

Mathematics

Grade 6

Learner Book



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Mathematics Learner Book Grade 6

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Authors:

Piet Human, Alwyn Olivier, Erna Lampen, Amanda le Roux,
Caroline Long, Chris Human

Contributors:

Malcolm Samuel Alexander, Joseph James Gordon, Funaki Junko,
John Bob Kolokoto, Nombuso Makhaza, Sylvia Sindiswa Mcosana, Thulisizwe Msomi,
Victor Siphon Mthombeni, Cebisa Faith Mtumtum, Basil Sakoor, David Sekao,
Nomvuyo Maureen Thobela, Patricia Whitten, Roselinah Sizane Zwane, Millard Zweni

Text design: Mike Schramm

Layout and typesetting: Anton Stark, Lebone Publishing Services

Illustrations and computer graphics:

Leonora van Staden; Piet Human; Lisa Steyn; Zhandré Stark;
Melany Pietersen (pp. 73, 74 and 85)

Cover design and illustration: Leonora van Staden and Piet Human

Photographer: Martin van Niekerk (unless stated otherwise)

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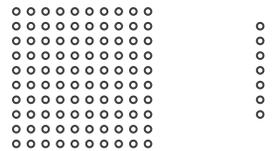
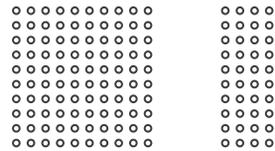
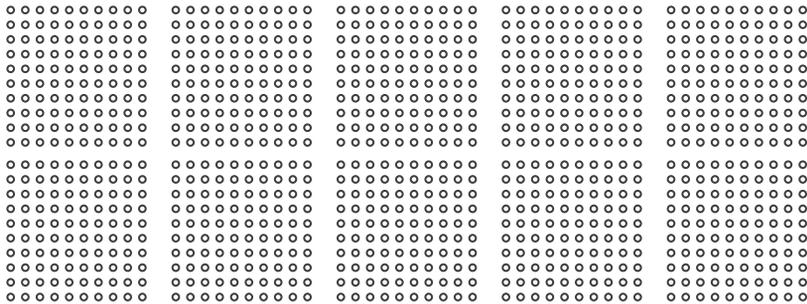
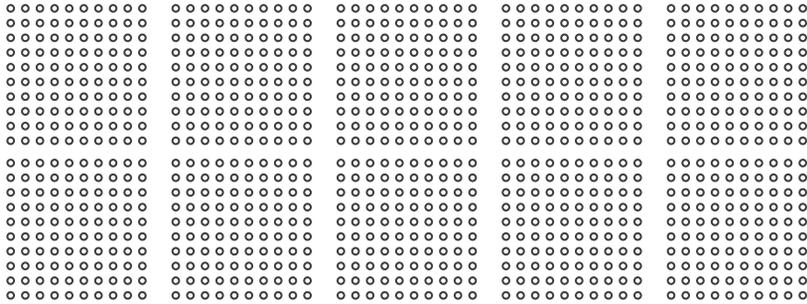
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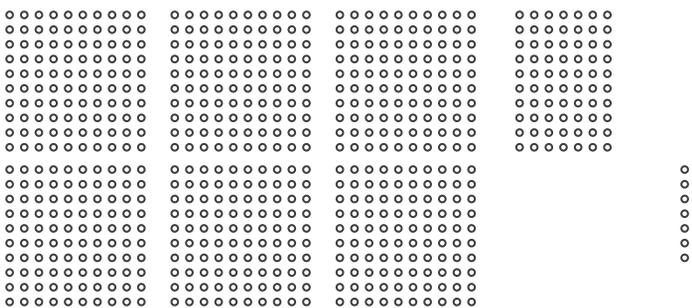
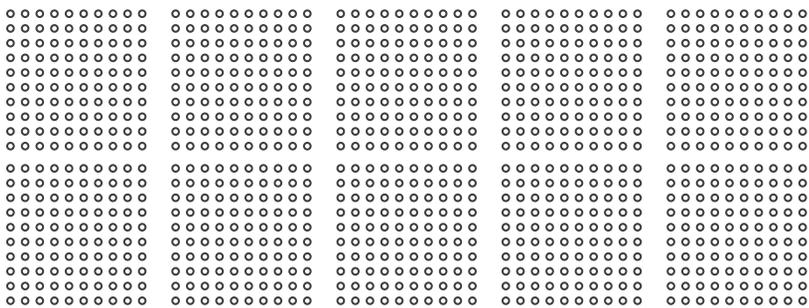
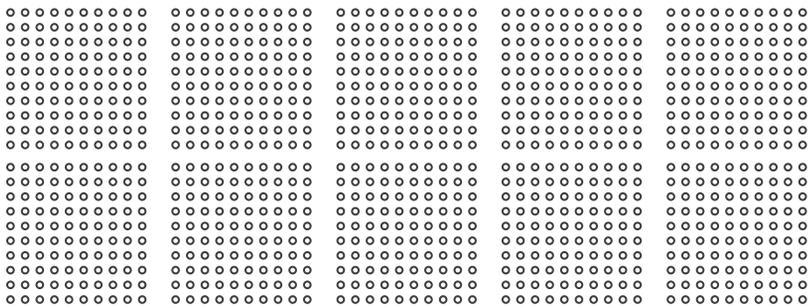
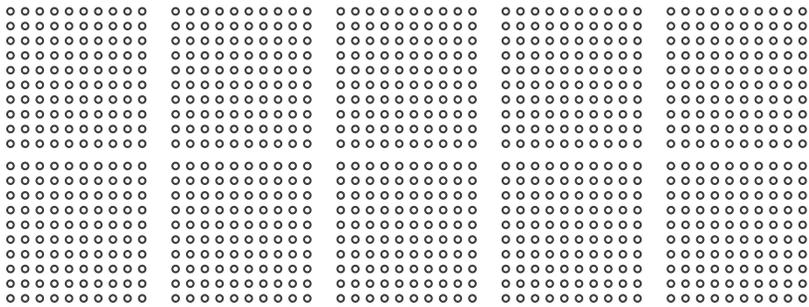
1.1 Count and represent numbers

1. How many rings are shown below?



- 2. How many more rings are needed to make up 3 000?
- 3. How many more rings are needed to make up 10 000?

4. How many rings are shown below?



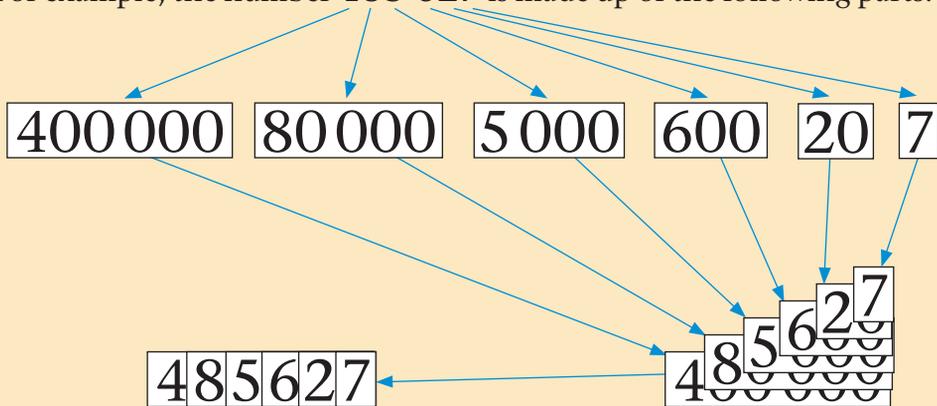
5. How many more rings are needed to make up 4 000?

6. How many more rings are needed to make up 10 000?

1.2 The place value parts of whole numbers

Whole numbers are made up of parts that may be called **place value parts**.

For example, the number 485 627 is made up of the following parts:



If you write the parts on pieces of paper, you can put them on top of each other to form the number symbol, as shown above. Notice how the zeros of the various parts are hidden in the **number symbol**.

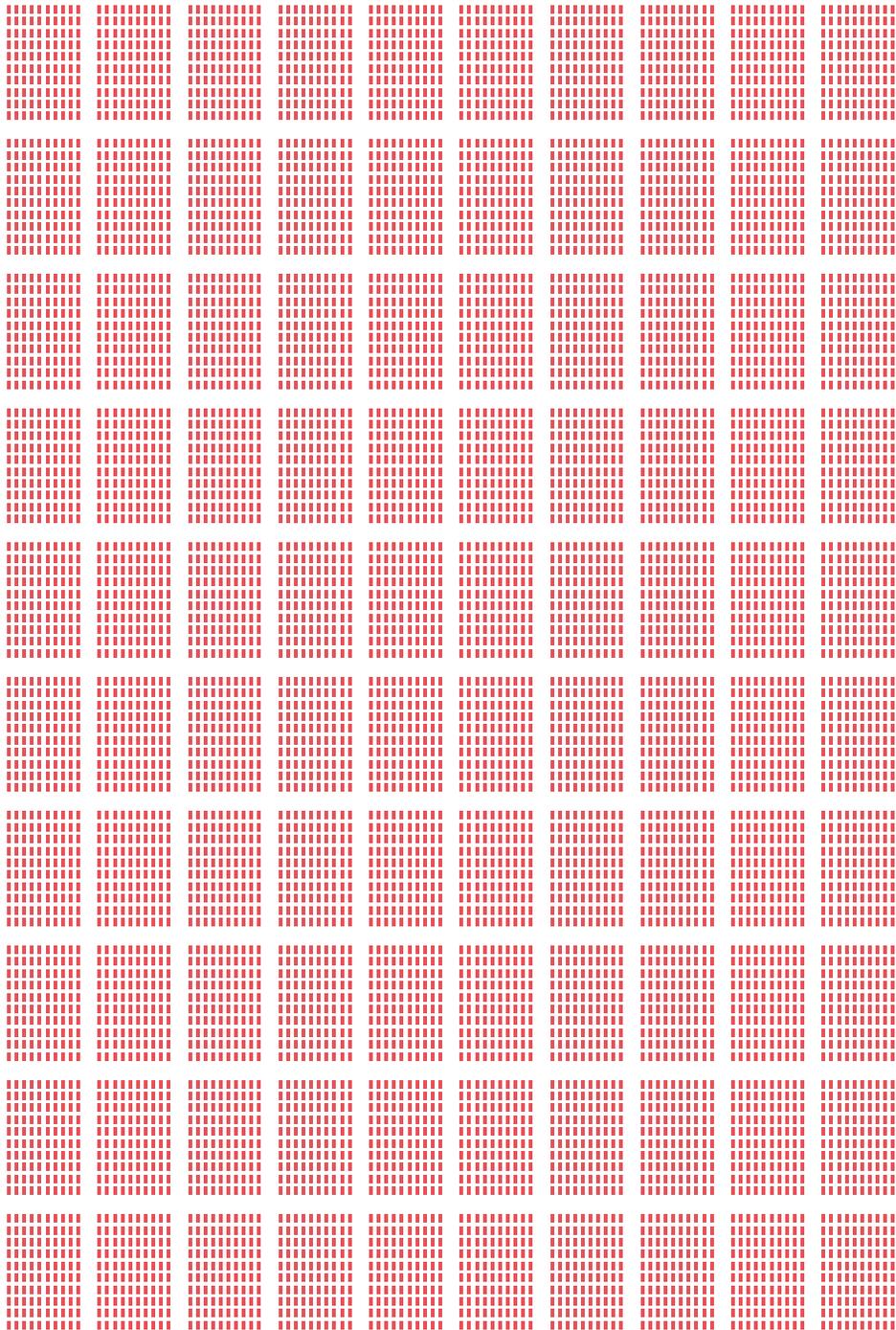
The place value parts are also used to make up the **number name**:

400 000 80 000 5 000 600 20 7
4 hundred and 85 thousand six hundred and twenty-seven

A number can also be expressed (written) as the sum of its place value parts:

$485\ 627 = 400\ 000 + 80\ 000 + 5\ 000 + 600 + 20 + 7$
400 000 + 80 000 + 5 000 + 600 + 20 + 7 is called the **expanded notation** or **place value expansion** of 485 627.

1. Write the number symbols and place value expansions for these numbers.
 - (a) one hundred and twenty-four thousand five hundred and sixty-five
 - (b) two hundred and ten thousand seven hundred and sixty-three
 - (c) four hundred and one thousand eight hundred and seven



1.3 Arrange numbers in order on number lines

1. Draw vertical number lines like these in your book. Do not draw the short marks; just use the lines in your book. Fill in the missing numbers on each number line. Make sure that you do this at the right places, so that the numbers are equally spaced and arranged from smallest to biggest as you go upwards.

(a)	(b)	(c)
240 000	230 000	800 000
239 000		795 000
		790 000
		785 000
		755 000
230 000	225 000	750 000
		745 000
221 000	220 500	705 000
	220 000	

1.4 Factors and multiples

1. Calculate:

(a) 2×3

(b) 1×17

(c) 5×7

(d) 7×11

(e) 11×13

(f) 13×17

(g) 7×13

(h) 7×17

(i) 11×11

(j) 11×17

2. The numbers below are the correct answers for question 1.

77 121 6 35 221

119 143 17 187 91

Match each answer to one of the calculations in question 1 and write the number sentence.

Example: $187 = 11 \times 17$

3. How much is each of the following? Your answers for questions 1 and 2 can be useful to answer this question.

(a) $91 \div 13$

(b) $91 \div 7$

(c) $121 \div 11$

(d) $143 \div 13$

(e) $221 \div 17$

(f) $119 \div 17$

$91 = 7 \times 13$. We say 91 is the **product** of 13 and 7.

We also say that 91 is a **multiple** of 13 and 91 is a multiple of 7.

13 and 7 are called **factors** of 91.

4. (a) What is the product of 13 and 17?

(b) Write two numbers that are factors of 187.

5. (a) Calculate 1×2 , 2×2 , 3×2 and 4×2 .

(b) Your answers for (a) are the first four multiples of 2.

Write down the next ten multiples of 2.

(c) Which of these numbers are *not* multiples of 2?

17 24 50 55

6. (a) Calculate 1×3 , 2×3 , 3×3 and 4×3 .
 (b) Your answers for (a) are the first four multiples of 3. Write down the next five multiples of 3.
7. Your teacher may hand out a page with this grid. If not, write all the whole numbers from 2 to 100 in a neat grid like this:

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- (a) Cross out all the numbers greater than 2 that are *multiples of 2*. What patterns do you notice?
- (b) Find the smallest number greater than 2 not yet crossed out. It is 3. Cross out all the numbers greater than 3 that are multiples of 3.
- (c) Find the smallest number greater than 3 not yet crossed out. It is 5. Cross out all the numbers greater than 5 that are multiples of 5.
- (d) The smallest number greater than 5 not yet crossed out should be 7. Cross out all the numbers greater than 7 that are multiples of 7.
- (e) Write down all the numbers that you did *not* cross out.

This method is called the **Sieve of Eratosthenes**, used in ancient Greece in about 240 BC. The numbers that the “sieve” catches (the remaining numbers) are called **prime numbers**. The sieve lets through all non-prime numbers.

These are the numbers that remain after implementing the above sieve:

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47
 53 59 61 67 71 73 79 83 89 97

We call these numbers the **prime numbers** smaller than 100.

The number 35 can be written as the product of two whole numbers in two ways: $35 = 1 \times 35$ and $35 = 5 \times 7$.

So 35 has four factors, namely 1, 5, 7 and 35.

Factors are the same as **divisors**: 1, 5, 7 and 35 are the only divisors of 35 because they are the only whole numbers that divide exactly into 35.

8. How many factors does each of these numbers have?

1 5 6 7 8 9 10 11 12 13 14 15 16 20 21 23 25

We can group numbers according to the number of factors they have:

- Numbers that have more than two factors are called **composite numbers**.
- Numbers that have only two different factors, namely 1 and itself, are called **prime numbers**.
- 1 is a special number because it is the only number that has only one factor. It is not prime and not composite.

9. Sort all the whole numbers 1, 2, 3, 4, ... , 100 into the three groups in the table.

Prime numbers	Composite numbers	Not prime or composite
2	4	1
3	6	⋮
⋮	⋮	

When a number is multiplied by 1, the answer is the number itself, for example $37 \times 1 = 37$.

This is called the **multiplicative property of 1**.

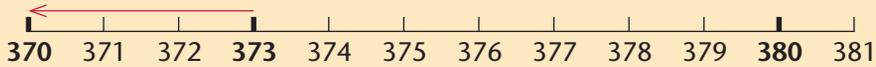
When 0 is added to a number, the answer is the number itself, for example $37 + 0 = 37$.

This is called the **additive property of 0**.

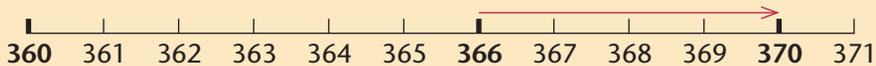
1.5 Rounding off

It is sometimes useful to **round numbers off**.

373 rounded off to the **nearest** multiple of 10 is 370, because 373 is closer to 370 than to 380:

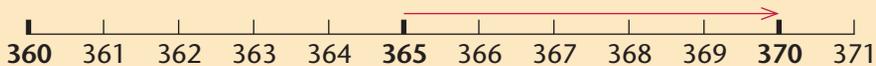


366 rounded off to the **nearest** multiple of 10 is also 370:

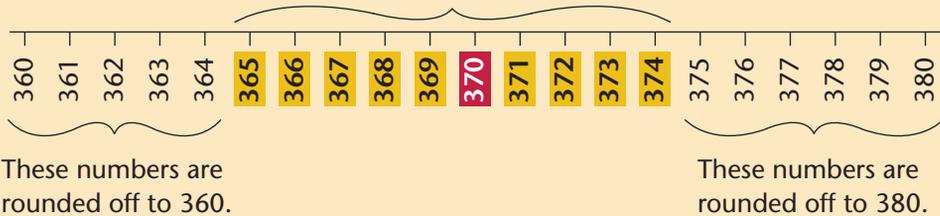


A number that ends in 5, such as 365, is equally far from the two nearest multiples of 10. When we round off to the nearest multiple of 10, a number that ends in 5 is rounded off to the **larger** one of the two nearest multiples of 10.

So, 365 is rounded off to 370 and not to 360.



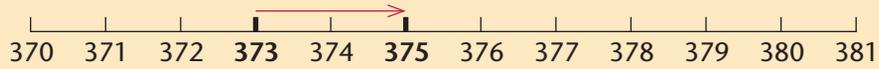
When you round off to the nearest 10,
all these whole numbers are rounded off to 370:



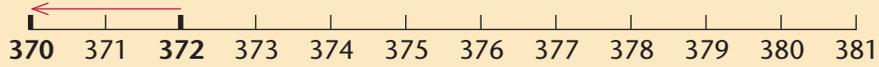
1. Round off each of the following numbers to the nearest 10.

- | | |
|-----------|-----------|
| (a) 724 | (b) 725 |
| (c) 734 | (d) 735 |
| (e) 2 736 | (f) 2 735 |
| (g) 2 734 | (h) 501 |
| (i) 5 011 | (j) 5 101 |

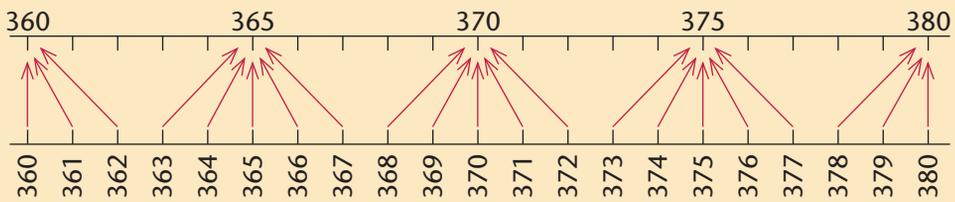
373 rounded off to the nearest multiple of 5 is 375, because 373 is closer to 375 than to 370:



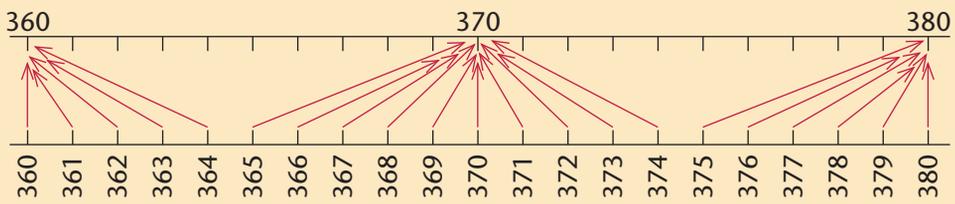
372 rounded off to the nearest multiple of 5 is 370:



The diagram below shows how different numbers are rounded off to the nearest multiple of 5.



The diagram below shows how the same numbers are rounded off to the nearest multiple of 10.

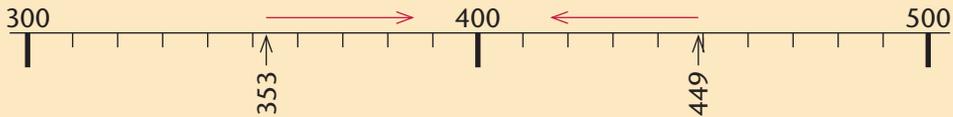


2. Round off each of the following numbers to the nearest 5.

- | | |
|---------|---------|
| (a) 272 | (b) 273 |
| (c) 274 | (d) 275 |
| (e) 276 | (f) 277 |
| (g) 278 | (h) 279 |
| (i) 280 | (j) 281 |
| (k) 282 | (l) 283 |
| (m) 873 | (n) 998 |

3. Round off the numbers in question 2 to the nearest 10.

353 rounded off to the nearest 100 is 400, because 353 is closer to 400 than to 300.



449 is also rounded off to 400, when you round off to the nearest 100.

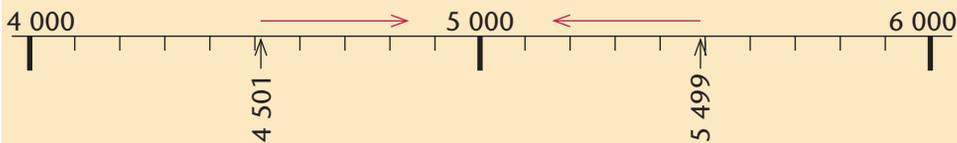
350 is rounded off to 400, and 450 is rounded off to 500.

When you round off to the nearest 100, 350 and all whole numbers bigger than 350 up to 399 are rounded off to 400.

4. Round these numbers off to the nearest 100.

- | | |
|-----------|-----------|
| (a) 349 | (b) 451 |
| (c) 749 | (d) 750 |
| (e) 849 | (f) 850 |
| (g) 1 643 | (h) 3 644 |
| (i) 2 645 | (j) 3 646 |
| (k) 8 647 | (l) 2 648 |
| (m) 3 649 | (n) 4 650 |

4 501 rounded off to the nearest 1 000 is 5 000, because 4 501 is closer to 5 000 than to 4 000.



4 500 is also rounded off to 5 000, when you round off to the nearest 1 000.

5 499 is rounded off to 5 000, and 5 500 is rounded off to 6 000.

When you round off to the nearest 1 000, 4 500 and all whole numbers bigger than 4 500 up to 5 499 are rounded off to 5 000.

5. Round these numbers off to the nearest 1 000.

- | | |
|-----------|-----------|
| (a) 2 500 | (b) 2 499 |
| (c) 1 799 | (d) 7 800 |

6. Complete this table, to show how the numbers in the left column should be rounded off to the nearest 5, 10, 100 and 1 000.

	to the nearest 5	to the nearest 10	to the nearest 100	to the nearest 1 000
753	755	750	800	1 000
796	795	800	800	1 000
998	1 000	1 000	1 000	1 000
3 997				
4 999				
2 992				
2 993				
2 994				
2 995				
2 996				
2 997				
2 998				
2 999				
4 444				
4 445				
4 446				
4 447				
4 448				
4 449				
4 450				
6 007				
6 008				
6 009				

2.1 Equivalence

Farmer Nyathi buys three goats for R423 each.

Here is a plan to calculate the total cost:

$$423 + 423 + 423$$

Here is another plan to calculate the total cost:

$$3 \times 400 + 3 \times 20 + 3 \times 3$$

We can write a **number sentence** to state our belief that these two calculation plans will give the same result:

$$423 + 423 + 423 = 3 \times 400 + 3 \times 20 + 3 \times 3$$

Two different calculation plans that produce the same result are called **equivalent** calculation plans.

- Complete the calculations below to check whether the plans $423 + 423 + 423$ and $3 \times 400 + 3 \times 20 + 3 \times 3$ are really equivalent.

423	
+ <u>423</u>	
6	
40	
<u>800</u>	$3 \times 400 = 1\ 200$
846	$3 \times 20 = \dots$
+ <u>423</u>	$3 \times 3 = \dots$
.....

The calculation plans $423 + 423 + 423$ and $3 \times 400 + 3 \times 20 + 3 \times 3$ produce the same result.

So, the two plans are equivalent.

Another way to state this is to write the number sentence

$$423 + 423 + 423 = 3 \times 400 + 3 \times 20 + 3 \times 3.$$

This is a **true** number sentence.

The number sentence $25 + 25 + 25 + 25 + 25 = 4 \times 20 + 4 \times 5$ is clearly not true. We say it is **false**.

2. Which of the number sentences below are *false*?

Replace each false sentence by a true sentence, by writing a different plan on the right-hand side.

(a) $7 \times 37 + 3 \times 37 = 10 \times 37$

(b) $14 \times 53 + 6 \times 53 = 24 \times 53$

(c) $47 + 28 = 50 + 25$

(d) $96 + 36 = 100 + 31$

(e) $14 \times 76 - 4 \times 76 = 10 \times 76$

(f) $683 + 683 + 683 + 683 + 683 = 5 \times 600 + 5 \times 80 + 5 \times 3$

3. For each of the *true* sentences in question 2, decide which of the two plans are the easiest. Then use the plan to find the answer.

4. Write an easier equivalent calculation plan for each of the following, and use your plan to find the answer.

(a) $4 \times 158 + 6 \times 158$

(b) $13 \times 47 - 3 \times 47$

(c) $134 \times 47 - 34 \times 47$

5. Calculate $7 \times 20 - 6 + 4 \times 2$.

Here are three different learners' answers for question 5:

Tom: $140 - 10 \times 2 = 130 \times 2 = 260$

He added the 6 and 4 to get 10, subtracted 10 from 140 and then multiplied by 2.

Zolani: $70 \times 2 = 140$

She also added 6 and 4 to get 10. Then she subtracted the 10 from 20. Then she calculated $7 \times 10 \times 2$.

Tshepo: $140 - 6 + 8 = 134 + 8 = 142$

He first calculated 7×20 and 4×2 , before he added and subtracted.



To avoid confusion like this when reading instructions to do calculations, people all over the world use the rules given on the next page.

When a calculation plan includes addition and subtraction only, the calculations are done from left to right.

Example: The calculation plan $30 - 6 + 8 - 5 + 7$ indicates that you have to do the following:

$$30 - 6 = 24 \qquad 24 + 8 = 32$$

$$32 - 5 = 27 \qquad 27 + 7 = 34$$

If you don't do the calculations from left to right, you will get a different answer.

For example, if you first calculate

$$6 + 8 = 14 \text{ and } 5 + 7 = 12 \text{ and then}$$

$$30 - 14 = 16 \text{ and then } 16 - 12, \text{ the answer is 4.}$$

That is why you should always follow the above rule, unless you replace the plan with an equivalent plan.

Suppose you want to state that 8 should be added to 6, and 7 to 5, and the two answers subtracted from 30. The use of brackets makes it possible to write such instructions.

Brackets are used to indicate that certain calculations should be done before others.

Example: The calculation plan $30 - (6 + 8) - (5 + 7)$ indicates that the following should be done:

$$6 + 8 = 14 \qquad 5 + 7 = 12 \qquad 30 - 14 = 16 \qquad 16 - 12 = 4$$

When a calculation plan includes multiplication, the multiplication is done first and the remaining calculations are done from left to right.

Example: The calculation plan $7 \times 20 - 6 + 4 \times 2$ indicates that the following should be done:

$$7 \times 20 = 140 \qquad 4 \times 2 = 8 \qquad 140 - 6 = 134 \qquad 134 + 8 = 142$$

Note that there is an equivalent plan that will produce the same result if it is performed *from left to right*, namely

$$30 + 8 + 7 - 6 - 5:$$

$$30 + 8 = 38$$

$$38 + 7 = 45$$

$$45 - 6 = 39$$

$$39 - 5 = 34$$

-
6. Write each of the sets of instructions below in symbols, for example $(20 + 5) \times 10 - 5 + 15$ for the instructions in (a).
- Add 5 to 20, multiply by 10, subtract 5 and add 15.
 - Multiply 20 by 10, add this to 5, subtract 5 and add 15.
 - Subtract 5 from 10, multiply this by 5, add the answer to 20, then add 15 to this answer.
 - Add 5 to 20, multiply the answer by 10 and write it down.
Add 5 and 15 and subtract this from the previous answer that you have written down.
7. For each *false* sentence below, make a true sentence by writing a different plan on the right-hand side.
- $(40 - 5) \times 6 = 40 \times 6 - 5 \times 6$
 - $37 \times (40 + 3) = 37 \times 40 + 3$
 - $24 \times (30 + 6) = 24 \times 30 + 24 \times 6$
 - $(400 + 60 + 3) + (300 + 20 + 5) = (300 + 60 + 5) + (400 + 20 + 5)$
 - $(400 + 60 + 3) - (300 + 20 + 5) = (300 + 60 + 5) - (400 + 20 + 5)$
 - $300 + 80 + 7 - (200 + 30 + 5) = 300 + 80 + 7 - 200 + 30 + 5$
 - $300 + 80 + 7 - (200 + 30 + 5) = 300 + 80 + 7 - 200 - 30 - 5$
 - $(300 + 80 + 7) - (200 + 30 + 5) = (300 + 200) - (80 + 30) - (7 + 5)$
 - $(500 + 70 + 6) - (200 + 40 + 2) = (500 - 200) + (70 - 40) + (6 - 2)$
8. Write an easier equivalent plan for each of the following sets of calculations.
- $(46 + 73) + (56 + 27)$
 - $(96 - 38) + (88 - 46)$
 - $46 \times 238 + 56 \times 238$
 - $46 \times 238 - 36 \times 238$
 - $(18 \times 23 + 17 \times 33) + (12 \times 23 - 7 \times 33)$

9. The number sentences below are about three numbers.

One of the numbers is hidden behind the red stickers. It is the same number behind each of the red stickers.

Likewise, another number is hidden behind each blue sticker, and another number behind each green sticker.

- (a) $\text{red} \times (\text{blue} + \text{green}) = \text{red} \times \text{blue} + \text{green}$
- (b) $\text{red} \times (\text{blue} + \text{green}) = \text{red} \times \text{blue} + \text{red} \times \text{green}$
- (c) $(\text{red} + \text{red}) \times (\text{blue} + \text{green}) = \text{red} \times \text{blue} + \text{red} \times \text{green}$

Which of the number sentences are *false*?

Give examples to show that your answer is right.

10. Which of these number sentences are true, and which are false?

- (a) $6 \times 37 = 6 \times 30 + 7$
- (b) $6 \times 37 = 6 \times 30 + 6 \times 7$
- (c) $26 \times 37 = 20 \times 30 + 6 \times 7$
- (d) $26 \times 37 = 20 \times 30 + 20 \times 7 + 6 \times 30 + 6 \times 7$

2.2 Writing number sentences

Here are some true number sentences that are made up with the numbers 20, 30, 40 and 50:

$$20 + 50 = 30 + 40 \quad 40 - 20 = 50 - 30 \quad 30 - 20 = 50 - 40$$

1. Write three true number sentences with each set of numbers:

- (a) 10, 30, 50 and 70
- (b) 400, 500, 600 and 700
- (c) 200, 400, 600 and 1 200
- (d) 1 000, 4 000, 7 000 and 10 000
- (e) 150, 250, 350 and 450
- (f) 220, 440, 660 and 880
- (g) 43, 56, 69 and 82

-
- Use the numbers 500, 200, 700 and a number of your own choice to write a true number sentence.
 - Use the numbers 800, 1 400 and any two numbers of your own choice to write a true number sentence.
 - Use any four numbers bigger than 100 to write a true number sentence.
 - Use any five numbers bigger than 100 of your own choice to write a true number sentence.

Different number sentences can be written with the numbers 2, 3, 20, 30 and 40, using all five numbers and some numbers more than once, for example:

$$2 \times 20 + 3 \times 30 = 2 \times 40 + 20 + 30 \text{ and } 3 \times 40 = 3 \times 20 + 2 \times 30$$

- Write two different number sentences with the numbers 2, 5, 20, 40 and 50, using each number at least once in each number sentence that you write.
- Write two different number sentences with the numbers 3, 4, 10, 50 and 100, using each number at least once in each number sentence that you write.

1 200 can be formed as a sum of three different multiples of hundred, in different ways, for example:

$$300 + 400 + 500 = 1\,200$$

$$200 + 400 + 600 = 1\,200$$

- Write number sentences for five different ways in which 1 000 can be formed as the sum of three different multiples of hundred.
 - Write number sentences for all the other ways in which 1 000 can be formed as the sum of three different multiples of hundred.
- Write number sentences for five different ways in which 3 700 can be formed as the sum of three different multiples of hundred.
- Write number sentences for all the different ways in which 10 000 can be formed as the sum of three different multiples of thousand.

400 can be formed by subtracting a multiple of 100 from another multiple of 100 in different ways, for example:

$$1\ 100 - 700 = 400 \qquad 1\ 500 - 1\ 100 = 400 \qquad 9\ 800 - 9\ 400 = 400$$

- Write five number sentences that show how 400 can be formed by subtracting one multiple of 100 from another multiple of 100.
- Write five number sentences that show how 3 000 can be formed by subtracting one multiple of 1 000 from another multiple of 1 000.

2.3 Solve and complete number sentences

Number sentences can be **open** or **closed**.

If some numbers are missing, a number sentence is called open, for example:

$$64 + \dots = 100$$

To complete this **open number sentence** you have to find out what you need to add to 64 to reach 100.

If all the numbers are given, a number sentence is called closed. For example $64 + 36 = 100$ is called a **closed number sentence**.

Open number sentences can be written in different ways:

$$700 + \dots = 1\ 000$$

$$700 + \square = 1\ 000$$

$$700 + ? = 1\ 000$$

$$700 + a\ number = 1\ 000$$

$$700 + \blacksquare = 1\ 000$$

$$700 + x = 1\ 000$$

- In each case, find the missing number that will make the number sentence true.
 - $100 + 1\ 100 = a\ number + 800$
 - $a\ number + 300 = 40 \times 40$
 - $300 + 500 = 100 + a\ number$
- Find the missing number in each sentence:
 - $\square \times 300 = 600 \times 100$
 - $700 + \square = 2\ 000 - 600$
 - $1\ 000 - 300 = 400 + \square$
 - $500 + 900 = \square + 700$
- Find the missing number in each sentence:
 - $18\ 000 - \square = 16\ 400 + 700$
 - $10\ 000 \div 100 = \square$
 - $5\ 000 - \square = 4\ 800 \div 2$
 - $300 \times 100 = \square$

3.1 Basic addition and subtraction facts and skills



Type A

Type B

Type C

There are 486 m of Type A fencing, 723 m of Type B fencing and 363 m of Type C fencing alongside a certain stretch of road.

Altogether, this is 1 572 m of fencing.

$$1\ 572 = 486 + 723 + 363$$

We say: 1 572 is the **sum** of 486, 723 and 363.

If 580 m of this fence is removed, there will be 992 m left.

We say: the **difference** between 1 572 and 580 is 992.

The difference between two numbers is found by subtraction:

$$1\ 572 - 580 = 992$$

1. Calculate.

(a) $900 + 600$

(b) $700 + 600$

(c) $90 + 60$

(d) $70 + 60$

(e) $9\ 000 + 6\ 000$

(f) $7\ 000 + 6\ 000$

(g) $500 + 800$

(h) $4\ 000 + 9\ 000$

(i) $1\ 300 - 400$

(j) $700 - 300$

(k) $57\ 000 + 8\ 000$

(l) $27\ 000 + 18\ 000$

(m) $21\ 000 + 4\ 000$

(n) $40\ 000 + 30\ 000$

(o) $4\ 000 + 39\ 000$

(p) $37\ 000 + 4\ 000$

(q) $34\ 000 + 10\ 000$

(r) $34\ 000 - 20\ 000$

(s) $31\ 000 + 9\ 000$

(t) $79\ 000 + 8\ 000$

(u) $29\ 000 + 8\ 000$

(v) $9\ 000 + 25\ 000$

(w) $27\ 000 + 18\ 000$

(x) $6\ 000 + 64\ 000$

The number name for 1 600 is one thousand six hundred. The name **sixteen hundred** can also be used.

To calculate $1\ 600 - 700$ you may think of it as **sixteen hundred minus seven hundred**, instead of one thousand six hundred minus seven hundred.

2. Write the number that is missing from each of these number sentences.

(a) $700 + \dots = 1\ 000$

(b) $1\ 000 - 700 = \dots$

(c) $1\ 000 - \dots = 700$

(d) $400 + \dots = 1\ 000$

(e) $10\ 000 - \dots = 7\ 000$

(f) $100\ 000 - \dots = 70\ 000$

(g) $800 + \dots = 1\ 000$

(h) $80 + \dots = 100$

(i) $\dots + 800 = 2\ 000$

(j) $\dots + 1\ 700 = 5\ 000$

(k) $10\ 000 = 7\ 500 + \dots$

(l) $20\ 000 = \dots + 16\ 000$

(m) $80\ 000 = 100\ 000 - \dots$

(n) $168 - 160 = \dots$

(o) $856 - 50 = \dots$

(p) $263 + 637 = \dots$

3. (a) How long is this line?



(b) How many millimetres long is each of the red parts of the line?

4. Do not use your ruler now.

(a) How many millimetres long is this line?



(b) How long are these two lines together?



5. In each case, state how long the two lines together are. Use number sentences such as $30 + 40 = 70$ to write your answers.



9 000 can be expressed as a sum of thousands in four different ways:
 $9\ 000 = 1\ 000 + 8\ 000 = 2\ 000 + 7\ 000 = 3\ 000 + 6\ 000 = 4\ 000 + 5\ 000$

6. Express each of the following numbers in four different ways as a sum of *hundreds*, *thousands*, *ten thousands* or *hundred thousands*.
- | | |
|-------------|---------------|
| (a) 90 000 | (b) 900 000 |
| (c) 80 000 | (d) 7 000 |
| (e) 600 000 | (f) 50 000 |
| (g) 40 000 | (h) 1 000 000 |

It is easy to know how much $8 + 7$ is, if you think of a number line:



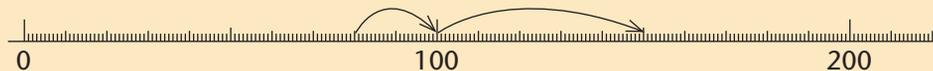
You can describe your thinking like this:

$$8 + 2 \rightarrow \mathbf{10} + 5 = 15$$

You need not draw a number line, just think of it.

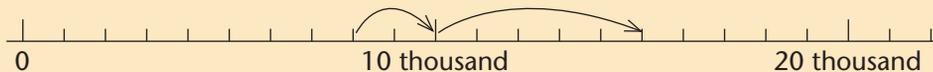
You can work in the same way with bigger numbers. For example:

To know how much $80 + 70$ is, you can think like this:

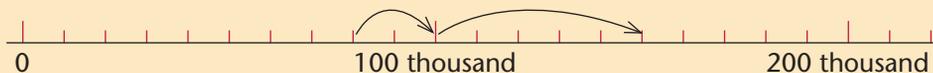


$$80 + 20 \rightarrow \mathbf{100} + 50 = 150$$

To know how much $8\ 000 + 7\ 000$ is, you can think like this:



To know how much $80\ 000 + 70\ 000$ is, you can think like this:



-
7. Copy the calculations for which you *cannot* give the answers quickly. You will work on them later.
- | | |
|-------------------------|--------------------------|
| (a) $500 - 200$ | (b) $500 + 200$ |
| (c) $800 + 700$ | (d) $8\ 000 + 7\ 000$ |
| (e) $80 + 70$ | (f) $80\ 000 + 70\ 000$ |
| (g) $5\ 000 + 7\ 000$ | (h) $15\ 000 + 9\ 000$ |
| (i) $7\ 000 + 9\ 000$ | (j) $70\ 000 + 90\ 000$ |
| (k) $60\ 000 + 80\ 000$ | (l) $140\ 000 - 80\ 000$ |
8. Do the calculations that you wrote down without answers when you did question 7.
9. Copy the calculations for which you *cannot* give the answers quickly. Then do the calculations.
- | | |
|---------------------------|---------------------------|
| (a) $400 + 700$ | (b) $30\ 000 + 80\ 000$ |
| (c) $800\ 000 + 500\ 000$ | (d) $8\ 000 + 9\ 000$ |
| (e) $47\ 000 + 7\ 000$ | (f) $800\ 000 - 200\ 000$ |
| (g) $40\ 000 + 80\ 000$ | (h) $30\ 000 + 90\ 000$ |
| (i) $130\ 000 + 90\ 000$ | (j) $6\ 000 + 8\ 000$ |
10. Jonas pays R20 000 for a trailer and R60 000 for a second-hand bakkie. He also buys a new engine for the bakkie for R70 000. How much money does he spend in total?
11. Geraldine bought a plot for R300 000. She then built a house on the plot for R600 000. How much did Geraldine pay altogether for the plot and the house?
12. A farmer already owns 700 hectares of farmland. He buys three more farms: one of 300 hectares, one of 700 hectares and one of 400 hectares. How many hectares of farmland does he now own?
13. Farmer Mphuthi owns 6 000 hectares of land and farmer MacBride owns 9 000 hectares of land. How much more land does farmer MacBride own than farmer Mphuthi?

-
- Use the “add on both sides” method to form another five addition facts, starting with $700 + 700 = 1\,400$.
 - Start from $20 + 30 = 50$ and use different methods to form ten different addition facts and twenty different subtraction facts.

The “**doubles**” are easy addition facts to know, for example $30 + 30 = 60$ and $4\,000 + 4\,000 = 8\,000$.

We can also say 3 tens + 3 tens = 6 tens and 4 thousands + 4 thousands = 8 thousands.

- How much is each of the following?
 - $6\,000 + 6\,000$
 - $900 + 900$
 - $70\,000 + 70\,000$
 - $80\,000 + 80\,000$

If you want to know how much $3\,000 + 5\,000$ is, you can start with the nearest double, which is $3\,000 + 3\,000 = 6\,000$, and add another $2\,000$ to get $3\,000 + 5\,000 = 8\,000$.

- Show how the answers for each of the following calculations can be found by first doubling one of the numbers.
 - $7\,000 + 8\,000$
 - $70 + 90$
 - $60\,000 + 80\,000$
 - $9\,000 + 6\,000$
 - $80\,000 + 90\,000$
 - $600 + 900$
- Start with $800 + 800$ and use different methods to form ten different addition facts and twenty different subtraction facts.
- Copy and complete these “number journeys” to practise filling up to multiples of 1 000 or 10 000.
 - $800 + \dots \rightarrow 1\,000 + \dots \rightarrow 1\,400 + \dots \rightarrow 2\,000 + \dots \rightarrow 3\,300$
 - $3\,800 + \dots \rightarrow 4\,000 + \dots \rightarrow 4\,600 + \dots \rightarrow 5\,000 + \dots \rightarrow 7\,400$
 - $7\,000 + \dots \rightarrow 13\,000 + \dots \rightarrow 20\,000 + \dots \rightarrow 100\,000$
 - $8\,000 + \dots \rightarrow 10\,000 + \dots \rightarrow 15\,000 + \dots \rightarrow 20\,000$
 - $7\,250 + \dots \rightarrow 8\,000 + \dots \rightarrow 10\,000 + \dots$

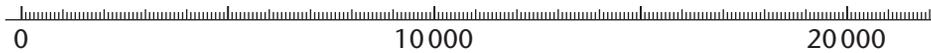
13. Show how each of the following can be calculated by first filling up to 10 000.

(a) $14\ 000 - 8\ 000$

(b) $12\ 000 - 5\ 000$

(c) $16\ 000 - 7\ 000$

(d) $15\ 000 - 9\ 000$



14. Show how each of the following can be calculated by first filling up to 10 000.

(a) $10\ 300 - 9\ 700$

(b) $10\ 200 - 5\ 700$

(c) $10\ 800 - 6\ 800$

(d) $12\ 300 - 9\ 600$



15. Copy the calculations for which you cannot find the answers quickly.

(a) $1\ 900 - 800$

(b) $1\ 300 - 900$

(c) $13 - 9$

(d) $170 - 60$

(e) $1\ 400 - 600$

(f) $14 - 6$

(g) $1\ 500 - 800$

(h) $150 - 70$

(i) $110 - 60$

(j) $16 - 8$

(k) $16 - 7$

(l) $900 - 500$

(m) $180 - 90$

(n) $1\ 800 - 800$

(o) $140 - 60$

(p) $1\ 700 - 800$

(q) $600 + 900$

(r) $170 - 90$

(s) $1\ 700 - 900$

(t) $1\ 600 - 800$

(u) $120 - 70$

16. Find the answers for the calculations that you wrote down in question 15.

When you calculate a sum such as $3\ 478 + 8\ 858 + 4\ 656 + 9\ 776$, you have to add up many multiples of thousand:

$$3\ 000 + 8\ 000 + 4\ 000 + 9\ 000.$$

You also have to add up many multiples of hundred and many multiples of ten:

$$400 + 800 + 600 + 700 \text{ and } 70 + 50 + 50 + 70.$$

17. Calculate the sum in each case.

(a) $70 + 80 + 90 + 30 + 60 + 80 + 60 + 90$

(b) $400 + 700 + 600 + 800 + 300 + 900$

(c) $8\ 000 + 5\ 000 + 7\ 000 + 4\ 000 + 6\ 000 + 8\ 000 + 7\ 000$

(d) $60\ 000 + 50\ 000 + 90\ 000 + 60\ 000 + 80\ 000 + 40\ 000 + 60\ 000$

18. Calculate the sum in each case.

(a) $60\ 000 + 70\ 000 + 30\ 000 + 60\ 000 + 80\ 000 + 80\ 000 + 70\ 000$

(b) $80\ 000 + 30\ 000 + 70\ 000 + 70\ 000 + 60\ 000 + 80\ 000 + 60\ 000$

(c) $70\ 000 + 60\ 000 + 80\ 000 + 60\ 000 + 80\ 000 + 30\ 000 + 70\ 000$

19. You should have obtained the same answer for each of the three sums in question 18. If you did not, identify where you went wrong and correct it.

20. Why are the answers the same for all three sums in question 18?

21. (a) Which do you think will be easier to calculate:

$700 + 500 + 800 + 300 + 600 + 200$ or

$700 + 300 + 800 + 200 + 600 + 500$?

(b) Explain why you think so.

22. Rearrange the numbers in each of the following sums so that it will be easier to calculate.

(a) $6\ 000 + 8\ 000 + 7\ 000 + 4\ 000 + 3\ 000 + 2\ 000 + 7\ 000$

(b) $7\ 000 + 500 + 40 + 7 + 4\ 000 + 800 + 30 + 8$

23. Find the missing number in each sentence.

(a) $36\ 000 + ? = 40\ 000$

(b) $5\ 700 + ? = 6\ 000$

(c) $5\ 740 + ? = 5\ 800$

(d) $5\ 740 + ? = 6\ 000$

(e) $5\ 740 + ? = 10\ 000$

(f) $5\ 740 + ? = 100\ 000$

(g) $36\ 400 + ? = 37\ 000$

(h) $36\ 470 + ? = 36\ 500$

(i) $63\ 680 + ? = 64\ 000$

(j) $63\ 680 + ? = 100\ 000$

(k) $63\ 680 + ? = 70\ 000$

(l) $63\ 680 + ? = 90\ 700$

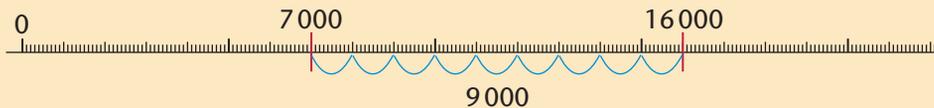
3.3 Subtraction and addition are inverses

If you know an addition fact, you also know a subtraction fact. For example if you know that $80 + 80 = 160$, you also know that $160 - 80 = 80$.

- Use each addition fact below to write two subtraction facts.
 - $250 + 450 = 700$
 - $367 + 633 = 1\ 000$
 - $2\ 480 + 2\ 520 = 5\ 000$
 - $64\ 753 + 27\ 538 = 92\ 291$
- Use each of these subtraction facts to write an addition fact.
 - $3\ 678 - 600 = 3\ 078$
 - $3\ 678 - 70 = 3\ 608$
 - $3\ 678 - 3\ 000 = 678$
 - $3\ 678 - 608 = 3\ 070$
- Use each of the subtraction facts in question 2 to write another subtraction fact, without doing any calculations.
- Try to find the following differences. Do as little work as possible.
 - $8\ 382 - 80$
 - $8\ 382 - 8\ 000$
 - $8\ 382 - 380$
 - $8\ 382 - 8\ 002$

To find differences between numbers, it is often useful to think of where the numbers are on a number line.

For example, to know what the difference between 16 000 and 7 000 is, the following picture in your mind may be useful.



- Find each of the differences below. If you do not know the answer immediately, you may think of movements on a number line or think in any other way.
 - $8\ 000 - 3\ 000$
 - $80\ 000 - 30\ 000$
 - $38\ 000 - 3\ 000$
 - $38\ 000 - 30\ 000$
 - $23\ 000 - 4\ 000$
 - $109\ 000 - 4\ 000$
 - $109\ 000 - 100\ 000$
 - $109\ 000 - 50\ 000$

-
6. Bert and Simanga both run a long-distance race. After some time Simanga has covered 18 000 m, and Bert has covered 13 000 m. How far is Simanga ahead of Bert?
 7. Manare is a rich fruit farmer in Limpopo. He already has 18 000 orange trees on his farm. He plants another 13 000 orange trees. How many orange trees does he have now?
 8. Gabieba saved R180 000 to make her shop bigger. She spends R130 000 on a veranda that can be used as a restaurant. How much money does she have left?
 9. Gert needs R180 to buy a book that he really wants. He has only R130. How much more money does he need?
 10. 1 800 houses in a township have electricity and 1 300 houses do not have electricity. How many houses are there altogether?
 11. Farmer Maleka has 1 800 chickens and farmer Engelbrecht has 1 300 chickens. How many more chickens does farmer Maleka have than farmer Engelbrecht?
 12. There are 1 800 learners in a school and 1 300 of them are girls. How many boys are there in the school?
 13. Which of the following do you think will have the same answer?
 - (a) $5\ 000 + 800 + 60 - 3\ 000 - 500 - 20$
 - (b) $(5\ 000 + 800 + 60) - (3\ 000 - 500 - 20)$
 - (c) $(5\ 000 + 800 + 60) - (3\ 000 + 500 + 20)$
 - (d) $5\ 000 - 3\ 000 + 800 - 500 + 60 - 20$
 - (e) $(5\ 000 - 3\ 000) + (800 - 500) + (60 - 20)$
 - (f) $60 + 5\ 000 - 20 - 3\ 000 - 500 + 800$
 - (g) $5\ 000 + 500 + 20 - 3\ 000 - 800 - 60$
 14. Do each set of calculations in question 13, and check your answers.

Brackets are used to indicate that the operations within the brackets are meant to be done first, unless the set of instructions is replaced with an equivalent set of instructions.

3.4 Rounding off and rearranging

The numbers 10, 20, 30, 40 and so on are called **multiples of 10**.

The numbers 1 000, 2 000, 3 000, 4 000 and so on are called **multiples of 1 000**.

The numbers 100 000, 200 000, 300 000, 400 000 and so on are called **multiples of 100 000**.

Mr Nene has to pay R1 424 for water and electricity. He also has to pay R2 783 for property rates, sanitation and waste removal. Which of the following statements will best describe Mr Nene's situation?

- Statement A: He needs more than R1 000.
- Statement B: He needs about R5 000.
- Statement C: He needs about R3 000.
- Statement D: He needs about R4 000.

To quickly answer the above question, it helps to notice that R1 424 is closer to **R1 000** than to R2 000, and R2 783 is closer to **R3 000** than to R2 000. So, we can quickly see that Mr Nene has to pay approximately R4 000.

To make estimates quickly, it is useful to round numbers off. For example, numbers can be rounded off to the nearest multiple of 10 or 100 or 500 or 1 000 or 1 million or whatever you may decide.

A number that is equally far from two multiples is normally rounded off to the higher multiple. For example, 1 500 would be rounded off to 2 000 not to 1 000, if it is rounded off to the "nearest" thousand.

When rounding off to the nearest 100, the number 3 450 would be rounded off to 3 500. When rounding off to the nearest 1 000, the number 3 450 would be rounded off to 3 000.

1. Round off each of the following numbers to the nearest thousand.
 - (a) 4 678 (b) 28 345 (c) 28 549 (d) 28 500
 - (e) 7 500 (f) 2 499 (g) 2 501 (h) 63 505
2. Round off each of the numbers in question 1 to the nearest 10 thousand.

3. Mrs Setati bought clothes for these amounts:

R768 R1 279 R2 877 R649 R750

- (a) Round off each amount to the nearest R1 000 to make an estimate of how much Mrs Setati has to pay in total.
- (b) Round off each amount to the nearest R100 to make an estimate of how much Mrs Setati has to pay in total.
4. Calculate $768 + 1\,279 + 2\,877 + 649 + 750$ to find out exactly how much Mrs Setati has to pay.
5. You have to calculate $2\,376 + 983 + 4\,874$.
- (a) Round off each number to the nearest 1 000 and use this to estimate the answer.
- (b) Do the calculation. Explain how you do the calculation.
6. First estimate $608 + 268 + 738 + 445$ by rounding off to the nearest 100, then do an exact calculation.
7. Estimate $7\,234 + 43\,875 + 4\,543$ to the nearest 1 000, then calculate it accurately.
8. Mr Samson bought a car for R78 749. He paid R47 535 in cash. Approximately how much does he still owe to the nearest
- (a) R10 000
- (b) R1 000?
9. Suppose you have to calculate $6\,000 + 700 + 50 + 3 + 3\,000 + 600 + 80 + 5$. Which of the following two arrangements of the numbers above will be easiest to use:
- (a) $6\,000 + 700 + 50 + 3 + 3\,000 + 600 + 80 + 5$ or
- (b) $6\,000 + 3\,000 + 700 + 600 + 50 + 80 + 3 + 5$

In order to estimate, we first have to round off to a convenient number.

$6\ 000 + 3\ 000 + 700 + 600 + 50 + 80 + 3 + 5$ can be calculated in different ways. One way is **to add on one number at a time**:

$$6\ 000 + 3\ 000 \rightarrow 9\ 000 + 700 \rightarrow 9\ 700 + 600 \rightarrow 10\ 300 + 50 \rightarrow 10\ 350 \\ 10\ 350 + 80 \rightarrow 10\ 430 + 3 \rightarrow 10\ 433 + 5 = 10\ 438$$

Another way is **to group each kind of multiple together**, like this:

$$\begin{array}{r} \underbrace{6\ 000 + 3\ 000} + \underbrace{700 + 600} + \underbrace{50 + 80} + \underbrace{3 + 5} \\ = 9\ 000 + 1\ 300 + 130 + 8 \\ = 9\ 000 + 1\ 400 + 30 + 8 \\ = 10\ 000 + 400 + 30 + 8 \\ = 10\ 438 \end{array}$$

We normally use brackets to indicate the decision to do certain calculations first, so the above plan can be described like this:

$$(6\ 000 + 3\ 000) + (700 + 600) + (50 + 80) + (3 + 5)$$

10. Rearrange each of the following sums so that they can be calculated by adding up each kind of multiple separately, as shown above. Use brackets to indicate which calculations you plan to do first.

(a) $4\ 000 + 700 + 60 + 5 + 9\ 000 + 600 + 80 + 7$

(b) $50\ 000 + 7\ 000 + 400 + 60 + 4 + 30\ 000 + 4\ 000 + 600 + 30 + 7$

(c) $400 + 30 + 6 + 300 + 80 + 5 + 900 + 30 + 4$

11. Implement the plans you made in question 10; in other words, do the calculations now.

12. Use your results for question 11 to state what the answers for the following will be.

(a) $4\ 765 + 9\ 687$

(b) $57\ 464 + 34\ 637$

(c) $436 + 385 + 934$

13. Which of the calculations in (a) do you think will have the same answer? Explain why you think so.

(a) $6\ 241 + 3\ 736$ $6\ 236 + 3\ 741$ $6\ 124 + 3\ 673$

(b) Do the calculations in (a) to check your prediction.

3. Lebogang wants to calculate $6\,231 - 2\,758$. She writes:

$$6\,231 = 6\,000 + 200 + 30 + 1$$

$$2\,758 = 2\,000 + 700 + 50 + 8$$

Write a suitable replacement for $6\,000 + 200 + 30 + 1$.

4. Estimate the answers by rounding off the numbers to the nearest thousand:
- (a) $27\,689 - 12\,324$ (b) $85\,324 - 52\,689$
(c) $64\,504 - 21\,286$ (d) $29\,679 - 15\,452$
5. In which cases in question 4 will it be necessary to make a replacement for the expansion of the first number, as was shown for $8\,246 - 3\,562$ on the previous page?

$8\,436 - 4\,787$ can be calculated by breaking the bigger number down like this: $8\,436 = 437 + 7\,999$

This makes it easy to subtract the parts of $4\,787$:

$$\begin{aligned} 7\,999 - 4\,787 &= (7\,000 - 4\,000) + (900 - 700) + (90 - 80) + (9 - 7) \\ &= 3\,000 + 200 + 10 + 2 \\ &= 3\,212 \end{aligned}$$

6. What must be added to $3\,212$ in the above example to get the correct answer for $8\,436 - 4\,787$?
7. Calculate $47\,235 - 32\,876$ and $49\,531 - 23\,845$ in the way you have just calculated $8\,436 - 4\,787$.
8. Calculate $88\,354 - 52\,768$ and $76\,423 - 52\,678$ in any way you prefer.
9. Do the calculations in question 4.
10. Do these calculations and use your work to check whether your answers for question 9 are correct.
- (a) $15\,365 + 12\,324$ (b) $32\,635 + 52\,689$
(c) $43\,218 + 21\,286$ (d) $14\,227 + 15\,452$

3.6 The vertical column notation for addition

The work to calculate $35\,526 + 16\,336 + 46\,719 + 54\,858$ can be written up in **expanded column notation** like this:

	100 000		20 000		2 000		100		20		
35 526 =		30 000	+	5 000	+	500	+	20	+	6	
16 336 =		10 000	+	6 000	+	300	+	30	+	6	
46 719 =		40 000	+	6 000	+	700	+	10	+	9	
54 858 =		<u>50 000</u>	+	<u>4 000</u>	+	<u>800</u>	+	<u>50</u>	+	<u>8</u>	
		150 000		23 000		2 400		130		29	
Total =	100 000	+	50 000	+	3 000	+	400	+	30	+	9
	= 153 439										

1. Calculate the sum of 76 548, 48 387, 54 674 and 66 075 and set out your work in expanded column notation as shown above.

The work to calculate $35\,526 + 16\,336 + 46\,719 + 54\,858$ can also be written up like this:

100 000											
20 000											
2 000											
100											
20											
35 526	=	30 000	+	5 000	+	500	+	20	+	6	
16 336	=	10 000	+	6 000	+	300	+	30	+	6	
46 719	=	40 000	+	6 000	+	700	+	10	+	9	
+ 54 858	=	<u>50 000</u>	+	<u>4 000</u>	+	<u>800</u>	+	<u>50</u>	+	<u>8</u>	
9		130 000		21 000		2 300		110		29	
30											
400											
3 000											
50 000											
+ 100 000											
153 439											

2. Calculate the sum of 26 367, 34 528 and 47 657 and set out your work as shown above.

You can write even less when you add, by just keeping the place value parts of the numbers in mind but not writing them down.

For example, you can write as shown on the right when you calculate $35\,526 + 16\,336 + 46\,719 + 54\,858$.

$$\begin{array}{r}
 100\,000 \\
 20\,000 \\
 2\,000 \\
 100 \\
 20 \\
 35\,526 \\
 16\,336 \\
 46\,719 \\
 + 54\,858 \\
 \hline
 9 \\
 30 \\
 400 \\
 3\,000 \\
 50\,000 \\
 + 100\,000 \\
 \hline
 153\,439
 \end{array}$$

3. Calculate $38\,264 + 23\,768 + 34\,526$ and set out your work as above.

It is possible to do even more of the work just in your mind when you do addition, as is shown on the right for $35\,526 + 16\,336 + 46\,719 + 54\,858$.

In fact, you can write even less, as shown here:

$$\begin{array}{r}
 100\,000 \\
 20\,000 \\
 2\,000 \\
 100 \\
 20 \\
 35\,526 \\
 16\,336 \\
 46\,719 \\
 + 54\,858 \\
 \hline
 153\,439 \\
 \\
 122\,120 \\
 35\,526 \\
 16\,336 \\
 46\,719 \\
 + 54\,858 \\
 \hline
 153\,439
 \end{array}$$

4. Calculate $52\,327 + 23\,538 + 18\,886$ and set out your work in the shortest way that is convenient for you.
5. Calculate $37\,546 + 23\,385 + 43\,824$ and $33\,825 + 27\,344 + 43\,586$. If your answers are not the same, you have made a mistake.

3.7 The vertical column notation for subtraction

You may write your work for calculating $52\,345 - 28\,857$ in **expanded column notation** like this:

$$\begin{aligned} 52\,345 &= 50\,000 + 2\,000 + 300 + 40 + 5 \\ &= 40\,000 + 11\,000 + 1\,200 + 130 + 15 && \text{(replacement)} \\ 28\,857 &= 20\,000 + 8\,000 + 800 + 50 + 7 \\ 52\,345 - 28\,857 &= 20\,000 + 3\,000 + 400 + 80 + 8 \\ &= 23\,488 \end{aligned}$$

This method is called the **borrowing method** or **transfer method**.

You can also calculate $52\,345 - 28\,857$ by making a different replacement than above, and write it in expanded column notation:

$$\begin{aligned} 52\,345 &= 2\,346 + 49\,999 \\ 49\,999 &= 40\,000 + 9\,000 + 900 + 90 + 9 \\ 28\,857 &= 20\,000 + 8\,000 + 800 + 50 + 7 \\ 49\,999 - 28\,857 &= 20\,000 + 1\,000 + 100 + 40 + 2 \\ \text{Add back } 2\,346 &= 2\,000 + 300 + 40 + 6 \\ 52\,345 - 28\,857 &= 20\,000 + 3\,000 + 400 + 80 + 8 \\ &= 23\,488 \end{aligned}$$

1. (a) Calculate $83\,532 - 37\,789$ in one of the above ways and set your work out in expanded column notation.
- (b) Calculate $83\,532 - 37\,789$ in the other way.
- (c) If your answers differ you must correct your mistakes.

If you calculate	49 999	You may write	49 999
$52\,345 - 28\,857$	$- \underline{28\,857}$	even less if you	$- \underline{28\,857}$
by replacing $52\,345$	2	wish, as shown	21 142
with $2\,346 + 49\,999$,	40	on the right.	$+ \underline{2\,346}$
you can write up your	100		23 488
work without writing	1 000		
the place value	$+ \underline{20\,000}$		
expansions of the	21 142		
numbers.	$+ \underline{2\,346}$		
	23 488		

2. Calculate $72\,564 - 28\,797$ by working in the above way.

In question 1 you calculated $83\,532 - 37\,789$ with the transfer method. You can write up your work as shown in the left column below, without writing the place value expansions of $83\,532$ and $37\,789$.

The numbers in grey show the parts of the replacement $70\,000 + 12\,000 + 1\,400 + 120 + 12$ for $80\,000 + 3\,000 + 500 + 30 + 2$.

The remarks in the second column explain how the different parts of the answer are obtained.

70 000	Two shorter ways of	70000	12000	1400	120	12
12 000	writing are shown on	8	3	5	3	2
1 400	the right.	3	7	7	8	9
120		4	5	7	4	3
12						
83 532					70	12
<u>- 37 789</u>					8	3
3	(12 - 9)				5	3
40	(120 - 80)				7	8
700	(1 400 - 700)				4	7
5 000	(12 000 - 7 000)				4	3
<u>40 000</u>	(70 000 - 30 000)					
45 743						

3. Calculate each of the following. Use the method that you prefer and write as little as possible.

- | | |
|-------------------------|----------------------------------|
| (a) $87\,452 - 23\,238$ | (b) $93\,231 - 37\,392$ |
| (c) $65\,394 - 28\,608$ | (d) $84\,678 - 23\,134$ |
| (e) $88\,786 - 62\,341$ | (f) $32\,329 + 5\,329 + 24\,342$ |
| (g) $26\,765 + 57\,684$ | (h) $27\,785 + 56\,664$ |

4. Calculate.

- | | |
|-----------------------------------|-----------------------------------|
| (a) $53\,325 + 24\,891 - 43\,456$ | (b) $43\,456 - 24\,891 + 53\,325$ |
| (c) $23\,567 - 41\,305 + 52\,827$ | (d) $52\,567 - 41\,305 + 23\,827$ |

5. First think about the amount of work, and then calculate the following in the quickest and easiest way you can think of.

$$81\,234 - 3\,467 - 7\,624 - 5\,784 - 3\,276 - 7\,776 - 3\,877 - 2\,659$$

3.8 Practise addition and subtraction

1. First estimate to the nearest thousand and write your estimates down, then calculate the following.
 - (a) $37\,466 + 8\,728$
 - (b) $78\,726 + 105\,834$
 - (c) $37\,728 + 8\,466$
 - (d) $78\,834 + 105\,726$
 - (e) $38\,768 + 7\,426$
 - (f) $175\,736 + 8\,824$
 - (g) $28\,768 + 17\,426$
 - (h) $108\,736 + 75\,824$
2. Your answers for 1(a), (c), (e) and (g) should be the same, and your answers for 1(b), (d), (f) and (h) should be the same. If they are not the same, you have made mistakes. In that case find and correct your mistakes.
3. In each case find the difference between the two numbers.
 - (a) 46 194 and 37 466
 - (b) 7 426 and 46 194
 - (c) 184 560 and 105 726
 - (d) 75 824 and 184 560
4. First estimate to the nearest thousand and write your estimates down, then calculate the following.
 - (a) $73\,426 - 25\,854$
 - (b) $89\,823 - 45\,776$
5.
 - (a) Calculate $3\,485 + 7\,583$. Add 8 575 to the answer. Subtract 3 485 from the answer. Subtract 7 583 from the answer.
(b) Should your final answer be 8 575? Explain.
(c) If you have made mistakes, find them and correct them.
6. For which of the following would you expect to get the same answers? Do not do the calculations now.
 - (a) $(8\,765 - 3\,638) - (1\,847 + 1\,386)$
 - (b) $8\,765 - (3\,638 - 1\,847) + 1\,386$
 - (c) $8\,765 - (3\,638 + 1\,847) + 1\,386$
 - (d) $(8\,765 - 3\,638) - (1\,847 - 1\,386)$
7. Do the calculations in question 6 and check your predictions.

3.9 Using a calculator

A calculator is a handy tool that can help you to calculate quickly and accurately, provided that you know how to press the correct keys.

Also, you need the right attitude when using a calculator.

Calculations like $9 + 3$, 5×6 , $30 + 7$ and $200 + 300$ can be done faster mentally than with a calculator. You should not use a calculator for such calculations.

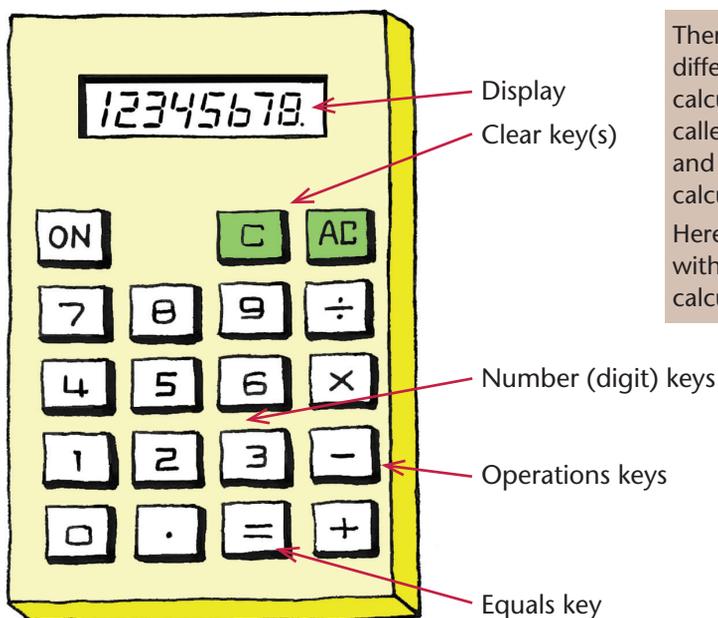
Learning calculator language

The calculator cannot think for you. It only does what you tell it to do.

So you must learn how to talk **calculator language**, so that the calculator can understand you!

If you want to use the calculator to solve a word problem, you must first translate from English into the language of arithmetic, and then into calculator language.

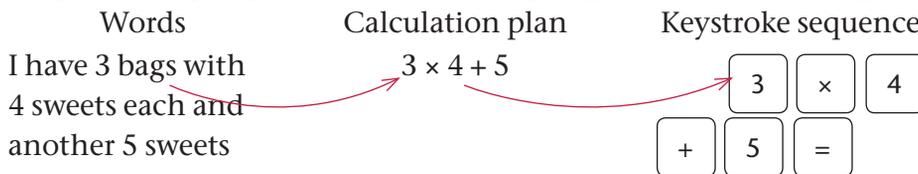
Calculator language is written as a **keystroke sequence** using the different kinds of keys on the calculator.



There are many different kinds of calculators. Some are called simple calculators and some scientific calculators.

Here we work only with a very simple calculator.

English language Arithmetic language Calculator language



We do simple basic operations on the calculator like this:

Calculation plan

Calculator keystroke sequence

$6 + 2$



$6 - 2$



6×2



$6 \div 2$



It is very important to understand that you must always use the

 key to tell the calculator to now do the operations that you typed.

Note that we are using *small* numbers here only to explain, and so that we can easily check the calculator mentally. But we should only use the calculator when calculating with *large* numbers.

- Use your calculator to calculate these. Check by estimating the answers.
 - 11×21
 - $150 \div 6$
 - $212 - 103$
 - $136 + 48$
 - 23×52
 - $1\,728 - 619$
- What is the biggest number that can be typed on your calculator?
Press 123456789 and see what happens. Can you type 100 000 000?
 - What is the biggest number that the calculator can show or display? Press 99999999  2  and see what happens.

3. Calculate using your calculator. How will you check that the answers are correct?

(a) $123\,456 + 234\,567$

(b) $1\,234\,567 + 7\,654\,321$

(c) $97\,531 - 57\,975$

(d) $7\,654\,321 - 779\,348$

(e) $7\,557 - 5\,975 + 7\,979$

(f) $879\,715 + 54\,021 - 176\,534$

Correcting mistakes

What if you make mistakes?

It is very easy to press wrong keys by accident, and then to get wrong answers, for example:

- You may press the wrong operation:

For example, you may press $\boxed{+}$ instead of $\boxed{-}$.

- You may press the wrong number:

For example, you may press 32 instead of 23.

It is a good habit to keep your eyes on the display, so that you can immediately see when you make a typing error.

Instead of then having to redo everything, you can learn shortcuts to correct different kinds of mistakes.

4. How do we correct the mistake of keying the wrong operation, for example pressing $\boxed{+}$ instead of $\boxed{-}$, except to start over again?

Do the following keystroke sequences on your calculator. Look at the display after every keystroke and try to explain how your calculator works. Try to *predict* the display before pressing each key.

(a) $\boxed{7} \boxed{+} \boxed{\times} \boxed{3} \boxed{=}$

(b) $\boxed{7} \boxed{\times} \boxed{+} \boxed{3} \boxed{=}$

(c) $\boxed{7} \boxed{+} \boxed{-} \boxed{3} \boxed{=}$

(d) $\boxed{7} \boxed{-} \boxed{+} \boxed{3} \boxed{=}$

(e) $\boxed{7} \boxed{+} \boxed{+} \boxed{3} \boxed{=}$

(f) $\boxed{7} \boxed{-} \boxed{\times} \boxed{3} \boxed{=}$

Describe a method to correct an incorrect operation entry on your calculator.

5. Your calculator will have a \boxed{C} (clear) key and maybe also an \boxed{AC} (all clear) key. Different calculators use these keys differently. On most calculators the \boxed{C} key clears only the last entry, and on some calculators pressing the \boxed{C} key twice deletes everything.

Find out how the correction (clear) key on your calculator works by typing these key sequences. Try to predict what the calculator will display after each keystroke.

(a) $\boxed{2} \boxed{+} \boxed{3} \boxed{C} \boxed{5} \boxed{=}$

(b) $\boxed{2} \boxed{+} \boxed{3} \boxed{C} \boxed{C} \boxed{2} \boxed{+} \boxed{5} \boxed{=}$

(c) $\boxed{2} \boxed{+} \boxed{3} \boxed{AC} \boxed{2} \boxed{+} \boxed{5} \boxed{=}$

(d) $\boxed{2} \boxed{\times} \boxed{3} \boxed{C} \boxed{5} \boxed{=}$

(e) $\boxed{2} \boxed{\times} \boxed{3} \boxed{AC} \boxed{2} \boxed{\times} \boxed{5} \boxed{=}$

6. Suppose you want to calculate $15 + 28 - 12 + 46$, but make the following mistakes. In each case type the given keystroke sequence, including the mistake. Then correct the mistake and complete the calculation. If you really make mistakes, correct them too!

(a) $15 \boxed{+} 29$

(b) $15 \boxed{+} 28 \boxed{+}$

(c) $15 \boxed{+} 28 \boxed{-} 21$

(d) $15 \boxed{+} 28 \boxed{-} 12 \boxed{-} 56$

7. If you discover that you typed a wrong operation only after you entered the next number, the mistake cannot be corrected in any of the above ways.

Ben has a bright idea: He wanted to calculate $35 + 89$, but typed

$$35 \boxed{-} 79.$$

$$\text{He corrects it like this: } 35 \boxed{-} 79 \boxed{+} 79 \boxed{+} 89 \boxed{=}$$

Explain why his method is correct.

Checking your work: estimate

It is very easy to press wrong keys by accident, and then to get wrong answers. You should develop the habit of always checking calculator answers.

8. Use your calculator to calculate $723 + 489$.
How do you know if the answer is correct?

Mary just types without thinking and did not see that she typed the $\boxed{\times}$ and not the $\boxed{+}$ key. She got the answer 353 547. Mary thought the answer was correct because she thinks the calculator is always right.

But Cyndi always first *estimates* the answer before she starts typing on the calculator. See if you understand her reasoning:

$$723 + 489 \text{ is more than } 700 + 400 = 1\ 100$$

$$723 + 489 \text{ is less than } 800 + 500 = 1\ 300$$

So the answer must be between 1 100 and 1 300.

Only then Cyndi typed on the calculator: $723 \boxed{\times} 489 \boxed{=}$ and just like Mary got the answer 353 547. But Cyndi immediately knew that the answer was wrong and that she must have made a mistake. Then she did it correctly and got 1 212. She was satisfied that the answer seemed reasonable because it is between 1 100 and 1 300. Do you agree?

9. In each case, first estimate the answer like Cyndi did. Then calculate the answer using your calculator, and decide if your answer seems about right.

(a) $3\,456 + 4\,567$

(b) $34\,567 + 45\,678$

(c) 34×56

(d) 678×234

(e) $123\,456 + 257\,257$

(f) $34\,527 + 426\,426$

Using the calculator to check the calculator

Because it is so easy to make mistakes, it is important that you check your calculator answers.

It usually is not a good idea to check a calculation by just repeating it, because you often make the same mistake again. It is better to check by using a different method the second time.

One way to check is to do the calculation in a different order.

10. Do the following calculations on your calculator in the given order and draw a conclusion.

(a) (1) $483 + 159 - 286$

(2) $483 - 286 + 159$

(b) (1) $276 + 288 + 951$

(2) $276 + 951 + 288$

(c) (1) $776 - 288 - 259$

(2) $776 - 259 - 288$

Two different keystroke sequences that give the same answer are called **equivalent sequences**.

You can check calculator results using the rearrangement principle: if you repeat the calculation with a different (but equivalent) keystroke sequence, you will get the same answer.

11. Use your calculator to calculate each of the following. Check the result by using the rearrangement principle.

(a) $15\,432 + 8\,786 + 3\,286$

(b) $15\,432 - 8\,786 + 3\,286$

(c) $15\,432 + 8\,786 - 3\,286$

(d) $15\,432 - 8\,786 - 3\,286$

(e) $15\,432 + 76\,894 + 32\,861$

(f) $15\,432 + 76\,894 - 32\,861$

Checking our work: inverses

12. Use your calculator to calculate each of the following. Draw a conclusion.

(a) $432 + 878 - 878$

(b) $5\,432 - 786 + 786$

(c) $1\,234 + 878 - 878$

(d) $54\,321 - 12\,786 + 12\,786$

(e) $1\,234 + 878 - 878 - 1\,234$

(f) $12\,786 - 12\,786 + 6\,787$

Calculator results can be checked by applying inverse operations to the result, in reverse order. You must then get the original input number as answer.

Sipho must calculate $2\,345 + 3\,214 - 2\,255$.

He uses this keystroke sequence:

$2\,345$ $+$ $3\,214$ $-$ $2\,255$ $=$ and gets $3\,304$.

To check, he continues with $3\,304$ $+$ $2\,255$ $-$ $3\,214$ $=$ and gets $2\,345$, and knows that the answer $3\,304$ must be right. Why?

13. Use your calculator to calculate each of the following.

Check the result by using inverse operations.

(a) $437 + 878$

(b) $837 - 378$

(c) $1\,234 + 878 - 978$

(d) $54\,321 - 12\,786 + 896$

(e) $67\,897 + 87\,834 - 35\,978$

(f) $54\,321 + 12\,786 + 49\,786$

Brackets

14. How can we do $2 \times 4 \times (5 + 6)$ on a calculator?

If you have a calculator with brackets, you can use the bracket keys to do calculations on the calculator just as they are written. If your calculator does not have brackets, you will have to make a plan!

Jane says that we must do the operation in brackets first:

→ 88

Do you agree that the answer is 88? Check on your calculator.

15. Calculate the following using your calculator.

- (a) $15\,432 - (8\,786 + 3\,286)$ (b) $15\,432 - (8\,786 - 3\,286)$
(c) $15\,432 + (8\,786 + 3\,286)$ (d) $15\,432 + (8\,786 - 3\,286)$
(e) $(786 + 289) \times 2 + 3\,456$ (f) $6\,789 - (5\,789 - 3\,276)$

3.10 Apply your knowledge

1. A local municipality has already spent R12 102 436 of its housing budget of R85 514 559. How much money is still available?
2. 253 492 of the 292 388 voters in a district are male. How many of the voters are female?
3. 253 476 new houses were built by the government in new settlements during a certain year. At the end of the year, there were 913 658 houses in the new settlements. How many houses were there at the beginning of that year?
4. During a previous election there were 863 458 registered voters in a certain city. During the next election there were 808 389 voters in the city. Did the number of voters increase or decrease? By how many?
5. 517 866 learners wrote the Grade 12 examinations in 2009, and 100 376 more wrote in 2014. How many learners wrote the examinations in 2014?

-
6. On a certain day, 238 756 ℓ of water from a water tank are used. At the end of the day, 688 782 ℓ are left. How much water was in the water tank at the beginning of the day?
 7. During an election, 398 065 people voted for the Family First Party and 397 676 people voted for the Fight Fair Party. By how many votes did the Family First Party win?
 8. A business buys a truck for R985 650 and a trailer for R398 950. How much do the truck and trailer cost together?
 9. The budgets of two schools are R874 800 and R978 500 for the year. How much is their combined budgets?
 10. A pre-primary school's budget is R964 500. This is R106 600 more than the previous year. What was the budget the previous year?
 11. During the first week of an arts festival, 104 475 people attended the festival. During the next week, 106 568 people attended. How many people went to the festival?
 12. (a) In a municipal election, 85 324 people voted for Mrs Dlamini and 52 689 people voted for Mr Brown. How many more votes did Mrs Dlamini get than Mr Brown?
(b) There are 27 689 learners at a big sports meeting. Only 12 324 T-shirts were delivered. How many more T-shirts should be brought, so that each learner can get a T-shirt?
 13. Bongani has a contract to repair 38 864 m of fencing along a road. He has already repaired 25 298 m. How many metres of fencing does he still have to repair?

4.1 Dividing into fraction parts

1. How much is each of the following?

(a) 3×8

(b) 4×6

(c) 2×12

(d) $24 \div 8$

(e) $24 \div 6$

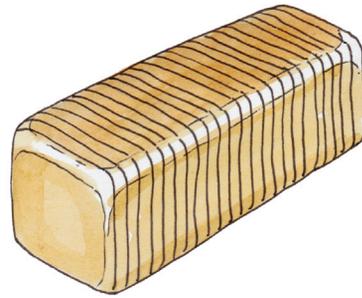
(f) $24 \div 3$

(g) $24 \div 4$

(h) $24 \div 12$

(i) $24 \div 2$

This loaf of bread is cut into 24 equal slices.



2. How many slices will each person get if the loaf of bread is **shared** equally between 3 people?

3. How much will each person get if R24 is shared equally between 3 people?

4. How much will each person get if R24 is shared equally between 4 people?

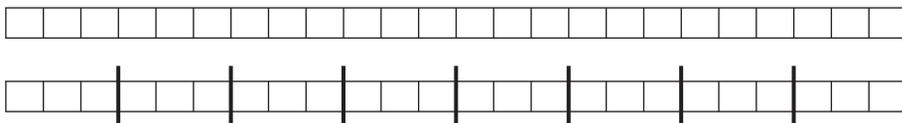
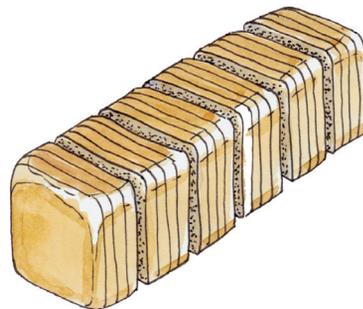
5. The 24 slices can be **grouped** into equal portions.

(a) How many portions of 4 slices each can be made up from the whole loaf?

(b) How many portions of 2 slices each can be made up from the whole loaf?

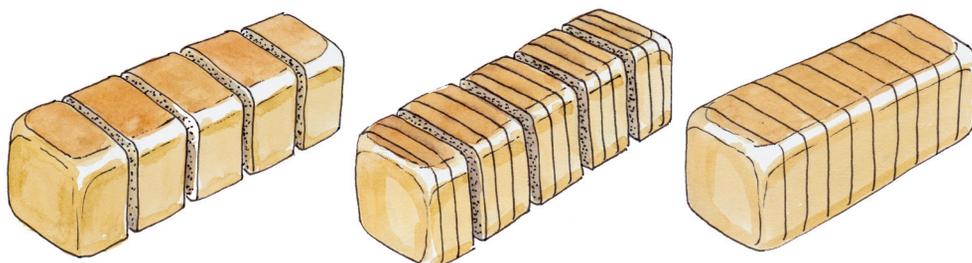
(c) How many portions of 8 slices each can be made up from the whole loaf?

(d) How many portions of 3 slices each can be made up from the whole loaf?



If something is divided into 5 equal parts, each part is called a **fifth** of the whole.

If something is divided into 15 equal parts, each part is called a **fifteenth** of the whole.



6. (a) Which is more, 1 fifth of a loaf or 4 fifteenths of a loaf?
- (b) Which is more, 1 fifth of a loaf or 2 fifteenths of a loaf?
- (c) Which is more, 1 fifth of a loaf or 3 fifteenths of a loaf?
- (d) Which is more, 3 fifths of a loaf or 10 fifteenths of a loaf?
- (e) Which is more, 2 fifths of a loaf or 4 tenths of a loaf?
- (f) Which is more, 1 fifth of a loaf or 1 sixth of a loaf?

This is a rough drawing to show what is meant by twelfths.



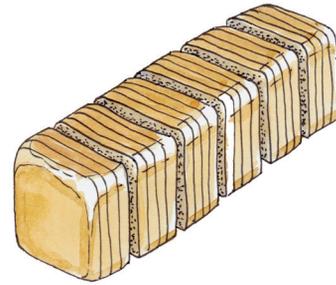
7. (a) Quickly make a neater and better rough drawing of what is meant by twelfths. Do not use a ruler.
 - (b) Draw a line inside each twelfth on your drawing, to roughly divide it into two equal parts.
 - (c) Into how many parts is your drawing now divided?
 - (d) What can each of the small parts be called?
8. (a) Make a rough drawing to show what is meant by eighths.
 - (b) Can you draw more lines on your drawing so that it shows what is meant by sixteenths?

This loaf is cut into 24 slices.

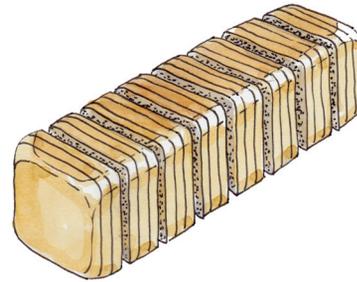
If the slices are equal, each slice is **one twenty-fourth** of the loaf.

In fraction notation, one twenty-fourth is written as $\frac{1}{24}$.

5 twenty-fourths is written as $\frac{5}{24}$.



9. (a) How many slices are there in one sixth of the loaf?
(b) How many slices are there in $\frac{5}{6}$ of the loaf?
(c) How many slices are there in $\frac{3}{8}$ of the loaf?
(d) How many thirds of the loaf is the same as $\frac{8}{24}$ of the loaf?



10. Write each of the following in fraction notation.

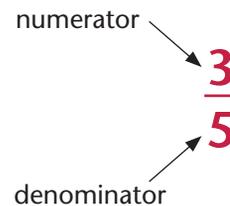
- (a) 6 twenty-fourths (b) 6 twentieths
(c) 7 tenths (d) 7 sixteenths

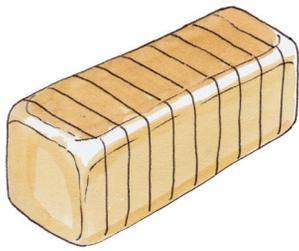
11. Write each of the following in words.

- (a) $\frac{7}{15}$ (b) $\frac{10}{50}$
(c) $\frac{5}{48}$ (d) $\frac{3}{8}$

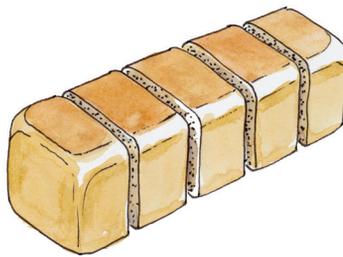
In fraction notation, the number below the line states the number of equal parts into which the whole is divided. It is called the **denominator**.

The number above the line states the number of equal parts. It is called the **numerator**.

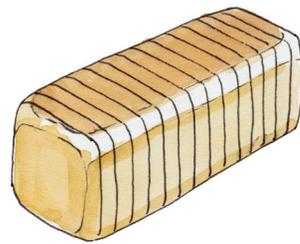




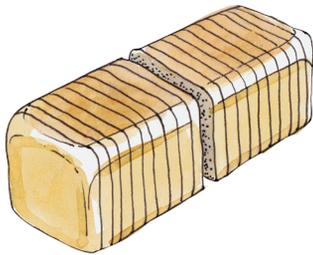
Loaf A



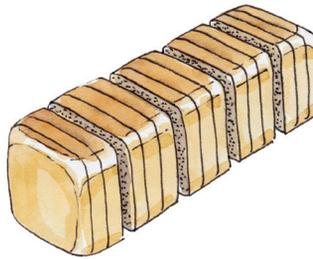
Loaf B



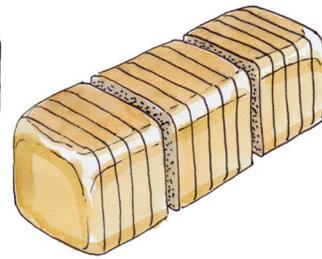
Loaf C



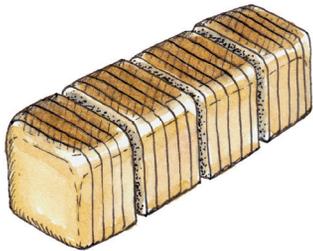
Loaf D



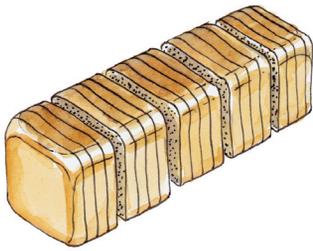
Loaf E



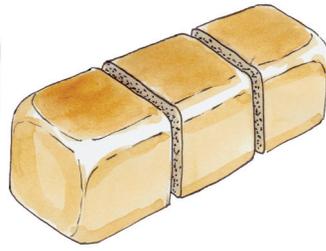
Loaf F



Loaf G



Loaf H



Loaf I

All the loaves above are exactly the same size, but they have been cut differently.

Loaf I has been cut into 3 thick slices and Loaf C into 15 thinner slices.

Loaf H has been cut into twentieths, and these slices are grouped into five equal portions.

Three slices of Loaf B is 3 fifths or $\frac{3}{5}$ of the loaf.

12. What part of a loaf is each of the following?

Write your answer in words and in fraction notation.

(a) 8 slices of Loaf H

(b) 4 slices of Loaf A

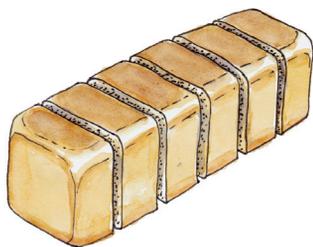
(c) 2 slices of Loaf B

(d) 6 slices of Loaf C

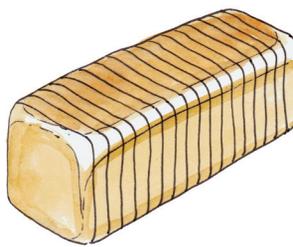
13. (a) Which loaf is cut into fifteenths?
 (b) Which loaf is cut into twentieths?
 (c) How many twentieths is 3 quarters?
14. (a) How many slices are there in 1 fifth of Loaf A?
 (b) How many slices are there in 2 fifths of Loaf A?
 (c) How many slices are there in 2 fifths of Loaf C?
 (d) How many slices are there in 2 fifths of Loaf H?
15. (a) Which is more, $\frac{2}{3}$ of a loaf or $\frac{2}{5}$ of a loaf?
 (b) Which is more, $\frac{2}{3}$ of a loaf or $\frac{3}{5}$ of a loaf?

It may help you to look at the pictures of Loaf E and Loaf F, and to count the slices.

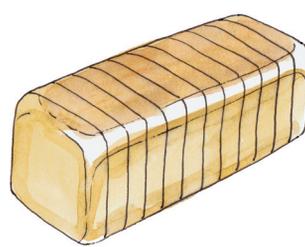
16. (a) Look at Loaf H. How many twentieths make up $\frac{3}{5}$ of a loaf?
 (b) Look at Loaf E. How many fifteenths make up $\frac{3}{5}$ of a loaf?
 (c) Is $\frac{12}{20}$ of a loaf the same amount of bread as $\frac{9}{15}$ of the same loaf?
17. What part of a loaf is each of the following?
 Write your answers in words and in fraction notation.
- | | |
|------------------------|-------------------------|
| (a) 5 slices of Loaf J | (b) 15 slices of Loaf K |
| (c) 8 slices of Loaf L | (d) 4 slices of Loaf J |
| (e) 2 slices of Loaf J | (f) 6 slices of Loaf K |



Loaf J



Loaf K



Loaf L

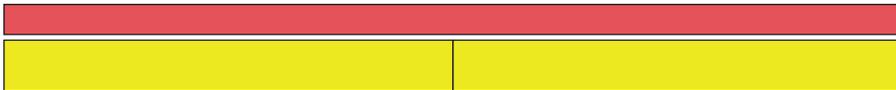
4.2 Measuring lengths accurately

In this section we will use a new “measuring unit”, the Yellowstick. You will find out how we can measure more accurately if we subdivide the unit into smaller fractional parts.

This is our unit, the Yellowstick:



The red strip below is exactly 2 Yellowsticks long:



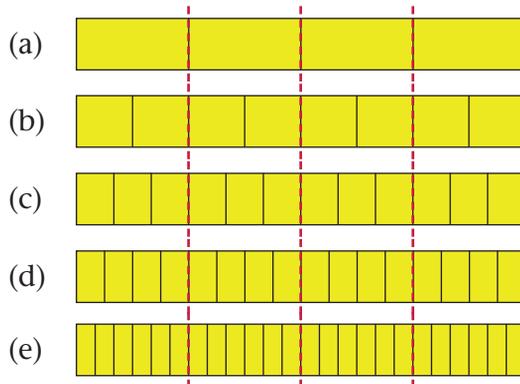
The green strip below is a bit longer than 1 Yellowstick, but shorter than 2 Yellowsticks.



To measure the green strip accurately, we need to find out what fraction of a Yellowstick the extra bit is. For that purpose we need Yellowstick rulers that are divided into smaller equal parts.

1. What shall we call the parts of each of these Yellowsticks?

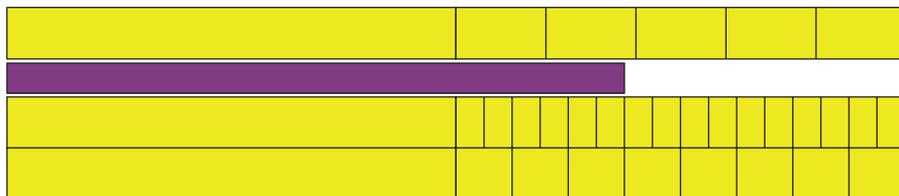
The broken lines may help you to count the number of equal parts.



2. (a) How many eighths of a Yellowstick are the same length as 3 quarters of a Yellowstick?
(b) How many twenty-fourths of a Yellowstick are the same length as 3 quarters of a Yellowstick?

We shall call a Yellowstick that is divided into sixths a sixths ruler. A Yellowstick that is divided into eighths is called an eighths ruler, etc.

3. (a) Is the purple strip below one and 2 fifths of a Yellowstick long?



- (b) Is the purple strip one and 3 eighths of a Yellowstick long?

- (c) Is the purple strip one and 6 sixteenths of a Yellowstick long?

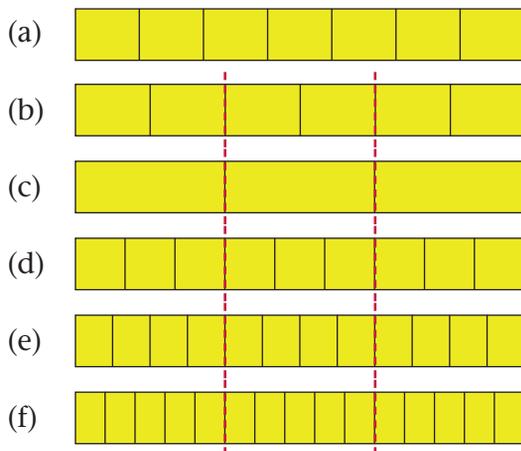
When two fractions describe the same quantity or length, we say they are **equivalent**.

Equivalent means having equal (the same) value.

3 eighths is equivalent to **6 sixteenths**.

We can write $\frac{3}{8} = \frac{6}{16}$.

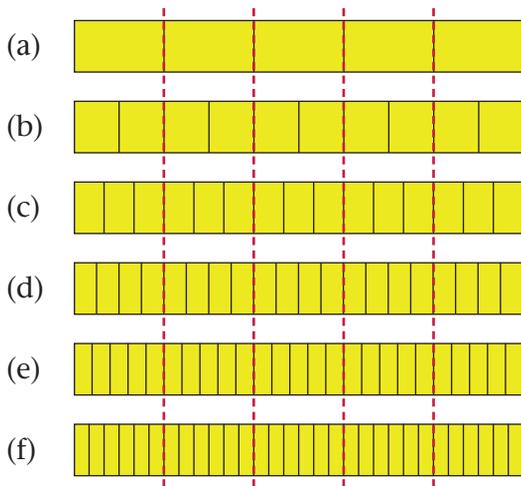
4. What shall we call the parts of each of these Yellowsticks?



5. (a) How many fifteenths of a Yellowstick are the same length as 2 thirds of a Yellowstick?
 (b) How many twelfths of a Yellowstick are the same length as 3 ninths of a Yellowstick?

6. (a) Name two fractions that are equivalent to $\frac{2}{3}$.
 (b) Name two fractions that are equivalent to $\frac{3}{4}$.

7. What shall we call the parts of each of these Yellowsticks?

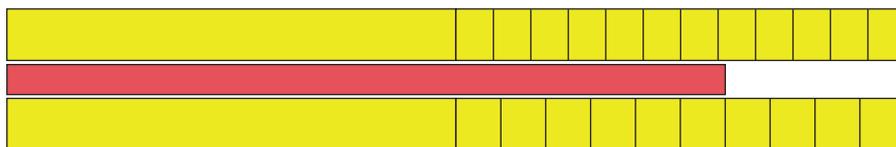


You can see in question 7 that $\frac{8}{20}$ is equivalent to $\frac{2}{5}$.
 We say: 2 fifths can be **expressed** in twentieths as 8 twentieths.

8. Express $\frac{3}{5}$ in
 (a) tenths (b) fifteenths
 (c) twentieths (d) twenty-fifths
 (e) thirtieths.

Write your answers in words and in fraction notation.

9. (a) How long is this red strip?

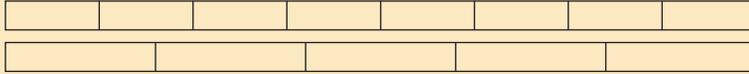


- (b) Express the length of the red strip in four different ways, with equivalent fractions.

4.3 Comparing and ordering fractions

Which is more, $\frac{5}{8}$ of a loaf of bread or $\frac{3}{5}$ of a loaf of bread?

You can make neat rough drawings to answer a question such as this.



Drawings like these are called **fraction strips**.

A fraction strip showing eighths is called an **eighths strip**.

A fraction strip that shows fifths is called a **fifths strip**.

- Make your own copy of the above fraction strips. Do not use a ruler, so that you can do it quickly. The two strips must have the same length.
 - Which is more, $\frac{5}{8}$ of a loaf or $\frac{3}{5}$ of a loaf?
 - Which is more, $\frac{3}{8}$ of a loaf or $\frac{2}{5}$ of a loaf?
 - Which is more, $\frac{6}{8}$ of a loaf or $\frac{3}{5}$ of a loaf?
 - Which is more, $\frac{3}{4}$ of a loaf or $\frac{4}{5}$ of a loaf?
- Draw more lines on your fraction strips so that you can answer the following questions.
 - Which is more, $\frac{5}{8}$ of a loaf or $\frac{7}{10}$ of a loaf?
 - Which is more, $\frac{3}{10}$ of a loaf or $\frac{5}{16}$ of a loaf?
 - Which is more, $\frac{5}{10}$ of a loaf or $\frac{8}{16}$ of a loaf?
- Quickly make fraction strips to find out which is bigger, $\frac{5}{8}$ of an object or $\frac{4}{6}$ of the same object. Do not use a ruler.
 - Which is bigger, $\frac{6}{8}$ of an object or $\frac{5}{6}$ of the same object?

4. In each case compare the two fractions. State which is the bigger of the two fractions of an object, or whether the two fractions describe the same part of the object. Try to do it without making drawings. You may make drawings if you are unsure of your answer. The drawings of fraction strips at the bottom of this page may also help you in some cases.

(a) $\frac{3}{10}$ or $\frac{5}{12}$

(b) $\frac{5}{12}$ or $\frac{3}{8}$

(c) $\frac{7}{20}$ or $\frac{3}{10}$

(d) $\frac{7}{20}$ or $\frac{3}{8}$

(e) $\frac{8}{15}$ or $\frac{11}{20}$

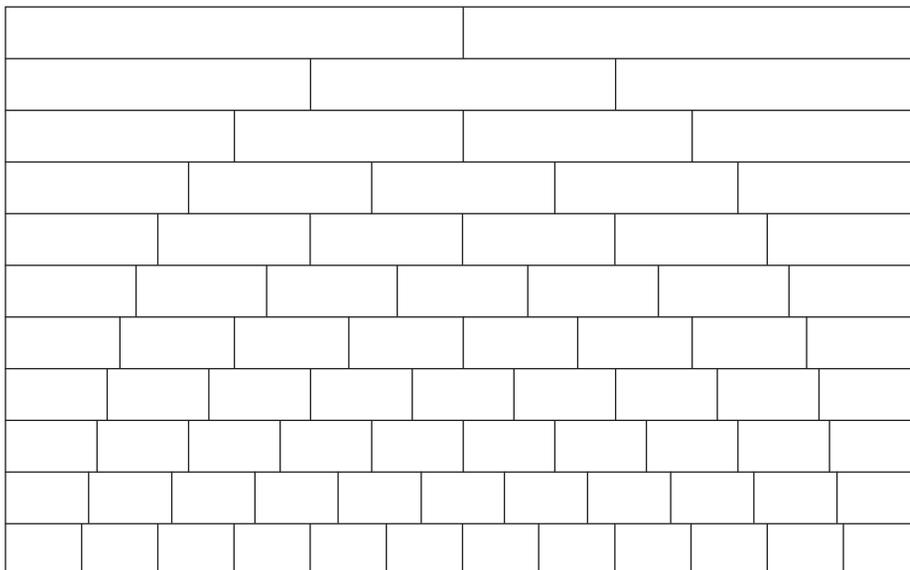
(f) $\frac{4}{9}$ or $\frac{7}{12}$

5. Order the fractions from smallest to biggest in each case.

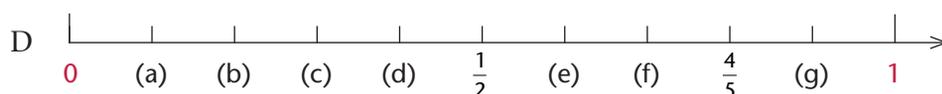
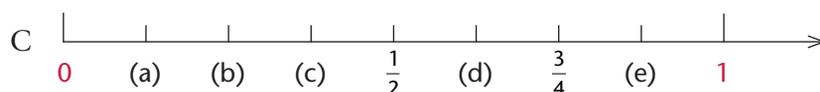
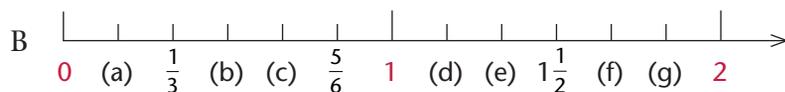
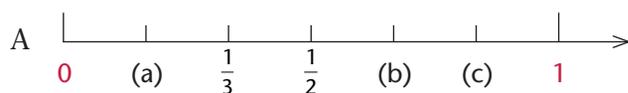
(a) $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{7}$, $\frac{9}{20}$

(b) $\frac{17}{20}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{7}{12}$

(c) $\frac{11}{20}$, $\frac{2}{3}$, $\frac{2}{7}$, $\frac{7}{15}$



6. Fill in the missing numbers on these number lines.



4.4 Hundredths

This fraction strip shows fifths. The strip is divided into five equal parts. We can call it a fifths strip.



The strip can be changed into a fifteenths strip, by dividing each fifth into three equal parts:

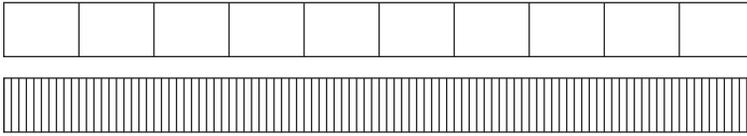


- (a) Describe how a fifths strip can be changed into a tenths strip. If you wish, you can make a rough drawing to help you do it.
(b) Describe how a fifths strip can be changed into a twentieths strip.

If something is divided into 10 equal parts, each part is called a **tenth** of the whole.

If something is divided into 100 equal parts, each part is called a **hundredth** of the whole.

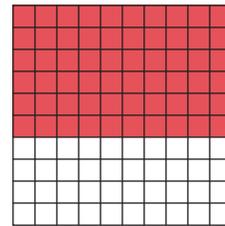
2. Describe how a tenths strip can be changed into a hundredths strip.



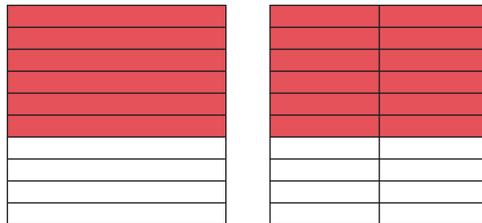
3. How many hundredths of each strip below are coloured? Explain your answers.



4. There are 100 square tiles on this floor. The two diagrams below may help you to find the answers to these questions.

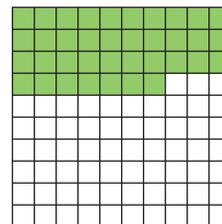


- (a) How many tenths of all the tiles are red?
 (b) How many hundredths of all the tiles are red?
 (c) How many twentieths of all the tiles are red?



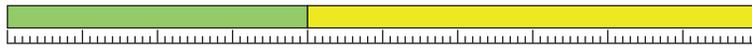
5. Are any of the following statements about the floor on the right false?

- (a) $\frac{37}{100}$ of the floor is green.
 (b) $\frac{2}{10} + \frac{17}{100}$ of the floor is green.
 (c) $\frac{3}{10} + \frac{7}{100}$ of the floor is green.

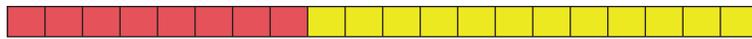


6. Describe in three different ways what part of the floor in question 5 is white.

7. (a) How many tenths of the strip below are green?



- (b) How many hundredths of the strip are green?
 (c) How many hundredths of the strip are yellow?
 (d) What part of the strip below is red?

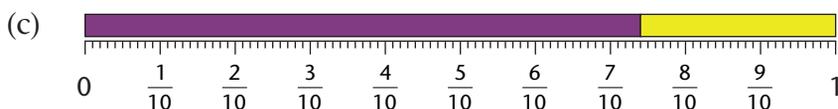
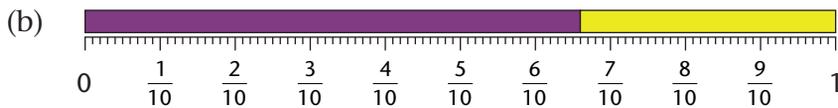
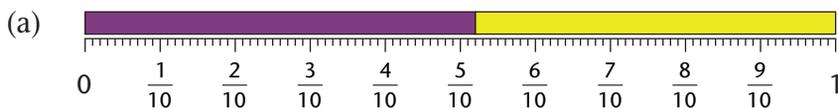


- (e) Will it be wrong to say that four-tenths of this strip is red?

8. The coloured strip below is 120 mm long.
 It is divided into 5 equal parts.



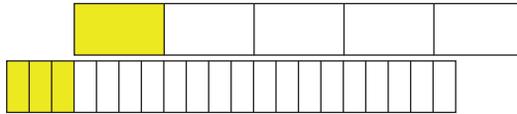
- (a) What fraction part of the whole strip is green?
 (b) Calculate how long the green part is, then check your answer by measuring it.
 (c) What fraction part of the whole strip is red?
 (d) How many hundredths of the strip are red?
9. What part of each strip below is purple? State each of your answers in at least two different ways.



10. What part of each strip in question 9 is yellow?

4.5 Adding and subtracting fractions

1. What part of a litre of milk will you get if you add a fifth of a litre to 3 twentieths of a litre? You may find the diagrams below helpful.



2. (a) How many twentieths are equal to one fifth?

(b) Is $\frac{1}{5} = \frac{5}{20}$ or is $\frac{1}{5} = \frac{4}{20}$?

(c) Is $\frac{1}{5} + \frac{3}{20} = \frac{4}{20} + \frac{3}{20}$?

3. Calculate:

(a) $\frac{1}{5} + \frac{3}{5}$

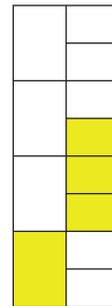
(c) $\frac{3}{5} + \frac{2}{5}$

(e) $\frac{1}{2} + \frac{1}{4}$

(b) $\frac{3}{12} + \frac{5}{12}$

(d) $\frac{3}{8} + \frac{5}{8}$

(f) $\frac{1}{4} + \frac{3}{8}$



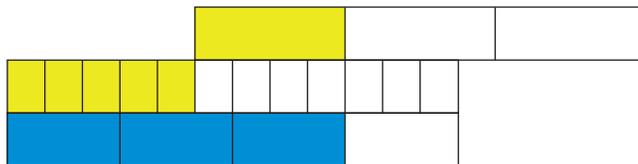
It is easy to add fractions that are expressed with the same denominator, like $\frac{5}{12}$ and $\frac{3}{12}$:

5 twelfths + 3 twelfths = 8 twelfths

To add fractions with different denominators, we have to use

equivalent fractions. For example, to calculate $\frac{5}{12} + \frac{1}{3}$, we have to replace $\frac{1}{3}$ with $\frac{4}{12}$:

$\frac{5}{12} + \frac{1}{3} = \frac{5}{12} + \frac{4}{12}$ and 5 twelfths + 4 twelfths is 9 twelfths.



4. Is it true that $\frac{5}{12} + \frac{1}{3} = \frac{3}{4}$?

5. Calculate each of the following:

(a) $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$

(b) $\frac{2}{3} + \frac{1}{6} + \frac{5}{6}$

(c) $\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

(d) $\frac{3}{8} - \frac{3}{8}$

(e) $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

(f) $\frac{7}{8} - \frac{3}{8}$

(g) $\frac{15}{16} - \frac{3}{16}$

(h) $\frac{15}{16} - \frac{3}{8}$

(i) $\frac{17}{20} + \frac{3}{10} - \frac{2}{5}$

(j) $\frac{7}{12} + \frac{3}{4}$

(k) $\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$

(l) $\frac{3}{5} + \frac{2}{15} + \frac{4}{5} - \frac{7}{15}$

(m) $\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

(n) $\frac{2}{10} + \frac{7}{100}$

(o) $\frac{74}{100} + \frac{3}{20} + \frac{3}{10} + \frac{9}{100}$

(p) $\frac{23}{100} + \frac{3}{10} + \frac{9}{10} + \frac{5}{100} + \frac{3}{20}$

(q) $\frac{35}{100} + \frac{3}{100} + \frac{70}{100} - \frac{2}{10}$

$7\frac{3}{10} - 3\frac{4}{5}$ can be calculated like this:

$$7\frac{3}{10} - 3 \rightarrow 4\frac{3}{10} - \frac{4}{5} \rightarrow 3\frac{1}{5} + \frac{3}{10} \rightarrow 3\frac{2}{10} + \frac{3}{10} \rightarrow 3\frac{5}{10} = 3\frac{1}{2}$$

Judy calculates $7\frac{3}{10} - 3\frac{4}{5}$ like this:

$$7 - 3 \rightarrow 4 - \frac{4}{5} \rightarrow 3\frac{1}{5} + \frac{3}{10} \rightarrow 3\frac{2}{10} + \frac{3}{10} \rightarrow 3\frac{5}{10} = 3\frac{1}{2}$$

Is Judy correct?

6. Try Judy's method or your own method to calculate $9\frac{1}{4} - 6\frac{3}{8}$.

7. It helps to be able to do certain calculations mentally. Try to calculate these in your head, without doing any writing.

(a) $\frac{1}{8} + \frac{3}{8}$

(b) $3 - \frac{3}{5}$

(c) $\frac{5}{8} + \frac{5}{8}$

(d) $3 - 2\frac{1}{4}$

(e) $3 + \frac{6}{5}$

(f) $5 - \frac{4}{7}$

(g) $\frac{7}{8} + \frac{5}{8}$

(h) $2\frac{3}{5} + \frac{4}{5}$

8. Calculate:

(a) $10\frac{1}{3} - 2\frac{5}{6}$

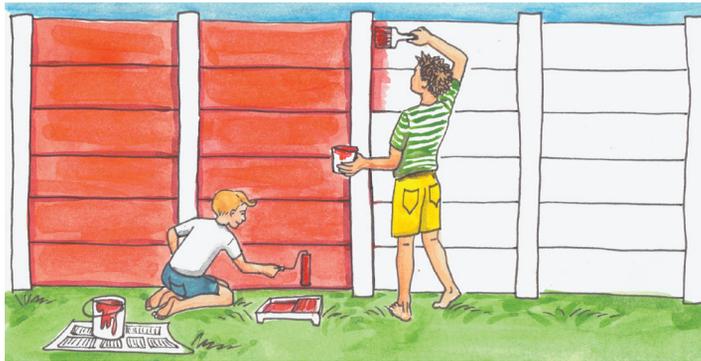
(b) $7\frac{3}{8} + 2\frac{3}{4} - \frac{1}{2}$

(c) $3\frac{7}{12} + 4\frac{5}{6} - \frac{1}{3}$

(d) $5\frac{1}{4} + 2\frac{1}{2} - \frac{7}{8}$

4.6 Problem solving

1. Bill cuts a piece of string which is $3\frac{5}{8}$ m long from a piece which is $10\frac{1}{4}$ m long. How long is the remaining piece of string?
2. Sarah has $8\frac{3}{4}$ m of lace. She cuts off three pieces which are each $\frac{3}{8}$ m long. What length is left?
3. Bongi wants to join three pipes of the same width. Their lengths are $3\frac{3}{4}$ m, $5\frac{1}{2}$ m and $4\frac{7}{8}$ m. How long will the pipe be?
4. Ben paints the garden wall red. The wall consists of 24 panels (divisions) of the same size.



- (a) On the first day, Ben painted $\frac{1}{3}$ of the wall. How many panels did he paint?
- (b) The following day he painted another $\frac{1}{6}$ of the wall. What fraction of the wall was then painted red?
- (c) On the third day, Ben painted another $\frac{1}{4}$ of the wall. His friend, Nick helped him and painted $\frac{1}{8}$ of the wall. What fraction of the wall did the two of them paint that day?
- (d) How many panels of the wall were still not painted red after three days?
- (e) What fraction of the whole wall is that?

5. 16 Vienna sausages are shared equally by a number of children. Each child gets $2\frac{2}{3}$ sausages. How many children are there?
6. A packet of Vienna sausages is shared equally by 9 children. Each child gets $4\frac{1}{3}$ sausages. How many sausages were there in the packet?
7. A chocolate slab is divided into 24 small blocks. Copy this table and write your answers to questions (a), (b), (c) and (e) below in the table.

The answers for 2 people sharing equally have been done for you.

Number of people who share	2	3	4	5	6	7	8
Number of blocks per person	12						
Fraction per person	$\frac{1}{2}$						
Fraction written in another way	$\frac{12}{24}$						

- (a) How many people can equally share this slab *easily*? Mark the numbers in the first row of the table.
 - (b) How many blocks will each person get in each case?
 - (c) What part (fraction) of the slab will each person get in each case?
 - (d) Did you find all possible answers to question (a)? How do you know?
 - (e) Try to write each fraction that you wrote in the third row of the table in another way. Do this in the last row.
8. There are 600 houses in Township A and 240 houses in Township B. 150 of the houses in Township A have running water, and 80 houses in Township B have running water.
 - (a) What fraction of the houses in each township don't have running water?
 - (b) In which township is the situation the best, with respect to the provision of running water?

5.1 Introduction

1. Read the text below.

The story goes that a long time ago a stranger reached a village and asked: “Will I be able to reach the next village before dark?”

A wise old woman looked at the sun and then instructed the man: “Please walk to the tree over there and back.”

The man thought it was strange, but did as he was asked. When he had returned to the old woman, she laughed: “Yes, yes you will definitely reach the next village before it is dark.”

“Thank you,” said the stranger, took his bag, and continued on his journey.

The next day, when the sun was more or less in the same position, one of the older villagers said: “I am going to walk to the next village.”

The wise old woman said: “I do not think you should go now because you will not reach the village before it is dark.”

2. Discuss: How did the wise old woman know that the stranger would reach the next village before dark?
3. Why did the woman think that the older villager would not reach the next village before dark?
4. Suppose one late afternoon your friend invites you to watch a sporting event at his home. Your parents say that you may go as long as you reach your friend’s house before it is dark.

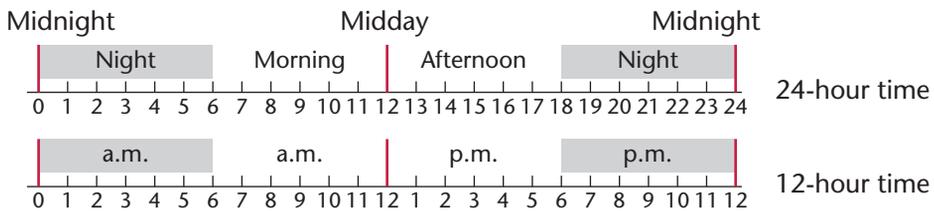
How would you determine whether you will reach your friend’s house before dark if it is about 4 km away?

5.2 Read, write and tell time

There are 24 hours in a day.

A **24-hour clock** tells us how much time has passed since *midnight*.

A **12-hour clock** tells us either how much time has passed since *midnight* or how much time has passed since *midday (noon)*.



What time does this **digital 24-hour clock** show?

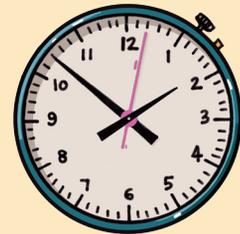
We *write* the time as 13:05 in 24-hour notation or as 1:05 p.m. in 12-hour notation.



We *say* the time is five minutes past one in the afternoon.

What time does this **analogue 12-hour clock** show?

If it is in the afternoon we *say* the time is eight minutes before two in the afternoon, ignoring seconds. We *write* it as 13:52 in 24-hour notation and as 1:52 p.m. in 12-hour notation.



If it is during the night we *say* the time is eight minutes before two in the night. We *write* it as 01:52 in 24-hour notation and as 1:52 a.m. in 12-hour notation.

- Write the times at which your school starts and ends in
 - words
 - 12-hour notation
 - 24-hour notation.
- Write these 24-hour times in 12-hour notation, in symbols and in words.

(a) 07:00	(b) 08:15	(c) 11:30	(d) 12:00
(e) 12:45	(f) 19:48	(g) 23:50	(h) 00:10

3. Write the time on each of these clock faces in 24-hour notation, in symbols and in words.

(a)



(b)



(c)



(d)



(e)



(f)



4. Write these 24-hour times in 12-hour notation, in symbols and in words.

(a)



(b)



(c)



(d)

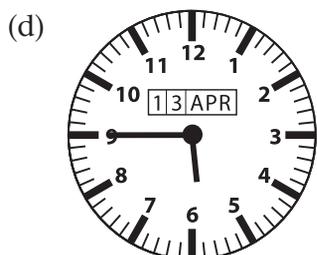
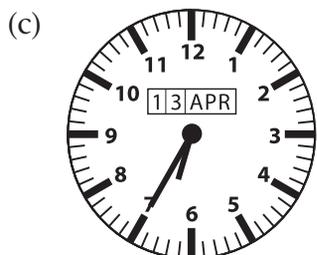
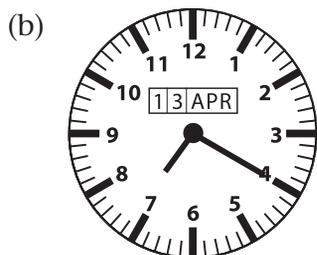
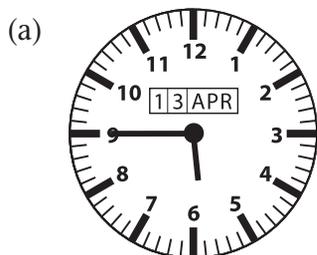


5.3 Time intervals

1. The clock faces below show the time when an activity started and when it ended. Calculate how long each activity lasted. Note that the clock faces show the time as well as the date (in the same year). Take the start times as morning and the end times as evening.

Start (morning)

End (evening)



2. Complete the table. Explain your methods.

Hour(s)	$\frac{1}{4}$	$\frac{1}{2}$		$1\frac{1}{2}$		$2\frac{1}{4}$		3
Minutes	15		45				150	
Seconds	900				7 200			

3. The table shows the times a train stops at the different stations along the Gauteng Metrorail route. How long does it take the train to travel from

- (a) Orlando to Longdale
 (b) New Canada to Mayfair
 (c) Croeses to Johannesburg
 (d) Orlando to Johannesburg?

Station	Time
Orlando	10:47
Mlamlankunzi	10:50
New Canada	10:53
Longdale	10:57
Croeses	10:58
Langlaagte	11:02
Grosvenor	11:05
Mayfair	11:07
Braamfontein	11:10
Johannesburg	11:15

4. Another train on the Orlando–Johannesburg route leaves Orlando station at 12:55. At what time does it arrive in Johannesburg?

5. Naledi travels by bus from Pretoria to Cape Town. The table shows the main route stops and times. How long does it take the bus to travel from

- (a) Pretoria to Bloemfontein
 (b) Johannesburg to Beaufort West
 (c) Pretoria to Worcester
 (d) Pretoria to Cape Town?

City/Town	Time
Pretoria	05:45
Johannesburg	06:45
Bloemfontein	11:00
Beaufort West	16:30
Worcester	20:15
Paarl	20:45
Bellville	21:15
Cape Town	21:30

6. A movie on television is 2 hours 55 minutes long and ends at 16:45. At what time did it start?

5.4 Time intervals on the stopwatch

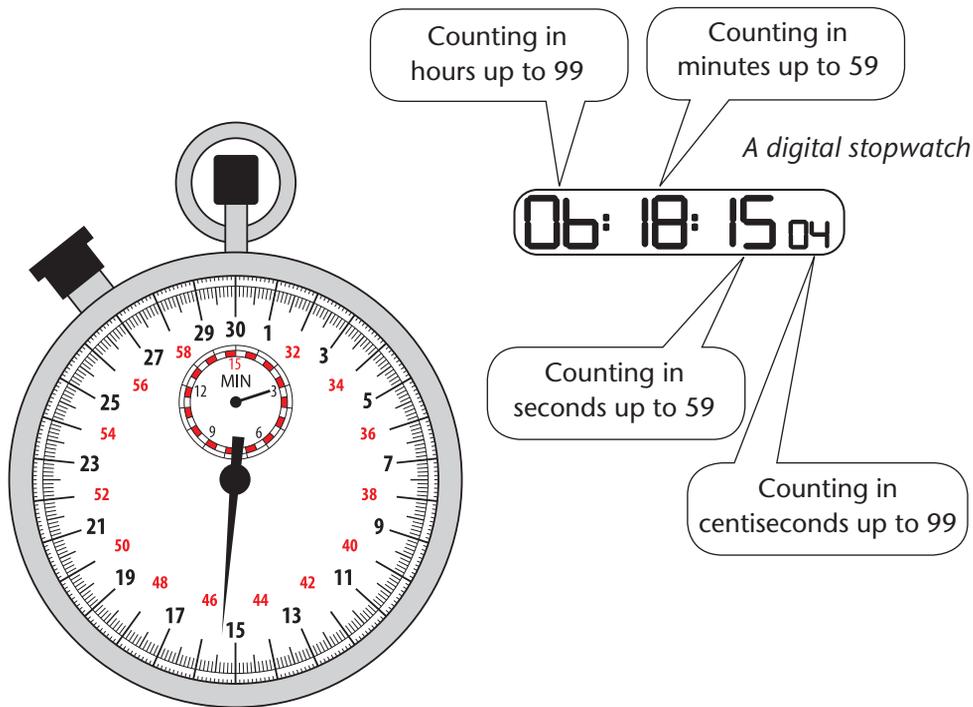
- Use a stopwatch (for example the stopwatch on a cell phone). Practise using it accurately. Then measure the following durations of time:
 - the time it takes your whole class to walk into the classroom in a neat row and to all sit down at your desks
 - the time it takes you to read a paragraph of about 12 lines in an English storybook
 - the time it takes you to tie your two shoe laces, or do up your buckles
- Order the times you measured in question 1 from short to long.
- In 2015, Caroline Wöstmann from South Africa won the Comrades women's marathon. Charné Bosman, who came second, is also from South Africa! Here are the times of the top four South African runners in the race:

Florence Griffith Joyner, from the United States, ran the 100 m race in 10,49 seconds in 1998. Her record still stands. Stopwatches are used to measure athletes' race times, because stopwatches can measure time very accurately from hours to fractions of a second.

Position	Runner	Stopwatch time
1st	Caroline Wöstmann	06:12:22
2nd	Charné Bosman	06:33:24
7th	Emmerentia Rautenbach	06:45:22
10th	Yolande Maclean	07:01:49

- The race started at exactly 05:30. At what time did Caroline Wöstmann finish the race?
- How much faster was Caroline Wöstmann than Charné Bosman?
- How much faster was Emmerentia Rautenbach than Yolande Maclean?

4. Tiko is the timekeeper of the soccer match at school. He has to make sure injury time is added to the final time after each half. Each half is 45 minutes long.



An analogue stopwatch

- (a) Do you think Tiko has to measure in centiseconds during this match? Why or why not?
- (b) Do you think one can score a goal in 30 seconds?
- (c) Should Tiko measure seconds accurately? Why?
- (d) The match started at exactly 15:00. After 12 minutes there was an injury that took 90 seconds to clear. At what time did the game resume?
- (e) Tiko measured a total of 4 minutes 24 seconds of injury time in the first half. At what time did he end the first half?

Did you know?
One centisecond is one hundredth of a second.

5.5 Years, decades and centuries

A normal calendar **year** has 365 **days**.

A year has 52 weeks.

A **decade** is a period of 10 years.

A **century** is a period of 100 years.

1. Do not look at a calendar.
 - (a) Make a list of the months of the year and write down the number of days in each month.
 - (b) How many months, weeks and days have passed since the beginning of the year until today?
 - (c) How many years and months will you still spend at school if you plan to attend school up to Grade 12?
 - (d) How old are you today in years, months, weeks and days?

We can say, “South Africa has been a democracy for more than two decades.” We can also say, “The decade of the 1980s was when East European countries became independent from the Soviet Union.”

1980s	1990s	2000s	2010s
Berlin Wall came down	South Africa became a democracy (1994)	Cell phones became popular and affordable	Soccer World Cup in South Africa (2010)
Soviet Union states became independent	Nelson Mandela became the president (1994)		

2. Find out about the decades during which you and your family have lived.
 - (a) What important things happened to your family in the last decade?
 - (b) Ask older people in your community what events they remember in each of the last three decades.

Centuries are traditionally counted with reference to the birth of Christ. Centuries before the birth of Christ are counted backwards. For example, the pyramids in Egypt were built about 3 000 years before the birth of Christ, written as 3000 **BC**.

We are now living in the **21st century**, meaning the 21st century After Christ (AC).

Nowadays people are increasingly using the expression “Common Era” (CE) instead of AC, and “Before the Common Era” (BCE) instead of BC because it is more neutral and inclusive of non-Christian people. Instead of 3000 BC we write 3000 **BCE**.

3. Mutodi says he does not understand why 2016 is in the 21st century and not in the 20th century. He thinks:
- 1960 (we read it as 19-sixty) should be in the 19th century, and
 - 2016 (20-sixteen) should be in the 20th century.

Investigate the situation so that you understand it, and then explain to someone why 2020 is in the 21st century.

4. Interesting inventions and discoveries were made in each of the last four centuries.

1700s 18th century	1800s 19th century	1900s 20th century	2000s 21th century
The steam engine and the spinning machine were invented	Electricity was discovered Trains were invented	Cars and aeroplanes were invented	Cell phones, computers and the internet came into wide use

Do research about the last five centuries. Work with a classmate and answer questions such as the following.

- (a) In which century were motor cars invented?
- (b) In which century was the printing press invented and were books printed for the first time?
- (c) In which century did the first people sail around the Earth?

5.6 A short history of calendars

Calendars are part of a complete timekeeping system: the date and time of day together specify an exact moment in time. This then makes it possible to calculate past or future time, for example to calculate how many days until a certain event takes place.

The **calendar year** (the number of days in the year) must be synchronised to the cycle of the seasons, so that the seasons start on the same dates every year. This means that the calendar year must be synchronised to the **solar year** (the exact time that it takes the Earth to move around the Sun once).

The problem with designing a calendar is that a calendar year must have a whole number of days (why?), but the solar year is not a whole number of days (it is about 365 days 5 hours 48 minutes 46 seconds). To solve the problem, we must *approximate* the solar year with a whole number of days, over a period of time. Throughout history, people tried to make better calendars by making better and useful approximations of the solar year. This is done by adding extra whole days in some years.

Roman Calendar

The Roman Calendar was invented by King Romulus at around 753 BCE. It was a lunar calendar, based on the phases of the moon. The year started in March and had 10 months with a total of 304 days, with 61 days in the winter not included.

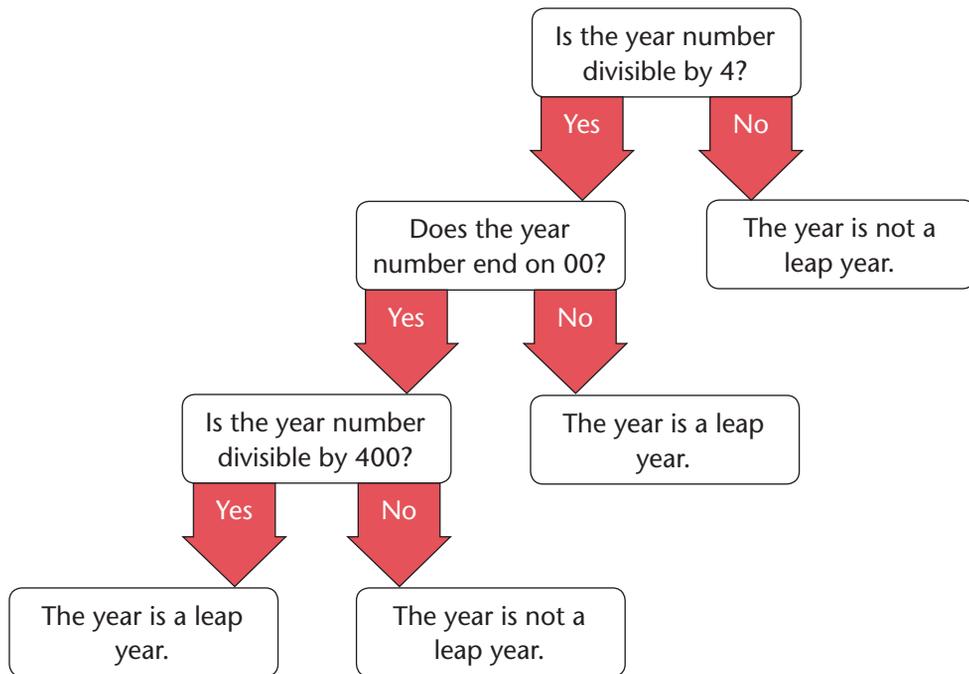
Around 700 BCE King Pompilius added the months of January and February to the calendar, increasing the calendar year to 355 days. The addition of two extra months meant that some of the months' names no longer agreed with their position in the calendar. For example, December was originally the 10th month (deci = tenth).

1. Do some research about the origins of the names of the months, and the names of the days of the week. Why July? Why Monday?

Julian Calendar

To create a more standardised calendar, the Roman Emperor Julius Caesar followed the advice of Sosigenes, an astronomer and mathematician from Alexandria in Egypt. He made some sweeping changes to the calendar. This calendar was named the Julian Calendar.

4. The flow diagram below is a recipe to find out if a year is a leap year or not in the Gregorian Calendar.



Which of the following years are leap years?

(a) 1600 1700 1800 1900 2000 2100 2200 2300 2400

Is it true that 3 out of every 4 century years are not leap years?

(b) 2010 2016 2017 2018 2019 2020 2024 2040 2044

- Can a leap year be an odd number? Explain.
- How many leap years can occur in a decade?
Are there decades with fewer than two leap years? Explain.
- Give all the leap years in the 21st century, that is, from 1 January 2001 to 31 December 2100.
- 1 January 2016 was on a Friday. What day of the week is 1 January 2017? What day of the week will 1 January 2030 be?
- Look at a calendar of the current year. On what day of the week is 1 July? On what day of the week will 1 July be 20 years from now?

5.7 Time zones

Why, at any given moment, is the time different in Tokyo, London and New York?

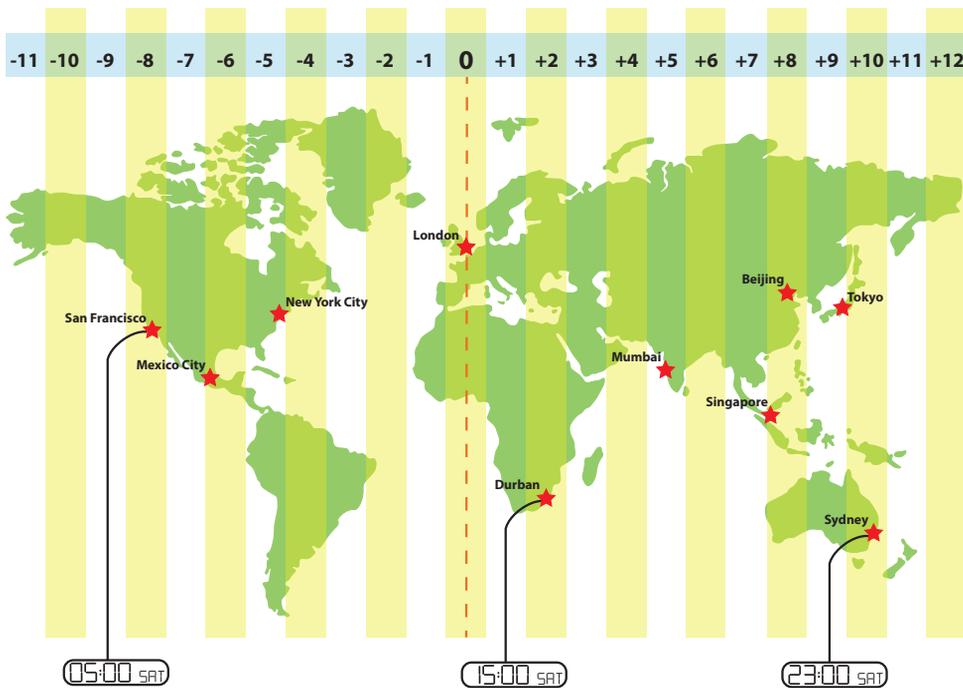
Wherever you are, the sun is at its highest point in the sky at exactly midday (noon).

However, midday (noon) is not at the same time everywhere across the world, because the Earth is round and rotates around its axis.

Midday is at the same time everywhere only **in the same time zone**. People who live in the same time zone set their watches to the same time. If you travel out of a time zone, you have to change the time on your watch to the new time in that time zone.

The time in each time zone differs by one hour from the time zone next to it.

The map below shows how the world is divided into time zones.



This time zone map is very basic and does not show two other aspects of time across the world:

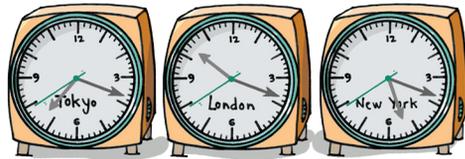
- Some countries have different time zones across the country (for example the USA). Other countries use just one time zone for the whole country. For example, South Africa uses the Durban time zone for the whole country.
- Many countries use daylight savings time, where the time is changed by an hour twice in the year to adapt for seasonal difference in sunlight, for example the time the sun rises. South Africa does not use daylight savings time.

1. The time zone map shows that when it is 15:00 in Durban, the time is 23:00 in Sydney and 05:00 in San Francisco. What is the time difference between
 - (a) Durban and Sydney
 - (b) Sydney and San Francisco?
2. Explain the meaning of this “number line” at the top of the map, and how to use it for time zone calculations:

... -3 -2 -1 0 1 2 3 ...

3. If it is 15:00 in South Africa, what is the time in
 - (a) Tokyo
 - (b) London
 - (c) New York?

4. If it is 10:18 in London, what is the time in
 - (a) Tokyo
 - (b) New York?



5. The duration of a flight by aeroplane from London to New York is approximately 7 hours 30 minutes. At what time (New York time) will a flight arrive in New York if it leaves London at
 - (a) 12:00
 - (b) 06:00
 - (c) 18:00?
6. The flight time of an aeroplane from London to Johannesburg is approximately 10 hours 45 minutes. At what time (local time) will a flight arrive in Johannesburg if it leaves London at
 - (a) 12:00
 - (b) 06:00
 - (c) 18:00?

6.1 Naming figures by the number of sides

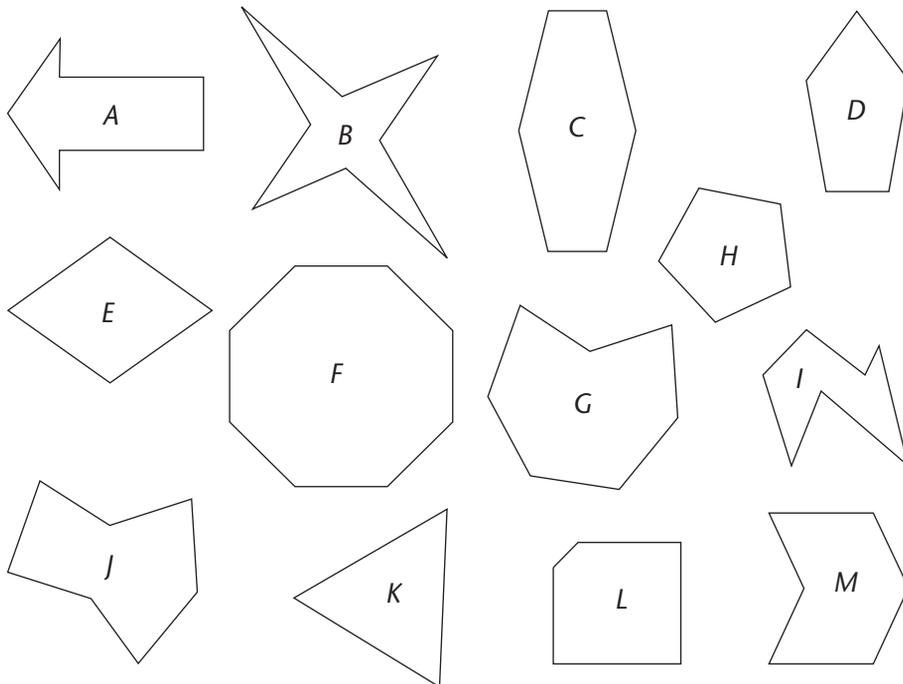
Closed figures with five straight sides are called **pentagons**.
“Penta” means five.

Closed figures with six straight sides are called **hexagons**.
“Hexa” means six.

Closed figures with seven straight sides are called **heptagons**.
“Hepta” means seven.

Closed figures with eight straight sides are called **octagons**.
“Octa” means eight.

1. Write down the letters of all the figures that have the shape of:
- | | |
|----------------|---------------------|
| (a) a triangle | (b) a quadrilateral |
| (c) a pentagon | (d) a hexagon |
| (e) a heptagon | (f) an octagon. |



6.2 Angles

1. (a) Construct these two lines on a clean sheet of paper.



- (b) Use your ruler and extend the two lines (make them longer).
The lines have to be extended on both sides up to the edge of your page.



2. Imagine that you are drawing and extending the two lines above on a very long white wall.

- (a) Do you think the lines on the wall will get closer to each other and meet somewhere, like the two lines below?



- (b) Use your ruler to draw two lines that meet, like the two lines above.

3. (a) Make two dots on the left-hand side of your page as shown below.



- (b) Use the two dots and draw two lines across the page so that they meet close to the right-hand edge of the page.

- (c) Make two new dots and draw two lines across the page so that they remain at the same distance from each other.



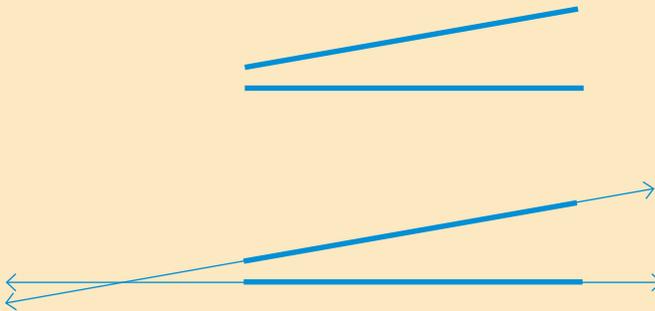
These two red lines have the same direction:



They will remain the same distance from each other, no matter how far you extend them.



The two blue lines have different directions. If you extend them, they will meet somewhere.



When two lines have different directions, we say the lines are **at an angle to each other**.



4. (a) Draw two lines that are at an angle to each other.
(b) Draw two lines that are not at an angle to each other.

5. Fold a strip of cardboard as shown in the photo.
The two arms are at an angle to each other.



6. Move the arms of the folded cardboard strip as shown in the photos below.



Photo A

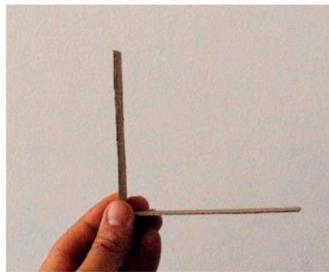


Photo B

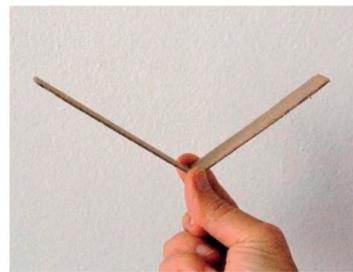


Photo C

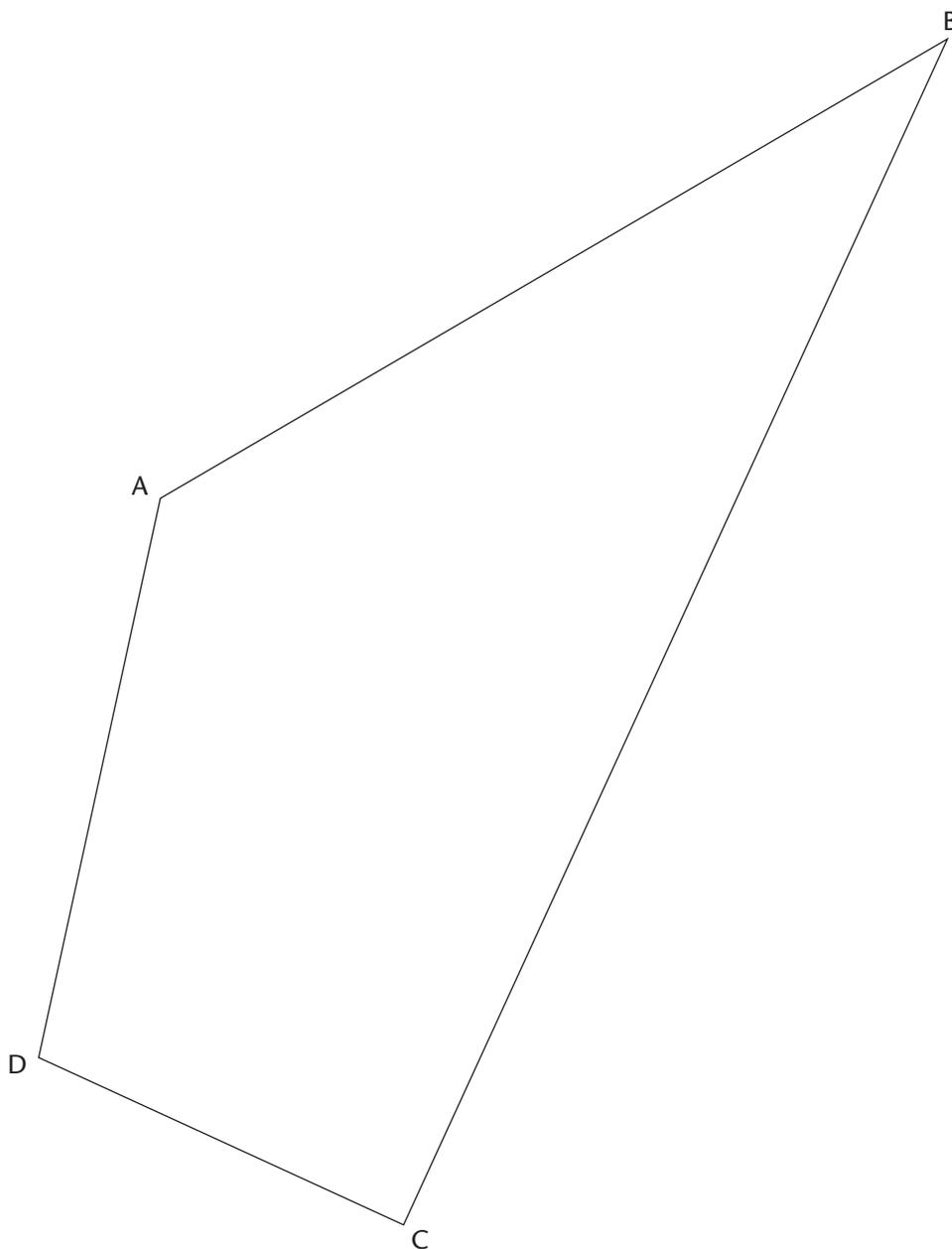
- (a) Are the arms wider apart in Photo A than in Photo B?
(b) In which photo are the arms widest apart?

The angle between the arms of the cardboard strip is smaller in Photo A than in Photo B.

The angle between the arms of the cardboard strip is bigger in Photo C than in Photo B.

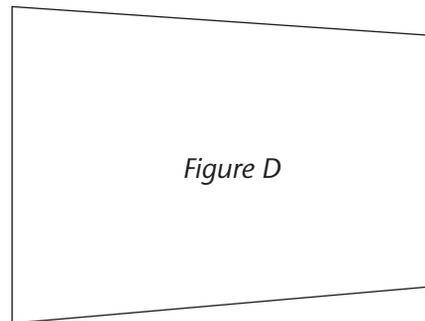
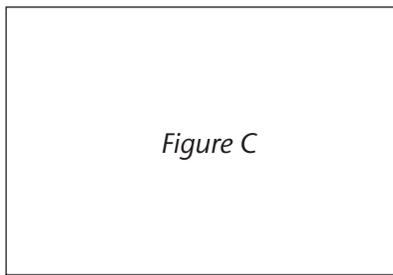
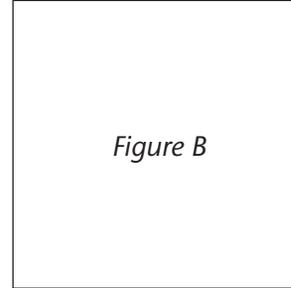
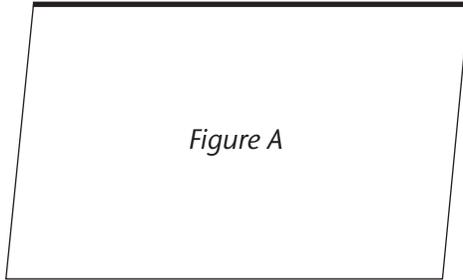
7. Use your ruler to draw two lines that form an angle:
- (a) like the angle in Photo A
(b) like the angle in Photo B
(c) like the angle in Photo C.

-
8. The quadrilateral below has vertices A, B, C and D. Use your folded cardboard strip to help you to compare the angles.



- (a) At which vertex is the angle between the two sides the biggest?
(b) At which vertex is the angle between the two sides the smallest?

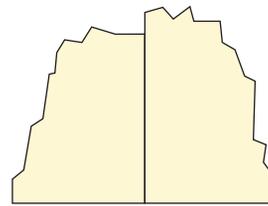
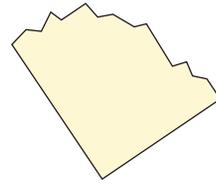
-
9. Use a loose A4 sheet of paper. Compare it to the figures below. Which figure has the same shape as the shape formed by the edges of the A4 sheet of paper?



10. Which of the above figures have shapes that differ from the shape of the sheet of paper? Describe what the difference is.
11. (a) Look at the angles between the sides at the two vertices at the top of each figure. Which angle is the biggest in each case?
- (b) Look at the angles between the sides at the two vertices on the right of each figure. Which angle is the biggest in each case?
12. Trace Figure A on your sheet of A4 paper. Place your tracing over Figure A again and then rotate your tracing so that the thick line is at the bottom. Are the angles at the top left and bottom right of Figure A equal?

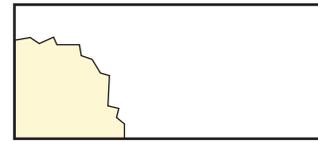
6.3 Angles of different sizes

- (a) Tear off a corner of your loose A4 sheet of paper, about the size shown on the right, to use as an angle measure.
(b) Place your angle measure at each of the other three corners of the sheet of paper from which you have made your angle measure. This is to check if the angles between the edges are the same at each corner.
(c) Tear off another corner. Put the two pieces next to each other, as shown on the right. Use a ruler to draw a straight line at the bottom edges of the two pieces of paper.

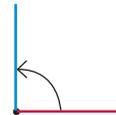


Bottom edge

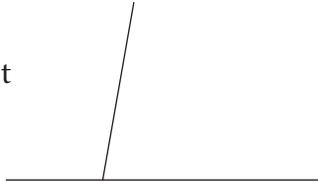
A piece of paper like one of the corners that you have torn off, is called a **right-angle template**. You can use it to check if an angle is a right angle or not.



All angles of the same size as the angles at the vertices of a rectangle are called **right angles**.



- Look again at Figure B in question 9 on the previous page. Use your right-angle template to check if any of the angles at the vertices are right angles.
- (a) Draw a straight line with your ruler.

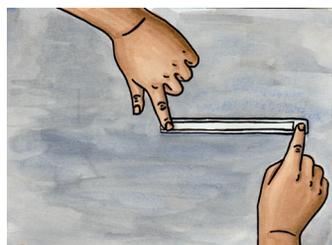
- (b) Draw another line as shown here, so that two angles are formed, one bigger than the other.

- (c) Which of your two angles is bigger than a right angle? Which one is smaller than a right angle? Indicate this on your sketch.

4. (a) Draw a new straight line. Then draw another line, as you did in 3(b), but draw it in such a way that the two angles are equal.
- (b) Check whether your two angles are right angles.

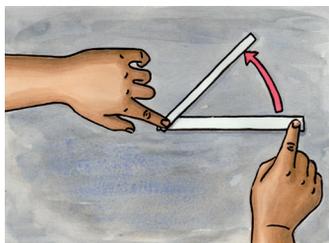
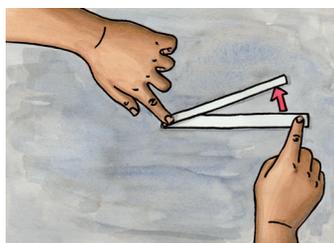
5. Work with two narrow strips of paper or cardboard.

- (a) Put the strips on top of each other. Hold one end of the bottom strip down with one finger.

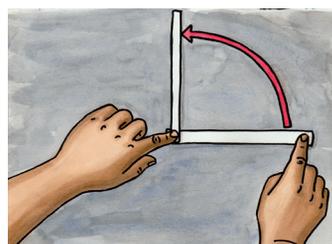
Hold a finger of your other hand lightly on the other end of the strip that lies on top.



- (b) Move your finger so that the upper strip turns to form an angle between the two strips, as shown below.

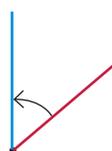


- (c) Move your finger a bit more so that a right angle is formed between the two strips.

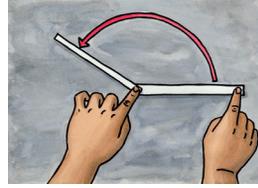
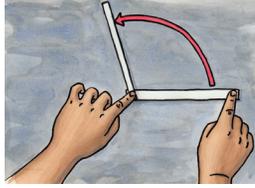


The two diagrams in 5(b) show angles smaller than a right angle.

An angle smaller than a right angle is called an **acute angle**.

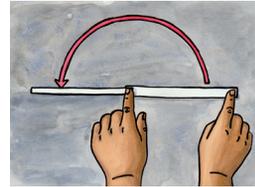


6. Turn the upper strip so that an angle bigger than a right angle is formed between the two strips, as shown below.



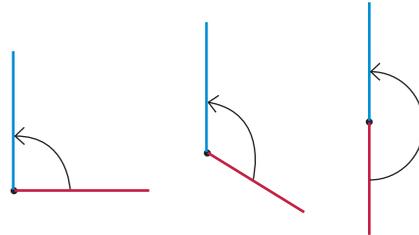
The angles in the two diagrams are called **obtuse angles**.

7. Continue to turn the upper strip until the two strips are in a straight line, as shown on the right.



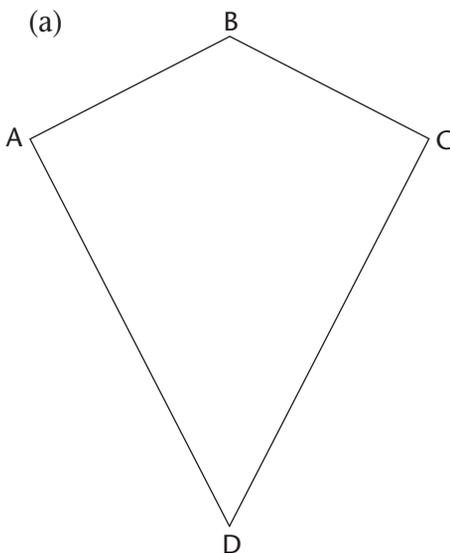
The angle indicated by the red arrow is called a **straight angle**.

An **obtuse angle** is bigger than a right angle but smaller than a straight angle.

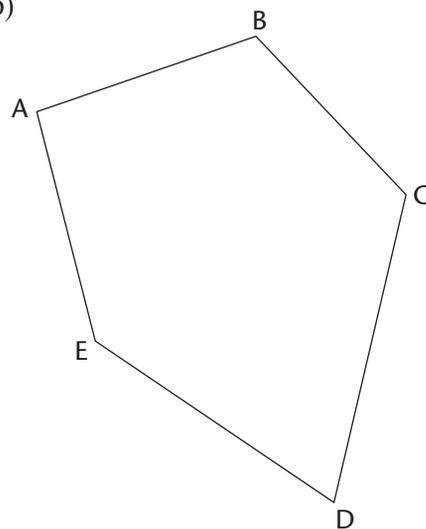


8. For each figure, state at which vertices the angle is a right angle, at which vertices the angle is obtuse, and at which vertices the angle is acute.

(a)



(b)



9. Put the two strips on top of each other as shown in Diagram A below. Turn one strip as shown in Diagram B.

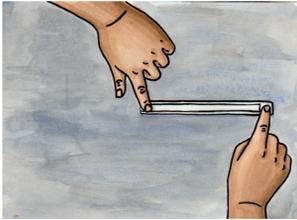


Diagram A

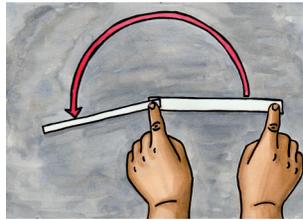


Diagram B

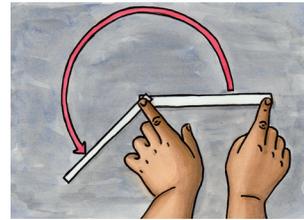


Diagram C

The angle through which you turned the strip is called a **reflex angle**.

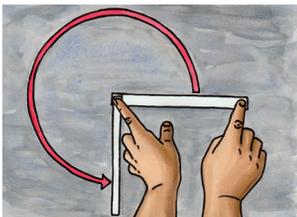


Diagram D

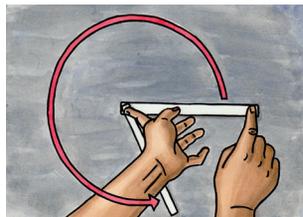


Diagram E

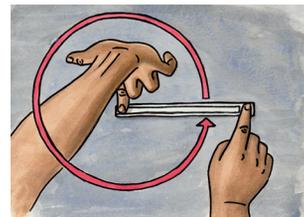
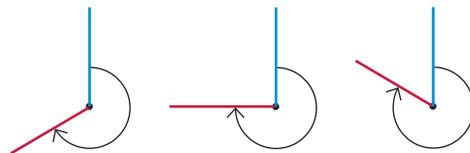


Diagram F

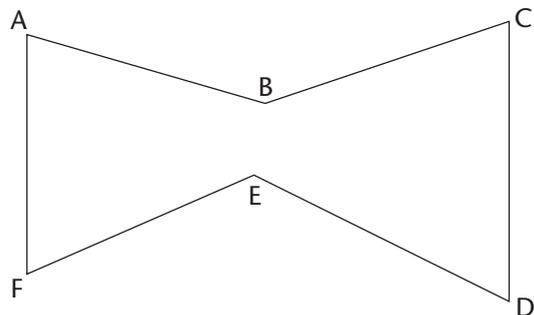
10. Continue to turn the strip as shown in Diagrams C, D, E and F.

Diagrams B, C, D and E all show reflex angles. Diagram F shows the strip completely turned around, so that it is in the same position again as in Diagram A. A full turn like this is called a **revolution**.

A **reflex angle** is bigger than a straight angle and smaller than a revolution.



11. At which vertices are the angles inside the figure reflex angles?



6.4 Parallelograms

1. (a) Trace this figure, then cut it out accurately along the edges.



- (b) Mark the vertices A, B, C and D.
(c) Put your cut-out figure on top of the above figure, so that vertex A of your cutout is at vertex C on the above figure.
(d) What do you notice about the angles at vertices A and C?
(e) What do you notice about the angles at vertices B and D?
(f) What do you notice about the length of the line from A to B, and the length of the line from D to C?
(g) What do you notice about the length of the line from A to D, and the length of the line from B to C?

A quadrilateral with equal opposite angles and equal opposite sides is called a **parallelogram**.

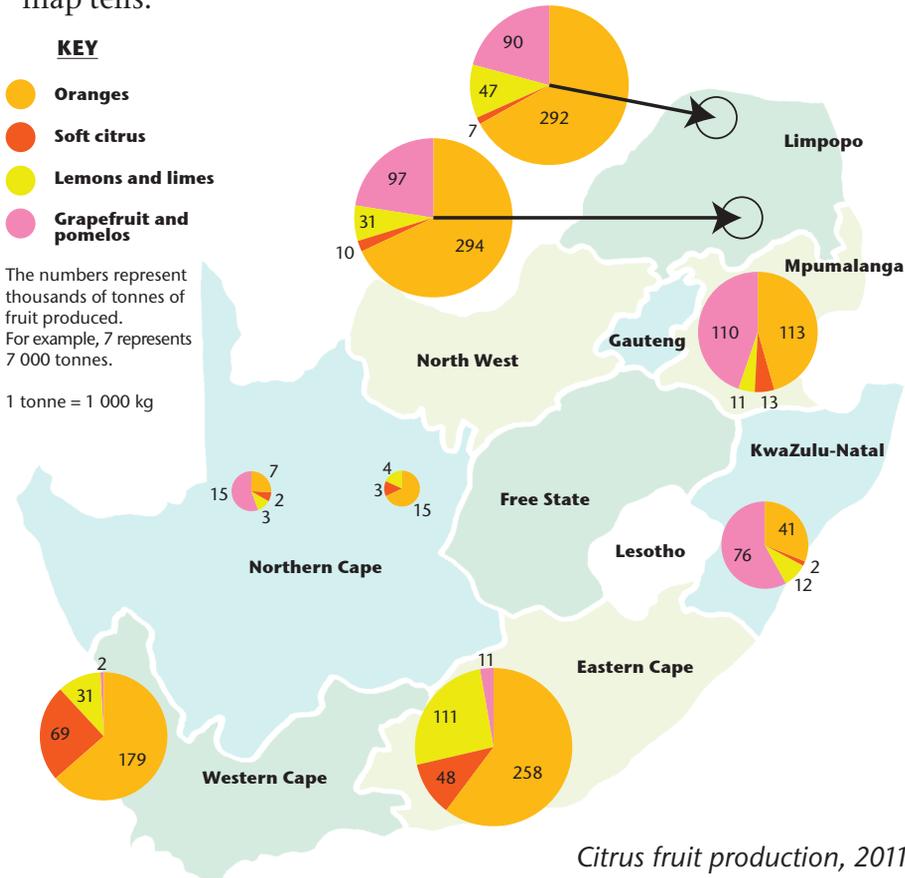
2. (a) Draw a rectangle.
(b) Are the opposite angles equal?
(c) Are the opposite sides equal?
(d) Is your rectangle a parallelogram?
(e) What makes your rectangle different from the above parallelogram?

In this unit you will investigate the farming of citrus fruit in South Africa. South Africa is well-known across the world for the oranges we export. Citrus fruit include oranges, lemons and limes, grapefruit and pomelos, as well as soft citrus.

The size of the fruit varies within a particular type. The varying sizes influence the number of fruit that can be packed into a box to be exported.

7.1 Understanding the data context

- The map shows *data* about citrus farming. Read the story that the map tells.



- (a) How many tonnes of each of the four groups of citrus fruit were produced in Mpumalanga? How many tonnes is this altogether?
- (b) How many tonnes of citrus fruit were produced in KwaZulu-Natal?
- (c) Why do you think the coloured circles on the map have different sizes?
- (d) Why do you think the slices on each coloured circle have different sizes?
- (e) In which province was the largest amount of lemons and limes produced?
- (f) In which province was the largest amount of grapefruit and pomelos produced?
- (g) In which provinces was the production of lemons and limes the smallest fraction of the total production for the province?
- (h) In which provinces is there no citrus fruit production?
- (i) Which province produces the most soft citrus?

“Soft citrus” is the name of the group of citrus fruit that includes naartjies as well as other citrus fruit that are easy to peel, such as Clementines and Mineolas.
There are different types of oranges, such as Navels and Valencias.

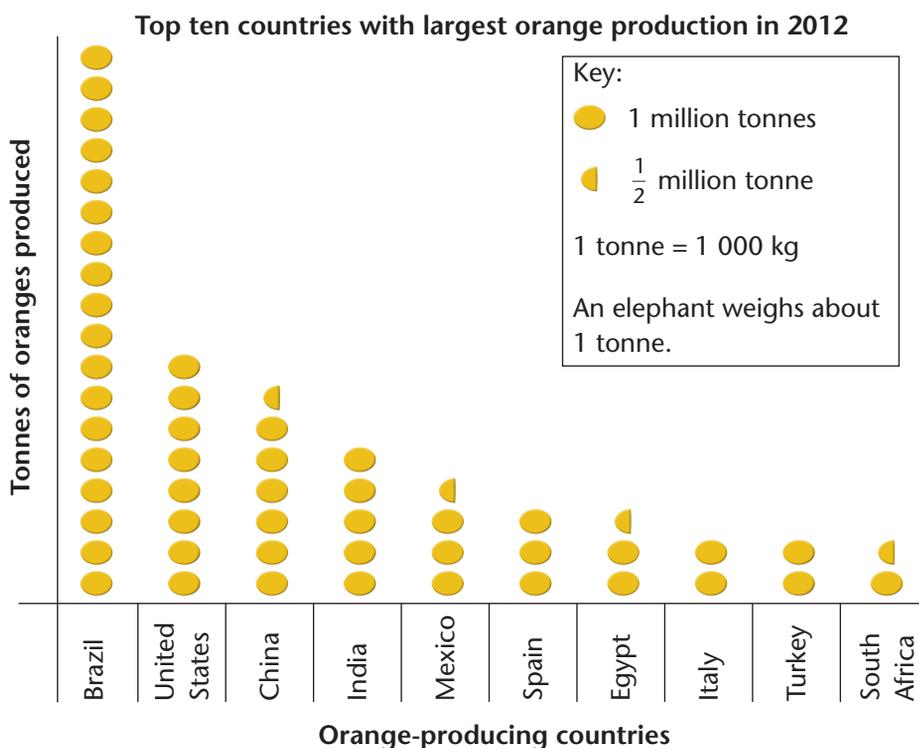
2. The table shows when different types of oranges are harvested in South Africa. The three columns under each month represent periods of approximately 10 days each.

Oranges	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov
Cara Cara	■	■	■	■				
Delta Seedless				■	■	■	■	
Midknights				■	■	■		
Navels	■	■	■	■	■	■	■	■
Tomangos			■	■				
Valencias			■	■	■	■	■	■

- (a) Which type has the longest harvesting period, Valencias or Navels? Explain.
- (b) During which months can you not buy freshly harvested oranges produced in South Africa?

7.2 Interpreting graphs

- South Africa is not the biggest producer of oranges in the world. Below is a pictograph with the top ten orange-producing countries in the world. What story does the pictograph tell? Write a paragraph in which you answer the following questions:
 - Which countries produce similar numbers of oranges?
 - Which countries produce more than double the number of oranges that South Africa produces?
 - Which countries produce more than ten times the number of oranges that South Africa produces?



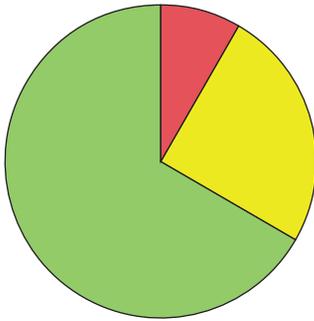
[Source: Wikipedia]

Numbers in graphs are usually not exact. For example, the pictograph shows that Brazil produces *about* 18 million tonnes of oranges, not exactly 18 million tonnes.

2. What story do these pie charts tell? Write a short paragraph and answer the following questions:

- Estimate the fraction of citrus fruit that is exported.
- Estimate the fraction of citrus fruit sold in South Africa.
- Compare the estimated fraction of citrus fruit that is sold for domestic use (eating) to the estimated fraction of citrus fruit that is processed.

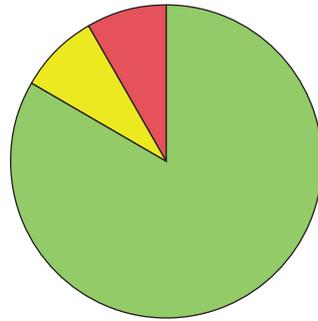
What happens to citrus fruit produced in South Africa?



Key:

■ Domestic use ■ Processed ■ Exported

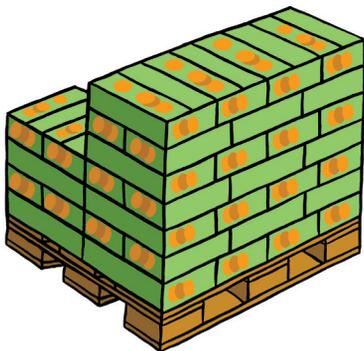
Processed citrus fruit



Key:

■ Juice ■ Jam ■ Other

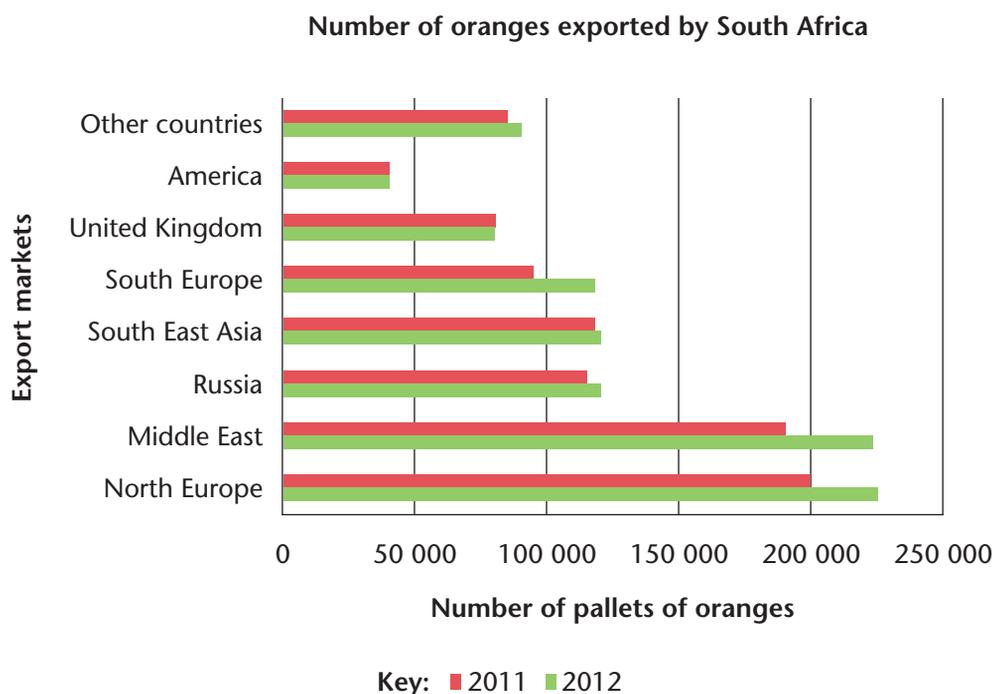
- Estimate the fraction of processed citrus fruit that is used to make juice.
- Out of every 100 oranges produced in South Africa, how many do you estimate are processed? How many of the 100 oranges do you estimate will be used to make jam?



A pallet with boxes of oranges

South Africa exports oranges to many countries. We are one of the biggest exporters of oranges in the world. Oranges are packed in boxes. The boxes are packed on pallets to be shipped. A pallet holds 80 boxes.

3. The double bar graph shows how many oranges South Africa exported in 2011 and 2012 and where the oranges were exported to.



Ask your teacher to show you on a map where these parts of the world are.

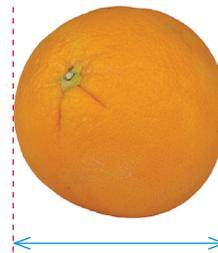
- To which parts of the world did South Africa export the largest number of oranges? Estimate the number of pallets for each year.
- About how many more pallets did we export to Russia in 2012 than to America?
- Compare exports in 2011 and 2012. To which parts of the world did our exports increase most? By how many pallets do you estimate the exports increased?
- Estimate the total number of pallets of oranges South Africa exported in 2012.

7.3 Organising data

In this section, you will organise data about the sizes of oranges.

A certain orange farm has two orange groves (orchards) with 500 trees in each grove. At the start of the harvesting season the managers gather data in order to estimate the number of oranges they can expect to export. They have to know how many oranges they can expect to harvest, how many boxes to order for packing, and how much space to book on ships to transport the oranges to different countries.

The **size** of an orange is measured by a machine. It measures the **width** of the orange.



The width of the orange is this length.

- The table below explains which sizes of oranges different export markets prefer and how many oranges are packed into a box.

Export market	Width (mm) of orange	No. of oranges per box
European Union	smaller than 60	more than 150
	60 to 62	150
Middle East	63 to 65	125
	66 to 69	105
	70 to 73	88
America	74 to 78	72
	79 to 82	64
	83 to 86	56
China	87 to 90	48
	91 to 99	40
	larger than 99	fewer than 40

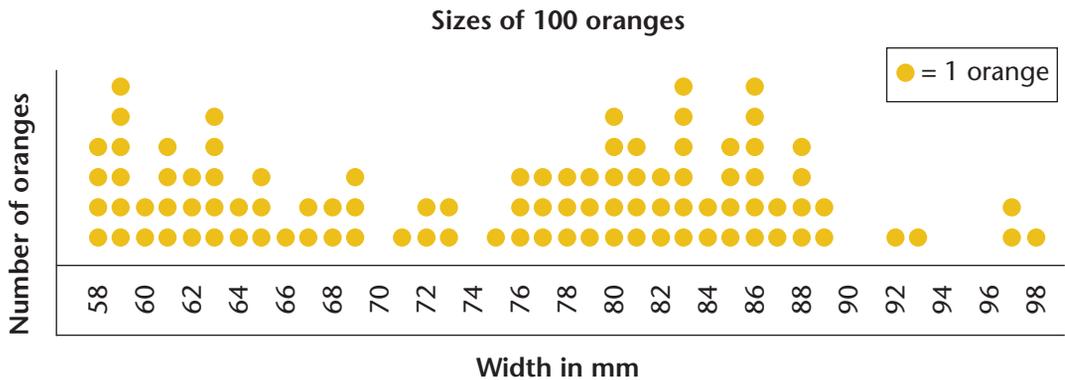
- What is the relationship between the width of the orange and the number of oranges per box?

(b) Discuss with a classmate:

How can the managers of the orange farm obtain information to estimate how many oranges of different sizes they can expect to harvest? Write down your ideas.

2. The pictograph below gives the widths of 100 oranges from two trees (one tree in each grove) on the orange farm. One of the managers of the farm investigated the groves and thinks the two trees are representative of the orange trees in each grove. We will use the oranges from these trees to get an idea of the different sizes of oranges they can expect to harvest.

If a tree is **representative** of all the trees in a grove, it means the *other trees are not much different* from this tree.



- (a) When one of the other managers of the farm saw the graph, she said: “It looks like the one tree has smaller oranges than the other tree.” Why do you think she said this? Say if you agree.
- (b) If you want to make two groups, namely *big oranges* and *small oranges*, which measurement would you choose to separate the oranges into the two groups? Explain how you decided on this measurement.
- (c) If you want to have *the same number* of oranges in the two groups in question (b), which measurement will you choose to separate the oranges into the two groups? Explain how you chose the measurement.

If you order the data from small to large, the measurement that separates the data into two groups with the same number of data is called the **median**.

So half of all the data (measurements) are bigger than the median, and half of the data are smaller than the median.

The median can only be used if your data are measurements.

- (d) Give one number that you can use to estimate the size of the small oranges. Say how you decided.
 - (e) Give one number that you can use to estimate the size of the big oranges. Say how you decided.
3. The manager says the trees from which the oranges in the pictograph were picked are representative of the groves. He expects all the trees will be similar to these trees. Say what you think:
- (a) If you look at two other trees from the groves, will you be surprised if there are many oranges that are larger than 90 mm?
 - (b) Will you be surprised if there are no oranges that are between 60 mm and 70 mm?
 - (c) What size do you expect most oranges to be?
 - (d) Write a short paragraph to say what you learnt about the sizes of the oranges in the two groves.

The **mode** and the **median** can be used to describe data that vary.

Use the **mode** if the measurement or category that occurs most often, occurs much more often than the others.

Use the **median** if you want to find the middle measurement.

- (e) Do you think the data set in the pictograph has a mode?
- (f) Do you think it makes sense to use the mode to summarise the sizes of the oranges in the pictograph?

4. The measurements in the table below are sizes of 100 oranges from two different trees, one from each grove.

Sizes of another 100 oranges (width in mm)									
87	80	80	88	58	81	73	62	82	63
59	83	58	60	85	59	73	63	75	80
76	86	83	88	64	78	63	77	58	62
86	58	63	66	69	61	83	83	89	59
97	67	85	88	78	72	84	68	83	97
85	80	72	86	76	79	82	79	88	77
60	81	81	65	69	77	64	63	59	89
83	63	62	92	80	61	98	65	98	86
86	69	71	93	63	63	59	88	61	81
76	78	87	81	67	86	79	85	68	59

We will use these measurements to work out what fraction of the harvest the managers can expect to export to different regions.

- (a) Copy the table below. Tally the data into the table.

Export market	Width (mm) of orange	Tallies	Frequency
European Union	less than 60		
	60 to 62		
Middle East	63 to 65		
	66 to 69		
	70 to 73		
America	74 to 78		
	79 to 82		
	83 to 86		
China	87 to 90		
	91 to 99		
	larger than 99		

- (b) Work out what fraction of the oranges is suitable for export to each of the export markets.

7.4 Project

In this project you will work together with your classmates to answer the following questions:

1. What kinds of citrus fruit (lemons, oranges, soft citrus etc.) are sold in your town in a particular month?
2. How does the number of oranges in a bag vary?
3. Pose your own question, for example:
How much juice can you squeeze from an orange?
Do all types of oranges produce the same amount of juice?

Step 1: Plan and collect data

1. Decide among your classmates where you will gather the data to answer question 1. Decide who will go to different shops and fruit vendors.

Do some research about the different types of citrus so that you can recognise the fruit if their names are not given in the shops.

2. For question 2, decide for how many bags of oranges (or another citrus fruit of your choice) each classmate should count how many oranges there are in a bag. Make sure that each bag is only counted once. If you gather information from an informal vendor, you may have to ask him or her how many oranges there were in the bag when he or she bought the bag.
3. Plan how to collect data to answer your own question (question 3).

Step 2: Organise and summarise the data

1. Use a tally table to summarise the kinds of citrus fruit sold in your area (question 1). How many shops or vendors sell each kind?
2. Use a tally table to summarise the numbers of oranges in all the bags of oranges checked by the class (question 2). The learners who gathered information from vendors may not have data to tally here.
3. Organise and summarise the data that you collected to answer your own question (question 3).

Step 3: Represent the data

1. Suitable graphs for question 1's data are pictographs and bar graphs. The horizontal axis will show categories of citrus.
2. A suitable graph for question 2's data is a pictograph. The horizontal axis will show a number line.
3. Choose a suitable graph to represent the data you collected to answer your own question (question 3).

Step 4: Analyse, interpret and report data

Share the work among classmates. Write up the story of your project and answer the questions. Use your knowledge of mode and median to interpret your data.

Think of ways to use the information.

Most mathematicians and scientists say,

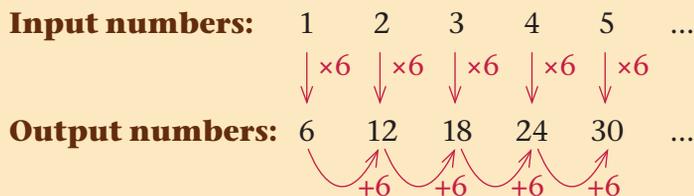
“Mathematics is the study of patterns.”

The more patterns you can see in mathematics, the better you are at mathematics!

You already know that in a **number sequence** like 6, 12, 18, 24, ... , although the numbers change (are *not the same*),

- there is some **horizontal pattern** that does not change (is always the same for all numbers) and
- there is a **vertical calculation plan (rule)** that does not change and is *the same* for all the input and output numbers.

Here are the horizontal and vertical patterns for 6, 12, 18, 24, ... :



We can describe and write the patterns in such sequences in different ways: in **words**, in a **table**, in a **flow diagram** or as a **calculation plan** (also called a **rule**).

These descriptions help us to solve problems like these:

1. To continue the sequence, in other words to find the next numbers in the sequence.
2. To calculate numbers further on in the sequence, for example the 100th number in the sequence. This is the same as calculating the output number if the input number is 100.
3. To find out the position of a number in the sequence, for example: Is 436 the 1st, 50th, ... 87th number in the sequence? This is the same as finding the input number if the output number is 436.
4. To decide if a number, for example 438, is in the sequence or not.

8.1 Revising sequences of multiples

1. Below are five *sequences of multiples*. For each sequence:
 - (a) Continue the sequence for the next five numbers.
 - (b) Calculate the 100th number in the sequence. Explain your method.
 - (c) 360 is a number in the sequence. Do you agree?
 - (d) What is the position of 360 in the sequence (for example, is it the 10th or 23rd)?
 - (e) Is 465 a number in the sequence? How do you know?

Sequence A: 3, 6, 9, 12, 15, 18, ...

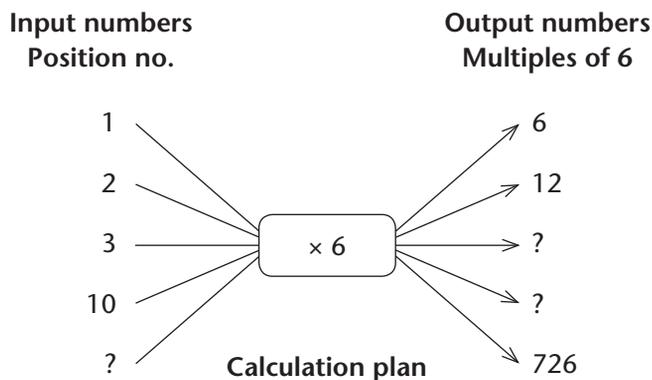
Sequence B: 4, 8, 12, 16, 20, 24, ...

Sequence C: 5, 10, 15, 20, 25, 30, ...

Sequence D: 9, 18, 27, 36, 45, 54, ...

Sequence E: 10, 20, 30, 40, 50, 60, ...

2. Complete all missing parts in this flow diagram and table for multiples of 6. What patterns do you notice?



Position no.	1	2	3	10	15	20	40	50	
Position no. × 6	6	12							726

8.2 Non-multiple sequences

We have studied sequences of multiples.

For example, what is the 100th multiple of 5 in 5, 10, 15, 20, 25, 30, ...?

Do you agree that it is easy: the 100th number is $100 \times 5 = 500$?

But what about sequences that are not multiples?

For example, what is the 100th number in 6, 11, 16, 21, 26, 31, ...?

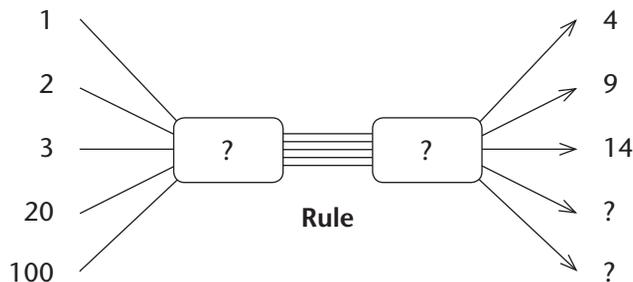
Let us now investigate this.

1. Study the three sequences in this table.

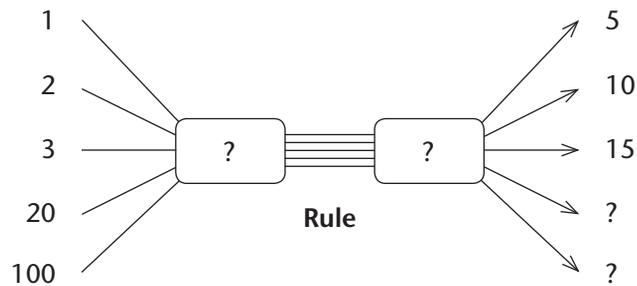
Position no.	1	2	3	4	5	6	20	100
Sequence 1	4	9	14	19	24			
Sequence 2	5	10	15	20	25			
Sequence 3	6	11	16	21	26			

- (a) Describe horizontal patterns for each of the sequences. How are they the same and how are they different?
 - (b) Describe vertical patterns for each of the sequences. How are they the same and how are they different?
 - (c) Complete the table. Describe and discuss your methods.
2. Below are three flow diagrams for the three sequences in question 1. How are the flow diagrams the same and how are they different? Complete all missing parts in the flow diagrams.

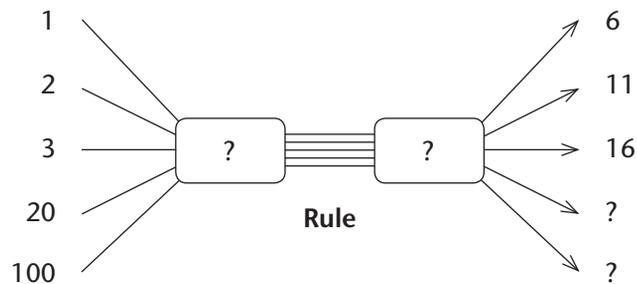
Sequence 1



Sequence 2



Sequence 3



3. Calculate the 100th number in 7, 12, 17, 22, 27, ...
4. Calculate the 100th number in 8, 13, 18, 23, 28, ...
5. (a) What is the same in Sequences A to D below?
(b) Calculate the 100th number in each sequence.

Sequence A: 6, 12, 18, 24, 30, 36, 42, ...

Sequence B: 7, 13, 19, 25, 31, 37, 43, ...

Sequence C: 9, 15, 21, 27, 33, 39, 45, ...

Sequence D: 4, 10, 16, 22, 28, 34, 40, ...

Every sequence of multiples has a family of sequences that are not multiples, but have **the same constant difference**. For example:

- | | | |
|--------------------------------|---|---------------------------------------|
| 4, 8, 12, 16, 20, 24, 28, ... | ← | these numbers are multiples of 4 |
| 5, 9, 13, 17, 21, 25, 29, ... | ← | these are 1 more than a multiple of 4 |
| 6, 10, 14, 18, 22, 26, 30, ... | ← | these are 2 more than a multiple of 4 |
| 3, 7, 11, 15, 19, 23, 27, ... | ← | these are 1 less than a multiple of 4 |

Problem: Find the 100th number in the sequence 10, 14, 18, 22, 26, 30, ...

Zukele does it like this:

My clue is that there is a constant difference of 4.

So then I know that it is family of the multiples of 4: 4, 8, 12, 16, ...

So I can see each number in 10, 14, 18, 22, ... is 6 more than 4, 8, 12, 16, ...

But I know that the 100th number in 4, 8, 12, 16, ... is $100 \times 4 = 400$

So I know that the 100th number in 10, 14, 18, 22, ... is $100 \times 4 + 6 = 406$

6. Calculate the 87th number in each of these sequences.

Also answer the questions.

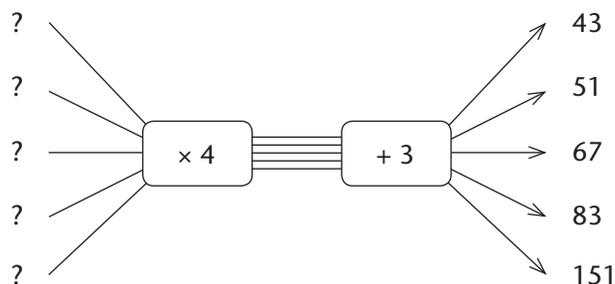
(a) 2, 5, 8, 11, 14, ... Is 623 a number in this sequence?

(b) 4, 7, 10, 13, 16, ... Is 334 a number in this sequence?

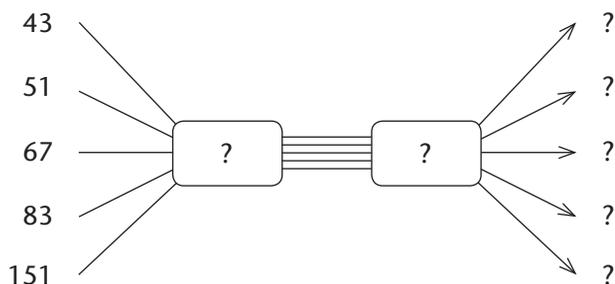
(c) 3, 6, 9, 12, 15, ... Is 334 a number in this sequence?

(d) 5, 8, 11, 14, 17, ... Is 623 a number in this sequence?

7. Find the missing input numbers:



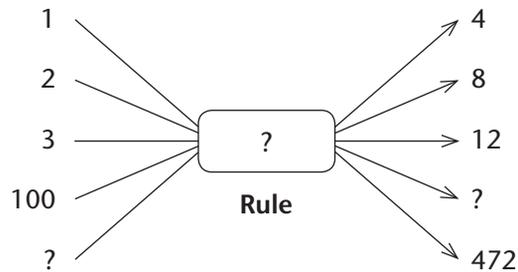
8. It will be easier to find missing input numbers if we rewrite the flow diagram in question 7 so that the known numbers become the input numbers. Complete all the missing parts in the flow diagram.



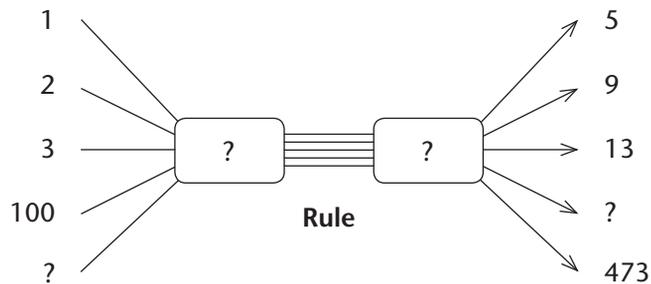
8.3 Flow diagrams and rules

1. Write the rule (calculation plan) for each of these sequences as a flow diagram. How are the flow diagrams different, and how are they the same? Also calculate all missing input and output numbers.

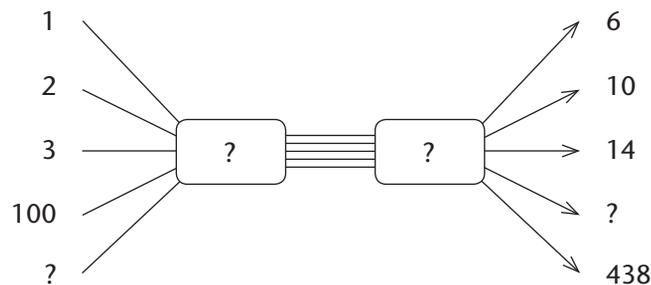
(a) 4, 8, 12, 16, 20, 24, 28, ...



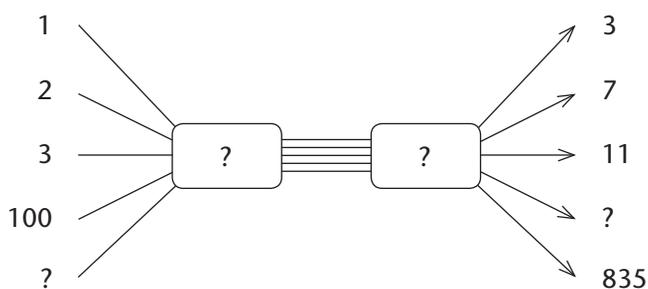
(b) 5, 9, 13, 17, 21, 25, 29, ...



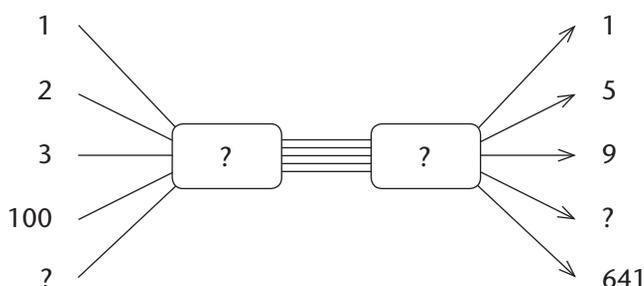
(c) 6, 10, 14, 18, 22, 26, 30, ...



(d) 3, 7, 11, 15, 19, 23, 27, 31, ...



(e) 1, 5, 9, 13, 17, 21, 25, 29, ...



2. Complete this table.

Position	1	2	3	4	5	6	30
$Position \times 4$	4	8	12				
$Position \times 4 + 1$	5	9					
$Position \times 4 + 2$	6						
$Position \times 4 + 3$							
$Position \times 4 + 4$							
$Position \times 4 + 5$							

(a) Describe and discuss your methods.

(b) Describe horizontal and vertical patterns in the table.

(c) What is the same in each sequence, and what is the same in each calculation plan (rule)?

8.4 Tables and rules

A computer uses a secret rule so that for every *input number* that you type in, it produces an *output number* using the same rule every time. Here are some examples of the computer's answers:

Input number	0	1	2	3	5	20
Output number	2	7	12	17	27	102

1. (a) Which one of these is the computer's rule (calculation plan)? Explain how you know, and how you can be sure.

Rule 1: $Output\ number = Input\ number + 6$

Rule 2: $Output\ number = Input\ number \times 6$

Rule 3: $Output\ number = Input\ number \times 5 + 2$

Rule 4: $Output\ number = (Input\ number + 2) \times 5$

None of these

- (b) What will the computer's output number be for each of these input numbers: 4, 6, 21, 25, 50, 100?

2. The computer also made tables using the other calculation plans (rules) in question 1. Which rule did the computer use for which table? Explain how you know, and how you can be sure.

Table 1

Input number	0	1	2	3	5	20
Output number	10	15	20	25	35	110

Table 2

Input number	0	1	2	3	5	20
Output number	6	7	8	9	11	26

Table 3

Input number	0	2	4	12	15	20
Output number	0	12	24	72	90	120

3. On two other occasions, the computer produced these tables:

Table 4

Input number	1	2	3	4	5	6	17	60
Output number	12	24	36	48				

Table 5

Input number	1	2	3	4	5	6	17	60
Output number	14	26	38	50				

- (a) Complete the tables.
- (b) Explain how you calculated *Output number 17* and *Output number 60* in each table.
- (c) How are Table 4 and Table 5 the same and how are they different?
Is there a connection (a link) between the two tables?
4. For each of Sequences A to F below:
- (a) Describe the pattern in the sequence.
- (b) Continue the sequence for another five numbers.
- (c) Calculate the 100th number.
- Sequence A: 7, 12, 17, 22, 27, 32, ...
- Sequence B: 8, 13, 18, 23, 28, 33, ...
- Sequence C: 9, 14, 19, 24, 29, 34, ...
- Sequence D: 7, 13, 19, 25, 31, 37, ...
- Sequence E: 8, 14, 20, 26, 32, 38, ...
- Sequence F: 1, 7, 13, 19, 25, 31, ...
5. Write down your own numerical sequence, ask your own questions, and then answer your questions.

Term Two

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1.1 Numbers bigger than a million

The symbol for ten thousand is 10 000.

The symbol for one hundred thousand is 100 000.

The symbol for 300 thousand is 300 000.

The symbol for a thousand thousands is 1 000 000.

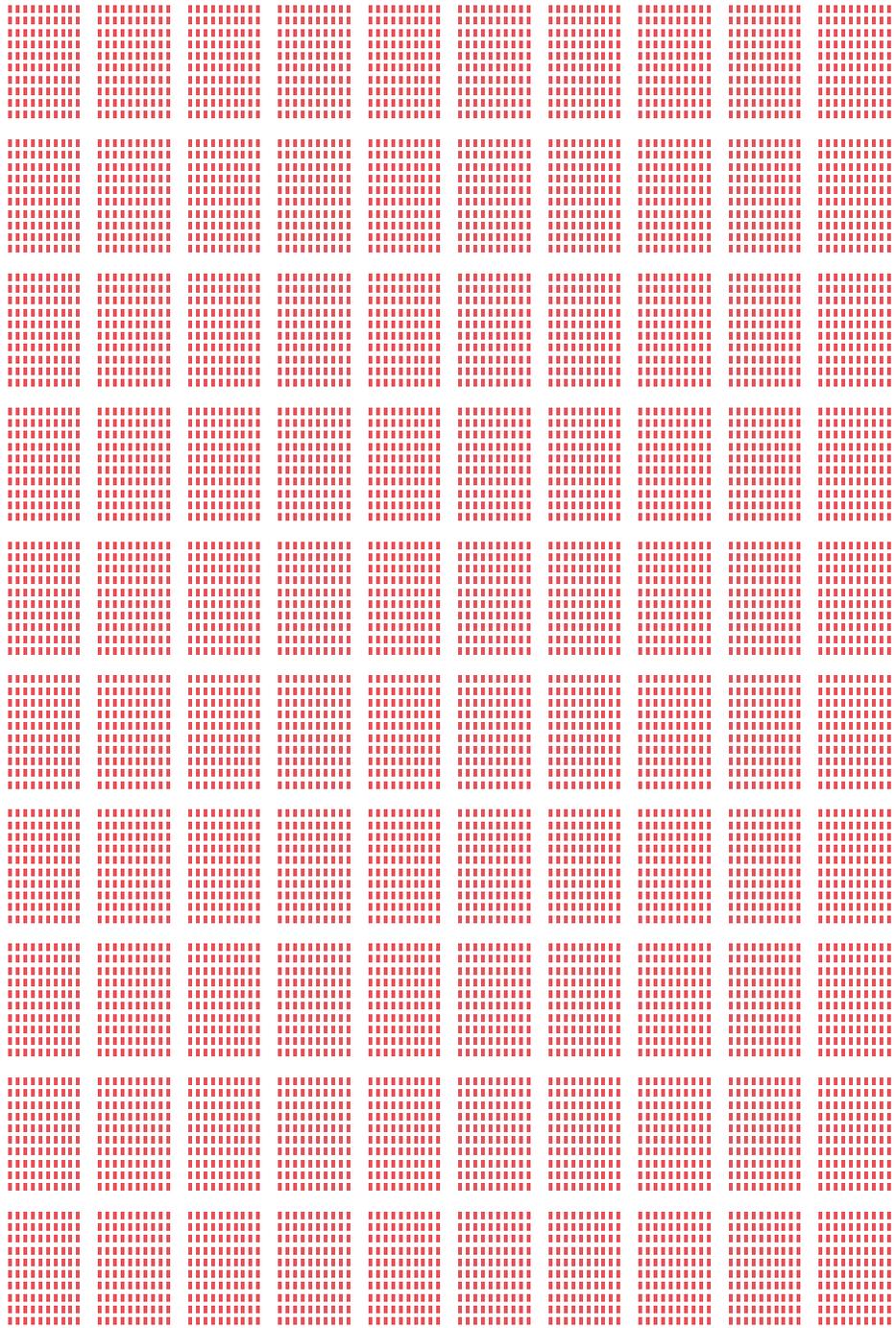
A thousand thousands is called 1 million.

The symbol for 10 million is 10 000 000.

One kilometre is 1 million millimetres.

The number of people who live in South Africa is about 55 million.

1. Approximately how many millimetres away from school is your home?
2. Take a look at the many short thick lines on the next page.
 - (a) How many lines are shown on the page?
 - (b) How many lines are there on ten pages like this?
 - (c) How many lines are there on a hundred pages like this?
 - (d) How many lines are there on a thousand pages like this?
 - (e) How many lines are there on ten thousand pages like this?
 - (f) How many lines are there on a hundred thousand pages like this?
3.
 - (a) How many thousands is 30 hundreds?
 - (b) How many thousands is 240 hundreds?
 - (c) How many thousands is 8 240 hundreds?
 - (d) How many thousands is 18 240 hundreds?
 - (e) How many millions is 4 000 thousands?
 - (f) How many millions is 40 000 thousands?
 - (g) How many millions is 400 000 thousands?
 - (h) How many millions is a thousand thousands?



4. Write the number symbols for these numbers.

- (a) nine hundred thousand
- (b) nine hundred and ninety thousand
- (c) nine hundred and ninety-nine thousand and ninety
- (d) nine hundred and ninety-nine thousand and ninety-nine

The symbol for two million is 2 000 000.

The symbol for two million five hundred thousand is 2 500 000.

The symbol for two million three hundred and forty thousand is 2 340 000.

The number name for 2 340 000 consists of two parts:

two million

three hundred and forty thousand

The **number of millions**
is stated first: 2 million

The **number of thousands**
is stated after the millions: 340 thousand

5. Write the number symbols for these numbers.

- (a) five million six hundred and seventy thousand
- (b) five million six hundred and seventy-five thousand
- (c) seventy million three hundred and twenty-eight thousand
- (d) seventy-three million three hundred and twenty-eight thousand
- (e) two hundred and seventy-three million three hundred and twenty-eight thousand

6. Write the number names for these numbers.

- (a) 6 400 000
- (b) 6 430 000
- (c) 6 437 000
- (d) 6 437 200
- (e) 6 437 230
- (f) 6 437 238
- (g) 6 403 238
- (h) 6 043 238
- (i) 8 070 050
- (j) 8 007 500
- (k) 8 700 005
- (l) 8 705 000

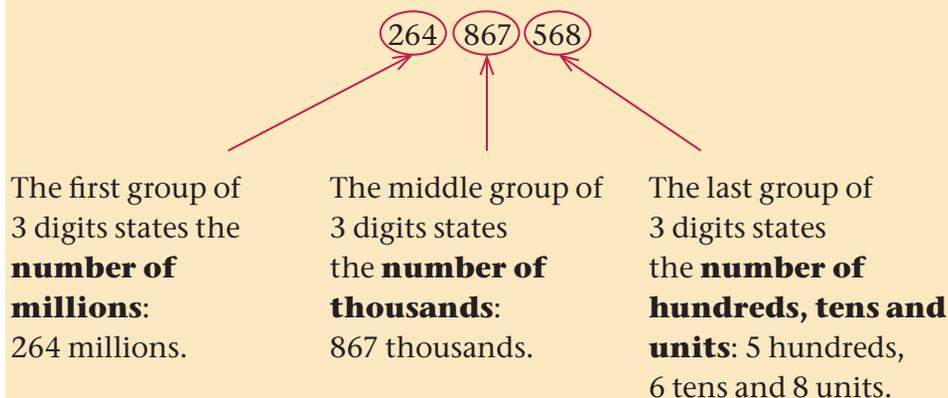
1.2 Count beyond 1 million

1. In each case, write the number symbols as you go along.
 - (a) Count in fifty thousands from eight hundred thousand to one million two hundred thousand.
 - (b) Count in two thousands from nine hundred and ninety thousand to one million and twelve thousand.
 - (c) Count in five hundred thousands from 100 000 up to three million six hundred thousand.
 - (d) Count in 250 000s from 4 million up to 6 million.
 - (e) Count in 250 000s from 41 million up to 43 million.
 - (f) Count in 250 000s from 423 million up to 425 million.
 - (g) Count in 5 millions from 621 million up to 651 million.
2. Write the number symbols as you go along.
 - (a) Count in 100 millions from 300 million up to 900 million.
 - (b) Count in 10 millions from 800 million up to 900 million.
 - (c) Count in millions from 890 million up to 900 million.
3. In each case, count backwards until you cannot go further down. Write the number symbols as you go along.
 - (a) Count backwards in 100 thousands from 2 million.
 - (b) Count backwards in 500 thousands from 10 million.
 - (c) Count backwards in 900 thousands from 10 million.
 - (d) Count backwards in 10 millions from 120 million.
4. In each case, write the number symbols as you go along.
 - (a) Count backwards in 100 thousands from 32 million to 31 million.
 - (b) Count backwards in 500 thousands from 230 million to 228 million.
 - (c) Count backwards in 200 thousands from 782 million to 779 million.

1.3 Represent and order 9-digit numbers

The number symbol for *two hundred and sixty-four million eight hundred and sixty-seven thousand five hundred and sixty-eight* is 264 867 568.

It consists of three parts:



So, we can think of 264 867 568 as 264 million, 867 thousand and 568.

The place value expansion (expanded notation) for 264 867 568 is

$200\,000\,000 + 60\,000\,000 + 4\,000\,000 + 800\,000 + 60\,000 + 7\,000 + 500 + 60 + 8$.

1. How much is each of the following? Write the number symbols.

- (a) $217\,458\,379 - 458\,000$
- (b) $217\,458\,379 - 379$
- (c) $217\,458\,379 - 217$ million
- (d) $217\,458\,379 -$ fifty thousand
- (e) $217\,458\,379 -$ three hundred
- (f) 300 million + 30 million + 30 thousand
- (g) 300 million + 3 million + 3 thousand
- (h) 300 million + 30 million + 3 thousand
- (i) 300 million + 30 thousand + 3 hundred
- (j) 30 million + 30 thousand + 3 hundred
- (k) 3 million + 30 thousand + 3 hundred

-
2. Write the number symbols for these numbers.
- (a) five million eight hundred and fifty thousand four hundred and fifty-six
 - (b) one hundred and one million fifty-four thousand three hundred and forty-eight
 - (c) thirty-two million forty thousand three hundred and seventy-five
 - (d) seven hundred and eighty-four million six hundred and eighteen thousand and thirteen
 - (e) seven million one hundred and ninety thousand and three
 - (f) nine hundred and sixty million eight-hundred and sixty-four thousand and ten
 - (g) one hundred and ten million one hundred and one thousand one hundred
3. Write the number symbols that you wrote for question 2 in ascending order (from smallest to biggest).
4. Arrange these eight numbers in descending order (from biggest to smallest).
- (a) 352 632 187
 - (b) 403 303 002
 - (c) 336 001 033
 - (d) 45 090 946
 - (e) 94 409 806
 - (f) 217 583 528
 - (g) 800 004 307
 - (h) 319 006 825
5. Now write the number names and place value expansions (expanded notation) for the numbers in question 4.
6. Round off each of the numbers in question 4:
- (a) to the nearest million
 - (b) to the nearest ten thousand
 - (c) to the nearest thousand.

2.1 Extending multiplication facts

To do calculations such as 56×73 and 254×78 , and to do multiplication with bigger numbers such as 357×472 and $7\,358 \times 573$, you need to know basic multiplication facts such as $50 \times 70 = 3\,500$ very well.

In this section, you will refresh your memory of multiplication facts.

1. For which of the following can you give the answers straight away? Write down *only* those answers that you know immediately. You can answer the others later.

- (a) 2×7 (b) 3×5 (c) 6×7 (d) 70×10
 (e) 8×90 (f) 6×4 (g) 6×8 (h) 6×9

A multiplication fact that you know can often help you to build knowledge of another multiplication fact. Here are some examples:

If you know that $2 \times 7 = 14$, you also know that $7 \times 2 = 14$.

You can easily see that

$$\begin{array}{l} 2 \times 70 = 140 \quad \text{and} \quad 70 \times 2 = 140 \\ 2 \times 700 = 1\,400 \quad \text{and} \quad 700 \times 2 = 1\,400 \\ 2 \times 7\,000 = 14\,000 \quad \text{and} \quad 7\,000 \times 2 = 14\,000. \end{array}$$

If you know that $2 \times 70 = 140$ and $70 \times 2 = 140$,

you can also easily see that

$$\begin{array}{l} 20 \times 70 = 1\,400 \quad \text{and} \quad 70 \times 20 = 1\,400 \\ 200 \times 70 = 14\,000 \quad \text{and} \quad 70 \times 200 = 14\,000 \\ 20 \times 700 = 14\,000 \quad \text{and} \quad 700 \times 20 = 14\,000 \\ 200 \times 700 = 140\,000 \quad \text{and} \quad 700 \times 200 = 140\,000, \text{ and so on.} \end{array}$$

You can double the answer of $2 \times 7 = 14$ to get $4 \times 7 = 28$ and then you also know that $7 \times 4 = 28$.

2. Continue to think of all the new multiplication facts that you can make from $2 \times 7 = 14$ and record them in a table like the one below.

Facts that can be easily formed by building on $2 \times 7 = 14$				
\times	7	70	700	7 000
2	14	140	1 400	14 000
20		1 400	14 000	
200		14 000	140 000	
2 000				
4	28			
40				
400				
4 000				

3. See which facts you can build from $3 \times 7 = 21$, and record your results in a table like the one above.
4. See which facts you can build from $4 \times 6 = 24$, and record your results in a table like the one above.

You saw earlier that in order to build new facts from $2 \times 7 = 14$, you can double the answer to get $4 \times 7 = 28$, and then build further from there.

Here is another way to build on a known fact:

If you know that $4 \times 7 = 28$, you can *add* another 7 to get $5 \times 7 = 35$.

If you know that $6 \times 8 = 48$, you can *add* another 8 to get $7 \times 8 = 56$.

5. Now use addition, as explained above, to build new facts from each of the following:
- (a) $7 \times 7 = 49$ (b) $5 \times 6 = 30$ (c) $2 \times 9 = 18$
 (d) $5 \times 7 = 35$ (e) $7 \times 9 = 63$ (f) $6 \times 4 = 24$
6. Look again at the work you did in question 1. Were there any questions that you could not answer? Try to work out those answers now.

2.2 Summarise and practise multiplication facts

1. Complete this table.

×	6	9	4	10	3	2	5	8	7
7	42								
3			12						
8									
5		45							
9									
2							10		
6									
4									
10									

2. Now complete this table.

×	4	90	60	7	30	8	5	20	10
50									
3									
6									
70									
40									
9									
80									
4									
10									

When you have completed the tables in questions 1 and 2, answer question 3.

-
1. What is the easiest way to calculate $2 \times 17 \times 5$?
 2. Ben buys 5 bags of bananas. Each bag has 4 bunches of bananas with 3 bananas in each bunch.
 - (a) How many bananas does Ben buy?
 - (b) Write down the calculation plan to calculate the number of bananas.
 3.
 - (a) Siba buys 4 boxes of beads. In each box there are 10 packets and every packet has 15 beads. How many beads does she buy? Write down your calculation plan to calculate the number of beads.
 - (b) Marie buys 10 boxes of beads. In each box there are 15 packets and every packet has 4 beads. How many beads does she buy? Write down your calculation plan to calculate the number of beads.
 - (c) Jeff buys 15 boxes of beads. In each box there are 10 packets and every packet has 4 beads. How many beads does he buy? Write down your calculation plan to calculate the number of beads.
 4. Compare your three calculation plans for question 3. What do you notice?

If we multiply three or more numbers we can rearrange the numbers to change the order in which we multiply. It does not change the answer. This is a **property of multiplication**.

When you have to multiply three or more numbers, you may rearrange the numbers to make the calculation easier.

5. Calculate the following. Rearrange the numbers to make it easier.
 - (a) $50 \times 37 \times 2$
 - (b) $4 \times 68 \times 25$
 - (c) $3 \times 74 \times 10$
 - (d) $5 \times 22 \times 8$

Now read this to refresh your memory about products and factors:

$$900 = 10 \times 6 \times 15$$

We say:

900 is the **product** of 10, 6 and 15.

10, 6 and 15 are **factors** of 900.

When a number is divided by one of its factors the remainder is 0.

- Are 10, 6 and 15 the only factors of 900?
- Write 900 as a product of *three* other numbers.
- Can you think of more ways in which you can write 900 as a product of *three* numbers? If you can, write them down.
- Can you write 900 as a product of *four or more* numbers? Try as many ways as you can.
- Have you found all the ways? Think of a way in which you would know whether you have found all the ways or not.

Read what Lindiwe did to find all the factors of 900.

Lindiwe started with $10 \times 6 \times 15$ as factors of 900.

She noticed that 10 can be written as 2×5

and that 15 can be written as 3×5 .

$$\begin{aligned} 900 &= 10 \times 6 \times 15 \\ &= 2 \times 5 \times 6 \times 3 \times 5 \quad (\text{Then she noticed that } 6 = 2 \times 3) \\ &= 2 \times 5 \times 2 \times 3 \times 3 \times 5 \end{aligned}$$

What Lindiwe did here was to **break a number down into factors**.

- When you combine any two of the three factors of 900 shown above, you can write 900 as a product of *five* numbers. Do this in two different ways.

12. Now break down each of the following numbers into factors. Start by writing the number as a product of two factors. Then see if you can break down these two factors into more factors, like Lindiwe did. Continue in this way until none of the factors can be broken down any further.

(a) 90

(b) 136

(c) 150

(d) 59

(e) 57

(f) 144

What happened when you wanted to break down the number 59 in question 12(d)?

You probably found $59 = 1 \times 59$ but could not continue. This is because some numbers, such as number 59, have no factors other than the number itself and 1.

Numbers with only two different factors, namely 1 and the number itself, are called **prime numbers**.

A factor that is a prime number, is called a **prime factor**.

13. Which of the following numbers are prime numbers?

15

51

59

57

43

91

42

27

101

14. Describe, in your own words, your *method* (what you did) to decide if the numbers in question 13 are prime numbers or not.

15. (a) Write down all the factors of 70.

(b) Write down all the prime factors of 70.

2.4 Multiplying with factors

Linda knows that it is easy to multiply by small numbers such as 2 and 3. Take a look at how she uses factors to multiply 687×42 .

Linda thinks of 42 as 6×7 and then of 6 as 2×3 .

She then rearranges the factors like this to make it easy:

Explanation of the steps:

$$\begin{aligned} 687 \times 42 &= 687 \times 6 \times 7 && \text{(because } 42 = 6 \times 7\text{)} \\ &= 687 \times 2 \times 3 \times 7 && \text{(because } 6 = 2 \times 3\text{)} \\ &= (687 \times 2) \times 3 \times 7 \\ &= (1\,374 \times 3) \times 7 && \text{(} 687 \times 2 = 1\,374\text{)} \\ &= 4\,122 \times 7 && \text{(} 1\,374 \times 3 = 4\,122\text{)} \\ &= (4\,000 + 100 + 20 + 2) \times 7 \\ &= 28\,000 + 700 + 140 + 14 \\ &= 28\,854 \end{aligned}$$

- Use Linda's method and use factors to calculate the following. You don't have to write any explanations next to your steps.
 - 12×67
 - 45×15
 - 51×16
 - 24×135
 - 21×72
 - $36 \times 4\,552$
- For which of the following will you *not* be able to use the "factors method"? Explain your answer.
 - 59×13
 - 29×31
 - 67×7
 - 79×11
 - 47×23
 - 89×57
 - 17×37
 - 63×9

2.5 Different ways of recording multiplication

You will now learn about other ways in which you can set out your work.

This is what Sarah wrote when she calculated 34×63 :

$$34 \times 63 = 34 \times 60 + 34 \times 3$$

$$34 \times 60 = 30 \times 60 + 4 \times 60$$

$$= 1\,800 + 240$$

$$34 \times 3 = 30 \times 3 + 4 \times 3$$

$$= 90 + 12$$

So, $34 \times 63 = 1\,800 + 240 + 90 + 12$

Then Sarah wrote this to do the adding up in the last step:

$$\begin{array}{r} 1\,800 \\ 240 \\ 90 \\ 12 \\ \hline 2 \\ 140 \\ 1\,000 \\ 1\,000 \\ \hline 2\,142 \end{array}$$

1. This is how Sarah started to do 42×57 :

$$42 \times 57 = 40 \times 57 + 2 \times 57$$

Complete the calculation in the way Sarah would do it.

2. How do you think Sarah will calculate these?

(a) 34×68

(b) 47×28

Indumiso uses the same method as Sarah, but he sets his work out in a slightly different way.

This is what he wrote when he calculated 34×63 :

$$34 \times 63 = 34 \times 60 + 34 \times 3$$

Here he calculated 34×60 :

$$\begin{array}{r} 34 \\ \times 60 \\ \hline 240 \quad (60 \times 4) \\ + 1\,800 \quad (60 \times 30) \\ \hline 2\,040 \end{array}$$

Here he calculated 34×3 :

$$\begin{array}{r} 34 \\ \times 3 \\ \hline 12 \quad (3 \times 4) \\ + 90 \quad (3 \times 30) \\ \hline 102 \end{array}$$

And then he added up the two totals:

$$\begin{array}{r} 2\,040 \\ + 102 \\ \hline 2\,142 \end{array}$$

3. Rewrite your work for question 1 in the way Indumiso sets his work out.

Indumiso's way of setting out his work for multiplication is called **expanded column multiplication**.

We also call this way of setting out the work the **expanded column notation**.

Later this year you will learn an even shorter way of setting out multiplication.

4. Calculate each of the following. Use the expanded column notation to set out your work.
- (a) 47×38
 - (b) 54×86
 - (c) 362×56
 - (d) 538×464

5. Another example of the **expanded column notation** is shown below. The number 2 800 at Step (a) was obtained by multiplying 400 by 7. For each of Steps (b) to (i), write down which two numbers were multiplied to obtain the number.

Example: (a) 400×7

Calculation of 473×587 :

$\begin{array}{r} 587 \\ \times 400 \\ \hline 2\,800 \end{array}$	$\begin{array}{r} 587 \\ \times 70 \\ \hline 490 \end{array}$	$\begin{array}{r} 587 \\ \times 3 \\ \hline 21 \end{array}$	$\begin{array}{r} 234\,800 \\ 41\,090 \\ + 1\,761 \\ \hline 277\,651 \end{array}$
(a)	(d)	(g)	
$32\,000$	$5\,600$	240	(h)
$+ 200\,000$	$+ 35\,000$	$+ 1\,500$	(i)
$234\,800$	$41\,090$	$1\,761$	

6. Calculate 769×239 .
7. Indira has to calculate $4\,385 \times 765$. She starts by setting her work out as follows:

$\begin{array}{r} 4\,385 \\ \times 700 \\ \hline \end{array}$	$\begin{array}{r} 4\,385 \\ \times 60 \\ \hline \end{array}$	$\begin{array}{r} 4\,385 \\ \times 5 \\ \hline \end{array}$
..... (a) (e) (i)
..... (b) (f) (j)
..... (c) (g) (k)
..... (d) (h) (l)

Where did Indira get the numbers 700, 60 and 5 from?

8. Which two numbers does Indira plan to multiply at Steps (a) to (l)?
9. Copy and complete Indira's work.
10. Calculate each of the following.
- $8\,374 \times 849$
 - $6\,357 \times 277$
 - $368 \times 7\,388$
 - $847 \times 4\,809$

2.7 Mental calculation versus the calculator!

A calculator is a handy tool that can help you to calculate quickly and accurately. But we need the right attitude in using the calculator.

Use **mental calculation** for facts that you should know or should be able to do faster in your head than on the calculator, for example 5×6 and 500×6 . We also use mental methods for **estimation** to check written or calculator calculations.

Use **written methods** when you need to explain your understanding of the mathematics, for example to explain how you do 349×56 .

Use the **calculator** for calculating with large numbers or for many repeated calculations when only the answer is important, for example to calculate the answer in word problems.

1. In this exercise use one calculator between two or three classmates. Compete to see who of you can calculate the fastest and correctly. One of you must use the calculator, and the others must calculate mentally.

Who wins for each of these calculations?

- | | |
|------------------------|-------------------------|
| (a) 345×45 | (b) $30 + 20$ |
| (c) 30×20 | (d) $20\,000 + 30\,000$ |
| (e) 25×4 | (f) $130 + 330$ |
| (g) $20\,000 \times 3$ | (h) 678×234 |

2. Say which kind of method (mental, written or calculator) you will use for each of these calculations. Why?

Find the answer.

- | | |
|----------------------|---------------------|
| (a) 345×45 | (b) 50×12 |
| (c) 300×200 | (d) 321×3 |
| (e) 20×234 | (f) 21×234 |

2.8 Use estimation to check the calculator

It is very easy to press wrong keys by accident, and then to get wrong answers. We should develop the habit of always checking calculator answers!

1. Use your calculator to calculate 723×489 .
How do you know if the answer is correct?

Mary just typed without thinking and did not see that she pressed the $\boxed{+}$ and not the $\boxed{\times}$ key. She got the answer 1 212. Mary thought the answer is correct because she thinks the calculator is always right!

Cyndi always **first estimates the answer before she starts typing on the calculator**. See if you understand her reasoning:

$$723 \times 489 \text{ is } \textit{more} \text{ than } 700 \times 400 = 280\,000$$

$$723 \times 489 \text{ is } \textit{less} \text{ than } 800 \times 500 = 400\,000$$

So the answer must be between 280 000 and 400 000

Only then did she type on the calculator: $723 \boxed{+} 489 \boxed{=}$ and just like Mary got the answer 1 212. But Cyndi immediately knew that the answer must be wrong, so she must have made a mistake. Then she did it correctly and got 353 547, and was satisfied that the answer seemed reasonable. Do you agree?

2. In each case, first estimate the answer like Cyndi did. Then calculate the answer with your calculator, and decide if your answer looks about right.
 - (a) $3\,456 \times 2\,345$
 - (b) $3\,456 \times 678$
 - (c) 34×567
 - (d) 678×234
 - (e) $12\,345 \times 357$
 - (f) $3\,452 \times 426$

2.9 Use equivalence to check the calculator

It is so easy to make mistakes on the calculator. So it is important that we always check our calculator answers.

You should not check a calculation by just repeating it, because we often make the same mistake again. It is better to check by using a different method the second time.

One way to check is to do the calculations in a different order.

1. Do the following calculations on your calculator in the given order and draw a conclusion.
 - (a) (1) $1\,716 \times 159 \div 286$
(2) $1\,716 \div 286 \times 159$
 - (b) (1) $276 \times 288 \times 959$
(2) $288 \times 959 \times 276$
 - (c) (1) $148\,896 \div 88 \div 94$
(2) $148\,896 \div 94 \div 88$

Two different calculation plans that give the same answer are called **equivalent calculation plans**.

We can check calculator results using the **rearrangement principle**: if we repeat the calculations in a different (but equivalent) order, we will get the same answer.

2. Calculate each of the following. Check the result by using the rearrangement principle.
 - (a) $543 \times 178 \times 86$
 - (b) $6\,545 \div 85 \times 28$
 - (c) $1\,536 \times 287 \div 328$
 - (d) $10\,976 \div 28 \div 14$
 - (e) $1\,543 \times 268 \times 128$
 - (f) $154 \times 768 \div 56$

2.10 Use inverses to check the calculator

1. Calculate each of the following with your calculator and draw a conclusion.

(a) $432 \times 878 \div 878$

(b) $432 \div 878 \times 878$

(c) $1\,234 \times 878 \div 878$

(d) $54\,321 \div 12\,786 \times 12\,786$

(e) $234 \times 187 \div 187$

(f) $12\,786 \div 127 \times 127$

If you start with a number, multiply it by a number and then divide by the same number, or the other way around, the start number remains unchanged.

We say multiplication and division are **inverse** operations, because the one undoes or cancels the other.

Explanation:

$$687 \times 42 \div 42 = (687 \times 42) \div 42$$

(one way of grouping)

$$= 687 \times (42 \div 42)$$

(an equivalent grouping)

$$= 687 \times 1$$

$$= 687$$

(a property of 1)

2. Sipho must calculate $234 \div 325 \times 225$.

He uses the keystroke sequence: $234 \div 325 \times 225 =$ and gets 162 as answer.

To check, he continues with $162 \div 225 \times 325 =$ and gets 234. Now he *knows* that the answer 162 *must* be right. Why?

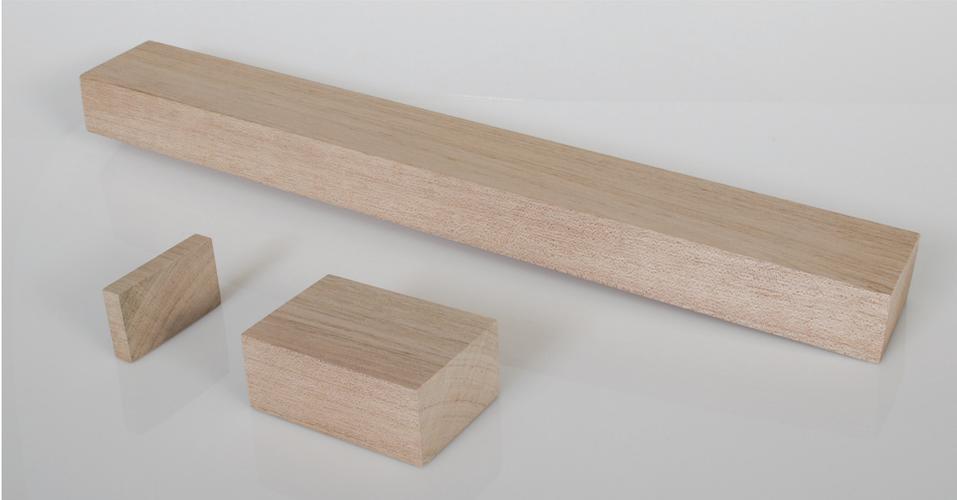
Calculator results can be checked by applying inverse operations to the result, in reverse order. You must then get the original input number as answer.

Explanation of the **inverse-in-reverse-order checking method**:

$$\begin{aligned} & 234 \div 325 \times 225 \div 225 \times 325 \\ = & 234 \times (325 \div 325) \times (225 \div 225) \quad (\text{re-order and re-group inverses}) \\ = & 234 \times 1 \times 1 \\ = & 234 \quad (\text{property of 1}) \end{aligned}$$

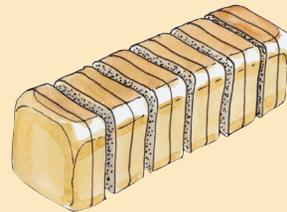
3. Calculate each of the following. Check the result by using inverse operations.
- | | |
|-------------------------------------|----------------------------------|
| (a) 437×878 | (b) $6\,804 \div 378$ |
| (c) $7\,654 \times 2\,748 \div 229$ | (d) $5\,432 \div 128 \times 496$ |
| (e) $6\,798 \times 76 \div 209$ | (f) $321 \times 62 \times 47$ |

3.1 Prisms



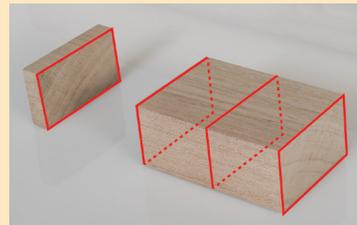
The wooden blocks are similar to a loaf of bread like the one shown on the right.

If the wooden blocks are cut into sections (slices) like the loaf of bread, the end faces are the same. This is shown with red lines on the photograph below.



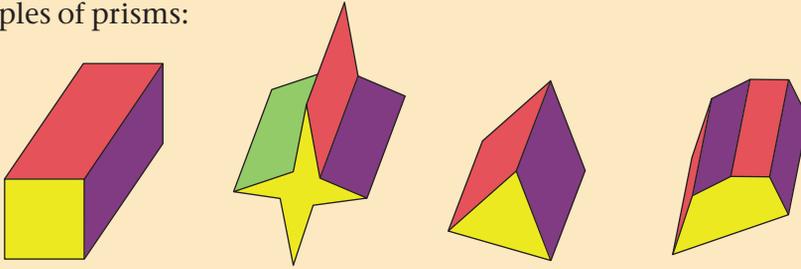
We say:

The **cross-section** of the wooden block (and the loaf of bread) remains the same along its length.



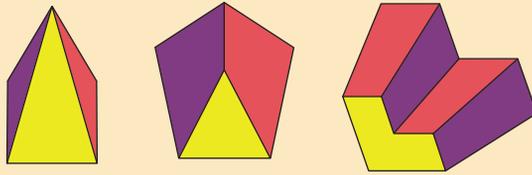
An object (like these wooden blocks) with identical ends, flat rectangular faces and a cross-section that remains the same along the length, is called a **rectangular prism**.

Examples of prisms:



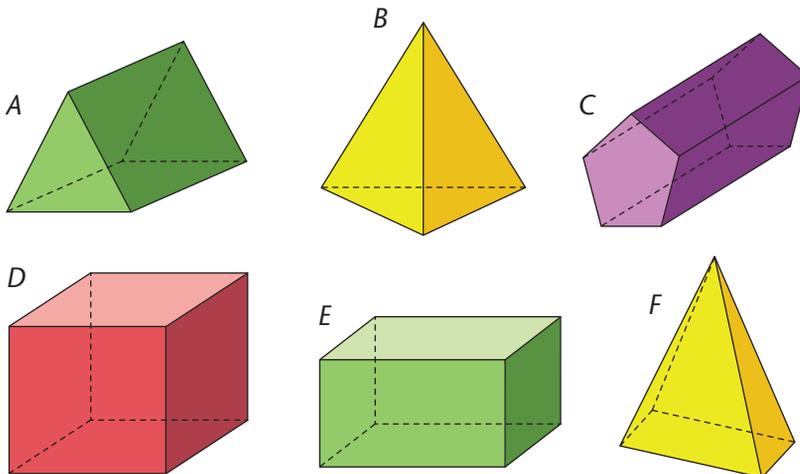
The prism on the left is a rectangular prism.

Examples of 3-D objects that are *not* prisms:



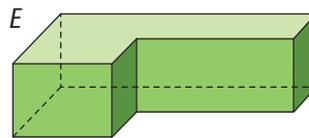
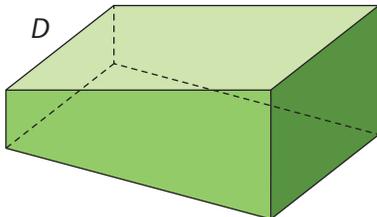
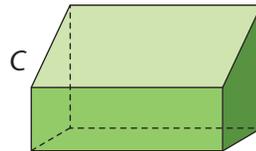
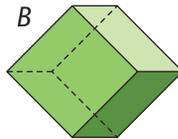
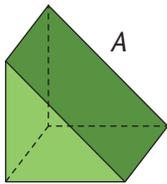
1. Match the descriptions with the objects shown below.

- The object has six faces. All faces are the same shape and size.
- The object has five faces. Two opposite faces are triangles that are the same shape and size.
- The object has seven faces. Five of the faces are rectangles that are the same shape and size.

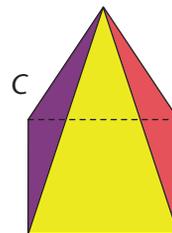
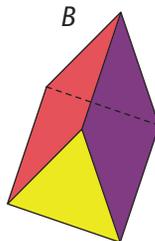
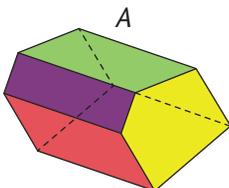


- Which two objects in question 1 are not prisms?
 - Which objects in question 1 are not rectangular prisms?

3. You cannot be sure that objects with the following properties are always rectangular prisms. Explain why not.
- The object has at least six faces.
 - The object has some rectangular faces.
 - The object has at least six faces that are rectangles.
 - The object has at least three faces that are rectangles.
 - The object has at least four faces that are rectangles.
4. Use the descriptions in question 3. Find the object below that fits each description. Decide if the object is a rectangular prism or not.



5. Object E is not a prism. Explain why not.
6. Describe the objects below in the way the objects are described in question 1.

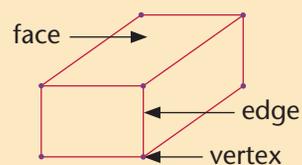


7. Which of the objects in question 6 is not a prism?

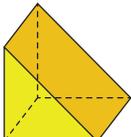
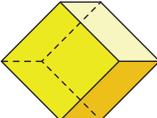
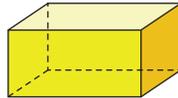
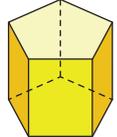
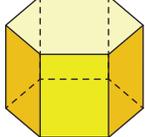
3.2 Faces, edges and vertices of prisms

When we form a prism out of polygons, two sides of two polygons are connected to form one **edge** of the prism.

The corners of three polygons are joined to form one **vertex** of the prism.



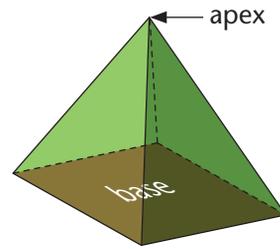
Copy and complete this table.

Prism	Shape of two opposite faces that are the same shape and the same size	Number of faces	Number of edges	Number of vertices
	triangles			
				
				
	pentagons			
	hexagons			

3.3 Pyramids

Objects like these are called **pyramids**.

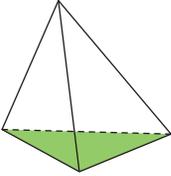
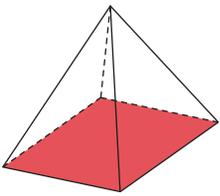
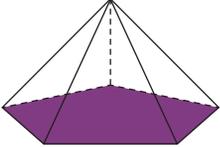
This is a square-based pyramid.

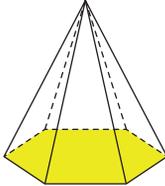
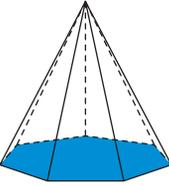
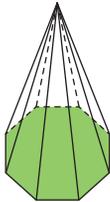


Square-based pyramid

1. (a) How many faces does a square-based pyramid have?
- (b) Describe the shapes of the faces.
- (c) How many edges does a square-based pyramid have?
- (d) How many vertices does a square-based pyramid have?

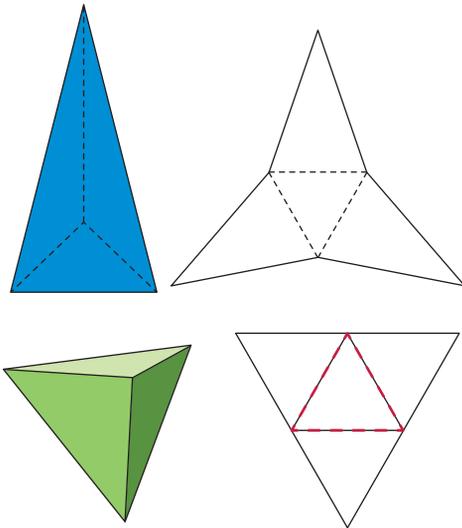
2. Complete the table by writing down the answers of (a) to (r).

Pyramid	Shape of the base	Number of faces	Number of edges	Number of vertices
	triangle	(a)	(b)	(c)
	square	(d)	(e)	(f)
	pentagon	(g)	(h)	(i)

Pyramid	Shape of the base	Number of faces	Number of edges	Number of vertices
	hexagon	(j)	(k)	(l)
	heptagon	(m)	(n)	(o)
	octagon	(p)	(q)	(r)

Pyramids that have only triangular faces are called **triangular pyramids**. Triangular pyramids can have many different shapes.

A triangular pyramid whose triangular faces are *all* the same shape and the same size is called a **tetrahedron**. All tetrahedrons have the same shape.



3.4 Build 3-D objects with straws or sticks

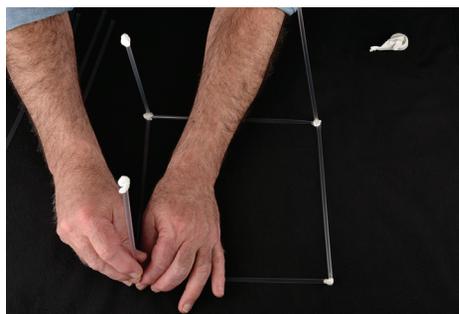
You can build skeletons of pyramids with clay or sticky putty, and straws or sticks.

Build skeletons of

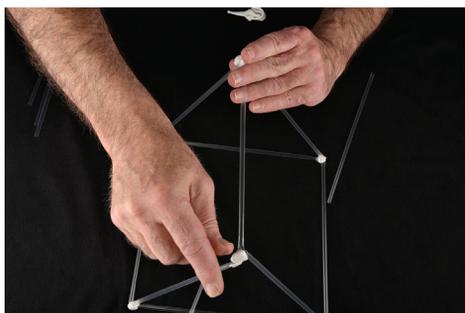
- (a) a square-based pyramid
- (b) a tetrahedron
- (c) a cube.



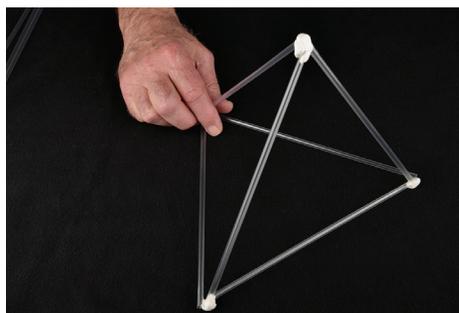
Starting with a building project



This may become the skeleton of a cube.



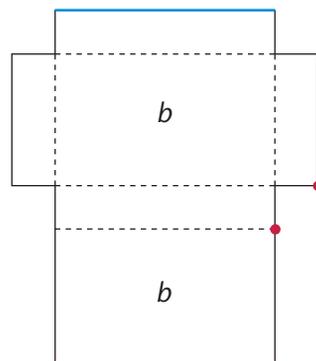
This skeleton of a prism is almost completed.



This skeleton of a pyramid is completed.

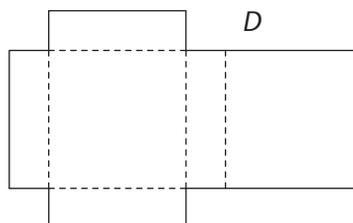
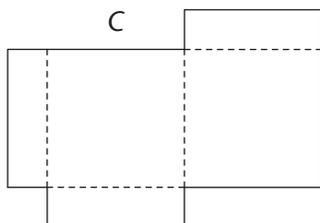
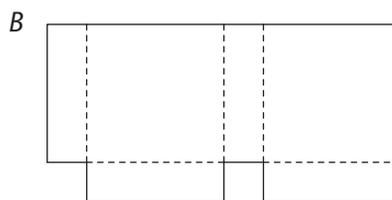
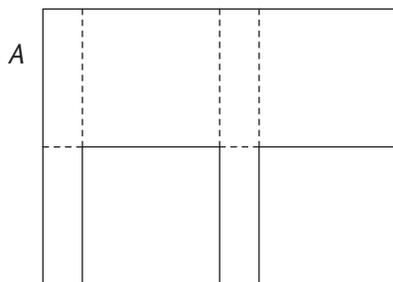
3.5 Nets of prisms and pyramids

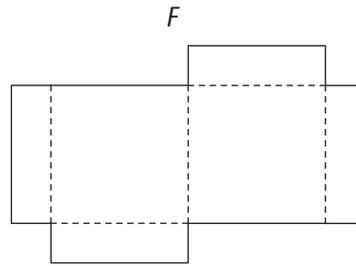
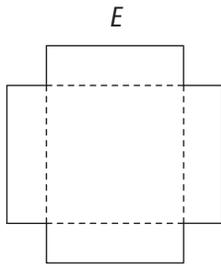
A flat figure which shows all the faces of an object is called a **net** of the object. This is a net of a rectangular prism.



- Copy the net. Label the faces on the net to explain which faces are opposite each other when the net is folded into a prism. Do this by writing the same letter on the pairs of opposite faces.
 - The blue sides of the net above will be joined to form one edge. Use matching colours on your net to show which sides will be joined to form the other edges of the prism.
 - The two red corners of the net above will form one vertex of the prism. Use matching colours on your net to show which corners of the faces will be connected to form the vertices of the prism.
- Which of the diagrams below and on the next page show nets for a rectangular prism?

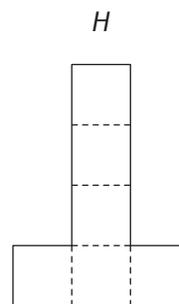
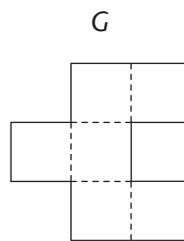
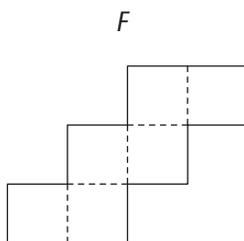
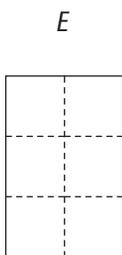
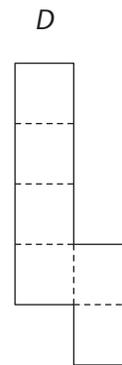
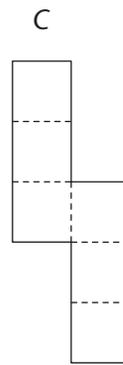
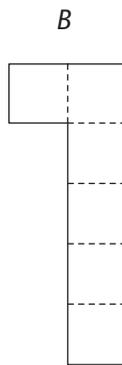
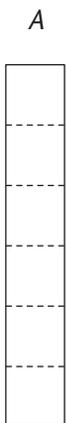
You may redraw the diagrams and use matching colours to show which sides will meet to form edges.





3. (a) Explain in your own words what a cube is.
- (b) Which of the diagrams below are nets of a cube?

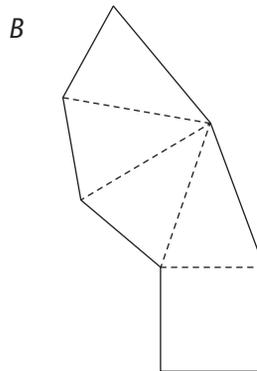
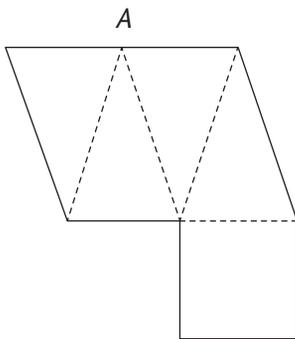
You may redraw the diagrams and use matching colours to show which sides will meet to form edges.



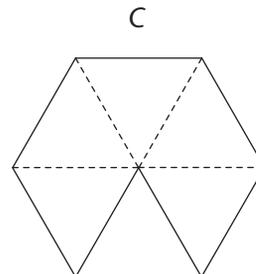
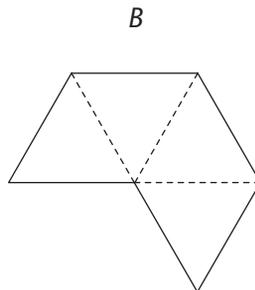
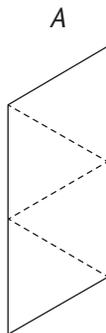
4. Imagine you cut out the diagrams below and fold them on the broken lines to form the faces for a square-based pyramid.

You may copy the diagrams and use matching colours to show which sides will meet to form edges.

- (a) Decide which diagram will not work as a net for a square-based pyramid. Explain why.



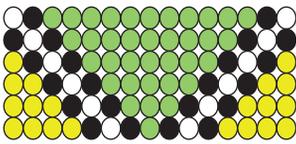
- (b) Draw a different net that can be folded to make a square-based pyramid. Cut out your net and test if it works.
- (c) Write to someone in another class. Explain how to make a net for a square-based pyramid. Make sure you say which sides of the polygons must be the same length.
5. Which of the diagrams below are nets for a tetrahedron? Explain why the other diagrams do not work.



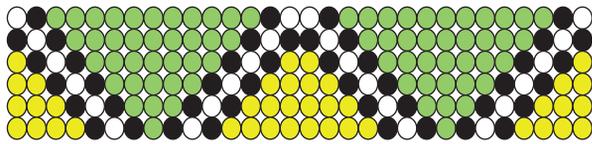
4.1 Making beautiful patterns

Busi makes beautiful bead bracelets of different designs and sizes. Size 1 and Size 2 for each design are shown below, but Busi can make bracelets of any size.

Design 1

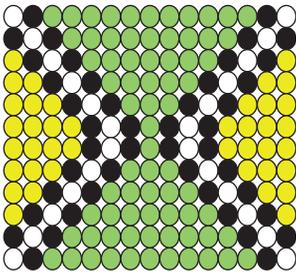


Size 1

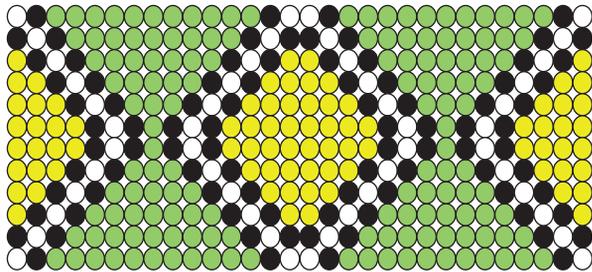


Size 2

Design 2

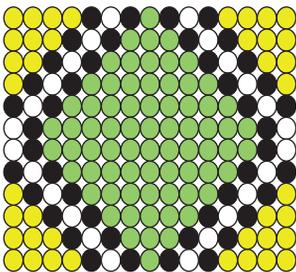


Size 1

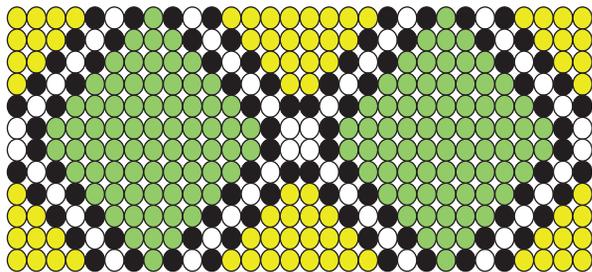


Size 2

Design 3



Size 1



Size 2

1. For Design 1:

- Describe in words how the design works.
- Complete this table. Do not count the beads in Size 1 and Size 2 one by one, but try to see bigger units and use calculation plans.

Size	1	2	3	4	5	30
No. of white beads						
No. of black beads						
No. of yellow beads						
No. of green beads						
Total no. of beads						

- Describe and discuss the methods you used to complete the table. Also describe and discuss patterns you see in the table.
 - Write down a calculation plan for the number of beads of each colour, and for the total number of beads.
 - Use your calculation plans to calculate the number of beads of each colour for Size 10, Size 20 and Size 100.
2. For Design 2, answer the same questions as for Design 1.
3. For Design 3, answer the same questions as for Design 1.

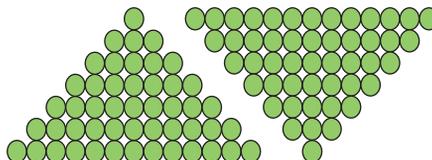
CHALLENGE

Did you see this pattern in the bracelets?

$$\text{No. of green beads} = 1 + 3 + 5 + 7 + 9 + 11$$

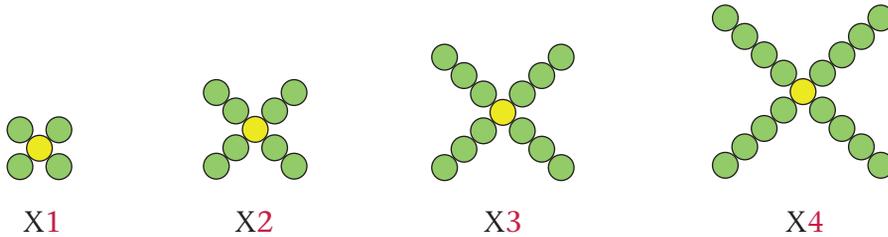
If this numeric pattern is continued, complete the table and discuss how patterns can make calculation easier.

No. of rows	No. of green beads
1	1
2	$1 + 3 = 4$
3	$1 + 3 + 5 = 9$
4	$1 + 3 + 5 + 7 = 16$
5	?
20	?



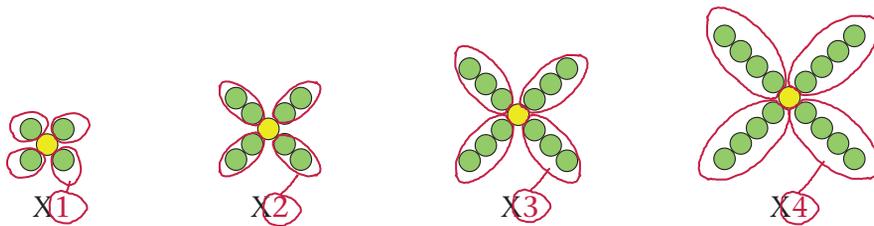
4.2 Writing calculation plans

1. Thabo uses beads to make a pattern of Xs like this:



If Thabo continues the pattern, how many beads will there be in X5, how many in X6, how many in X50 and how many in X60?

2. Mary uses clever counting to answer question 1! Try to follow her reasoning. Explain her plan to a classmate.



			Mary starts here:
		← ←	<i>I see four, four,</i>
			<i>four, four greens</i>
			<i>plus one yellow</i>
		Then here:	X4 = 4 × 4 + 1
	← ←		
	Then here:	<i>Four threes</i>	
		<i>plus 1</i>	
← ←		X3 = 4 × 3 + 1	
X1 = 4 × 1 + 1	X2 = 4 × 2 + 1		

$$X_{\text{number}} = 4 \times \text{number} + 1$$

It means “multiply the *number* by 4, then add 1”.

$$\text{So } X5 = 4 \times 5 + 1$$

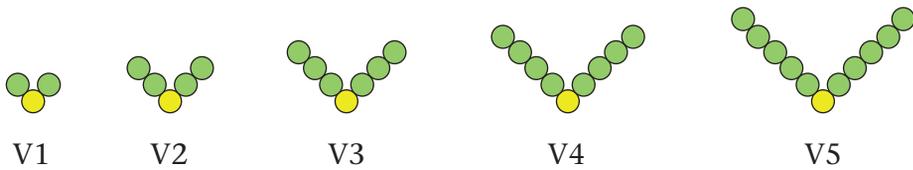
$$\text{So } X6 = 4 \times 6 + 1$$

$$\text{So } X50 = 4 \times 50 + 1$$

$$\text{So } X60 = 4 \times 60 + 1$$

Mary writes a **calculation plan (rule)**:
 $X_{\text{number}} = 4 \times \text{number} + 1$
 Now she can calculate X_{number} for any number.

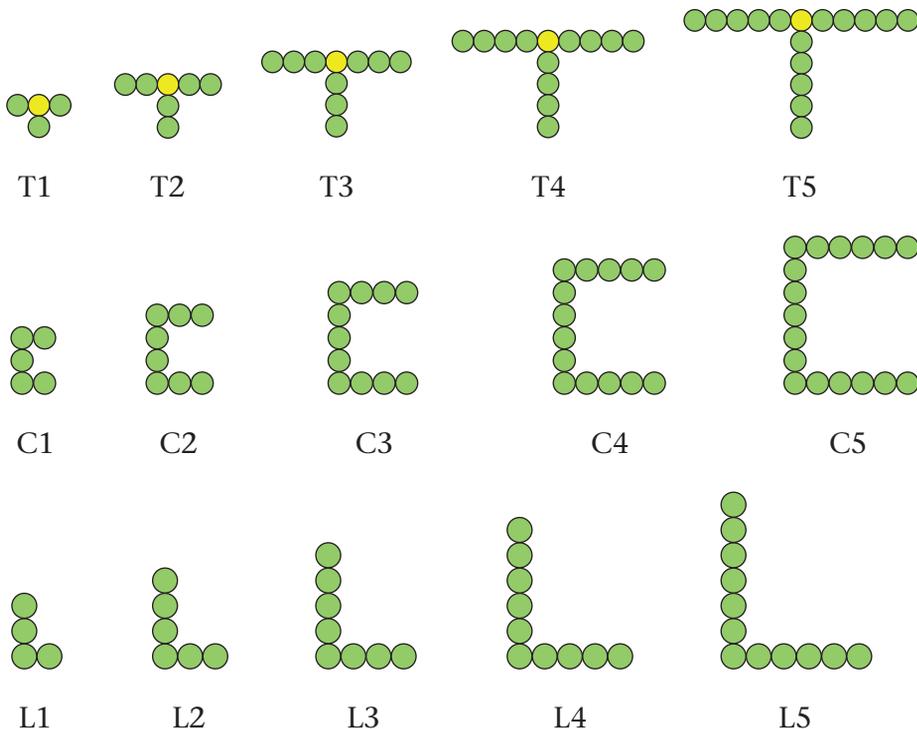
3. Suzi uses beads to make this growing V-pattern:



- Describe V6, V60 and V87 in words.
- Write your plan as a *flow diagram* and then calculate the number of beads in V6, V60 and V87.
- Write down your calculation plan, and then use it to calculate the total number of beads in V6, V60 and V87.
- What is the biggest V-number that can be made with 100 green beads and one yellow bead? How many beads are left over?

4. Sam uses beads to make these alphabet patterns.

Answer the same questions as in question 3 for these T, C and L patterns.

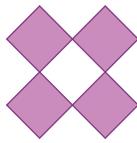


4.3 Describing patterns

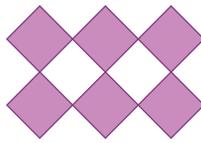
Purple tiles and white tiles are arranged to make this growing pattern:



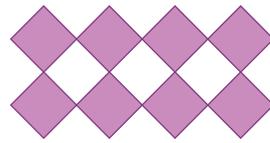
Size 1



Size 2



Size 3



Size 4

- Complete the table. Describe your methods.

Size	1	2	3	4	5	6	30
No. of purple tiles	2	4	6				
No. of white tiles	0	1	2				
Total no. of tiles	2	5	8				

- Describe *horizontal* numeric patterns (number patterns) for the purple tiles, for the white tiles and for the total number of tiles in the table.

How can you use these horizontal patterns to calculate the number of purple tiles, the number of white tiles and the total number of tiles?

“Horizontal” means from left to right;
“vertical” means from top to bottom.

- Describe *vertical* numeric patterns for the purple tiles, for the white tiles and for the total number of tiles in the table.

How can you use these patterns to calculate the number of purple tiles, the number of white tiles and the total number of tiles?

- How many purple tiles are there in a Size 50 pattern?
- How many white tiles are there in a Size 50 pattern?
- How many tiles are there in total in a Size 50 pattern?

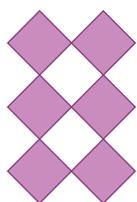
-
7. Here are three other growing geometric patterns made with purple and white tiles.

Answer the same questions as in questions 1 to 6 for each tile pattern.

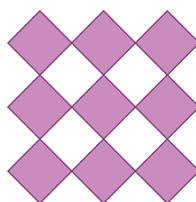
Pattern X



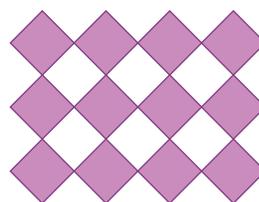
Size 1



Size 2



Size 3

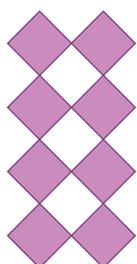


Size 4

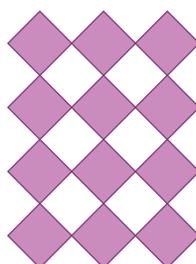
Pattern Y



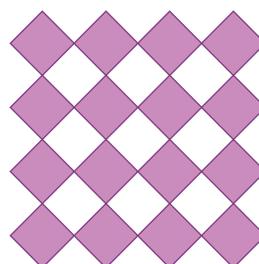
Size 1



Size 2



Size 3

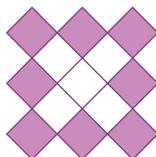


Size 4

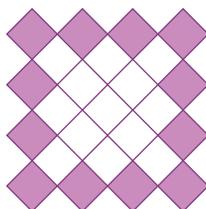
Pattern Z



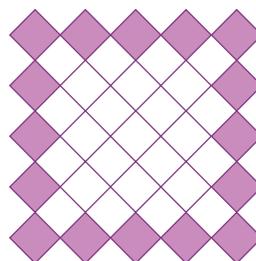
Size 1



Size 2



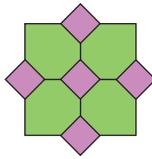
Size 3



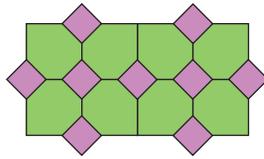
Size 4

4.4 From pictures to tables

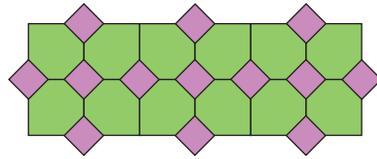
In this tile pattern, Size 1 is made of 4 green tiles and 5 smaller purple tiles. The pattern is then continued as shown.



Size 1



Size 2



Size 3

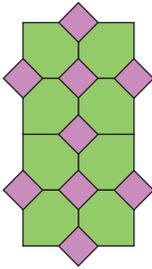
- Complete this table and describe your methods.

Size	1	2	3	4	5	30
No. of green tiles	4					
No. of purple tiles	5					

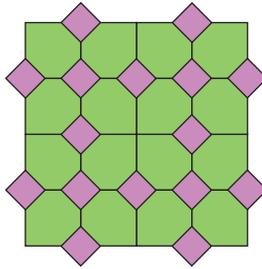
- Describe *horizontal* numeric patterns for the green and for the purple tiles in the table.
How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
- Describe *vertical* numeric patterns for the green and for the purple tiles in the table.
How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
- Write down a calculation plan (rule) to calculate the number of green tiles instead of counting them.
How many green tiles are there in a Size 50 pattern?
- Write down a calculation plan (rule) to calculate the number of purple tiles.
How many purple tiles are there in a Size 50 pattern?

4.5 More pictures and tables

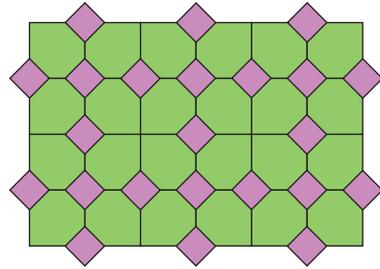
In this tile pattern, Size 1 is made of 8 green tiles and 9 smaller purple tiles. The pattern is then continued as shown.



Size 1



Size 2



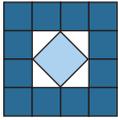
Size 3

1. Complete this table and describe your methods.

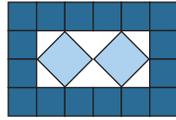
Size	1	2	3	4	5	30
No. of green tiles	8					
No. of purple tiles	9					

2. Describe *horizontal* numeric patterns for the green tiles and for the purple tiles in the table.
How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
3. Describe *vertical* numeric patterns for the green tiles and for the purple tiles in the table.
How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
4. Write down a calculation plan (rule) to calculate the number of green tiles instead of counting them.
How many green tiles are there in a Size 50 pattern?
5. Write down a calculation plan (rule) to calculate the number of purple tiles.
How many purple tiles are there in a Size 50 pattern?

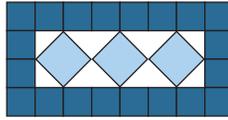
6. This growing pattern of light blue, dark blue and white tiles is used for a large supermarket floor.



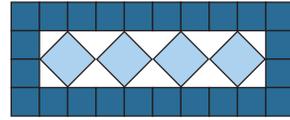
Size 1



Size 2



Size 3



Size 4

Complete the table.

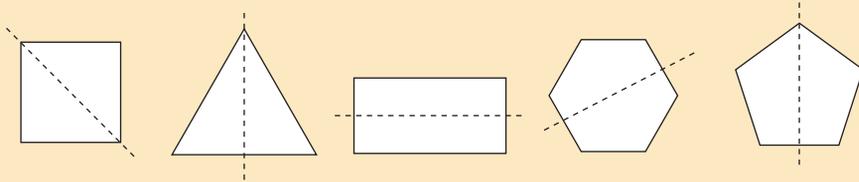
Describe your method, and describe the patterns that you see in the table.

Size	1	2	3	4	5	6	10	30
No. of light blue tiles	1	2						
No. of dark blue tiles	12							

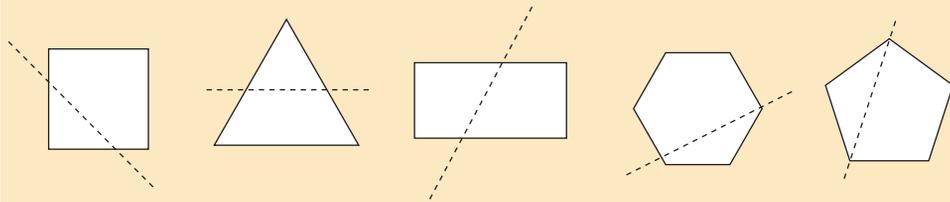
7. Make your own growing geometric pattern with squares, ask your own questions, and then answer your questions.

5.1 Lines of symmetry

The lines across these figures are **lines of symmetry** for the figures:



The lines across these figures are *not* lines of symmetry:



1. What is the difference between the two groups of figures above in the way the lines relate to the figures?
2. Which statement below explains what it means to say a figure has **line symmetry**?

Statement A:

A figure has line symmetry if you can fold it into two parts that are exactly the same size and shape.

or

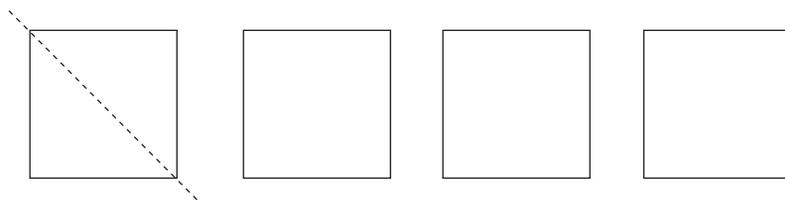
Statement B:

A figure has line symmetry if you can fold it into two parts that are exactly the same size and shape and the two parts fold exactly onto each other.

5.2 Many lines of symmetry

1. It is possible to find four different lines of symmetry for a square.

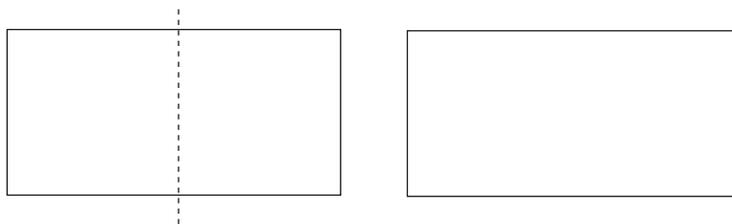
- (a) Draw four identical squares. Draw a different line of symmetry for each square. Here is one line of symmetry.



- (b) Explain how you know that you are right.

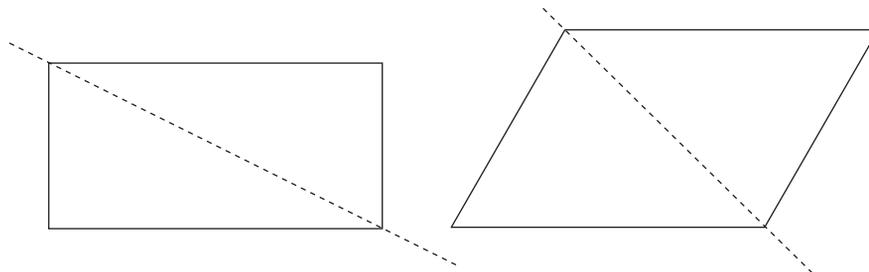
2. A rectangle that is not a square has exactly two lines of symmetry.

- (a) Draw two identical rectangles with a different line of symmetry in each one. Here is one of the lines of symmetry.



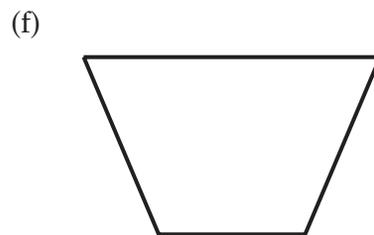
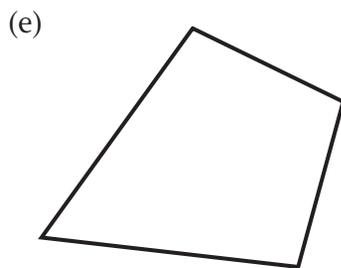
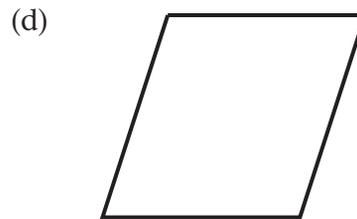
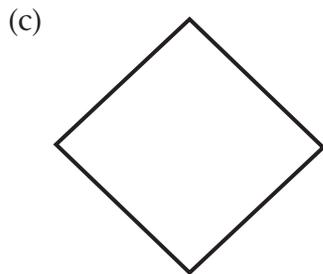
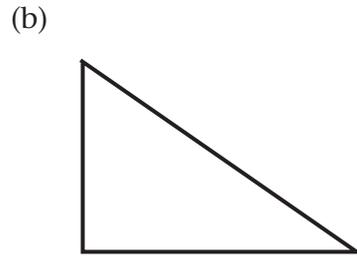
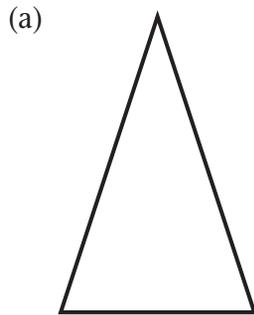
- (b) Explain how you know that you are right.

3. Explain clearly why these lines are *not* lines of symmetry for the quadrilaterals.



-
4. How many lines of symmetry do these polygons have? Draw the polygons and show the lines of symmetry.

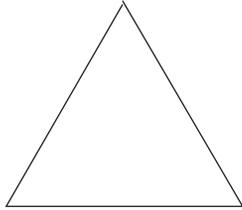
If you struggle to draw the polygons, put a clean page over this one, mark the corners of the polygons with dots and then connect the dots with straight lines.



Polygons of which the sides are all the same length, and the angles are all the same size are called **regular polygons**.

5. Say how many sides and how many lines of symmetry each of the regular polygons below have.

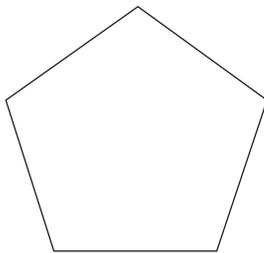
(a)



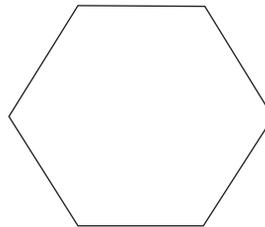
(b)



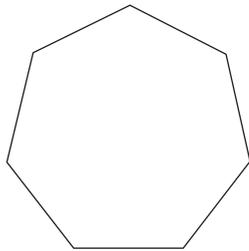
(c)



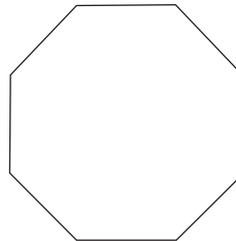
(d)



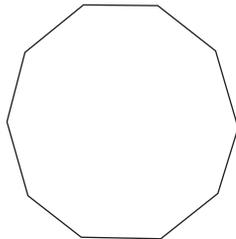
(e)



(f)

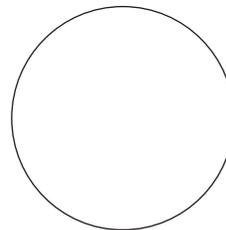


(g)



6. How many lines of symmetry does a circle have?

Explain why you say so.



6.1 What is division?

To answer any of the following questions, you have to do division.

- A. How many pieces of 34 cm each can you cut from 7 894 cm of rope on a roll?
- B. How much will each person get if R7 854 is shared equally between 34 people?
- C. A house is 34 times as big as its drawing on the building plan. In the actual house, one of the walls is 7 888 mm long. How long is the line that shows this wall on the plan?
- D. A wall is 34 mm long on the building plan. The actual wall in the house is 7 888 mm long. How many times bigger than the plan is the actual house?
- E. For what number will the sentence $34 \times \dots = 7\,888$ be true?

1. Read question A again. Think about the situation. Then answer these questions:
 - (a) Do you think you can cut 1 000 pieces of 34 cm each from a roll with 7 894 cm of rope?
 - (b) Can you cut 100 pieces of 34 cm each from the roll?
2. Read question B again, think about it and then answer these questions:
 - (a) Do you think each person can get at least R200?
 - (b) Do you think each person can get R300?
3. Read question C again. Then answer this question:

If a wall is shown by a 200 mm line on the building plan, how long is the wall in the actual house?

-
4. Read question D again. Then answer this question:
If the house is 200 times as big as the drawing on the plan, how long is the wall shown by the 34 mm line in the actual house?
 5. Read question E again and then answer these questions:
 - (a) Can the number that will make the sentence true be bigger than 300?
 - (b) How much is 34×250 ?
 - (c) How much is 34×230 ?
 6. What number will make the sentence $57 \times \dots = 4\,731$ true?

You will study a method of division in the next section.

To do division, you have to be good at **forming multiples** of the numbers that you divide by, for example the number 64 in $3\,829 \div 64$.

The number by which you divide another number is called the **divisor**.

To form a multiple of a number, you multiply the number by another number. For example:

10×64 is 640, so 640 is a multiple of 64.

100×64 is 6 400, so 6 400 is a multiple of 64.

Doubling may be used in some cases to find multiples.

For example, if you know that $40 \times 53 = 2\,120$, you can double 2 120 to find 80×53 .

Halving may also be useful to find multiples.

For example, if you know that $100 \times 68 = 6\,800$, you can halve 6 800 to find 50×68 .

7. To do division you need to be able to answer questions like these. Note that you can use your answers for (a) to easily find the answers for (b).
 - (a) How much are 100×73 and $53 \times 1\,000$?
 - (b) How much are 50×73 and 53×500 ?
 - (c) How much are 25×73 and 53×250 ?
 - (d) How much are 125×73 and 53×750 ?

8. Practise forming multiples. Also keep in mind what you have just read about doubling and halving! You will find both techniques very useful.

Form nine multiples of each number below, by multiplying it with 10, 100, 50, 30, 40, 60, 70, 80 and 90.

- | | |
|--------|--------|
| (a) 37 | (b) 76 |
| (c) 98 | (d) 43 |
| (e) 38 | (f) 55 |

6.2 Dividing by building up

$6\ 150 \div 73$ can be calculated like this:

Thinking	Writing	Thinking
		$100 \times 73 = 7\ 300$
Half of that:	$50 \times 73 = 3\ 650$	3 650 is more than 1 000 away from 6 150.
	$10 \times 73 = 730$	$60 \times 73 = 3\ 650 + 730 = \mathbf{4\ 380}$ So there is room for 730 more.
	$10 \times 73 = 730$	$70 \times 73 = 4\ 380 + 730 = \mathbf{5\ 110}$ There is room for 730 more.
	$10 \times 73 = 730$	$80 \times 73 = 5\ 110 + 730 = \mathbf{5\ 840}$ Still 160 + 150 to go!
	$3 \times 73 = 219$	$83 \times 73 = 5\ 840 + 219 = \mathbf{6\ 059}$ So I can add another 73.
	$\frac{1 \times 73}{\quad} = \frac{73}{\quad}$	$84 \times 73 = 6\ 059 + 73 = \mathbf{6\ 132}$
Altogether:	$84 \times 73 = 6\ 132$	

$6\ 150 - 6\ 132 = 18$, so $6\ 150 \div 73 = 84$ remainder 18.

In the above method, multiples of 73 are added up until the distance from 6 150 is less than 73.

The work can be set out more briefly by leaving out the descriptions of the thinking:

$50 \times 73 = 3\ 650$	$3\ 650$
$10 \times 73 = 730$	$4\ 380$
$10 \times 73 = 730$	$5\ 110$
$10 \times 73 = 730$	$5\ 840$
$3 \times 73 = 219$	$6\ 059$
$\underline{1} \times 73 = \underline{73}$	$6\ 132$
$84 \times 73 = 6\ 132$	

$6\ 150 - 6\ 132 = 18$, so $6\ 150 \div 73 = 84$ remainder 18.

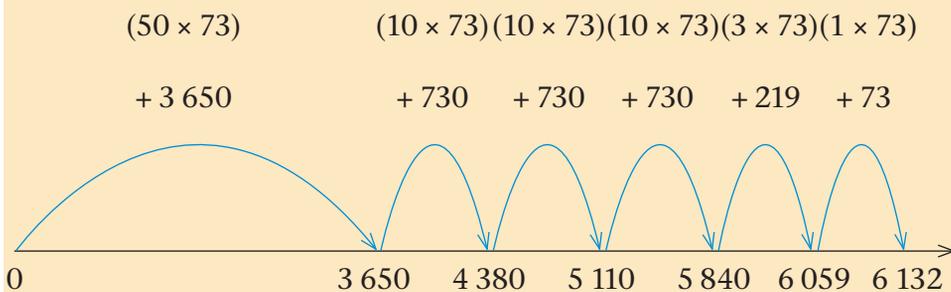
The work to calculate $6\ 150 \div 73$ can also be summarised as follows:

$$3\ 650 + 730 \rightarrow 4\ 380 + 730 \rightarrow 5\ 110 + 730 \rightarrow 5\ 840 + 219 \rightarrow 6\ 059 + 73 \rightarrow 6\ 132$$

or

$$(50 \times 73) + (10 \times 73) + (10 \times 73) + (10 \times 73) + (3 \times 73) + (1 \times 73) = 6\ 132$$

We may also think of the division work as movements on a number line. This is shown here for $6\ 150 \div 73$.

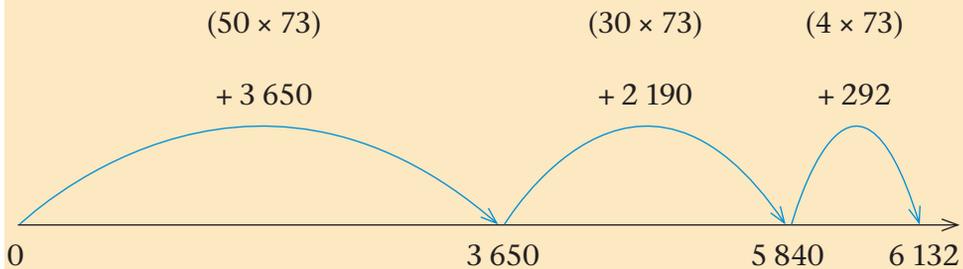


$6\ 150 - 6\ 132 = 18$, so $6\ 150 \div 73 = 84$ remainder 18.

If you can estimate well, you can do it in fewer steps. For example:

$$\begin{array}{r}
 50 \times 73 = 3\,650 \\
 30 \times 73 = 2\,190 \\
 \underline{4 \times 73 = 292} \\
 84 \times 73 = 6\,132
 \end{array}
 \qquad
 \begin{array}{r}
 3\,650 \\
 5\,840 \\
 6\,132
 \end{array}$$

$6\,150 - 6\,132 = 18$, so $6\,150 \div 73 = 84$ remainder 18.



$6\,150 - 6\,132 = 18$, so $6\,150 \div 73 = 84$ remainder 18.

You can check the answer by multiplying, and you may use a calculator to do so:

$$84 \times 73 = 6\,132$$

Remember to add the remainder to this:

$$6\,132 + 18 = 6\,150$$

Work out the answers to these questions.

- Calculate $950 \div 64$. Use as many steps as you need.
 - Investigate how you could have done it using fewer steps.
 - Multiply to check your answer.
- Calculate $5\,700 \div 64$. Use as many steps as you need.
 - Investigate how you could have done it using fewer steps.
 - Multiply to check your answer.

3. (a) Calculate $3\,450 \div 93$. Use as many steps as you need.
(b) Check your answer. Show how you do it.
4. A computer factory builds 2 784 computers every day. If the factory operates 24 hours a day, how many computers are built in one hour?
5. Peppy wants to buy a skateboard that costs R1 875. He washes cars in the neighbourhood and earns R28 for every car he washes. How many cars must he wash to earn enough money so that he can buy the skateboard?
6. A supermarket donates 4 698 boxes of wax crayons to nursery schools. The boxes of wax crayons are divided equally between 27 nursery schools. How many boxes of wax crayons does each nursery school get?

Some people find it useful to subtract every now and again when doing division. They do this in order to know more accurately what the remainder is.

For example, while calculating $6\,150 \div 73$, first 3 650 and later 5 840 are subtracted from 6 150. The blue frames below show you where this is done:

		Remainder
$50 \times 73 = 3\,650$	$3\,650$	$6\,150 - 3\,650 = 2\,500$
$20 \times 73 = 1\,460$	$5\,110$	
$10 \times 73 = 730$	$5\,840$	$6\,150 - 5\,840 = 310$
$\underline{4 \times 73 = 292}$	$6\,132$	$6\,150 - 6\,132 = 18$
$84 \times 73 = 6\,132$		

$6\,150 - 6\,132 = 18$, so $6\,150 \div 73 = 84$ remainder 18.

7. Use the above technique to calculate the following.
 - (a) $5\,068 \div 36$
 - (b) $9\,274 \div 26$

6.3 Practice

- Mr Nkosi can transport 26 bundles of wood per load with his bakkie. How many loads will he need to transport 3 300 bundles of wood?
- (a) 8 000 roof tiles have to be made up in 32 equal stacks. How many tiles will there be in a stack?
(b) There are 248 bricks on one pallet. How many pallets do you have to buy if you need 8 000 bricks?
- Captain Hook and his 35 pirates discovered a chest with gold coins. They shared the 4 752 coins equally amongst them. How many coins did each of the 36 men get?
- The school's drama club is putting on a play. Each club member has to sell 26 tickets. The members have to sell a total of 1 404 tickets. How many members does the drama club have?
- A number of soccer players practised kicking goals. In total, 1 470 kicks were made. Each player kicked 35 times. How many players took part in this practice session?
- Calculate.
 - $1\ 176 \div 28$
 - $1\ 176 \div 42$
 - $3\ 060 \div 36$
 - $3\ 072 \div 32$

6.4 The long division method

You have now often done division by *adding up* multiples of the divisor.

Below is an example of how to use the method when we have to calculate $8\ 649 \div 34$. The divisor is 34.

		Remainder
$200 \times 34 =$	6 800	6 800
$50 \times 34 =$	1 700	8 500
$\underline{4} \times 34 =$	<u>136</u>	8 636
$254 \times 34 =$	8 636	13

So $8\ 649 \div 34 = 254$ remainder 13.

Here is another way of doing division. Instead of adding up the multiples of the divisor, we can *subtract* them from the number that is divided into parts.

This method is shown below, again for $8\,649 \div 34$.

	8 649	Remainder	Explanation
$200 \times 34 =$	6 800	1 849	$8\,649 - 6\,800 = 1\,849$
$50 \times 34 =$	1 700	149	$1\,849 - 1\,700 = 149$
$\underline{4} \times 34 =$	<u>136</u>	13	$149 - 136 = 13$
$254 \times 34 =$	8 636		

So $8\,649 \div 34 = 254$ remainder 13.

Here is a shorter way of recording this:

8 649	Explanation	Explanation
$\begin{array}{r} 8\,649 \\ - 6\,800 \\ \hline 1\,849 \end{array}$	200×34	$8\,649 - 6\,800 = 1\,849$
$\begin{array}{r} 1\,849 \\ - 1\,700 \\ \hline 149 \end{array}$	50×34	$1\,849 - 1\,700 = 149$
$\begin{array}{r} 149 \\ - 136 \\ \hline 13 \end{array}$	$\underline{4} \times 34$	
	254	$200 + 50 + 4 = 254$

So $8\,649 \div 34 = 254$ remainder 13.

- Use the above method to do the following calculations. You may leave out the explanation column that shows the subtractions.
 - $7\,814 \div 42$
 - $9\,638 \div 28$
- Now do the calculations in question 1 by adding up multiples of the divisor, as you did previously.
- Use any method to calculate the following.
 - $2\,444 \div 47$
 - $4\,205 \div 29$
 - $1\,856 \div 32$
 - $7\,922 \div 34$

A piece of history

In the past, people used the following way to record their work when doing division. The explanations were normally left out.

$\begin{array}{r} 254 \\ 34 \overline{) 8\,649} \\ \underline{6\,800} \\ 1\,849 \\ \underline{1\,700} \\ 149 \\ \underline{136} \\ 13 \end{array}$	<p>Explanation</p> <p>200×34</p> <p>50×34</p> <p>4×34</p> <p>254</p>
--	--

So $8\,649 \div 34 = 254$ remainder 13.

The work was done in stages as shown below.

The zeros were not written, to keep the space for the other figures.

Stage 1	Stage 2	Stage 3
$\begin{array}{r} 200 \\ 34 \overline{) 8\,649} \\ \underline{6\,800} \\ 1\,849 \end{array}$	$\begin{array}{r} 250 \\ 34 \overline{) 8\,649} \\ \underline{6\,800} \\ 1\,849 \\ \underline{1\,700} \\ 149 \end{array}$	$\begin{array}{r} 254 \text{ remainder } 13 \\ 34 \overline{) 8\,649} \\ \underline{6\,800} \\ 1\,849 \\ \underline{1\,700} \\ 149 \\ \underline{136} \\ 13 \end{array}$
200×34	200×34 50×34	200×34 50×34 4×34

If you wish, you may also do and record your division work like this.

6.5 Practice

- 8 028 books are wrapped in bundles of 36 for distribution to schools. How many bundles of 36 books will there be?
 - A school has R9 200 available to buy books at R88 each. How many books can the school buy?
- A water tank has a capacity of 150 ℓ. The capacity of a small measuring cup is 100 ml. How many full measuring cups will fill the tank (provided that no water is spilled)?
- To make a chocolate drink, 10 ml of chocolate powder has to be used for every 200 ml of milk used.
 - How much milk should be used with 5 ml chocolate powder?
 - How much chocolate powder do you need for $\frac{1}{2}$ ℓ of milk?
 - If 3 ℓ of chocolate drink is shared equally among 8 children, how much does each child get? Answer in millilitres.
- 1 728 small *cubic* building blocks are stacked to form a bigger *cube*. If the height of the bigger cube is 12 blocks, how many blocks are needed for the length and how many are needed for the width?
- A total of 1 400 square tiles are laid in the shape of a square and a rectangle. The square consists of 144 tiles.
 - How many rows of tiles are there in the square, and how many tiles are there in one row?
 - How many tiles are there in the rectangle?
 - The short side of the rectangle consists of 8 tiles. How many rows of 8 tiles each are there?
- A special box of sweets has 1 080 sweets! The sweets are packed in neat rows and in more than one layer.
 - In each layer, there are 18 sweets in a row. If there are 216 sweets in one layer, how many rows are there in one layer?
 - How many layers of sweets are there in the box?

6.6 Dividing with the calculator

It is easy to divide with the calculator. For example, to calculate $6\,804 \div 324$, the keystroke sequence $6\,804 \div 324 =$ gives 21.

1. Calculate each of the following using your calculator.
How can you be sure your answer is correct?
Describe and use different methods to check your calculator answer.
 - (a) $6\,804 \div 324$
 - (b) $6\,318 \div 234$
 - (c) $32\,136 \div 78$
 - (d) $5\,408 \div 13$
 - (e) $7\,353 \div 9$
 - (f) $9\,963 \div 123$

Vusi and Busi check their written and calculator answers by using **estimation** before or after calculation.

Their method is to round off the numbers, so that they can easily calculate the estimate *mentally*. For example, to check $6\,804 \div 324$ they work as follows:

- Vusi rounds $6\,804 \div 324$ to $6\,000 \div 300$ and calculates it *mentally* as 20.
- Busi rounds $6\,804 \div 324$ to $6\,600 \div 300$ and calculates it *mentally* as 22.

Then the calculator answer $6\,804 \div 324 = 21$ seems about right.

2. Complete this table by first doing some mental calculation before finding the calculator answer:

Calculation	Mental estimation	Calculator answer
$6\,804 \div 324$	$6\,000 \div 300 = 20$	21
$4\,248 \div 236$		
$675 \div 15$		
$3\,584 \div 32$		
$5\,705 \div 163$		
$5\,781 \div 47$		
$8\,118 \div 66$		

6.7 Broken keys: estimate and improve

Cliffy wants to calculate $731 \div 17$ on his calculator, but the $\boxed{\div}$ is broken! Can you help him to get the answer on the calculator?

Cliffy reasons that if $731 \div 17 = \square$, then $17 \times \square = 731$, and then he uses the $\boxed{\times}$ key to calculate \square with an estimate-and-improve method:

Calculation	Estimate	= 731?
$731 \div 17$	$17 \times 12 = 204$	No, 12 is far too small.
	$17 \times 30 = 510$	No, 30 is too small.
	$17 \times 40 = 680$	No, 40 is too small.
	$17 \times 45 = 765$	No, 45 is too big. \square is between 40 and 45.
	$17 \times 42 = 714$	42 is too small. \square is between 42 and 45.
	$17 \times 43 = 731$	Yes! So $731 \div 17 = 43$.

Use Cliffy's estimate-and-improve multiplication method to calculate each of the following on your calculator. Remember, you may not use the $\boxed{\div}$ key.

1. $871 \div 13$
2. $1\,334 \div 23$
3. $1\,462 \div 34$
4. $9\,717 \div 123$
5. $6\,873 \div 87$
6. $14\,508 \div 62$

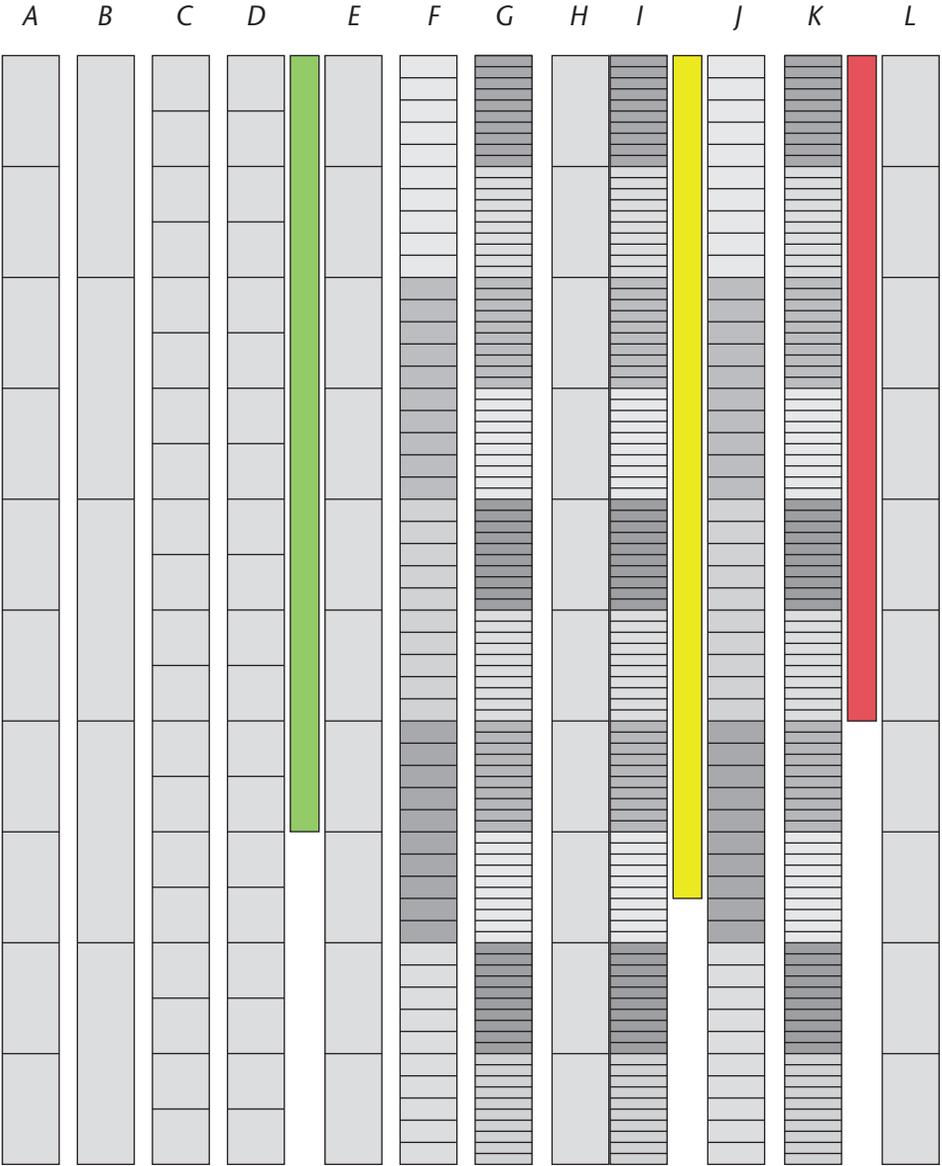
7.1 Fifths and tenths and hundredths

In this unit you will measure lengths with Greysticks. Because the Greystick is longer than the Yellowstick, we can divide it into many more smaller parts than we could divide a Yellowstick. This makes it possible to measure lengths more accurately.

We shall focus on fifths, tenths and hundredths and will learn a different notation for fractions.

Answer the questions below. The strips and Greysticks are given on the next page.

1. What can we call the small parts in Greysticks A, B and C?
2. How long is the green strip? Write your answer in more than one way.
3. What do we call the small parts in Greystick F?
4. What do we call the small parts in Greystick G?
5. How long is the yellow strip?
6. How long is the red strip? Give two or more possible answers.
7. Write these fractions as tenths:
 - (a) two fifths
 - (b) three fifths
 - (c) eight twentieths
 - (d) five fiftieths
8. Write these fractions as hundredths:
 - (a) two fifths
 - (b) three fifths
9. Add the following and give your answers in hundredths:
 - (a) 6 tenths + 7 tenths
 - (b) 23 hundredths + 5 hundredths
 - (c) 35 hundredths + 73 hundredths
 - (d) 14 tenths + 3 hundredths
 - (e) 123 tenths + 42 hundredths



7.2 A different notation for fractions

You can write the number $2\frac{3}{10}$ as 2,3 and the number $1\frac{1}{2}$ as 1,5.

1. If $2\frac{3}{10}$ is written as 2,3, why do you think $1\frac{1}{2}$ is written as 1,5?
Discuss this with one or two of your classmates.

$2\frac{3}{10}$ and 2,3 are two different notations for the same number.

2,3 is the **decimal notation**.

$\frac{3}{10}$ has no whole number part and so it is written as 0,3.

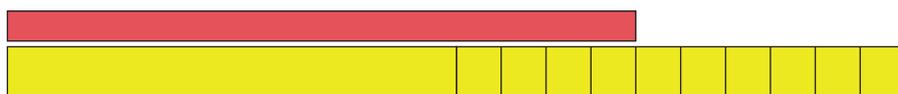
A comma separates the whole number part from the fraction. The first position after the comma indicates the number of tenths in the number. The second position is for the hundredths.

The number $1\frac{1}{2}$ can be written as 1,5 because 1,5 is 1 and 5 tenths.

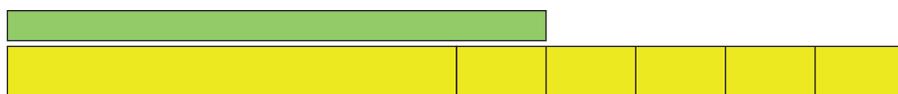
2. Write the length of each of these strips in fraction notation and in decimal notation. Measure in Yellowsticks. This is one Yellowstick:



(a)

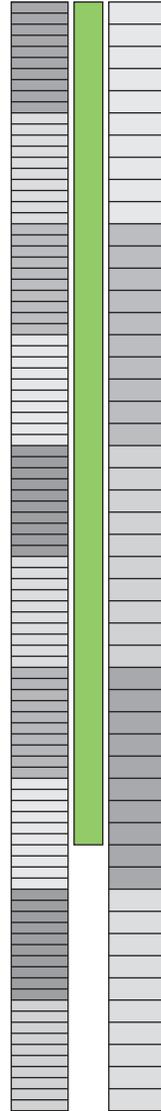


(b)



3. How can you turn a tenths ruler into a fiftieths ruler?
4. How can you turn a tenths ruler into a hundredths ruler?

5. (a) On the right is a green strip between two Greysticks. Write the length of the green strip in fraction notation. Give two answers.
- (b) Write the length of the green strip in decimal notation.



6. Write the following fractions in decimal notation:

(a) $\frac{7}{10}$

(b) $\frac{72}{100}$

(c) $3\frac{7}{100}$

(d) $1\frac{70}{100}$

(e) $\frac{3}{100}$

(f) $\frac{27}{10}$

7. Write the following in fraction notation:

(a) 2,57

(b) 0,3

(c) 1,04

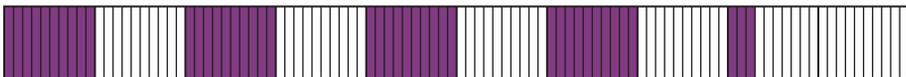
(d) 0,03

(e) 5,30

(f) 1,22

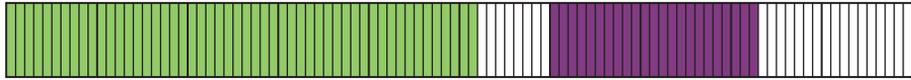
8. (a) What fraction of this rectangle is purple?
- (b) What fraction of this rectangle is white?

Give your answers in fraction notation as well as in decimal notation.



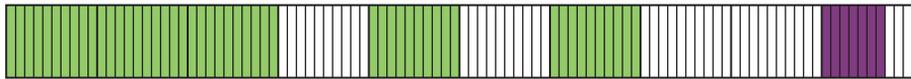
9. What fraction of this rectangle is
 (a) green (b) purple (c) white?

Give your answers in fraction notation as well as in decimal notation.



10. What fraction of this rectangle is
 (a) green (b) purple (c) white?

Give your answers in fraction notation as well as in decimal notation.



11. What fraction of this rectangle is
 (a) green (b) purple (c) white?

Give your answers in fraction notation as well as in decimal notation.



7.3 Place value parts and number names

We write $300 + 50 + 6 + \frac{7}{10} + \frac{2}{100}$ as 356,72.

This notation, $300 + 50 + 6 + \frac{7}{10} + \frac{2}{100}$, is called the **expanded notation** or **place value expansion** of 356,72.

- Write down *in words* how you would read the number 356,72 aloud.
- Simon says 356,72 is three hundred and fifty-six comma seventy-two.
 - Is Simon correct?
 - Explain your answer.

We can also call $300 + 50 + 6 + \frac{7}{10} + \frac{2}{100}$ the **place value parts** of 356,72.

We read 356,72 as three hundred and fifty-six comma seven two.

The **number name** of 356,72 is three hundred and fifty-six and seven tenths and two hundredths.

The digit 3 in 354,76 tells us that there are 3 hundreds in the number.

The digit 3 in 534,76 tells us that there are 3 tens in the number.

The digit 3 in 543,76 tells us that there are 3 units in the number.

The digit 3 in 547,36 tells us that there are 3 tenths in the number.

The digit 3 in 547,63 tells us that there are 3 hundredths in the number.

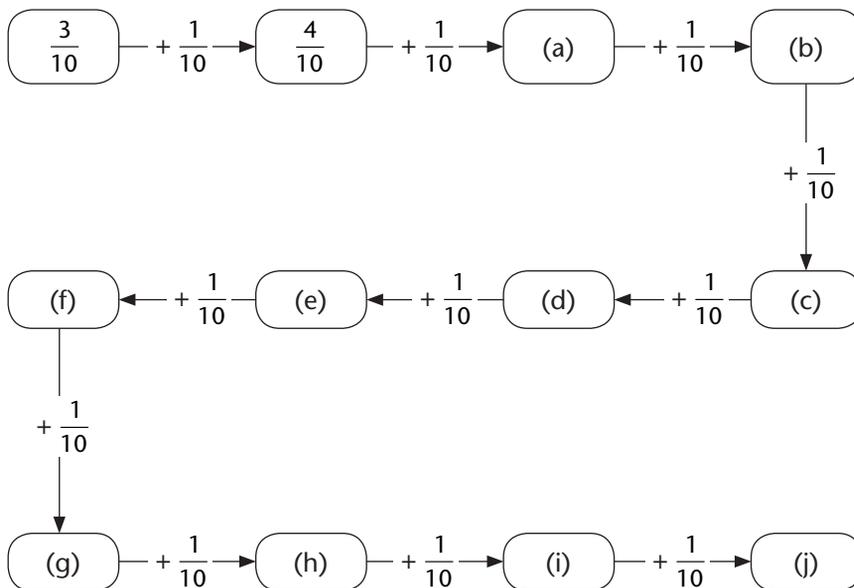
This table shows how the above numbers are made up of place value parts. The table also shows the different numbers that are indicated by the digit 3 in different positions.

	Hundreds	Tens	Units	Tenths	Hundredths
354,76	3	5	4	7	6
534,76	5	3	4	7	6
543,76	5	4	3	7	6
547,36	5	4	7	3	6
547,63	5	4	7	6	3

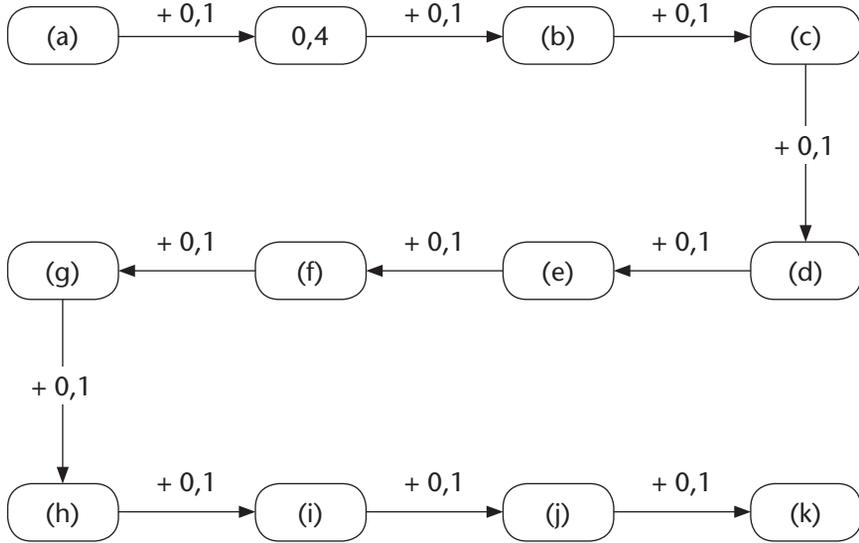
3. Write the number name and the place value parts of each of the following numbers:
- (a) 362,74
 - (b) 1 208,50
 - (c) 70,36
 - (d) 154,12
 - (e) 592,04
 - (f) 735,83

7.4 Counting in tenths in both notations

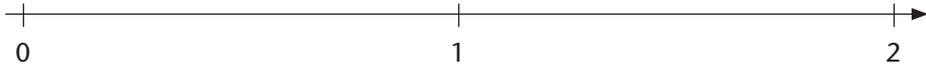
- Write the next *ten* numbers in each sequence:
 - $\frac{1}{10}; \frac{2}{10}; \frac{3}{10}; \dots$
 - 0,1; 0,2; 0,3; ...
- Calculate.
 - What is $\frac{9}{10} + \frac{1}{10}$?
 - What is $10 - 0,1$?
- Write the next *ten* numbers in each sequence:
 - 99,5; 99,6; 99,7; ...
 - 11,4; 11,3; 11,2; ...
 - 9,8; 9,6; 9,4; ...
 - 11,4; 11,3; 11,2; ...
 - 5,7; 5,5; 5,3; ...
 - 3,9; 3,6; 3,3; ...
- Follow the arrows and count in tenths in this flow diagram. Find the numbers for (a), (b), (c) etc. and write them in a list.



5. Follow the arrows and count in 0,1s. Find the numbers for (a), (b), (c) etc. and write them in a list.



6. Draw an open number line like the one below. Measure carefully and write the 0, 1 and 2 at the correct places below the line.



Now, without making any measurements, place the following numbers carefully on your number line. Estimate where they should be:

1,2; 0,3; 0,9; 1,5; 0,75

7. Complete the sequences:

(a) 0,2; 0,4; 0,6; ___; ___; ___; ___; ___; ___; ___; ___.

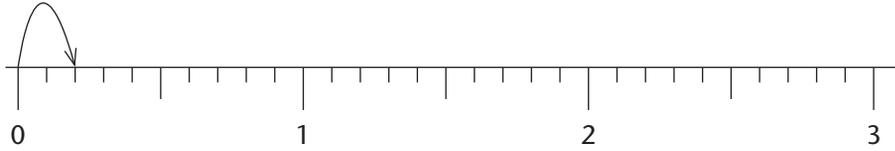
(b) 0,3; 0,6; 0,9; ___; ___; ___; ___; ___; ___; ___; ___.

(c) 0,4; 0,8; 1,2; ___; ___; ___; ___; ___; ___; ___; ___.

(d) 0,5; 1; 1,5; ___; ___; ___; ___; ___; ___; ___; ___.

(e) 0,6; 1,2; 1,8; ___; ___; ___; ___; ___; ___; ___; ___.

8. Complete the sequences below. Copy the number line if you need to draw arrows on it to help you find the numbers.



- (a) 0,2; 0,4; 0,6; ___; ___; ___; ___; ___; ___; ___; ___.
- (b) How many 0,2s are in 1?
- (c) 0,3; 0,6; 0,9; ___; ___; ___; ___; ___; ___; ___.
- (d) 0,4; 0,8; ___; ___; ___; ___; ___; ___; ___; ___; ___.
- (Adding on in 0,4s)
- (e) How many 0,4s are in 2?
- (f) 0,5; ___; ___; ___; ___; ___; ___; ___; ___; ___; ___.
- (Adding on in 0,5s)
- (g) 0,6; ___; ___; ___; ___; ___; ___; ___; ___; ___; ___.
- (Adding on in 0,6s)

7.5 Counting in hundredths in both notations

1. Write the next *ten* numbers in each sequence:

(a) $\frac{1}{100}$; $\frac{2}{100}$; $\frac{3}{100}$; ...

(b) 0,01; 0,02; 0,03; ...

(c) $\frac{5}{100}$; $\frac{10}{100}$; ...

2. (a) How many groups of 5 hundredths are there in 1?

(b) What is $1 - \frac{1}{100}$?

(c) What is $1 - 0,01$?

3. Write the next *ten* numbers in each sequence:

(a) 101,05; 101,04; 101,03; ...

(b) 11,04; 11,03; 11,02; ...

(c) 9,05; 9,07; 9,09; ...

(d) 10,07; 10,06; 10,05; ...

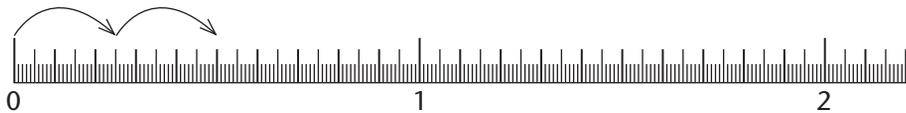
(e) 7,13; 7,16; 7,19; ...

(f) 6; 5,96; 5,92; ...

4. Use the given number lines, if you need to, to help you to complete the sequences.

(a) Count in 0,25s from 0,25 to 2,5.

0,25; 0,50; ...



(b) Count in 0,05s from 0,05 to 1,1.

0,05; 0,1; 0,15; ...

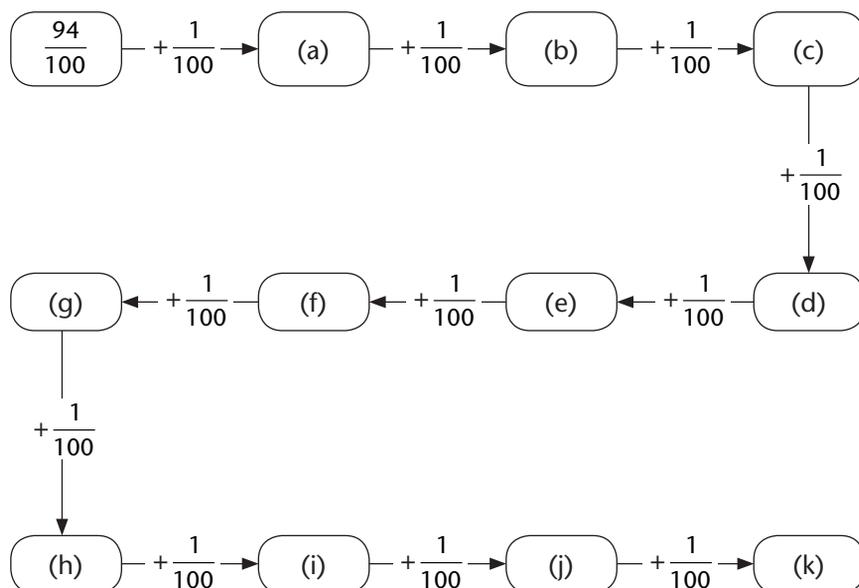


(c) Count in 0,15s from 0,15 to 1,5.

0,15; 0,30; 0,45; ...



5. Count in hundredths in this flow diagram. Find the correct numbers for (a), (b), (c) etc. and write them in a list.



7.6 From fractions to decimals to fractions

- We have to write a fraction as tenths or hundredths in order to be able to write it as a decimal fraction to two decimal places.
 - Which other fractions, besides tenths and hundredths, are easy to write as decimals?
 - Explain how you will go about writing each of these fractions as decimals.
- Write the following numbers in decimal notation.

(a) $2\frac{1}{10}$	(b) $5\frac{7}{10}$
(c) $4\frac{1}{5}$	(d) $\frac{8}{10}$
(e) $124\frac{1}{2}$	(f) $17\frac{1}{4}$
(g) $23\frac{13}{100}$	(h) $4\frac{7}{100}$

3. Write the following numbers in expanded fraction notation to show the place value parts of each number.

(a) 3,2

(b) 4,27

(c) 7,53

(d) 12,03

(e) 50,30

(f) 3,25

(g) 56,20

(h) 20,50

(i) 11,75

(j) 0,8

4. First complete each sequence in decimals, and then rewrite the sequence in fraction notation.

(a) 10; 9,8; 9,6; ___; ___; ___; ___; ___; ___; ___

10; $9\frac{8}{10}$; $9\frac{6}{10}$; ___; ___; ___; ___; ___; ___; ___

(b) 0,15; 0,3; 0,45; ___; ___; ___; ___; ___; ___

$\frac{15}{100}$; $\frac{3}{10}$; ___; ___; ___; ___; ___; ___; ___

7.7 Comparing decimals

1. Below are the results of two events at an athletics championship. For each event, arrange the names in order, starting with the winner.

(a) Boys under 19: 100 m sprint (*time in seconds*)

Temba Tshembe 11,9 s Con September 11,59 s

Gavin Solomon 11,63 s NoahTshabalala 11,23 s

Ivan Williams 11,4 s Manfred Ngcobo 11,57 s

(b) Girls under 19: Long jump (*distance in metres*)

Kato Zuma 4,23 m Jane Sithole 4,51 m

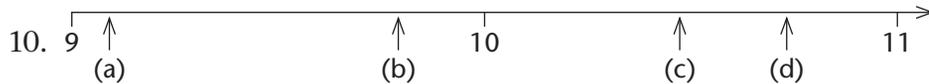
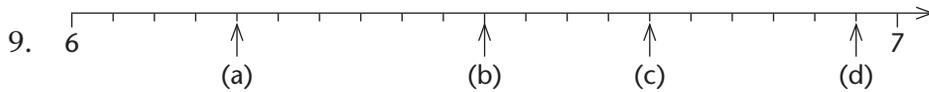
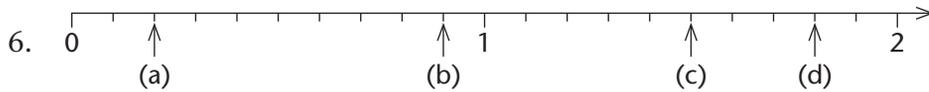
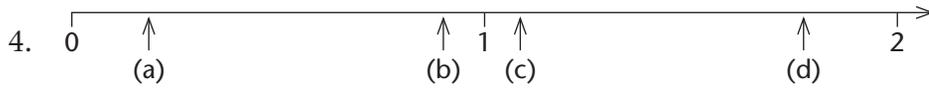
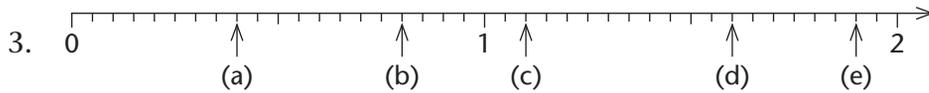
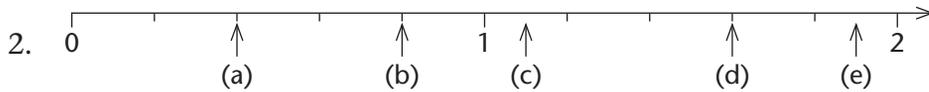
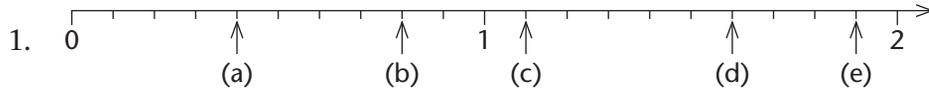
Lindi Xolani 4,5 m Pumla Makae 4,7 m

Nthabi Faku 4,07 m Denise Galant 4,72 m

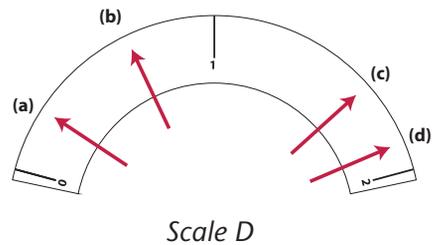
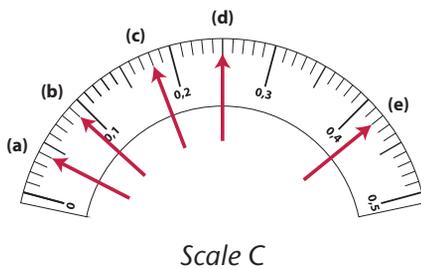
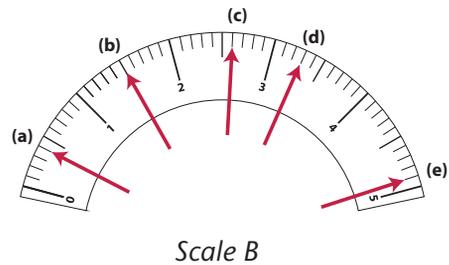
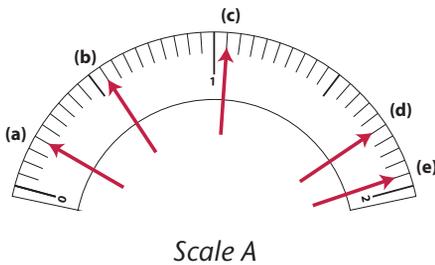
-
2. In each case, say which decimal you think is bigger and *why*.
- (a) 0,6 or 0,06
 - (b) 4,6 or 4,60
 - (c) 0,3 or 0,43
 - (d) 0,3 or 0,23
 - (e) 7,42 or 7,24
 - (f) 5,6 or 5,57
 - (g) 0,4 or 0,40
 - (h) 3,45 or 3,5
3. Sometimes we can take away a zero in a number and it does not change the value of the number. But sometimes the value of the number does change if the zero is removed.
In each case, say whether or not we can take the zero away without changing the value of the number. Give a reason for your answer.
- (a) 3,08
 - (b) 72,40
 - (c) 20,56
 - (d) 2,05
 - (e) 23,60
 - (f) 0,43
4. In each case, give a number that is *between* the two given numbers.
- (a) 4,5 and 4,7
 - (b) 3,9 and 3,11
 - (c) 7,8 and 7,9
 - (d) 14 and 14,1
 - (e) 0 and 0,1
5. How many numbers are between 7,5 and 7,6?

7.8 Reading scales

Read the value indicated by each of the arrows on the number lines. For some of them you have to estimate as accurately as possible.



11. Read the value indicated by each of the arrows on the scales. For some of them you have to estimate as accurately as possible.



7.9 Addition of decimals

Our number system is a decimal system. Ten is the basis of our number system. We count 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

The next number, 11, is $10 + 1$. We extend the set of whole numbers to form the **rational numbers**, which include numbers less or smaller than 1.

Addition and subtraction of fractions written in decimal notation works in the same way as addition and subtraction of whole numbers.

1. In each case, add the fractions and then rewrite all of the fractions in decimal notation.

(a) $\frac{3}{10} + \frac{4}{10}$

(b) $\frac{4}{10} + \frac{7}{100}$

(c) $\frac{36}{100} + \frac{53}{100}$

(d) $\frac{6}{100} + \frac{8}{100}$

2. First write all of the numbers in expanded notation. Then add the numbers and write the answer in decimal notation.

(a) $14,35 + 23,41$

(b) $12,14 + 324,7$

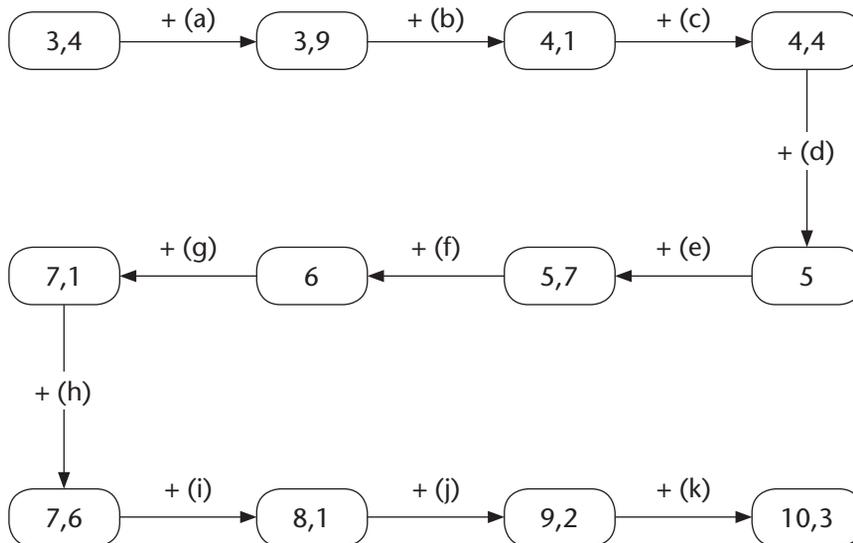
(c) $56,05 + 32,67$

(d) $41,30 + 18,77$

(e) $276,54 + 13,86 + 103,29$

(f) $532,66 + 81,92 + 202,43 + 47,64$

3. In this question you have to find the numbers that must be added to get to the target. Write your answers for (a), (b), (c) etc. in a list.



Now use your calculator to check your answers.

When you have to add many numbers it may help to arrange them with the corresponding place value parts directly below one another as shown here:

$$\begin{array}{r}
 532,66 = 500 + 30 + 2 + \frac{6}{10} + \frac{6}{100} \\
 81,92 = + 80 + 1 + \frac{9}{10} + \frac{2}{100} \\
 202,43 = 200 + + 2 + \frac{4}{10} + \frac{3}{100} \\
 47,64 = + 40 + 7 + \frac{6}{10} + \frac{4}{100} \\
 532,66 + 81,92 + 202,43 + 47,64 = 700 + 150 + 12 + \frac{25}{10} + \frac{15}{100} \\
 = 700 + 150 + 12 + \frac{26}{10} + \frac{5}{100} \\
 = 700 + 150 + 14 + \frac{6}{10} + \frac{5}{100} \\
 = 700 + 160 + 4 + \frac{6}{10} + \frac{5}{100} \\
 = 800 + 60 + 4 + \frac{6}{10} + \frac{5}{100} \\
 = 864,65
 \end{array}$$

4. (a) Estimate the answer of $34,27 + 187,45 + 98,36 + 241,83$ to the nearest ten.
- (b) Now calculate the answer of $34,27 + 187,45 + 98,36 + 241,83$ accurately. You may write the numbers in columns as shown above to make it easier to keep track of the place value parts.

7.10 Subtraction with decimals

1. Calculate each of the following:

$$(a) \left(7 + \frac{6}{10} + \frac{5}{100}\right) - \left(4 + \frac{5}{10} + \frac{3}{100}\right) \quad (b) \left(4 + \frac{2}{10} + \frac{6}{100}\right) - \left(2 + \frac{3}{10} + \frac{7}{100}\right)$$

$$(c) \left(7 + \frac{4}{100}\right) - \left(3 + \frac{6}{10}\right) \quad (d) 5,68 - 2,53$$

Some of the calculations in question 1 were quite easy and some may have given you problems. We shall now look at how to do subtraction by breaking numbers down into their place value parts.

For example, $79,56 - 45,24$ can be calculated like this:

$$79,56 = 70 + 9 + \frac{5}{10} + \frac{6}{100}$$

$$45,24 = 40 + 5 + \frac{2}{10} + \frac{4}{100}$$

$$\begin{aligned} 79,56 - 45,24 &= 30 + 4 + \frac{3}{10} + \frac{2}{100} \\ &= 34,32 \end{aligned}$$

In the case of $34,62 - 27,95$ the parts that are circled cause difficulties:

$$34,62 = 30 + 4 + \frac{6}{10} + \frac{2}{100}$$

$$27,95 = 20 + 7 + \frac{9}{10} + \frac{5}{100}$$

$$34,62 - 27,95 = 10 +$$

We cannot subtract 7 from 4. We also cannot subtract 0,9 from 0,6 and 0,05 from 0,02. The difficulties can be resolved by rewriting

$$30 + 4 + \frac{6}{10} + \frac{2}{100} \text{ as } 20 + 13 + \frac{15}{10} + \frac{12}{100}.$$

We can do this because the two numbers above are identical in value, although they are written differently. Now answer question 2.

-
2. Explain why $30 + 4 + \frac{6}{10} + \frac{2}{100}$ can be replaced by $20 + 13 + \frac{15}{10} + \frac{12}{100}$.
Discuss this with a classmate:

$$34,62 = 30 + 4 + \frac{6}{10} + \frac{2}{100}$$

$$34,62 = 20 + 13 + \frac{15}{10} + \frac{12}{100}$$

$$27,95 = 20 + 7 + \frac{9}{10} + \frac{5}{100}$$

$$\begin{aligned} 34,62 - 27,95 &= 0 + 6 + \frac{6}{10} + \frac{7}{100} \\ &= 6,67 \end{aligned}$$

3. Nare wants to calculate $712,34 - 563,57$.

He writes the place value expansion for both numbers:

$$712,34 = 700 + 10 + 2 + \frac{3}{10} + \frac{4}{100}$$

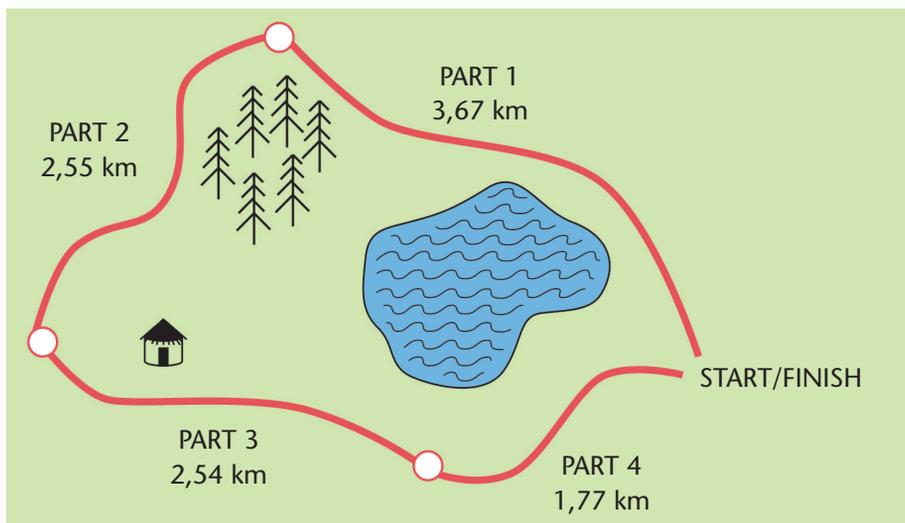
$$563,57 = 500 + 60 + 3 + \frac{5}{10} + \frac{7}{100}$$

Write a suitable replacement for $700 + 10 + 2 + \frac{3}{10} + \frac{4}{100}$, that will make it easy to calculate $712,34 - 563,57$.

4. Estimate the answers by rounding off the numbers to the nearest ten.
- (a) $53,68 - 22,34$
 - (b) $351,65 - 203,46$
 - (c) $546,37 - 238,15$
 - (d) $569,34 - 286,77$
5. In which cases in question 4 will it be necessary to make a replacement for the expansion of the first number as shown above?
6. Do the calculations in question 4.

7.11 Problem solving with decimals

1. Tsheko wants to know what the thickness of one sheet of very thin paper is. He measures the thickness of 100 sheets, which is only 3 mm. It means the thickness of each sheet of paper is $\frac{3}{100}$ mm.
 - (a) Write this number as a decimal.
 - (b) If you use a calculator to get the answer, what will your instructions to the calculator be?
 - (c) What is the thickness of a stack of 200 pages?
2. Simon's time for the 100 m sprint is 12,13 seconds. Julius's time for the same race is 11,56 seconds. Who won the race and by how much did he win?
3. A box of 100 balloons weighs 272 g and costs R35,00.
 - (a) If the mass of the empty box is 22 g, what is the mass of the 100 balloons?
 - (b) What is the mass of one balloon?
 - (c) How much does one balloon cost?
4. A relay race consists of four parts. The distances are shown on the map below. What is the total distance of the race?



7.12 Using the calculator to understand decimals

The calculator is not only a calculating device. It is also very useful to do investigations and to discover aspects of numbers and decimals in particular.

1. You can set up your common (not scientific) calculator to be a counting machine to count in 0,1s.

Press $\boxed{0}$ $\boxed{.}$ $\boxed{1}$ and then press the $\boxed{+}$ key *twice* and then press $\boxed{=}$.

Keep on pressing the $\boxed{=}$ key.

What do you notice?

The calculator keeps on counting in 0,1s. It shows 0.1; 0.2; 0.3; ...

This is useful if you want to check whether you have completed a sequence of numbers correctly.

For example, if you had to complete the sequence 0,3; 0,6; 0,9; ...

press $\boxed{0}$ $\boxed{.}$ $\boxed{3}$ $\boxed{+}$ $\boxed{+}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$

and there you go!

You can even count backwards, by pressing the number that you want to subtract each time, followed by $\boxed{-}$ $\boxed{-}$ and then $\boxed{=}$. Next, press the number from which you want to count back and then press $\boxed{=}$ $\boxed{=}$. If you want the calculator to repeatedly multiply by the same number, press the number you want to multiply by, followed by $\boxed{\times}$ $\boxed{\times}$ and $\boxed{=}$. Any number that you then press, will be multiplied by that number.

You can even divide repeatedly by pressing the number (that is the divisor, the one that you want to divide by) and $\boxed{\div}$ $\boxed{\div}$ $\boxed{=}$.

Play around with your calculator. Most of the common calculators work like this and it is most useful to know this function.

Note that the calculator uses a point (.) and not a comma to separate whole number part and fraction part.

Remember: What you do when you key in the instructions, is to send a message to your calculator.

The moment that you press a function key, such as $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, or the CANCEL function \boxed{C} , that message is cancelled. So, if you want the calculator to start counting at a certain number, do not press the \boxed{C} key, simply enter the number and $\boxed{=}\boxed{=}\boxed{=}$.

2. Set up your calculator to count in 0,3s, starting at 20,1.
3. (a) Set up you calculator to count in 0,1s.
(b) Enter 1111,11. What is the value of each of the 1s in the number?
(c) Now keep on pressing the $\boxed{=}$ key. Describe what you notice.
(d) Can you explain what you see?
4. Try the following on your calculator:

Enter the number 74 653.

You have to “shoot down” each one of the digits, which means you must replace that digit by 0. You can only do this by subtracting a number that will leave 0 in the place of the digit you are aiming at.

Example:

Number entered: 74 653

If you want to shoot the 3 down, you can subtract 3 on your calculator. The number on the screen will now be 74 650.

- (a) Now shoot down the 4. What must you subtract?
- (b) Shoot down the rest of the digits (that is, the 6, 7 and 5) until you have only 0 on your screen.
- (c) Shoot down the digits of the number 67 452,13, in any order. Write down what you subtract every time.
- (d) Now shoot down the digits of the number 354 168,27. Shoot them down in ascending order, starting with the 1, then the 2, and so on. Write down what you subtract every time.

-
5. You can also set up your calculator to be a multiplying machine.

Press 10 and then \times \times , followed by $=$.

So your key sequence is 10 \times \times $=$.

Then press any number and if you press $=$ again, that number will be multiplied by 10.

Set up your calculator to multiply by 10. Then type in a decimal number, for example 123,45, followed by $=$.

What do you notice? Explain what you see.

6. Enter this: 100 \times \times $=$

Then press any number and if you press $=$ again, that number will be multiplied by 100.

Set up your calculator to multiply by 100. What do you expect to see if you now type in 38,43 followed by $=$? Explain your expectation.

UNIT 8

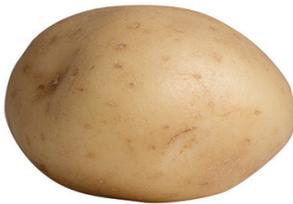
CAPACITY AND VOLUME

8.1 The difference between capacity and volume

This measuring jug has space for 500 ml of water, up to the 500 ml mark. We say the **capacity** of the jug is 500 ml.

You can see that the water takes up 275 ml of the space in the jug. We say the **volume** of the water is 275 ml.

1. Estimate the volume of the potato.



To know what the volume of the potato is we need to know how much space it takes up. We can do that by putting the potato in the jug with water as shown here.

2. Compare the water level in the jug without the potato, and with the potato. Can you now say what the volume of the potato is?



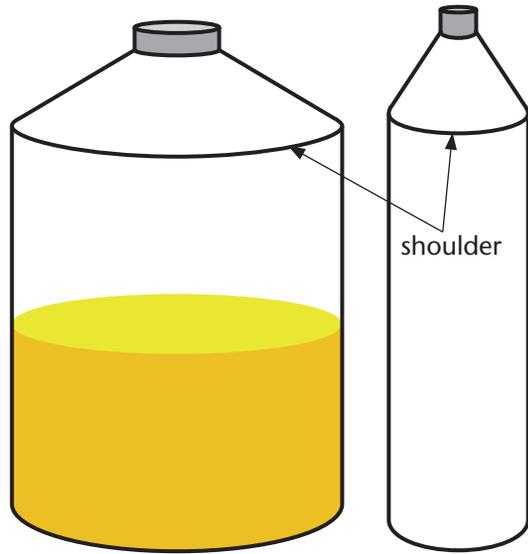
Objects such as cups, glasses, jugs, buckets, bottles and cartons are called **containers**.

The wide bottle on the left will hold 120 ml of liquid (or sugar, or flour or other material) when it is filled up to its shoulder.

The capacity of the wide bottle up to its shoulder is 120 ml.

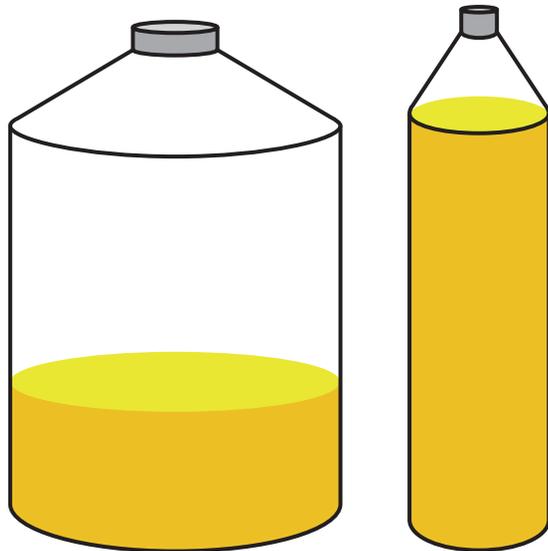
The wide bottle in the picture contains 60 ml of oil. The volume of oil in the bottle is 60 ml.

The capacity of the narrow bottle up to its shoulder is 20 ml.

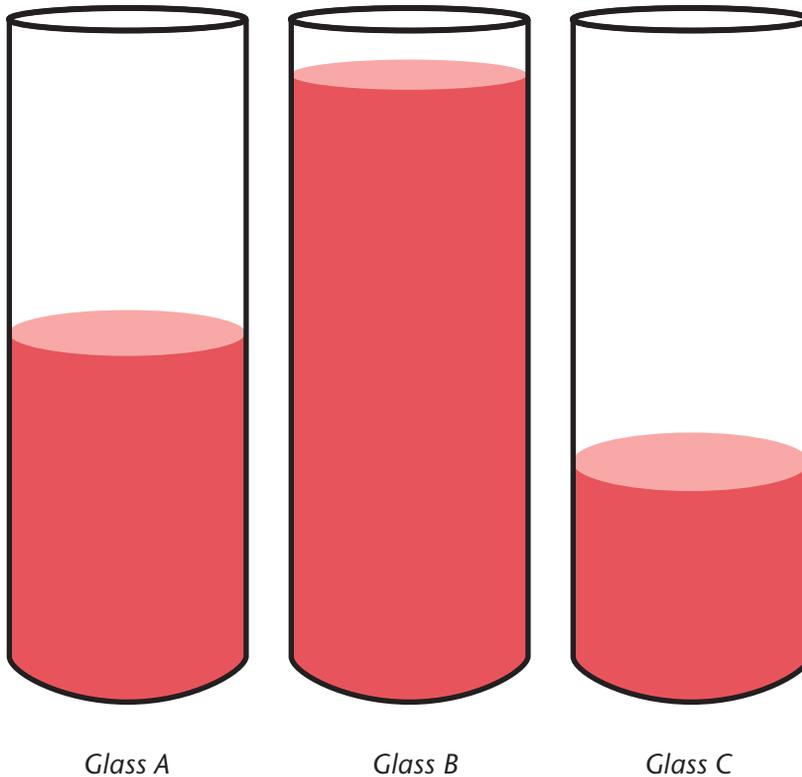


20 ml of oil is poured from the wide bottle into the narrow bottle.

3. What is the volume of the oil in the wide bottle now?
4. What is the capacity of the wide bottle up to its shoulder?
5. How much oil must now be added to fill the wide bottle up to its shoulder?



-
6. Each of these glasses can hold 100 ml of juice if it is filled right to the top. Approximately how much juice is shown in each glass?



7. The above glasses, with scales printed on them, are shown again on page 212. Turn to that page to check how good your estimates in question 6 were.
8. (a) Pour some water into a measuring jug and take the volume reading as in question 1.
- (b) Estimate how many millilitres of sand you can hold in your hand, and write your estimate down.
- (c) Pour one handful of sand into the water in the jug and take a reading again so that you can find out what the volume of the sand really is.

8.2 Containers and measurements

If the largest volume of water that can be held in a container is 1 litre, we say the container has a capacity of 1 litre. Both volume and capacity are often measured in millilitres, litres or kilolitres.

1 000 ml = **1 litre**

The official symbols for litre are L and l. Because the letter l is easily confused with the number 1, we often write ℓ instead of l.

1 kilolitre = 1 000 ℓ

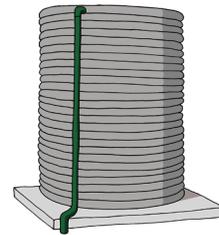
The official symbols for kilolitre are kl and kL.

In everyday life you will come across the following notations:

Name	Symbols
litre	l, L or ℓ
millilitre	ml, mL or mℓ
kilolitre	kl, kL or kℓ

- (a) How many millilitres are 1 kl?
(b) How many litres are 0,5 kl?
(c) How many millilitres are 0,1 kl?

Many of the water tanks used in towns and on farms are 1 kl tanks; this means tanks with a capacity of 1 kl.



Doctors, nurses and other people who take care of sick people often have to measure out small volumes of medicine. In some cases they use measuring spoons; in other cases they use syringes.

The largest volume that can be accurately measured is normally stated as the capacity of a container.

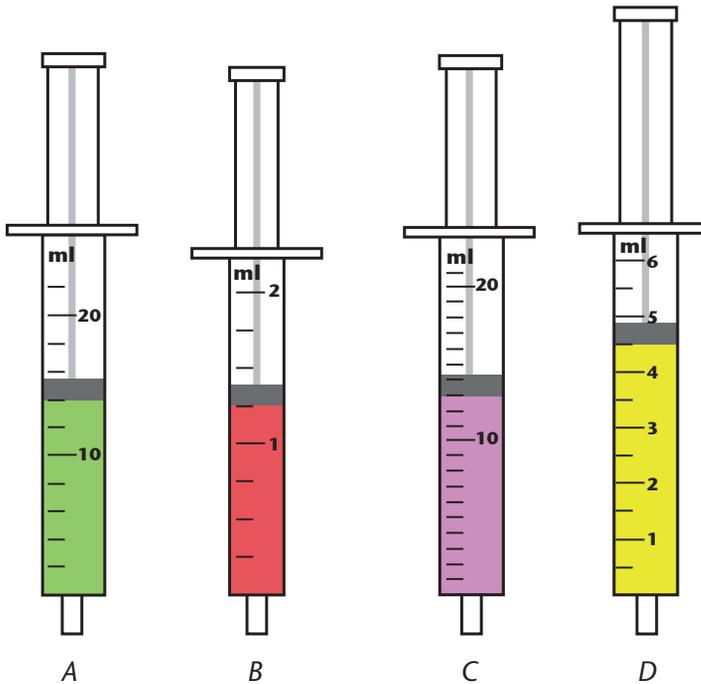




2. The above picture shows the actual size of a small syringe.
 - (a) What do you think the capacity of this syringe is?
 - (b) How much medicine is in the syringe?
3. The pictures below do not show the actual sizes of the syringes. The bottom part of each syringe, up to the plunger, is filled with medicine. All the syringes are marked in millilitres.

There is 14 ml of medicine in Syringe A.

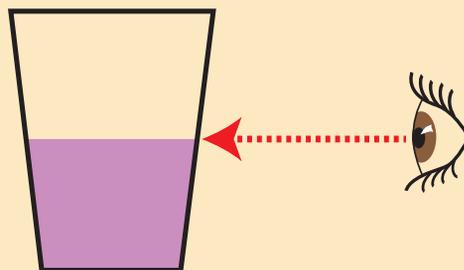
What volume of medicine is in each of the other syringes?



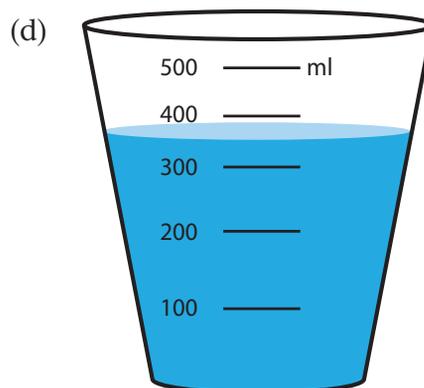
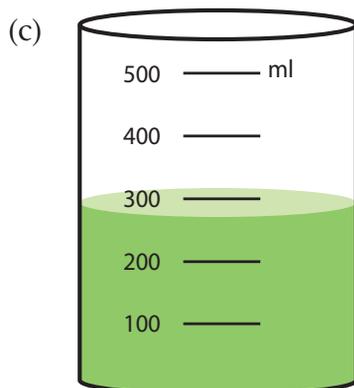
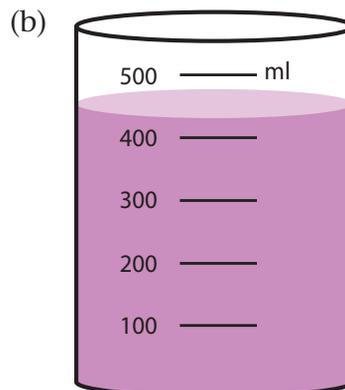
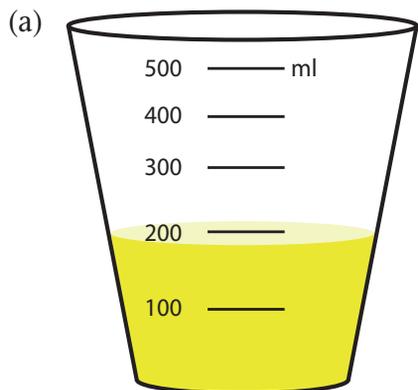
4.
 - (a) What is the measuring capacity of each syringe?
 - (b) For each syringe, state how much more medicine can be drawn in to fill it up to its measuring capacity.
 - (c) Which syringe contains the most medicine?

When you take a reading on a measuring jug, it is important to have your eyes at the same height than the level of the liquid.

Why do you think this is important?



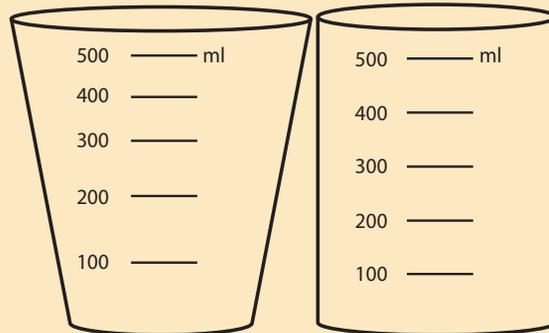
5. What is the volume of liquid in each of the measuring cups below, and what is the capacity of each cup?



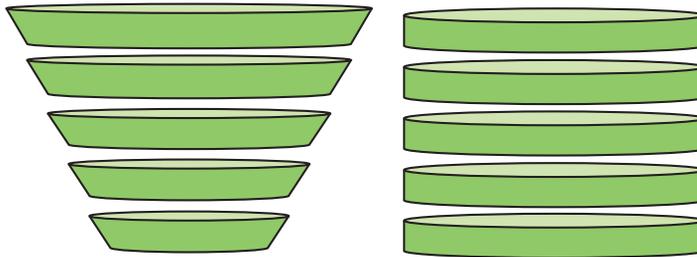
These pictures of two 500 ml measuring cups are much smaller than the actual cups.

The measuring cup on the left has the shape of part of a cone.

The cup on the right has the shape of a cylinder.



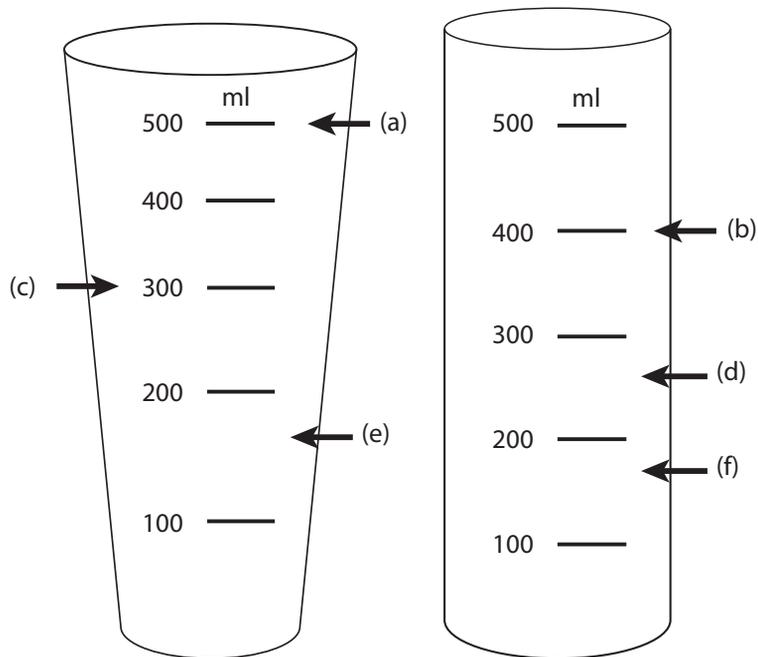
6. Why are the intervals on the cone-shaped cup above not spaced equally? Think about it and write your thoughts in a short paragraph. You may find these pictures helpful to guide your thoughts:



7. (a) Which spoon will you use to measure 30 ml of medicine?
 (b) Which combination of spoons will you use to measure 20 ml of medicine accurately?
 (c) Which combination of spoons will you use to measure 10 ml of medicine accurately?



8. A tablespoon has a capacity of about 15 ml. How many tablespoons of water do you need to fill a cup with a capacity of 250 ml?
9. Imagine that measuring jugs such as the ones below have some juice in them. State the volume of juice indicated by each arrow. In the cases where the juice level is not at a mark, you have to estimate the volume.

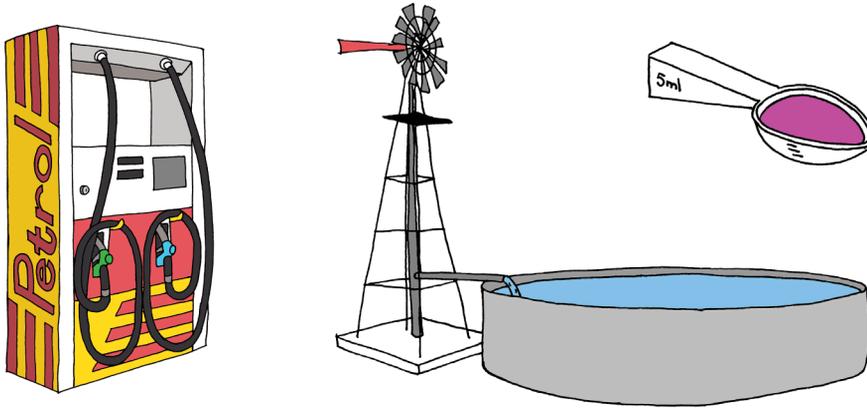


10. Make rough sketches of the following:
- (a) two containers with the same height, but with different capacities
 - (b) two containers with the same capacities, but with different heights
11. Does an empty container have a volume?

8.3 Work with different units of measurement

Small quantities that a person may drink or eat, such as medicine, salt, sugar and milk, are normally measured in millilitres.

Larger quantities, such as petrol and paint, are normally measured in litres. Very large quantities, such as water in tanks or dams, are normally measured in kilolitres.



Remember:

- ml is a symbol for millilitre.
- ℓ is a symbol for litre.
- kl is a symbol for kilolitre.
- 1 000 ml is the same as 1 ℓ.
- 1 000 ℓ is the same as 1 kl.

1. With which unit (ml, ℓ or kl) will you measure the following?
 - (a) salt for dough of 10 loaves of bread
 - (b) water for the coffee flask
 - (c) petrol for the car
 - (d) water for the bathtub
 - (e) water in the Vaal dam
 - (f) a dose of cough mixture

2. (a) How many cups of 250 ml each do you need to fill a 5 ℓ bucket with water?

(b) How many buckets of 5 ℓ each can you fill with water from a full 2 kl water tank?

(c) How many 20 ℓ tanks can be filled from a dam that holds 6 kl?

To find the answers for questions 2 and 3, it will help you to keep in mind that 1 ℓ = 1 000 ml, so 5 ℓ = 5 000 ml, and that 1 kl = 1 000 ℓ, so 6 kl = 6 000 ℓ.

3. (a) How many 5 ml spoonfuls will fill a 250 ml cup?

(b) A 1 ℓ container holds 1 000 ml. How many 250 ml measuring cupfuls will fill the container?

(c) How many 5 ml spoonfuls do you need to fill a 1 ℓ jug?

4. Write these volumes as fractions of 1 ℓ.

Example: 2 750 ml = $2\frac{3}{4}$ ℓ

(a) 250 ml

(b) 800 ml

(c) 750 ml

(d) 100 ml

(e) 50 ml

(f) 1 500 ml

(g) 1 ℓ + 500 ml

(h) 3 050 ml

5. You know by now that decimals are just another way of expressing fractions. Therefore you can also write the above volumes in decimal notation as litres. Try to do that!

6. Write each of the following in millilitres.

Example: 0,5 ℓ = $\frac{5}{10}$ ℓ = 500 ml

(a) 0,1 ℓ

(b) 0,6 ℓ

(c) 0,9 ℓ

(d) 1,4 ℓ

(e) 5,3 ℓ

(f) 10 ℓ

(g) 100 ℓ

(h) 500 ℓ

(i) one tenth of a kilolitre

(j) five tenths of a kilolitre

(k) 1 kl

(l) 1,5 kl

(m) 2,7 kl

(n) 0,25 kl

7. (a) During a drought, 1 kl of water is to be equally shared between 50 people. How much water will each person get?
- (b) How much water will each person get if 1 kl is to be equally shared between 100 people?
- (c) How much water will each person get if 1 kl is to be equally shared between 1 000 people?

When you do question 8, it will help you to keep in mind that fractions can be written in decimal notation.

For example, $\frac{3}{10} + \frac{2}{100}$ can be written as $0,3 + 0,02$ which is $0,32$.

8. Write each of the following in litres.
- | | |
|---------------------------|----------------------------|
| (a) one tenth of 1 kl | (b) 0,1 kl |
| (c) one hundredth of 1 kl | (d) one thousandth of 1 kl |
| (e) 0,01 kl | (f) 3,07 kl |
| (g) 0,11 kl | (h) 2,5 kl |
| (i) 2,11 kl | (j) 3,25 kl |
| (k) 4,35 kl | (l) 10,05 kl |
| (m) 600 kl | (n) 6 000 ml |

1 000 ℓ = 1 kl. So 500 ℓ is half of 1 kl, which means that 500 ℓ = 0,5 kl.

250 ℓ is a quarter of 1 kl, which means that 250 ℓ = 0,25 kl.

100 ℓ is one tenth of 1 kl, which means that 100 ℓ = 0,1 kl.

300 ℓ is three tenths of 1 kl, which means that 300 ℓ = 0,3 kl.

10 ℓ is one hundredth of 1 kl, which means that 10 ℓ = 0,01 kl.

70 ℓ is seven hundredths of 1 kl, which means that 70 ℓ = 0,07 kl.

460 ℓ is forty-six hundredths of 1 kl.

We can also say it is 4 tenths and 6 hundredths of 1 kl.

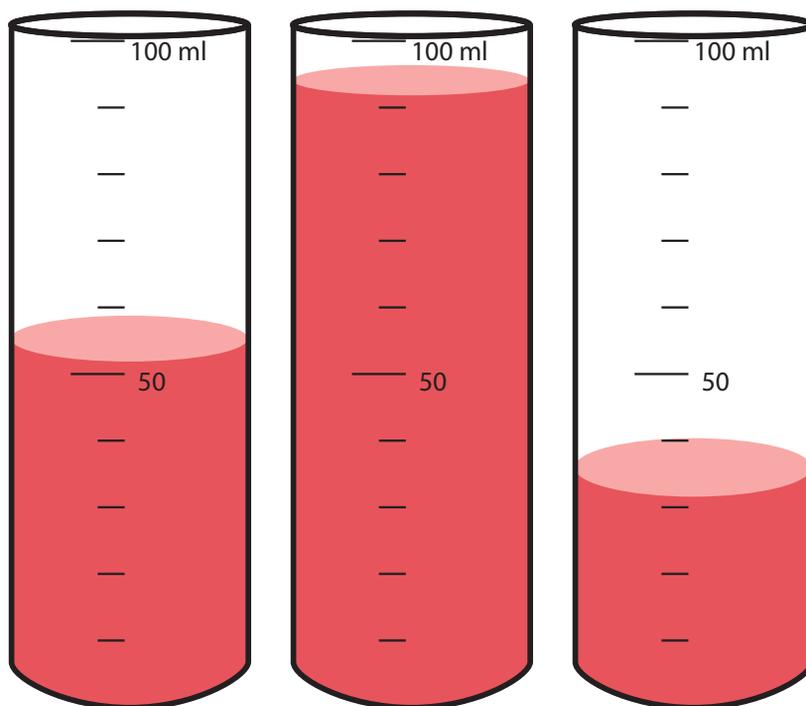
This means that 460 ℓ = 0,4 kl + 0,06 kl which is 0,46 kl.

9. (a) How many tenths of a kl is 400 ℓ? Write it in decimal notation.
- (b) How many hundredths of a kl is 360 ℓ? Write it in decimal notation.

When we write $320 \ell = 0,32 \text{ kl}$,
we can say we **express** 320ℓ in kl.

10. Express each of the following in kl, as a fraction in common fraction notation and in decimal notation.
- | | |
|--------------------------|---------------------------|
| (a) 250ℓ | (b) $1\,250 \ell$ |
| (c) $2\,750 \ell$ | (d) 650ℓ |
| (e) 150ℓ | (f) $12\,500 \ell$ |
| (g) 370ℓ | (h) $6\,830 \ell$ |
| (i) $80\,000 \text{ ml}$ | (j) $600\,000 \text{ ml}$ |
11. (a) Write in ascending order:
 639ℓ $2,54 \text{ kl}$ $45\,100 \text{ ml}$ $7,33 \ell$ 8 kl
- (b) Write in descending order:
 $87\,420 \text{ ml}$ $0,25 \text{ kl}$ $125\frac{1}{2} \ell$ $1\frac{1}{4} \text{ kl}$ $6,89 \ell$
12. Thuli adds 250 ml of concentrated fruit juice to 2ℓ of water, to make drinks for the athletes in a long-distance race.
- (a) How much concentrated juice should she add to 5ℓ of water?
- (b) How many athletes can she provide with 400 ml of juice each, with the juice she made by adding concentrate to 5ℓ of water?
13. Diesoline is used to generate electricity at a small power station. The power station uses 684ℓ of diesoline each day. For how many days can the power station operate if there is a stock of $9\,765 \ell$ of diesoline available?
14. The following volumes of milk are produced on a dairy farm on the first 10 days of November:
- | | | | | |
|---------------|---------------|---------------|---------------|---------------|
| $1\,287 \ell$ | $1\,321 \ell$ | $1\,108 \ell$ | $1\,234 \ell$ | $1\,276 \ell$ |
| $1\,117 \ell$ | $1\,198 \ell$ | $1\,223 \ell$ | $1\,298 \ell$ | $1\,201 \ell$ |
- Approximately how much milk, in total, do you think will be produced over the next 6 days?
- Give detailed reasons for your estimate.

These glasses refer to page 202.



Glass A

Glass B

Glass C

Term Three

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1.1 Quiz

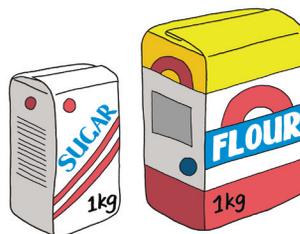
Choose the correct answer to find out what you understand about mass.

- The mass of an object tells us if the object is:
(a) big or small (b) long or short (c) heavy or light
- We measure mass in fractions or multiples of:
(a) metres (b) kilograms (c) litres
- If you want to measure your mass, what kind of scale will you use?
(a) a kitchen scale (b) a bathroom scale (c) a balance scale
- A litre of pure water has a mass of about:
(a) 1 g (b) 1 m (c) 1 kg
- A good estimate of the mass of a box of matches is:
(a) about 3 g (b) about 3 kg (c) about 3 l
- A good estimate of the mass of a schoolbag is:
(a) about 3 g (b) about 3 kg (c) about 3 l
- 1 kg of sugar has exactly the same mass as:
(a) 1 l of sugar (b) 1 000 ml of sugar (c) 1 000 g of sugar
- The mass of 10 oranges that are about the same size:
(a) is about 10 times more than the mass of 1 orange
(b) cannot be estimated from the mass of 1 orange
(c) is the same as the mass of 1 orange
- A cupful of water (about 250 ml) without the cup has a mass of about:
(a) 500 g (b) 100 g (c) 250 g

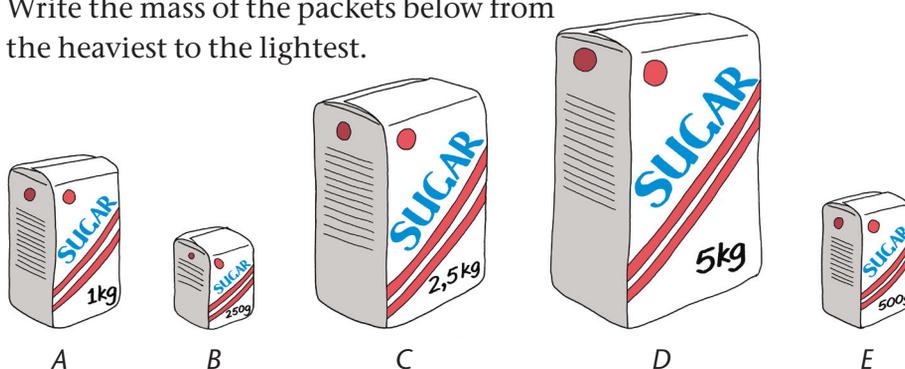
1.2 Comparing mass measurements

1. Compare a 1 kg packet of sugar and a 1 kg packet of flour.

- Does the mass of the two packets differ?
- Does the size of the two packets differ?
- Explain why it is that a 1 kg packet of sugar and a 1 kg packet of flour have different sizes.



2. Write the mass of the packets below from the heaviest to the lightest.



- How many of Packet B have the same mass as Packet A?
- What fraction of the sugar in Packet A has the same mass as Packet B?
- Which packet holds half as much sugar as Packet A?
- Which packet holds $2\frac{1}{2}$ times as much sugar as Packet A?

The **kilogram (kg)** is the **SI unit** for mass. A kilogram is divided into 1 000 parts called **grams (g)**. So there are 1 000 g in 1 kg.

500 g is half of 1 000 g. $500 \text{ g} = \frac{1}{2} \text{ kg} = 0,5 \text{ kg}$

250 g is a quarter of 1 000 g. $250 \text{ g} = \frac{1}{4} \text{ kg} = 0,25 \text{ kg}$

$2,5 \text{ kg} = 2 \text{ kg}$ and $500 \text{ g} = 2\frac{1}{2} \text{ kg}$

$100 \text{ g} = \frac{100}{1000} \text{ kg} = 0 + \frac{1}{10} \text{ kg} = 0,1 \text{ kg}$

$50 \text{ g} = \frac{50}{1000} \text{ kg} = 0 + \frac{5}{100} \text{ kg} = 0,05 \text{ kg}$

3. (a) Match the common fractions below with the mass of the illustrated items.

$$\frac{1}{4} \text{ kg}; \frac{1}{2} \text{ kg}; \frac{3}{4} \text{ kg}; 2\frac{1}{2} \text{ kg}; 1\frac{1}{2} \text{ kg}$$



A



B



C



D



E

- (b) Write the given common fraction notation masses in decimal notation.

Kitchen scales can be used to measure small quantities of food, usually up to 5 kg. Most bathroom scales can measure mass up to 120 kg.

4. Estimate the mass of the following objects. Then use an appropriate scale to check your estimates.
- (a) a cupful of sugar
 - (b) a cupful of rice
 - (c) a cupful of sand
 - (d) a cupful of stones
 - (e) a cupful of tea
 - (f) your own mass
 - (g) the mass of a chair in the classroom
 - (h) the mass of your Mathematics textbook
 - (i) the mass of one page of your Mathematics textbook

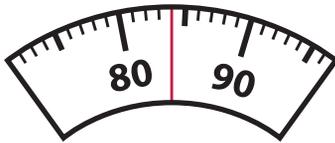
If we work with **estimates** of measurements, our answers must always say "about so much". We say this is the **approximate measurement**.

5. Give the equivalent mass in grams of the following:
- (a) a baby with a mass of 2,8 kg
 - (b) a book with a mass of 0,5 kg
 - (c) a brick with a mass of 1,5 kg
 - (d) a bag of dog food with a mass of 20 kg
 - (e) a person with a mass of 60 kg
6. Give the equivalent mass in kilograms of the following:
- (a) a puppy of 2 000 g
 - (b) a 250 g bag of flour
 - (c) an apple of 100 g
 - (d) a 750 g stone
 - (e) a schoolbag of 5 500 g
 - (f) a 3 250 g fish

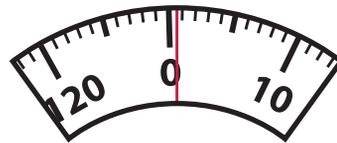
1.3 Reading mass in grams and kilograms

1. Write down each mass in kilograms.

(a)



(b)

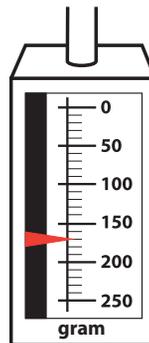


2. Convert each mass in question 1 to grams.
3. Read each scale and write down the mass. Describe how you thought in order to find the answers.

(a)

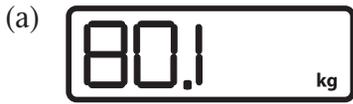


(b)



4. Round off each mass in question 3 to the nearest kilogram.

5. Write each mass in common fraction notation.



6. Round off the mass in questions 5(a) and (d) to the nearest kilogram.

7. Convert each mass in question 5 to grams.

1.4 Solving problems about mass and quantity

1. 500 large paper clips have a total mass of 1 kg. Calculate the mass of:

- (a) 50 large paper clips
- (b) 10 large paper clips
- (c) 1 large paper clip.



2. A bag of 30 oranges, all of the same mass, weighs 5 kg. Calculate the mass of:

- (a) 15 oranges
- (b) 5 oranges
- (c) 1 orange.



3. Look back at questions 1 and 2.

- (a) Will all the paper clips have exactly the same mass?
- (b) Will all the oranges have exactly the same mass?

4. At the fresh produce market, you can buy vegetables per kilogram. Onions sell for R12 per kilogram. If a single large onion has a mass of 150 g, what will the following cost?

- (a) 2 000 g of onions
- (b) 20 onions
- (c) 300 g of onions
- (d) 1 onion

Sometimes when we calculate we get exact answers. Sometimes our answers are approximations. For example, if four Grade 5 learners have a mass of 140 kg, we cannot be sure that each learner has a mass of exactly 35 kg.

5. Suppose the table shows the mass of different animals on a farm and the mass of the food they eat per day.

Animal	Mass of animal	Mass of food per day
Pigeon	500 g	500 g
Duck	2,8 kg	280 g
Chicken	1,9 kg	190 g
Sheep	64 kg	1 600 g
Pig	200 kg	4 000 g

- (a) How much more is the mass of a duck than the mass of a pigeon?
- (b) How much more is the mass of a duck than the mass of a chicken?
- (c) Which animals eat a mass of food equal to their own mass per day?
- (d) Which animal eats about one tenth of its own mass in food per day? Why do you say so?
- (e) What fraction of its own mass does a pig eat per day?

When you compare one mass to another, don't forget to check in which units the measurements are given. You may first have to write one mass in the same unit as the other one.

We say we **convert** a mass from one unit to another.

6. Jenna is a dog breeder. She has 27 puppies that are now 2 months old. She must feed the puppies until they reach 7 months. The table shows how much food each puppy must get per day.

Daily serving					
Age	2 months	3 months	4 months	5 months	6 months
Grams	355 g	475 g	525 g	530 g	530 g

- (a) Work out how much food Jenna needs every month.
- (b) She buys food in large bags of 25 kg. Work out how many of these bags are enough for the whole period.
- (c) One bag of 25 kg puppy food costs R189,90. How much will 150 kg of puppy food cost?

2.2 Represent 6-digit to 9-digit numbers

- Write the number symbols for these numbers.
 - three hundred and sixty-four million two hundred and thirty-four thousand five hundred and sixty-seven
 - eighty-nine million seven hundred and five thousand nine hundred and fifteen
 - six hundred and four million nine hundred and ninety-seven thousand one hundred and twenty-two

28 387 rounded off to the nearest 5 is 28 385, and rounded off to the nearest 10 it is 28 390.

28 384 rounded off to the nearest 5 is 28 385, and rounded off to the nearest 10 it is 28 380.

28 384 rounded off to the nearest 100 is 28 400, and rounded off to the nearest 1 000 it is 28 000.

- Round 42 368 and 50 233 off to the nearest:
 - 5
 - 10
 - 100
 - 1 000
- Write the number symbols for these numbers.
 - $10\,000\,000 + 5\,000\,000 + 600\,000 + 10\,000 + 2\,000 + 900 + 50 + 2$
 - $300\,000\,000 + 7\,000\,000 + 200\,000 + 30\,000 + 400 + 2$
 - $40\,000\,000 + 6\,000\,000 + 100\,000 + 50\,000 + 3\,000 + 500 + 60 + 4$
 - $4\,000\,000 + 500\,000 + 3\,000 + 200 + 80 + 7$
 - $100\,000\,000 + 60\,000\,000 + 400\,000 + 600\,000 + 8\,000 + 600 + 70 + 8$
- Write the numbers in expanded notation.
 - 790 538 209
 - 32 679 895
 - 435 034 975
 - 206 905 196
 - 76 004 781
 - 14 752 893

2.3 Multiples and factors

When two or more numbers are multiplied, another number is formed, for example $3 \times 5 = 15$ and $15 \times 20 = 300$.

3 and 5 are called **factors** of 15, and 15 is called the **product** of 3 and 5.

15 and 20 are called factors of 300, and 300 is called the product of 15 and 20.

15 can also be obtained by multiplying 1 and 15: $1 \times 15 = 15$. So, apart from 3 and 5, 1 and 15 are also factors of 15.

300 can be obtained in many other ways by multiplying numbers, for example:

$$1 \times 300 = 300 \quad 3 \times 100 = 300 \quad 6 \times 50 = 300 \quad 2 \times 3 \times 5 \times 2 \times 5 = 300$$

So 1, 2, 3, 5, 6, 50, 100, 300, ... are also factors of 300.

1. (a) Which of the numbers below are factors of 40?
Justify each answer by writing a number sentence like $4 \times 10 = 40$, which shows that 4 is indeed a factor of 40.

1 2 3 4 5 6 7 8 9 10 20 40 42

- (b) Which of the above numbers are factors of 42?

2. (a) Which of the numbers below are factors of 17?

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20

- (b) Which of the above numbers are factors of 18?

- (c) Which of the above numbers are factors of 19?

- (d) Which of the numbers 17, 18 and 19 have the property described below?

The number can be written as the product of two factors in only one way (ignoring order). In other words, it has only two different factors.

A number that can be written as the product of two whole numbers in only one way (if the order does not matter) is called a **prime number**. A prime number has only two different factors, namely 1 and itself.

3. Find *all* the factors of these numbers: 13, 31, 23, 32, 39, 93.
Which of the numbers are prime?

3.1 Revision

1. Approximately 1 million people are expected to attend a Youth Day celebration on the 16th of June.
 - (a) It is expected that about 100 000 of the people will be 25 years old or older. About how many people are expected to be younger than 25 years?
 - (b) In the previous year about 740 000 people attended the celebration. How many more are expected this year?
2. Write each of the following as a single number. Write the number symbols, for example 605 080.
 - (a) 30 thousand + 7 hundred
 - (b) 300 thousand + 7 hundred
 - (c) 3 million + 7 hundred
 - (d) 300 thousand + 70
 - (e) 4 ten thousands + 6 hundreds + 5 units
 - (f) 4 hundred thousands + 6 thousands + 5 tens
 - (g) 4 hundred thousands + 5 ten thousands + 5 tens

204 870 can be written in words:

two hundred and four thousand eight hundred and seventy

300 000 + 400 000 can be written in symbols *and* words:

300 thousand + 400 thousand

3. Write each of the following in symbols *and* words, and then calculate.
 - (a) 30 000 + 70 000
 - (b) 300 000 + 7 000
 - (c) 180 000 + 400 000
 - (d) 70 000 + 80 000
 - (e) 230 000 – 80 000
 - (f) 630 000 – 80 000

-
4. In each case, first estimate the answer to the nearest ten thousand. Then calculate the answer using your calculator. Also calculate how far your estimate is from the actual answer.

(a) $273\,456 + 354\,567$

(b) $534\,512 - 255\,678$

(c) $873\,456 + 75\,456$

(d) $734\,560 + 54\,567$

(e) $734\,560 + 545\,670$

(f) $734\,560 + 154\,567$

(g) $435\,456 + 213\,257$

(h) $734\,560 - 154\,567$

(i) $362\,527 + 282\,426 - 363\,229$

(j) $267\,566 + 19\,123 - 74\,234 + 67\,762 - 38\,658 + 57\,235 - 13\,273$

5. Calculate each of the following without using a calculator. Then use your calculator to check the answer by doing the calculations in a different order.

(a) $145\,132 + 38\,786 + 433\,286$

(b) $(645\,132 - 318\,786) + 533\,674$

(c) $354\,317 + 328\,786 - 433\,286$

(d) $615\,432 - 238\,786 - 163\,286$

(e) $115\,432 + 376\,894 + 432\,861$

(f) $315\,432 + 176\,894 - 132\,861$

6. Do questions (a) and (b) without using a calculator, but use a calculator to check your answer for each question as you go along.

- (a) There is 50 m of thin copper cable on a roll.

How much copper cable will be left on the roll, after *all* the following lengths of cable have been cut off?

380 cm; 1 324 cm; 345 cm

- (b) Another piece of cable is cut off and now 21,45 m of cable is left. How long is the piece of cable that was cut off?

- (c) How much copper cable will be left on the roll, after the following lengths have all been cut off?

8 234 mm; 236 cm; 38,4 cm; 289 mm

3.2 Addition and subtraction in financial contexts

The income and expenses during 2013 of some departments of a large municipality are given in rands in the table below.

	Income (R)	Expenses (R)
Health	23 765 488	58 459 303
Traffic	1 386 457	1 856 487
Electricity	336 349 543	152 357 388
Taxes	9 273 243	1 237 378
Buildings	874 598	31 276 387
Water	134 567 343	183 453 499
Sport	276 388	9 458 256

- Which department had the lowest (smallest) income and which department had the highest (biggest) income?
 - Which department had the lowest expenses and which department had the highest expenses?
- In which departments was the income higher than the expenses?
 - In each case state how much higher the income was than the expenses.
 - Add up the amounts that you calculated in question (b).
- In which departments was the income lower than the expenses?
 - In each case state how much lower the income was than the expenses.
 - Add up the amounts that you calculated in question (b).
- Calculate the total income of the seven departments.
 - Calculate the total expenses of the seven departments.
 - Calculate the difference between the total income and the total expenses.
- Use your answers for questions 2(c) and 3(c) to check your answer for question 4(c).

The monthly income and expenses of a small business, over a period of 12 months, are given in rands in the table below.

	Jan	Feb	March	April	May
Income (R)	196 348	187 326	165 388	199 203	157 772
Expenses (R)	162 342	167 438	166 329	173 298	164 373

June	July	August	Sept	Oct	Nov	Dec
167 326	228 548	171 223	163 265	193 472	152 398	225 251
167 295	176 922	165 237	166 487	174 398	166 398	186 326

Questions 6, 7 and 8 below are about this small business. You may use your calculator where you believe it will be helpful.

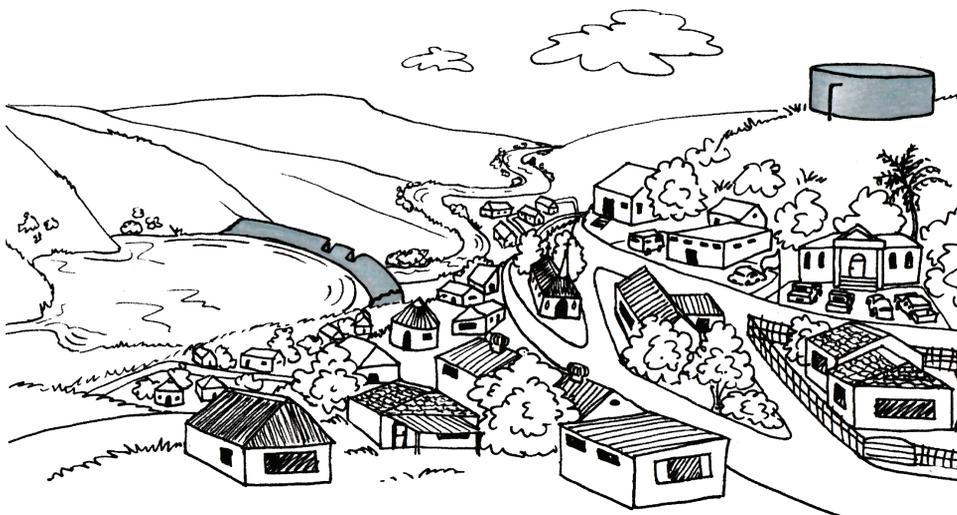
6. As you can see in the table, both the income and the expenses changed from month to month.
 - (a) Which changed the most from month to month, the income or the expenses?
 - (b) What is the difference between the highest monthly income and the lowest monthly income?
 - (c) What is the difference between the highest monthly expenses and the lowest monthly expenses?

From January to February, the expenses increased by R5 096 from R162 342 to R167 438. From January to February, the income decreased by R9 022, from R196 348 to R187 326.

7.
 - (a) From which month to which month did the biggest increase in income occur, and what was this increase?
 - (b) From which month to which month did the biggest decrease in income occur, and what was this decrease?
8. At the beginning of January the small business had R234 765 in cash. During the year, all the income was added to this amount, and all the expenses were paid out of this amount. How much cash did the business have at the end of December?

3.3 Add and subtract measurements

The residents of a certain village get their household water from a reservoir on a hilltop. Water is pumped into the reservoir from a large dam in a nearby river.



The following quantities are measured at 12:00 each day:

- the amount of water pumped into the reservoir over the last 24 hours (the “inflow”)
- the amount of water used by the residents over the last 24 hours (the “consumption”)
- the volume of water in the reservoir.

Some of the measurements over a number of days are given in the table below. All the measurements are in kilolitres.

	Day 1	Day 2	Day 3	Day 4
Inflow	98 743	107 589	106 222	97 342
Consumption	128 236	132 675	123 763	108 228
Volume in reservoir	956 378	931 292		

1. What should the measurements for the volume of water in the reservoir on Days 3 and 4 be, if there are no leakages from the reservoir?

The records for the next six days are not complete.

Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
110 237	131 809	(a)	96 284	105 638	110 547
113 678	102 563	121 073	(b)	(c)	128 345
899 424	928 670	931 975	901 512	(d)	857 428

2. What should the missing measurements (a), (b), (c) and (d) be?
3. What is the total amount of water used by the residents over the period of 10 days?
4. (a) On which of the ten days was the consumption higher than the inflow?
(b) Does the volume of water in the reservoir increase or decrease when the consumption is higher than the inflow?

The records for the next six days are given in the table below.

Day 11	Day 12	Day 13	Day 14	Day 15	Day 16
123 452	128 547	131 267	128 769	127 226	132 387
112 765	115 238	112 347	116 385	118 376	114 285
868 115	881 424	900 137	911 532	916 367	909 536

5. The manager of the water system suspects that water has started to leak from the reservoir.

Do you see any evidence of leakage in the records for Days 11 to 16?

Give reasons for your answer and write a detailed report on the matter.

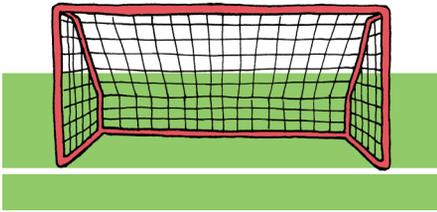
In your report, indicate how much water is possibly leaking, and whether the leakage gets worse or remains stable.

3.4 Calculations using a calculator

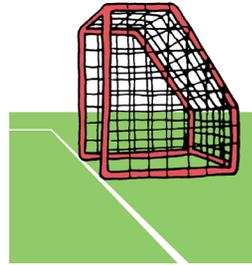
1. Vusi bought a house for R904 400. He borrowed the money from a bank and he has already paid back R105 000. How much does he still have to pay?
2. In a certain year, 297 673 learners passed Grade 10. Two years later, 100 584 more learners passed. How many learners passed their Grade 10 exams in that year?
3. Thandeka is buying a business for R946 300. She has already paid the owner R450 450. How much does she still have to pay?
4. A fruit export company exported 130 375 boxes of fruit during the first six months of the year. At the end of the year they had exported 504 250 boxes of fruit. How many boxes did they export during the second half of the year?
5. The captain of a large passenger liner sailed 604 773 sea miles during his first 15 years as a captain. He wants to sail at least 1 million sea miles before his retirement. How many sea miles does he still have to sail to reach his goal?
6. After a drought, 150 605 tonnes of maize had to be imported. The next year things looked up and only 105 057 tonnes had to be imported. What was the difference between the number of tonnes that had to be imported that year and the year before it?
7. After a severe drought, rangers counted 298 700 antelopes of different species in a game park. Before the drought there were 418 900 antelopes. How many perished?
8. During the winter, 500 202 people in the city caught flu. In spring the number dwindled to 100 984. By how much did the number decrease?
9. A car dealer bought a good second-hand 4×4 vehicle for R255 785 and sold it for R105 300 more. What was the selling price of the vehicle?

4.1 Different views of the same object

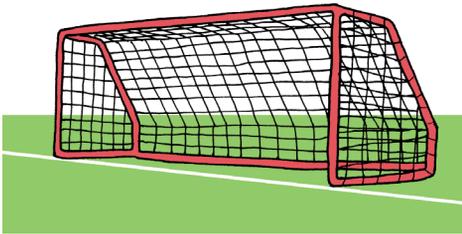
Four friends are on a soccer field.
These pictures all show the same goalpost.



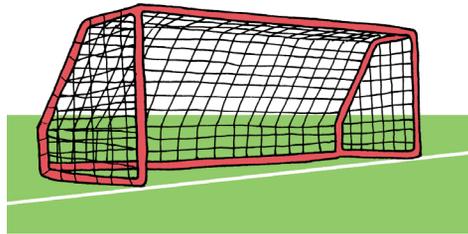
Picture A shows what Janet sees.



Picture B shows what Lebogang sees.



Picture C shows what Elsbeth sees.



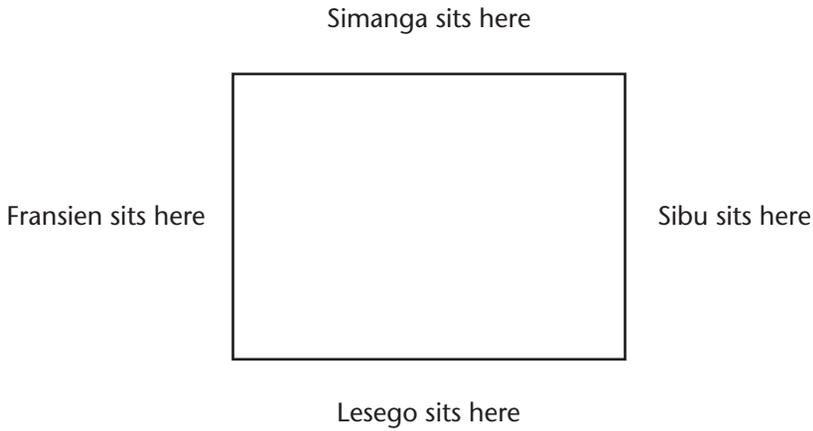
Picture D shows what Thuni sees.

Make rough sketches of two goalposts on a sheet of paper. Imagine that the sheet is the soccer field on which the four friends are playing. Write their names on the paper to show where they are standing when they see what is in Pictures A to D.



4.2 Different views of a collection of objects

Simanga, Fransien, Sibü and Lesego are sitting around a table.



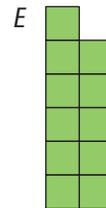
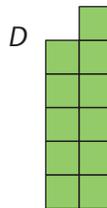
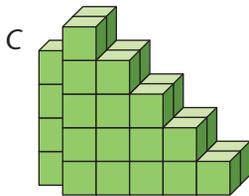
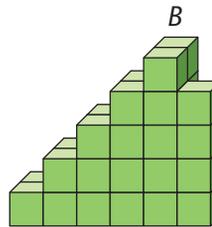
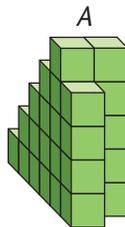
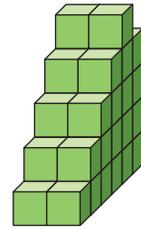
Photograph A shows what Simanga sees on the table.

1. Which photograph shows what Sibü sees?
2. Which photograph shows what Lesego sees?
3. Which photograph shows what Fransien sees?

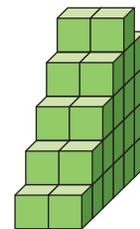


4.3 Different views of a stack of cubes

1. Which of the drawings below shows what you will see if you look at this stack of cubes from the left, as the eye shows?



2. Which of the above drawings shows what you will see if you look at the stack of cubes from the back, as the eye shows?



3. Which of the above drawings shows what you will see if you look at the stack of cubes:
- from the right
 - from above
 - from below?

4.4 Different views of composite objects

This colourful object was made by combining four rectangular prisms.

Zweli looks at the object from this side



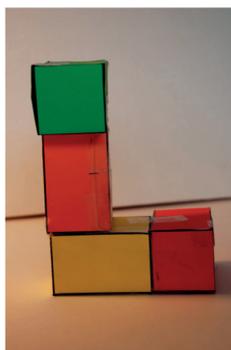
Jabu looks at the object from behind

Esther looks at the object from this side

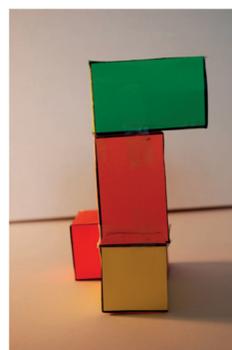
1. (a) Which of the following photographs shows what Jabu sees?
(b) Which photograph shows what Esther sees?
(c) Which photograph shows what Zweli sees?



A



B



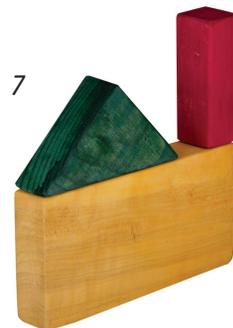
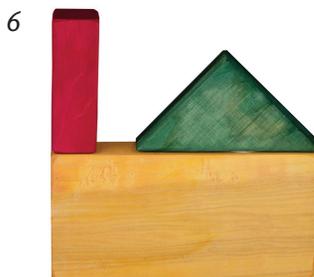
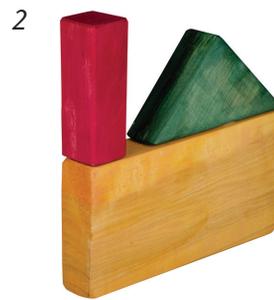
C

2. A photographer placed this combination of three prisms in the middle of a table.

She then walked *once* around the table and stopped at seven places to take photographs.

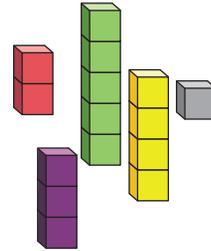
She first took Photograph 1 below, and then Photograph 2.

In which order did she take the other five photographs?

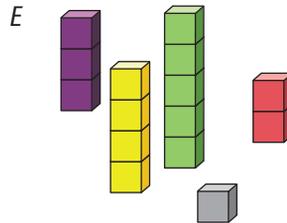
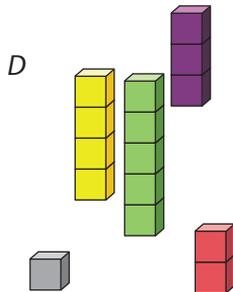
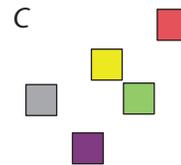
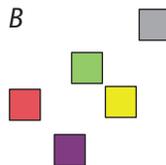
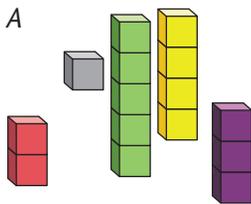


4.5 Different views of more stacks of cubes

Imagine that these stacks of cubes were placed in the middle of a small square table. This is how you are seeing them from one side of the table.



1. Imagine that you are leaning forward and are looking at them directly from above. Which picture shows what you are seeing?

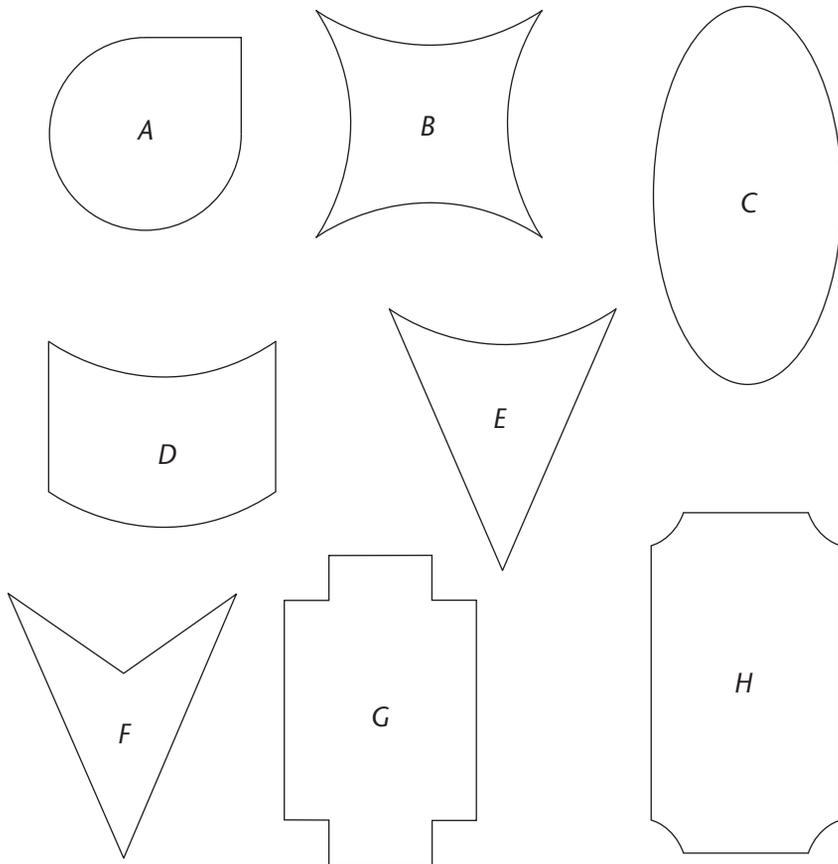


In question 2 you have to imagine that you are looking at the cubes from the other three sides of the table: the side on your left, the side on your right, and the side opposite you.

2. (a) Which picture shows what you will see from the side opposite you?
(b) Which picture shows what you will see from the side on your left?
(c) Which picture shows what you will see from the side on your right?

5.1 Some revision

1. (a) For each figure below, state whether it has straight sides only, straight and curved sides, or curved sides only.

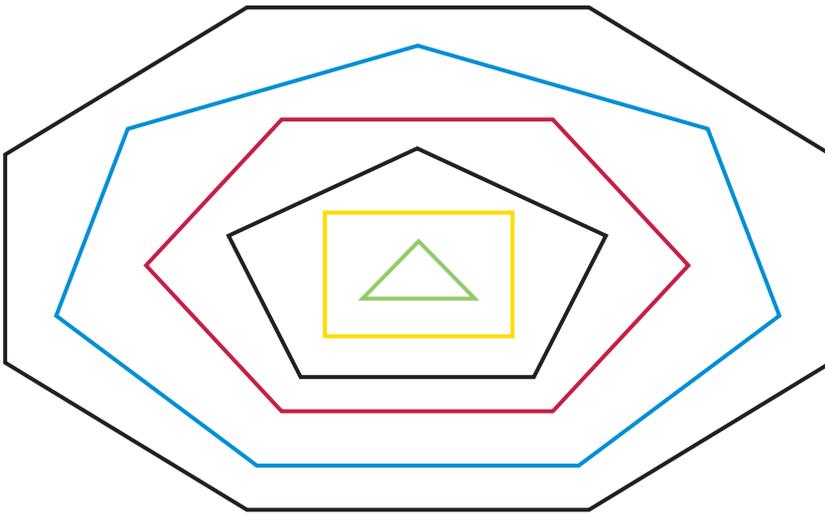


- (b) What is the number of straight sides in each figure?
 (c) What is the number of curved sides in each figure?
2. (a) How many reflex angles are inside Figure F?
 (b) How many reflex angles are inside Figure G?
 (c) How many right angles are inside Figure G?

5.2 Polygons

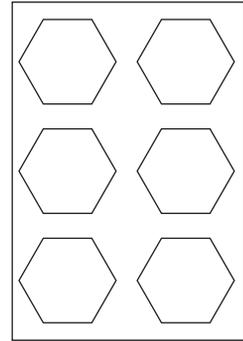
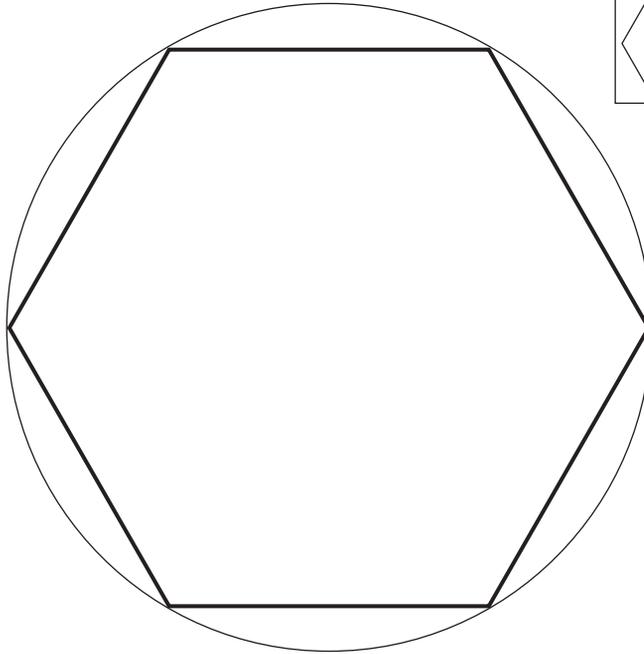
Polygons are named according to their number of sides:

- A polygon with 8 sides is called an octagon.
- A polygon with 7 sides is called a heptagon.
- A polygon with 6 sides is called a hexagon.
- A polygon with 5 sides is called a pentagon.
- A polygon with 4 sides is called a quadrilateral.
- A polygon with 3 sides is called a triangle.

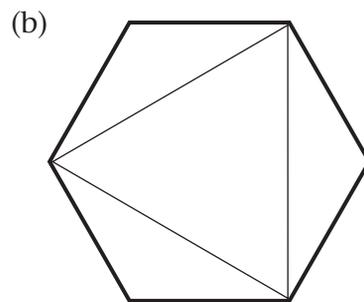
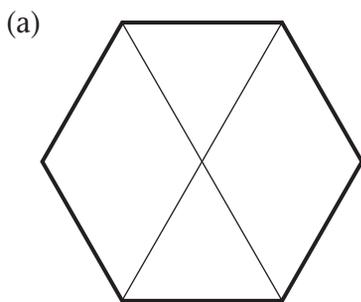


1. What is the colour of each of these polygons in the above diagram?
 - (a) the quadrilateral
 - (b) the heptagon
 - (c) the octagon
 - (d) the pentagon
 - (e) the hexagon
 - (f) the triangle
2. In which polygons in the diagram above are all the angles
 - (a) bigger than right angles
 - (b) right angles
 - (c) smaller than right angles?

3. Put a clean sheet of paper on top of the diagram below, and trace the hexagon. Shift your sheet of paper to trace more copies of the hexagon. Trace six copies of the hexagon altogether, as shown on the right.



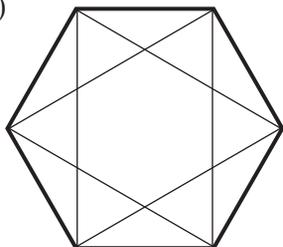
4. Draw lines inside two of your hexagons, as shown below.



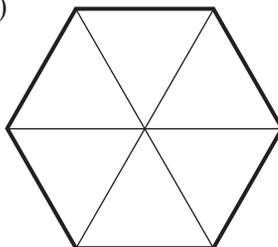
5. (a) Make the sides of one quadrilateral darker in your Figure 4(a), and in your Figure 4(b).
 (b) Shade both triangles in your Figure 4(a), and shade a pentagon in your Figure 4(b).

6. Draw lines as shown below inside two of your hexagons.

(a)



(b)



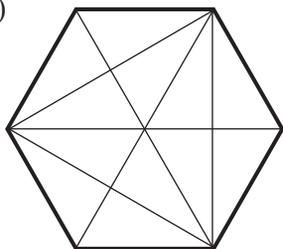
7. (a) Shade all the triangles inside your Figure 6(a).

(b) What kind of polygon is not shaded in your Figure 6(a)?

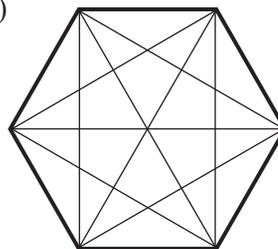
(c) Shade any three of the triangles in your Figure 6(b).

8. Draw lines as shown below inside two of your hexagons.

(a)



(b)



9. (a) There is a quadrilateral with three angles smaller than right angles in your Figure 8(a). Shade this quadrilateral.

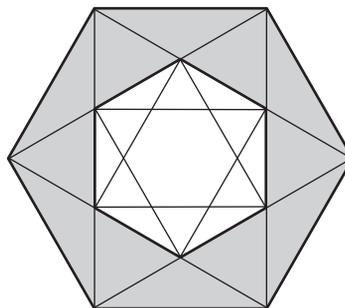
(b) Darken the sides of the heptagon in your Figure 8(b).

(c) Lightly shade an octagon in your Figure 8(b).

(d) There is at least one pentagon inside your Figure 8(b) that has two right angles. Shade one such pentagon dark.

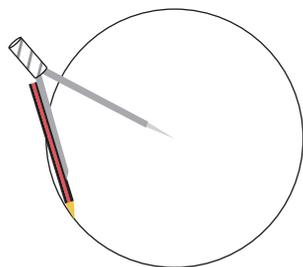
10. (a) Draw lines as shown here in the unshaded part of your drawing of Figure 6(a).

(b) Shade the small hexagon in the middle of the diagram.

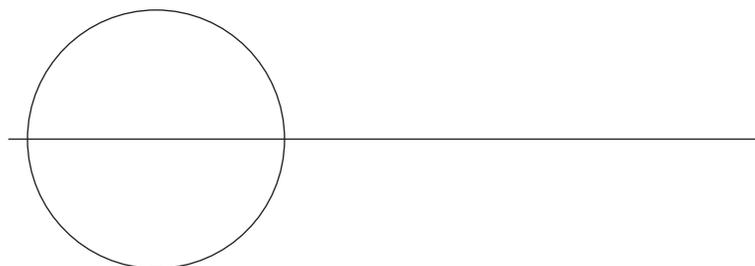


5.3 Drawing circles and patterns in circles

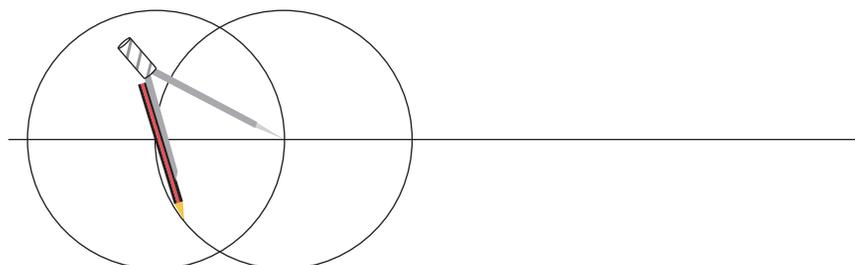
- Set your compasses so that the sharp tip and the pencil tip are about 3 cm apart.
 - Draw a circle on the left side of a clean sheet of paper.



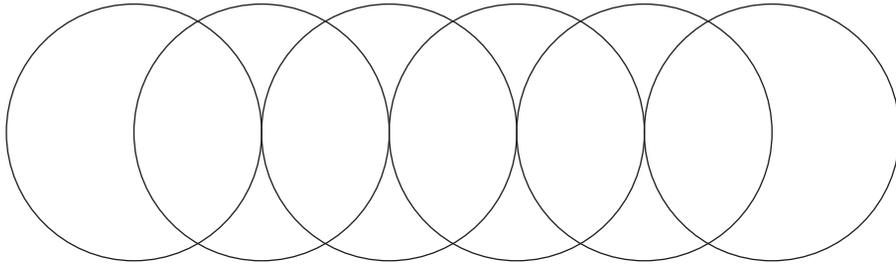
- Draw a line through the centre of your circle, from left to right across the page.



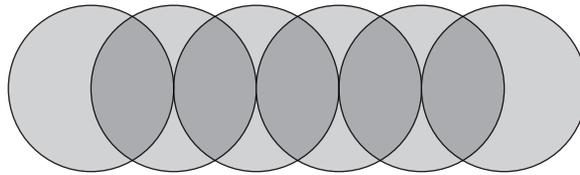
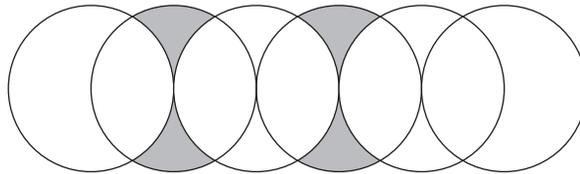
- Put the sharp tip of the compasses at the point where the line and the circle cross, and draw another circle. Your compasses must have the same setting as in question 1.



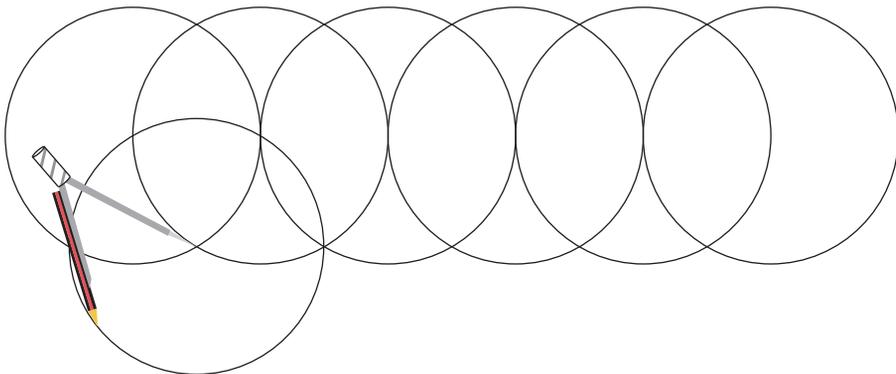
- Draw four more circles in the same way, to make a pattern with **interlocking** circles as shown on the next page.



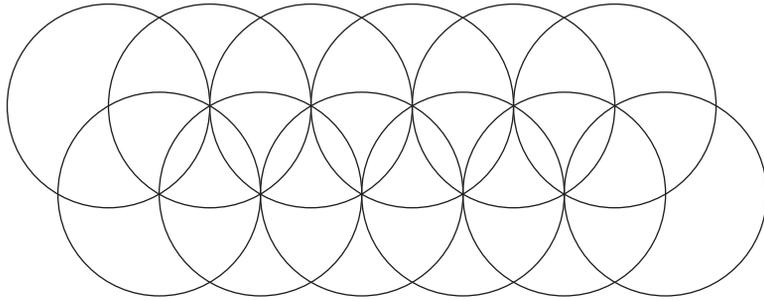
More patterns can be made by shading parts of a diagram like the one above. Two examples are shown here.



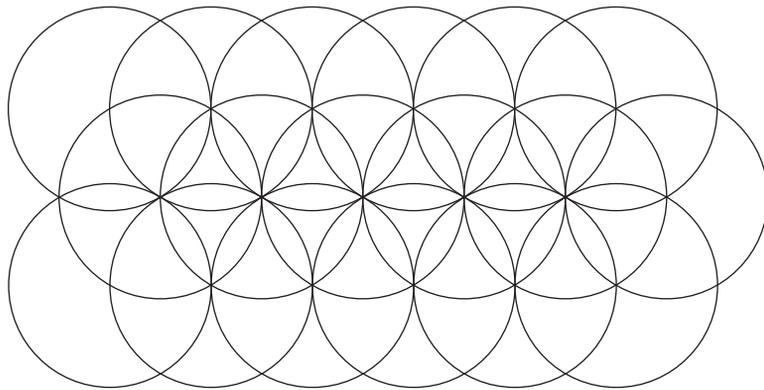
3. You can add a second row of circles to a pattern like the one you drew in question 2:



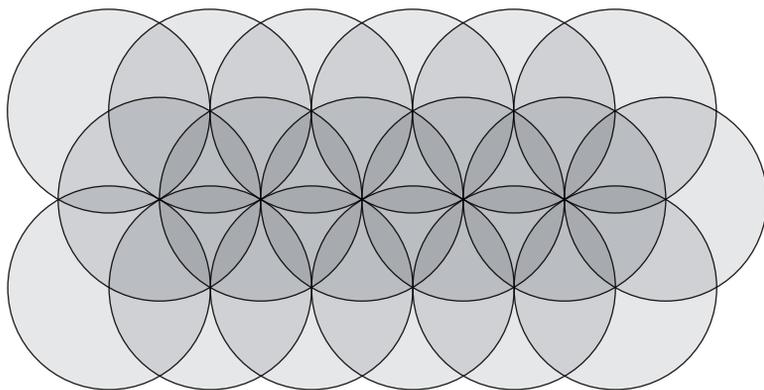
Make a diagram with two rows of interlocking circles, as shown at the top of the next page.



4. Add another row of circles to your drawing, to make a pattern like this:

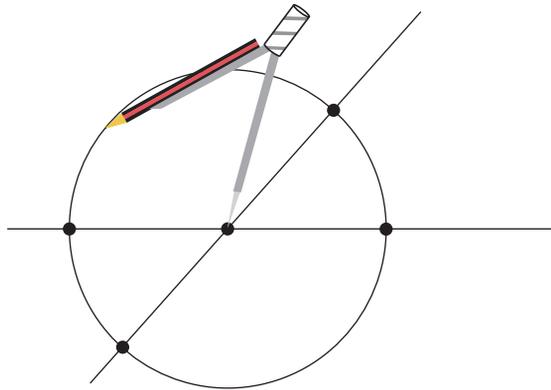


A pattern like the one above can be shaded in different ways.

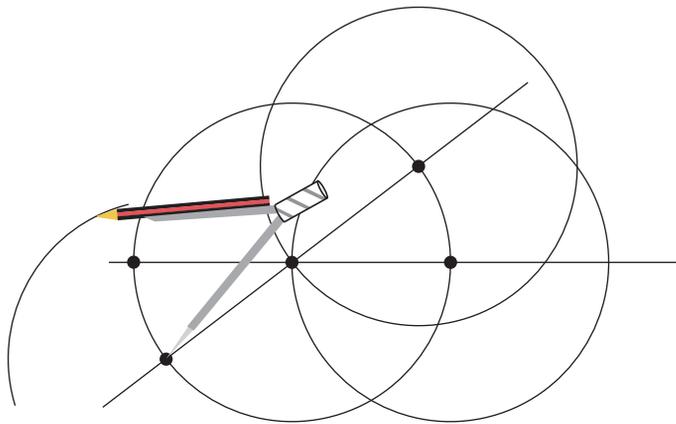


5.4 Patterns with circles

- (a) Draw two lines that cross each other as shown below.
Draw a circle with its centre at the point where the two lines cross.

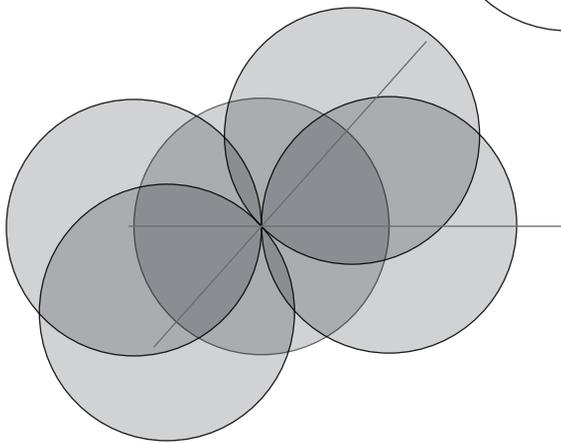
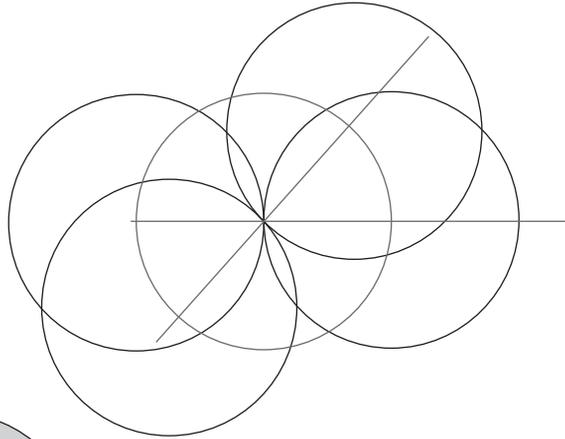


- (b) Make small dots at the points where the circle crosses the lines you have drawn.
- (c) Keep your compasses at the same setting and draw four circles, with their centres at the dots that you have made.

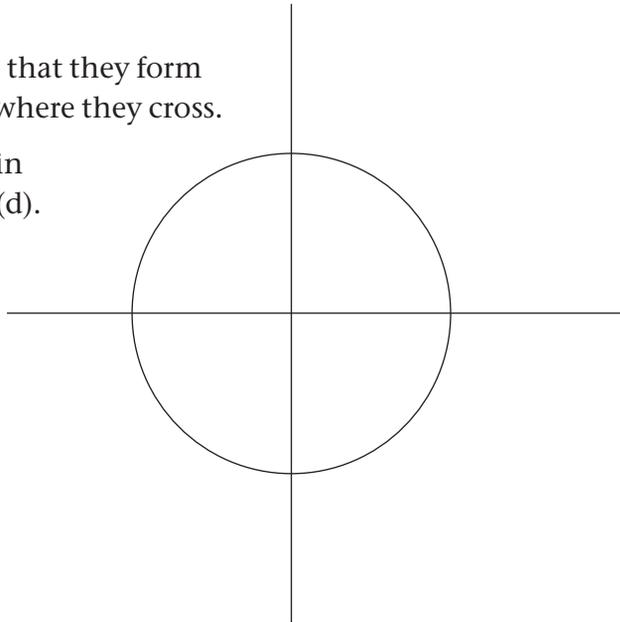


- (d) When you have finished, your drawing should look as shown at the top of the next page.

A drawing like this can be shaded in different ways, to show the patterns.

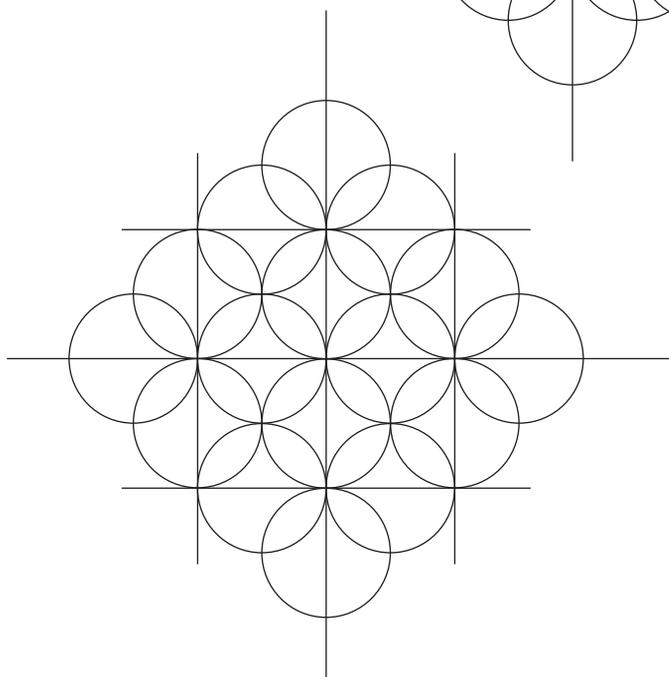
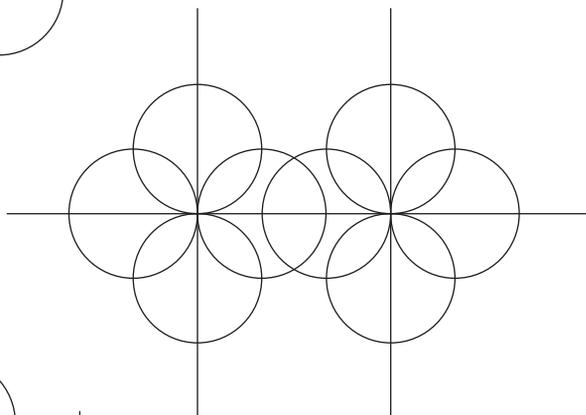
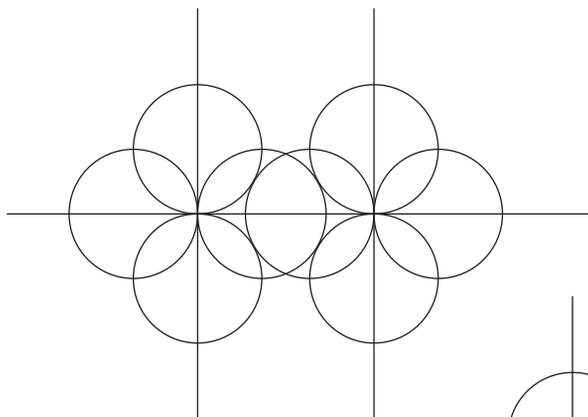
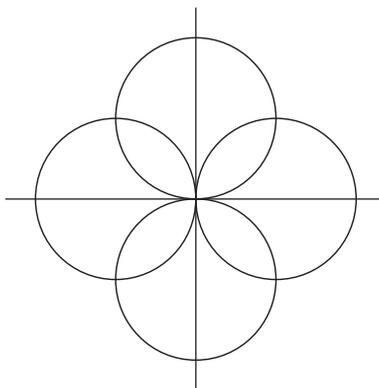


2. (a) Draw two lines so that they form four right angles where they cross.
(b) Do what you did in questions 1(a) to (d).



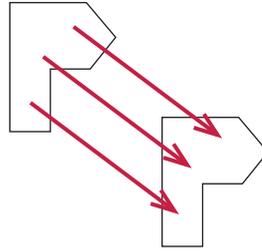
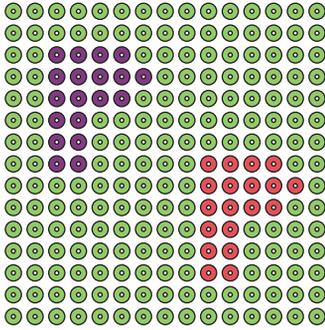
The drawing you made in question 2 should look like this:

Some other patterns that you can make by drawing circles with compasses are shown below.

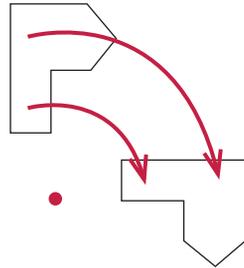
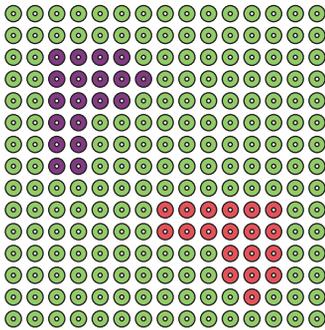


6.1 Rotations, reflections and translations

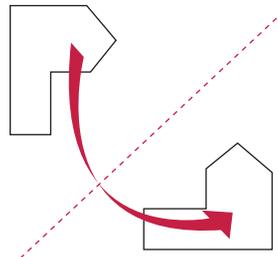
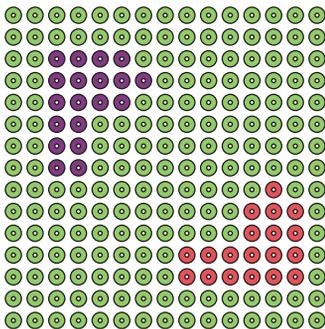
The red figure on this bead mat is a **translation** of the purple figure.



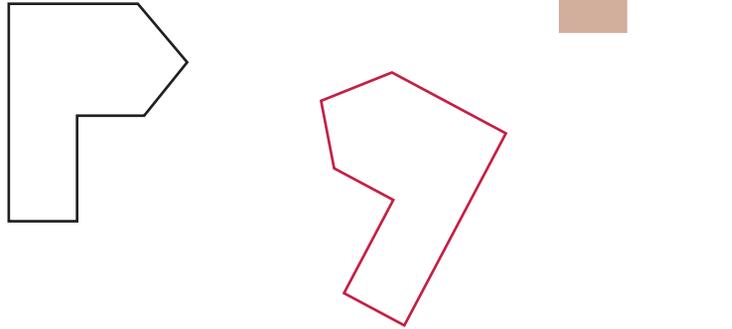
The red figure on this bead mat is a **rotation** of the purple figure.



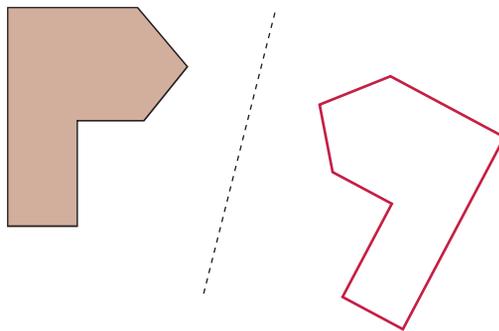
The red figure on this bead mat is a **reflection** of the purple figure.



Mzwi traced the figure on the right onto brown cardboard. He then cut it out, and used it as a template to draw diagrams consisting of two figures each.



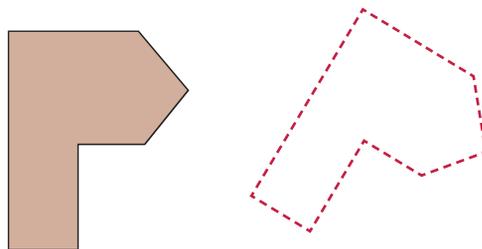
To draw the above diagram, Mzwi first put the template in the position on the left, and traced around it in black.



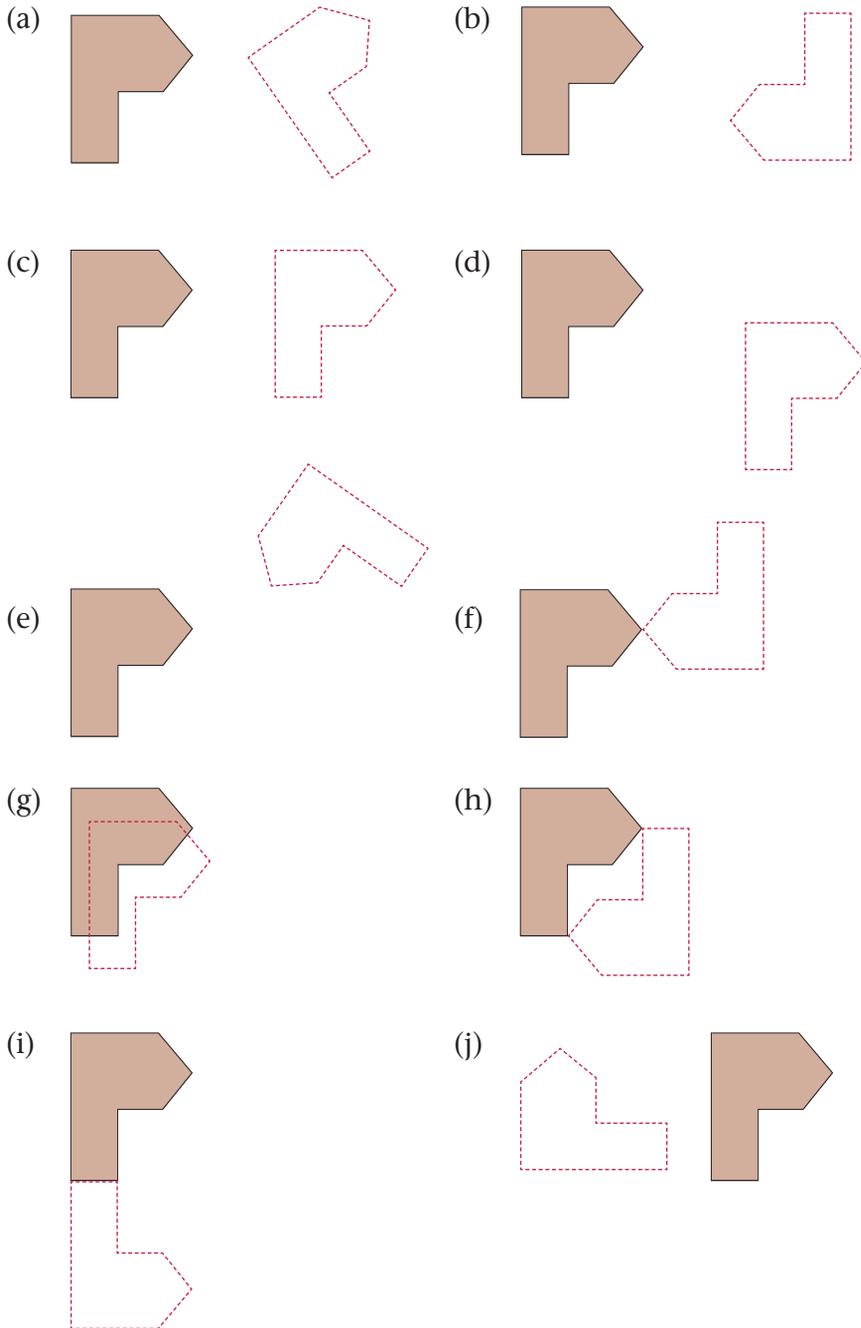
Then he **reflected** the template to the position on the right, and traced around it in red.

1. The diagram below shows the template on the black position.

- (a) Can the template be moved to fit on the red position just by translating it?
- (b) How must the template be moved to fit on the red position?



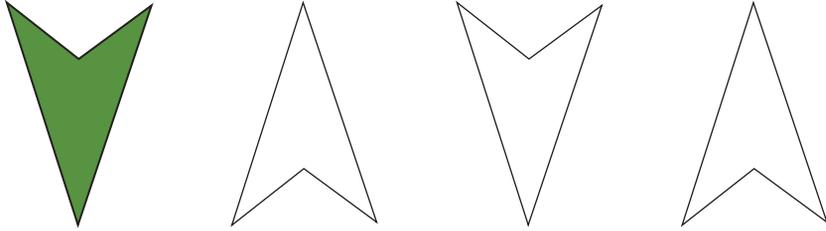
2. The diagrams below show the brown template on the black position. In each case state whether the template should be rotated, reflected or translated to move it to fit on the red position.



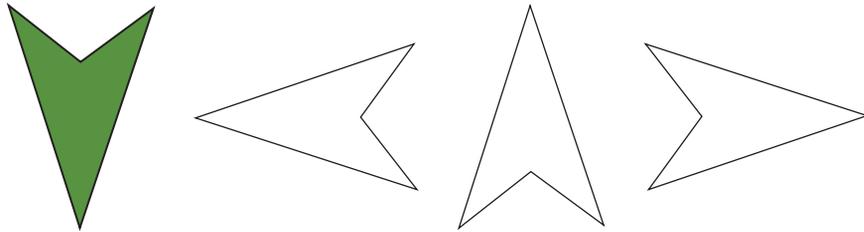
3. In the patterns below, a green template is shown on the first position. In each case state whether the template should be rotated, reflected or translated to move it from one position to the next.



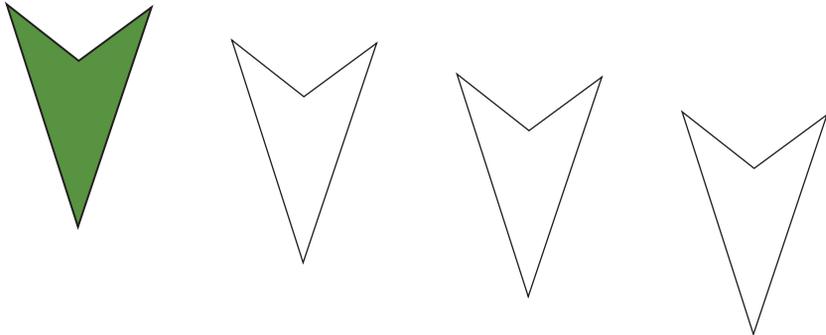
(a)



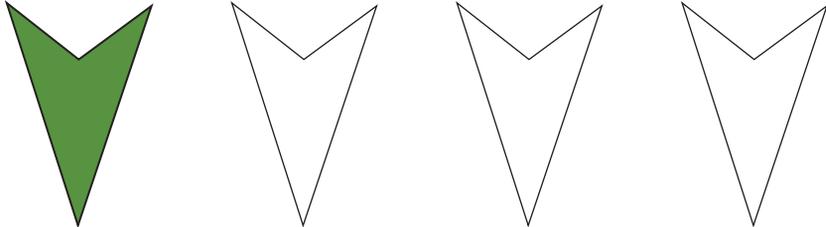
(b)



(c)



(d)



6.2 Describing patterns

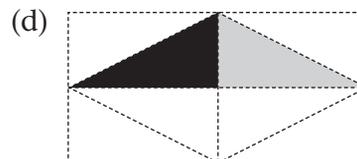
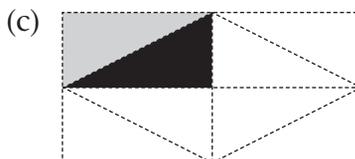
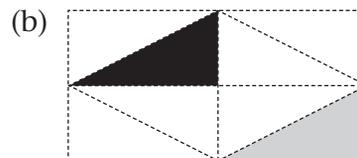
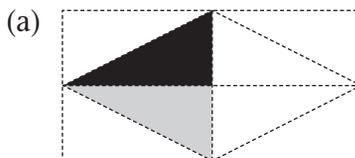


The design with triangles on the above wall is often used in Ndebele art. There are many rotations, reflections and translations in this design.

- Which of these arrangements form part of the above design, and which do not?



- In each case below, state whether the grey triangle is a translation, rotation or reflection of the black triangle.



6. In Pattern 1, each red triangle is a translation of any other red triangle. Is this also true for Pattern 2?

Pattern 1

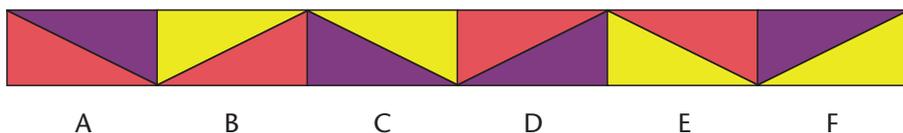


Pattern 2



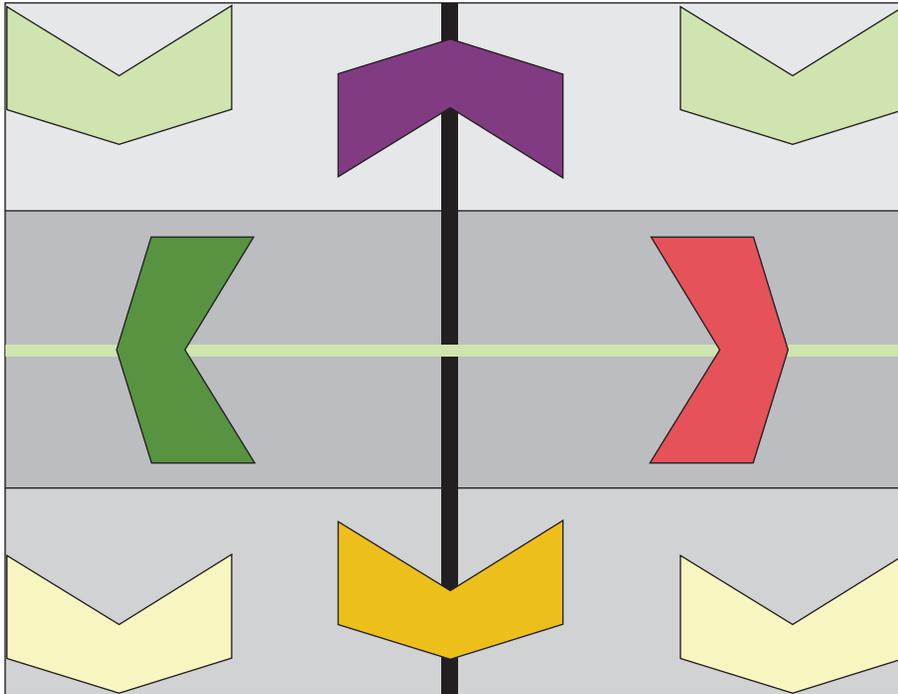
7. Each of the parts A, B, C, D, E and F of Pattern 2 consists of two triangles.
- Is the red triangle in part A of Pattern 2 a translation, rotation or reflection of the red triangle in part E?
 - Which yellow triangles in Pattern 2 are rotations of the yellow triangle in part E?
 - Are there any examples of reflection in Pattern 1 or Pattern 2?
8. Choose ONE word to describe how Pattern 3 differs from Pattern 2: *translation, reflection* or *rotation*.

Pattern 3

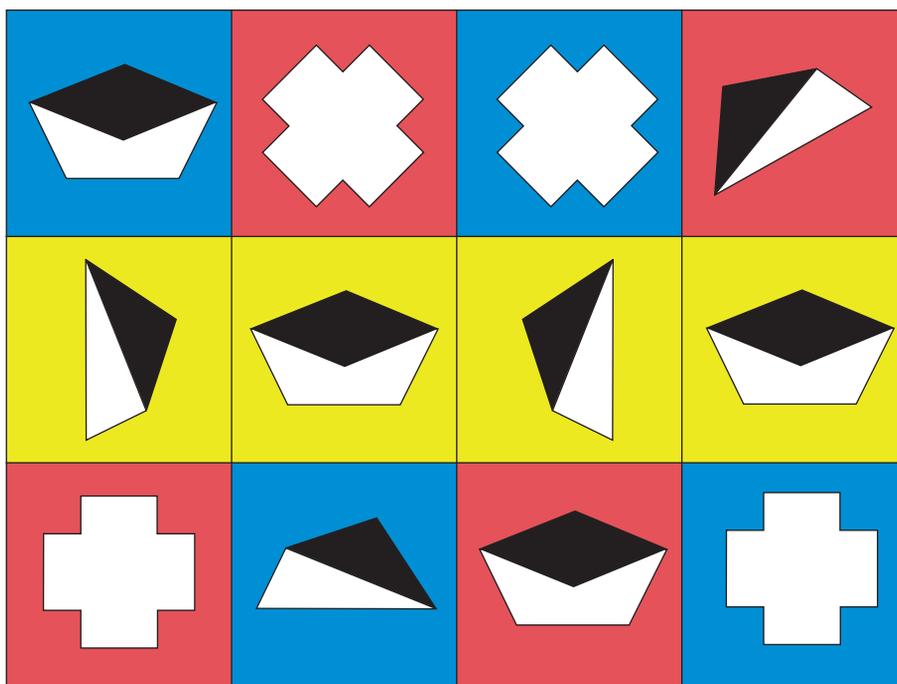


9. If you move your eyes from A to F on Pattern 3, you will see that the red triangle is first reflected, then rotated and then reflected again. Describe in the same way what you see about each of the following:
- the purple triangles in Pattern 3
 - the purple triangles in Pattern 2
 - the purple triangles in Pattern 1

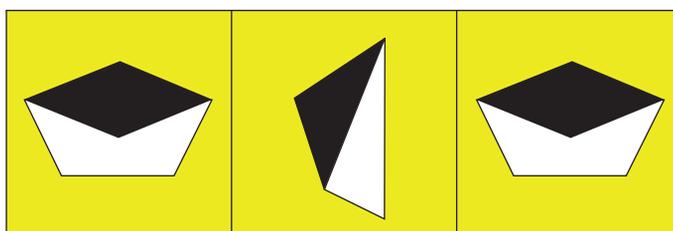
6.3 Symmetry in patterns



1. Which of the following statements about the above diagram are true, and which are false?
 - (a) The red hexagon is a rotation of the purple hexagon.
 - (b) The light green hexagons and the cream-coloured hexagons are translations of the golden-yellow hexagon.
 - (c) The thick light green line is a line of symmetry of the whole diagram, if colour is ignored.
 - (d) The golden-yellow hexagon is a rotation of the purple hexagon.
 - (e) The golden-yellow hexagon is a reflection of the purple hexagon.
 - (f) The thick black line is a line of symmetry of the whole diagram, if colour is ignored.
2. Make five other true statements about reflections, translations and rotations in the above diagram.



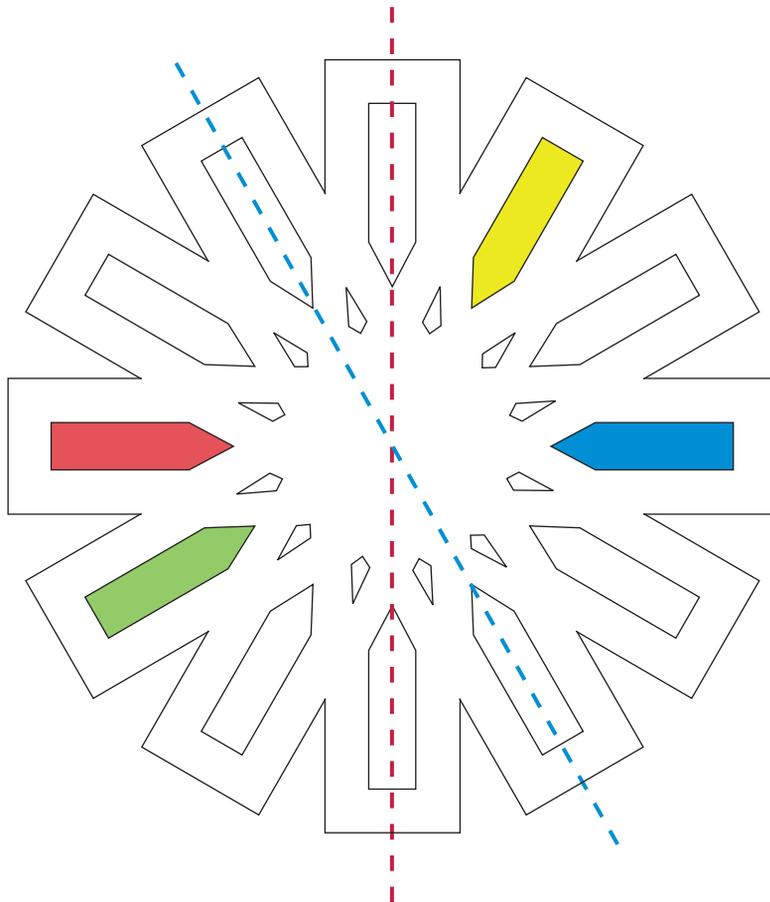
3. The diagram below shows a part of the above diagram that *is not* symmetrical.



Make a rough drawing of a part of the above diagram that *is* symmetrical. Show the line of symmetry with a broken line.

4. *In the diagram at the top of the page, the white cross can be rotated to move it from the one red square to the other red square.*
- Write five more statements like this about that diagram.
 - How can the quadrilateral be moved from the yellow square on the left, to the red square at the top right?
5. Describe two reflections in the diagram at the top of the page.

6. In the design below, some parts are coloured and two broken lines are used so that you can easily make statements about the design.



- Use your knowledge of rotations, translations, reflections and lines of symmetry to write five statements about this diagram.
- Which other arrows are reflections of the yellow arrow?
- Which other arrows are rotations of the yellow arrow?
- How many lines of symmetry does the above diagram have, if you ignore the colours?

-
7. (a) Make a rough sketch of the placemat including only the *basic* design element(s) for which this placemat has lines of symmetry. Show the lines of symmetry with broken lines on your sketch.



- (b) Try to see how the placemat maker used rotation in the design. Make sketches to show this.
- (c) Make a sketch to show how reflection is used in the design.

7.1 The Celsius scale and medical thermometers

We use a thermometer to measure temperature. The scale we use is the **Celsius scale**, constructed by Anders Celsius, a Swedish astronomer in the 18th century.

The Celsius scale is based on the fact that pure water at sea level freezes at about 0°C (zero degrees Celsius) and boils at about 100°C (hundred degrees Celsius).

We say *about* because the freezing point and boiling point is influenced by many other factors.

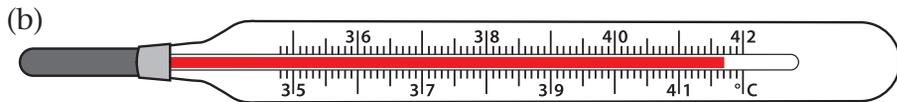
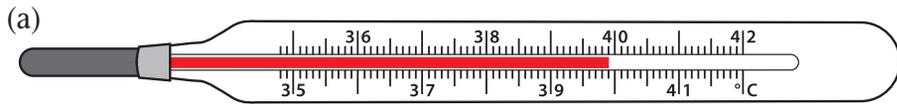
1. The medical thermometer below was designed to measure the body temperature of humans.



An analogue medical thermometer

- (a) What is the highest temperature that this thermometer can measure?
- (b) What is the lowest temperature that this thermometer can measure?
- (c) Why do you think this thermometer was designed to measure temperature only between these numbers?
- (d) The distance between 35°C and 36°C is divided into ten smaller units. Find the line that shows $35,5^{\circ}\text{C}$.
- (e) Why do you think 37°C is written in red on the thermometer?

2. Write the temperature shown by each of the thermometers.



3. A healthy person has a body temperature that is no more than one degree Celsius lower or higher than 37°C . Say whether the patients below have healthy body temperatures or not.

- (a) Patient A: $36,5^{\circ}\text{C}$ (b) Patient B: $37,4^{\circ}\text{C}$
 (c) Patient C: $34,6^{\circ}\text{C}$ (d) Patient D: $38,6^{\circ}\text{C}$

4. $38,6^{\circ}\text{C}$ rounded off to the nearest degree Celsius is 39°C . Round off the temperature shown on this digital medical thermometer to the nearest degree Celsius.



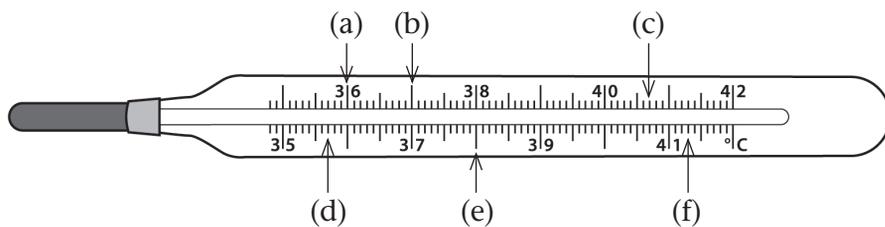
A digital medical thermometer

5. Order these temperatures from the highest to the lowest:

$39,7^{\circ}\text{C}$ $37,4^{\circ}\text{C}$ $40,8^{\circ}\text{C}$ $38,9^{\circ}\text{C}$ $41,25^{\circ}\text{C}$ $38,35^{\circ}\text{C}$

6. Round the temperatures in question 5 off to the nearest degree Celsius.

7. Write down the temperature readings at (a) to (f).



8. Tom is ill. His mom says his temperature is about 39°C . Give two possible temperature readings that her digital thermometer may have shown for her to say this.

7.2 Daily temperature

1. The minimum (lowest) and maximum (highest) temperatures for towns across South Africa are reported every day on the TV and the radio.
 - (a) What do you think the reason is for giving these reports?
Discuss with a few classmates.
 - (b) Estimate the temperature on a hot summer's day where you live.
 - (c) Estimate the temperature on a cold winter's night where you live.
 - (d) Consult the weather report on the radio, TV, the internet or in a newspaper. Find out what the forecasted maximum and minimum temperatures for your region for this day are. Adjust your estimates in questions (b) and (c) if you need to.
2. This information is from the website of the South African Weather Service:

The highest worldwide temperature was recorded in Al Aziziya, Libya measuring $57,7^{\circ}\text{C}$ on 13 September 1922.

The lowest worldwide temperature was recorded in Vostok, Antarctica at $-89,2^{\circ}\text{C}$ on 21 July 1983.

- (a) Imagine what it is like to live in a place like Al Aziziya. Think about the effect of a temperature of over 50°C on plants and animals and on food farming.

Ask your Natural Sciences or Social Sciences teacher about life in extreme temperatures.

- (b) Write a paragraph about your ideas.

The lowest temperature in South Africa was recorded at Buffelsfontein near Molteno (Eastern Cape) measuring $-18,6^{\circ}\text{C}$ on 28 June 1996.

The highest temperature in South Africa was recorded at Dunbrody (Sundays River Valley in Eastern Cape) measuring 50°C on 3 November 1918.

3. The table gives the minimum and maximum temperatures measured in Molteno in the week 18 to 22 May 2015.

	18 May	19 May	20 May	21 May	22 May
Minimum	4 °C	5 °C	5 °C	5 °C	4 °C
Maximum	22 °C	23 °C	23 °C	21 °C	19 °C

- (a) Work out the difference between the minimum and maximum temperatures each day.
- (b) How will you dress in Molteno in May?
- (c) How does the temperature in Molteno compare to the temperature where you live? Write a short paragraph to explain.
4. This table shows the minimum and maximum temperatures measured in Letaba (Limpopo) in the week 18 to 22 May 2015.

	18 May	19 May	20 May	21 May	22 May
Minimum	18 °C	18 °C	19 °C	19 °C	21 °C
Maximum	23 °C	23 °C	23 °C	24 °C	26 °C

- (a) Work out the difference between the minimum and maximum temperatures each day.
- (b) How would you dress in Letaba in May?
- (c) How does the temperature in Letaba compare to the temperature in Molteno?
- (d) How does the temperature in Letaba compare to the temperature where you live? Write a short paragraph to explain.
5. When Elizabeth left her home at 07:00 it was 3,5 °C. At 13:00 it was 15 °C. How much warmer was it at 13:00 than at 07:00?
6. On Monday the minimum temperature was 4,5 °C. Monday's maximum temperature was 18 degrees higher. What was the maximum temperature?

Letaba has an average annual maximum temperature of around 35 °C. This means that a maximum temperature of 35 °C in winter is not unusual.

8.1 Working with hundredths

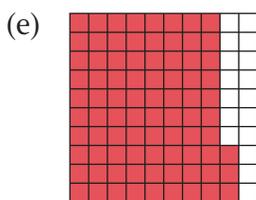
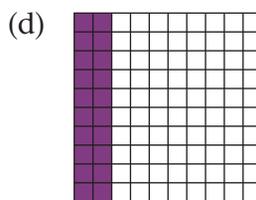
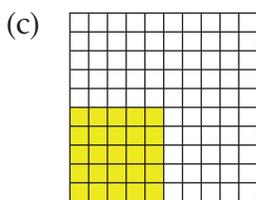
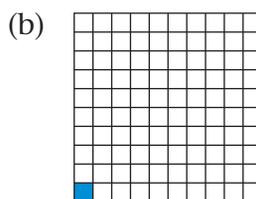
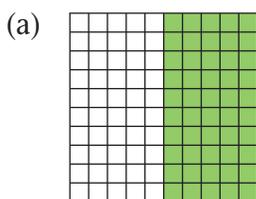
1. What does the word “percentage” mean to you? Think about it for a minute. Do you remember getting 50% in a test?

“Percentage” is another word for “hundredths”.

23% means $\frac{23}{100}$, which is the same as $\frac{2}{10} + \frac{3}{100}$ or 0,23.

Instead of saying 23 hundredths we can say 23%.

2. What fraction part of each square below is shaded?
Give your answers in percentage notation, fraction notation and decimal notation.



3. What part of each of the above figures is not shaded? Give your answers in percentage notation, fraction notation and decimal notation.

4. Write each of the decimals as percentages.

(a) 0,45

(b) 0,7

(c) 0,03

(d) 0,95

(e) 0,20

(f) 2,5

5. Write each of the fractions as percentages.

(a) $\frac{2}{5}$

(b) $\frac{7}{10}$

(c) $\frac{3}{4}$

(d) $2\frac{1}{2}$

(e) $\frac{13}{20}$

(f) $1\frac{11}{50}$

(g) $\frac{14}{25}$

(h) $\frac{6}{5}$

8.2 Finding percentages of whole numbers

Now that we have answered the questions above, we can say:

Finding a percentage of a whole number is similar to finding a fraction of a whole number. We can also say:

A **percentage** is a **fraction** written in a different notation.

So, to find 6% of 65 is the same as finding $\frac{6}{100}$ of 65.

This requires dividing by 100. It can easily be done mentally. We need to practise the skill.

1. If you have a calculator available, use it to do the following:

(a) $123 \div 10$

(b) $123 \div 100$

(c) $123,4 \div 10$

(d) $1\ 234 \div 100$

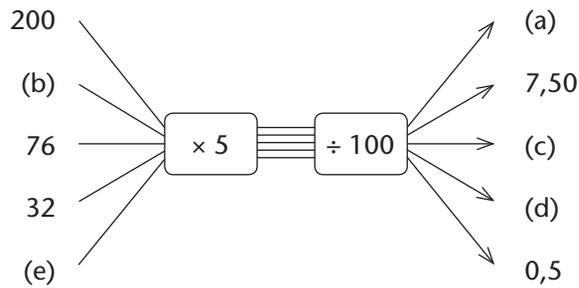
2. Use your calculator and revisit the activity we did before (page 197).

Set up your calculator to divide by 10 like this:

enter 10 $\boxed{\div}$ $\boxed{\div}$ $\boxed{=}$ and then enter 12345 $\boxed{=}$

Describe what you see, and explain.

7. Write down the numbers that can replace the letters (a) to (e) to complete this flow diagram.



8. Calculate:

- | | |
|----------------|-----------------|
| (a) 6% of 65 | (b) 20% of 300 |
| (c) 12% of 450 | (d) 25% of 244 |
| (e) 3% of 60 | (f) 14% of R150 |

8.3 Apply your knowledge

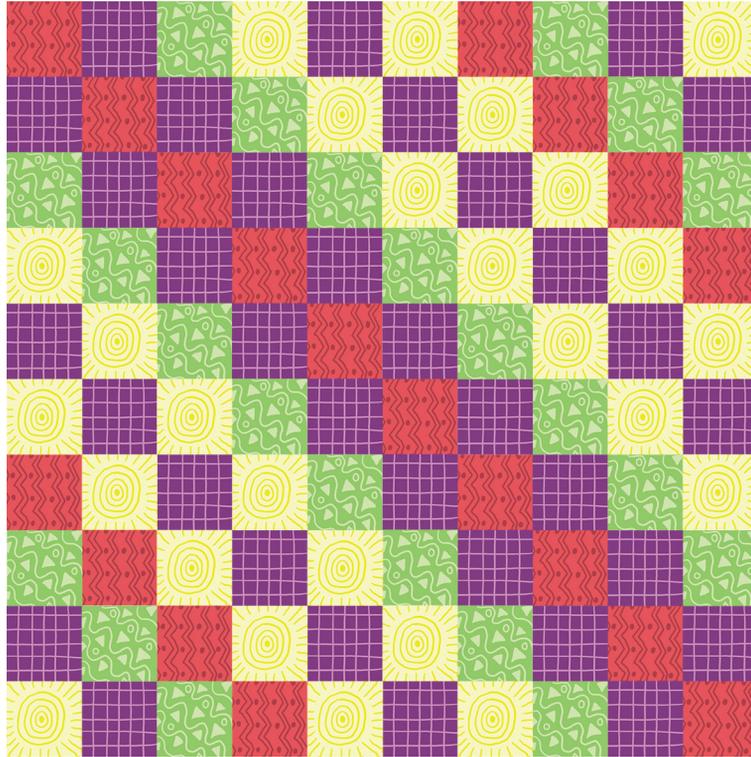
1. A fashion retailer has a 20% off sale. This means that the clothes on sale will sell for 20% less than the normal price. Calculate what the sale price will be if the original price is:

- (a) R400
- (b) R120
- (c) R150
- (d) R60
- (e) R70
- (f) R1 250



-
2. Nomsa plays netball. During her last match she tried to score a goal 10 times. She was successful 6 of the times she tried.
- (a) What fraction of her attempts to score a goal was successful?
 - (b) What percentage of her attempts was successful?
 - (c) What percentage of her attempts was not successful?
3. Andiswa got 21 out of 30 for her Mathematics test. What percentage did she get?
4. Many children had flu in winter. One day during this time, 120 out of 800 children were absent from school. What percentage was absent?
5. John spends R50 in this way:
- | | | |
|--------------------|---------------------|-----------------------|
| R3 for an apple | R6 for a bus ticket | R8 for a tin of juice |
| R13 for a meat pie | R12 for a taxi | R8 for milk |
- What percentage of the money did he spend on:
- (a) travel
 - (b) drinks
 - (c) food?
6. Mother bought a box of apples. Of the 60 apples in the box, 15 were bad. What percentage of the box of apples was bad?
7. Fundi used about three-quarters of the paint in the can. What percentage of the paint did she use?
8. Peter scored 78% in a test. The test was out of 150. What was Peter's mark?
9. Mimi bought a camera that was marked R850 in the shop. She got 20% discount. How much did she pay for the camera?
10. Miss Pula could enter the top 30% of her Mathematics learners for a competition. There are 46 learners in her class. How many learners could she enter? (Use your common sense when you give the answer!)

11. This is a patchwork quilt (bed cover) that Maggie made. She used square pieces of material of different colours and patterns.



- (a) How many squares are there in the quilt?
- (b) How did you find your answer? Did you count all the squares or did you make a clever plan?
- (c) What fraction of the quilt is red?
- (d) What fraction is green?
- (e) What fraction is yellow?
- (f) What fraction is purple?
- (g) Write the fraction of each of the colours as a percentage.

9.1 Representing data

FIFA has regulations about the height that a soccer ball must bounce. A Size 4 ball must bounce at least 110 cm high at 5 °C.

The data in this table are from an official test. A machine was used to drop the ball from a height of exactly 2 m, and the bounce height was measured with a laser beam.

The test was done with a Size 4 ball.

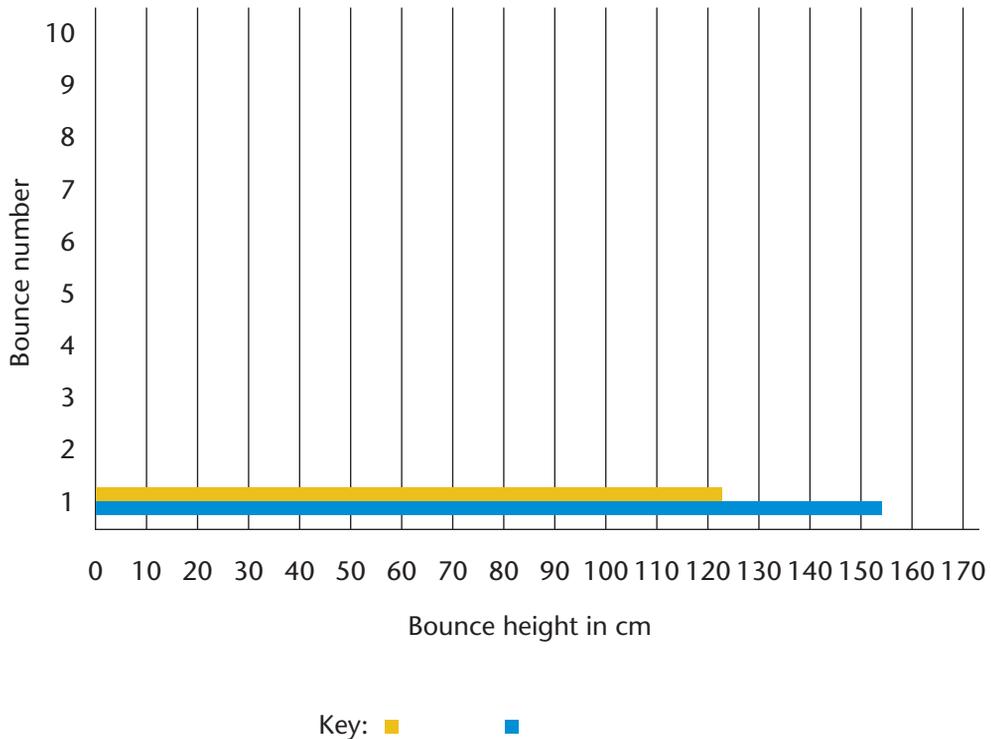
Bounce test: Size 4 ball

20 °C	5 °C
155 cm	123 cm
160 cm	127 cm
120 cm	121 cm
150 cm	120 cm
158 cm	119 cm
158 cm	121 cm
133 cm	119 cm
147 cm	130 cm
128 cm	126 cm
131 cm	128 cm

You can test the soccer balls at your school too. Make sure the ball is inflated properly. Drop the ball 10 times from a height of exactly 2 m onto a hard surface. You have to place a measuring tape against the wall before the time, and find a way to mark how high the ball bounces each time. However, you will struggle to measure accurately and your data may be invalid.

- Look at the data in the table.
 - How many times was the ball bounced at each temperature?
 - What questions can you ask about the data?

- (c) Draw a double bar graph of the bounce results. Redraw the axes below and show the data. (The first bounce at each temperature has been drawn in.) Give your graph a heading and a key.



- (d) The ball was tested at two temperatures, 20 °C and 5 °C. How do you think the temperature influences the bounce height of a soccer ball? Explain why you think so.

There is only one height that is the lowest bounce height, and one height that is the highest bounce height. The lowest and highest values do not help us to better understand how high the other eight bounces were. We must look for a number that will tell us what is going on *between* the lowest and highest measures.

- (e) Which bounce height is a good height to tell the story of the results of the bounce test at 20 °C? Say how you chose the number.

- (f) The middlemost value in a data set is called the **median**. Find the median of the bounce heights at 20 °C.

How to find the median

Place the 10 bounce heights on a number line so that they are arranged from lowest to highest. Make a mark on the number line exactly halfway between the fifth and the sixth bounce height. The median is the number that you read off at the mark.

- (g) Write a sentence in which you use the median to tell the story of the bounce heights at 20 °C.
2. Do you agree with this report of the bounce height of the ball at 5 °C? Compare the data on the graph to decide.

At 5 °C the ball bounced between about 119 cm and about 130 cm high. The lowest bounce height is well above the FIFA requirement of 110 cm. The median bounce height is 122 cm. So the high bounces are those that bounced between 122 cm and 130 cm.

3. Think critically about this situation:

Schools usually practise soccer in the afternoon when the temperature is around 20 °C. However, they play matches on Saturday mornings, when the temperature is often much lower than 20 °C.

How will you advise the team that will use this ball to play a match on a cold day? How much difference must they expect in the bounce height compared to warmer conditions?

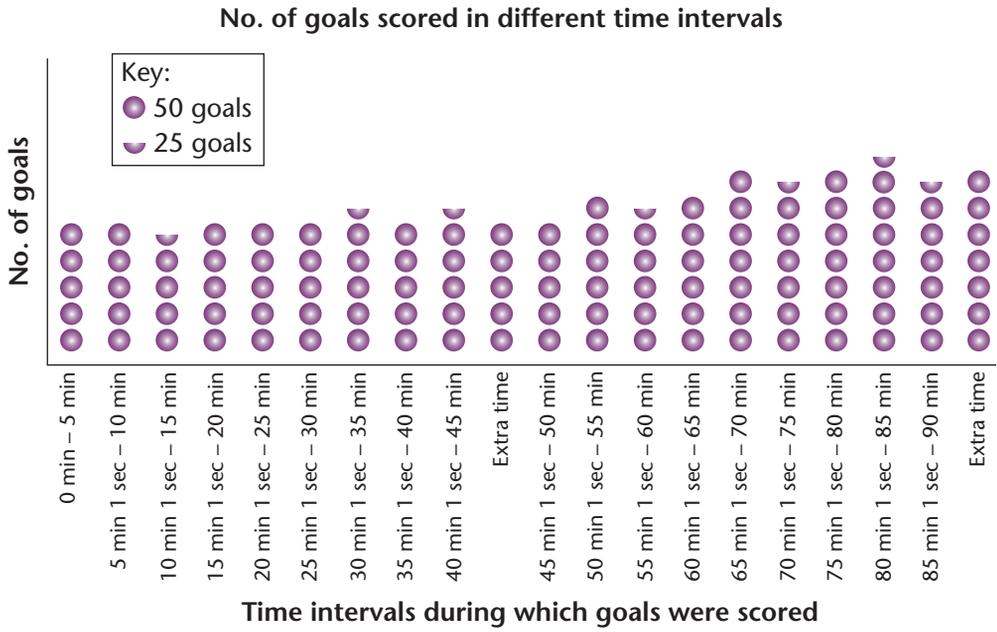
9.2 Analysing and interpreting data in a pictograph

Fikile is a big soccer fan. He wonders: When during soccer matches are the most goals scored? Maybe during the first half? Or maybe in the last 5 minutes? Or maybe there is no pattern?

What do you think?

1. What do you think about Fikile's question? When do you think the most goals are scored during games? Give reasons why you think so.

Fikile searched the internet for data to help him to answer the question. He found this **pictograph** of goals scored in the English Premier League over an eight-year period. Help Fikile to understand what the graph says.



[Adapted from www.soccerstatistically.com]

- Look at the key. Do you think this type of graph is accurate?
- During which half of the games were the most goals scored?
- During which quarter of the games where the most goals scored?
- When during the first half were the most goals scored?
- When during the second half were the most goals scored?
- During which 5-minute interval were the most goals scored?
- Can we now from this data answer Fikile’s question: when during soccer matches are the most goals scored?

A soccer match has two halves of 45 minutes each, plus extra time after each half.

If there is a time interval with more goals than in other times, we call it the **mode** interval.

9.3 Interpreting and reporting data

Statistics South Africa is the government agency that gathers data and writes reports about many aspects of South African life. The data help the government and business to plan ahead.

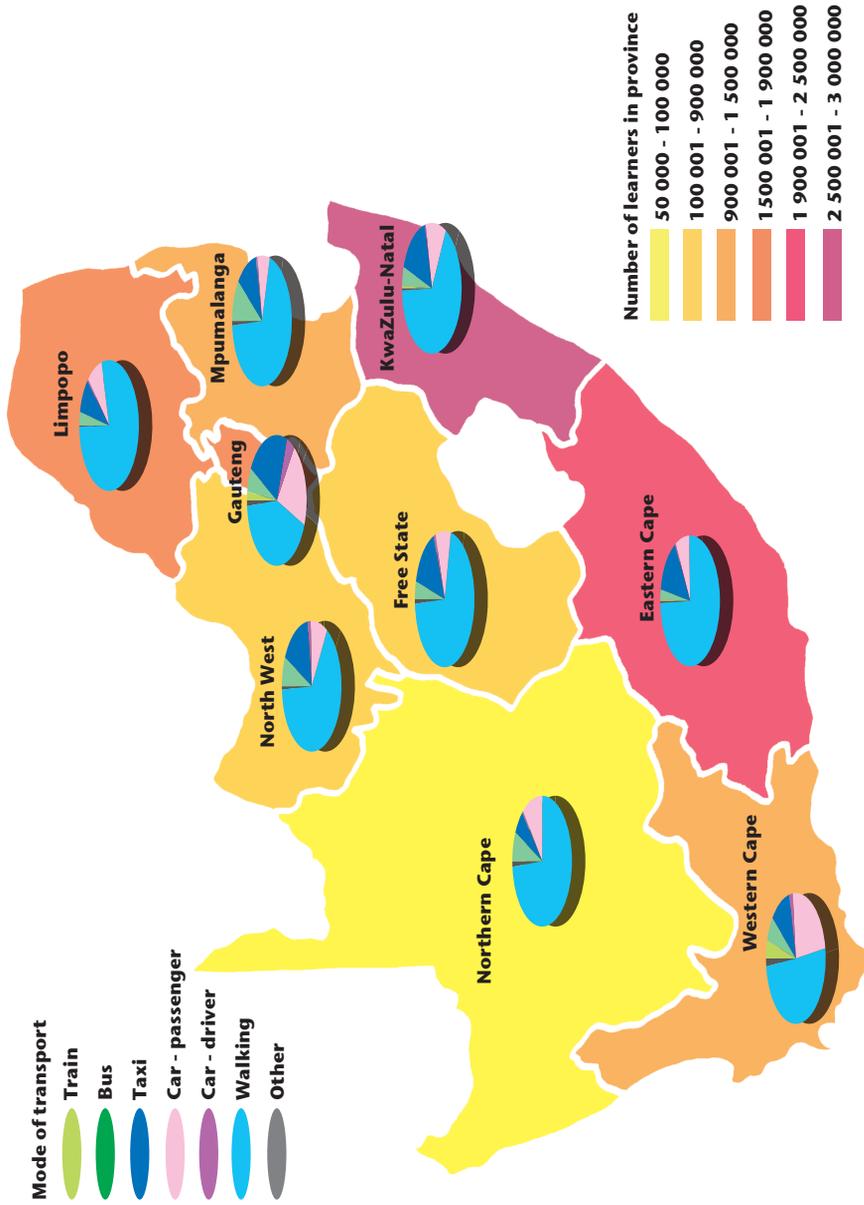
The data we will use in this section were published by Statistics South Africa in the *National Household Travel Survey* of 2013.

1. Study the map on the next page to answer the questions.
 - (a) Read the heading. Write in your own words what situation is described by the map.
 - (b) Read the key. Explain in your own words what information we can get from the colours used on the map and in the pie charts.
 - (c) Different provinces have different numbers of learners. List the provinces in order, from the province with the smallest number of learners to the province with the largest number of learners.
 - (d) The pie charts are too small to be accurate, but they still tell a story. Use your own words to explain what the message of the pie charts is.
2.
 - (a) Study the map and then decide if the following statements are true. Explain why you say so.

Statement A: *Of all the learners who live in Limpopo, more than three quarters walk to school.*

Statement B: *More than three quarters of all the learners that walk to school live in Limpopo.*
 - (b) Write a sentence to explain what fraction of learners in Gauteng walk to school.
 - (c) The pie charts of two provinces tell a different story than the pie charts of the other seven provinces about the way their learners travel to school. Which provinces are they?
 - (d) What is the difference between the two provinces you named in question (c), and all the other provinces?
 - (e) The map shows data from 2013. Do you think the situation is much different this year? Why do you say so?

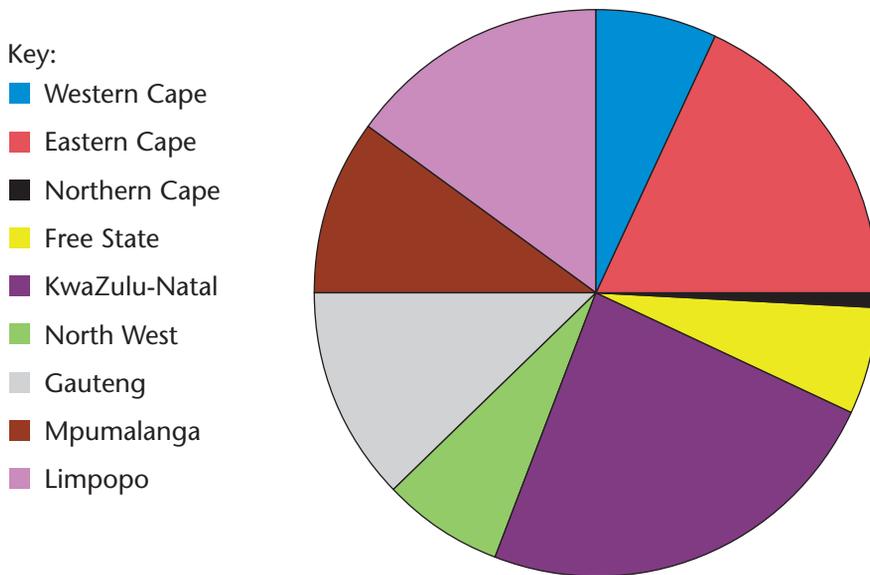
Number of learners attending school per province and main mode of travel used, 2013



[Source: www.statssa.gov.za]

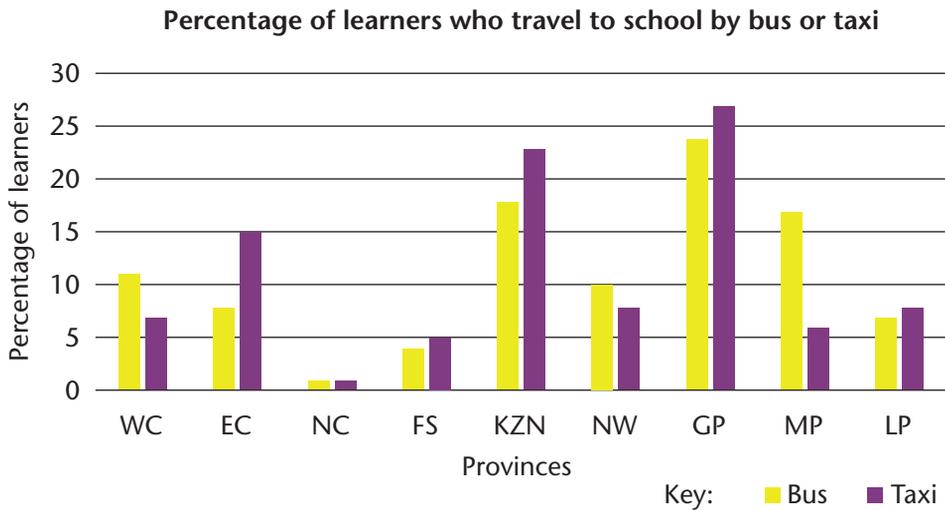
3. This pie chart tells the story from a different point of view. It shows what fraction (percentage) of all the learners who walk to school live in each of the provinces.

Where learners who walk all the way to school live



- (a) In which province does the largest percentage of all South Africa's learners who walk to school live?
- (b) Estimate the percentage of all learners who walk to school and live in Gauteng. Say how you estimated the percentage.
- (c) The percentages for three of the provinces are about the same. Which provinces are they?
- (d) Estimate the percentage for the provinces you named in question (c).
(You may compare fractions first: is it about one eighth or one sixteenth of all learners who walk to school?)

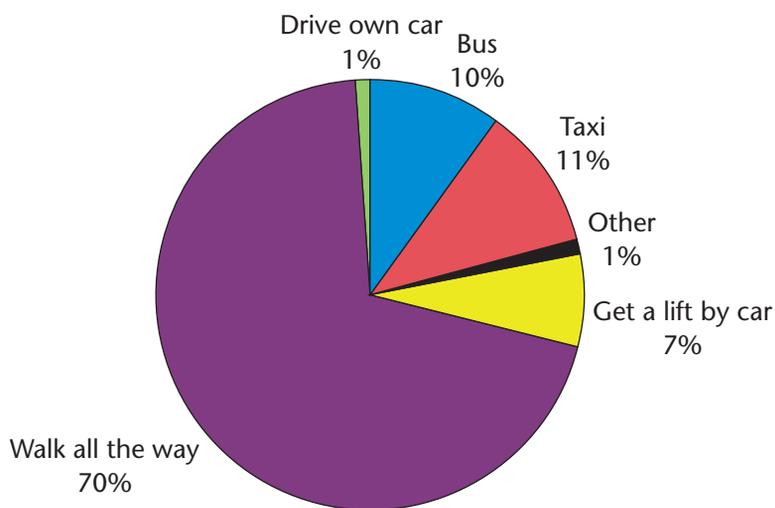
4. The double bar graph compares the percentages of learners who travel to school by taxi and by bus, and where they live.



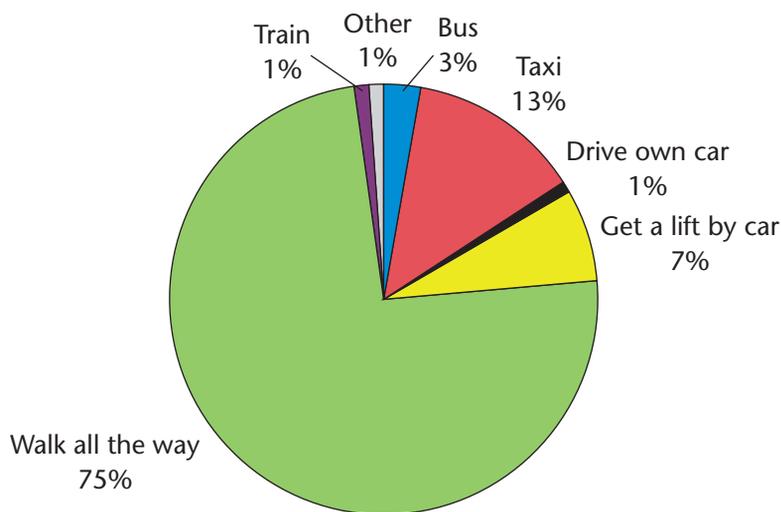
- In which provinces do more learners travel to school by taxi than by bus?
 - In which provinces do more learners travel to school by bus than by taxi?
 - In one province the number of learners who travel by taxi is about double the number of learners who travel by bus. Which province is this?
 - In one province the number of learners who travel by bus is about three times the number of learners who travel by taxi. Which province is this?
5. Look at the pie charts on the next page. Compare how learners in Mpumalanga travel to school to how learners in the Eastern Cape travel to school.
- In which province does a larger percentage of learners walk to school?
 - In which province does a larger percentage of learners travel to school by train? Why do you say so?

- (c) Compare the percentage of learners in the two provinces who travel by taxi. Is it about the same, or very different? Why do you say so?
- (d) Compare the percentage of learners in the two provinces who travel to school by bus. Is it about the same, or very different? Why do you say so?

How Mpumalanga's learners travel to school



How Eastern Cape's learners travel to school



9.4 Project

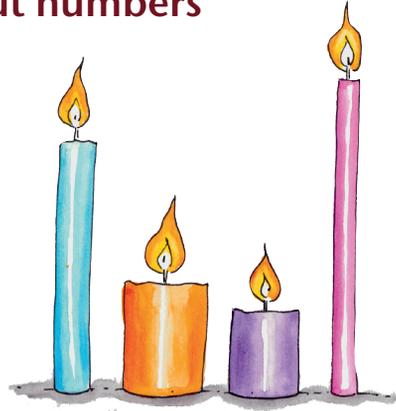
Work together with your classmates to answer the three questions below. Gather data from all learners in your school about the way they travel to school. Make sure you find out about all the types of transport.

- **Question 1:** Of all the learners in your school, who makes use of what kind of transport?
 - **Question 2:** Of all the learners that make use of a certain kind of transport (for example walking), who are in which grade?
 - **Question 3:** Ask your own question, for example: How long does it take to get to school with the different kinds of transport?
1. Think of the questions you have to ask to get the information. For example, for question 3 you may want to know how far the learners travel as well as the kind of transport they use. Or you may want to know at what time they leave their homes to be in time for school.
 2. Use your questions in point 1 and the kinds of transport you have learnt about earlier in this unit to make a questionnaire to gather data.
 3. Share the work between all the learners in your class. Decide who will gather the data from each grade. Each learner must only be interviewed once.
 4. Work together to tally the information in the questionnaires and calculate frequencies. For example, work with the questionnaires answered by Grade 1 learners and organise the data about the numbers of learners that use each kind of transport. Then do the same for each of the other grades. Think of other ways to make categories to organise the data.
 5. Draw bar graphs to show the information in the data.
 6. With each graph also write down the question you wanted to answer; then say how you read the graph to answer the question.
 7. Compare the information between grades to make conclusions, for example: Do Grade 1 to 3 learners tend to use different kinds of transport than Grade 6 and 7 learners? You can also compare the times at which learners using different kinds of transport leave for school in the morning.

10.1 Finding input and output numbers

A candle manufacturer claims that their new candles burn for at least 16 hours.

To test the claim, a Grade 6 Natural Sciences class did an experiment: they lit four different candles and measured their lengths every hour for four hours and then stopped. Here are their results:



Candle A

Time (hours)	0	1	2	3	4	5	10
Length (cm)	36	34	32	30	28		

Candle B

Time (hours)	0	1	2	3	4	5	10
Length (cm)	16	15	14	13	12		

Candle C

Time (hours)	0	1	2	3	4	5	10
Length (cm)	12	11,5	11	10,5	10		

Candle D

Time (hours)	0	1	2	3	4	5	10
Length (cm)		44	42	40	38		

1. The class did not continue after 4 hours. But if they did, can you say how long each candle was after 5 hours and after 10 hours? (Complete the tables.)

Explain and discuss your methods.

2. The class forgot to fill in the length of Candle D before they lit it. How long was it?
3. Which calculation plan (rule or formula) belongs with which table? How do you know?

Rule 1: $Length = 46 - 2 \times Time$

Rule 2: $Length = 16 - Time$

Rule 3: $Length = 12 - 0,5 \times Time$

Rule 4: $Length = 36 - 2 \times Time$

4. Use the rules in question 3 to calculate how long each of the candles will be after 12 hours and after 15 hours.
5. (a) After how many hours will Candle A be 10 cm long? Explain your method.
(b) After how many hours will each of the other candles be 10 cm long?
6. (a) Is the manufacturer's claim that all the candles will burn for more than 16 hours true? How do you know?
(b) How many hours will Candle A burn before it is burnt out?
(c) How many hours will each of the other candles last?
(d) Which candle will burn the longest? How long? Explain!
7. The manufacturer's newest "monster candle" is 48 cm long and burns at 3 cm per hour. Complete this table of the candle's length over time. How many hours will it burn before it is burnt out?

Time (hours)	0	1	2	3	4	10	15
Length (cm)							

10.2 Using patterns to solve problems

1. Mario sells small doughnuts at a stall in a shopping mall. He does not want to do calculations every time he sells some doughnuts. So he started to prepare the following table:

Number of doughnuts	1	2	3	4	5	6	7	8	9	10	100
Total cost (in cents)	25	50	75	100	125						

- (a) Complete Mario's table.
- (b) How much will 25 doughnuts cost?
- (c) Describe your rule for calculating the cost of any number of doughnuts.
- (d) How do you know that your rule is correct?
- (e) A customer pays Mario R5,50. How many doughnuts does she buy?



2. The Natural Sciences class measured the growth of a seedling over a two-week period. They recorded the following information:

Day number	0	2	4	6	8	10	12	14
Height (mm)	0	3	6	9	12	15	18	21

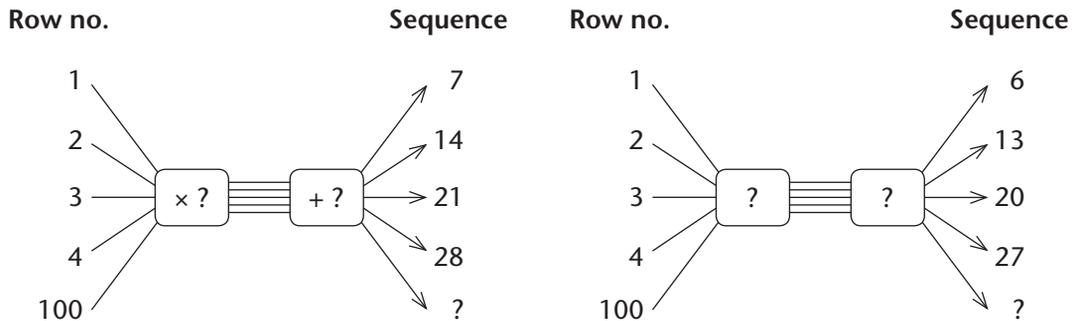
- (a) What was the daily growth of the seedling?
- (b) When was the seedling 10,5 mm high?
- (c) What was the height of the seedling after 11 days?
- (d) Explain how the age and the height of the seedling are related.
- (e) If the seedling continues to grow at the same rate, when will it be 60 mm high?
- (f) Do you think the seedling will continue to grow at this rate? Explain your answer.

10.3 From tables to rules

The whole numbers are arranged in columns like this.

	Column 1 ↓	Column 2 ↓	Column 3 ↓	Column 4 ↓	Column 5 ↓	Column 6 ↓	Column 7 ↓
Row 1 →	1	2	3	4	5	6	7
Row 2 →	8	9	10	11	12	13	14
Row 3 →	15	16	17	18	19	20	21
Row 4 →	22	23	24	25	26	27	28
	⋮	⋮	⋮	⋮	⋮	⋮	⋮

1. Discuss what patterns you see in the grid.
2. If the grid is continued downwards, what will Row 100 look like? Write it down.
3. In which Row and which Column is 256?
4. What are the calculation plans (rules) for Column 7 and Column 6? In other words, what rule will give these input and output values in these flow diagrams?

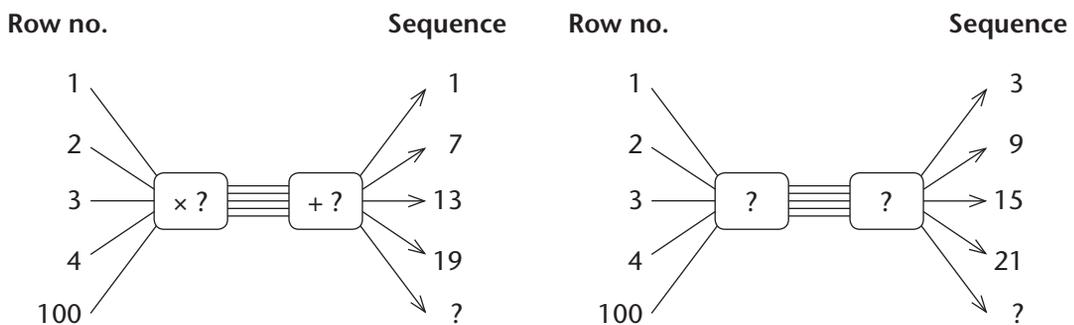


5. Write down rules for each of Columns 1 to 7.

6. Now study this arrangement of numbers.

	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
	↓	↓	↓	↓	↓	↓
Row 1 →	1	2	3	4	5	6
Row 2 →	7	8	9	10	11	12
Row 3 →	13	14	15	16	17	18
Row 4 →	19	20	21	22	23	24
	⋮	⋮	⋮	⋮	⋮	⋮

- (a) Discuss what patterns you see in the grid.
- (b) If the grid is continued downwards, what will Row 100 look like? Write it down.
- (c) In which Row and which Column is 256?
- (d) What are the calculation plans (rules) for Column 1 and Column 3? In other words, what rule will give these input and output values in these flow diagrams?



7. Write down rules for each of Columns 1 to 6.

10.4 Adding sequences

What happens if we *add* the numbers in two sequences? Let's investigate.

We will start by adding the 1st numbers in each of the two sequences, then the 2nd numbers, then the 3rd numbers and so on. The answers will give us a new sequence, with a new pattern.

Here is an example:

Sequence 1: 2, 4, 6, 8, 10, ...

Sequence 2: 3, 6, 9, 12, 15, ...

Sequence 1 + Sequence 2: 5, 10, 15, 20, 25, ...

1. It seems from the example above that if we add multiples of 2 and multiples of 3, the result is multiples of 5. Do you agree?
2. Investigate what happens if you add these sequences. In each case, continue the new sequence for another five numbers, and then calculate the 20th and 100th number in the new sequence.
 - (a) Sequence 1: 2, 4, 6, 8, 10, ...
Sequence 2: 4, 8, 12, 16, 20, ...
 - (b) Sequence 1: 3, 6, 9, 12, 15, ...
Sequence 2: 4, 8, 12, 16, 20, ...
 - (c) Sequence 1: 2, 4, 6, 8, 10, ...
Sequence 2: 5, 10, 15, 20, 25, ...
 - (d) Sequence 1: 4, 7, 10, 13, 16, ...
Sequence 2: 6, 11, 16, 21, 26, ...
3. Calculate the 20th and 100th number in the new sequence if you
 - (a) add the sequences of multiples of 3 and multiples of 8
 - (b) add the sequences of multiples of 4 and multiples of 7.

10.5 Multiplying sequences

Investigate what happens if we multiply the numbers in two sequences. Here is an example:

Sequence 1: 2, 4, 6, 8, ...
Sequence 2: 3, 6, 9, 12, ...
Sequence 1 \times Sequence 2: 6, 24, 54, 96, ...

1. In each case below, form a new sequence by multiplying the two sequences. Then continue the new sequence for another five numbers, and calculate the 20th and 100th number in the new sequence.

(a) Sequence 1: 2, 4, 6, 8, 10, ...

Sequence 2: 2, 4, 6, 8, 10, ...

(b) Sequence 1: 1, 2, 3, 4, 5, ...

Sequence 2: 2, 4, 6, 8, 10, ...

(c) Sequence 1: 2, 4, 6, 8, 10, ...

Sequence 2: 3, 6, 9, 12, 15, ...

(d) Sequence 1: 2, 4, 6, 8, 10, ...

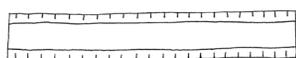
Sequence 2: 4, 8, 12, 16, 20, ...

(e) Sequence 1: 3, 6, 9, 12, 15, ...

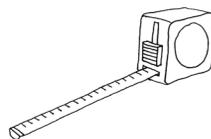
Sequence 2: 4, 8, 12, 16, 20, ...

2. Calculate the 20th and 100th number in the new sequence if you
 - (a) multiply the sequences of multiples of 2 and multiples of 5
 - (b) multiply the sequences of multiples of 4 and multiples of 5.

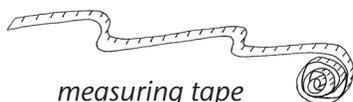
11.1 Estimate, measure, compare and order



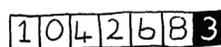
ruler



builder's tape measure



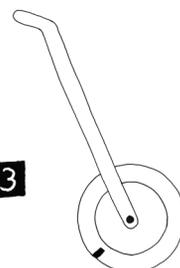
measuring tape



odometer



metre stick



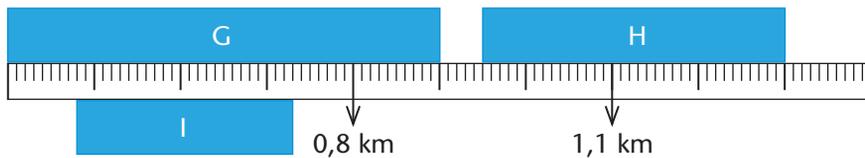
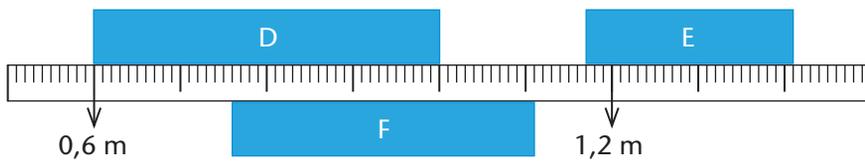
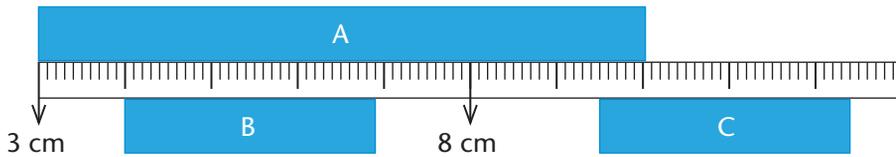
trundle wheel

1. There are different measuring instruments we can use, for example a measuring tape, a metre stick, a trundle wheel, a ruler, a builder's tape measure and an odometer (in a vehicle).

Write down which one of these measuring instruments you will use to measure each of the following, and why:

- (a) the width of a street
 - (b) the width of your chair's seat
 - (c) the width of your pinkie nail
 - (d) your height
 - (e) the width of a rugby field or soccer field
 - (f) the distance from the windowsill to the floor in your classroom
 - (g) the length of your table or desk in your classroom
 - (h) the thickness of your upper arm
 - (i) the distance between two towns
2. *Estimate* the lengths in questions 1(a) to (f). Write them down.
 3. *Measure* the lengths in questions 1(a) to (f). Write the measurements next to your estimates in question 2.
 4. How far out were your estimates? Compare your answers in questions 2 and 3 by subtracting the smaller measurement from the bigger one.

5. Can everything in question 1 be measured exactly with the measuring instrument you chose? Give a reason for your answer.
6. Below are rulers with strips above and below them.



- (a) Measure the lengths of the strips. If the measurement is not a whole number of units, give your answer as a fraction and as a decimal.
- (b) What is the total length of Strips A, B and C?
- (c) What is the total length of Strips D, E and F?
- (d) What is the total length of Strips G, H and I?
- (e) What is the total length of 10 Strip Cs? Give your answer in mm and cm.
- (f) What is the total length of 100 Strip Es? Give your answer in m.
- (g) What is the total length of 1 000 Strip Hs? Give your answer in km.

-
7. The world long jump record is held by Mike Powel, who jumped 8,95 m in 1991.
- (a) Do you think this is longer or shorter than the width of your classroom? Explain your answer.
 - (b) Measure this distance. What instrument did you use? Explain your choice.
 - (c) The South African long jump record is held by Khotso Mokoena, who jumped 8,50 m in 2009. How much shorter was Khotso's jump than Mike's jump?
 - (d) Do you think the difference in jump lengths is about the length of your exercise book, twice the length or 1,5 times the length of your exercise book? Explain your choice.
 - (e) Measure out this difference in jump lengths and compare it to the length of your exercise book.
 - (f) Jump as far as you can. First estimate and then measure how far you jumped.
 - (g) Make a graph that shows the lengths of all the jumps done by your class.
8. Ram Sing Chauchan holds the record for the world's longest mustache. His mustache is 4,29 m.
- (a) If you and your friends lay end to end, would 4,29 m be about the length of 2 friends, 3 friends, 4 friends or 5 friends? Explain your answer.
 - (b) Work in pairs. Show how far you estimate 4,29 m to be.
 - (c) Now measure the distance. What instrument did you use? Explain your choice.
9. Write in descending order.
- (a) 54,9 km; 45,09 km; 450,9 km
 - (b) 704,6 m; 76,04 m; 76,4 m
10. Write in ascending order.
- (a) 547,2 km; 72,54 km; 275,4 km
 - (b) 65,23 m; 653 m; 236,6 m

11.2 Write in different units

We have been working with kilometres, metres, centimetres and millimetres since Grade 4. In Grade 5 we saw that there were other metric units that we seldom use in everyday life:

Kilometre (km)	Hectometre (hm)	Decametre (dam)	Metre (m)	Decimetre (dm)	Centimetre (cm)	Millimetre (mm)
1	10	100	1 000	10 000	100 000	1 000 000

Units on the right are smaller units than units on the left, for example a millimetre is smaller than a centimetre. Each unit in the table is ten times the size of the unit on its right, for example $100\ 000\text{ cm} = 10 \times 100\ 000\text{ mm} = 1\ 000\ 000\text{ mm}$. Because the metric system is based on tens it is called a **decimal system** of measurement.

This table can make converting between units very easy. To convert between units, you can write the number you want to convert under the unit you are converting from. Mark the unit you are converting to.

If you are converting from a bigger unit to a smaller unit, then you *multiply by 10* each time as you move from column to column to a lower unit, for example:

- Write 25 km as m.
 $25\text{ km} \rightarrow 25 \times 10\text{ Hm} \rightarrow 25 \times 10 \times 10\text{ Dm} = 25 \times 10 \times 10 \times 10\text{ m} = 25\ 000\text{ m}$
- Write 3,25 m as mm.
 $3,25\text{ m} \rightarrow 3,25 \times 10\text{ dm} \rightarrow 3,25 \times 10 \times 10\text{ cm} = 3,25 \times 10 \times 10 \times 10\text{ mm} = 3\ 250\text{ mm}$

If you are converting from a smaller unit to a bigger unit, then you *divide by 10* each time you move from column to column to a higher unit, for example:

- Write 4 000 mm as m.
 $4\ 000\text{ mm} \rightarrow 4\ 000 \div 10\text{ cm} \rightarrow 4\ 000 \div 10 \div 10\text{ dm} = 4\ 000 \div 10 \div 10 \div 10\text{ m} = 4\text{ m}$
- Write 500 m as km.
 $500\text{ m} \rightarrow 500 \div 10\text{ Dm} \rightarrow 500 \div 10 \div 10\text{ Hm} = 500 \div 10 \div 10 \div 10\text{ km} = \frac{5}{10}\text{ km} = \frac{1}{2}\text{ km}$

-
- Convert the following lengths to the given units.
 - 1,2 m to cm
 - 13 478 mm to cm
 - $3\frac{1}{2}$ m to mm
 - 639,2 cm to mm
 - 4 593 cm to m
 - 4 071 mm to cm
 - Write 12,46 m in:
 - centimetres
 - millimetres
 - Write 8,87 km in:
 - metres
 - centimetres
 - Write 3 890 mm in:
 - centimetres
 - metres
 - Write 4 460 cm in:
 - millimetres
 - metres
 - Write 290,84 m in:
 - centimetres
 - millimetres
 - Write as kilometres. (When there is a fraction part in your answer use the common fraction form.)
 - 8 000 m
 - 3 500 m
 - 7 482 m
 - 100 m
 - Write each length as a combination of the units given. Look at the following example:

Write 1,54 m as m and cm and mm.

$$1,54 \text{ m} = 1 \text{ m and } 50 \text{ cm and } 4 \text{ mm}$$
 - 658 mm as cm and mm
 - 2,34 m as m and cm and mm
 - 45,6 cm as m and mm
 - Use the signs $>$, $=$, $<$ to compare these lengths and distances:
 - 500 m 0,05 km
 - 3,3 m 303 mm
 - 743 cm 7,45 mm
 - $\frac{7}{8}$ m 875 mm
 - 12,75 km $12\frac{3}{4}$ m
 - 549,5 cm 5 km

11.3 Calculations

1. Calculate.

(a) $10\frac{1}{3}$ km $-$ $2\frac{5}{6}$ km

(b) $7\frac{3}{8}$ cm $+$ $2\frac{3}{4}$ cm $-$ $\frac{1}{2}$ cm

(c) $3\frac{7}{12}$ m $+$ $4\frac{5}{6}$ m $-$ $\frac{1}{3}$ m

(d) $5\frac{1}{4}$ km $+$ $2\frac{1}{2}$ km $-$ $\frac{7}{8}$ km

2. Complete by writing the sign of operation and the missing length in order to get the required length.

Examples: 26 m $\boxed{+}$ 24 m = 50 m 7,8 m $\boxed{-}$ 2,4 m = 5,4 m

(a) 48 mm $\boxed{\quad}$ = 70 mm

(b) 78 cm $\boxed{\quad}$ = 1 m

(c) 884 mm $\boxed{\quad}$ = 90 cm

(d) 5 641,02 m $\boxed{\quad}$ = 10 km

(e) $13\frac{1}{2}$ km $\boxed{\quad}$ = 11 000 m

(f) 1 764 cm $\boxed{\quad}$ = 15 m

3. Musi measured the length and width of a soccer field and then made a drawing of the soccer field. He decided to use 1 mm on his ruler to represent 3 m on the soccer field. If his measurement of the width of the soccer field was 63 m, what is the width of his drawing?
4. Jabes travelled 3 406 km in 8 days. If he travelled the same distance every day, how many kilometres did he travel per day?
5. Nomsa cuts a piece of fabric which is $3\frac{5}{8}$ m long from a roll which has $32\frac{1}{4}$ m fabric on it. What length of fabric is left on the roll?
6. (a) How many one and a half metre pieces of rope can be cut from a 20 m roll?
- (b) How much rope will be left over?

11.4 Rounding off

In everyday life, we often round off measurements. Distances are often rounded to the nearest kilometre. If we buy material for sewing it is usually to the nearest metre. A carpenter can ask for planks to be cut to the nearest centimetre.

When we work with lengths, it is convenient to work with full units (km, m, cm, mm).

If a length is given in a smaller unit, we often round it off to a bigger unit. If you round off to the nearest 100 cm, it is the same as rounding off to the nearest metre and so on.

28 mm to the nearest centimetre is 3 cm, because 28 mm is $2\frac{8}{10}$ cm. It works like this: when you have a fraction that is less than half, you ignore it. If, however, your fraction is half or more than half, you add one whole. We say 54 cm is more than half a metre and should be rounded up to 1 m.

For tenths:

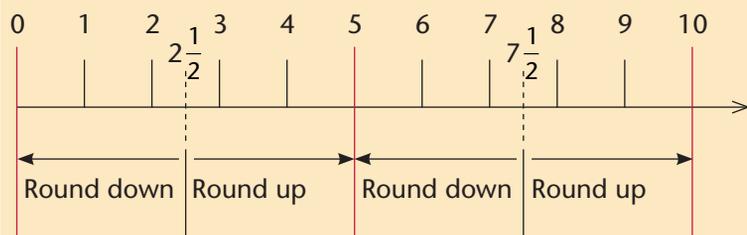


Here are some more examples:

$7\frac{4}{8}$ is rounded up to 8 and $7\frac{2}{5}$ is rounded down to 7.

We can also round off to other numbers, for example to the nearest 5.

The number 5 and its multiples then become your base:



Rounded to the nearest 5:

8 becomes 10; 42 becomes 40; 64 becomes 65; 67 becomes 65.

-
- Round off each length to the nearest centimetre.
 - 8,2 cm
 - 3,6 cm
 - 45 mm
 - $78\frac{3}{4}$ cm
 - Do this calculation mentally: $4\frac{7}{8}$ m + 36,34 m + 27 m.
 - How did you make the calculation easier? Write down in your own words what you did.
 - Join one or two classmates and discuss what you did to do the calculation mentally.
 - Refloee changed the calculation in question 2(a) to this:
 5 m + 36,5 m + 27 m
How far was her answer from the exact answer to question 2(a)?
 - When is it sensible to round off lengths and when not? Discuss this with some of your classmates.

When you buy material such as fabric, rope, wire or planks, you will usually *round up* instead of down to the nearest multiple of 5, 10, 100 or 1 000 of the measuring unit. For example, if you need 73 cm ribbon, you will not round 73 cm off to 70 cm; you will round it up to 75 cm or to 80 cm – otherwise you will buy too little.

- Round the following measurements up or down to the nearest whole number.
 - $16\frac{6}{10}$ cm
 - 9 422,48 mm
 - $220\frac{1}{4}$ cm
 - 1 329,93 km
 - $209\frac{4}{6}$ mm
 - $999\frac{4}{8}$ cm
- Round off to the nearest 5 of the given unit.
 - 36 mm
 - 43,6 cm
 - 22,5 m
 - $25\frac{1}{2}$ m
 - 599 mm
 - $12\frac{2}{8}$ km

-
7. Calculate.
- (a) Calculate in centimetres and round off your answer to the nearest 100 mm:
 $6\text{ m} + 157\text{ cm} - 1\,145\text{ mm}$
- (b) Calculate in millimetres and round off your answer to the nearest 1 000 mm:
 $23 \times (1\,380\text{ mm} - 78\text{ cm})$
- (c) Calculate in metres and round off your answer to the nearest kilometre:
 $5,4\text{ km} - 204\text{ m} \times 14$
8. The circumference of the Earth around the equator is 40 075,16 km. At the poles it is 67,16 km shorter.
- (a) What is the circumference of the Earth at the poles?
- (b) Round off both circumferences to the nearest 100 km.

11.5 Problem solving

1. Selina and Zinzi each have to make 15 aprons for the bazaar. They need 73 cm of material per apron.
- (a) Selina decides to round off the amount of material per apron to the nearest 10 cm. How much material does she buy?
- (b) Zinzi rounds off the amount of material to the nearest 5 cm. How much material does she buy?
- (c) Who has too much material and who has too little?
- (d) By how much is the material too much or too little?
- (e) What did you learn from this about rounding off?
2. David measured the distance between two trees with a stick. He found that the distance was $12\frac{1}{3}$ sticks. When he measured the stick with his ruler, it was 78 cm long. What was the distance between the two trees? Give your answer in centimetres and metres.
3. How many 85 cm lengths can be cut from a roll of material that is 16 m long?

-
4. A soccer field is 101 m long and 69 m wide. The soccer team has to run around the field 12 times during their practice. How far do they have to run?
 5. Heinrich had to make 7 trips from East London to Cape Town and back. The distance between the two cities is 756 km. What is the total distance that he travelled?
 6. The governing body of the school decides to build a fence in front of the tuck shop. The total length of the fence is 15 m. The builders plant the poles 150 cm apart.

They plant each pole so that 25 cm is under the ground and 1 m above the ground.

- (a) How many poles do they plant? (Hint: Make a sketch first!)
 - (b) How many long poles of 3 m each do they need to saw enough poles? (Hint: How long is one pole? Remember they cannot be joined together.)
 - (c) What percentage of each pole is under the ground?
 - (d) Three lengths of wire are strung between every two of the poles. How many metres of wire do they need? Allow for an extra 50 cm per wire.
7. What is the perimeter of a rectangle with length 2,4 cm and width 17 mm?
 8. If an aircraft flies 14 km in 1 min, how many minutes will it take for it to fly 413 km?
 9. If Chuck cycled 147 km in 14 days, how many kilometres did he cycle in 5 days if he cycled the same distance every day?
 10. A car travels at a speed of 105 km per hour. How far will it travel in 5 hours if it travels more or less at the same speed all the time?

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1.1 Represent, order and compare big numbers

- Write the number symbol for each number.
 - three hundred million and five thousand
 - three hundred million and five hundred thousand
 - three hundred million and fifty thousand
 - three hundred million and five hundred
- Write the number name for each number.
 - 700 400 030
 - 700 040 300
 - 700 004 030
 - 700 043 000
 - 704 000 030
 - 700 004 300
- Round each of the numbers in question 2 off to the nearest
 - hundred
 - million
 - thousand
 - ten thousand
 - hundred thousand.
- Round each number off to the nearest 5, and to the nearest 10.

(a) 27	(b) 124
(c) 309	(d) 796
- Write the number symbol for each number.
 - two hundred and three million five hundred and seventy-nine thousand one hundred and seventeen

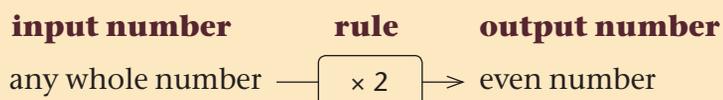
- (b) five hundred and seventy-eight million one hundred and twenty-three thousand four hundred and sixty-seven
- (c) ninety-eight million fifty thousand six hundred and eighteen
- (d) nine million eight hundred and seventy-six thousand five hundred and forty-three
- (e) nine hundred and seven million seven hundred and seventeen thousand and fourteen
6. Now rewrite the number symbols you wrote in question 5 in descending order (from highest to lowest).
7. In each case, write =, > or < between the two numbers.
- (a) 3 492 897 and 3 940 289
- (b) 6 374 294 and 6 374 294
- (c) 102 901 890 and 201 899 013
- (d) 1 000 010 and 1 000 010

1.2 Investigate even, odd and prime numbers

For *any* whole number as input number, the output number of this flow diagram is an *odd* number:



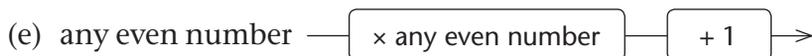
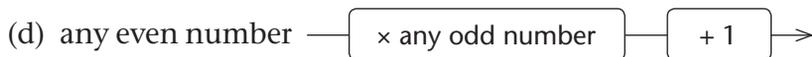
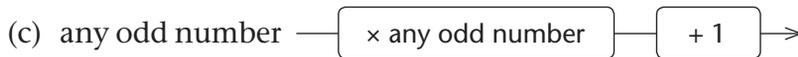
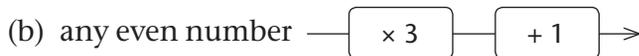
For *any* whole number as input number, the output number of this flow diagram is an *even* number:



1. Use the flow diagrams above to make 5 odd numbers and 5 even numbers.

2. For each flow diagram below, investigate whether the output numbers of the flow diagram will be
- odd numbers in all cases or
 - even numbers in all cases or
 - odd numbers in some cases, even numbers in other cases.

Give examples to support your answers.



3. In each case, investigate whether the statement is true or false. Give examples to demonstrate your answers.

- An odd number times an odd number is always an odd number.
 - An even number times an even number is always an even number.
 - An even number times an odd number is always an odd number.
 - Any multiple of an even number is even.
 - Any multiple of an odd number is odd.
- How many of all the multiples of 3, smaller than 1 000, are odd numbers?
 - How many of all the multiples of 7 are odd numbers?
 - Write all the prime numbers bigger than 60 but smaller than 70.
 - Write all the prime numbers bigger than 40 but smaller than 50.
 - Investigate whether the statement below is true. Then write a paragraph that will convince the reader that what you say is true.
If an odd number smaller than 100 is not a prime number, it is a multiple of 3 or 5 or 7.

5. This is how Cassius explains his method when calculating $4 \times 2\,367$:

Step 1: $2\,367$ is $2\,000 + 300 + 60 + 7$

Step 2: Therefore $4 \times 2\,367 = 4 \times (2\,000 + 300 + 60 + 7)$

Step 3: This is the same as $4 \times 2\,000 + 4 \times 300 + 4 \times 60 + 4 \times 7$

Step 4: And this is $8\,000 + 1\,200 + 240 + 28 = 9\,468$

Explain why Cassius can say “This is the same as...” in Step 3.

2.2 A shorter way of setting out multiplication

1. Calculate 578×43 .

It is possible to do calculations like the one you did in question 1 by writing one column only. Here is an example.

Writing without columns

$$\begin{aligned}
 &284 \times 378 \\
 &= 200 \times 378 + 80 \times 378 + 4 \times 378 \\
 &= 4 \times 378 + 80 \times 378 + 200 \times 378 \\
 &= 4 \times 8 + 4 \times 70 + 4 \times 300 \\
 &\quad + 80 \times 8 + 80 \times 70 + 80 \times 300 \\
 &\quad + 200 \times 8 + 200 \times 70 + 200 \times 300 \\
 &= 32 + 280 + 1\,200 \\
 &\quad + 640 + 5\,600 + 24\,000 \\
 &\quad + 1\,600 + 14\,000 + 60\,000 \\
 &= 107\,352
 \end{aligned}$$

Writing in one column

378	Reason:	
× 284		
32	(4 × 8)	
280	(4 × 70)	
1 200		
640		
5 600		
24 000		
1 600		
14 000		
60 000		
107 352		

Writing in several columns

378	378	378	1 512
× 4	× 80	× 200	+ 30 240
32	640	1 600	+ 75 600
280	5 600	14 000	107 352
1 200	24 000	60 000	
1 512	30 240	75 600	

2. Rewrite the example for writing in one column on the previous page, and include all the reasons for the part answers.
3. Try to set out your work for the following calculations in one column, in the way shown on the previous page.

(a) 238×69	(b) 564×382
(c) $5\,639 \times 94$	(d) $7\,694 \times 268$

2.3 An even shorter way to set out your work

On the left below, you can see the calculation for 378×284 as it was shown on the previous page. On the right you can see how you can set out the work more briefly.

Calculation of 378×284 :

$ \begin{array}{r} 378 \\ \times 284 \\ \hline 32 \\ 280 \\ 1\,200 \\ 640 \\ 5\,600 \\ 24\,000 \\ 1\,600 \\ 14\,000 \\ + 60\,000 \\ \hline 107\,352 \end{array} $	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%;">378</td> <td style="width: 50%;"></td> </tr> <tr> <td>$\times 284$</td> <td></td> </tr> <tr> <td>$\hline 1\,512$</td> <td>(4×378)</td> </tr> <tr> <td>$30\,240$</td> <td>(80×378)</td> </tr> <tr> <td>$+ 75\,600$</td> <td>(200×378)</td> </tr> <tr> <td>$\hline 107\,352$</td> <td></td> </tr> </table>	378		$\times 284$		$\hline 1\,512$	(4×378)	$30\,240$	(80×378)	$+ 75\,600$	(200×378)	$\hline 107\,352$	
378													
$\times 284$													
$\hline 1\,512$	(4×378)												
$30\,240$	(80×378)												
$+ 75\,600$	(200×378)												
$\hline 107\,352$													

1. Try to write as little as possible when you do the following calculations.

(a) 23×76	(b) 38×53	(c) 457×46
(d) 583×454	(e) $3\,856 \times 267$	(f) $2\,638 \times 387$
2. Compare your work with the work of two of your classmates. Correct your work if necessary.

4. Last summer, The Little Corner Shop ordered 2 453 boxes of fruit juice. In each large box there were 144 small boxes of 125 ml fruit juice each. How many small boxes of fruit juice were ordered?
5. A small national airliner made 1 273 trips last year. If it carried 167 passengers each time, how many people made use of this airliner last year?
6. A T-shirt factory produces 2 745 T-shirts a day. If the factory has a five-day work week, how many T-shirts are manufactured in 46 weeks?
7. Hendrik's fishing licence allows him to catch 1 255 kg of fish every day. How many kilograms of fish is he allowed to catch in 124 days?
8. A technician has worked 4 838 hours on a building project. His rate of pay is R286 per hour. How much should he be paid in total?
9. A shop bought 697 printers at R1 090 each and sold them for R2 394 each. Calculate the difference between what they paid in total and what they received.
10. A dairy farmer sells his 437 Jersey cows at R6 378 each. What is the total value of this transaction?



2.5 Use your calculator. But check the answer!

It is easy to get answers using the calculator. But we easily make mistakes. So you should always check your calculator answers.

$$134 \times 327 = ?$$

134×327 is bigger than $100 \times 300 = 30\,000$

134×327 is smaller than $200 \times 400 = 80\,000$

1. In each case, first estimate the answer. Use the strategy shown at the bottom of the previous page to find useful smaller and bigger possible answers. Then calculate the answer using your calculator, and decide if your answer looks about right.

(a) 134×327

(b) $345\ 136 \times 88$

(c) 444×666

(d) 578×34

(e) $43\ 545 \times 213$

(f) $6\ 252 \times 82 \times 63$

$27 \times 56 \div 18$ and $27 \div 18 \times 56$ are **equivalent** – they have the same answer.

2. Calculate each of the following using your calculator. Then use your calculator to check the answer by doing the calculations in a different (but equivalent) order.

(a) $4\ 513 \times 878 \times 332$

(b) $4\ 513 \times 187 \times 86$

(c) $5\ 435 \times 252 \div 315$

(d) $66\ 444 \div 678 \div 49$

(e) $1\ 543 \times 768 \times 32$

(f) $154 \times 768 \div 528$

$8\ 137 \times 328 \div 328 = 8\ 137$

3. Calculate each of the following using your calculator. Then check the result by using inverse operations.

(a) $8\ 137 \times 328$

(b) $345\ 136 \div 88$

(c) $3\ 105 \times 654 \div 345$

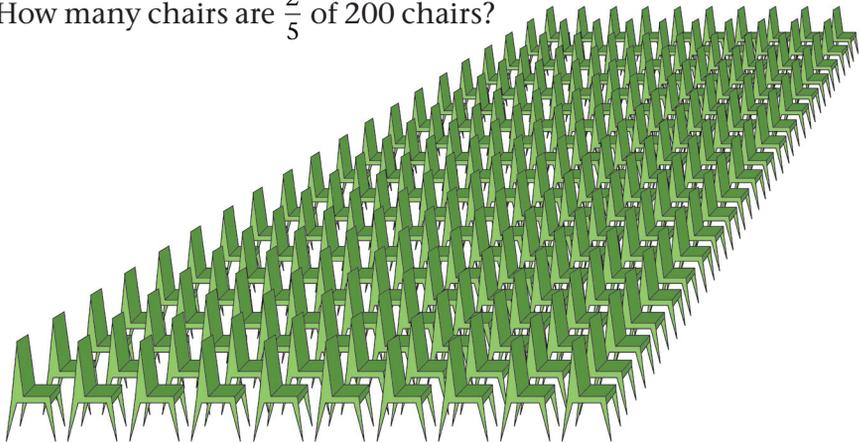
(d) $4\ 321 \div 125 \times 625$

(e) $2\ 805 \times 784 \times 43$

(f) $12\ 342 \div 121 \div 6$

3.1 Fractions of collections

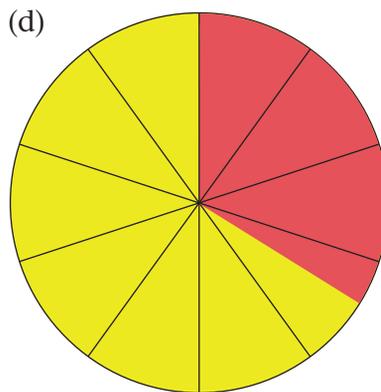
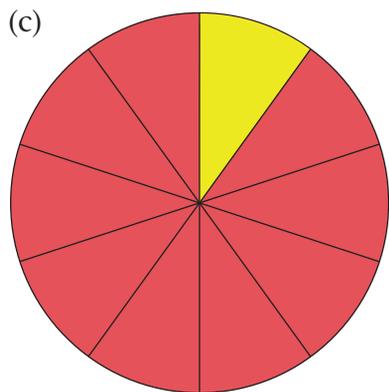
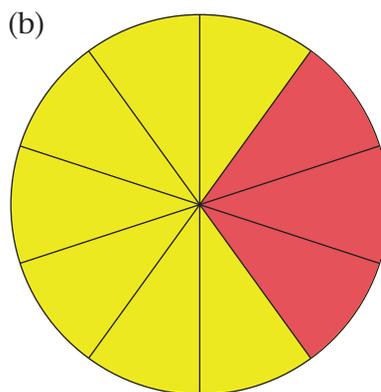
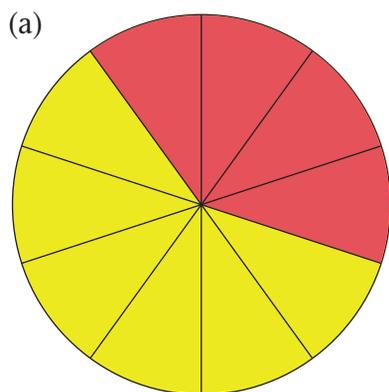
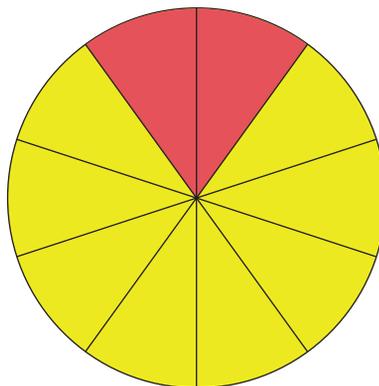
- Calculate $25 + 25 + 25 + 25 + 25 + 25 + 25 + 25 + 25 + 25$.
- How much is each of the following?
 - $\frac{1}{10}$ of 250
 - $2\frac{1}{2}$ hundreds
 - $\frac{1}{100}$ of 250
 - $\frac{10}{100}$ of 250
 - $\frac{4}{100}$ of 250
 - $\frac{1}{25}$ of 250
- Calculate:
 - $200 \div 10$
 - $200 \div 5$
 - $200 \div 20$
 - $200 \div 40$
- 200 chairs must be put on a sports field for a meeting. The task of bringing the chairs is shared equally by 10 people, so each person must bring $\frac{1}{10}$ of the 200 chairs.
 - How many chairs are $\frac{1}{10}$ of 200 chairs?
 - How many chairs are $\frac{1}{20}$ of 200 chairs?
 - How many chairs are $\frac{4}{10}$ of 200 chairs?
 - How many chairs are $\frac{2}{5}$ of 200 chairs?



3.2 Writing the same number in different forms

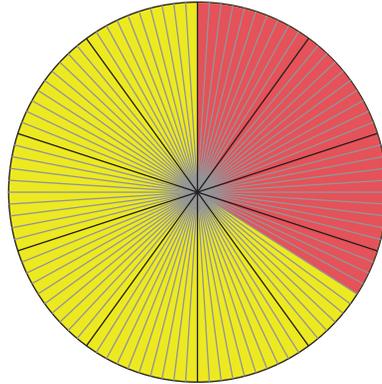
Two tenths of the circle on the right are coloured red. We can also say 20 hundredths or one fifth of the circle is red.

1. What part of this circle is yellow?
2. What part of each circle below is coloured red? Write each of your answers in two different ways. For the circle in question (d), you can only give an approximation.



This is the same circle as in question 1(d).

- Can you now say what part of this circle is coloured red?
- Is it correct to say that 17 fiftieths of this circle is red?
- Which of the following are correct ways of stating what part of this circle is red?



- | | |
|---|------------------------------------|
| (a) $\frac{30}{100} + \frac{4}{100}$ | (b) 3,4 |
| (c) $\frac{34}{100}$ | (d) 34% |
| (e) 0,34 | (f) $\frac{3}{10} + \frac{4}{100}$ |
| (g) $\frac{2}{10} + \frac{14}{100}$ | (h) 2,14 |
| (i) $\frac{1}{5} + \frac{7}{50}$ | (j) 0,17 |
| (k) $\frac{1}{5} + \frac{1}{10} + \frac{1}{25}$ | (l) $\frac{17}{50}$ |

A fraction, for example 34 hundredths, can be expressed in three different ways:

In **common fraction notation**:

$$34 \text{ hundredths} = \frac{34}{100}$$

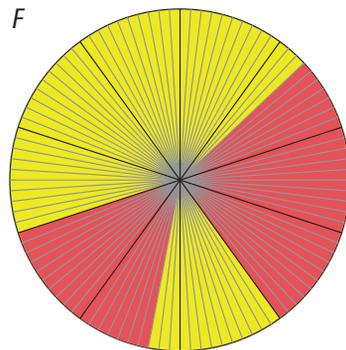
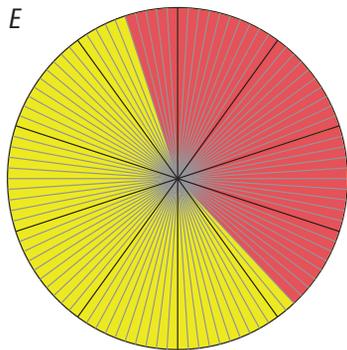
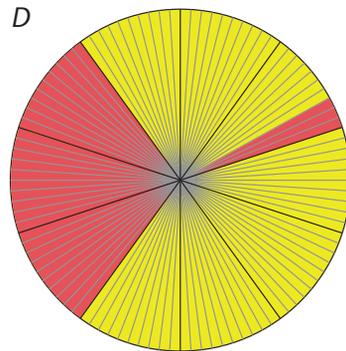
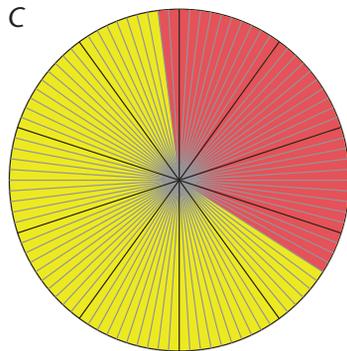
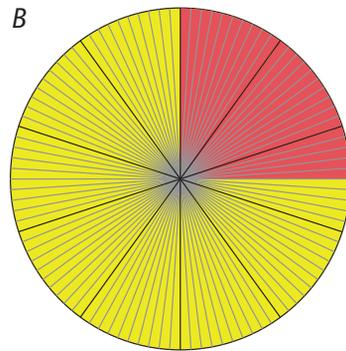
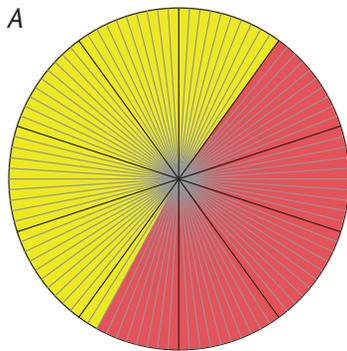
In **percentage notation**:

$$34 \text{ hundredths} = 34\%$$

In **decimal notation**:

$$\begin{aligned} & 34 \text{ hundredths} \\ &= \frac{3}{10} + \frac{4}{100} = 0,34 \end{aligned}$$

6. For each of Circles A to F below, state what part of the circle is red. Do this in five different ways:
- as a sum of tenths and hundredths
 - as hundredths
 - as a decimal
 - as a percentage
 - in one other way



3.3 Equivalent fractions

When you have to add fractions you often have to replace a fraction with an equivalent fraction.

For example, if you have to calculate $\frac{2}{5} + \frac{7}{20}$, you have to replace $\frac{2}{5}$ with $\frac{8}{20}$ so that you have two fractions with the same denominator:

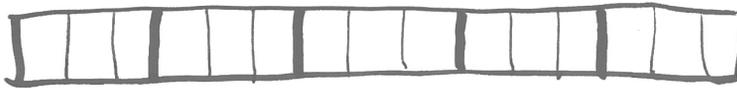
$$\frac{2}{5} + \frac{7}{20} = \frac{8}{20} + \frac{7}{20} = \frac{15}{20}$$

You can draw fraction strips to find equivalent fractions, as described in the activities below.

- (a) To find fractions that are equivalent to $\frac{3}{5}$, you can start by drawing a fraction strip that shows fifths. You need not do this accurately. Do not use a ruler so that you can work quickly.



- (b) Divide each fifth on your strip into three approximately equal parts:



- (c) What fraction of the whole strip is each of the smaller parts you have just drawn?
 - (d) How many of these smaller parts are in three fifths of the whole strip?
 - (e) How many fifteenths are equivalent to 3 fifths?
- (a) Draw three more strips that show fifths, like you did in question 1(a).
 - (b) Use one of your strips to show that $\frac{6}{10} = \frac{3}{5}$.
 - (c) Use one of your strips to show that $\frac{12}{20} = \frac{3}{5}$.
 - (d) Use one of your strips to show that $\frac{24}{40} = \frac{3}{5}$.

3. Write three equivalent fractions for each of the fractions below. You may draw fraction strips to support your thinking.

(a) $\frac{3}{8}$ (b) $\frac{3}{10}$ (c) $\frac{5}{12}$ (d) $\frac{2}{7}$
(e) $\frac{2}{6}$ (f) $\frac{4}{9}$ (g) $\frac{8}{20}$ (h) $\frac{6}{8}$

4. (a) Draw a fraction strip that shows eighths, across the full width of a page.



- (b) Divide each eighth into equal smaller parts so that the strip shows fortieths.
(c) Into how many equal parts did you have to divide each eighth of the strip to get fortieths?
(d) Into how many equal parts would you have to divide each eighth of the strip to get twenty-fourths?
5. A rectangular strip is divided into sixths. Into how many equal parts do you have to divide each sixth to get
(a) eightieths? (b) thirtieths?

3.4 Practice

1. Write down the next three numbers in the sequence as decimals.

(a) 0,4; 0,8; 1,2; ... (b) 0,92; 0,94; 0,96; ...
(c) 1,13; 1,12; 1,11; ... (d) 22,27; 22,28; 22,29; ...
(e) 1,6; 0,8; 0,4; ...

2. Arrange in ascending order (from smallest to biggest).

(a) $\frac{1}{4}$; $\frac{7}{10}$; 0,5; 40%; $\frac{3}{5}$; 72%; $\frac{9 \times 7}{100}$; 0,07
(b) $2 + \frac{1}{10} + \frac{37}{100}$; $1 + \frac{13}{10} + \frac{17}{100}$; $2 + \frac{4}{10} + \frac{7}{100}$; $1 + \frac{14}{10} + \frac{7}{100}$

3. Calculate.

(a) $1 - \frac{1}{100}$

(c) $1 - \frac{9}{100}$

(e) $1 - \frac{99}{100}$

(g) $0,99 + 0,02$

(i) $0,1 + 3,9$

(k) $12,83 - 0,1$

(b) $\frac{99}{100} + \frac{3}{100}$

(d) $\frac{99}{100} + \frac{1}{10}$

(f) $1 - \frac{9}{10}$

(h) $1,95 + 0,1$

(j) $1,06 - 0,1$

(l) $17 - 0,01$

4. Calculate.

(a) $2\frac{3}{5} + 1\frac{4}{5}$

(c) $3\frac{7}{8} + \frac{1}{4} + 2\frac{3}{8}$

(e) $\frac{3}{5} + \frac{7}{10}$

(b) $2\frac{3}{5} - 1\frac{4}{5}$

(d) $1\frac{3}{10} - \frac{4}{5}$

(f) $\frac{7}{9} - \frac{1}{3}$

5. Copy this table and complete it.

Tenths and hundredths in words	Hundredths in words	Tenths and hundredths in fraction notation	Two equivalent fractions	Decimal fraction	%
3 tenths and 2 hundredths	32 hundredths	$\frac{3}{10} + \frac{2}{100}$	$\frac{32}{100}; \frac{16}{50}$		32%
					75%
				0,45	
	6 hundredths				
				0,60	
		$\frac{7}{10} + \frac{8}{100}$			
	66 hundredths				

6. Calculate.

(a) $4,25 + 0,1$

(b) $15,83 - 0,1$

(c) $0,1 + 0,09$

(d) $0,1 + 4,9$

(e) $0,1 + 5,98$

(f) $1,04 - 0,1$

3.5 Using fractions to compare quantities

1. Mrs Daku is cooking jam. She first cooks the syrup separately and then she cooks the fruit in the syrup to make the jam. Each type of jam gets a syrup with its own recipe. Some types of jam need more sugar in the syrup than others. In Mrs Daku's recipe book is this table to guide her:

	Water	Sugar
Type A:	2 cups	2 cups
Type B:	3 cups	2 cups
Type C:	4 cups	2 cups

- Which type of syrup will be the sweetest?
- Why did you choose this type as the sweetest?
- For the syrup of a Type B jam, Mrs Daku uses 2 cups of sugar for every 3 cups of water.

If she wants to make syrup with 9 cups of water, how many cups of sugar should she add to the syrup?

2. Mrs Bester uses 2 cups of sugar for every 3 cups of water for syrup.

Say whether each of the following is true or false:

- $\frac{2}{3}$ of the syrup consists of sugar.
- $\frac{2}{5}$ of the syrup consists of sugar.
- There is $\frac{2}{3}$ as much sugar as water.
- There is $1\frac{1}{2}$ times as much water as sugar.

3. For every 3 steps that Jody takes, his father takes 1 step.
- (a) What fraction of his father's step is Jody's step?
- (b) Make a copy of this number line. Measure the distance so that the dots are spaced equally.



Draw two lines to show Jody's step compared to his father's step.

- (c) Compare the father's step to Jody's step by completing this sentence:

The father's step is _____ times Jody's step.

- (d) Compare Jody's step to his father's step by completing this sentence:

Jody's step is _____ of his father's step.



4. There are 60 minutes in one hour.
- (a) Copy the tables and complete them.

Hour	Minutes	Hour	Minutes	Hour	Minutes
$\frac{1}{3}$		$\frac{2}{10}$		$\frac{1}{5} + \frac{1}{2}$	
$\frac{2}{3}$		$\frac{4}{6}$		$\frac{7}{10}$	
$\frac{1}{5}$		$\frac{2}{5}$		$\frac{8}{10}$	
$\frac{1}{6}$		$\frac{5}{6}$		$\frac{1}{3} + \frac{1}{2}$	
$\frac{1}{10}$		$\frac{4}{10}$		$\frac{4}{5}$	

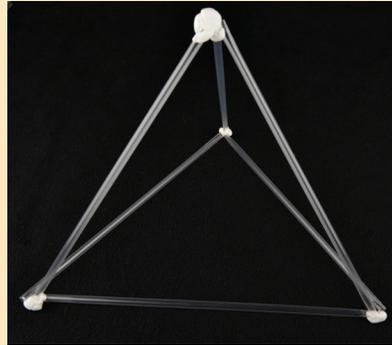
- (b) Examine your tables. Explain where you see that different fractions of an hour have the same number of minutes.

4.1 Skeleton models of 3-D objects

We can build skeleton models of 3-D objects using drinking straws or sticks for the edges, and clay for the vertices.

The picture on the right shows the skeleton of a **tetrahedron**. It has six edges, all the same length. It has four vertices and four triangular faces.

Some more skeleton models are shown below.

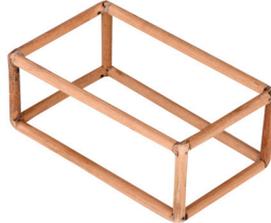


1. How many faces, edges and vertices does each of these 3-D objects have?

(a)



(b)



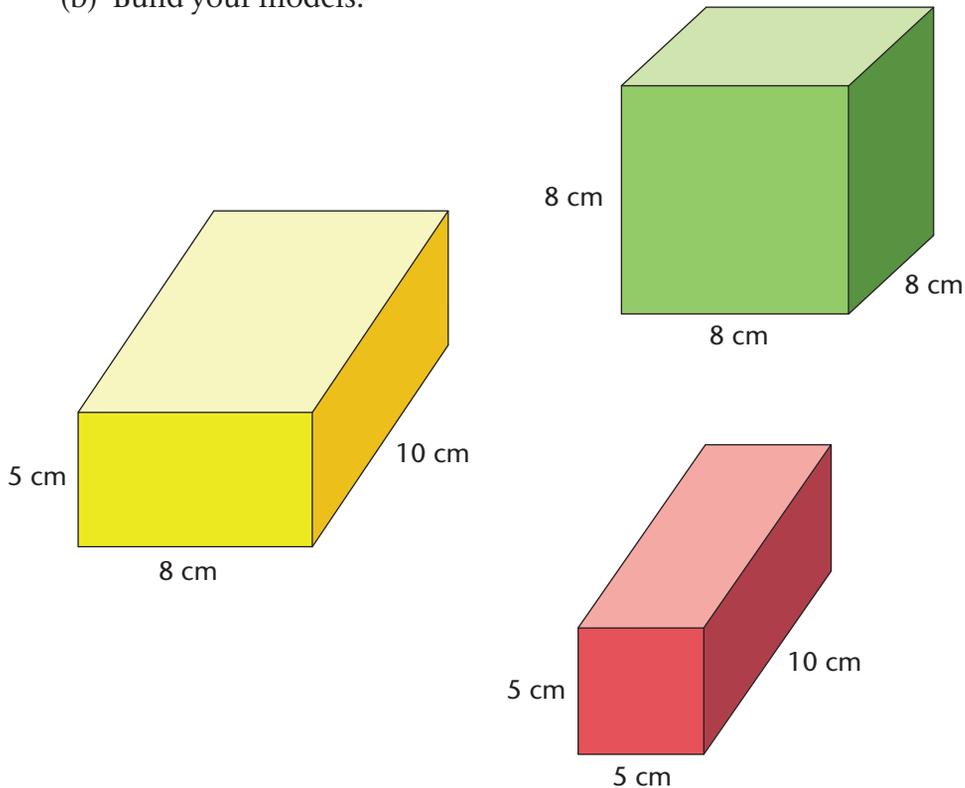
(c)



(d)



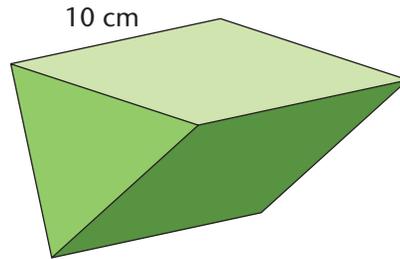
2. (a) Prepare to build skeleton models of the rectangular prisms below. For each rectangular prism, state how many straws or sticks of certain lengths you will need, and how many pieces of clay you will need to join the straws.
- (b) Build your models.



A rectangular prism with six square faces is called a **cube**. All the edges of a cube are the same length.

3. (a) How many faces, how many edges and how many vertices does each of your three skeleton models have?
- (b) Think of other rectangular prisms. How many edges, how many faces and how many vertices do they have?
- (c) Is it true that all rectangular prisms have 6 faces, 8 vertices and 12 edges?

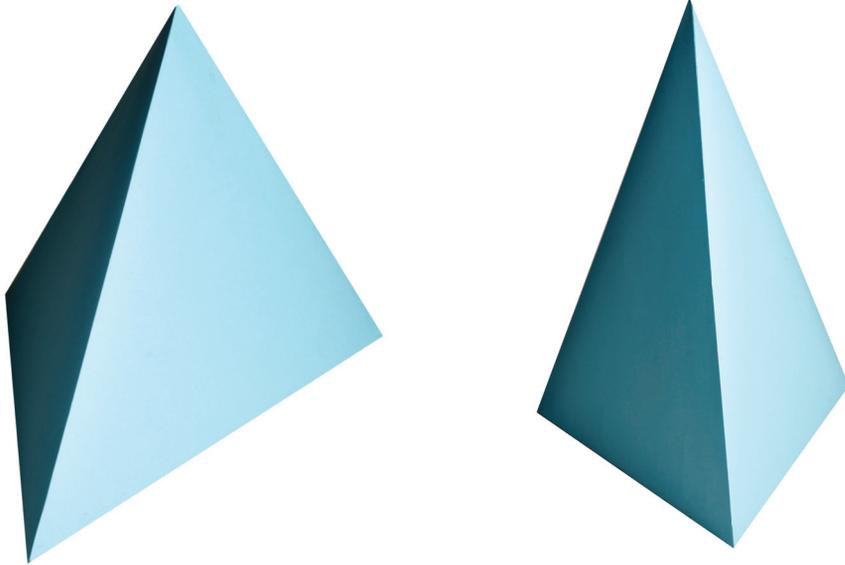
4. The sides of the triangular faces of this prism are all 5 cm long.



- (a) How many 5 cm long straws do you need to build a skeleton model of the prism?
- (b) How many 10 cm long straws do you need?
- (c) How many pieces of clay do you need to join the straws?
- (d) Build a skeleton model of this triangular prism.
- (e) How many rectangular faces does this prism have?
- (f) How many vertices does this prism have?
5. Decide how many straws of each length you need to build skeleton models of the following prisms. Build each prism.
- (a) a prism with two pentagonal faces: all the edges of the pentagonal faces are 5 cm long; all the other edges are 10 cm long
- (b) a cube: each edge is 5 cm long
6. Decide how many straws of each length you need to build skeleton models of the following pyramids. Build each pyramid.
- (a) a tetrahedron: the edges are all 5 cm long
- (b) a pentagonal pyramid: the sides of the base are all 3 cm long; all the other edges are 8 cm long
- (c) a square pyramid: the sides of each triangular face are 8 cm, 8 cm and 5 cm long
7. (a) Is a tetrahedron a triangular pyramid?
- (b) How does a tetrahedron differ from other triangular pyramids?

4.2 Drawings and pictures of pyramids

1. These are pictures of triangular pyramids.



- How many faces of each pyramid can you not see in the picture?
- Are the hidden faces also triangles?
- How many edges of each pyramid can you not see in the picture?

2. This is a picture of a square pyramid.

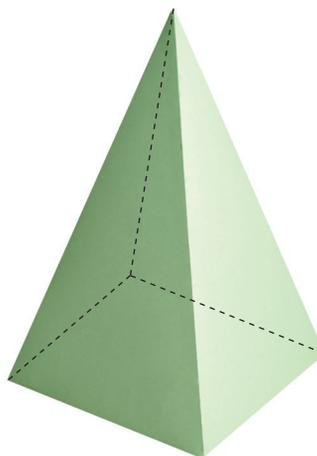
- How many faces can you not see in the picture?
- What are the shapes of the hidden faces?
- How many edges and vertices of this pyramid can you not see in the picture?
- Can this also be a picture of a triangular or a pentagonal pyramid?



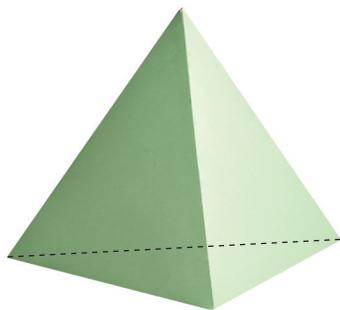
3. The broken lines show the edges that are hidden in this picture of a rectangular pyramid.

In the pictures below, the broken lines also indicate the hidden edges.

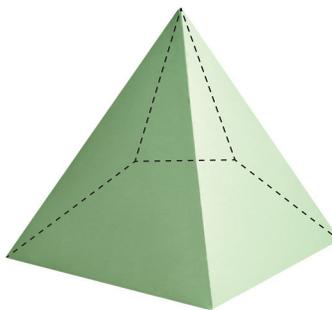
In each case, state what kind of pyramid it is and how many triangular faces, how many vertices and how many edges it has.



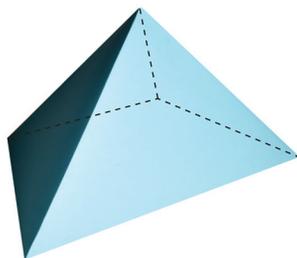
(a)



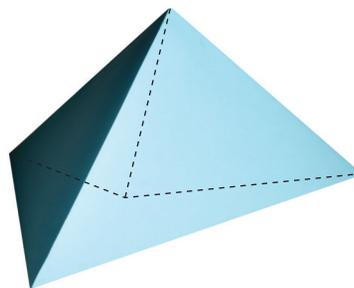
(b)



(c)



(d)



4. If you want to take on a challenge, you can build a skeleton model for the pyramid in question 3(d).

-
5. These are two pictures of the same hexagonal pyramid.

How many faces, edges and vertices are hidden in each of the pictures?



Picture A

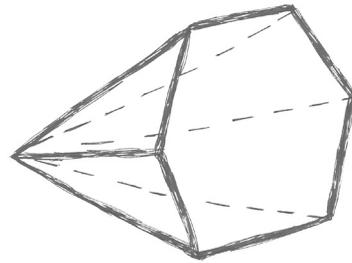


Picture B

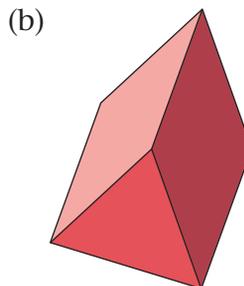
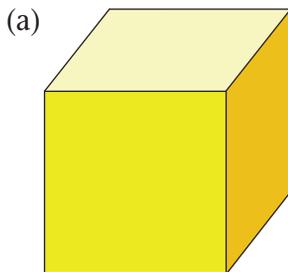
6. This is a rough drawing of the skeleton of the hexagonal pyramid in Picture A.

The broken lines show the edges that you cannot see in the picture.

Make a drawing like this of the skeleton of the hexagonal pyramid, as you see it in Picture B.



7. Make drawings of the skeletons of the objects below.



4.3 Faces, vertices and edges of 3-D objects

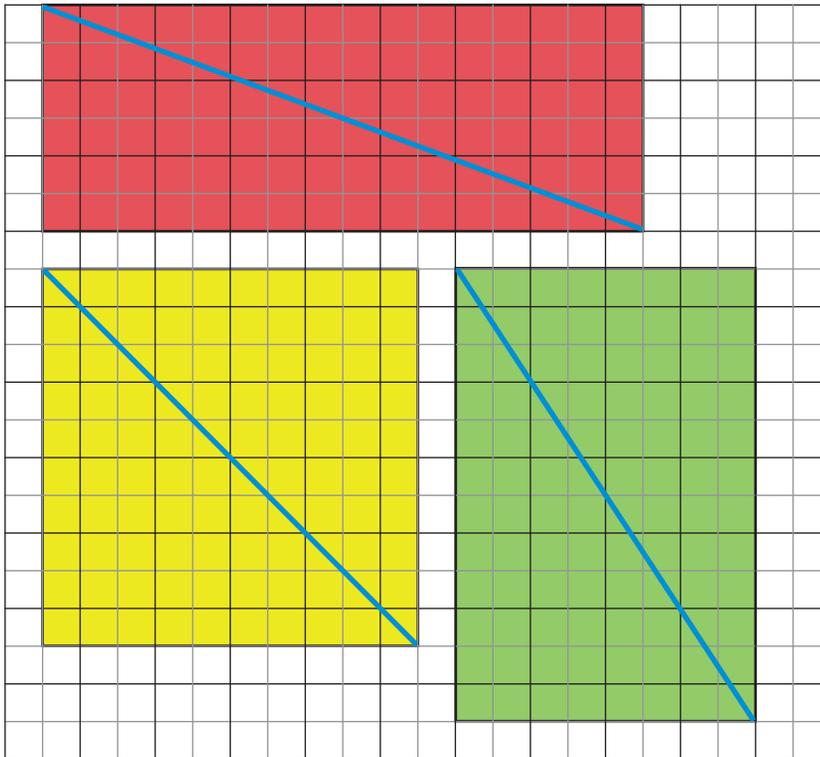
- Name three 3-D objects that have at least three triangular faces each.
 - Name a 3-D object that has eight vertices.
 - Name a 3-D object that has seven vertices.
 - Name a 3-D object that has four vertices.
 - Name a 3-D object that has square faces only.
 - Name a 3-D object that has only one square face.
 - Name a 3-D object that has only four faces, and the faces are all exactly the same.
 - Name a 3-D object that has only six faces, and the faces are all exactly the same.
- For each object, state how many faces it has and what the shapes of the faces are. You may do question 5 before you do question 2.
 - a cube
 - a rectangular prism that is not a cube
 - a tetrahedron
 - a triangular pyramid that is not a tetrahedron
 - a square pyramid
 - a pentagonal pyramid
 - a hexagonal pyramid
- For each object in question 2, state how many edges it has and how many of the edges have the same length.
- For each object in question 2, state how many vertices it has.
- Draw the skeleton of each of the objects in question 2.

5.1 Perimeter and area

The grid below includes small and large squares. Each large square covers an area of 1 square centimetre.

The length of each side of the small squares is equal to 0,5 cm.

- How many small grid squares cover one large grid square?

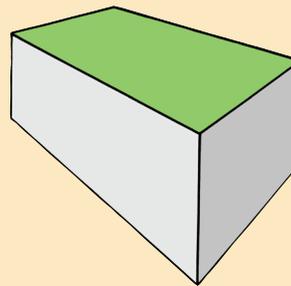
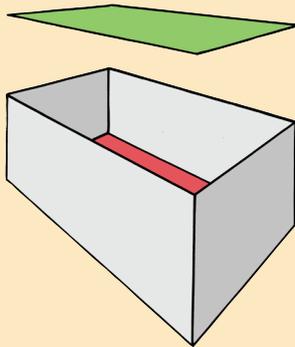
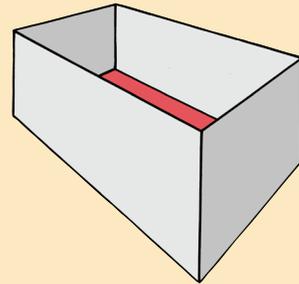


- Which coloured figure looks the biggest to you?
 - Which figure has the longest edge, all around it?
 - Which figure covers the most large grid squares?
 - Is the blue line inside each figure equal to a side of the figure? Use your ruler to measure the sides and the blue lines.

Your classroom has four walls. The picture on the right does not show the windows, the door and the roof.

Most classrooms have the shape of a rectangular prism.

When you are inside a classroom and you look up, you may see the roof. Or, you may see the ceiling which is on top of the walls underneath the roof. Ceilings are normally painted white, but in these pictures the ceiling is green.



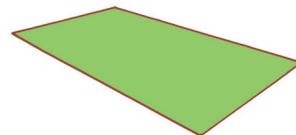
The ceiling of the classroom is like the top face of a rectangular prism.

Builders sometimes put a moulding round the wall of a room just below the ceiling, to close the small gap between the wall and the ceiling.

This is called a cornice.

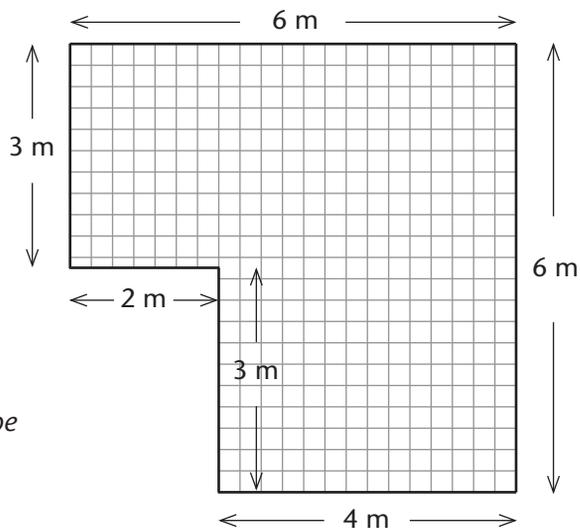


3. Estimate the total length of moulding that would be needed to put a new cornice around the ceiling of your classroom.



The **perimeter** of a figure is the total distance around the edge of a figure.

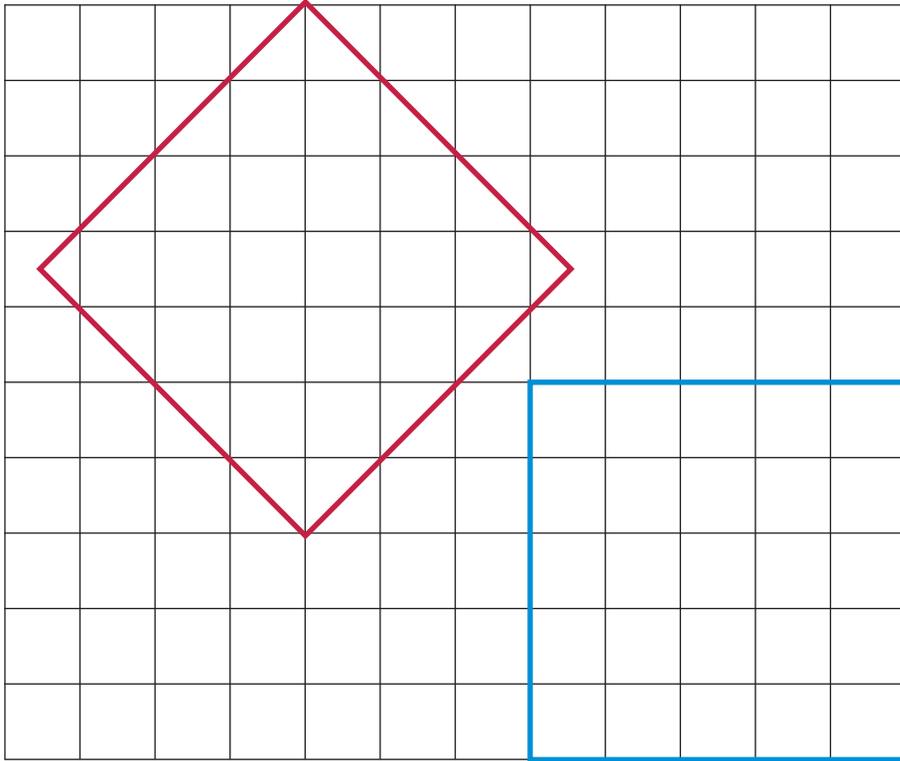
- Use a ruler or a measuring tape to measure the edge of the top of your desk. State the perimeter to the nearest centimetre.
- Work in a team and measure the perimeter of your classroom. Use a measuring tape.



This sketch shows the floor shape and dimensions of a room.

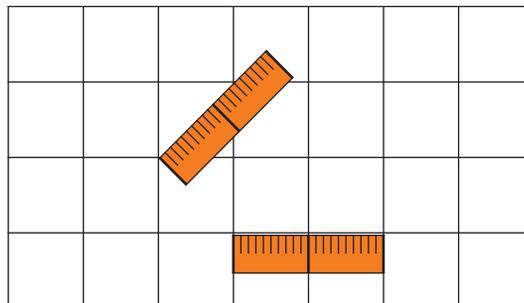
- What is the perimeter of this room?
- A school is built on a plot of land in the shape of a quadrilateral. The lengths of the sides of the quadrilateral are 346 m, 423 m, 298 m and 372 m.
 - A fence must be built around the school grounds, on the edges of the plot of land. How long will this fence be, in total?
 - If 1 m of fence costs R47, what will it cost to put up the fence around the school grounds?
- A game reserve has the shape of a pentagon with sides 3,82 km, 6,14 km, 5,23 km, 1,43 km and 4,44 km.
 - What is the perimeter of the game reserve?
 - What will it cost to put up a fence around the game reserve, at R47 per metre of fencing?

9. Measure the perimeter of each quadrilateral in two ways:
- by using the grid lines, which are 1 cm from each other
 - with a ruler.

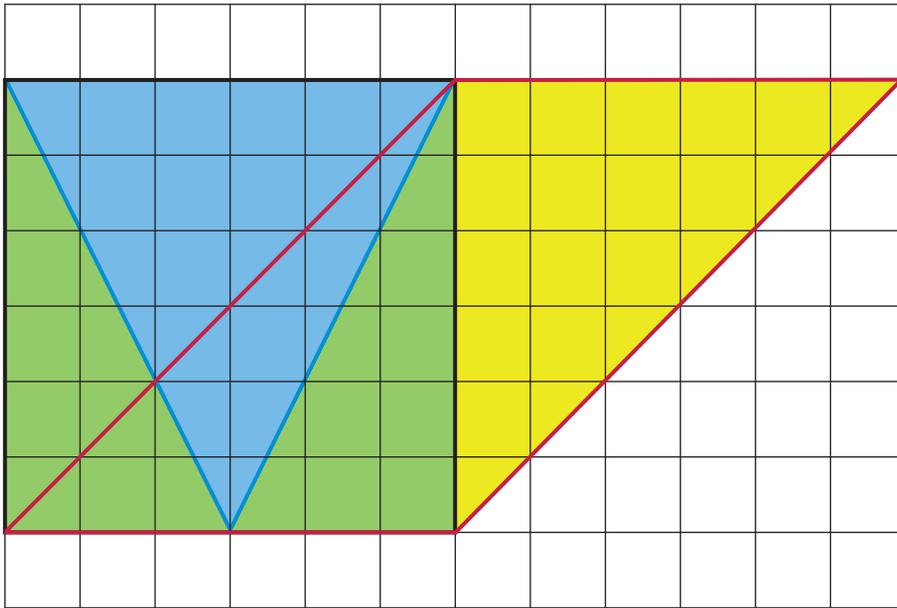


10. Some people get different answers when they measure the perimeter of the red square with the grid and with their rulers.
- Why do they get two different answers?
 - Which of the two answers is correct?

11. This diagram may be helpful if you want to improve the explanation you wrote in question 10(b).



12. (a) Use a ruler and measure the sides of the square below.
 (b) Measure the length of the red line that divides the square into two triangles.



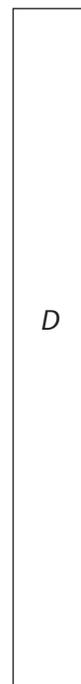
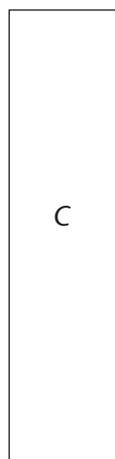
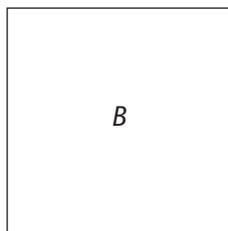
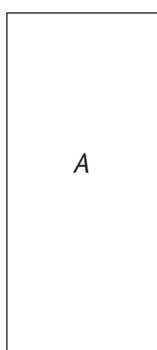
- (c) What is the perimeter of the square?
 (d) What is the perimeter of the parallelogram (the figure with the red sides)?
 (e) What is the perimeter of the blue triangle?
 (f) How many small squares are needed to cover the area inside the square?
 (g) How many full squares, and how many half squares, are needed to cover the area inside the parallelogram?

The **surface area** of a closed figure can be measured by counting how many equal squares are needed to cover it.

13. State the area of each of the following in the drawing in question 12:
 (a) the square
 (b) the parallelogram
 (c) the blue triangle
 (d) the yellow triangle
 (e) the two green triangles together

5.2 Area and perimeter

1. Which of these rectangles do you think has the smallest area, and which has the smallest perimeter?
Write down your answers.



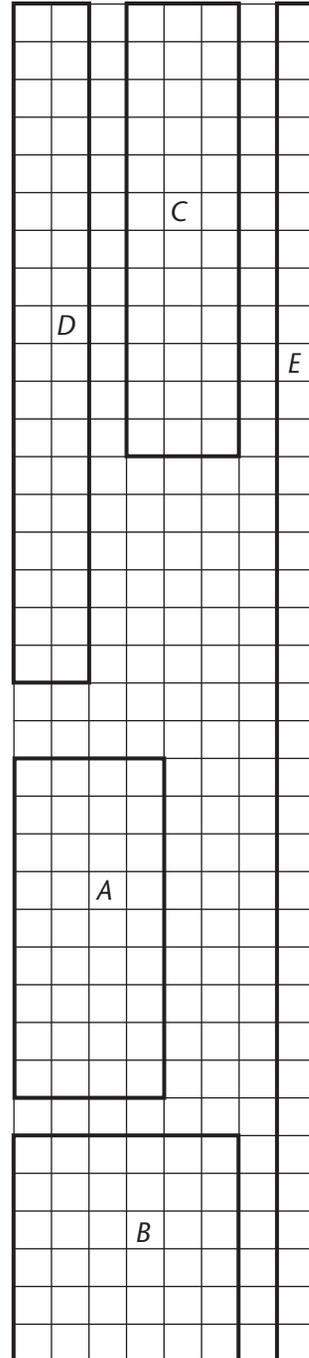
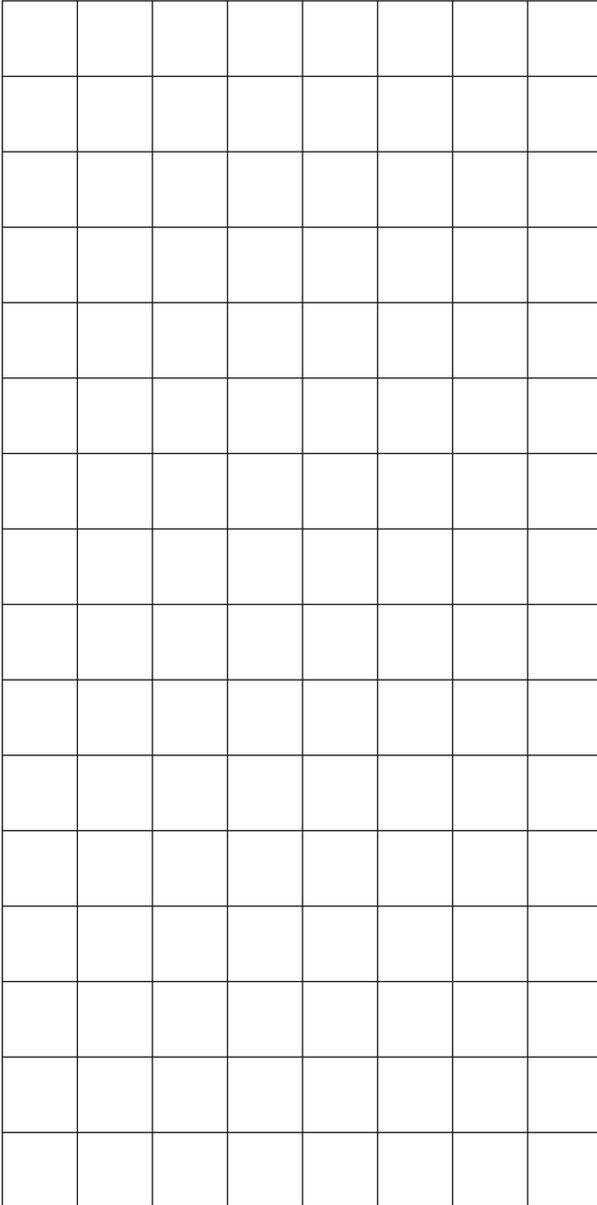
2. Do not measure now.
- (a) *Estimate* how many squares with a side length of 0,5 cm are needed to cover each of the rectangles. Write your estimates in the first row of the table below.

	A	B	C	D	E
Area					
Perimeter					

- (b) *Estimate* the perimeter of each of the rectangles in centimetres. Write your estimates in the second row of the table.
3. The same rectangles (A to E) are shown on a 0,5 cm grid on the next page. Find the area and perimeter of each rectangle.
Make a new table like the one in question 2 and enter your measurements in the table.

4. Work on a grid such as the one shown below. Draw three different rectangles that each have an area of 24 squares.

Find the perimeter of each of your rectangles.

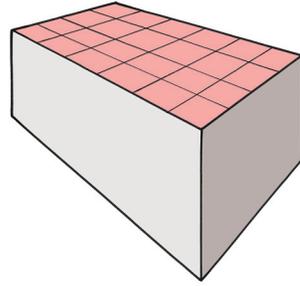


Try to do some of the questions below without making drawings and counting squares.

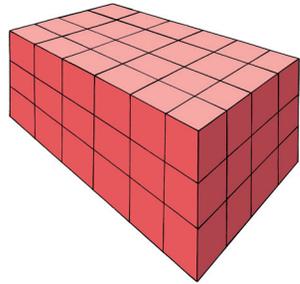
5. In this question, only consider rectangles in which each side is a whole number of centimetres.
 - (a) Of the different possible rectangles with a perimeter of 32 cm, which one has the biggest area?
 - (b) Also investigate this for rectangles with a perimeter of 28 cm, and rectangles with a perimeter of 24 cm.
6. Jonas wants to make a fowl-run for his chickens. He has 24 m chicken mesh and wants to make the biggest possible rectangular fowl-run. What should the length and width of the fowl-run be?
7. The area of a certain rectangle is 40 grid squares.
 - (a) If there are 8 squares in one row of squares, how many rows of squares are there in the rectangle?
 - (b) How many squares are there in each row, if there are 10 rows?
8. Imagine you have to draw figures that each cover 100 grid squares.
 - (a) If you want the figure to be a square, how many grid squares should be in one row and how many rows should be there?
 - (b) How can you draw a rectangle with an area of 100 grid squares that is not a square? Give two possibilities.
 - (c) How will you draw a rectangle with an area of 100 grid squares and the smallest possible perimeter?
9.
 - (a) You have 48 small square tiles. Describe all the different ways to arrange them so that they form a rectangle.
 - (b) Do what you did in (a) for 50 tiles.
 - (c) Do what you did in (a) for 46 tiles.
 - (d) Do what you did in (a) for 47 tiles.
 - (e) What numbers of tiles up to 100 can be arranged into squares?
10. Two equal squares are joined at one side to form a rectangle.
 - (a) Is the area of the rectangle twice the area of one of the squares?
 - (b) Is the perimeter of the rectangle twice the perimeter of one of the squares?

5.3 Volume and capacity

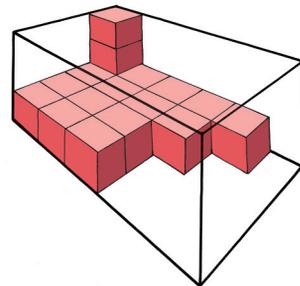
The **capacity** of a box can be measured by counting how many cubes can be packed into it.



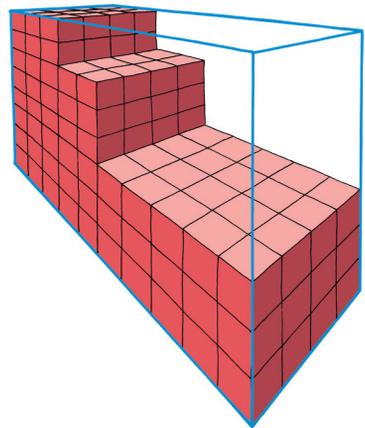
The **volume** of a stack of cubes can be measured by counting the cubes.



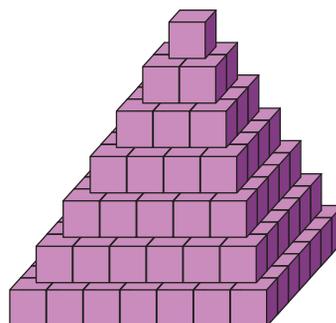
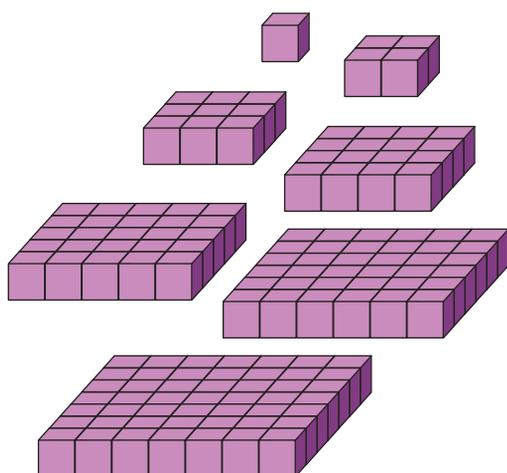
1. Some cubes were packed in the box on the right. How many such cubes can be packed into this box, in total?



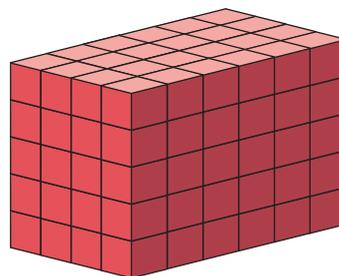
2. (a) What is the volume of the stack inside the blue box?
(b) What is the capacity of the blue box?
(c) How many more cubes must be put into the box to fill it completely?



3. The stack on the right has 7 layers, which are shown separately below. What is the volume of the stack?



4. (a) How many cubes make up the length of this stack?
 (b) How many cubes make up the width of this stack?
 (c) How many cubes are there in one layer?
 (d) How many layers are there altogether?
 (e) How many cubes were used to build the stack?



5. The inner measurement of a box is 6 cm long and 4 cm wide. The box is 5 cm deep.
- (a) How many cubes with sides of 1 cm do you need to cover the bottom of the box?
 (b) How many layers of cubes can you pack in the box?
 (c) How many cubes will fit into the box?

Measurement was among one of the first intellectual achievements of the early humans. People learnt to measure centuries before they learnt how to write and it was through measurement that people learnt to count.

Think back about four thousand years. Imagine that you are somewhere in Egypt close to the Nile River. The annual flood has just receded and you have to measure out your land.

You also want to build a new house, square in shape, so that you use as little material as possible for a sizeable room.

You have to use your body parts as measuring tools and you have the following measuring units at your disposal:

cubit: length of the forearm, from the bottom of the elbow to the tip of the middle finger

hand: length between the tip of the little finger and the tip of the thumb

palm: four fingers across

finger: width of a finger

The length of the walls of your house must be eight cubits each. You and your father start in one corner, one building the northern wall, and the other the eastern wall.

When you have finished, the walls are not the same length! Why is that?

To measure your piece of land after the flood, you use another commonly used measuring tool: a length of rope tied in knots at regular intervals.

Do you think your neighbours will necessarily be satisfied with the outcome? Why or why not?

Of course, these units varied from person to person and this created many difficulties!

1. Check it out for yourself. Use body parts as measuring tools, like the Egyptians did. Form groups of four and measure:
 - (a) one wall of the classroom in cubits (it must be the same wall)
 - (b) the length of your desk in hands (everyone must measure the same size desk or table)
 - (c) the length of your exercise book in palms
 - (d) the height of a stack of four Mathematics textbooks in fingers.Write the outcomes of the measurements of the different groups in columns on the chalk board. (Keep it there for question 4.)
2. Now have a class discussion about ancient measurement methods and the tools available.
3. What would be the effect on trade and the economy these days if each country manufactured mechanical, electrical and other goods according to their own specific measuring units?



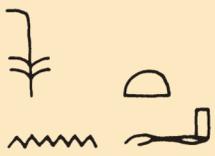
Old Egyptian houses

How was it possible that the ancient Egyptians could build pyramids, palaces and tombs with such differences regarding units of measurement?

They standardised them. By 2500 BCE, a royal cubit of black granite had become the master (standard) cubit. All measuring sticks (cubit sticks) used in Egypt had to be the same length as the master cubit and this was checked regularly.

The table on the next page shows how long the Egyptian units were in modern metric lengths. Use these measurements to answer question 4.

Ancient Egyptian units of length

Unit	Egyptian name	Equivalent Egyptian values	Metric equivalent
Finger	 <i>djeba</i>	1 finger = $\frac{1}{4}$ palm	1,88 cm
Palm	 <i>shesep</i>	1 palm = 4 fingers	7,5 cm
Hand	 <i>drt</i>	1 hand = 5 fingers	9,38 cm
Greek cubit	 <i>meh nedjes</i>	1 short cubit = 6 palms = 24 fingers	45 cm
Royal cubit	 <i>meh niswt</i>	1 royal cubit = 7 palms = 28 fingers	52,4 cm
Rod of cord	 <i>khēt</i>	1 rod of cord = 100 cubits	52,5 m

[Source: https://en.wikipedia.org/wiki/Ancient_Egyptian_units_of_measurement]

4. As a class, decide on the most common lengths for the four measurements in question 1 (written on the chalk board). Round them off and then complete the table. Use your calculator.

Object	Egyptian length	Metric length
Length of classroom wall		
Length of desk or table		
Length of exercise book		
Height of stack of four Mathematics textbooks		

As you can see in the table on the previous page, there was also a Greek cubit. A Greek cubit, also called “short cubit” or “small cubit”, was 4 fingers (1 palm) shorter than a royal cubit. With Greece being one of the southern European countries, the Greeks not only traded with the Egyptians but they also exchanged ideas on mathematics.

Most countries, however, had their own ways of measuring. Through many years, and through a complicated transformation, it seems that the measurements inch, foot, and yard evolved from the ancient Egyptian units. Interestingly enough, a length of about one foot could be found in the length measurement of most countries.

More than 200 years ago the metric system was adopted by France. It was the beginning of the **international metric system**. Instead of having a large number of units of different sizes, it was decided to use multiples of 10 for longer measurements and decimal fractions for smaller measurements.

Today the metric system is used all over the world, except in America and a few small countries. The metre is the “cornerstone” of the metric system. The word “metre” comes from a Greek word which means “a measure”.

7.1 Explore division with bigger numbers

- How much is each of the following?
 - 4×25
 - 40×25
 - $10 \times 4 \times 25$
 - $10 \times 40 \times 25$
 - 8×125
 - 80×125
 - 4×250
 - 40×250
 - 50×200
 - $8 \times 1\ 250$
- 10 000 new chairs are ready to be delivered to schools but each school must get the same number of chairs.
 - How many chairs can each school get if there are 5 schools?
 - How many chairs can each school get if there are 10 schools?
 - How many chairs can each school get if there are 20 schools?
 - How many chairs can each school get if there are 25 schools?
 - How many chairs can each school get if there are 50 schools?
 - How many chairs can each school get if there are 125 schools?
- Do not do accurate calculations to find answers for these questions.
 - Estimate* how many goats at R320 each a farmer can buy with R10 000.
 - Estimate* how many lambs at R197 each a farmer can buy with R10 000.
 - Estimate* how many calves at R720 each a farmer can buy with R10 000.
- Multiply the prices with your estimates for question 3 to check how good your estimates were.
- Now do calculations to find the exact answers for question 3.

7.2 Two methods of division

Two ways in which division may be performed are shown here. You have already learnt about them in Term 2.

Division may be performed by *adding up multiples* of the divisor.

For example, to calculate $7\,283 \div 183$ you will work as follows:

	Multiples		Remainder
	$10 \times 183 = 1\,830$	1 830	
<i>Doubling</i>	$20 \times 183 = 3\,660$	5 490	1 793
<i>Halving</i>	$5 \times 183 = 915$	6 405	878
	$\underline{4} \times 183 = 732$	7 137	146
	39		

So $7\,283 \div 183 = 39$ remainder 146.

Division may be performed by *subtracting multiples* of the divisor.

For example, to calculate $7\,283 \div 183$ you will work as follows:

$$\begin{array}{r} 7\,283 \\ 10 \times 183 = \underline{1\,830} \\ 5\,453 \\ 20 \times 183 = \underline{3\,660} \\ 1\,793 \\ 5 \times 183 = \underline{915} \\ 878 \\ \underline{4} \times 183 = \underline{732} \\ 39 \qquad \qquad 146 \end{array}$$

So $7\,283 \div 183 = 39$ remainder 146.

Use any method to calculate the following.

- (a) $4\,200 \div 123$ (b) $4\,205 \div 145$
- (a) $7\,888 \div 232$ (b) $7\,888 \div 32$

7.3 Apply and use your knowledge

- A special training leash is needed to train a guide dog. One leash costs R237. How many guide dogs are to be trained if the dog trainer paid R2 844 for leashes?
- A farmer delivers 872 kg pumpkins to the market and receives R6 104 for it. How much money does he get for 1 kg?
- A multi-storey parking garage has 3 375 parking bays. Each floor has exactly 375 parking bays. How many floors does the parking garage have?
- A taxi charges R1 284 to take passengers from Cape Town to Worcester. How many passengers share the cost if each of them pays R321?
- Linus needs 2 120 drawing pins for a project. Drawing pins come in 120 per box. How many boxes must he buy?
- Cedric bought 3 packets of sweets, each with 75 sweets in it. He divides the sweets evenly among his 7 friends and keeps the remaining sweets for himself. How many sweets does Cedric keep for himself?
- A store ordered 108 boxes of baked beans. Each box had 18 cans. After all the boxes were unpacked, the shopkeeper stacked the cans in 27 rows with the same number of cans in each row. How many cans were in each row?
- Kate and Jane have to empty a box with 360 golf balls. They take turns to take out the golf balls. Kate takes out 3 balls at a time. Jane takes out 5 balls at a time. How many balls will each of the girls have taken out when the box is empty?

-
9. A store ordered oranges from the fruit market. Twenty-one of the 3 485 oranges are rotten and cannot be sold.
- (a) How many oranges can the store owner put in one pocket if he wants to make 130 equal pockets?
 - (b) How many oranges will be left over (excluding the rotten ones)?
10. The music store has a special offer of R99 for three CDs. Carmen made use of this opportunity to get more of her favourite music. She spent R1 089. How many CDs did she get?
11. Martin bought a tablet for R7 336. He borrowed the money from his mother and promised to pay her back R262 each month. How long will it take Martin to pay back the full amount? Give your answer in years and months.
12. A hotel spent R9 792 on new bath towels. Each bath towel cost R144. How many towels did they buy?

7.4 Ratio

Is it always fair to divide quantities into equal parts? Read this story and decide for yourself.

30 learners from a school are going on a school camp. They are divided into two groups: one group has 20 learners and the other group has 10 learners.

When the food is handed out, the group of 20 learners is given a box with 60 apples, and the group of 10 learners is also given a box with 60 apples.

Do you think this is fair?

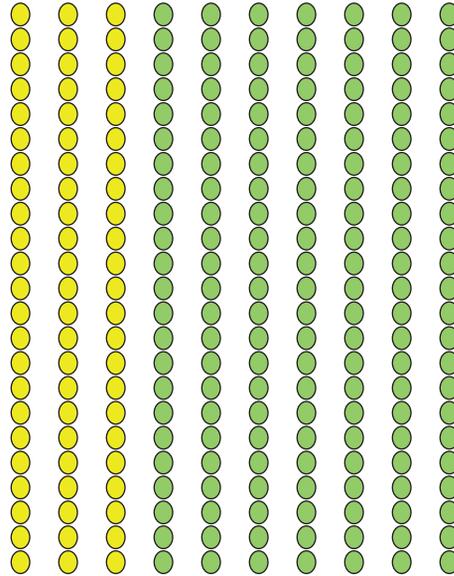
1. Now that you have read the story and thought about it, answer these questions.
 - (a) How many apples will each learner get, if each group is given one box of apples?
 - (b) How do you think the apples should have been divided between the two groups?

2. Each row in this diagram has 3 yellow beads and 7 green beads.
(The rows run from left to right.)

- (a) If the diagram is continued in the same way so that there are 80 rows, how many green beads will there be, and how many yellow beads?

- (b) If there are 9 000 beads in the diagram, how many of them will be green?

- (c) If there are 5 600 green beads in the diagram, how many yellow beads will there be?

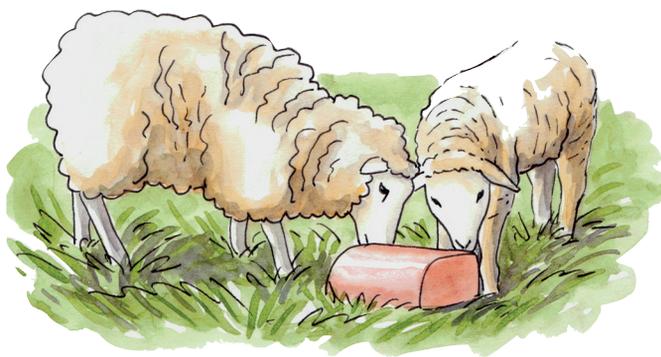


3. In a diagram similar to the one in question 2, there are 5 yellow beads and 8 green beads in each row.
- (a) If there are 6 500 beads in total in the diagram, how many of them are green?
- (b) If there are 2 000 yellow beads in total in the diagram, how many green beads are there?
4. A farmer keeps his goats in two camps. He keeps 20 goats in Camp A and 30 goats in Camp B.
- (a) One day, he has 100 scoops of food for the goats. How many scoops of food should he put in each camp? Explain.
- (b) On another day he has 150 scoops of food for the goats. How many scoops of food should he now put in each camp?
- (c) One day the farmer has 5 buckets of food for his goats. How many buckets of food should he put in each camp?
- (d) On another day the farmer puts 180 scoops of food in Camp B. How many scoops of food should he put in Camp A?

-
5. When Hilary bakes bread, she always uses the same recipe. So she always mixes 5 cups of white flour with 2 cups of wholewheat flour.
- (a) How many cups of wholewheat flour must she mix with 20 cups of white flour?
 - (b) How many cups of wholewheat flour must she mix with 35 cups of white flour?
 - (c) How many cups of white flour must she mix with 20 cups of wholewheat flour?
 - (d) How many cups of white flour must she mix with 70 cups of wholewheat flour?
 - (e) If Hilary uses 42 cups of flour in total to bake bread, how many cups of white flour does she use?
 - (f) If Hilary buys 240 kg of wholewheat flour, how much white flour should she buy?

7.5 More practice

1. A sheep farmer keeps her sheep in two large camps. She has 300 sheep in Camp A and 450 sheep in Camp B.

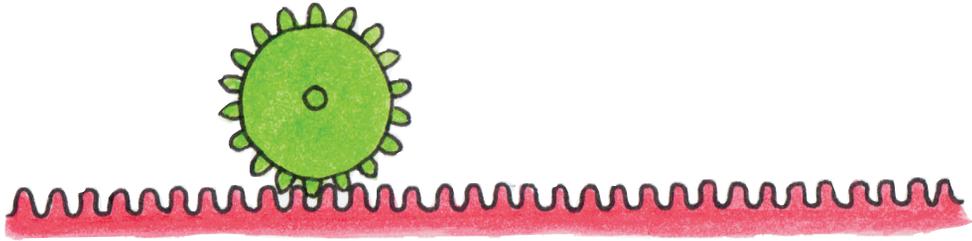


- (a) She has 7 500 kg salt lick that she must divide between the two camps so that each sheep gets the same amount. How should she divide it?
 - (b) She adds 63 bales of lucerne to Camp B as extra feeding. How many bales of lucerne should she put in Camp A so that every sheep gets the same amount?
2. Ahmed bought 8 235 pears from a farmer. He sells the pears to a dealer in crates with 125 pears in each crate. How many crates can he fill?

-
3. Mrs Naidoo needs 172 cm of fabric to make one skirt. How many skirts can she make if she has 12,5 m of fabric?
 4. String is provided in rolls of 2 250 cm.
 - (a) How many lengths of 128 cm can be cut from the roll?
 - (b) What length of string remains on the roll?
 5. Two numbers give 3 720 when multiplied.
 - (a) Find the two numbers.
 - (b) Find another two numbers that give 3 720 when multiplied. Try to find numbers that are as close to each other as possible.
 - (c) Describe the plans that you tried out to solve this problem.
 6. Jody's father will lend him R9 750 to buy a laptop. Jody has agreed to pay his father back R195 per month. How long will it take to pay back the full amount? Answer in years and months.
 7. Faizal buys paint. The price is R134 for one can.
 - (a) How many cans does he buy if he pays R8 978 in total?
 - (b) How many cans could he buy for R89 780?
 8. There is R9 167 available to pay a bonus to 103 workers. How much will each worker receive if the money is divided equally?
 9. Yuko mixed 835 ml of lemon syrup with 3 290 ml of water and then poured the same amount of juice in each of 15 glasses. How much juice was in each glass?
 10. Use any method to calculate the following.

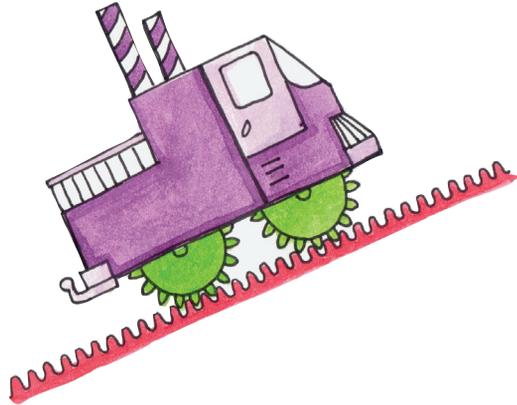
(a) $5\,796 \div 12$	(b) $5\,796 \div 144$
(c) $9\,588 \div 17$	(d) $9\,588 \div 282$
(e) $11\,250 \div 450$	(f) $11\,650 \div 282$
 11. A school wants to raise R11 700 to buy two computers. The learners plan to ask businesses in their community to each donate R650. How many businesses will they have to ask for donations?

7.6 Rate



1. How many teeth does this cogwheel have?
2. How many notches will the locomotive in the diagram move forward when the wheels turn around once?

3. (a) How many notches will the locomotive move forward when the wheels turn around 6 times?
(b) How many notches will the locomotive move forward when the wheels turn around 54 times?



4. How many full turns of the wheels are needed so that the locomotive will move forward by each of the following numbers of notches?
(a) 1 800 notches
(b) 6 372 notches
(c) 6 381 notches
(d) 6 390 notches
(e) 801 notches
(f) 1 602 notches

8.1 Statements of equivalence

$$\begin{aligned} 10 \times 10 - 5 \times 5 \\ = 100 - 25 \\ = 75 \end{aligned}$$

$$\begin{aligned} (10 + 5) \times (10 - 5) \\ = 15 \times 5 \\ = 75 \end{aligned}$$

Two different sets of calculations with 10 and 5 produce the same result.

We can say:

The calculations $10 \times 10 - 5 \times 5$ and $(10 + 5) \times (10 - 5)$ are **equivalent**.

- Calculate $20 \times 20 - 10 \times 10$ and $(20 + 10) \times (20 - 10)$.
 - Calculate $8 \times 8 - 3 \times 3$ and $(8 + 3) \times (8 - 3)$.
 - Calculate $5 \times 5 - 2 \times 2$ and $(5 + 2) \times (5 - 2)$.
- Suppose you have to find out how much $18 \times 18 - 12 \times 12$ and $53 \times 53 - 47 \times 47$ and $505 \times 505 - 495 \times 495$ are.
 - Do you think 30×6 will produce the right answer for $18 \times 18 - 12 \times 12$? Investigate.
 - Find out how much $53 \times 53 - 47 \times 47$ and $505 \times 505 - 495 \times 495$ are. Use a calculator and check your answers.
 - Do you *think* that $2 \times 69\,570$ will give the answer for $34\,786 \times 34\,786 - 34\,784 \times 34\,784$? Explain your thinking.
- Which of these number sentences are true, and which are false?
 - $3 \times 5 + 3 \times 7 = 3 \times 12$
 - $3 \times 5 + 3 \times 7 = 6 \times 12$
- Michael firmly believes the following:

$$4 \times 6 + 4 \times 9 = 8 \times 15$$

$$3 \times 5 + 3 \times 7 = 6 \times 12$$
 - What do you think Michael will believe about $6 \times 4 + 6 \times 8$ and $10 \times 5 + 10 \times 7$?
 - Write a letter to Michael. Explain to him why what he believes about addition and multiplication is wrong.

8.2 Substitution, trial and improvement

Suppose we want to find out what number will make this number sentence true:

$$5 \times \text{the number} + 4 = 64 - 3 \times \text{the number}$$

The number in the left-hand part of the number sentence must be *the same* as the number in the right-hand part of the number sentence.

We can try the number **10**:

$$5 \times \mathbf{10} + 4 = 50 + 4 = 54 \text{ and}$$

$$64 - 3 \times \mathbf{10} = 64 - 30 = 34,$$

so the number is not 10.

If the number is **10**,

$5 \times \text{the number} + 4$ is **20 bigger** than $64 - 3 \times \text{the number}$.

We can try the number **20**:

$$5 \times \mathbf{20} + 4 = 100 + 4 = 104 \text{ and}$$

$$64 - 3 \times \mathbf{20} = 64 - 60 = 4,$$

so the number is not 20.

If the number is **20**,

$5 \times \text{the number} + 4$ is **100 bigger** than $64 - 3 \times \text{the number}$.

We can try a number smaller than 10. Let's try the number **5**:

$$5 \times \mathbf{5} + 4 = 25 + 4 = 29 \text{ and}$$

$$64 - 3 \times \mathbf{5} = 64 - 15 = 49,$$

so the number is not 5.

Now $5 \times \text{the number} + 4$ is **smaller** than $64 - 3 \times \text{the number}$.

We can summarise the work that we did in a table:

Trial number	10	20	5
$5 \times \text{the number} + 4$	54	104	29
$64 - 3 \times \text{the number}$	34	4	49
Difference	20	100	-(20)

1. Try the number 6 in $5 \times \text{the number} + 4$ and in $64 - 3 \times \text{the number}$.
If the results still differ, try some other numbers until you know for which number the two calculation plans give the same result.

-
2. Find the numbers that make the number sentences true.
- (a) $15 \times \square - 11 = 11 \times \square + 1$
- (b) $100 - 5 \times \square = 3 \times \square - 28$
- (c) $10 \times \square + 1\,500 = 20 \times \square + 1\,250$
3. (a) Write five numbers for which $10 \times \square + 1\,500$ is bigger than $20 \times \square + 1\,250$.
- (b) Write five numbers for which $10 \times \square + 1\,500$ is smaller than $20 \times \square + 1\,250$.
4. Find the numbers that make the number sentences true.
- (a) $10 \times \square + 1\,500 = 20 \times \square - 2\,000$
- (b) $20 \times (\square - 100) = 10 \times (\square + 150)$
5. Explain why the number sentences $10 \times \square + 1\,500 = 20 \times \square - 2\,000$ and $20 \times (\square - 100) = 10 \times (\square + 150)$ are true for the same number.
6. Find the numbers that make the sentences true.
- (a) $10 \times \square + 1\,500 = 20 \times \square + 1\,470$
- (b) $10 \times \square + 1\,500 = 20 \times \square - 6\,500$
- (c) $10 \times \square + 1\,500 = 20 \times \square + 300$
- (d) $10 \times \square + 1\,500 = 20 \times (\square + 15)$
- (e) $10 \times (\square + 150) = 20 \times \square + 300$
7. (a) Try to find the number that makes this sentence true:
 $10 \times (\square + 150) = 10 \times \square + 1\,500$
- (b) Compare your experience with some classmates.
Try to find an explanation for what you experienced.
8. (a) Try to find the number that makes this sentence true:
 $10 \times (\square + 150) = 10 \times \square + 150$
- (b) Compare your experience with some classmates.
Try to find an explanation for what you experienced.

8.3 Use number sentences when needed

The production rate at a brick factory is 128 000 bricks per day. Bricks are made seven days of the week. On the morning of 1 September, there is a stock of 2,4 million bricks at the factory. Assume that no bricks are sold during September.

1. How many bricks will be in stock at the end of the day on 2 September?
2. How many bricks will be in stock at the end of the day on 10 September?
3. How many bricks will be in stock at the end of the day on 16 September?
4. At the end of which day in September will the stock level reach 6,24 million?
5. Which of the following are correct plans to calculate the stock level at the end of the day at the factory on \square September?

You may use a calculator.

Plan A: $2\,400\,000 \times \square + 128\,000$

Plan B: $128\,000 \times \square + 2,4$

Plan C: $128\,000 \times \square + 2\,400\,000$

Plan D: $6,24 = 128\,000 + 2,4 + \square$

Plan E: $16 \times \square + 128\,000$

Plan F: $128 \times \square + 2\,400$

6. Which of these number sentences show the situation in question 4?

Number sentence A: $128\,000 \times \square + 2,4 = 6,24$

Number sentence B: $128\,000 \times \square + 6\,240\,000 = 2\,400\,000$

Number sentence C: $128 \times \square + 2\,400 = 6\,280$

Number sentence D: $128\,000 \times \square + 2\,400\,000 = 6\,240\,000$

7. (a) Copy and complete the table below to show the stock levels at the end of the day at the brick factory on different days in September.

Day of September	1	7	14	21	28
Stock level					

- (b) On what day in September will the stock level pass the 3 million mark?
- (c) On what day will it pass the 4 million mark?
- (d) On what day will the stock level be 5,344 million?



A large truck is used to deliver cement to building sites. The mass of the empty truck is 2 360 kg.

8. Pockets of cement with a mass of 90 kg each are loaded onto the truck.
- (a) What is the total mass of the truck with the load, if 144 pockets of cement are loaded?
- (b) How can the total mass of the truck with the load be calculated, for *any* number of pockets of cement?
- (c) If the total mass of the truck and the load is 12 710 kg, how many pockets of cement are loaded?
- (d) How many pockets of cement are loaded if the total mass of the truck and load is 8 480 kg?
9. The same truck is used to transport roof sheets that weigh 50 kg each.
- (a) What is the total mass of the truck with the load, if 76 roof sheets are loaded?

-
- (b) What is the total mass of the truck with the load, if 42 roof sheets and 65 pockets of cement are loaded?
- (c) The total mass of the truck with a load of 60 roof sheets and some pockets of cement is 9 680 kg. How many pockets of cement are loaded?
- (d) The total mass of the truck with a load of roof sheets and pockets of cement is 11 940 kg. How many pockets of cement and how many roof sheets are loaded?
This is not meant to be an easy question. You will have to do some trial and improvement. If you feel like giving up, it may help to do the following questions first.
- (e) Calculate $9\,580 - 90 \times \square$ for different values of \square (in other words, different numbers in the place of the \square) until you find a value of \square for which $9\,580 - 90 \times \square$ is a multiple of 50.
- (f) The total mass of the truck with a load of roof sheets and pockets of cement is 12 940 kg. How many pockets of cement and how many roof sheets are loaded?

9.1 Making larger copies of figures

- (a) How do the three figures differ?
(b) What is the same about the three figures?

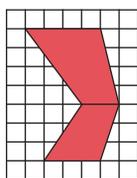


Figure A

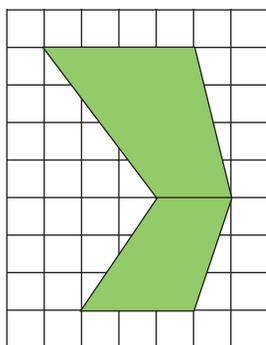


Figure B

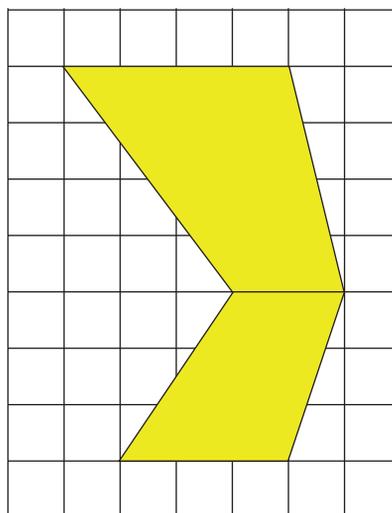


Figure C

Figure B is 2 times as large as Figure A.

Figure C is 3 times as large as Figure A.

Figure C is $1\frac{1}{2}$ times as large as Figure B.

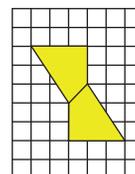
Note that each figure is a combination of two quadrilaterals.

Figures B and C are called **enlargements** of Figure A.

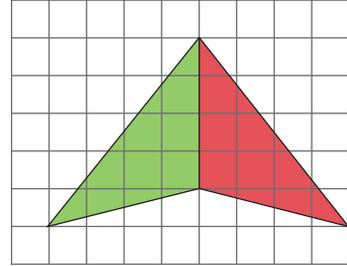
We can also say:

- Figure A is **enlarged by a scale factor of 2** to make Figure B.
- Figure A is **enlarged by a scale factor of 3** to make Figure C.
- Figure B is **enlarged by a scale factor of 1,5** to make Figure C.

- (a) Can you think of a way to enlarge this figure by a scale factor of 2, in other words to accurately draw it twice as large? Describe your plan.
(b) Can you think of a way to enlarge it by a scale factor of 4? Describe your plan.

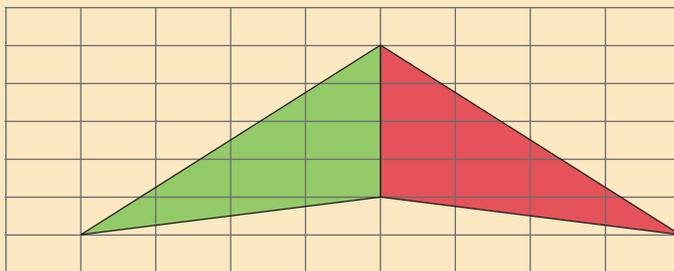
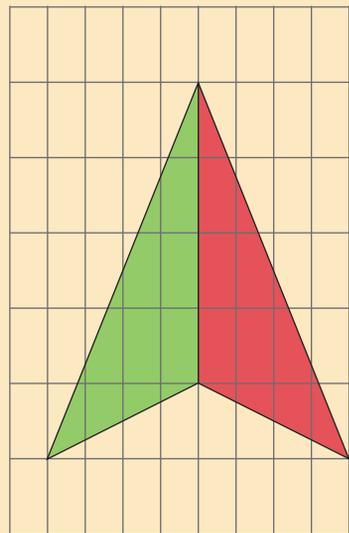


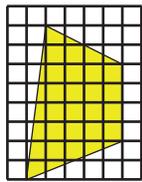
This kite was drawn on a 0,5 cm grid.
To enlarge the kite by a scale factor of 3,
you can draw it on 1,5 cm grid paper.



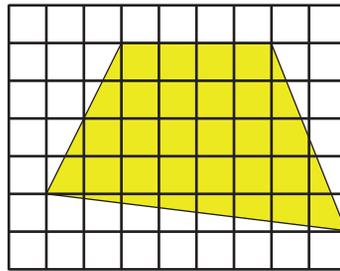
3. (a) Put a clean sheet of paper over the 1,5 cm grid on the next page, and use your ruler to copy the grid.
- (b) Enlarge the above kite by a scale factor of 3 by drawing it on your 1,5 cm grid. You may look at Figures A, B and C on the previous page to see how this can be done.
- (c) Find a grid on the next two pages that you can use to enlarge the above kite by a scale factor of 2. Copy the grid and draw the enlargement.

These figures are *not* called enlargements of the kite at the top right of the page, because the shapes of these kites are different than the shape of the one at the top.

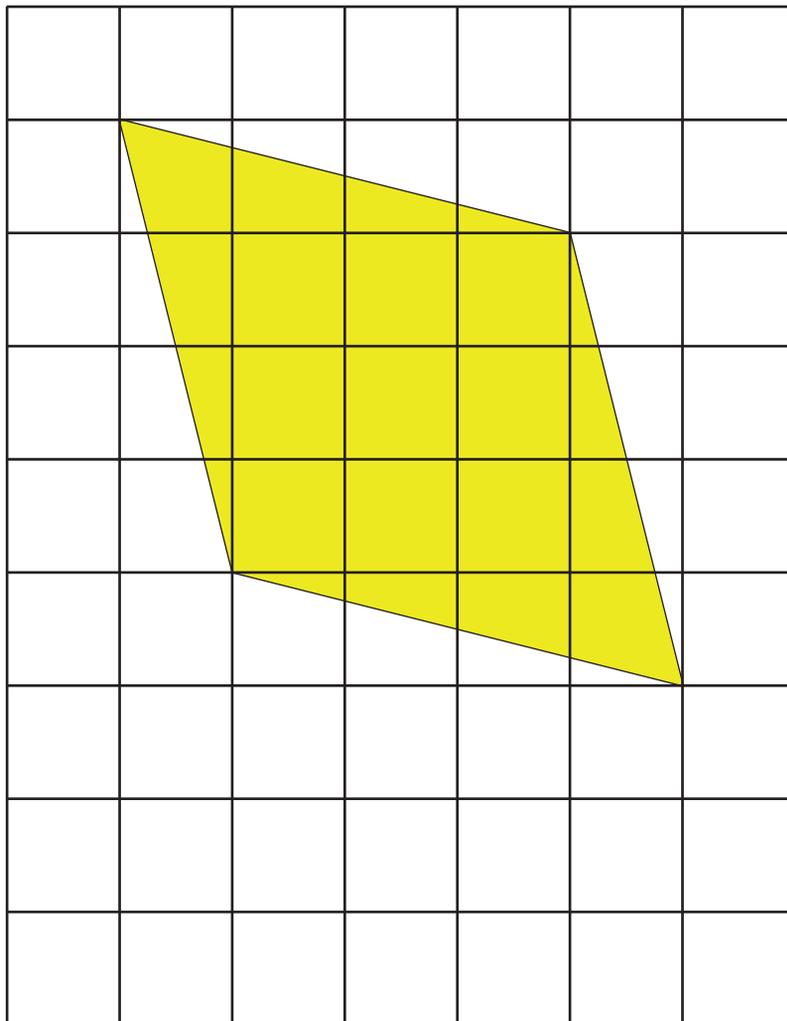




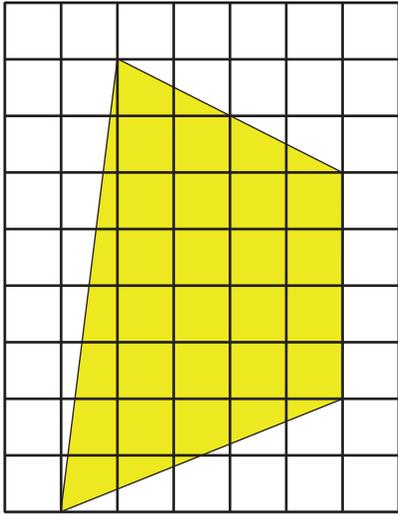
0,25 cm grid



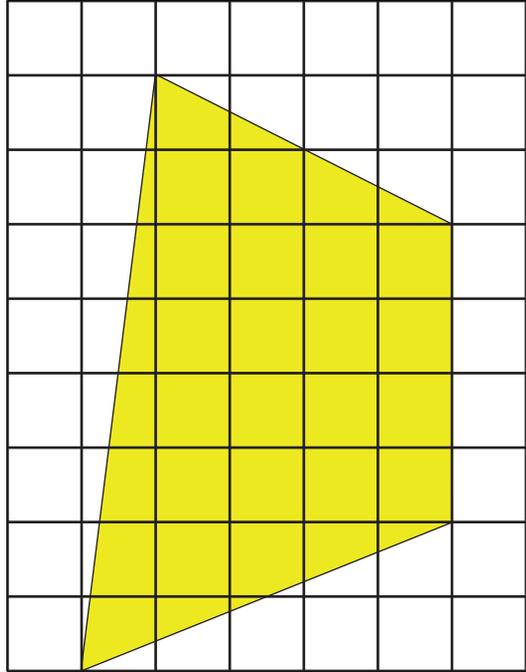
0,5 cm grid



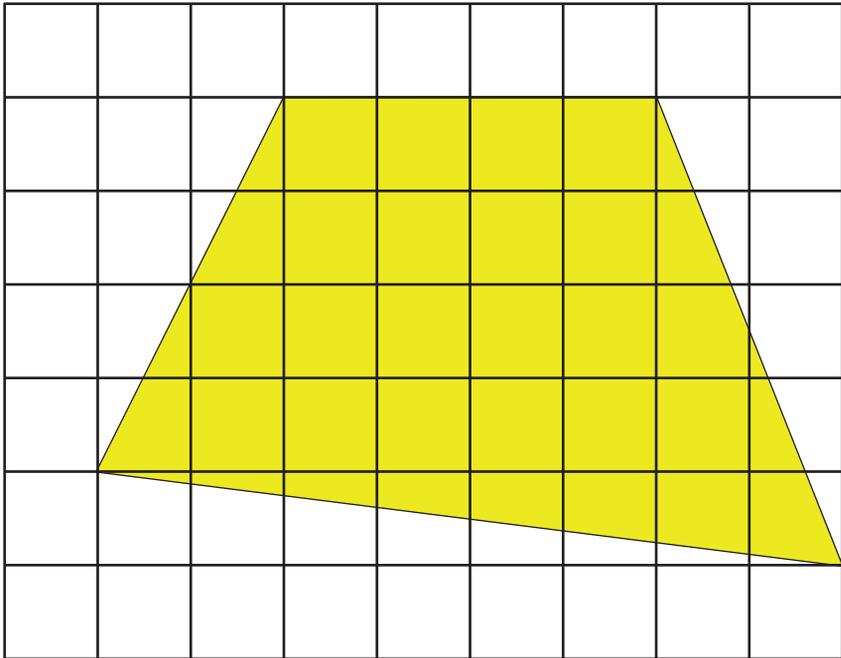
1,5 cm grid



0,75 cm grid

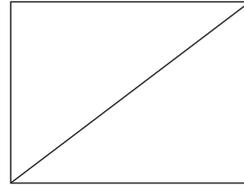


1 cm grid



1,25 cm grid

4. (a) Use your ruler to accurately draw a rectangle with sides of 6 cm and 8 cm on square grid paper. Make sure that your rectangle is “square” and not skew like the red quadrilateral.



- (b) Draw a straight line between two vertices (corners) of your rectangle. This line is called a “diagonal”, and it divides your rectangle into two triangles.



- (c) Measure the length of the diagonal.

5. The side lengths of some rectangles are given in (a) to (d) below. Which of these rectangles do you think are enlargements of the rectangle you have just drawn?

- (a) 9 cm and 11 cm (b) 9 cm and 12 cm
(c) 14 cm and 16 cm (d) 12 cm and 16 cm

6. Accurately draw rectangles with the above dimensions. In each case draw a diagonal as well, and measure the length of the diagonal. Check the prediction you made in question 5. *This will be easier and quicker to do if you work on 1 cm grid paper.*

7. (a) Use your results to the above questions to complete the table below. Predict what the lengths of the diagonals will be in the two rectangles that you have not drawn as yet, namely D and E.

	A	B	C	D	E
Length of rectangle	8	12	16	20	24
Width of rectangle	6	9	12	15	18
Length of diagonal					

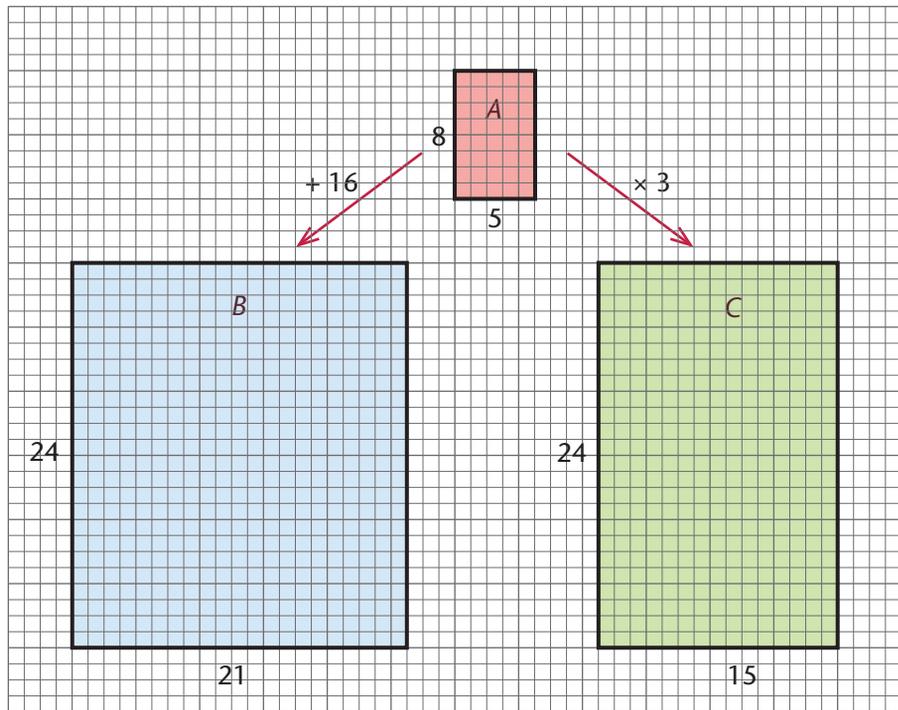
- (b) Draw Rectangles D and E accurately, and measure the diagonals to check your predictions.
8. This question is again about the rectangles you have drawn.
- (a) Which rectangle (B, C, D or E) is 2,5 times as large as A?
(b) Which rectangle is one third as large as E?
(c) Which rectangle is 0,5 times as large as E?

9.2 Enlargements and reductions

1. Nkhangweleni says: *To make an enlargement of a polygon, you just add the same length to all the side lengths, for example 16 units.*

Rebecca disagrees: *No, if you do that the shape will change as well, not just the size. To make an enlargement that keeps the shape you have to multiply all the side lengths by the same number, for example by 3.*

Write your opinion on this matter. You may refer to this diagram.



Rectangle C above is called an **enlargement** of Rectangle A, because it has the *same shape* as Rectangle A.

Rectangle B is larger than Rectangle A, but it is not called an enlargement, because it has a *different shape*.

Rectangle A is called a **reduction** of Rectangle C.

2. The yellow quadrilateral on the 1,5 cm grid on page 352 is called a rhombus. Draw two reductions of the rhombus: one on 1,25 cm grid paper, and one on 0,5 cm grid paper.

Figure A on page 350 is **a reduction by a factor of 3** of Figure C. This means Figure A is **one third as large** as Figure C.

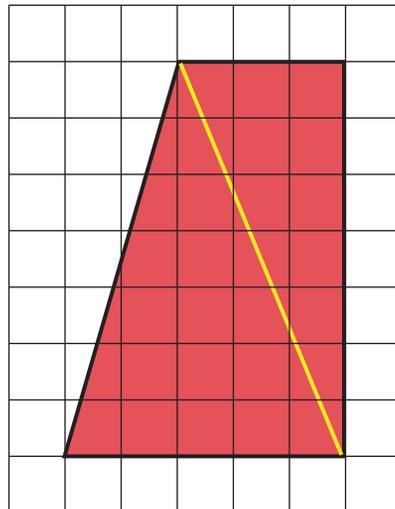
3. This question is about Figures A, B and C on page 350.
 - (a) Is Figure B three times as large as Figure A?
 - (b) Is Figure B two times as large as Figure A?
 - (c) Is Figure B two-thirds the size of Figure C?

4. This question is about the quadrilaterals on pages 352 and 353.
 - (a) Which grid shows a reduction by a factor of 5, of the quadrilateral on the 1,25 cm grid?
 - (b) On which grid is the quadrilateral $\frac{3}{4}$ the size of the one on the 1 cm grid?
 - (c) On which grid is it $\frac{3}{5}$ the size of the one on the 1,25 cm grid?
 - (d) On which grid is it $\frac{5}{4}$ as large as on the 1 cm grid?
 - (e) On which grid is it $\frac{4}{3}$ as large as on the 0,75 cm grid?

5. Draw the following enlargements and reductions of the quadrilateral on the right.

- (a) 1,5 times as large
- (b) $\frac{2}{3}$ the size
- (c) $1\frac{2}{3}$ as large

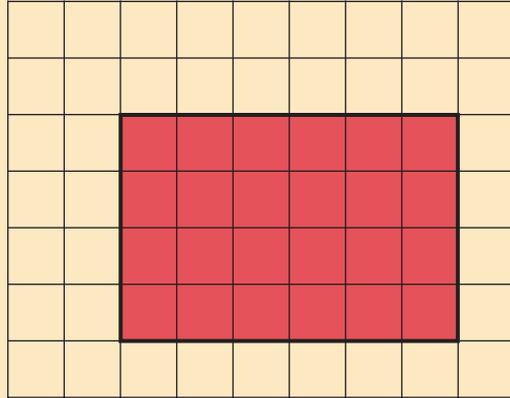
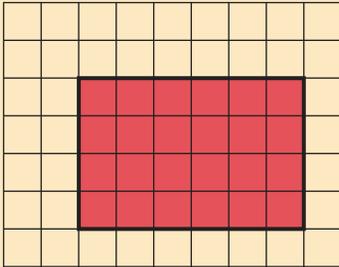
6. Measure all the sides and the yellow diagonal of the quadrilateral on the right, and on each of the enlargements and reductions you have drawn.



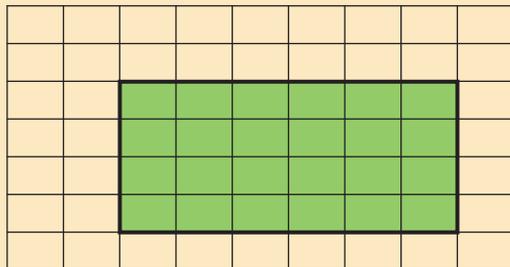
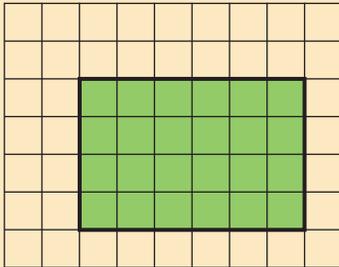
Is it true that the length of each side is increased or decreased by the same scale factor as the figure as a whole?

9.3 Increasing the lengths of two sides only

To make an enlargement or reduction of a rectangle, the lengths of all four sides can be multiplied by the same scale factor.



You can also do something different, namely multiply the lengths of only two opposite sides by a scale factor:



We can say the rectangle is here **stretched** in one direction only.

1. Draw rectangles with the following lengths and widths.
Draw one diagonal in each of the rectangles, and measure the diagonals.
 - (a) 3 cm and 5 cm
 - (b) 4 cm and 5 cm
 - (c) 3 cm and 6 cm
 - (d) 4 cm and 6 cm
2. (a) Draw four new rectangles, by stretching the lengths of each of your rectangles by a factor of 2.
(b) Investigate whether the diagonals also get stretched by a factor of 2.

10.1 Locate positions on a grid

The plan for a new garden at a public building is given on the grid on the next page.

On the plan, benches are indicated in Cells B12 and H6.

- What is indicated in each of the following cells on the plan?

(a) C12	(b) H2	(c) A6
(d) D5	(e) G9	(f) B3
(g) B11	(h) D4	(i) F12
(j) H3	(k) G11	(l) B6
(m) G7	(n) C4	(o) F1
(p) C3	(q) G3	(r) F2
(s) F10	(t) E8	(u) E5
(v) A5	(w) F11	(x) G2
- Where will the water fountain be? Write down the cell number(s).
- Walking through the flower beds and shrub beds will not be allowed in the garden. Through which cells will you have to walk, if you want to take the shortest route from the bench in B12 to the toilet in F1?
- Draw a grid like the one on the next page but with more cells, with Columns A to O, and Rows 1 to 15. Use square grid paper.
 - Shade the following cells on your grid:
E7, O7, F6, N6, F8, N8, G5, M5, G9, M9
 - Which other cells do you have to shade to form a square?

12		bench	tree			pond		
11		tree				flower bed		
10						flower bed		
9		water fountain					tree	
8				tree				
7							tree	
6	pond	pond						bench
5	pond			shrubs	shrubs			
4			shrubs	shrubs				tree
3		shrubs	shrubs				tree	tree
2						shed	shed	shed
1						toilet		
	A	B	C	D	E	F	G	H

10.2 Giving directions using a map

A map of a certain area is given on the next page.

The thick blue lines indicate highways.

The two highways are called Great North Road and Link Road.



The red lines indicate tarred roads.

The tarred roads are numbered R31 and R88.



The brown lines indicate gravel roads.

The gravel roads are numbered G98, G54 and G42.



The blue broken line indicates a river.



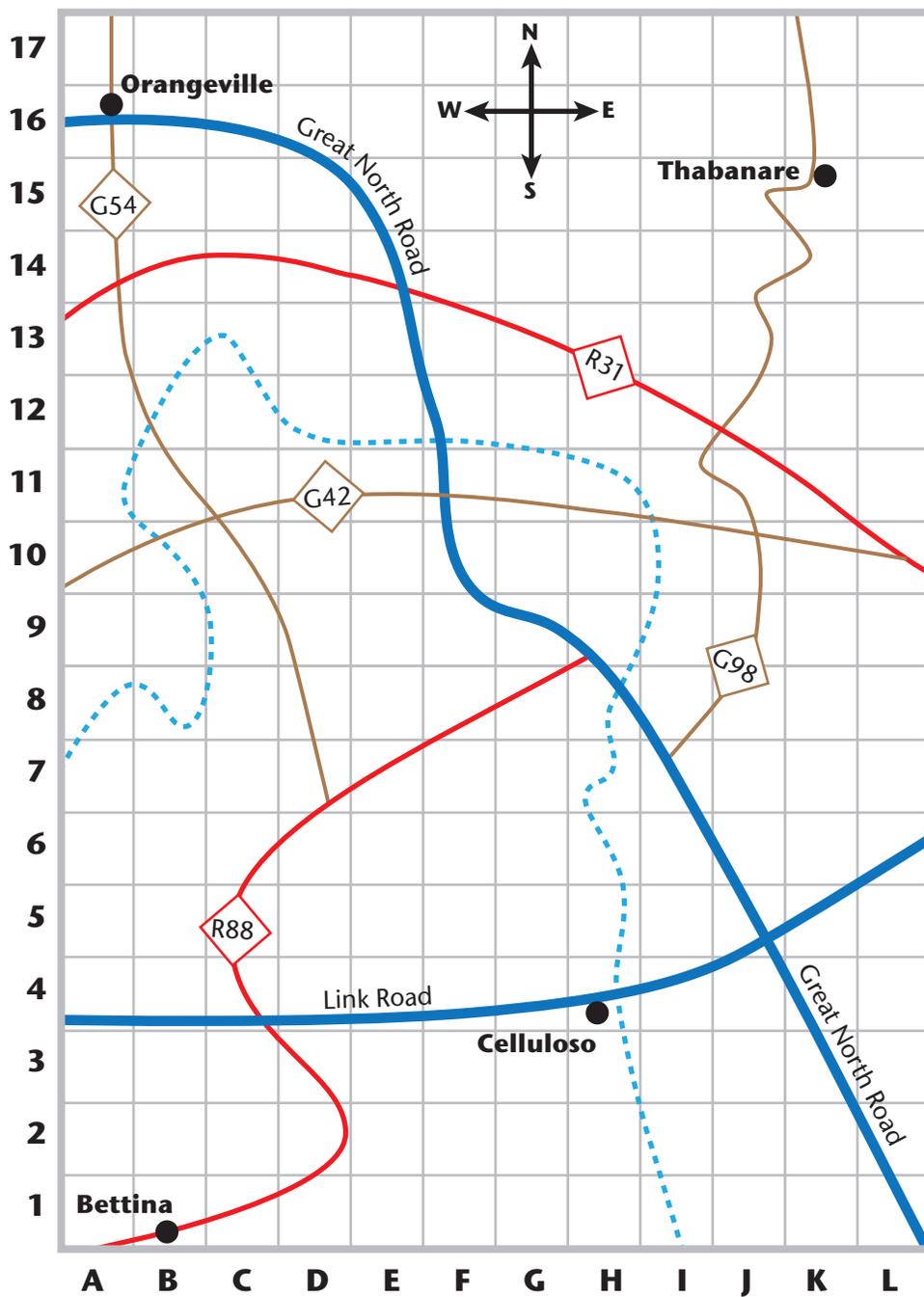
The two highways cross in Cell J5 on the map.

1. In which cells does the Great North Road cross the river?
2. In which cell does the Link Road highway cross the river?
3. Which road crosses the river in Cell B10?
4. Which roads cross in Cell J10?

Each cell is an area of 10 km by 10 km.

The town Orangeville is in Cell A16.

5. Approximately how far is it from Orangeville to the crossing of the two highways?



To travel from the town Bettina in Cell B1 to the town Thabanare in Cell K15 you could follow these directions:

Travel northwards on the R88, cross the Link Road highway and continue until you meet the Great North Road. Turn left onto the Great North Road. Pass the crossing with the gravel road G42, cross the river and continue until you get to the R31. Turn right onto the R31 and keep going in a southeasterly direction up to the crossing with the G98. Turn left and travel northwards until you get to Thabanare, which is on the right side of the road.

6. If you follow the directions above, through which cells will you pass between the G54 turnoff and the Great North Road?
7. If you travel from Bettina to Thabanare, but turn right in Cell C4 onto the Link Road highway and continue, which route can you follow to Thabanare? Describe the route in a similar way to the description given at the top of this page.
8. The town Celluloso is in Cell H4. Your friend wants to travel from Thabanare to Celluloso. He does not have a map. Write instructions to tell him how he should travel, using the shortest road.
9. Your friend tells you that he wants to travel as little gravel road as possible, because the G98 is in a very poor condition. Describe an alternative route that he can take, even if it is longer.

The farm Ijabuna is in Cell E9. You can also describe this location in the following way, by referring to the roads:

Ijabuna is situated between the G54 and Great North Road, and between the G42 and the R88.

10. Gabriel owns a farm in Cell I10. Describe the location of his farm in a way similar to the way above, without referring to cells.
11. Describe the shortest route from Orangeville to Bettina.
12. Describe the shortest route from Orangeville to Gabriel's farm (I10).
13. Eric has a farm close to the G54 bridge over the river. Describe a route from Eric's farm to Thabanare.

11.1 Tossing a coin

1. (a) Imagine you are throwing a ball against a wall and trying to catch it when it bounces back. Do you think you will catch it more often than failing to catch it?
 - (b) Write a short paragraph to explain why you think so.
 - (c) Talk to two classmates. Tell each other what you answered for (a) and (b).

2. (a) Imagine you are tossing a coin many, many times. Do you think you will get “heads” more often than “tails”?
 - (b) Write a short paragraph to explain why you think so.
 - (c) Talk to two classmates. Tell each other what you answered for (a) and (b).

3. Work with a classmate. Make a table like the one below to record your results. Take turns to toss the coin 10 times. Use tallies in Columns A and B.

When we say “**heads**” we mean the side of the coin that shows our country’s coat of arms. When we say “**tails**” we mean the side that shows the value of the coin.

	A	B	C	D
	“Heads”	“Tails”	Total “heads”	Total “tails”
First 10 trials				
Second 10 trials				
Third 10 trials				
Fourth 10 trials				
Fifth 10 trials				

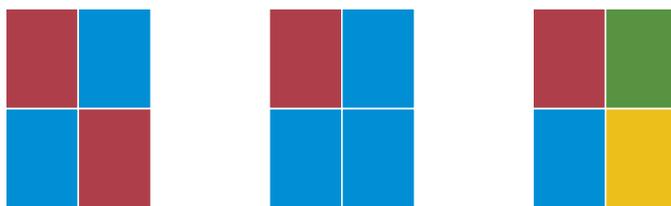
11.2 Spinner experiments

Make your own spinner. Look at the photograph below.

Take a square piece of cardboard and make a hole in the centre. Put your pencil through the hole. Then make a dot or mark at the centre of each of the sides of the square.



Prepare three sheets of A4 paper. Make sure the parts with different colours meet at a central point.



Examples of Sheets 1, 2 and 3

Sheet 1: Colour two quarters of the sheet red and two quarters blue.

Sheet 2: Colour one quarter of the sheet red and three quarters of the sheet blue.

Sheet 3: Colour one quarter of the sheet red, one quarter blue, one quarter green and the last quarter yellow.

Prepare to gather the spinner data

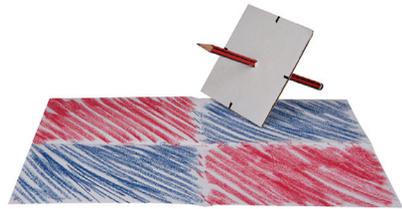
Use three sheets of *squared paper*. On each sheet draw a 10 by 10 square. Label the sheets: Experiment 1, Experiment 2, Experiment 3. You also need coloured pencils in each of the colours you used to prepare the experiment sheets, that is, red, blue, green, yellow.

You are now ready to gather data with the spinner experiments. Work with a classmate. Take turns to spin and record the data.

Experiment 1

Put the spinner on the central point of Experiment sheet 1 and spin it repeatedly.

Each time the dot lands on a red area, shade a block in the 10 by 10 square RED. Each time the dot lands on a blue area, shade a block BLUE. Make sure you *shade the blocks from left to right, row after row*, until all 100 blocks are shaded.



- (a) Don't count yet. What fraction of the 100 squares do you expect to be red? And blue? Why do you say so?
- (b) Just look at your data sheet by holding it at the end of your arm. Does it look as if your expectation was correct?
- (c) Count the number of red squares and write the number of red squares as a fraction out of 100. Is the fraction close to what you expected?
- (d) Are you surprised that you got this number of red squares? Why do you say that?
- (e) Compare your data sheet with the data sheets of other classmates. Are their results similar to yours?
- (f) Is there any pattern in the colours of the blocks, or do you think the pattern is random?
- (g) What is your longest run of red blocks? How many long runs of red are there in your data?

A run of a colour means the same colour is repeated without another colour in between.

-
- (h) What is your longest run of blue blocks? How many long runs of blue are there in your data?
 - (i) Compare your data about long runs of a colour with that of your classmates. What is the most common long run? What is the most unusual long run?
2. Do you think the different colours have the same chance in this experiment? Why do you say so?
 3. Imagine gathering data by colouring squares; but instead of using a spinner, you spin a coin. If it lands “heads” up you colour a square RED, and if it lands “tails” up you colour a square BLUE. Do you think you will get a similar set of data as in Experiment 1 or a very different set? Why do you say so?

Experiment 2

Spin the spinner on Experiment sheet 2. Each time the dot lands on a red area, shade a block in the 10 by 10 square RED. Each time the dot lands on a blue area shade a block BLUE. Make sure you shade the blocks from left to right, row after row, until all 100 blocks are shaded.

1. Answer all the questions asked in Experiment 1, but answer them with data from Experiment 2.
2. Can you think of an experiment with a die that will give similar data as this experiment? Explain your answer.

Experiment 3

Spin the spinner on Experiment sheet 3. Shade the 100 blocks in the colour determined by the spinner as you spin each time.

1. Answer all the questions asked in Experiment 1, but answer them with data from Experiment 3.
2. Can you think of an experiment with coins or dice that will give similar data as this experiment? Explain your answer.

11.3 The Subtraction Game

In this section and the next, you will play and analyse simple probability games with two dice, and use mathematics to work out if the rules of the games are fair or not.



- (a) Imagine you are rolling two dice, a blue die and a red die. The blue die may show 1 and the red die may show 6, and we may write (1;6) to represent this outcome. Write down all the other possible outcomes.
(b) Suppose you subtract the smaller number from the bigger number in each case. What are all the possible results when you subtract the numbers on the two dice?

How to play the Subtraction Game

Play with a classmate. You each need a die. Each player chooses a set of three numbers from the possible results, for example:

0, 4, 5 or 1, 2, 3

Rules: Each player rolls his or her die. Look at the numbers on the dice and subtract the smaller number from the bigger number. If the difference is 0, 4 or 5, the player who chose these numbers scores one point. If the difference is 1, 2 or 3, the player who chose these numbers scores the point.

The game ends after 12 rounds; that is, after you have rolled your dice 12 times. The player with the most points wins the game.

2. Play the Subtraction Game 10 times.
 - (a) Do you think the rules of the game are fair? Why do you say so?
 - (b) Which result do you get most often when you play the Subtraction Game? Why do you think this happens?

Rules that are fair give both players an equal chance to win the game.

3. Analyse all the possible outcomes for the Subtraction Game.

- (a) Copy the table below onto squared paper.
- (b) Complete the table to show all the possible outcomes if you subtract the smaller number from the bigger number when you roll two dice.

Example: Player A rolls a 2 and Player B rolls a 5. Thus: $5 - 2 = 3$. Find the block where Player A's Column 2 and Player B's Column 5 meet and write 3 in it.

		Player A's die					
		1	2	3	4	5	6
Player B's die	1						
	2						
	3						
	4						
	5						
	6						

There are six possible outcomes for each die. They are 1, 2, 3, 4, 5 and 6.

In this Subtraction Game there are six possible results. They are 0, 1, 2, 3, 4 and 5. There are **36 different ways** in which the two dice can combine to give these six results. The 36 ways are the **possible outcomes** of the game.

- (c) Think again about the rules of the Subtraction Game. Can you now explain why the rules are not fair?
 - (d) How many chances do you have altogether to get 0, 4 or 5?
 - (e) How many chances do you have altogether to get 1, 2 or 3?
 - (f) If you play the Subtraction Game 30 times, what fraction of the games do you expect to win if you choose 0, 4, 5 as your numbers?
4. How can you change the rules of the Subtraction Game to make the game fair? Write down the new rules.

11.4 The Addition Game

1. Imagine you roll two dice and add the two numbers. Write down the possible results that you could get.
2. If you roll one die 10 times, what numbers do you think you will get? Write down why you say so.
3. Make a table like the one you made for the Subtraction Game to find all the possible outcomes when you roll two dice and add the numbers.
4. Copy the frame below. Use the table you completed in question 3 to make a pictograph that shows the number of ways in which each of the eleven results (totals) can be obtained. For example, the result 5 can be obtained in four different ways.

There are 11 possible results (totals) in the Addition Game. They are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

There are 36 different ways to get the results. The 36 ways are the **possible outcomes** of the game.

			x							
			x							
			x							
			x							
2	3	4	5	6	7	8	9	10	11	12
Possible outcomes: roll two dice and add the numbers										

5. Work with a classmate. Make fair rules for an addition game and write them down.
6. Play your addition game 10 times to test it out. Are 10 times enough times to play to decide if your rules are fair? Why do you say so?

Rules that are fair give both players an equal chance to win the game each time they play.

