authentic	Data authenticity unclear	Data NOT authentic
Chart	The chart was structured well and contains all the information in a clear format	The chart was properly structured but not all information is included	The chart structure is fairly good but with unclear information	No chart was presented and if any it had a very poor structure with lots of missing information
Presentation of results	Presentation was well done with explanations very clear	Presentation was fairly done with some unclear explanations	Presentation was not properly done and explanations unclear	Presentation poorly done and no explanations at all

[20]

NAME: ______________________________



GRAND TOTAL = 75 MARKS

QUESTION 1

1.4 **Data table**

Table heading ✓	(1)
Column heading ✓	(1)
Row heading ✓	(1)
Values evenly spread (from R10 000 to R 100 000) ✓ ✓ ✓	(3)
Correct data as from evidence ✓ ✓	(2)
Neatness of table ✓ ✓	(2)
	[10]

1.5 **Line graph**

Graph heading ✓

Correct plotting/ differences clearly shown/ neatness/ ✓ ✓ ✓ ✓ [5]

1.6 The conclusion is very relevant and summarises the graph ✓ ✓ ✓ ✓ ✓ (5)

OR

The conclusion is relevant but not summarising the graph ✓ ✓ ✓ (3)

OR

The conclusion is very irrelevant with little information from the graph ✓ (1)

[5]

QUESTION 2

2.1 Total value of computers = R 4999 x 20 = R 9998.00 ✓ (2)

Now scrap value = $\frac{20}{100}$ x R 9998.00 = R 19996.00 ✓

2.2 Money that needs to be borrowed = R 9998.00 - R 19996.00 = R 79984.00 ✓ ✓ (2)

Identifying the rate bracket in which R79984.00 falls ✓ ✓ (2)

Calculations of the loan repayment value for each financial institution.

The learner should calculate the total amount that the school is going to pay after 5 years ✓✓✓✓✓✓ (6)

Generalising the solution. (All percentages to be simplified to decimals) ✓✓ (2)

[12]

2.3

2.3.1 Value after 5 years = P $(1 - i)^n$ ✓

= R 4999.00 $(1 - 0.12)^5$ = R2 638.13 ✓✓ (3)

2.3.2 Value of new computer after 5 years

= R 4999.00 (1=5(0.08)) =R6998.60 ✓✓✓ (3)

2.3.3 ***(Sinking Fund = Price of New Computers after 5 Years – Price of Computers after 5 Years)***

= (R 69998.60 x 20) - (R 2638.13 x 20)
= R 87209.40

✓✓✓✓(4)

2.3.4 Money that needs to be invested = R87 209.40 ✓✓(2)
Calculations involving investment value for each financial institution.
Calculations should determine the institution that offers the best interest rates and will reap the required money within 5 yrs.

Generalising the solution. (All percentages to be simplified to decimals) ✓✓✓✓✓✓

(6)

2.5 Any meaningful conclusion that ranges from determining the cheapest way of investing money. ✓✓✓ (3)

[35]

2.5 Use the following as your marking guidelines for authenticity of data and data presentation.

[20]

	[5]	**[3]**	**[2]**	**[0]**
Evidence of data collection	Evidence clear and in line with results	Evidence clear but not inline with results	Not very clear evidence	No evidence
Data authenticity	Data is clearly authentic	Data is partly authentic	Data authenticity unclear	Data NOT authentic
Chart	The chart was structured well and contains all the information in a clear format	The chart was properly structured but not all information is included	The chart structure is fairly good but with unclear information	No chart was presented and if any it had a very poor structure with lots of missing information
Presentation of results	Presentation was well done with explanation very clear	Presentation was fairly done with some unclear explanation	Presentation was not properly done and explanations unclear	Presentation poorly done and no explanations at all

GRAND TOTAL = 75 MARKS

Grade 12

A. <u>INVESTIGATION</u>

<u>MARKS: 100</u>

<u>NUMBER PATTERNS</u>

<u>QUESTION 1</u>

1.1 Investigate the relationship between the Common difference, d and the Difference (D) between (the product of the first and the third terms) and (the square of the middle term) of any three consecutive numbers of a linear sequence.

<u>NOTE:</u> If the sequence is given by: *a*; *b*; *c*; . . . the ***Difference (*D) =** $\mathbf{b}^2 - \boldsymbol{a} \times \boldsymbol{c}$

- Use **FIVE** different linear sequences with a common difference, d of 1 and calculate the Difference, D as stated above.
  Step 1 (3)
- Write down your conjecture in words.
  Step 2 (2)
- Repeat Step 1: change the common difference, d between the terms to 2.
  Step 3 (3)
- Repeat Step 1: change the common difference, d between the terms to 3.
  Step 4 (3)
- Repeat Step 1: change the common difference, d between the terms to a number of your choice (NOT 1, 2 or 3).
  Step 5 (3)
- Write down your conjecture in words.
  Step 6 (2)
- Is this true in general? Prove your conjecture ! ! !
  Step 7 (4)

Overall presentation and variety of sequences used.

(10)

[30]

1.2 Investigate the Difference (D) between (the sum of the first and the third terms) and (two times the second/middle term) of any three consecutive numbers of a quadratic sequence.

Given: (1; 3; 6; . . . / 2; 4; 8 . . . / 2; 6; 12; 20; . . . / 3; 4; 8; 15; . . .)

<u>NOTE:</u> If the sequence is given by: a ; b ; c ;... the Difference **(D) = (a + c) – 2 x b**

- Use the given sequences and determine the Difference, D in each case.
  Step 1 (4)

- Compare your answers in Step 1 with the second difference for each sequence.
  Step 2 (4)

- What is your conjecture?
  Step 3 (2)

- Prove that this is **TRUE** in general.
  Step 4 (5)

[15]

<u>QUESTION 2</u> – PATIOS

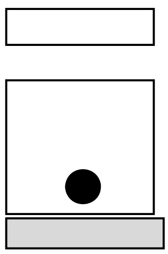
A company that paves patios uses the following types of marble blocks:

Light rectangles (B_1)

Square slabs (B_2)

Circular pebble (B_3)

Dark rectangle (B_4)

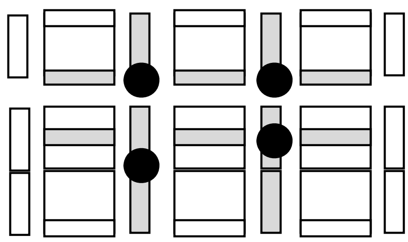


2.1 SQUARE PATIOS (*n* x *n*)

Example: The diagram alongside shows such a **3 x 3 square patio**.

This patio required:

12 light rectangles
9 square slabs
4 circular pebbles
12 dark rectangles



Clearly show all your work/sketches to investigate how many of each type of block will be required to lay a:

You can use more marble blocks to establish a pattern for various types of square patios.

2.1.1 2 x 2 square patio
2.1.2 4 x 4 square patio
2.1.3 5 x 5 square patio

- **Sketches of 2 x 2, 3 x 3, 4 x4, 5 x 5, *n* x *n* patios.**
  5 x (2) = (10)
- **Record your findings in the table provided.**
  (4)

2.1.4 Use the data recorded in the table, or otherwise, to determine the formulae that will represent the number of each block that will be required for a $n \times n$ square patio. **(12)**

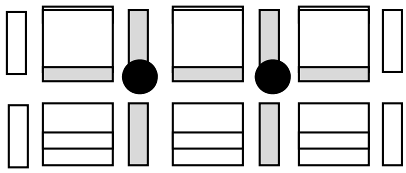
2.1.5 How many of each of the types of blocks will be required to make a 10 x 10 square patio? **(4)**

[30]

2.2 RECTANGULAR PATIOS (*m* x *n*)

Example: The diagram alongside shows such a **2 x 3 rectangular patio**.
This patio required:

10 light rectangles
6 square slabs
2 circular pebbles
7 dark rectangles



Clearly show all your work/sketches to investigate how many of each type of block will be required to lay a:

2.2.1 2 x 3 rectangular patio
2.2.2 3 x 5 rectangular patio
2.2.3 5 x 8 rectangular patio

- **Sketches of 2 x 3, 3 x 5, 5 x 8, *m* x *n* patios. 4 x (2) = (8)**
- **Record your findings in the table provided. (4)**

2.2.4 Use the data recorded in the table, or otherwise, to determine the formulae that will represent the number of each block that will be required for a $m \times n$ rectangular patio. **(8)**

2.2.5 How many of each of the types of blocks will be required to make a 8 x 13 rectangular patio? **(5)**

[25]

<u>Total 100……………………………………………….</u>

CONSOLIDATION SHEET

Complete the following table for the number of each type of block required for every type of square patio.

TABLE (n x n)

Block type	Patio type				
	2 × 2	3 × 3	4 × 4	5 × 5	$n \times n$
Light rectangle blocks B_1		.... = 12			
Square Slabs B_2		.... = 9			
Circular Pebbles B_3		.... = 4			
Dark rectangle blocks B_4		.... = 12			

TABLE (m x n)

Block type	Patio type			
	2 × 3	3 × 5	5 × 8	$m \times n$
Light rectangle blocks B_1	.... = 10			
Square Slabs B_2	.... = 6			
Circular Pebbles B_3	.... = 2			
Dark rectangle blocks B_4	.... = 7			

MARKING TOOL FOR INVESTIGATION
Question 1.1 RUBRIC

Marks →	1	2	3	4	5
Layout	Poor presentation	Average presentation	Good presentation with some logic	Very good presentation with clear logic	Excellent presentation with clear logic
Sequences used	Increasing sequence, using positive numbers	Increasing sequence, using positive numbers including larger numbers	Increasing sequence, using positive and negative numbers	Increasing sequence, using positive, negative numbers and fractions	Decreasing sequence, using positive / negative / fractions

(10)

A. *** Note that the learner's sequences might/will differ from those in the marking tool.***

*** Learner may use ANY five linear sequences. ***
*** Will be penalised in rubric for lack of variety. ***

Step 1 – any five linear sequences with a common difference of 1 ✓✓✓ (3)

e.g. 2 ; 3 ; 4 ; . . . ==> d = 1 and ==> $D = 3^2 - (2\times4)$ = 1

15 ; 16 ; 17 ; . . . ==> d = 1 and ==> $D = 16^2 - (15\times17)$ = 1

-5 ; - 4 ; -3 ; . . . ==> d = 1 and ==> $D = (-4)^2 - (-5)\times(-3)$ = 1

$\frac{5}{2}$; $\frac{7}{2}$; $\frac{9}{2}$; … ==> d = 1 and ==> $D = \left(\frac{7}{2}\right)^2 - \left(\frac{5}{2}\right)\left(\frac{9}{2}\right)$ = 1

8 ; 7 ; 6 ; . . . ==> d = - 1 and ==> $D = 7^2 - (8\times6)$ = 1

Step 2 – Common difference, d is the same as the calculated difference, D and equals to 1,
except for decreasing sequence. Then it seems that $D = -d$ or $D = d^2$.
✓✓ (2)

Step 3 – any five linear sequences with a common difference of 2 ✓✓✓(3)

e.g. 2 ; 4 ; 6 ; . . . ==> d = 2 and ==> $D = 4^2 - (2\times6) = 4$

15 ; 17 ; 19 ; . . . ==> d = 2 and ==> $D = 17^2 - (15\times19) = 4$

-5 ; -3 ; -1 ; . . . ==> d = 2 and ==> $D = (-3)^2 - (-5)\times(-1) = 4$

$\frac{3}{2}$; $\frac{7}{2}$; $\frac{11}{2}$; ... ==> d = 2 and ==> $D = \left(\frac{7}{2}\right)^2 - \left(\frac{3}{2}\right)\left(\frac{11}{2}\right) = 4$

12 ; 10 ; 8 ; . . . ==> d = -2 and ==> $D = 10^2 - (12\times8) = 4$

Step 4 – any five linear sequences with a common difference of 3 ✓✓✓(3)

e.g. 2 ; 5 ; 8 ; . . . ==> d = 3 and ==> $D = 5^2 - (2\times8) = 9$

15 ; 18 ; 21 ; . . . ==> d = 3 and ==> $D = 18^2 - (15\times21) = 9$

-5 ; -2 ; 1 ; . . . ==> d = 3 and ==> $D = (-2)^2 - (-5)\times(1) = 9$

$\frac{1}{3}$; $\frac{10}{3}$; $\frac{19}{3}$; ... ==> d = 3 and ==> $D = \left(\frac{10}{3}\right)^2 - \left(\frac{1}{3}\right)\left(\frac{19}{3}\right) = 9$

12 ; 9 ; 6 ; . . . ==> d = -3 and ==> $D = 9^2 - (12\times6) = 9$

Step 5 – any five linear sequences with a common difference of your choice (not 1, 2 or3) ✓✓✓(3)

e.g. 2 ; 7 ; 12 ; . . . ==> d = 5 and ==> $D = 7^2 - (2\times12) = 25$

15 ; 20 ; 25 ; . . .==> d = 5 and ==> $D = 20^2 - (15\times25) = 25$

-12 ; -7 ; -2 ; . . .==> d = 5 and ==> $D = (-7)^2 - (-12)\times(-2) = 25$

$\frac{2}{5}$; $\frac{7}{5}$; $\frac{12}{5}$; ... ==> d = 5 and ==> $D = \left(\frac{7}{5}\right)^2 - \left(\frac{2}{5}\right)\left(\frac{12}{5}\right) = 25$

17 ; 12 ; 7 ; . . . ==> d = -5 and ==> $D = 12^2 - (17\times7) = 25$

Step 6: $D = d^2$. The calculated difference, D is equal to the square of the common difference, d. ✓✓(2)

Step 7: Yes

$a;\ a+d;\ a+2d$ **(if you see the general sequence)** ✓

Substitution of the general term the formula: $L = b^2 - ac$

$D = (a+d)^2 - a(a+2d)$

$D = a^2 + 2ad + d^2 - a^2 - 2ad$

✓ **Substitution;** ✓ **Removing**

$D = d^2$

Yes

✓ d^2

(4)

[30]

B. Step1:

For	**1; 3; 6; ....**	$D = (1+6) - 2(3) = 1$	✓	**(1)**
For	**2; 4; 8; ....**	$D = (2+8) - 2(4) = 10 - 8 = 2$	✓	**(1)**
For	**2; 6; 12; 20; ....**	$D = (2+12) - 2(6) = 14 - 12 = 2$	✓	**(1)**
For	**3; 4; 8; 15; ....**	$D = (3+8) - 2(4) = 11 - 8 = 3$	✓	**(1)**

B. Step 2:

For **1 3 6**

2 3

1 ✓ **(1)**

For **2 4 8**

2 4

2 ✓ **(1)**

For **2 6 12 20**

4 6 8

2 2 ✓ **(1)**

For **3 4 8 15**

1 4 7

3 3 ✓ **(1)**

Step 3:
For each sequence, the calculated difference (D) is the same as the second difference of the sequence.

✓✓ **(2)**

Step 4:

For $a + b + c$; $4a + 2b + c$; $9a + 3b + c$

$$D = [(a + b + c) + (9a + 3b + c)] - 2(4a + 2b + c) \quad ✓✓$$
$$D = 10a + 4b + 2c - 8a - 4b - 2c$$
$$D = 2a$$

Furthermore, generally for a quadratic sequence:
$T_n = an^2 + bn + c$

	First differences	Second differences
$T_1 = a + b + c$		
	$3a + b$	
$T_2 = 4a + 2b + c$		$2a$
	$5a + b$	✓✓
$T_3 = 9a + 3b + c$		$2a$
	$7a + b$	
$T_4 = 16a + 4b + c$		

Yes, it is true. ✓ **(5)**

[15]

SOLUTION TO PATIOS

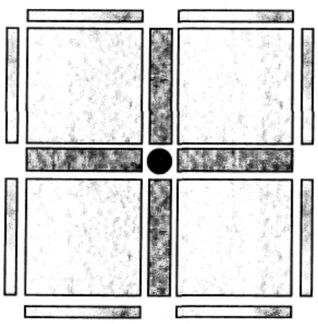
Block type	Patio type 2 × 2	3 × 3	4 × 4	5 × 5	$n \times n$
Light rectangle blocks B_1	2 x 4 = **8**	3 x 4 = **12**	4 x 4 = **16**	5 x 4 = **20**	n x 4 = **4n**
Square Slabs B_2	2 x 2 = 2^2= **4**	3 x 3 = 3^2 = **9**	4 x 4 = 4^2 = **16**	5 x 5 = 5^2= **25**	n x n^2 = n
Circular Pebbles B_3	$(2 - 1)^2$ = **1**	$(3 - 1)^2$= **4**	$(4 - 1)^2$ = **9**	$(5 - 1)^2$= **16**	$(n - 1)^2$
Dark rectangle blocks B_4	2[2(2 – 1)] = **4**	2[3(3 – 1)] = **12**	2[4(4 – 1)] = **24**	2[5(5 – 1)] = **40**	$2[n(n - 1)]$ = $2(n^2 - n)$ = $2n^2 - 2n$

(✓) (✓) (✓) (✓)
(1) (1) (1) (1)

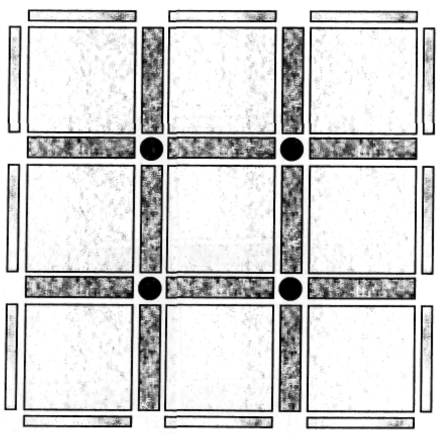
Patio type		
$\boldsymbol{n \times n}$		
$n \times 4 = \mathbf{4}\boldsymbol{n}$	(✓)(✓)	**(2)**
$n \times n = \boldsymbol{n}^2$	(✓)(✓)	**(2)**
$(n - 1)^2$	(✓)(✓)(✓)(✓)	**(4)**
$2[n(n - 1)]$ $= 2(n^2 - n)$ $= \mathbf{2}\boldsymbol{n}^2 - \mathbf{2}\boldsymbol{n}$	(✓)(✓)(✓)(✓)	**(4)**

SKETCHES ***(two marks depends on neatness and correctness of shapes)***

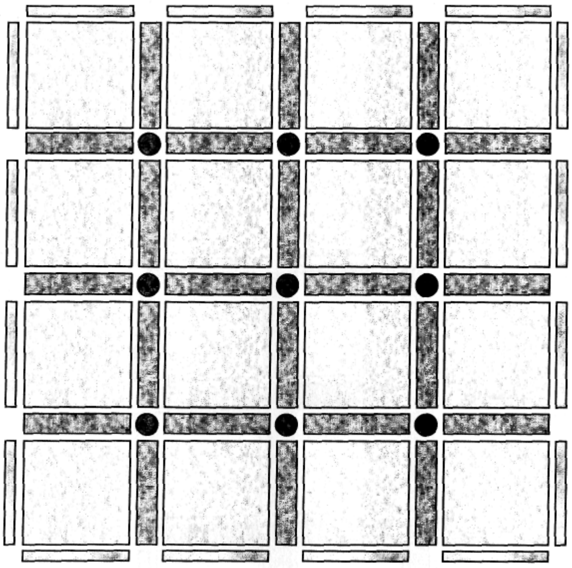
2 x 2 (✓✓) (2)



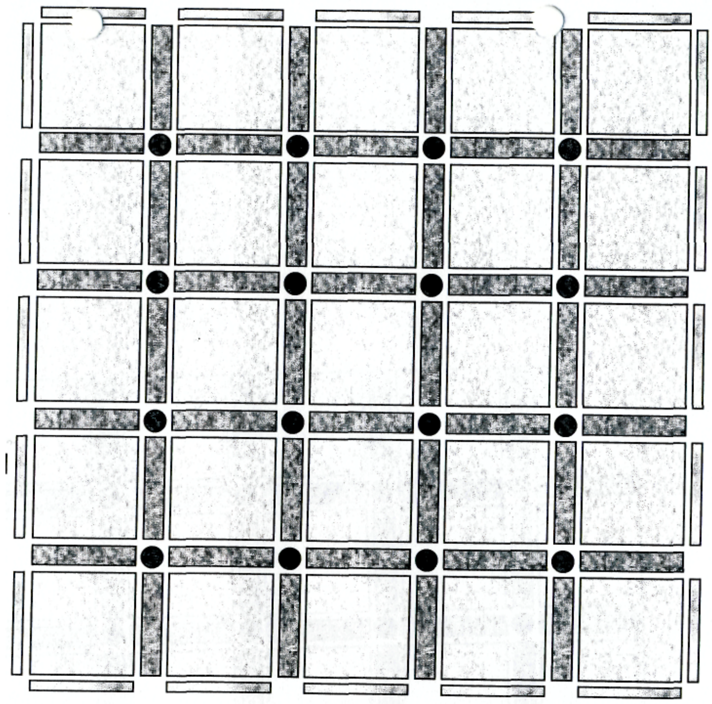
3 x 3 (✓✓) (2)



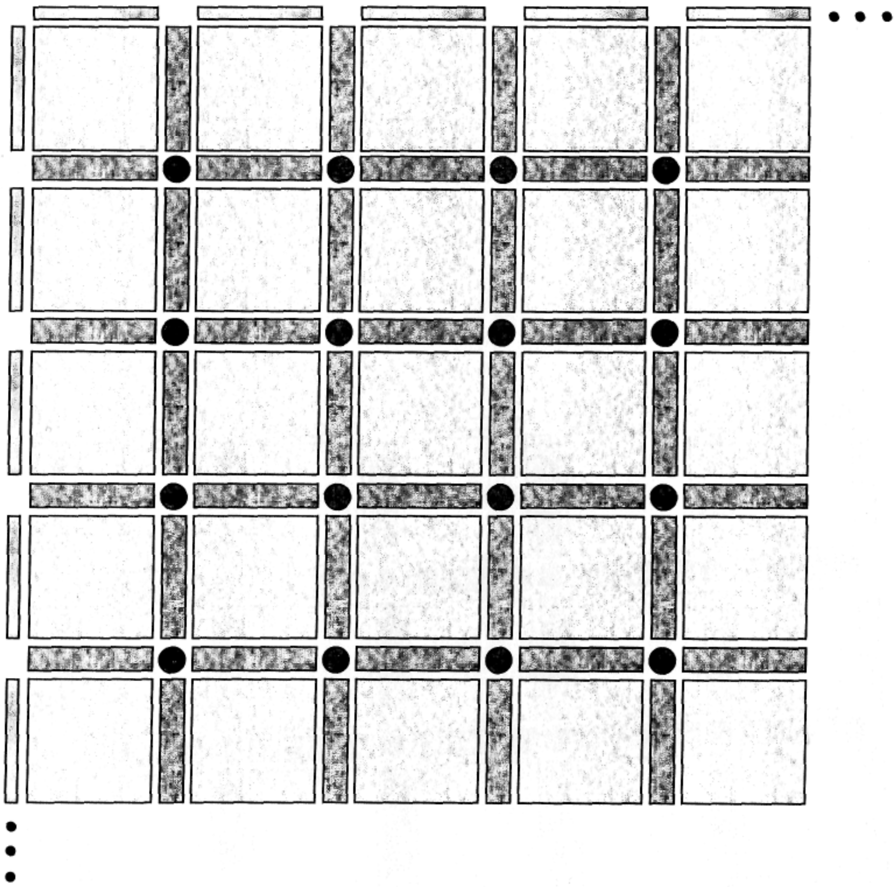
4 x 4 (✓✓) (2)



5 x 5 (✓✓) (2)



n* x *n (✓✓) (2)



2.5

Light rectangle blocks in 10 x 10 square patio = $4(10) = 40$ (✓) (1)

Square slabs in a 10 x 10 square patio = $(10)^2 = 100$ (✓) (1)

Circular pebbles in a 10 x 10 square patio = $(10 - 1)^2 = 81$ (✓) (1)

Dark rectangle blocks in a 10 x 10 square patio = $2(10)^2 - 2(10) = 180$ (✓) (1)

[30]

B. INVESTIGATION (COMPOUND ANGLE IDENTITY)

If α and β are angles, the expression $\alpha - \beta$ represents the difference between angles. Investigate whether the $\cos(\alpha - \beta)$ is distributive under subtraction and derive its compound angle identity.

Part 1

Question 1.

Calculate $\cos(180^\circ - 150^\circ)$ without using a calculator

a) <u>Mpho's cancelled solution</u>

$\cos(180^\circ - 150^\circ)$

$= \cos 30^\circ$ step 1

$= \frac{\sqrt{3}}{2}$ step2

b) <u>Mpho's second solution</u>

$\cos(180^\circ - 150^\circ)$

$= \cos 180^\circ - \cos 150^\circ$ step 3

$= -1 - \cos(90^\circ + 60^\circ)$ step 4

$= -1 - (-\sin 60^\circ)$ step 5

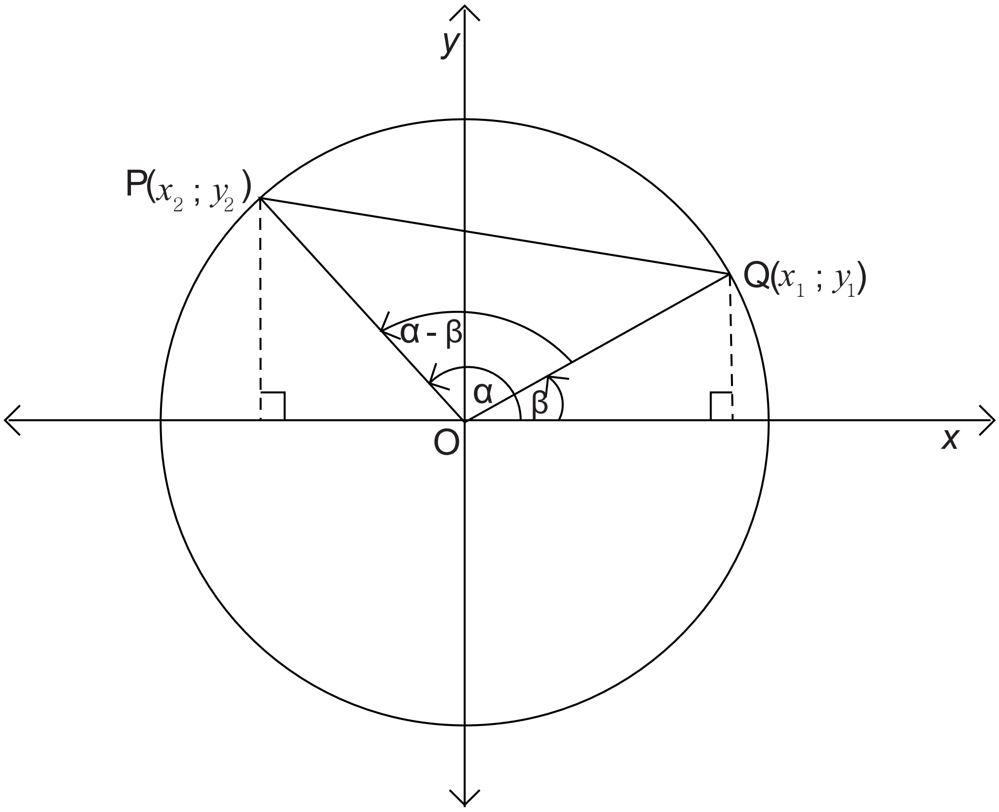
$= -1 + \frac{\sqrt{3}}{2}$ step 6

1. Which of the two solutions is correct? (1)
2. Identify the error in the incorrect solution. (1)
3. What can you conclude in relation to the distributive property in trigonometric ratio $\cos(\alpha - \beta)$? (2)

Part 2

Derivation of compound angle identity for $\cos(\alpha - \beta)$

4. Now consider the diagram below which is a unit circle with α and β the angles formed with the positive *x*-axis



4.1	Write the coordinates of Q in terms of β	(2)
4.2	Write the coordinates of P in terms of α	(2)
4.3	Use the distance formula to calculate the length of PQ in terms of α and β	(4)
4.4	Use the cosine rule in ΔPOQ to calculate the length of PQ in terms of α and β	(4)
4.5	Equate the value of PQ in 4.3 and 4.4 and make $\cos(\alpha - \beta)$ the subject of the formula.	(3)
4.6	Use the results obtained in 4.5 to show that: $\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$	(4)
4.7	Use co-function and expansion of $\cos[(90^\circ - \alpha) - \beta]$ to show that: $\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$	(4)
4.8	Use 4.7 to show that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$	(3)
4.9	Use 4.6 to expand $\cos 2\alpha$	(3)
4.10	Use 4.7 to expand $\sin 2\beta$	(3)

Part 3

5. Use double-angle identities and special angles to evaluate the following without using a calculator

5.1 $\cos(45^\circ - 30^\circ)$ (3)

5.2 $\sin 105^\circ$ (3)

5.3 $\cos^2 225^\circ - \sin^2 225^\circ$ (3)

6. Simplify: $\cos 20^\circ \cos 320^\circ - \sin 160^\circ \sin 40^\circ$ without using a calculator (5)

TOTAL 50 Marks

SOLUTIONS (COMPOUND ANGLE IDENTITY)

Part 1

1. Solution no 2 ✓ (1)
2. Error is in step 3 $\cos(180^\circ - 150^\circ) = \cos 180^\circ - \cos 150^\circ$ ✓(1)
3. $\cos(\alpha - \beta) \neq \cos\alpha - \cos\beta$ ✓✓ (2)

Part 2

4.1 $x_1 = \cos\beta$ ✓

$y_1 = \sin\beta$ ✓ (2)

$\therefore Q(\cos\beta\ ;\ \sin\beta)$

4.2 $x_2 = \cos\alpha$ ✓

$y_2 = \sin\alpha$ ✓ (2)

$\therefore P(\cos\alpha\ ;\ \sin\alpha)$

4.3 $PQ = \sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2}$ ✓

$= \sqrt{\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta}$ ✓

$= \sqrt{(\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$

$= \sqrt{1 + 1 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$ ✓

$= \sqrt{2 + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$ ✓ (4)

4.4 $PQ^2 = PO^2 + OQ^2 - 2\,PO \cdot OQ\ \cos(\alpha - \beta)$ ✓

$= 1 + 1 - 2(1)(1)\cos(\alpha - \beta)$ ✓

$= 2 - 2\cos(\alpha - \beta)$ ✓

$PQ = \sqrt{2 - 2\cos(\alpha - \beta)}$ ✓ (4)

4.5 $\sqrt{2 - 2\cos(\alpha - \beta)} = \sqrt{2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)}$ ✓

$\therefore 2 - 2\cos(\alpha - \beta) = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$ ✓

$\therefore \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ ✓ (3)

4.6 $\cos(\alpha + \beta) = \cos(\alpha - (-\beta))$ ✓

$= \cos\alpha\ \cos(-\beta) + \sin\alpha\sin(-\beta)$ ✓

$= \cos\alpha\cos\beta + \sin\alpha(-\sin\beta)$ ✓

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$ ✓ (4)

4.7 $\cos[(90^\circ - \alpha) - \beta] = \cos(90^\circ - (\alpha - \beta)) = \sin(\alpha - \beta)$ ✓

$\sin(\alpha - \beta) = \cos(90^\circ - \alpha)\cos\beta + \sin(90^\circ - \alpha)\sin(-\beta)$ ✓

$= \sin\alpha\cos\beta + \cos\alpha(-\sin\beta)$ ✓

$= \sin\alpha\cos\beta - \cos\alpha\sin\beta$ ✓ (4)

4.8 $\sin(\alpha-\beta)=\sin(\alpha+(-\beta))$ ✓

$=\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta)$

$=\sin\alpha\cos\beta+\cos\alpha(-\sin\beta)$ ✓

$=\sin\alpha\cos\beta-\cos\alpha\sin\beta$ ✓ (3)

4.9 $\cos 2\beta=\cos(\beta+\beta)$ ✓

$=\cos\beta\ \cos\beta-\sin\beta\sin\beta$ ✓

$=\cos^2\beta-\sin^2\ \beta$ ✓ (3)

4.10 $\sin 2\alpha=\sin(\alpha+\alpha)$ ✓

$=\sin\alpha\cos\alpha+\cos\alpha\sin\alpha$

$=2\sin\alpha\cos\alpha$ ✓ (3)

Part 3

5.1 $\cos(45^\circ-30^\circ)=\cos 45^\circ\cos 30^\circ+\sin 45^\circ\sin 30^\circ$ ✓

$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}$ ✓

$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}$

$=\frac{1+\sqrt{3}}{2\sqrt{2}}$ ✓ (3)

5.2 $\sin 105^\circ=\ \sin(60^\circ+45^\circ)$ ✓

$=\sin 60^\circ\cos 45^\circ+\cos 60^\circ\sin 45^\circ$

$=\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}+\frac{1}{2}\times\frac{1}{\sqrt{2}}$ ✓

$=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}$

$=\frac{1+\sqrt{3}}{2\sqrt{2}}$ ✓ (3)

Or

$\sin 105^\circ=\sin(180^\circ-75^\circ)$ ✓

$=\sin 75^\circ=\sin(45^\circ+30^\circ)$

$=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$

$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}$ ✓

$$=\frac{1+\sqrt{3}}{2\sqrt{2}}$$ ✓(3)

5.3 $\cos^2 22{,}5^\circ - \sin^2 22{,}5^\circ = \cos 22{,}5^\circ \cdot \cos 22{,}5^\circ - \sin 22{,}5^\circ \cdot \sin 22{,}5^\circ$ ✓

$= \cos(22{,}5^\circ + 22{,}5^\circ)$ ✓

$= \cos 45^\circ = \frac{1}{\sqrt{2}}$ ✓ (3)

6

$\cos 20^\circ \cos 320^\circ - \sin 160^\circ \sin 40^\circ = \cos 20^\circ \cdot \cos(360^\circ - 40^\circ) - \sin(180^\circ - 20^\circ) \cdot \sin 40^\circ$ ✓

$= \cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ$ ✓

$= \cos(20^\circ + 40^\circ)$ ✓

$= \cos 60^\circ$ ✓

$= \frac{1}{2}$ ✓ (5)

Total **50** Marks

C. INVESTIGATION GR. 12 (Sum & Product of roots).

To investigate the relationship between the roots of a quadratic equation and the equation

1. **Relationship between a and the roots, y-intercept and gradient of the equation** $y = ax^2 + bx + c$

1.1 Draw the following quadratic equations on the same system of axes.

1.1.1 $y = x^2 - 1$
1.1.2 $y = 2(x^2 - 1) = 2x^2 - 2$
1.1.3 $y = 3(x^2 - 1)$
1.1.4 $y = 0{,}5(x^2 - 1)$

1.2 Draw and solve the following quadratic equations on the same system of axes.

1.2.1 $y = x^2 + x - 6$
1.2.2 $y = 2(x^2 + x - 6) = 2x^2 + 2x - 12$
1.2.3 $y = 3(x^2 + x - 6)$
1.2.4 $y = 0{,}5(x^2 + x - 6)$

CONCLUSION (in your own words)

…………………………………………………………………………………………
…………………………………………………………………………………………
…………………………………………………………………………………………

…………………………………………………………………………………………
…………………………………………………………………………………………

2. **Relationship between sum of roots, product of roots and coefficients of the quadratic equation.**

2.1 Solve the equations below and complete the following table:

	Equation	Roots	Sum of roots	Product of roots
2.1.1	$x^2 + 7x + 12 = 0$			
2.1.2	$x^2 + 5x + 6 = 0$			
2.1.3	$x^2 + x - 20 = 0$			

2.1.4	$2x^2 - 5x - 3 = 0$			
2.1.5	$3x^2 + 5x + 1 = 0$			
2.1.6	$ax^2 + bx + c = 0\ (a \neq 0)$			

2.2 If the roots of a quadratic equation are α and β, then the sum would be ……….. and the product …………

2.3 If the quadratic equation is $ax^2 + bx + c = 0,\ a \neq 0$, then in terms of the roots it can also be written as $a(x-\alpha)(x-\beta) = 0, a \neq 0$

2.4 $\therefore\ ax^2 + bx + c = a(x-\alpha)(x-\beta)$

$$= a(x^2 - \alpha x - \beta x + \alpha\beta)$$
$$= ax^2 - a\alpha x - a\beta x + a\alpha\beta$$
$$= ax^2 - a(\alpha+\beta)x + a.\alpha\beta$$

$\therefore b = -a(\alpha+\beta)$ and $c = a.\alpha\beta$

$(\boldsymbol{\alpha} + \boldsymbol{\beta}) = -\frac{\boldsymbol{b}}{\boldsymbol{a}}$ $\qquad \boldsymbol{\alpha\beta} = \frac{\boldsymbol{c}}{\boldsymbol{a}}$

3. Applications

3.1 Write ***a*** quadratic equation whose roots are -3 and $\frac{1}{2}$. Write another equation with the same roots.

3.2 Find a quadratic equation with roots 2 and -5. (Note: there are infinity many solutions to this problem.)

3.3 The roots of the equation $3x^2 - 10x - 8 = 0$ are α and β.

3.3.1 Find the values of $\alpha + \beta$ and $\alpha\beta$.

3.3.2 Find a quadratic equation with roots 3α and 3β.

3.3.3 Find a quadratic equation with roots $\alpha + 2$ and $\beta + 2$

3.4 The roots of the equation $x^2 - 7x + 15 = 0$ are α and β.
Find the quadratic equation with roots α^2 and β^2.

4. What about **polynomials such as** $f(x) = ax^4 + bx^3 + cx^2 + ...$

$$f(x) = a(x-p)(x-q)(x-r)\ ...$$

Then p, q, r, etc. are the roots (where the polynomial equals zero)

Expand the factors:

$$a(x-p)(x-q)(x-r) = ax^3 - a(p+q+r)x^2 + a(pq+pr+qr)x - a(pqr)$$

Cubic: ax^3 $+ bx^2$ $+ cx$ $+ d$

Expanded: $ax^3 - a(p+q+r)x^2 + a(pq+pr+qr)x - a(pqr)$

$$\therefore\ -a(p+q+r) = b$$

$$\therefore p+q+r = -\frac{b}{a}$$

and $-apqr = d,$ so $pqr = -\frac{d}{a}$

CONCLUSION:

Adding the roots gives ………………. complete (exactly the same as the Quadratic)

Multiplying the roots gives ……………….. (similar to $+\frac{c}{a}$ for quadratic)

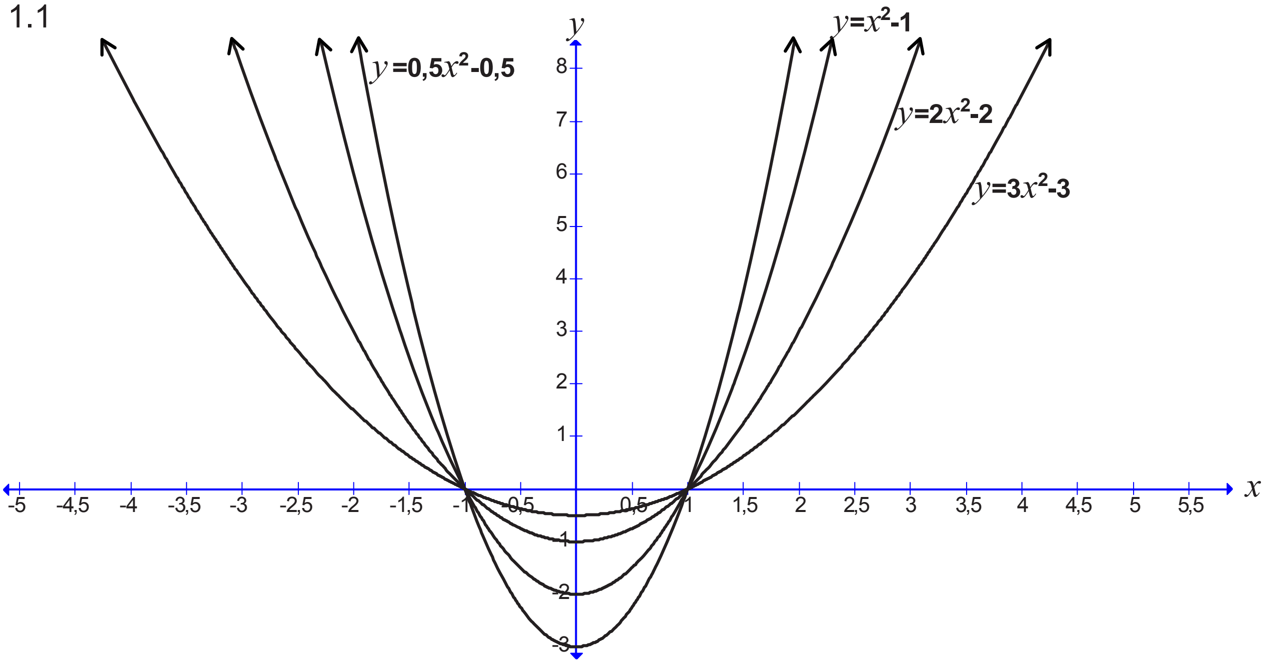
We also get $pq + pr + qr = \frac{c}{a}$

Higher polynomials: In general:

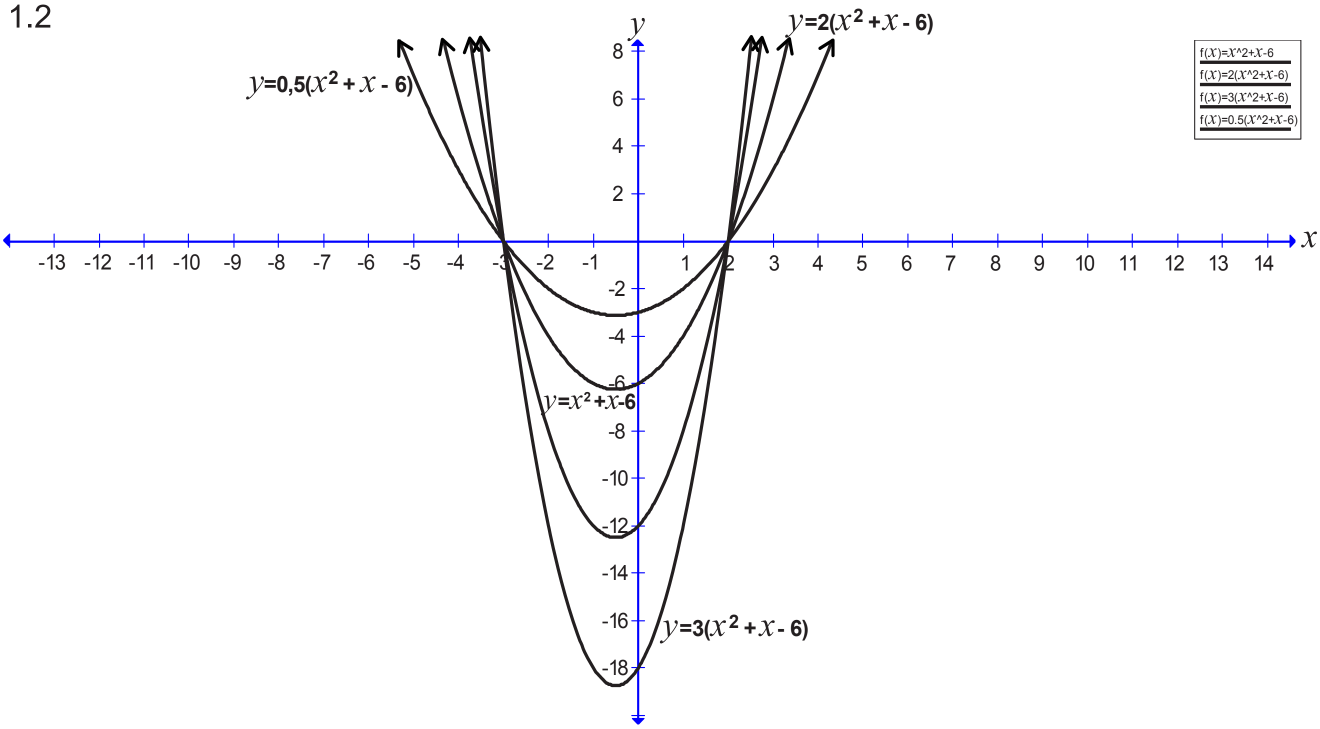
- **Adding the roots gives**: $-\frac{b}{a}$
- **Multiplying the roots gives (where z is the constant at the end):**
  $\frac{z}{a}$ **(for even degree polynomials like quadratics)**

INVESTIGATION GR. 12 Sum and product of roots (MEMO)

1.1



1.2



1.3 CONCLUSION:……………………………………………………………………………………

……………………………………………………………………………………………………

2.1

	Equation	Roots	Sum of roots	Product of roots
2.1.1	$x^2 + 7x + 12 = 0$	$-4;\ -3$	-7	$+12$
2.1.2	$x^2 + 5x + 6 = 0$			
2.1.3	$x^2 + x - 20 = 0$			
2.1.4	$2x^2 - 5x - 3 = 0$			
2.1.5	$3x^2 + 5x + 1 = 0$			
2.1.6	$ax^2 + bx + c = 0\ (a \neq 0)$			

2.2 Sum of roots: Product of roots:

3.1 $x^2 + \frac{5}{2}x - \frac{3}{2} = 0$ or $2x^2 + 5x - 3 = 0$

3.2 $x^2 + 3x - 10 = 0$ or $2x^2 + 6x - 20 = 0$

3.3.1 $\alpha + \beta = \frac{10}{3}$ and $\alpha\beta = -\frac{8}{3}$

3.3.2 $x^2 - 10x - 24 = 0$ or $2x^2 - 20x - 48 = 0$

3.3.3 $3x^2 - 22x + 24 = 0$

3.4 $x^2 - 19x + 225 = 0$

4. Conclusion: ………………………………………………………………………..

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