MATHEMATICS

Grade 7

CAPS

Learner Book

2017 Edition

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Chapter 1
Working with whole numbers

1.1 Revision

Do not use a calculator at all in section 1.1.

build numbers up and break numbers down

1. Write each of the following sums as a single number:
   (a) 4 000 + 800 + 60 + 5
   (b) 8 000 + 300 + 7
   (c) 40 000 + 9 000 + 200 + 3
   (d) 800 000 + 70 000 + 3 000 + 900 + 2
   (e) 8 thousands + 7 hundreds + 8 units
   (f) 4 hundred thousands + 8 ten thousands + 4 hundreds + 9 tens

The word **sum** is used to indicate two or more numbers that have to be added.

The answer obtained when the numbers are added, is also called the **sum**. We say: 20 is the sum of 15 and 5.

2. What is the sum of 8 000 and 24?

3. Write each of the numbers below as a sum of units, tens, hundreds, thousands, ten thousands and hundred thousands, like the numbers that were given in question 1(e) and(f).
   (a) 8 706 (b) 449 203
   (c) 83 490 (d) 873 092

When a number is written as a sum of units, tens, hundreds, thousands etc., it is called the **expanded notation**.

4. Arrange the numbers in question 3 from smallest to biggest.

5. Write the numbers in expanded notation (for example, 791 = 700 + 90 + 1).
   (a) 493 020 (b) 409 302 (c) 490 032 (d) 400 932

6. Arrange the numbers in question 5 from biggest to smallest.

7. Write each sum as a single number.
   (a) 600 000 + 40 000 + 27 000 + 100 + 20 + 34
   (b) 320 000 + 40 000 + 8 000 + 670 + 10 + 5
   (c) 500 000 + 280 000 + 7 000 + 300 + 170 + 38
   (d) 4 hundred thousands + 18 ten thousands + 4 hundreds + 29 tens + 5 units

8. Write each sum as a single number.
   (a) 300 000 + 70 000 + 6 000 + 400 + 80 + 6
   (b) 400 000 + 20 000 + 2 000 + 500 + 10 + 3
   (c) 500 000 + 40 000 + 7 000 + 300 + 60 + 6
   (d) 800 000 + 90 000 + 7 000 + 800 + 90 + 8
   (e) 300 000 + 110 000 + 12 000 + 400 + 110 + 3

9. In each case, add the two numbers. Write the answer in expanded form and also as a single number.
   (a) The number in 8(a) and the number in 8(b)
   (b) The number in 8(c) and the number in 8(b)
   (c) The number in 8(c) and the number in 8(a)
   (d) The number in 8(d) and the number in 8(a)

10. (a) Subtract the number in 8(b) from the number in 8(d).
   (b) Are the numbers in 8(b) and 8(e) the same?
   (c) Subtract the number in 8(a) from the number in 8(b).

11. Write each of the following products as a single number:
   (a) 2 × 3
   (b) 2 × 3 × 5
   (c) 2 × 3 × 5 × 7
   (d) 2 × 3 × 5 × 7 × 2
   (e) 2 × 3 × 5 × 7 × 2 × 2

The word **product** is used to indicate two or more numbers that have to be multiplied.

The answer obtained when numbers are multiplied, is also called the **product**. We say: 20 is the product of 2 and 10.

12. (a) What is the product of 20 and 500?
   (b) Write 1 000 as a product of 5 and another number.
   (c) Write 1 000 as a product of 50 and another number.
   (d) Write 1 000 as a product of 25 and another number.
   (e) What is the product of 2 500 and 4?
   (f) What is the product of 250 and 40?

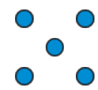
13. In the table on the right, the number in each yellow cell is formed by adding the number in the red row above it to the number in the blue column to its left. Copy the table and fill the correct numbers in all the empty yellow cells.

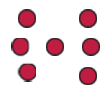
14. The table on the next page is formed in the same way as the table on the right. Copy the table and fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

+	2	3	4	5
10				
20				
30			34	
40				
50				
60		63		65
70				

+	8	5	4	9	7	3	6	18	36	57
7										64
3						6				
9										
5										
8										
6										
4										

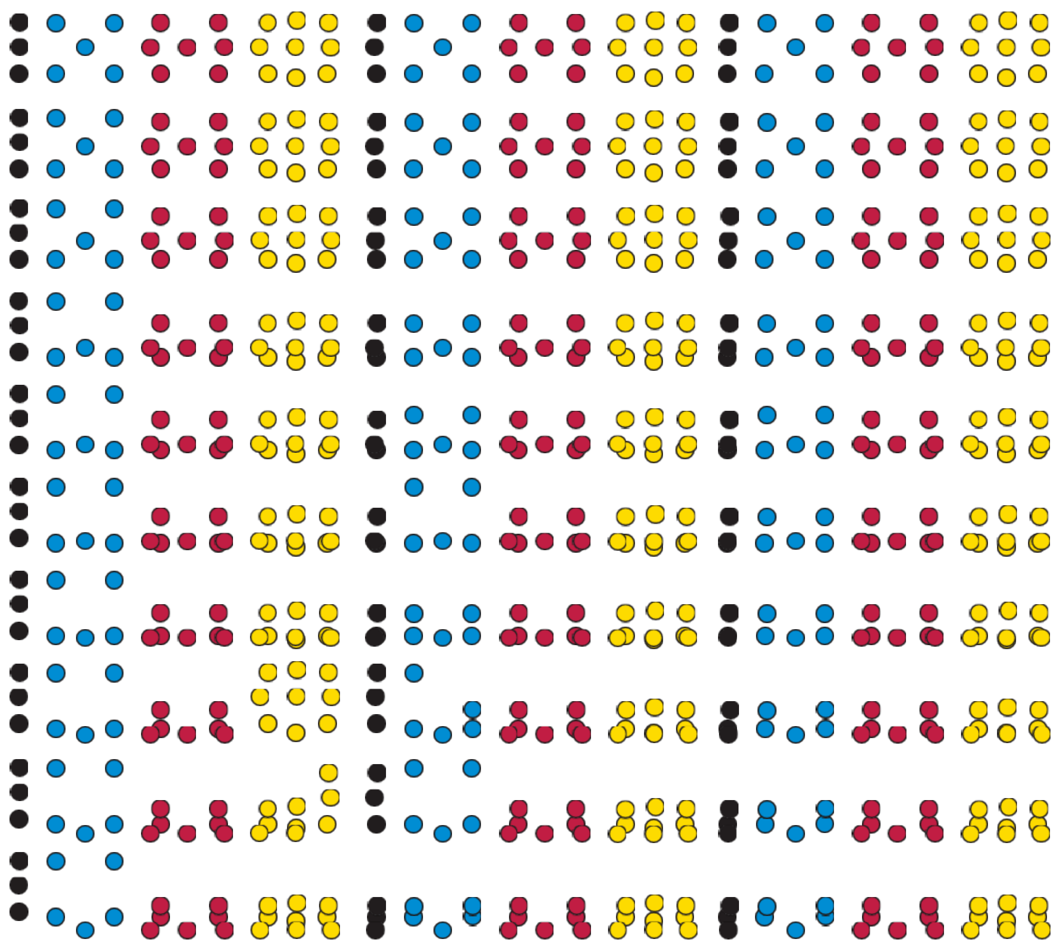
multiples

1. In the arrangement below, the blue dots are in groups likethis: 

   The red dots are in groups like this: 

   (a) How would you go about finding the number of blue dots below, if you do not want to count them one by one?

   (b) Implement your plan, to find out how many blue dots there are.



Suppose you want to know how many black dots there are in the arrangement on page 3. One way is to **count in groups** of three. When you do this, you may have to point with your finger or pencil to keep track.

The counting will go like this: *three*, *six*, *nine*, *twelve*, *fifteen*, *eighteen* . . .

Another way to find out how many black dots there are is to **analyse** the arrangement and **do some calculations**. In the arrangement, there are ten rows of threes from the top to the bottom, and three columns of threes from left to right, just as in the table alongside.

3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3

One way to calculate the total number of black dots is to do 3 × 10 = 30 for the dots in each column, and then 30 + 30 + 30 = 90. Another way is to add up each row (3 + 3 + 3 = 9) and then multiply by 10: 10 × 9 = 90. A third way is to notice that there are 3 × 10 = 30 groups of three, so the total is 3 × 30 = 90.

2. When you determined the number of blue dots in question 1(b), did you count in fives, or did you analyse and calculate, or did you use some other method? Now use a different method to determine the number of blue dots and check whether you get the same answer as before. Describe the method that you used.

3. The numbers that you get when you count in fives are called **multiples** of five. Copy the table below and draw circles around all the multiples of 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

4. How many red dots are there in the arrangement on page 3? Describe the method that you use to find this out.

5. (a) Underline all the multiples of 7 in the table you drew for question 3.
   (b) Which multiples of 5 in the table are also multiples of 7?

A number that is a multiple of 5, and also a multiple of 7, is called a **common multiple** of 5 and 7.

6. How many yellow dots are there in the arrangement on page 3? Describe the method that you use to find this out.

7. (a) Cross out all the multiples of 9 in the table you drew for question 3.
   (b) Which numbers in the table in question 3 are common multiples of 7 and 9?

8. (a) Look at the numbers in the yellow cells of the table below. How are these numbers formed from the numbers in the red row and the numbers in the blue column?
   (b) Copy the table below and fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

×	8	5	4	9	7	3	6	2	10	20
7										
3										
9									90	
5			20							
8										
6										
4										
2										
10		50								
20						60				

9. Copy the table below and write down the first thirteen multiples of each of the numbers in the column on the left. The multiples of 4 are already written in, as an example.

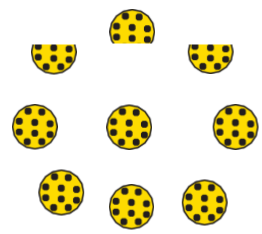
1	2	3	4	5	6	7	8	9	10	11	12	13
2												
3												
4	8	12	16	20	24	28	32	36	40	44	48	52
5												
6												
7												
8												
9												
10												
11												
12												
13												

10. Copy and complete this table. For some cells, you may find your table of multiples on the previous page helpful.

×	6	2	7	9	4	5	3	8	10	50
8										
6										
7										
9										
5										
3										
4										
2										

11. Go back to the table you drew for question 8(b). If you can easily fill in the numbers in some of the open spaces now, do it.

12. Suppose there are 10 small black spots on each of the yellow dots in the arrangement on page 3. How many small black spots would there be on all the yellow dots together, in the arrangement on page 3?

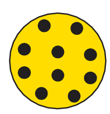


multiples of 10, 100, 1 000 and 10 000

1. How many spotted yellow dots are there on page 7? Explain what you did to find out.

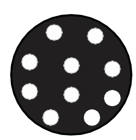
2. How many learners are there in your class? Suppose each learner in the class has a book like this. How many spotted yellow dots are there on the same page (that is, on page 7) of all these books together?

3. Each yellow dot has 10 small black spots, as you can see on this enlarged picture.



   (a) How many small black spots are there on page 7?
   (b) How many small black spots are there on page 7 in all the books in your class?

4. Here is a very big enlargement of one of the black spots on the yellow dots. There are 10 very small white spots on each small black spot. How many very small white spots are there on all the black spots on page 7?



5. (a) How many very small white spots are there on 10 pages like page 7?
   (b) How many very small white spots are there on 100 pages like page 7?

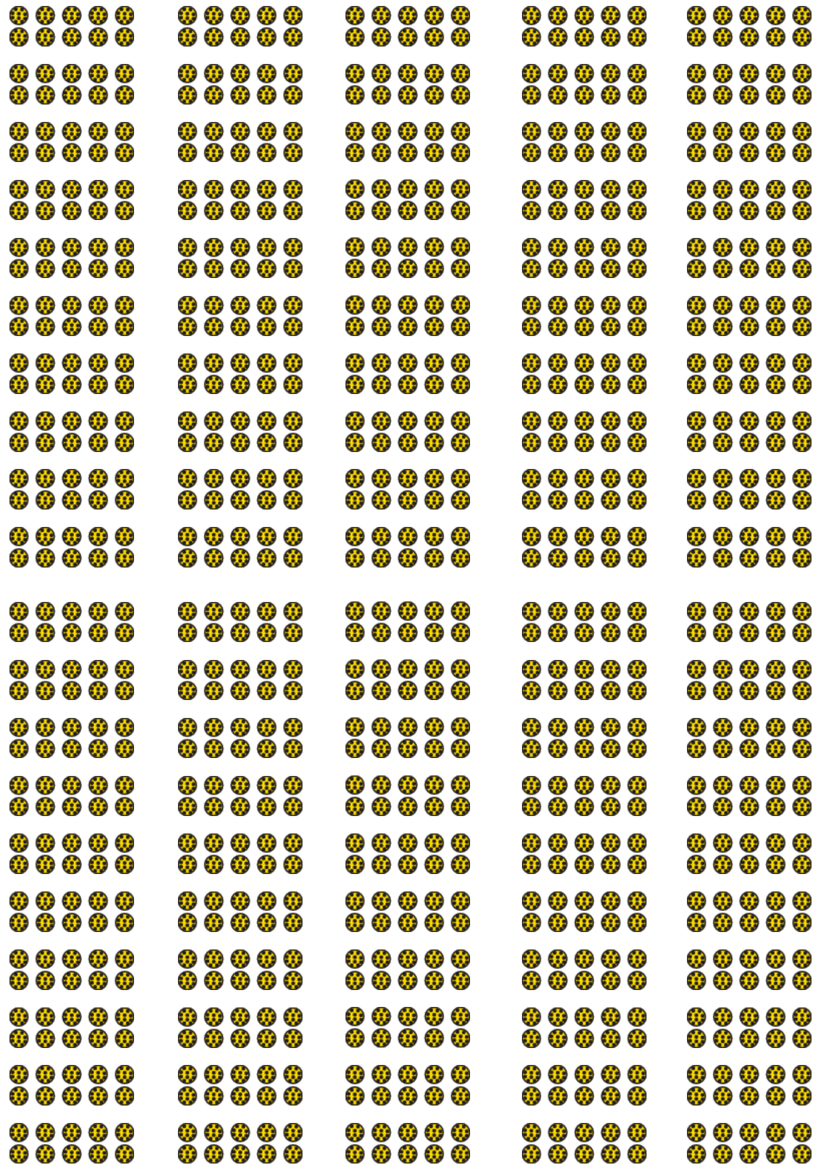
10 tens are a **hundred**: 10 × 10 = 100

10 hundreds are a **thousand**: 10 × 100 = 1 000

10 thousands are a **ten thousand**: 10 × 1 000 = 10 000

10 ten thousands are a **hundred thousand**: 10 × 10 000 = 100 000

10 hundred thousands are a **million**: 10 × 100 000 = 1 000 000



6. (a) Write 7 000 + 600 + 80 + 4 as a single number.
   (b) Write 10 times the number in (a) in expanded notation and as a single number.
   (c) Write 100 times the number in (a) in expanded notation and as a single number.

7. Write each of the following numbers in expanded notation:

(a) 746	(b) 7 460	(c) 74600
(d) 746 000	(e) 7 460 000	

8. (a) Write 10 000 as a product of 10 and one other number.
   (b) Write 10 000 as a product of 100 and one other number.
   (c) Write 100 000 as a product of 10 and one other number.
   (d) Write 100 000 as a product of 1 000 and one other number.
   (e) Write 1 000 000 as a product of 1 000 and one other number.

9. Copy the table below and fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

×	10	20	30	40	50	60	70	80	90	100
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										

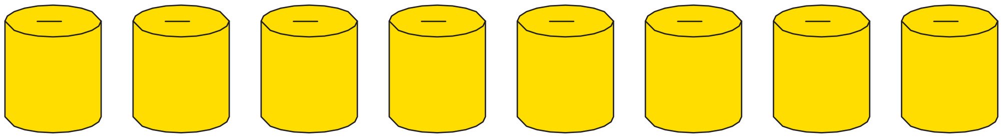
10. Copy the table below and fill in all the cells in the table for which you know the answers immediately. Leave the other cells open for now.

×	100	200	300	400	500	600
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						

11. How many multiples of 10 are smaller than 250?
    (a) Estimate.
    (b) Check your estimate by writing down the multiples.

12. In each case, first estimate, then check by writing all the multiples down and counting them.
    (a) How many multiples of 100 are smaller than 2 500?
    (b) How many multiples of 250 are smaller than 2 500?
    (c) How many numbers smaller than 2 500 are multiples of both 100 and 250?
    (d) How many numbers smaller than 2 500 are multiples of both 250 and 400?

13. In each of the tins below, there are three R10 notes, three R20 notes, three R100 notes and three R200 notes.



    Zain wants to know what the total value of all the R10 notes in all the tins is. He decides to find this out by counting in 30s, so he says: *thirty*, *sixty*, *ninety* ... and so on while he points at one tin after another.
    (a) Complete what Zain started to do.
    (b) Count in 300s to find the total value of all the R100 notes in all the tins.

14. (a) How much money is there in total in the eight yellow tins in question 13?
    (b) Join with two classmates and tell them how you worked to find the total amount of money.

15. (a) Investigate what is easiest for you, to count in twenties or in thirties or in fifties, up to 500.
    (b) Many people find it easier to count in fifties than in thirties. Why do you think this is so?

16. What do you expect to be the most difficult, to count in forties or in seventies or in nineties? Investigate this and write a short report.

Here is some advice that can make it easier to count in certain counting units, for example in seventies.

It feels easier to count in fifties than in seventies because you get to multiples of 100 at every second step:

fifty, **hundred**, one hundred and fifty, **two hundred**, two hundred and fifty, **300**, 350, **400**, 450, **500** ... and so on.

When you count in seventies, this does not happen:

seventy, one hundred and forty, two hundred and ten, two hundred and eighty ...

It may help you to cross over the multiples of 100 in two steps each time, like this:

70 + *30* → **100** + *40* → **140** + *60* → **200** + *10* → **210** + *70* → **280** ...

30 + *40* = *70* *60* + *10* = *70*

In this way, you make the multiples of 100 act as "stepping stones" for your counting.

17. (a) Count in forties up to 1 000. Try to use multiples of 100 as stepping stones. You can write the numbers while you count.
    (b) Write down the first twenty multiples of 80.
    (c) Write down the first twenty multiples of 90.
    (d) Write down the first ten multiples of 700.

18. Copy and complete this table.

×	60	50	70	90	40	20	30	80
8								
6								
7								
9								
5								
3								
4								
2								
70								
30								
60								
80								
40								
90								
50								
20								

doubling and halving

1. Write the next eight numbers in each pattern:
   (a) 1 2 4 8 16 32
   (b) 3 6 12 24
   (c) 5 10 20 40
   (d) 5 10 15 20
   (e) 6 12 24 48

2. Which pattern or patterns in question 1 are *not* formed by **repeated doubling**?

The pattern 3 6 12 24 48 ... may be called the **repeated doubling pattern** that starts with 3.

3. Copy the table. Write the first nine terms of the repeated doubling patterns that start with the numbers in the left column of the table. The pattern for 13 has been completed as an example.

2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13	26	52	104	208	416	832	1 664	3 328
14								
15								
16								
17								
18								
19								

Doubling can be used to do multiplication.

For example, 29 × 8 can be calculated as follows:

8 doubled is 16, so 16 = 2 × 8 (step1)

16 doubled is 32, so 32 = 4 × 8 (step2)

32 doubled is 64, so 64 = 8 × 8 (step3)

64 doubled is 128, so 128 = 16 × 8 (step 4). Doubling again will go past 29 × 8.

16 × 8 + 8 × 8 + 4 × 8 = (16 + 8 + 4) × 8 = 28 × 8.

So 28 × 8 = 128 + 64 + 32 which is 224. So 29 × 8 = 224 + 8 = 232.

4. Work as in the above example to calculate each of the following. Write only what you need to write.

   (a) 37× 21 (b) 17 × 41

5. Continue each repeated halving pattern as far as you can:
   (a) 1 024 512 256 128
   (b) 64 000 32 000 16 000 8 000

Halving can also be used to do multiplication.

For example, 37 × 28 can be calculated as follows:

100 × 28 = 2 800. Half of that is 50 × 28, which is half of 2 800, that is 1 400.

Half of 50 × 28 is half of 1 400, so 25 × 28 is 700.

10 × 28 = 280, so 25 × 28 + 10 × 28 = 980, so 35 × 28 = 980.

2 × 28 = 2 × 25 + 2 × 3 = 56, so 37 × 28 is 980 + 56 = 1 036.

6. 80 × 78 = 6 240. Use this information to work out each of the following:
   (a) 20 × 78
   (b) 37 × 78

If chickens cost R27 each, how many chickens can you buy with R2 400? A way to use halving to work this out is shown below.

100 chickens cost 100 × 27 = R2 700. That is more than R2 400. Fifty chickens cost half as much, that is R1 350.

So I can buy 50 chickens and even more.

Half of 50 is 25, and half of R1 350 is R675.

So 75 chickens cost R1 350 + R675, which is R2 025. So there is R375 left.

Ten chickens cost R270, so 85 chickens cost R2 025 + R270 = R2 295. There is R105 left.

Three chickens cost 3 × R25 + 3 × R2 = R81.

I can buy 88 chickens and that will cost R2 376.

	Total cost	Thinking
100	R2 700	
50	R1 350	half of R2 700
25	R675	half of R1 350
75	R2 025	50 + 25 chickens
10	R270	10 × R27
85	R2 295	75 + 10 chickens
3	R81	3 × R27
88	R2 376	85 + 3 chickens

7. Use halving as in the above example to work out how many books at R67 each a school can buy with R5 000. Copy and use table on left to show your calculations.

	Total cost	Thinking

using multiplication to do division

1. R7 500 must be shared between 27 netball players. The money is in R10 notes, and no small change is available.
   (a) How much money will be used to give each player R100?

(b) Do you think there is enough money to give each player R200?
(c) Do you think there is enough money to give each player R300?
(d) How much of the R7 500 will be left over, if each player is given R200?
(e) Is there enough money left to give each player R50 more, in other words a total of R250 each?
(f) What is the highest amount that can be given to each player, so that lessthan R270 is left over? Remember that you cannot split up the R10 notes.

2. Work like you did in question 1 to solve this problem:

   There is 4 580 m of string on a big roll. How many pieces of 17 m each can be cut from this roll?

   Hint: You may start by asking yourself how much string will be used if you cut off 100 pieces of 17 m each.

3. Work like you did in questions 1 and 2 to solve this problem:

   A shop owner has R1 800 available with which he can buy chickens from a farmer. The farmer wants R26 for each chicken. How many chickens can the shop owner buy?

What you actually did in questions 1, 2 and 3 was to calculate 7 500 ÷ 27, 4 580 ÷ 17 and 1 800 ÷ 26. You solved division problems. Yet most of the work was to do multiplication, and a little bit of subtraction.

When you had to calculate 1 800 ÷ 26 in question 3, you may have asked yourself:

With what must I multiply 26, to get as close to 1 800 as possible?

Division is called the **inverse** of multiplication.
Multiplication is called the **inverse** of division.
Multiplication and division are **inverse operations**.

1.2 Ordering and comparing whole numbers

how far can you count, and how far is far?

1. How long will it take to count to a million? Let us say it takes one second to count each number. Find out how long is one million seconds. Work in your exercise book. Give your final answer in days, hours and seconds.

2. Write 234 500 320 in words.

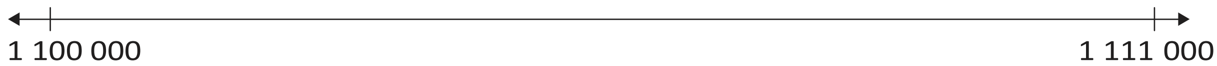
3. In each case write one of the symbols > or < to indicate which number is the smaller of the two.

(a) 876 243		876 234	(b) 534 616		543 016
(c) 701 021		698 769	(d) 103 232		99 878

4. Copy the number lines. In each case, place the numbers on the number line as carefully as you can.

   (a) 185 000; 178 000; 170 900; 180 500

   (b) 1 110 000; 1 102 900; 1 100 500; 1 105 050



The first row in the table shows the average distances of the planets from the sun. These distances are given in **millions of kilometres**.

One million kilometres is **1 000 000 km**.

The distances from the sun are called average distances, because the planets are not always the same distance from the sun. Their orbits are not circles.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance from the sun	58 million km	108 million km	150 million km	228 million km	778 million km	1 427 million km	2 870 million km	4 497 million km
Equatorial diameter	4 880 km	12 102 km	12 756 km	6 794 km	142 800 km	120 000 km	52 400 km	49 500 km

5. Which planet is the second farthest planet from the sun?
6. How does Mars' distance from the sun compare to that of Venus? Give two possible answers.
7. Arrange the planets from the smallest to the biggest.

Sometimes we do not need to know the exact number or exact amount. We say a loaf of bread costs about R10, or a bag of mealie meal costs about R20. The loaf of bread may cost R8 or R12 but it is close to R10. The mealie meal may cost R18 or R21 but it is close to R20.

When you read in a newspaper that there were 15000 spectators at a soccer game, you know that that is not the actual number. In the language of mathematics we call this process **rounding off** or **rounding**.

rounding to 5s, 10s, 100s and 1 000s

To round off to the **nearest 5**, we round numbers that end in 1 or 2, or 6 or 7 **down** to the closest multiple of 5. We round numbers that end in 3 or 4, or 8 or 9 **up** to the closest multiple of 5.

For example, 233 is rounded down to 230, 234 is rounded up to 235, 237 is rounded down to 235 and 238 is rounded up to 240.

1. Round the following numbers to the nearest 5 by checking the **unit value**:

   (a) 612 (b) 87 (c) 454 (d) 1 328

To round off to the **nearest 10**, we round numbers that end in 1, 2, 3 or 4 **down** to the closest multiple of 10 (or decade). We round numbers that end in 5, 6, 7, 8 or 9 **up** to the closest multiple of 10.

For example, if you want to round off 534 to the nearest 10, you have to look at the units digit. The units digit is 4 and it is closer to 0 than to 10. The rounded off number will be 530.

2. Round the following numbers to the nearest 10 by checking the **unit value**:

   (a) 12 (b) 87 (c) 454 (d) 1 325

When **rounding to the nearest 100**, we look at the last **two digits** of the number. If the number is less than 50 we **round down** to the lower 100. If the number is 50 or more we **round up** to the higher 100.

3. Copy and complete the table.

	Round to the nearest 5	Round to the nearest 10	Round to the nearest 100
681			
5 639			
5 361			
12 458			

When **rounding to the nearest 1 000**, we look at the hundreds. Is the hundreds value less than, equal to or greater than 500? If less than 500, round down (the thousands value stays the same), if equal to 500 round up, and if greater than 500 round up too.

When **rounding to the nearest 10 000**, we look at the thousands. Is the thousands value less than, equal to or greater than 5 000? If less than 5 000, round down (the ten thousands value stays the same), if equal to 5 000 or greater than 5 000 round up.

4. Copy and then complete the table.

	Round to the nearest 1 000	Round to the nearest 10 000
142 389		
343 621		
356 552		
100 489		

1.3 Factors, prime numbers and common multiples

different ways to produce the same number

The number 80 can be produced by multiplying 4 and 20: 4 × 20 = 80.

The number 80 can also be produced by multiplying 5 and 16: 5 × 16 = 80.

1. In what other ways can 80 be produced by multiplying two numbers?

The number 80 can also be produced by multiplying 2, 10 and 4:

2 × 10 = 20 and 20 × 4 = 80 or 10 × 4 = 40 and 40 × 2 = 80.

We can use brackets to describe what calculation is done first. So instead of writing "2 × 10 = 20 and 20 × 4 = 80", we may write (2 × 10) × 4. Instead of writing "10 × 4 = 40 and 40 × 2", we may write 2 × (10 × 4).

2. Show how the number 80 can be produced by multiplying four numbers. Describe how you do it in two ways: without using brackets and by using brackets.

3. Show three different ways in which the number 30 can be produced by multiplying two numbers.

4. (a) Can the number 30 be produced by multiplying three whole numbers? Which three whole numbers?

(b) Can the number 30 be produced by multiplying four whole numbers that do not include the number 1? If you answered "yes", which four numbers?

The number 105 can be produced by multiplying 3, 5 and 7, hence we can write $105 = 3 \times 5 \times 7$. Mathematicians often describe this by saying "105 is the **product** of 3, 5 and 7" or "105 can be **expressed as the product** $3 \times 5 \times 7$".

5. Express each of the following numbers as a product of three numbers.

   (a) 248 (b) 375

The whole numbers that are multiplied to form a number are called **factors** of the number. For example, 6 and 8 are factors of 48 because $6 \times 8 = 48$.

But 6 and 8 are not the only numbers that are factors of 48. 2 is also a factor of 48 because $48 = 2 \times 24$. And 24 is a factor of 48. The numbers 3 and 16 are also factors of 48 because $48 = 3 \times 16$.

6. Describe all the different ways in which 48 can be expressed as a product of two factors.

The number 36 can be formed by $2 \times 2 \times 3 \times 3$. Because 2 is used twice, it is called a **repeated factor** of 36. The number 3 is also a repeated factor of 36.

7. (a) Express 48 as a product of three factors.
   (b) Express 75 as a product of three factors.

8. (a) Can 36 be expressed as a product of three factors? How?
   (b) Can 36 be expressed as a product of five factors? How?

9. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   (a) 300 (b) 310 (c) 320
   (d) 330 (e) 340 (f) 350

prime numbers

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.

   (a) 36 (b) 37 (c) 38
   (d) 39 (e) 40 (f) 41
   (g) 42 (h) 43 (i) 44
   (j) 45 (k) 46 (l) 47
   (m) 48 (n) 49

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product $1 \times$ *the number itself*?

A number that cannot be expressed as a product of two whole numbers, except as the product $1 \times$ *the number itself*, is called a **prime number**.

3. (a) Which of the numbers in question 1 are prime numbers?
   (b) Which numbers between 20 and 30 are prime numbers?
   (c) Are 11and 17 prime numbers?

Eratosthenes, a Greek mathematician who lived a long time ago, designed a method to find the prime numbers. The process is called "the sieve of Eratosthenes".

4. Copy the table on the right.

   Follow the steps to find all the prime numbers up to 100.

   Step 1: Cross out 1.

   Step 2: Circle 2, and then cross out all the multiples of 2.

   Step 3: Circle 3, then cross out all the multiples of 3.

   Step 4: Find the next number that has not been crossed out and cross out all its multiples.

   Continue like this.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. (a) What is the smallest number that can be formed as a product of three prime numbers, if the same factor may be repeated?
   (b) What is the smallest number that can be formed as a product of three prime numbers, if no repeated factors are allowed?

6. Manare did a lot of work, and found out that 840 can be formed as the product of 2, 2, 2, 3, 5 and 7. Check whether Manare is correct.

We can say that Manare **found the prime factors** of 840, or Manare **factorised 840 completely**.

We can write:

$2 \times 2 \rightarrow 4 \times 2 \rightarrow 8 \times 3 \rightarrow 24 \times 5 \rightarrow 120 \times 7 = 840.$

7. The prime factors of some numbers are given below. What are the numbers?
   (a) 3, 5, 5 and 11 (b) 3, 3, 5 and 7 (c) 2, 7, 11 and 13

8. Investigate which of the following statements you agree with. Give reasons for your agreement or disagreement in each case.
   (a) If a number is even, 2 is one of its prime factors.
   (b) If half an even number is also even, 2 is a repeated prime factor.
   (c) If a number is odd, 3 is one of its prime factors.
   (d) If a number ends in 0 or 5, then 5 is one of its prime factors.

Here is a method to find the prime factors of a number:
If the number is even, divide it by 2. If the answer is even, divide by 2 again. Continue like this as long as it is possible. If the answer is odd, divide by 3, if it is possible. Continue to divide by 3 as long as it is possible. Then switch to 5. Continue like this by each time trying to divide by the next prime number.

9. Find all the prime factors of each of the following numbers. Write only your answers below.
   (a) 588 (b) 825
   (c) 729 (d) 999
   (e) 538 (f) 113

10. Find at least three prime numbers between 800 and 850.

highest common factor and lowest common multiple

1. (a) Factorise 195 and 385 completely.
   (b) Is 7 a factor of both 195 and 385?
   (c) Is 5 a factor of both 195 and 385?

When a number is a factor of two or more other numbers, it is called a **common factor** of the other numbers. For example, the number 5 is a common factor of 195 and 385.

The factors of a certain number are 2; 2; 5; 7; 7; 11 and 17. The factors of another number are 2; 3; 3; 7; 7; 11; 13 and 23. The common prime factors of these two numbers are 2; 7; 7 and 11.

The biggest number that is a factor of two or more numbers is called the **highest common factor (HCF)** of the numbers.

2. Find the HCF of the two numbers in each of the following cases.
   (a) 2 × 2 × 5 × 7 × 7 × 11× 17 and 2 × 3 × 3 × 7 × 7 × 11× 13 × 23
   (b) 24 and 40
   (c) 8 and12
   (d) 12 and 20
   (e) 210 and 56

3. Write five different numbers, all different from 35, that have 35 as a highest common factor.

4. Copy pattern A and pattern B. Write the next seven numbers in each pattern:

   A: 12 24 36 48

   B: 15 30 45 60

The numbers in pattern A are called the **multiples** of 12. The numbers in pattern B are called the multiples of 15. The numbers, for example 60 and 120, that occur in both patterns, are called the **common multiples** of 12 and 15. The smallest of these numbers, namely 60, is called the **lowest common multiple** (LCM) of 12 and 15.

5. Continue writing multiples of 18 and 24 until you find the LCM:

   18 36

   24 48

6. Find the HCF and LCM of the given numbers in each case below:
   (a) 5 and 7
   (b) 15 and 14
   (c) 20and 30
   (d) 10 and 100
   (e) 8 and 9
   (f) 25 and 24
   (g) 8 and 12
   (h) 10 and 18

1.4 Properties of operations

order of operations and the associative property

Suppose you want to tell another person to do some calculations. You may do this by writing instructions. For example, you may write the instruction 200 – 130 – 30. This may be called a **numerical expression**.

Suppose you have given the instruction 200 – 130 – 30 to two people, whom we will call Ben and Sara.

This is what Ben does: 200 – 130 = 70 and 70 – 30 = 40.

This is what Sara does: 130 – 30 = 100 and 200 – 100 = 100.

To prevent such different interpretations or understandings of the same numerical expression, mathematicians have made the following agreement, and this is followed all over the world:

In a numerical expression that involves **addition and subtraction only**, the operations should be performed **from left to right**, **unless otherwise indicated** in some way.

An agreement like this is called a **mathematical convention**.

1. Who followed this convention, Ben or Sara?

2. Follow the above convention and calculate each of the following:

   (a) 8 000 + 6 000 – 3 000

   (b) 8 000 – 3 000 + 6 000

   (c) 8 000 + 3 000 – 6 000

3. Follow the above convention and calculate each of the following:

   (a) R25 000 + R30 000 + R13 000 + R6 000

   (b) R13 000 + R6 000 + R30 000 + R25 000

   (c) R30 000 + R25 000 + R6 000 + R13 000

In question 3, all your answers should be the same. When three or more numbers are added, the order in which you perform the calculations makes no difference. This is called the **associative property of addition**. We also say that: **addition is associative**.

4. Investigate whether multiplication is associative. Use the numbers 2, 3, 5 and 10.

5. What must be added to each of the following numbers to get 100?

   73 56 66 41 34 23 88

6. Calculate each of the following. Note that you can make the work simple by being smart in deciding which additions to do first.

   (a) 73 + 54 + 27 + 46+ 138

   (b) 34 + 88 + 41 + 66 + 59 + 12 +127

the commutative property of addition and multiplication

1. (a) What is the total cost of 20 chairs at R250 each?

   (b) What is the total cost of 250 exercise books at R20 each?

   (c) R5 000 was paid for 100 towels. What is the price for one towel?

   (d) R100 was paid for 5 000 beads. What is the price for one bead?

2. Which of the following calculations will produce the same answer? Copy the calculations and mark those that will produce the same answers with a ✓ and those that won't with a ✗.

   (a) 20 × 250 and 250 × 20

   (b) 5 000 ÷ 100 and 100 ÷ 5000

   (c) 730 + 270 and 270+ 730

   (d) 730 – 270 and 270 –730

25 + 75 and 75 + 25 have the same answer. The same is true for any other two numbers. We say: addition is **commutative**; the numbers can be swopped around.

3. Demonstrate each of your answers with two different examples.
   (a) Is subtraction commutative?
   (b) Is multiplication commutative?
   (c) Is division commutative?

more conventions and the distributive property

1. Do the following:
   (a) Multiply 5 by 3, then add the answer to 20.
   (b) Add 5 to 20, then multiply the answer by 5.

Mathematicians have agreed that **unlessotherwise indicated, multiplication and division should be done before addition and subtraction**. According to this convention, the expression 20 + 5 × 3 should be taken to mean "multiply 5 by 3, then add the answer to 20" and not "add 5 to 20, then multiply the answer by 3".

2. Follow the above convention and calculate each of the following:

(a) 500 + 20 × 10	(b) 500 − 20× 10	(c) 500 + 20 −10
(d) 500 − 20+ 10	(e) 500 + 200 ÷ 5	(f) 500 − 200 ÷ 5

   If some of your answers are the same, you have made mistakes.

The above convention creates a problem. How can one describe the calculations in question 1(b) with a numerical expression, without using words?

To solve this problem, mathematicians have agreed to use brackets in numerical expressions. **Brackets are used to specify that the operations within the brackets should be done first.** Hence the numerical expression for 1(b) above is (20 + 5) × 5, and the answer is 125.

If there are **no brackets** in a numerical expression,it **means that multiplication and division should be done first, and addition and subtraction onlylater**.

If you wish to specify that addition or subtraction should be done first, that part of the expression should be enclosed in brackets.

examples

The expression 12 + 3 × 5 means "multiply 3 by 5, then add 12". It *does not* mean "add 12 and 3, then multiply by 5".

If you wish to say "add 5 and 12, then multiply by 3", the numerical expression should be 3 × (5 + 12) or (5 + 12) × 3. They mean the same.

3. Keep the various mathematical conventions about numerical expressions in mind when you calculate each of the following:

(a) $500 + 30 \times 10$	(b) $(500 + 30) \times 10$
(c) $100 \times 500 + 30$	(d) $100 \times (500 + 30)$
(e) $500 - 30 \times 10$	(f) $(500 - 30) \times 10$
(g) $100 \times 500 - 30$	(h) $100 \times (500 - 30)$
(i) $(200 + 300) \div 20$	(j) $200 \div 20 + 300 \div 20$
(k) $600 \div (20 + 30)$	(l) $600 \div 20 + 600 \div 30$

4. Calculate the following:

(a) $50 \times (70 + 30)$	(b) $50 \times 70 + 50 \times 30$
(c) $50 \times (70 - 30)$	(d) $50 \times 70 - 50 \times 30$

Your answers for 4(a) and 4(b) should be the same.

Your answers for 4(c) and 4(d) should also be the same.

5. Do not do calculations A to I below. Just answer these questions about them. You will check your answers later.
   (a) Will A and B have the same answers?
   (b) Will G and H have the same answers?
   (c) Will A and D have the same answers?
   (d) Will A and G have the same answers?
   (e) Will A and F have the same answers?
   (f) Will D and E have the same answers?

A: $5 \times (200 + 3)$	B: $5 \times 200 + 3$
C: $5 \times 200 + 5 \times 3$	D: $5 + 200 \times 3$
E: $(5 + 200) \times 3$	F: $(200 + 3) \times 5$
G: 5×203	H: $5 \times 100 + 5 \times 103$
I: $5 \times 300 - 5 \times 70$	

6. Now do calculations A to I. Then check the answers you gave in question 5.

7. (a) Choose three different numbers between 3 and 11, and write them down like this:

   Your first number: ___ Your second number: ___ Your third number: ___

   (b) Add your first number to your third number. Multiply the answer by your second number.
   (c) Multiply your first number by your second number. Also multiply your third number by your second number. Add the two answers.
   (d) If you worked correctly, you should get the same answers in (b) and (c). Do you think you will get the same result with numbers between 10 and 100, or any other numbers?

The fact that your answers for calculations like those in 7(b) and 7(c) are equal, for any numbers that you may choose, is called the **distributive property of multiplication over addition**.

> It may be described as follows:
>
> first number × second number + first number × third number
>
> = first number × (second number + third number).
>
> This can be described by saying that **multiplication distributes over addition**.

8. Check whether the distributive property is true for the following sets of numbers:
   (a) 100, 50 and 10
   (b) any three numbers of your own choice (you may use a calculator to do this)
9. Use the numbers in question 8(a) to investigate whether multiplication also distributes over subtraction.

It is quite fortunate that multiplication distributes over addition, because it makes it easier to multiply.

For example, 8 × 238 can be calculated by calculating 8 × 200, 8 × 30 and 8 × 8, and adding the answers: 8 × 238 = 8 × 200 + 8 × 30 + 8 × 8 = 1 600 + 240 + 64 = 1 904.

10. Check whether 8 × 238 is actually 1 904 by calculating 238 + 238 + 238 + 238 + 238 + 238 + 238 + 238, or by using a calculator.

1.5 Basic operations

a method of addition

To add two numbers, the one may be written below the other.

For example, to calculate 378 539 + 46 285 the one number may be written below the other so that the units are below the units, the tens below the tens, and so on.

```
378 539
 46 285
```

Writing the numbers like this has the advantage that:

- the units parts (9 and 5) of the two numbers are now in the same column,
- the tens parts (30 and 80) are in the same column,
- the hundreds parts (500 and 200) are in the same column, and so on.

This makes it possible to work with each kind of part separately.

We only write this:	In your mind you can see this:					
378 539	300 000	70 000	8 000	500	30	9
46 285		40 000	6 000	200	80	5

The numbers in each column can be added to get a new set of numbers:

378 539	300 000	70 000	8 000	500	30	9
46 285		40 000	6 000	200	80	5
14						14
110					110	
700				700		
14 000			14 000			
110 000		110 000				
300 000	300 000					
424 824	It is easy to add the new set of numbers to get the answer.					

Note that you can do the above steps in any order. Instead of starting with the units parts as shown above, you can start with the hundred thousands, or any other parts.

Starting with the units parts has an advantage though as it makes it possible to do more of the work mentally and to write less, as shown below:

378 539 46 285 424 824	To achieve this, only the units digit 4 of the 14 is written in the first step. The 10 of the 14 is remembered and added to the 30 and 80 of the tens column, to get 120.

We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 120: only the digit "2" is written (in the tens column, so it means 20), and the 100 is carried to the next step.

1. Calculate each of the following:
   (a) 237 847 +87 776 (b) 567 298 + 392 076 (c) 28 387 + 365 667

A municipal manager is working on the municipal budget for a year. He has to try to keep the total expenditure on new office equipment below R800 000. He still has to budget for new computers that are badly needed, but this is what he has written so far:

74 new office chairs	R 54 020
42 new computer screens	R 100800
12 new printers	R 141 600
18 new tea trolleys	R 25 740
8 new carpets for senior staff offices	R 144000
108 small plastic filing cabinets	R 52 380
new table for the boardroom	R 48 000
18 new chairs for the boardroom	R 41 400
	R ___

2. How much has the municipal manager budgeted for printers and computer screens together?

3. How much, in total, has the municipal manager budgeted for chairs and tables?

4. Work out the total cost of all the items the municipal manager has budgeted for.

5. Calculate.

   (a) 23 809 + 2 009+ 23 (b) 320 293 + 16 923 + 349 + 200 323

methods of subtraction

There are many ways to subtract one number from another. For example, R835 234 – R687 885 can be calculated by "filling up" from R687 885 to R835 234:
687 885 + <u>15</u> → 687 900 + <u>100</u> → 688 000 + <u>12 000</u> → 700 000 + <u>135 234</u> → 835 234

The difference between R687 885 and R835 234 can now be calculated by adding up the numbers that had to be added to 687 885 to get 835 234.

So R835 234 – R687 885 = R147 349.

15
100
12 000
<u>135 234</u>
147 349

Another easy way to subtract is to **round off and compensate**. For example, to calculate R3 224 – R1 885, the R1 885 may be rounded up to R2 000. The calculation can proceed as follows:

- Rounding R1 885 up to R2 000 can be done in two steps: 1 885 + 15 = 1 900, and 1 900 + 100 = 2 000. In total, 115 was added.
- 115 can now be added to 3 224 too: 3 224 + 115= 3 339.

Instead of calculating R3 224 – R1 885, which is a bit difficult, R3 339 – R2 000 may be calculated. This is easy: R3 339 – R2 000 = R1 339.

This means that R3 224 – R1 885 = R1 339, because
R3 224 – R1 885 = (R3 224 + R115) – (R1 885 + R115).

To do question 1, you may use any one of the above two methods, or any other method you may know and prefer. Do not use a calculator, because the purpose of this work is for you to come to understand how subtraction may be done. What you will learn here, will later help you to understand **algebra**.

1. Calculate each of the following:

   (a) 6 234 – 2 992 (b) 76 214 – 34867
   (c) 134 372 – 45 828 (d) 623 341 – 236 768

2. Check each of your answers in question 1 by doing addition, or by doing subtraction with a different method than the method you have already used.

Another method of subtraction is to think of the numbers in **expanded notation**. For example, to calculate R835 234 − R687 885, which was already done in a different way on the previous page, we could work like this:

We may write this:	In your mind you can see this:					
835 234	800 000	30 000	5 000	200	30	4
687 885	600 000	80 000	7 000	800	80	5

Unfortunately, it is not possible to subtract in the columns now. However, the parts of the bigger number can be rearranged to make the subtraction in each column possible:

835 234	700 000	120 000	14 000	1100	120	14
687 885	600 000	80 000	7 000	800	80	5
	100 000	40 000	7 000	300	40	9

The answer is now clearly visible; it is 147 349.

The rearrangement, also called "borrowing", was done like this:
10 was taken from the 30 in the tens column, and added to the 4 in the units column. 100 was taken from the 200 in the hundreds column, and added to the 20 that remained in the tens column. 1 000 was taken from the 5 000 in the thousands column, and added to the 100 that remained in the hundreds column.

3. Describe the other rearrangements that were made in the above work.

It is not practical to write the expanded notation and the rearrangements each time you do a subtraction. However, with some practice you can learn to do it all in your mind without writing it down. Some people make small marks above the digits of the bigger number, or even change the digits, to keep track of the rearrangements they make in their minds.

835 234
<u>687 885</u>
147 349

4. Calculate the difference between the two car prices in each case.
   (a) R73 463 and R88 798 (b) R63 378 and R96889

5. In each case, first estimate the answer to the nearest 100 000, then calculate.
   (a) 238 769 − 141 453 (b) 856 333 − 439 878

6. In each case, first estimate the answer to the nearest 10 000, then calculate.
   (a) 88 023 − 45 664 (b) 342 029 − 176553

7. Look again at the municipal budget on page 25. How much money does the municipal manager have left to buy new computers?

8. Calculate.
   (a) 670 034 − 299 999 (b) 670 034 − 300 000
   (c) 376 539 − 175 998 (d) 376 541 − 176 000

a method of multiplication

6 × R3 258 can be calculated in parts, as shown below.

6 × R3 000 = R18 000

6 × R200 = R1 200

6 × R50 = R300

6 × R8 = R48

$$\begin{array}{r} 3\,2\,5\,8 \\ \times\ 6 \\ \hline 4\,8 \\ 3\,0\,0 \\ 1\,2\,0\,0 \\ 1\,8\,0\,0\,0 \\ \hline 1\,9\,5\,4\,8 \end{array}$$

The four partial products can now be added to get the answer, which is R19 548. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown on the right above.

In fact, if you are willing to do some hard thinking you can produce the answer with even less writing. You can achieve this by working from right to left to calculate the partial products, and by "carrying" parts of the partial answers to the next column, as you can do when working from right to left in columns. It works like this:

$$\begin{array}{r} 3\,2\,5\,8 \\ \times\ 6 \\ \hline 1\,9\,5\,4\,8 \end{array}$$

When 6 × 8 = 48 is calculated, only the "8" is written down, in the units column. The "4" that represents 40 is not written. It is kept "on hold" in your mind.

When 6 × 50 = 300 is calculated, the 40 from the previous step is added to 300 to get 340. Again, only the "4" that represents 40 is written. The 300 is kept on hold or "carried" to add to the answer of the next step. The work continues like this.

1. Calculate each of the following. Do not use a calculator.
   (a) 8 × 786 (b) 9 × 3 453 (c) 60 × 786 (d) 60 × 7 860

2. You may use a calculator to check your answers for question 1. Repeat the work if your answers are not correct, so that you can learn where you make mistakes. Then put your calculator away again.

3. Use your answers for questions 1(a) and (c) to find out how much 68 × 786is.

To calculate 36 × 378, the work can be broken up in two parts, namely 30 × 378 and 6 × 378.

4. Calculate 36 × 378.

A complete write-up of calculating 76 × 2 348 in columns is shown on the right.

5. (a) Explain how the 240 in row B was obtained.
   (b) Explain how the 560 in row E was obtained.
   (c) Explain how the 21 000 in row G was obtained.

$$\begin{array}{lrrrrrr} & & & 2 & 3 & 4 & 8 \\ & & & & \times & 7 & 6 \\ \hline \text{A} & & & & & 4 & 8 \\ \text{B} & & & & 2 & 4 & 0 \\ \text{C} & & & 1 & 8 & 0 & 0 \\ \text{D} & & 1 & 2 & 0 & 0 & 0 \\ \text{E} & & & & 5 & 6 & 0 \\ \text{F} & & & 2 & 8 & 0 & 0 \\ \text{G} & & 2 & 1 & 0 & 0 & 0 \\ \text{H} & 1 & 4 & 0 & 0 & 0 & 0 \\ \hline & 1 & 7 & 8 & 4 & 4 & 8 \end{array}$$

A short write-up of calculating 76 × 2 348 in columns is shown on the right.

You may try to do the calculations in question 6 in this way. If you find it difficult, you may first write some of them up completely, and then try again to write less when you multiply.

$$\begin{array}{r} 2\,3\,4\,8 \\ \times\ 7\,6 \\ \hline 1\,4\,0\,8\,8 \\ 1\,6\,4\,3\,6\,0 \\ \hline 1\,7\,8\,4\,4\,8 \end{array}$$

6. Calculate each of the following.
   (a) 53 × 738 (b) 73 × 3 457

7. Calculate.
   (a) 64 × 3 478 (b) 78 × 1 298 (c) 37 × 3 428 (d) 78 × 7 285

8. Use a calculator to check your answers for question 7. Redo the questions that you had wrong, so that you can learn to work correctly.

9. Use your correct answers for question 7 to give the answers to the following, without doing any calculations:
   (a) 101 244 ÷ 1 298 (b) 568 230 ÷ 7 285

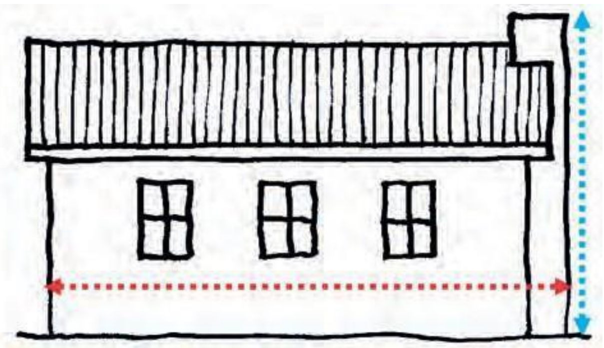
10. Calculate, without using a calculator.
    (a) 3 659 × 38 (b) 27 × 23 487 (c) 486 × 278 (d) 2 135 × 232

a process called long division

You may use a calculator to do questions 1 to 6.

1. You want to buy some live chickens at R37 each and you have R920 available. How many live chickens can you buy in total?
2. R880 is to be shared equally among 34 learners? How many full rands can each learner get?
3. You want to buy live chickens at R47 each. You have R1 280 available. How many live chickens can you buy?
4. 42 equal bags of rice weigh a total of 7 560 g. How much does one bag weigh?
5. The number 26 was multiplied by a secret number and the answer was 2 184. What was the secret number?

This is an accurate sketch of the back of a house. The red line on the sketch is 70 mm long and it shows the width of the house. The blue line on the sketch indicates the height of the chimney. **Do not measure the blue line now.**



The width of the actual house is 5 600 mm, and the height of the chimney is 3 360 mm.

6. (a) How many times is the house bigger than the sketch? Describe what you can do to find this out.
   (b) Calculate how long the blue line on the sketch should be.
   (c) Now measure the blue line to check your answer for (b).

Division is used for different purposes:

> In question 1 you knew that the amount is split into equal parts. You had to **find out how many parts there are** (how many chickens). This is called **grouping**.
>
> In question 2 you knew that the amount was split into 34 equal parts. You needed to **find out how big each part is** (how much money each learner will get). This is called **sharing**.

7. (a) What does question 3 require, sharing or grouping?
   (b) What does question 4 require, sharing or grouping?

In question 6 division was done for a different purpose than sharing or grouping.

Put your calculator away now. It is very important to be able to solve division problems by using your own mind. The activities that follow will help you to do this better than before. While you work on these activities, you will often have to **estimate** the product of two numbers. If you can estimate products well, division becomes easier to do. Hence, to start, do question 8, which will provide you with the opportunity to practise your product estimation skills.

8. (a) What do you think is closest to 4 080: 10 × 74 or 30 × 74 or 50 × 74 or 70 × 74 or 90 × 74?
   (b) Calculate some of the products to check your answer.
   (c) What do you think is closest to 9 238: 30 × 38 or 50 × 38 or 100 × 38 or 150 × 38 or 200 × 38 or 250 × 38 or 300 × 38?
   (d) Calculate some of the products to check your answer.
   (e) What do you think is closest to 9 746: 10 × 287 or 20 × 287 or 30 × 287 or 40 × 287 or 50 × 287 or 60 × 287 or 70 × 287?
   (f) Calculate some of the products to check your answer.
   (g) By what multiple of 10 should you multiply 27 to get as close to 6 487 as possible?

9. A principal wants to buy T-shirts for the 115 Grade 7 learners in the school. The T-shirts cost R67 each, and an amount of R8 500 is available. Do you think there is enough money to buy T-shirts for all the learners? Explain your answer.

10. (a) How much will 100 of the T-shirts cost?
    (b) How much money will be left if 100 T-shirts are bought?
    (c) How much money will be left if 20 more T-shirts are bought?

The principal wants to work out exactly how many T-shirts, at R67 each, she can buy with R8 500. Her thinking and writing are described below.

Step 1

What she writes:	What she thinks:
$67\overline{)\,8\,500}$	*I want to find out how many chunks of 67 there are in 8 500.*

Step 2

What she writes:	What she thinks:
100	*I think there are at least 100 chunks of 67 in 8 500.*
$67\overline{)\,8\,500}$	
$\underline{6\,700}$	*100 × 67 = 6 700. I need to know how much is left over.*
1 800	*I want to find out how many chunks of 67 there are in 1 800.*

Step 3 (She has to rub out the one "0" of the 100 on top, to make space.)

What she writes:	What she thinks:
120	*I think there are at least 20 chunks of 67 in 1 800.*
$67\overline{)\,8\,500}$	
$\underline{6\,700}$	
1 800	
$\underline{1\,340}$	*20 × 67 = 1 340. I need to know how much is left over.*
460	*I want to find out how many chunks of 67 there are in 460.*

Step 4 (She rubs out another "0".)

What she writes:	What she thinks:
125	*I think there are at least five chunks of 67 in 460.*
$67\overline{)\,8\,500}$	
$\underline{6\,700}$	
1 800	
$\underline{1\,340}$	
460	
$\underline{335}$	*5 × 67 = 335. I need to know how much is left over.*
125	*I want to find out how many chunks of 67 there are in 125.*

Step 5 (She rubs out the "5".)

What she writes:	What she thinks:
126	*I think there is only one more chunk of 67 in 125.*
$67\overline{)\,8\,500}$	
$\underline{6\,700}$	
1 800	
$\underline{1\,340}$	
460	
$\underline{335}$	
125	
$\underline{67}$	*I wonder how much money will be left over.*
58	*So, we can buy 126 T-shirts and R58 will remain.*

Do not use a calculator in the questions that follow. The purpose of this work is for you to develop a good understanding of how division can be done. Check all your answers by doing multiplication.

11. (a) Selina bought 85 chickens, all at the same price. She paid R3 995 in total. What did each of the chickens cost? Your first step can be to work out how much Selina would have paid if she paid R10 per chicken, but you can start with a bigger step if you wish.

    (b) Anton has R4 850. He wants to buy some young goats. The goats cost R78 each. How many goats can he buy?

12. Calculate the following without using a calculator:

    (a) 7 234 ÷ 48

    (b) 3 267 ÷ 24

    (c) 9 500 ÷ 364

    (d) 8 347 ÷ 24

13. (a) A chocolate factory made 9 325 chocolates of a very special kind one day. The chocolates were packed in small, decorated boxes, with 24 chocolates per box. How many boxes were filled?

    (b) A farmer sells eggs packed in cartons to the local supermarkets. There are 36 eggs in one carton. One month, the farmer sold 72 468 eggs to the supermarkets. How many cartons is this?

1.6 Problem solving

rate and ratio

You may use a calculator for doing the work in this section.

1. The people in a village get their water from a nearby dam. On a certain day the dam contains 688 000 litres of water. The village people use about 85 000 litres of water each day. For how many days will the water in the dam last, if no rains fall?

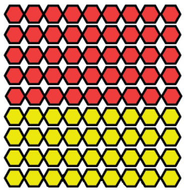
> Instead of saying "85 000 litres each day" or "8 cm each hour", people often say "**at a rate of** 85 000 litres **per day**" or "**at a rate of** 8 cm **per hour**".

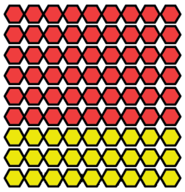
2. During a period of very heavy rain, the water level in a certain river increases at a rate of 8 cm each hour. If it continues like this, by how much will the water level increase in 24 hours?

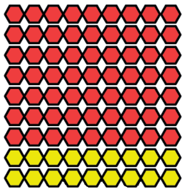
3. A woman is driving from Johannesburg to Durban. Her distance from Durban decreases at a rate of about 95 km per hour. How far does she travel, approximately, in four hours?

4. The number of unemployed people in a certain province increases at a rate of approximately 35 000 people per year. If there were 860 000 unemployed people in the year 2000, how many unemployed people will there be, approximately, in the year 2020?

5. In pattern A below, there are five red beads for every four yellow beads.
   Describe patterns B and C in the same way.


Pattern A


Pattern B


Pattern C

In a certain food factory, two machines are used to produce tins of baked beans. Machine A produces at a rate of 800 tins per hour, and machine B produces at a rate of 2 400 tins per hour.

6. (a) Copy and complete the following table, to show how many tins of beans will be produced by the two machines, in different periods of time.

Number of hours	1	2	3	5	8
Number of tins produced by machine A	800	1 600	2 400	4 000	
Number of tins produced by machine B	2 400	4 800			

   (b) How much faster is machine B than machine A?
   (c) How many tins will be produced by machine B in the time that it takes machine A to produce 30 tins?
   (d) How many tins will be produced by machine B in the time that it takes machine A to produce 200 tins?
   (e) How many tins will be produced by machine B in the time that it takes machine A to produce one tin?

The patterns in question 5 can be described like this:

> In pattern A, the **ratio** of yellow beads to red beads is 4 to 5. This is written as 4 : 5.

In pattern B, the ratio between yellow beads and red beads is 3 : 6, and in pattern C the ratio is 2 : 7. In question 6, machine A produces one tin for every three tins that machine B produces. This can be described by saying that the ratio between the production speeds of machines A and B is 1 : 3.

7. Two huge trucks are travelling very slowly on a highway. Truck A covers 20 km per hour, and truck B covers 30 km per hour. Both trucks keep these speeds all the time.
   (a) What distance will truck B cover in the same time that truck A covers 10 km?
   (b) In the table below, the distances that truck A covers in certain periods of time are given. Copy and complete the table to show the distances covered by truck B, in the same periods of time.

Distance covered by truck A	10 km	18 km	50 km	100 km	30 km
Distance covered by truck B					

   (c) What distance will truck B cover in the same time that truck A covers 1 km?
   (d) What is the ratio between the speed at which truck A travels and the speed at which truck B travels?

8. R240 will be divided between David and Sally in the ratio 3: 5. This means Sally gets R5 for every R3 David gets. How much will David and Sally each get in total?

9. How much will each person get, if R14 400 is shared between two people in each of the following ways?
   (a) In the ratio 1 : 3
   (b) In the ratio 5 : 7

financial mathematics

A man borrows R12 000 from a bank for one year. He has to pay 15% interest to the bank. This means that, apart from paying the R12 000 back to the bank after a year, he has to pay 15 hundredths of R12 000 for the privilege of using the money that actually belongs to the bank.

One hundredth of R12 000 can be calculated by dividing R12 000 by 100. This amount can then be multiplied by 15 to get 15 hundredths of R12 000.

15% is read as **15 per cent**, and it is just a different way to say **15 hundredths**.

The money paid for using another person's house is called **rent**. The money paid for using another person's money is called **interest**.

Do not use a calculator when you do the following questions.

1. Calculate 12 000 ÷ 100, then multiply the answer by 15.

2. Calculate:
   (a) 12% of R8 000
   (b) 18% of R24 000

3. In each case below, calculate how much interest must be paid.
   (a) An amount of R6 000 is borrowed for one year at 9% interest.
   (b) An amount of R21 000 is borrowed for three years at 11% interest peryear.
   (c) An amount of R45 000 is borrowed for ten years at 12% interest per year.

A car dealer buys a car for R60 000 and sells it for R75000. The difference of R15000 is called the **profit**. In this case, the profit is a quarter of R60 000, which is the same as 25 hundredths or 25%. This can be described by saying "the car dealer made a profit of 25%".

4. Calculate the amount of profit in each of the following cases. The information is about a car dealer who buys and sells used vehicles.
   (a) A car is bought for R40 000 and sold for R52 000.
   (b) A small truck is bought for R100 000 and sold at a profit of 28%.
   (c) A bakkie is bought for R120 000 and sold at a profit of 30%.

A shop owner bought a stove for R2 000 and sold it for R1 600. The shop owner did not make a profit, he sold the stove at a **loss** of R400.

5. (a) How much is one hundredth of R2 000?
   (b) How many hundredths of R2 000 is R400?
   (c) How much is 20% of R2 000?

Notice that by doing question 5(b) you have worked out at what percentage loss the shop owner sold the stove.

6. The shop owner also sold a fridge that normally sells for R4 000 at a **discount** of 20%. This means the customer paid 20% less than the normal price. Calculate the discount in rands and the amount that the customer paid for the fridge.

Chapter 2
Exponents

2.1 Quick squares and cubes

again and again

1. How much is each of the following?

(a) 2×2 (b) 3×3 (c) 4×4 (d) 5×5 (e) 6×6 (f) 7×7
(g) 8×8 (h) 9×9 (i) 10×10 (j) 11×11 (k) 12×12 (l) 1×1

Instead of saying "ten times ten", we may say "ten squared" and we may write 10^2.

2. Copy and complete these tables.

2×2			12×12		8×8
2^2	5^2			4^2	
2 squared		10 squared			
	25	100			64

		1×1	9×9		
	7^2		9^2		
11 squared				3 squared	
121					36

3. 8 squared is 64, and 9 squared is 81.

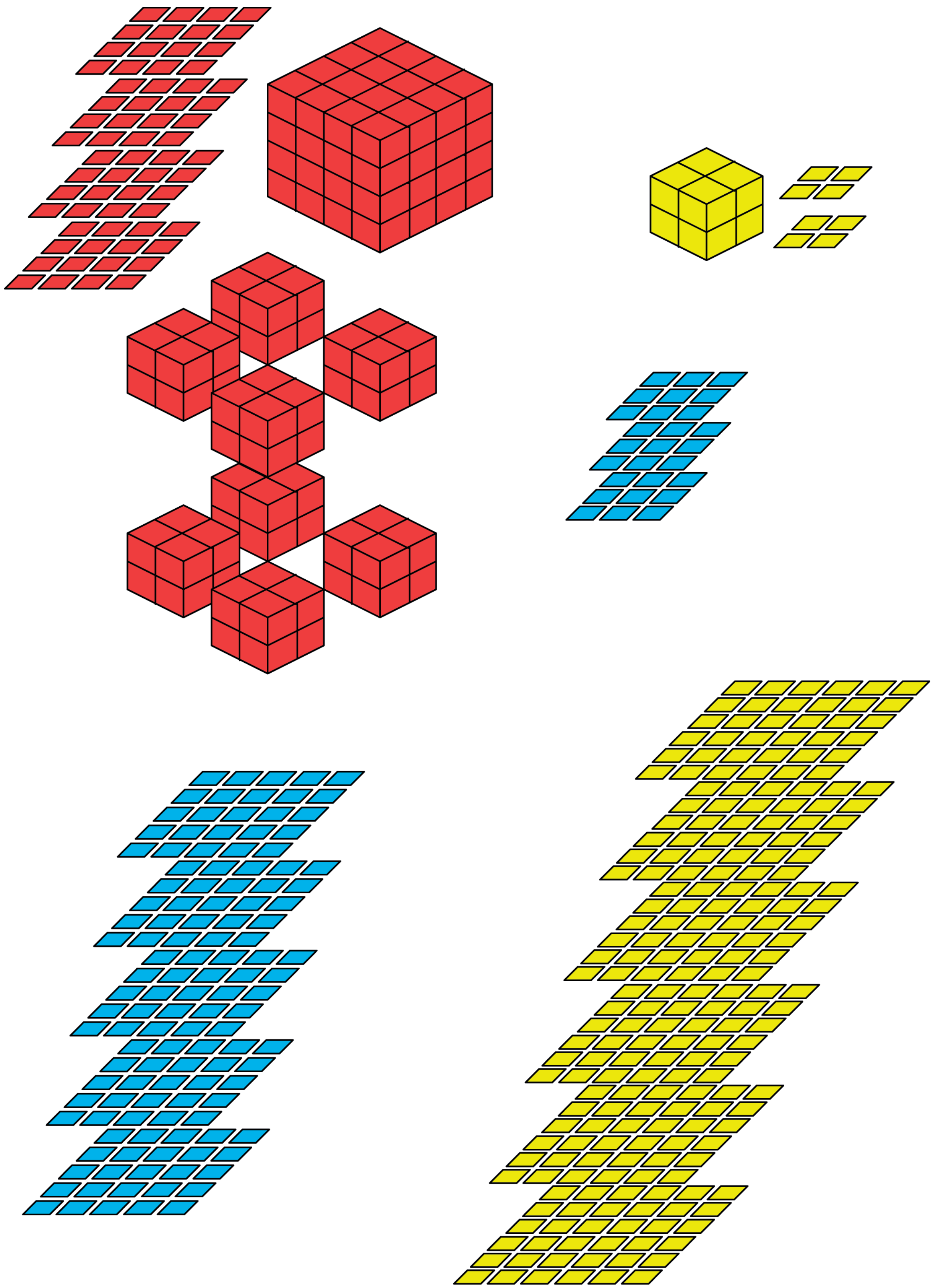
(a) What number squared is 25? (b) What number squared is 100?
(c) What number squared is 64? (d) What number squared is 36?

4. Calculate:

(a) $10^2 + 5^2 + 2^2$ (b) $5 \times 10^2 + 7 \times 10 + 3$
(c) $7 \times 10^2 + 3 \times 10 + 6$ (d) $2 \times 10^2 + 9 \times 10 + 6$

5. How much is each of the following?

(a) $2 \times 2 \times 2$ (b) $3 \times 3 \times 3$ (c) $4 \times 4 \times 4$ (d) $5 \times 5 \times 5$ (e) $6 \times 6 \times 6$
(f) $7 \times 7 \times 7$ (g) $8 \times 8 \times 8$ (h) $9 \times 9 \times 9$ (i) $10 \times 10 \times 10$
(j) $11 \times 11 \times 11$ (k) $12 \times 12 \times 12$ (l) $13 \times 13 \times 13$ (m) $1 \times 1 \times 1$



Instead of saying "10 times 10 times 10", we may say "10 cubed" and we may write 10^3.

6. Copy and complete the tables.

$4 \times 4 \times 4$	$7 \times 7 \times 7$				
4^3		11^3			
4 cubed			2 cubed		
64				216	1 000

$8 \times 8 \times 8$					
			9^3		
	12 cubed			3 cubed	
		1			125

7. 5 cubed is 125, and 9 cubed is 729.
   (a) What number cubedis 27?
   (b) What number cubed is 1 000?
   (c) What number cubed is 8?
   (d) What number cubed is 1?
   (e) What number cubed is 216?
   (f) What number cubed is 343?

8. Calculate:
   (a) $3 \times 10^3 + 7 \times 10^2 + 5 \times 10 + 6$
   (b) $7 \times 10^3 + 7 \times 10^2 + 7 \times 10 + 7$
   (c) $8 \times 10^3 + 1 \times 10^2 + 4 \times 10 + 2$
   (d) $4 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 9$
   (e) 10×10^2
   (f) $10^2 \times 10^2$

9. Can you think of two numbers, so that the square of the one number is equal to the cube of the other number?

10. Can you think of two numbers, so that when you add their squares, you get the square of another number?

2.2 The exponential notation

repeated multiplication with the same number

1. Express each number below as a product of prime factors.
   Example: $250 = 2 \times 5 \times 5 \times 5$
   (a) 35
   (b) 70
   (c) 140
   (d) 280
   (e) 81
   (f) 625

5 is a **repeated factor** of 250. It is repeated three times.

2. Which numbers in question 1 have repeated factors? In each case, state what number is repeated as a factor and how many times it is repeated.

A number that can be expressed as a product of one repeated factor is called a **power** of that number.

Examples:

32 is a power of 2, because 32 = 2 × 2 × 2 × 2 × 2

100 000 is a power of 10, because 10 × 10 × 10 × 10 × 10 = 100 000

3. Express each number as a power of 2, 3, 5 or 10.

(a) 125 (b) 64

(c) 100 (d) 1 000

4. Calculate each of the following. You can use each answer to get the next answer.

(a) 2 × 2 × 2× 2 (b) 2 × 2 × 2 × 2 × 2

(c) 2 × 2 × 2 × 2 × 2 × 2 (d) 2 × 2 × 2 × 2 × 2 × 2 ×2

(e) 2 × 2 × 2 × 2 × 2 × 2 × 2× 2 (f) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

(g) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

(h) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

(i) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 ×2

(j) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 ×2

Because the factor 2 is repeated five times, 32 is called the **fifth power of 2**, or **2 to the power 5.**

Similarly, 125 is the third power of 5.

125 can also be called "5 to the power 3" or "5 cubed".

5. The seventh power of 2 is shown in question 4(d).
What power of 2 is shown in each of the following parts of question 4?

(a) 4(j) (b) 4(i)

(c) 4(h) (d) 4(f)

6. What power of what number is shown in each case below?

(a) 15 × 15 × 15 × 15 × 15 × 15 × 15 × 15

(b) 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12

Instead of writing "5 to the power 6" we may write 5^6.
This is called the **exponential notation**.
5^6 means 5 × 5 × 5 × 5 × 5 × 5.
5 × 6 means 6 + 6 + 6 + 6 + 6.

7. Write each of the numbers in question 3 in exponential notation.

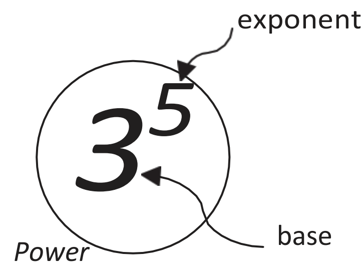
8. Write each of the numbers in question 4 in exponential notation.

9. In each case write the number in exponential notation.
   (a) The fifth powerof 5
   (b) The sixth power of 5
   (c) The third powerof 4
   (d) 6 to the power 4
   (e) 4 to the power 6
   (f) 5 to the power 15

3^5 means $3 \times 3 \times 3 \times 3 \times 3$.
The repeating factor in a power is called the **base**.

The number of repetitions is called the **exponent** or **index**.

3^1 means 3. The base is 3 but there is no repetition. Any number raised to the power 1 equals the number itself.



10. In each case below some information about a number is given. Each number can be expressed as a power. What is the number in each case?
    (a) The base is 5 and the index is 3.
    (b) The base is 10 and the exponent is 4.
    (c) The base is 20 and the exponent is 3.

11. Calculate each of the following:
    (a) $5 \times 5 \times 5$
    (b) $5 \times 5 \times 5 \times 5 \times 5$
    (c) $5 + 5 + 5$
    (d) $5 + 5 + 5 + 5 + 5$
    (e) 5×3
    (f) 5^3

powers of different numbers

1. Copy and complete this table of powers of 2.

Exponent	1	2	3	4	5	6	7	8	9
Power of 2	2	4	8	16					

Exponent	10	11	12	13	14
Power of 2					

2. (a) Calculate each of the following:
   $2^2 - 2^1$ $\quad 2^3 - 2^2$ $\quad 2^4 - 2^3$ $\quad 2^5 - 2^4$
   $2^6 - 2^5$ $\quad 2^7 - 2^6$ $\quad 2^8 - 2^7$
   (b) Describe what you notice about the differencesbetweenconsecutivepowersof2.

Numbers that follow on each other in a pattern are called **consecutive numbers**.

3. Suppose you calculate the differences between consecutive powers of 3. Do you think these differences will be the consecutive powers of 3 again?

4. Copy and complete this table of powers of 3.

Exponent	1	2	3	4	5	6	7	8	9
Power of 3	3	9							

Exponent	10	11	12	13	14
Power of 3					

5. (a) Calculate each of the following:

   $3^2 - 3^1$ $\quad$ $3^3 - 3^2$ $\quad$ $3^4 - 3^3$ $\quad$ $3^5 - 3^4$ $\quad$ $3^6 - 3^5$ $\quad$ $3^7 - 3^6$ $\quad$ $3^8 - 3^7$

   (b) How do these numbers differ from what you expected when you answered question 3?

   (c) Divide each of your answers in 5(a) by 2.

   (d) If you observe anything interesting, describe it.

6. In questions 1 to 5 you have investigated the differences between consecutive powers of 2 and 3. You have observed certain interesting things about these differences. You will now investigate, in the same way, the differences between consecutive powers of 4.

   (a) Before you investigate, think a bit. What do you expect to find?

   (b) Copy the table below and do your investigation. Write a short report on what you find.

Exponent								
Power of 4								

7. Do what you did in question 6, but now for powers of 10.

Exponent							
Power of 10							

2.3 Squares and cubes

The number 9 is called the **square** of 3 because $3 \times 3 = 9$. The number 3, called the base, is multiplied by itself. $\mathbf{3^2}$ is read as **three squared** or **three to the power 2**.

The number 27 is called the **cube** of 3 because $3 \times 3 \times 3 = 27$. The base, the number 3, is multiplied by itself and again by itself. $\mathbf{3^3}$ is read as **three cubed** or **three to the power 3**.

calculating squares and cubes

Squaring the number 2 means that we must multiply 2 by itself. It means we have to calculate 2 × 2, which has a value of 4, and we write 2 × 2 = 4.

1. In (a) to (f) below, the numbers in set B are found by squaring each number in set A. Copy the table and write down the numbers that belong to set B in each case.

	Set A	Set B
(a)	{1; 2; 3; 4; 5; 6; 7; 8}	
(b)	{1; 3; 5; 7; 9; 11; 13}	
(c)	{10; 20; 30; 40; 50}	
(d)	{2; 4; 6; 8; 10; 12; 14}	
(e)	{5; 10; 15; 20; 25}	
(f)	{15; 12; 9; 6; 3}	

Cubing the number 2 means that we must multiply 2 by itself, and again. It means we have to calculate 2 × 2 × 2, which has a value of 8, and we write 2 × 2 × 2 = 8.

2. (a) Cube 1. Also cube 2 and 3.
   (b) Cube 5. Also cube 10 and 4.

3. In (a) and (b) below, the numbers in set B are found by cubing each number in set A. Write down the numbers that belong to set B in each case.

   (a) Set A: {1; 2; 3; 4; 5; 6; 7; 8}
   Set B:

   (b) Set A: {10; 20; 30; 40; 50}
   Set B:

4. (a) Write down the squares of the first 15 natural numbers.
   (b) What do you observe about the last digit of each square number?
   (c) Give an example of a number that ends in one of the digits you have written above that is not a square.

The number 64 can be written both as a square and a cube.

$64 = 8^2$ and $64 = 4^3$

The number 17 is neither a square nor a cube.

5. Are the following numbers squares, cubes, both or neither? Just write *square, cube, both* or *neither*. Compare your answers with the answers of two classmates.
   (a) 64 (b) 1 (c) 121
   (d) 1 000 (e) 512 (f) 400
   (g) 65 (h) 216 (i) 169

2.4 The square root and the cube root

The inverse to finding the square of a number is to find its **square root**.

The question, "What is the square root of 25?" is the same as the question, "What number, when squared, equals 25?"

The answer to the question is 5 because 5 × 5 = 25.

determining what number was squared

1. What number, when squared, equals 9? Explain.
2. What is the square root of49? Explain.
3. What number, when squared, equals 81?Explain.
4. What number, when squared, equals 225? Explain.
5. What is the square root of121? Explain.
6. What number must be squared to get 169? Explain.
7. Copy and complete the diagrams below.

$$14 \longrightarrow \boxed{\text{squared}} \longrightarrow ?$$

$$? \longleftarrow \boxed{\text{square root}} \longleftarrow 196$$

The inverse operation to finding the cube of a number is to find its **cube root**.

The question, "What number, when cubed, equals 125?" is the same as the question, "What is the cube root of 125?"

The answer to the question above is 5 because 125 = 5 × 5 × 5.

determining what number was cubed

1. What number, when cubed, equals 27?Explain.
2. What is the cube root of 343? Explain.
3. What number, when cubed, equals 8? Explain.
4. What is the cube root of 1 000? Explain.
5. What number, when cubed, equals 512?Explain.
6. What number produces the same answer when it is squared and when it is cubed?

7. Copy and complete the diagrams below.

? → cubed → 1 000

10 ← cube root ← ?

calculating square roots and cube roots

1. Copy and complete the table. The first one has been done for you.

	Number	Cube root	Check your answer
(a)	8	2	2 × 2 × 2 = 8
(b)	27		
(c)	64		
(d)	125		
(e)	216		
(f)	1 331		
(g)	1 000		
(h)	512		
(i)	8 000		

2. Copy and complete the table. The first one has been done for you.

	Number	Square root	Check your answer
(a)	9	3	3 × 3 = 9
(b)	1 600		
(c)	144		
(d)	196		
(e)	625		
(f)	900		
(g)	16		
(h)	400		
(i)	121		

The symbol $\sqrt{25}$ can be used to indicate thesquare root of 25. So we can write $\sqrt{25} = 5$.

The symbol $\sqrt[3]{125}$ can be used to indicate thecube root of 125. So we can write $\sqrt[3]{125} = 5$.

3. What mathematical symbol can be used to indicate each of the following?
   (a) The square root of 169
   (b) The cube root of 343
   (c) The square root of 2 500
   (d) The cube root of 729
   (e) The cubeof 25
   (f) The square of 25

By agreement amongst mathematicians, the symbol $\sqrt{\ }$ means the square root of the number that is written inside the symbol. So we normally write $\sqrt{4}$ instead of $\sqrt[2]{4}$.

For the cube root, however, the number 3 outside of the root sign $\sqrt[3]{\ }$ must be written in order to distinguish the cube root from the square root.

4. Copy the table. Find the values of each of the following. The first one has been done for you. Check your answers.

		Value	Check your answer
(a)	$\sqrt{64}$	8	$8 \times 8 = 64$
(b)	$\sqrt{49}$		
(c)	$\sqrt{36}$		
(d)	$\sqrt{784}$		
(e)	$\sqrt{2\ 025}$		
(f)	$\sqrt{324}$		

5. Copy the table. Find the values of each of the following. The first one has been done for you. Check your answers.

		Value	Check your answer
(a)	$\sqrt[3]{8}$	2	$2 \times 2 \times 2 = 8$
(b)	$\sqrt[3]{64}$		
(c)	$\sqrt[3]{512}$		
(d)	$\sqrt[3]{1}$		
(e)	$\sqrt[3]{216}$		
(f)	$\sqrt[3]{125}$		

2.5 Comparing numbers in exponential form

bigger, smaller or equal?

1. Which is bigger?
   (a) 2^5 or 5^2
   (b) 3^4 or 4^3
   (c) 2^3 or 6^1

We can use mathematical symbols to indicate whether a number is bigger, smaller or has the same value as another number.

We use the symbol > to indicate that the number on the left-hand side of the symbol is bigger than the one on the right-hand side. **The number 5 is bigger than 3 and we express this in mathematical language as 5 > 3.**

The symbol < is used to indicate that the number on the left-hand side of the symbol is smaller than the number on the right-hand side. **The number 3 is smaller than 5 and we express this mathematically as 3 < 5.**

When numbers have the same value, we use the equal sign (=). **The numbers 2^3 and 8 have the same value and we write this as $2^3 = 8$.**

2. Use the symbols =, < or > to make the following true. Check your answers.

(a) $\sqrt[3]{64}$ ☐ $\sqrt{16}$	(b) 3^3 ☐ 4^2
(c) 6 ☐ $\sqrt{36}$	(d) $\sqrt[3]{125}$ ☐ $\sqrt{100}$
(e) 3^3 ☐ $\sqrt[3]{216}$	(f) 2^4 ☐ 3^4
(g) 2^3 ☐ 3^2	(h) $\sqrt[3]{1}$ ☐ $\sqrt{1}$
(i) 9 ☐ 3^3	(j) 100 ☐ 15^2

3. Which is bigger, 1^{100} or 100^1? Explain.

4. What is the biggest number you can make with the symbols 4 and 2?

5. Two whole numbers that follow on each other, like 4 and 5, are called consecutive numbers. Is the difference between the squares of two consecutive whole numbers always an odd number?

be smart when doing calculations

Our knowledge of squares can help us to do some calculations much quicker. Suppose you want to calculate 11×12.

11^2 has a value of 121. We know that $11 \times 11 = 121$.
11×12 means that there are 12 elevens in total.
So $11 \times 12 = 11 \times 11 + 11$
$= 121 + 11$
$= 132$

Suppose you want to calculate 11×17.

11×17 = 17 elevens in total = 11 elevens + 6 elevens
Well, we know that $11 \times 11 = 121$
So $11 \times 17 = 11 \times 11 + 6 \times 11$
$= 121 + 66$
$= 187$

Now do the following calculations in your exercise book, using your knowledge of square numbers.

1. 11×19
2. 13×16
3. 15×18
4. 12×18

arranging numbers in ascending and descending order

The numbers 1, 4, 9, 16, 25, ... are arranged from the smallest to the biggest number. We say that the numbers 1, 4, 9, 16, 25, ... are arranged in **ascending order**.

The numbers 25, 16, 9, 4, 1, ... are arranged from the biggest to the smallest number. We say that the numbers 25, 16, 9, 4, 1, ... are arranged in **descending order**.

1. In questions (a) to (d), arrange the numbers in ascending order:

   (a) $\sqrt[3]{64}$; 3^2; $\sqrt{64}$; $\sqrt{36}$

   (b) $\sqrt{225}$; $\sqrt[3]{729}$; $\sqrt[3]{1\,000}$; 2^2

   (c) $\sqrt[3]{1}$; 0; 100; 10^3

   (d) 1^2; 2^3; 4^2; 5^2

2. In questions (a) to (d), arrange the numbers in descending order:

   (a) $\sqrt[3]{216}$; $\sqrt[3]{10^3}$; 2^5; 20

   (b) 10^3; $\sqrt[3]{20^3}$; $\sqrt{144}$; 12^2

   (c) $\sqrt{121}$; $\sqrt[3]{125}$; 11^2; 5^3

   (d) 1^5; 2^4; 7^2; 6^3; 5^3

2.6 Calculations

the order of operations

When a numerical expression includes more than one operation, for example both multiplication and addition, what you do first makes a difference.

If there are no brackets in a numerical expression, it means that **multiplication and division must be done first, and addition and subtraction only later**. For example, the expression $12 + 3 \times 5$ means "multiply 3 by 5; then add 12". It does *not* mean "add 12 and 3; then multiply by 5".

If you wish to specify that addition **should be done first**, that part of the expression should be **put in brackets.** For example, if you wish to say "add 5 and 12; then multiply by 3", the numerical expression should be $3 \times (5 + 12)$ or $(5 + 12) \times 3$.

Here is another example: The expression $10 - 6 \div 3$ means "divide 6 by 3; then subtract the answer from 10". It does *not* mean "subtract 6 from 10; then divide by 3". If you wish to specify that subtraction should be done first, that part of the expression should be put in brackets. The numerical expression $(10 - 6) \div 3$ means "subtract 6 from 10; then divide the answer by 3".

It is important to know the **correct order** in which operations in a numerical expression should be done.

writing numerical expressions in words

1. Write each of the following numerical expressions in words:

(a) $5 \times 2^2 + 3$ (b) $5^2 \times (2 + 3)^2$ (c) $\sqrt{36 + 64} + 3^3$

(d) $\sqrt{16} + \sqrt{9}$ (e) $10^3 - 9^3$ (f) $(18 \div \sqrt{9})^2$

(g) $\frac{26}{6}\ \sqrt{4}$

calculations with exponents

Do these calculations without using a calculator.

1. Calculate:

(a) $2^4 + 1^4$ (b) $(2 + 1)^4$ (c) $2^3 + 3^3 + 4^3$

(d) $2^3 + 5^3 \times 3$ (e) $12^2 \div 2^3$ (f) $\dfrac{12 + 2 \times 3^2}{4^2 - 1^3}$

2. Do the calculations below and then say which expression has the same value as 2^5.
   (a) $2^3 + 2^2$ (b) $2^3 \times 2^2$

3. Do the calculations below and then say which expression has the same value as 5^4.
   (a) $5^3 + 5^1$ (b) $5^3 \times 5^1$

4. Which of the expressions below has the same value as 8^4?
   (a) $2^4 \times 4^4$ (b) $8^3 \times 8$

5. Calculate the following:
   (a) $4^2 + 3^2$ (b) $12^2 + 5^2$

6. (a) Continue this list to find the values of the "powers of 2" from 2^1 to 2^{12}:
   $2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$;
   (b) Do you notice a pattern in the last digit of the numbers? Write down the pattern in your own words.
   (c) Use the pattern to predict the *last digit* of the following values. (You should not need to actually calculate the values in full.)
   (i) 2^{20} (ii) $2^{1\,002}$

calculations involving square roots and cube roots

1. Calculate each of the following without using a calculator:
   (a) $\sqrt{64} + \sqrt{36}$ (b) $\sqrt{9 + 16}$
   (c) $\sqrt{25}$ (d) $\sqrt{100}$
   (e) $\sqrt{64 + 36}$ (f) $\sqrt{9} + \sqrt{16}$

2. Say whether each of the following is true or false. Explain your answer.
   (*Note*: ≠ in question (d) means "is not equal to")
   (a) $\sqrt{64 + 36} = \sqrt{64} + \sqrt{36}$ (b) $\sqrt{16} + \sqrt{9} = \sqrt{16 + 9}$
   (c) $\sqrt{100} = \sqrt{64} + \sqrt{36}$ (d) $\sqrt{25} \neq \sqrt{9} + \sqrt{16}$
   (e) $\sqrt{9 \times 9} = 9$ (f) $\sqrt[3]{2 \times 2 \times 2} = 2$
   (g) $\sqrt{169} - \sqrt{25} = 8$ (h) $\sqrt{169 - 25} = 12$

3. Calculate each of the following without using a calculator:
   (a) $2 + \sqrt[3]{8} + (3 + 2)^2$ (b) $2 + \sqrt[3]{8} + 3^2 + 2^2$
   (c) $2 + \sqrt[3]{8} + 2^5 - 2^3$ (d) $\frac{5 + 4 \times (\sqrt{169} - 2^3)}{5}$
   (e) $(15 - \sqrt{25})^3$ (f) $\frac{28 - 24}{(\sqrt[3]{27} + 1)^2} \sqrt{4}$

Worksheet

1. Write in expanded form:

   6^6

2. Write in exponential form:

   14 to the power 9

3. Rewrite the numbers from the smallest to the biggest: 3^4; 2^5; 4^3; 10

4. Say whether each of the following is true or false. Explain your answer.

   (a) $\sqrt{64 + 36} = \sqrt{64} + \sqrt{36}$

   (b) $\sqrt{25} + \sqrt{9} = \sqrt{59 + 5}$

5. Calculate:

   (a) $3^3 \times 2^2$

   (b) $\sqrt{144} + \sqrt{81}$

   (c) $11^2 + 5^2 - \sqrt{144}$

   (d) $(14 - 12)^4 \div \sqrt[3]{8}$

   (e) $9^2 - 4^2 \times 3$

   (f) $7 + \sqrt[3]{125} + 1^5 - 2^3$

   (g) $(\sqrt[3]{27} + \sqrt{64})^2$

   (h) $(\sqrt{16 + 9} \div 5^1) \times 93$

   (i) $\dfrac{9^2 + 12^2 + 5^3 + 650}{\sqrt[3]{125} \times 10^2}$

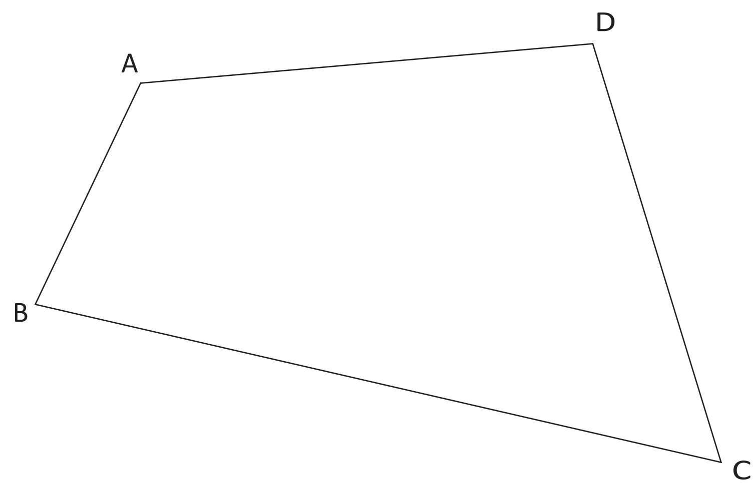
   (j) $\dfrac{6^3 - (\sqrt{169})^2 + \sqrt[3]{8}}{7^2 \times 1^9}$

Chapter 3
Geometry of straight lines

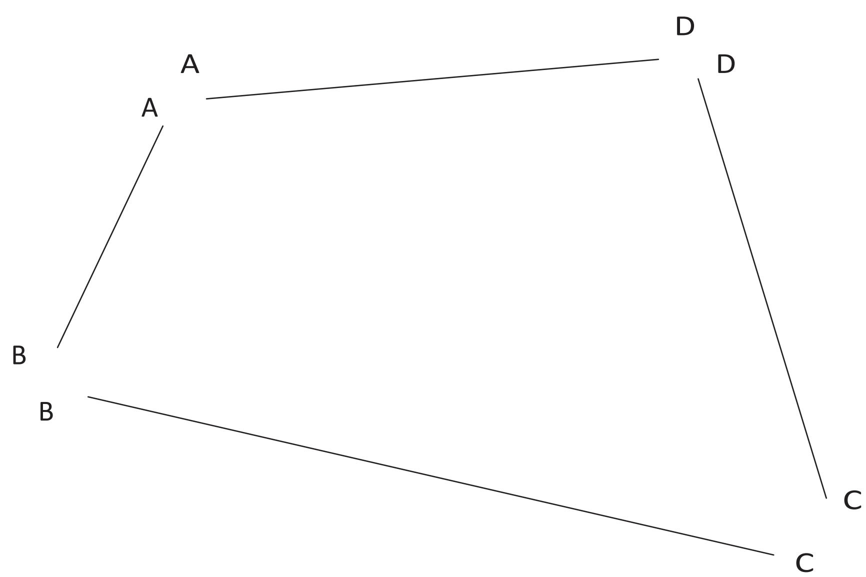
3.1 Line segments, lines and rays

line segments

1. Measure each side of this quadrilateral. Copy the quadrilateral and write the measurements at each side.



Each side of a quadrilateral is a **line segment**.

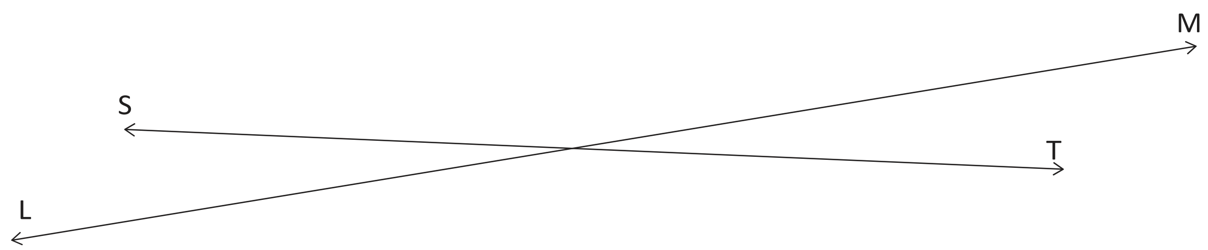


A **line segment** has a definite starting point and a definite endpoint. We can draw and measure line segments.

2. Draw a line segment that is 12 cm long.

lines and rays

We can think of lines that have no ends, although we cannot draw them completely. We draw line segments to represent lines. When we draw a line segment to represent a line, we may put arrows at both ends to show that it goes on indefinitely on both sides.



The word **line** is used to indicate a line that goes on in both directions. We can only see and draw part of a line. A line cannot bemeasured.

1. Draw line AB.

2. Did you draw the whole ofline AB? Explain.

We can also think of a line that has a definite starting point but goes on indefinitely at the other end. This is called a half-line or a **ray**.

We can draw the starting point and part of a ray, using an arrow to indicate that it goes on at the one end.

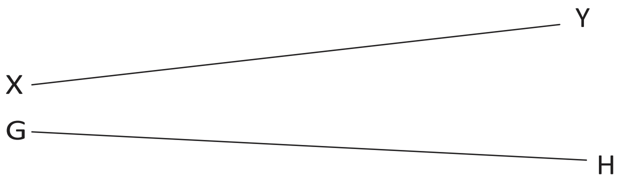
Ray PQ goes on towards the right:

Ray DC goes on towards the left:

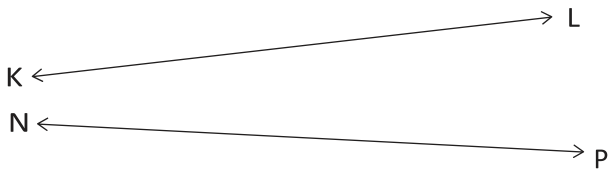
3. Draw ray EF.

4. Did you draw the whole of ray EF? Explain.

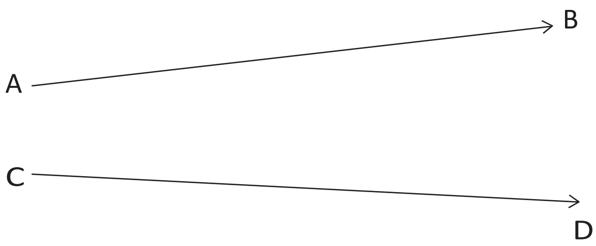
5. Do line segments XY and GH meet anywhere?



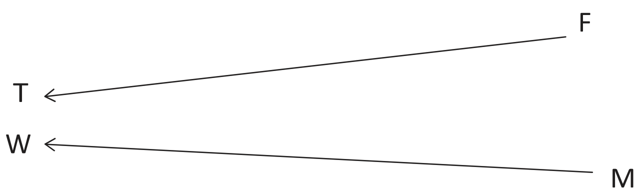
6. Do lines KL and NP meetanywhere?



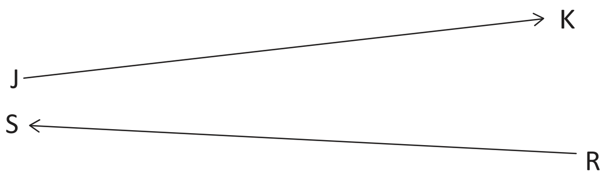
7. Do rays AB and CD meetanywhere?



8. Do rays FT and MW meet anywhere?



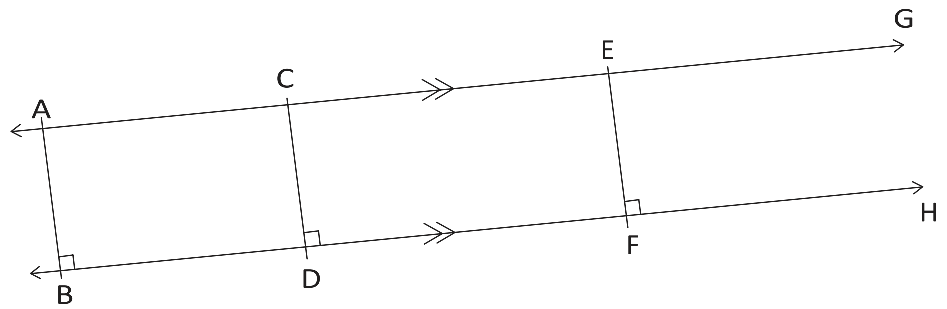
9. Do rays JK and RS meet anywhere?



3.2 Parallel and perpendicular lines

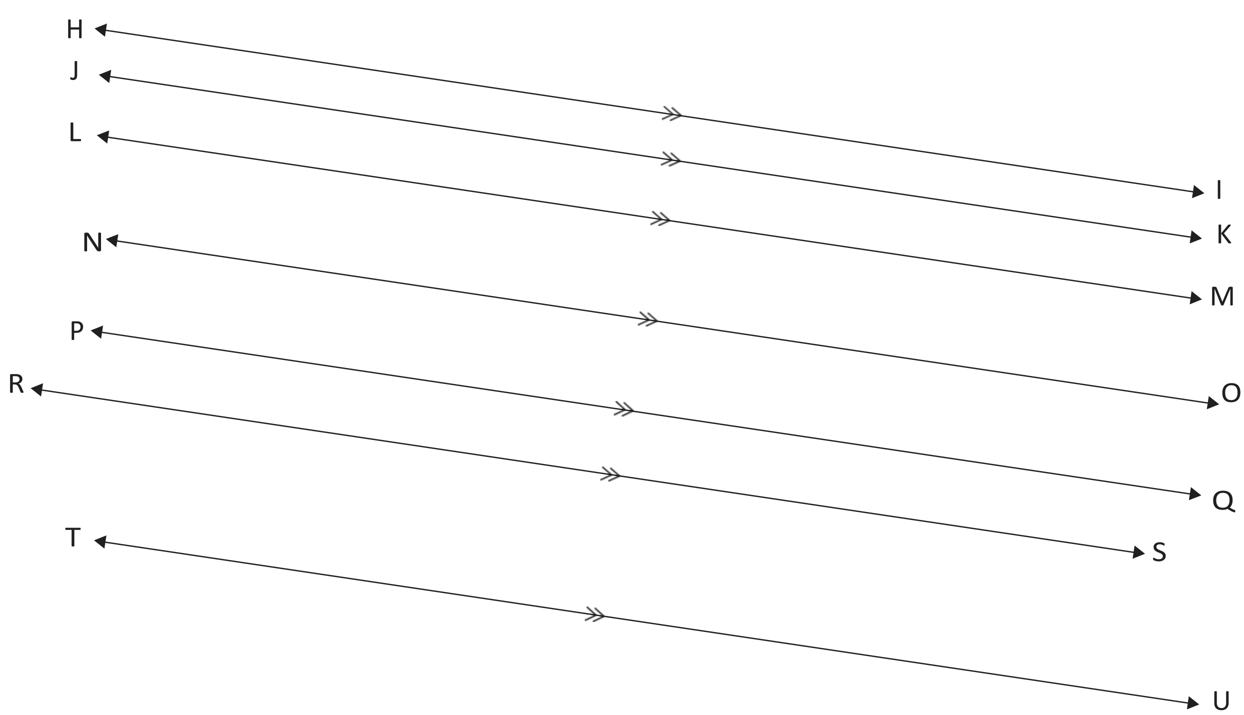
parallel lines

Two lines that are a constant distance apart are called **parallel lines**. Lines AG and BH below are parallel. The symbol // is used to indicate parallel lines. We write: AG//BH.



1. Measure the distance between the two lines:
   (a) at A and B
   (b) at C and D
   (c) at E and F

Here are some more parallel lines:

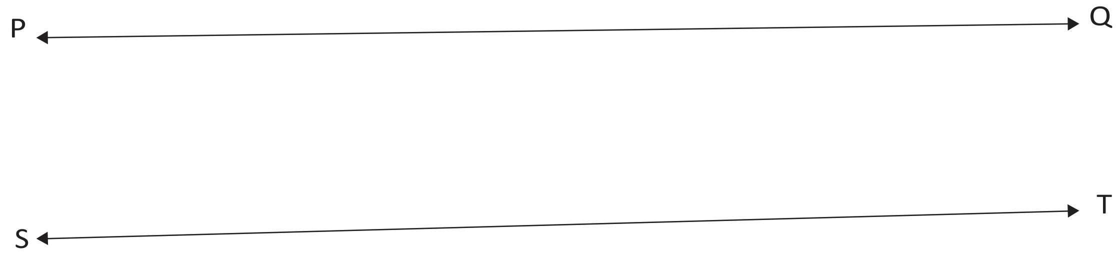


2. Draw two parallel lines.

3. Draw three lines that are parallel to each other.

4. Will parallel lines meet somewhere?

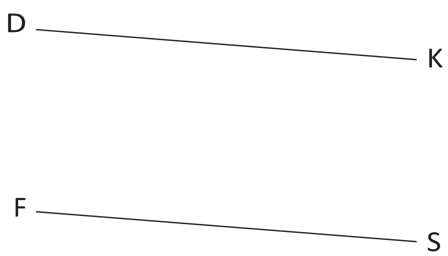
5. Do you think lines PQ and ST are parallel? How can you check?



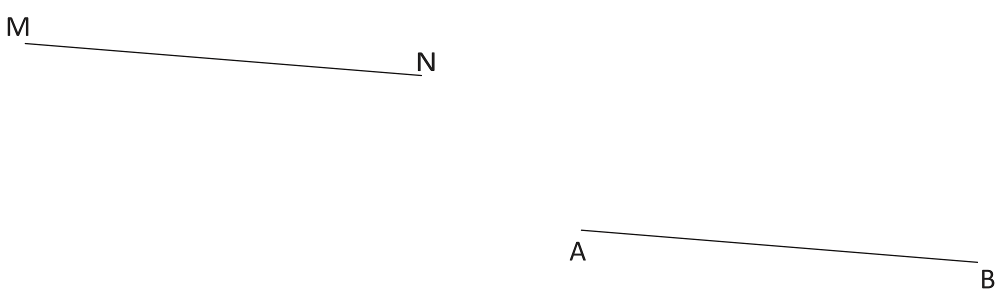
6. (a) Draw two lines that are almost parallel, but not quite.
   (b) Describe what you did to make sure that your two lines are not parallel.

7. Can two line segments be parallel?

8. Are line segments DK and FS parallel?

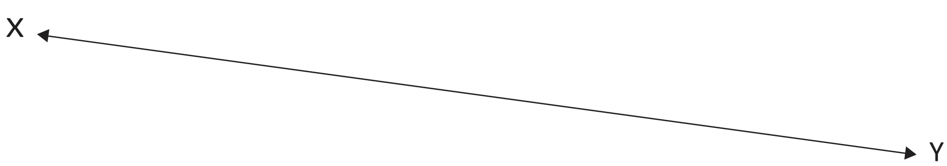


9. Are line segments MN and AB parallel?



10. What can you do so that you will be better able to check whether the above two line segments are parallel or not?

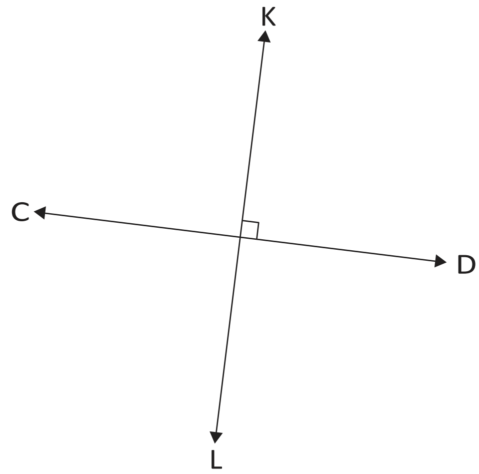
11. Can a line be parallel on its own?



12. Copy line XY above. Then draw a line parallel to it.

perpendicular lines

Lines CD and KL below are perpendicular to each other. The symbol ⊥ is used to indicate perpendicular lines. We write: CD ⊥KL.



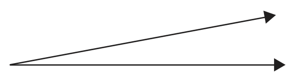
1. How many angles are formed at the point where the above two lines meet?

Two lines that form right angles are **perpendicular** to each other.

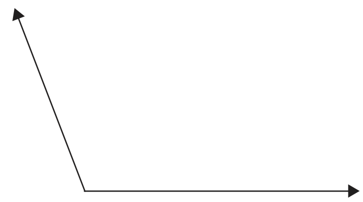
2. Draw two rays that have the same starting point.
3. Draw two rays that are perpendicular to each other and have the same starting point.
4. Draw two rays that meet, but not at their starting points.
5. Draw two rays that meet but not at their starting points, and that are perpendicular to each other.
6. Can you draw two rays that have the same starting point, and are parallel to each other?

Chapter 4
Construction of geometric figures

4.1 Angles revision



small angle

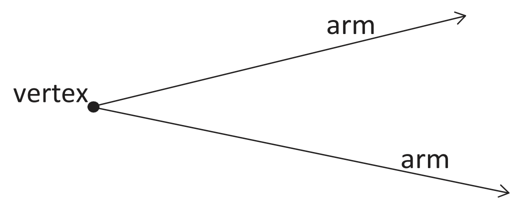


big angle

When two lines point in different directions, we say they are **at an angle** to each other. If the directions are almost the same, we say the **angle** between them is small. If the directions are very different, we say the angle between them is big.

Words we use to describe angles:

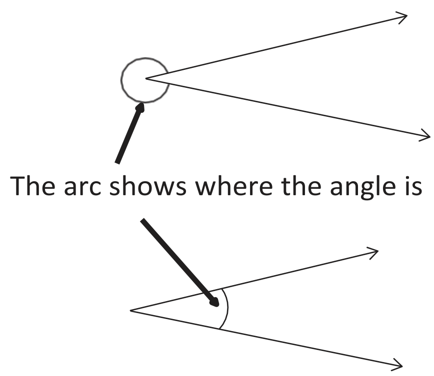
- **Arms of the angle:** the two lines that are at an angle to each other
- **The vertex:** the point where the two arms meet
- **Vertices:** plural of "vertex"



Symbols to describe angles:

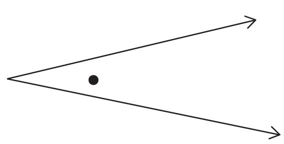
Arrowheads on the lines mean that the lines keep on going. The length of an angle's arms does not change the size of the angle. Whether the arms are long or short, the angle size stays the same.

There are **two angles at a vertex** so it is important to show which one we are talking about.

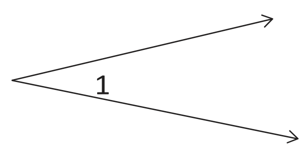


Labelling angles:

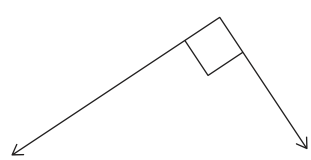
There are many different ways to label angles. Look at the examples below:



Using a dot or a star

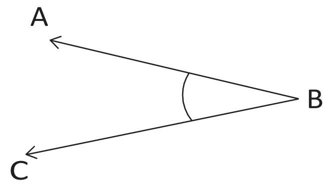


Angle 1



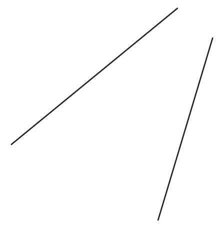
Right angle (90°)

You can name the angle on the right in different ways: you can say $A\hat{B}C$ or $C\hat{B}A$ or just $\hat{B}$. The "hat" on the letter shows where the angle is.



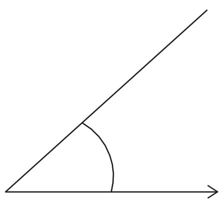
revision: seeing angles and describing angles

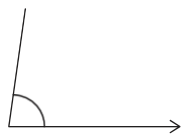
1. Look at the drawing on the right.



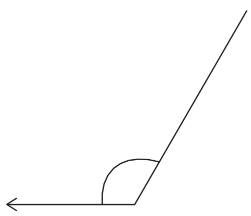
   (a) Are these lines at an angle to each other? Do the lines have to meet to be at an angle?
   (b) Copy the lines. Use a pencil and your ruler to draw the lines a bit longer so they meet. Did you change the angle between the lines when you extended them?

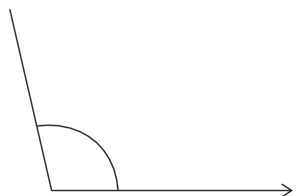
2. Arrange the angles from biggest to smallest. Just write the letters (a) to (f) in the correct order.

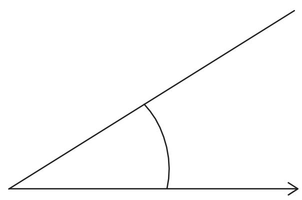
(a)


(b)


(c)

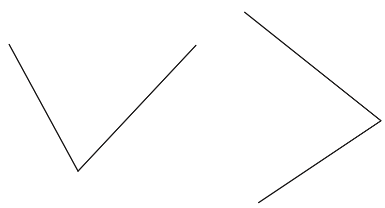

(d)


(e)


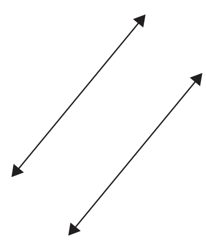
(f)


3. How can you check that an angle is a right angle without using any special mathematics equipment? (*Hint*: Think about where you can find right angles around you.)

4. Are these two angles the same size? Describe how you found your answer. (*Hint*: A piece of scrap paper may help!)

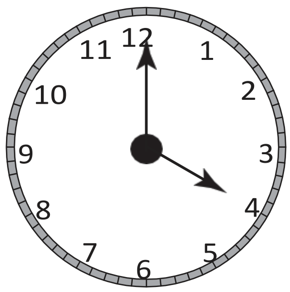


5. Two lines are drawn by holding down a ruler and drawing lines on both sides. What can you say about the two lines?



6. Look at the analogue clock face on the next page. The minute hand and the hour hand make an angle. Focus on the smaller angle for now.

(a) Explain why the angle between the hands at 8 o' clock is the same size as the angle at 4 o' clock.
(b) Compare the angle at 2 o' clock with the angle at 4 o' clock. What do you notice? Why is thisso?
(c) Is the angle at 3 o' clock the same as the angle at a quarter past 12? Explain.



7. When you open the cover of a hardcover book you can make different angles. Can you think of at least five other situations in everyday life where objects are turned through angles? Say what the arms and the vertices are in each of your examples.

4.2 The degree: a unit for measuring angles

Imagine if we didn't have units for measuring length. *How would tailors make clothes to the right size without a tape measure? How could an architect design a safe and beautiful house without a ruler? How could we lay out a professional soccer field without being able to measure accurately in metres?*

We need units and measuring instruments in many situations. You know that we use metres, centimetres, kilometres, millimetres, etc. for measuring lengths.

We should also have units for measuring angles. The units we use for measuring angles are very ancient. No one today is completely sure why, but our ancestors decided many thousands of years ago that a revolution should be divided into 360 equal parts. We call these parts degrees. The symbol for a degree is °.

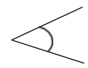
some familiar angles in degrees

1. Copy and complete the table by filling in the size of each angle described.

Angle (in words)	Angle (degrees)
right angle	90°
straight angle	
revolution	360°
half a right angle	
a third of a right angle	
a quarter of a right angle	22,5°
half a straight angle	
three quarters of a revolution	
a third of a revolution	

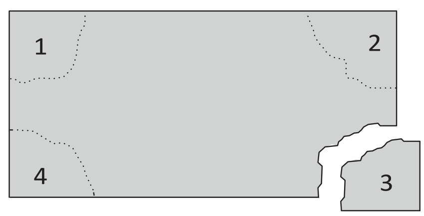
2. Look at the clock shown. How many degrees does:
   (a) the minute hand move in an hour?
   (b) the hour hand move in an hour?

3. In Grade 6 you learnt that angles are classified into types. Copy and complete the table. The first one has been done as an example for you.

Angle	Size of the angle	Sketch of the angle
Acute angle	Between 0° and 90°	
Right angle		
Obtuse angle		
Straight angle		
Reflex angle		
Revolution		

comparing angles using a4 paper

You need a sheet of A4 paper. At the corners you have four right angles. Number them and tear the corners off as shown in the diagram. Do not make them too small.



Now use your right angles to investigate the following situations:

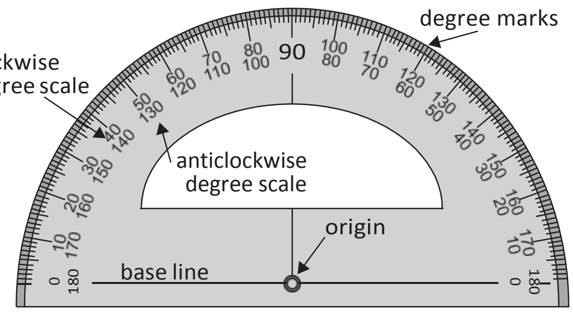
1. Show that a straight angle is two right angles. You can sketch what you have done in your book.
2. Show that a revolution is four right angles. You can sketch what you have done in your book.
3. Create a right angle using three of your corners. You can sketch what you have done in your book.
4. Describe how you can use one of your corners to check if an angle is acute, right or obtuse.
5. (a) Fold corner 1 so that you can use it to measure 45°.
   (b) Fold corner 2 so that you can use it to measure 30°.
   (c) Fold corner 3 so that you can use it to measure 22,5°.
   (d) Which is bigger: a right angle *or* half a right angle + a third of a right angle + a quarter of a right angle? Can you do a calculation to show that?

Important: Keep your folded pieces of paper for the next lesson!

4.3 Using the protractor

We have a special instrument for measuring angles. It is called a **protractor**. Look at the picture of a typical protractor with its important parts labelled.

Protractors can be big or small but they all measure angles in exactly the same way. The size of the protractor makes no difference to an angle's size.



measuring some familiar angles

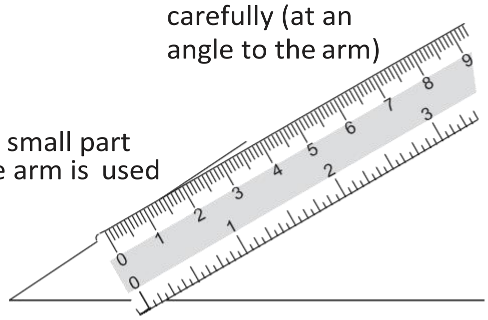
You need the four folded angles from the previous activity on page 60. If you didn't do that activity, then go back now and follow the instructions in question 5.

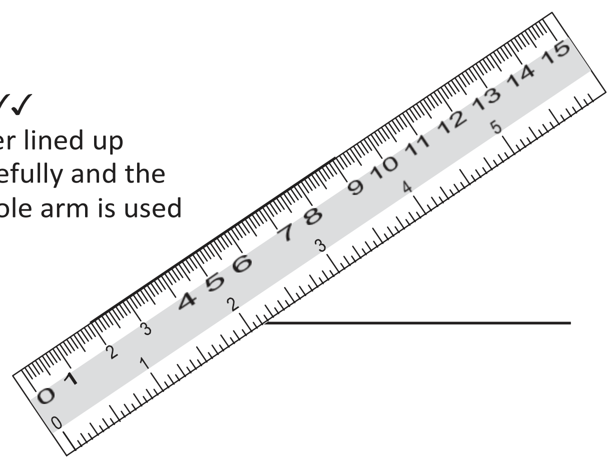
1. In a group of three or four, use your protractor to measure the angles that you made: 90°; 45°; 30° and 22,5°.
2. Did you measure the correct sized angle? If not, then ask yourself the following questions:
   - Did you put the vertex of the angle at the origin of the protractor?
   - Is the bottom arm of your angle lined up with the base line?
   - Did you fold your corners correctly?

how to use a protractor to measure an angle

Step 1: Are the angle arms long enough?

The angle arms must be a little longer than the distance from the origin of the protractor to its edge. If they are too short, use a sharp pencil and a ruler to make them longer. Be careful to line the ruler up with the arm.





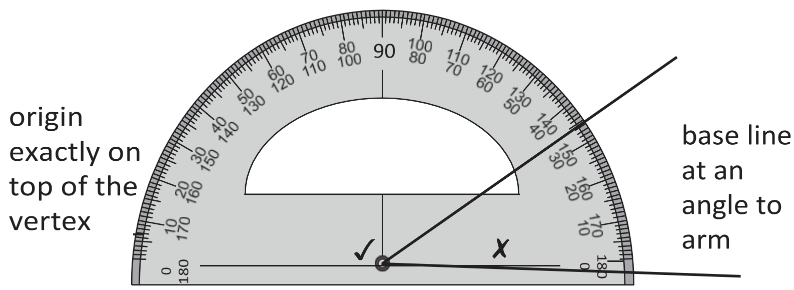
Now you are ready to start measuring your angle.

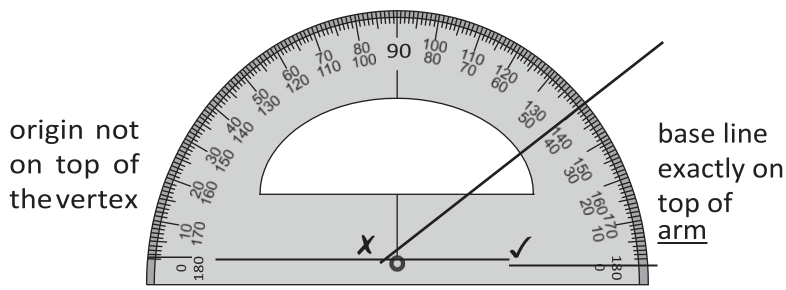
Step 2: Line up the angle and your protractor

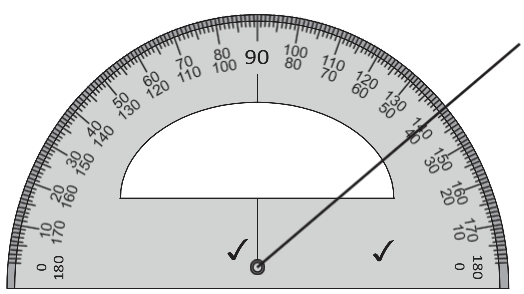
Place your protractor on top of the angle. Make sure of the following:

- the origin is exactly on the vertex of the angle, and
- the base line is exactly on top of one of the arms of the angle.

Keep adjusting the position of the protractor until the origin and the base line are exactly lined up.





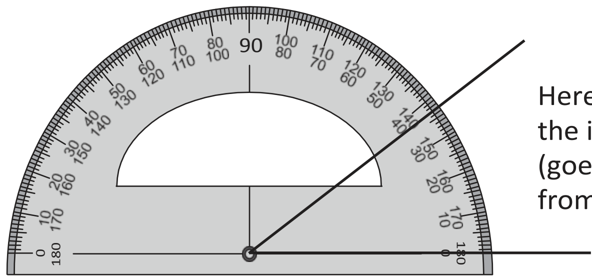


origin exactly on top of the vertex and base line exactly on top of arm

Once your protractor is in the correct place, keep a finger on the protractor to stop it from moving. If it moves … start again! You are now ready to make a measurement.

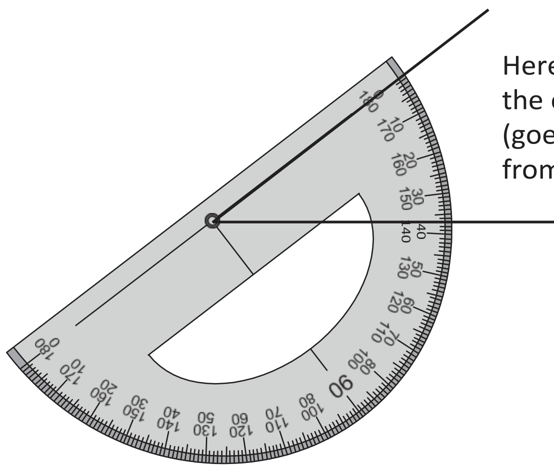
Step 3: Measure the angle

A protractor gives a clockwise degree scale and an anticlockwise degree scale. You choose the correct scale by finding the one that starts with 0° on the angle arm. Look at where the other angle arm passes under the degree scale. That is where your measurement is.



Here we must use the inner scale (goes anticlockwise from the base line).

You can also place the protractor on the angle using the other arm. Then the correct position looks like this:



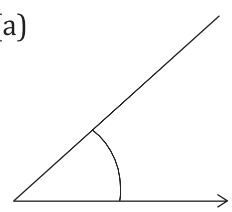
Here we must use the outer scale (goes clockwise from the base line).

The angle in the pictures above is 37°. Do you agree? Do you see that there are two ways to measure an angle?

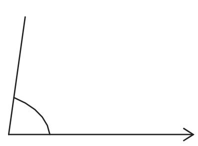
practise measuring with a protractor

1. Measure the angles and copy and complete the table on the next page. If you need to, copy the drawing and extend the arms.

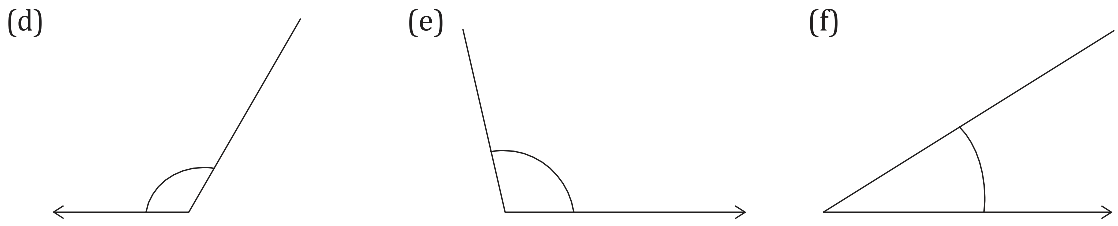
(a)



(b)



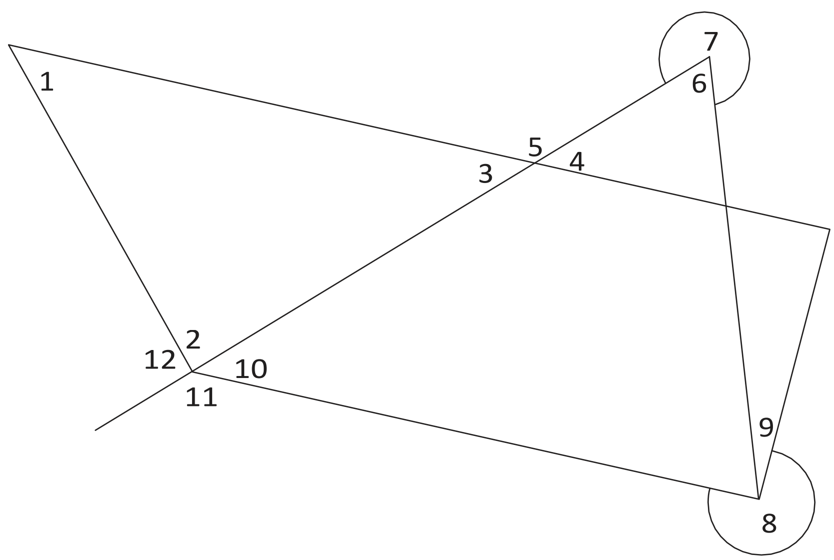
(c)



Angle	(a)	(b)	(c)	(d)	(e)	(f)
Angle size in degrees						

2. Copy the table below. Measure all the numbered angles in the following figure. Some angles can be measured directly, others not. Your protractor cannot measure reflex angles like angles 7 and 8. So you will have to make a plan!

Angle	Size
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	



3. Write a short note for yourself about measuring reflex angles.

some things to think about

Look at your answers in question 2 above.

1. How do angles 3 and 4 compare?
2. What about angles 6 and 7?
3. What about angles 4 and 5?
4. There are some interesting ideas here. Try to do some further investigation and show your teacher what you discover.

4.4 Using a protractor to construct angles

constructing angles to a given line

Work together with a partner on this activity. You need your protractor, a sharp pencil and a straight ruler.

1. Your first challenge is to copy the line below and construct a line at exactly right angles to it. Begin by choosing a point on the line. You must mark this point clearly and neatly with a small dot. Then use your understanding of a protractor to draw a 90° angle.

2. Now copy Steps 2 to 4 and fill in the missing words.

   Step 1: Choose a point anywhere on the line. Make a small mark on the line. (You don't always have a choice here. Sometimes you must use a specific point on the line.)

   Step 2: Place the protractor with its ____________ on the line and its origin exactly on top of the ____________

   Step 3: Make a small, clear mark at the ____________

   Step 4: Use a ruler to line up the two ____________ and draw a straight line that passes exactly through them.

3. Copy the line below and use it to construct the angles listed below. The line below will be one arm of the angles you are going to construct. The vertex for each of your angles is the point labelled O where the tiny vertical line cuts the long horizontal one. Your angles must be measured *anticlockwise* from the line.

   (a) 23° (b) 45° (c) 65° (d) 79° (e) 90°
   (f) 121° (g) 154° (h) 180° (i) 200° (j) 270°
   (k) 300°

angle direction
The line you have been given below is called a **reference line**.
Mathematicians usually measure angles **anticlockwise** from the reference line.

O

4. Copy the line on the right. Then at each end, draw lines at an angle of 60° to form a triangle. What sort of triangle is this?

5. Copy and complete the quadrilateral below. The angle at P must be 52° and the one at Q, must be 23°.

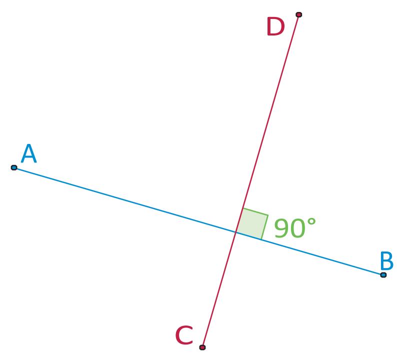
4.5 Parallel and perpendicular lines

Perpendicular lines meet each other at an angle of 90°.

The sketch shows two perpendicular lines.

We say: AB is perpendicular to DC.

We write: AB ⊥DC



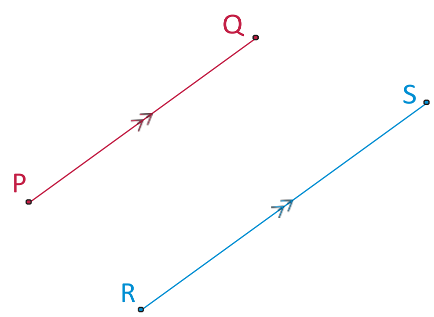
Parallel lines never meet each other. They are an equal distance apart. They have the same direction.

The sketch shows two parallel lines.

We say: PQ is parallel to RS.

We write: PQ //RS

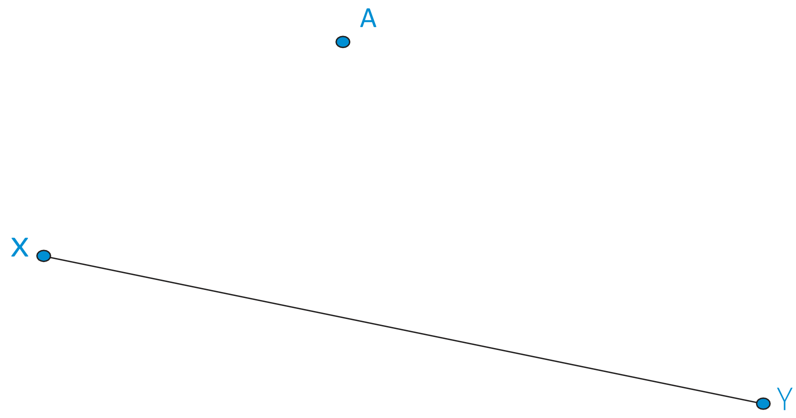
The arrows on the middle of the lines show that the lines are parallel to each other.



constructing perpendicular and parallel lines

When constructing parallel lines, remember that the lines always stay the same distance apart. Follow the steps on page 67 to draw perpendicular and parallel lines using a protractor and a ruler.

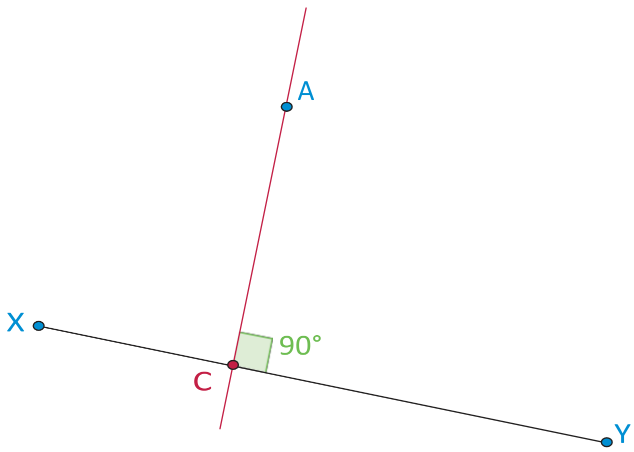
1. We want to draw a line that is parallel to XY and that passes through point A.



Step 1: Draw a perpendicular line between A and XY.

Copy the line XY above. Use your protractor to draw a line that goes through A and is at 90° to XY. Labelthepoint Cwhereyour new line touches XY.

Look at the sketch on the right if you get stuck.

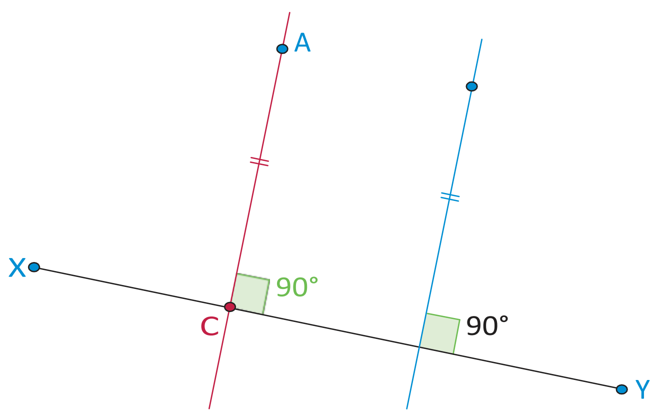


Step 2: Measure the perpendicular distance between the point and the line.

Write down the length of AC.

Step 3: Draw a point that is the same distance from the line.

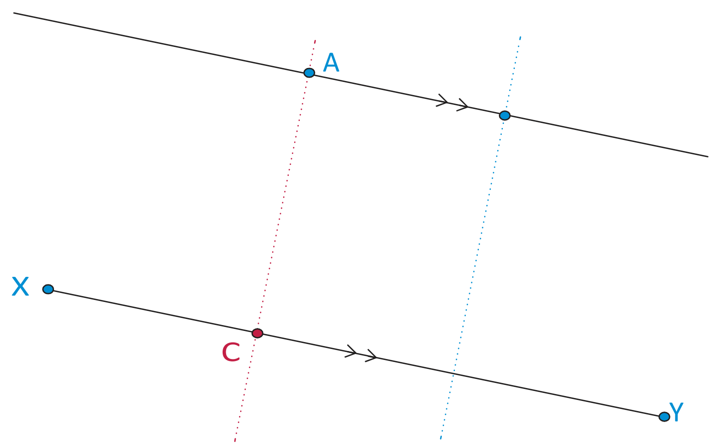
Then draw another line that is perpendicular to line XY. Mark off the same length as AC on that line. The sketch below shows what you must do.



Step 4: Draw the parallel line.

Join A with the new point that is an equal distance away from XY.

You now have a parallel line.



2. Practise constructing perpendicular and parallel lines using a protractor and a ruler.

4.6 Circles are very special figures

And now for something slightly different ... let us have a look at **circles**.

a circle with string

You may need to work with a partner here. You need two sharp pencils and a short length of string, an A4 sheet of paper and a ruler.

1. Tie the string to both pencils with double knots. The knots must be firm but not tight. The string must swing easily around the pencils without falling off. Once you have tied your string, the distance between the pencils when the string is tight should not be more than 8 cm.
2. Your partner must hold one pencil *vertically* with its point near the centre of the sheet of paper.
3. Now carefully move the tip of the other pencil around the middle one, drawing as you go. Try to keep the string *stretched* and the pencil *vertical* as you draw.

   If you have been careful, you will have drawn a circle (well, hopefully something pretty close to a circle). You can swop now so your partner also has a turn drawing while you hold the centre pencil.
4. Mark three points on the circle line. Measure the distance between the point and the centre of the circle for each. If you have a circle you should find that the distances are the same.

Circles are special for many reasons. The most important reason is the following:

The distance from the centre of a circle to the edge is the same in any direction.

This distance is called the **radius**.

We pronounce this: “ray-dee-us”.

The plural of **radius** is **radii**.

We pronounce this: “ray-dee-eye”.

think about it

Can you think of any other figure where the distance between the centre and the edge is constant in all directions?

- A square?
- A hexagon?
- What about an oval shape (ellipse)?

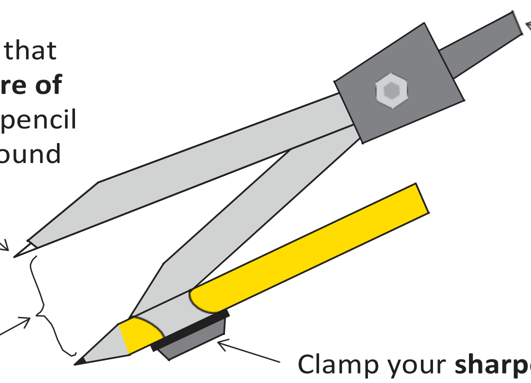
do some investigation to see what you can find.

Do you agree that the two pencils and string are not a good way to draw circles? The string is stretchy. It is difficult to change the radius. And, the drawing pencil can wander off course and make a spiral or a wobbly curve. We need something better.

4.7 Using the compass

We need a special instrument for drawing circles. It must have a pointy tip, like the centre pencil. It must also have a drawing tip, like the pencil you moved. If you can set the distance between these two tips, you can draw circles of any radius. This instrument is called a **pair of compasses**, or often just a **compass**.

This is the point that goes at the **centre of your circle**. The pencil tip must turn around this point.



Here is the handle of the compass. You hold it between your forefinger and thumb. Only use one hand when you draw a circle. **Turn the compass by twisting it between your forefinger and thumb**.

This gap will be the **radius** of your circle. You can set the gap to the correct radius using your ruler. Change the gap by changing the angle between the arms of the compass.

Clamp your **sharpened** pencil tightly in here. The tip of the pencil must be next to the tip of the turning point when you push the arms together.

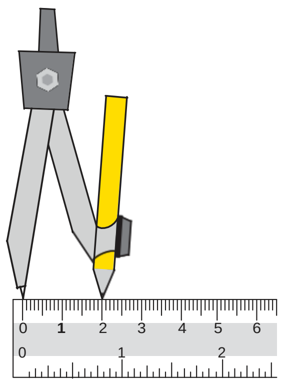
constructing circles with a compass

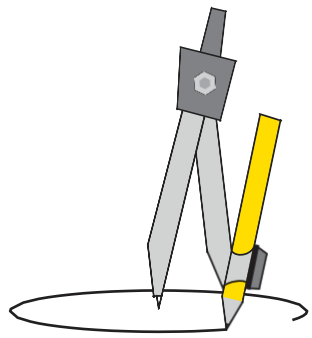
1. You will see a point labelled A below. Follow the steps below and on the next page to draw a circle with a radius of 2 cm. The centre must be at A.

   Step 1: Place the pointed tip on the zero line of your ruler. Carefully widen the angle between the arms. Move the pencil tip until it is exactly at 2 cm. Make sure that the pointed tip is still on zero. Be careful not to change the gap once it is set to 2 cm.

   Step 2: Mark point A in the centre of your page. Gently push the pointed tip into point A. Push just deep enough into the paper to keep it in place. This will be the centre of your circle.

Step 1





Step 4

● A

   Step 3: Hold the handle between the forefinger and thumb of your writing hand. Keep your other hand out of the way. Use only one hand when you draw a circle with a compass.

   Step 4: Twist the handle between your thumb and forefinger. If you are right-handed, it is easiest to twist the compass clockwise. If you are left-handed, turn the compass anticlockwise. Let the pencil tip *drag* over the paper. Don't push down too hard on the pencil. Rather, push down lightly on the pointed arm as you draw. The pencil tip must move smoothly and easily.

2. Then draw concentric circles at centre A with radii of 3 cm, 4 cm, 5 cm and 6 cm. Set the gap carefully each time. Write the radius on the edge of each circle.

concentric circles have the same midpoint.

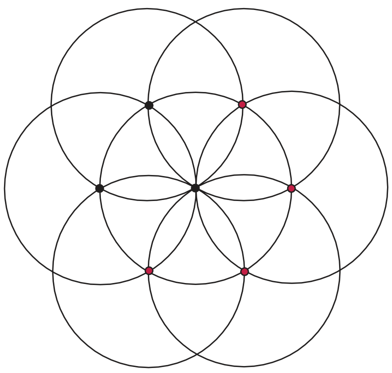
Learning to use a compass is like learning to ride a bicycle. It takes co-ordination and practice. Don't be embarrassed if it goes wrong. With practice you will get very good at it. If your circles end up being all wobbly lines, just begin again!

Here are some tips for drawing circles:

- If your circles are turning into spirals it is because the arms of your compass have moved. Check their width again against a ruler.
- If the arms of your compass won't stay in the position you set them at, it is because the nut at the hinge below the handle is loose. Ask your teacher to help you if you can't tighten it yourself.
- If you can't do the twist, imagine you have a small piece of soft clay between your thumb and forefinger and you are trying to roll it into a small strip. The twist for turning your compass uses the same type of sliding movement. Let the compass hang from your hand in the air and twist the handle. Then try it on scrap paper a few times until you can turn the compass easily.

circles on circles

It's time to have some fun with the compass while getting better at using it. Follow these instructions to draw the beautiful pattern shown on the right in your exercise book.



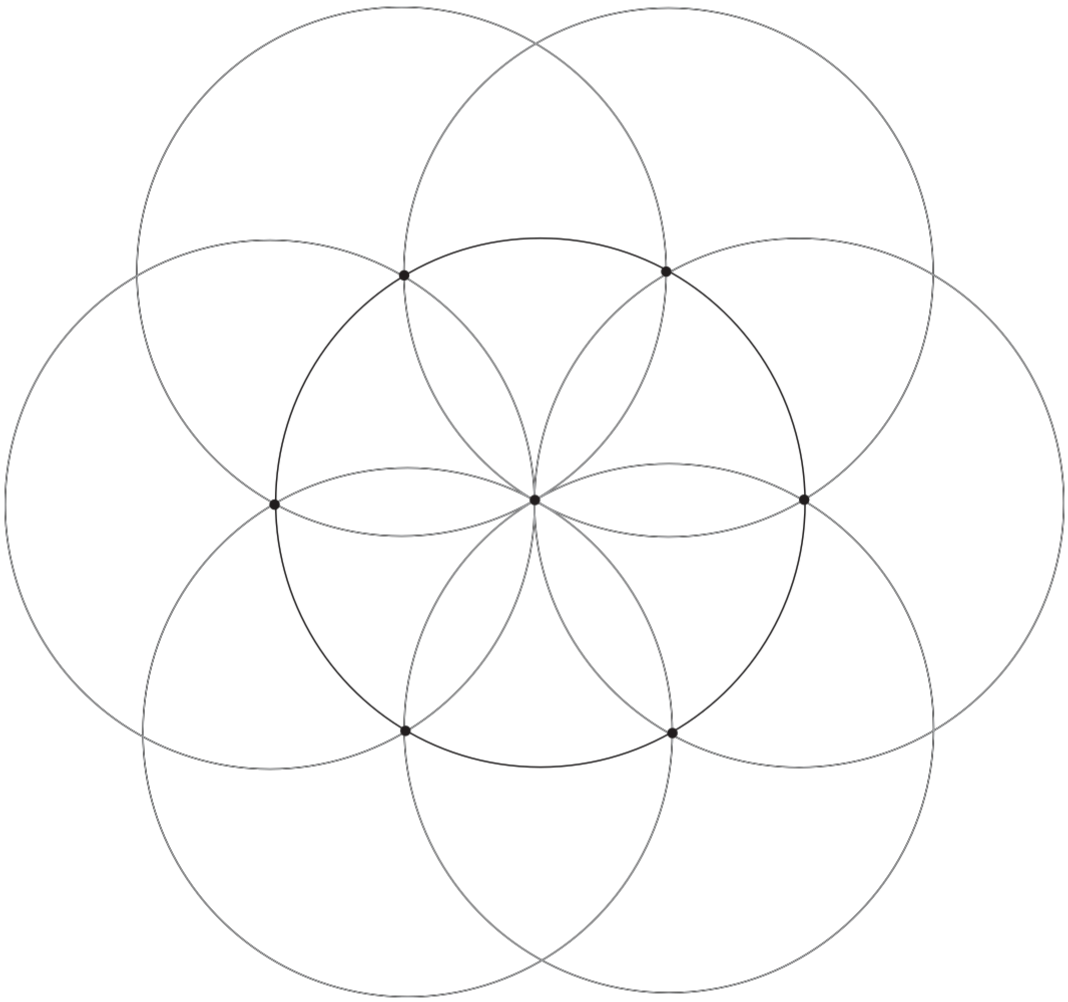
1. Make sure your pencil is sharp; then place it in the compass.
2. Set the radius to 4 cm. Draw a circle at the centre of your page. Important: your radius must stay the same for the whole activity.
3. Put your compass point anywhere on the circle edge. Draw another circle. This circle should pass through the centre of your first circle (they have the same radius).
4. Your second circle cuts the first circle at two points. Choose one of these points. Place your compass point at this point. Draw another circle of radius 4 cm.
5. Repeat step 3 with your third circle, fourth circle etc. You should end up with six circles on your first circle. That is, seven circles in total.
6. Decorate it as you please. (You can decorate your pattern further by adding more circles or joining points with straight lines, and so on. See what patterns and shapes you can discover among all the circles.)

4.8 Using circles to draw other figures

geometric figures hiding in the circles

Below is a set of seven circles like the one you drew. Sit with a partner and try to find hidden polygons.

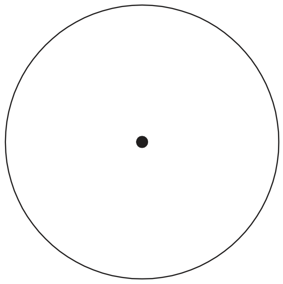
You will find these polygons by joining the points where the circles cut each other. The points will be the vertices of the polygons. Look carefully. There are triangles, quadrilaterals, pentagons and hexagons. When you can see them, neatly and carefully rule in their sides with a pencil. If there is not enough space on the set of circles below, redraw the circles on a separate piece of paper and show the figures there. If you wish, you can measure the angles at each vertex and the lengths of the sides.



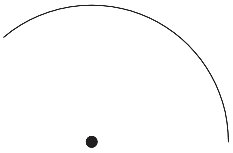
Arcs of circles

We do not have to draw whole circles to construct figures. We are only really interested in the points where the circles cross each other, so we could just draw arcs where they cross. Next year, you will use arcs in your geometric constructions.

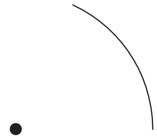
An **arc** is a small part of a circle. We use the term **circumference** when we refer to the distance around a circle or around any other curved shape.



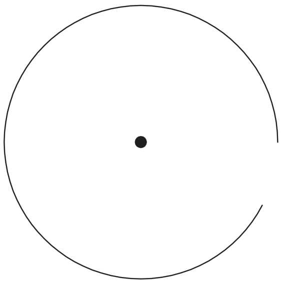
This is a full circle.



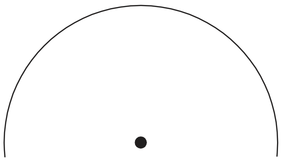
This is only part of a circle. A part of a circle like this is called an arc.



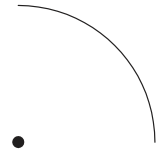
This arc is smaller.



This arc is almost a full circle.



This arc is just more than a semi-circle.



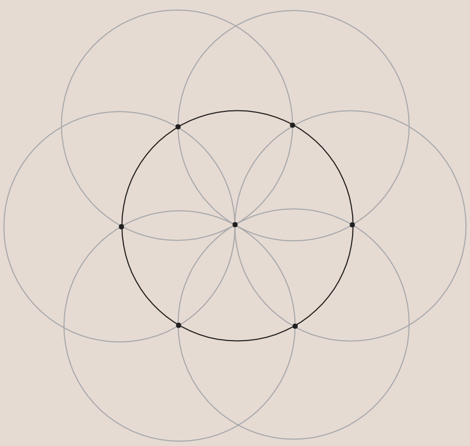
This arc is a quarter circle.

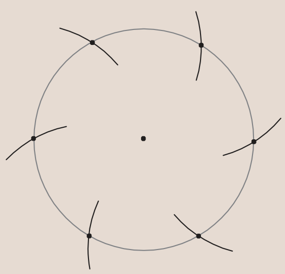
Do the following in your exercise book:

1. Draw an arc using a radius of 3 cm.
2. Draw an arc bigger than a quarter circle, using a radius of 5 cm.
3. Draw an arc smaller than a quarter circle, using a radius of 5 cm.

enrichment

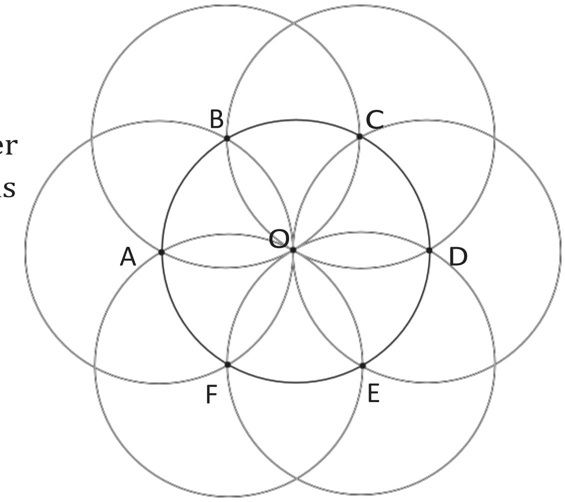
Once you have finished the work in section 4.8, experiment with drawing only the arcs that you need in various constructions. Here is an example to show how to construct a regular hexagon with only arcs:





familiar figures in the seven-circle pattern

For this activity you need five seven-circle sets like the ones drawn in the previous two activities. Start by drawing these on blank pieces of paper. Don't make your radius bigger than 4 cm. Number your sets figure 2 to figure 6. Label each figure as shown on the right.



1. Copy the figure above right by following the instructions below.
   - Figure 1: Draw lines connecting AB, BC, CD, ... up to FA.
   - Figure 2: Draw lines connecting A, O and B.
   - Figure 3: Draw lines connecting B, F and D.
   - Figure 4: Draw lines connecting BC, CE, EF and FB.
   - Figure 5: Draw lines connecting CD, DE, EF and FC.
   - Figure 6: Draw lines connecting AB, BC, CE and EA.
2. Copy and complete the table below. It shows the name of each figure and its properties.

   Figure 1 (on the right) has been done as an example.

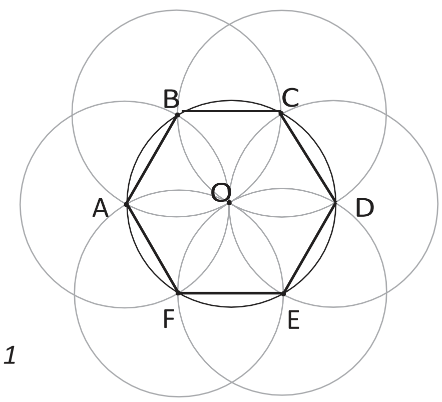


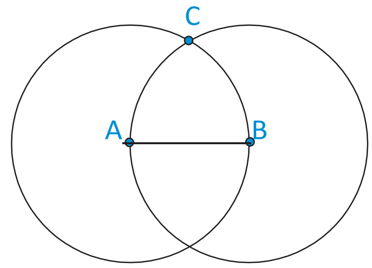
Figure 1

Figure	Name of figure	Properties
1	Regular hexagon	six-sided figure. All the sides are equal. All the interior angles are equal.
2		
3		
4		
5		
6		

construct some more figures

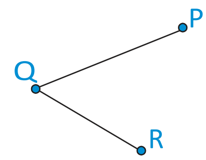
Read the instructions carefully and follow them exactly.

1. (a) Draw a line in your exercise book. The line should be between 3 cm and 6 cm long. Draw it in the middle of your page.
   (b) Label the ends A and B.
   (c) Place the point of your compass at point A. Carefully set the radius of your compass to the distance between A and B.
   (d) Draw a circle with the compass point at A.
   (e) Draw another circle with the compass point at B without changing the radius width.
   (f) The circles cross at two points. Choose one of these points. Label it C. Check that you are on the right track by comparing your sketch to the one on the right.



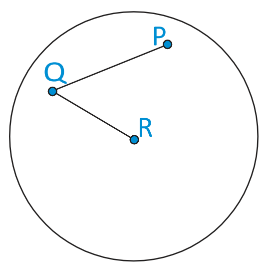
   (g) Carefully rule the lines AC and BC.
   (h) What sort of figure is ABC? Check this by measuring angles. Why do you think this happened?

2. (a) Draw two lines PQ and QR in your exercise book.
   - The lines meet and form an angle at Q.
   - You can make your angle any size.
   - Make your line lengths different.
   - Do not make your lines longer than 6 cm each.

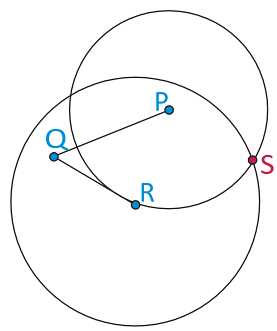


Step (a)

   (b) Place your compass point at point Q. Set the radius of your compass to the distance QP. Place the compass point at R. Draw a circle.
   (c) Place the compass point back at Q. Set the radius to the length QR. Place the compass point at P. Draw a circle.



Step (b)



Steps (c) and (d)

   (d) The two circles cross at two points. Decide which point will be the vertex of a parallelogram. Call this point S.
   (e) Join the lines SP and SR. Is PQRS a parallelogram?

something to think about
Why does this method form a parallelogram?

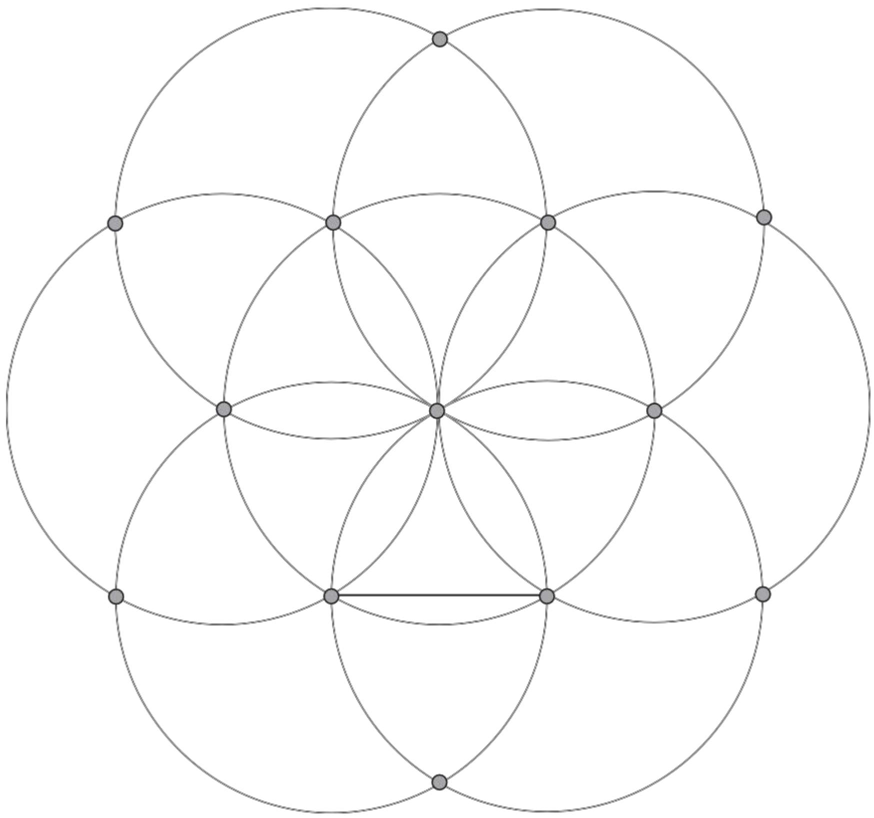
4.9 Parallel and perpendicular lines with circles

parallel and perpendicular

1. Revision: Copy and complete these definitions.
   (a) When one line is parallel to another line, the lines …
   (b) When one line is perpendicular to another line, the lines …

2. Copy the seven-circle figure below. The intersection points have been marked. A line segment has been drawn in. Use a ruler and pencil to join pairs of points so that the lines are:
   (a) parallel to the line segment
   (b) perpendicular to the line segment.

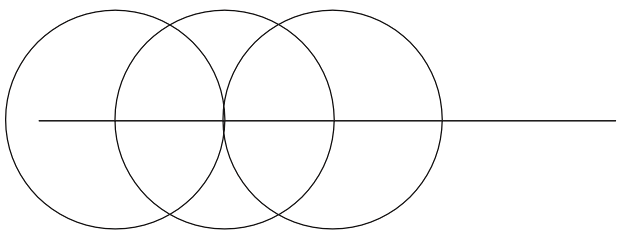
When two lines (or arcs) cross each other we say they **intersect**.
The **intersection point** is the place where they meet.



You should have drawn seven lines (two parallel and five perpendicular to the line segment).

Compare your lines with a friend's lines. Do you agree?

3. Draw a few circles with the same radius along a line. Start by drawing a line. Then use your compass to draw a circle with the midpoint on the line.



Keep your compass the same width and draw another circle with the centre where the first circle crossed the line. Repeat as many times as you wish. In the example at the bottom of the previous page, only three circles have been drawn.

(a) Can you find that example in the seven-circle figure? Look carefully until you see it.

(b) Can you see where you can construct lines that are perpendicular to the given line? Draw them carefully with a pencil and your ruler.

(c) Can you see the two lines that are parallel to the given line? Draw them in too.

4. Copy the line below. Use circles to construct a line that is perpendicular to the line below.

5. Copy the line below. Use circles to construct a line that is parallel to the line below.

extension

1. Write P•, as in the example below. Set your compass at a certain distance, for example 3 cm, and investigate points that are the same distance from a fixed point, P.

• P

2. Write two points, A and B, as shown below. Use your compass and investigate all the points

Chapter 5
Geometry of 2D shapes

5.1 Triangles, quadrilaterals, circles and others

decide which is which and draw some figures

A **triangle** is a closed figure with three straight sides and three angles.

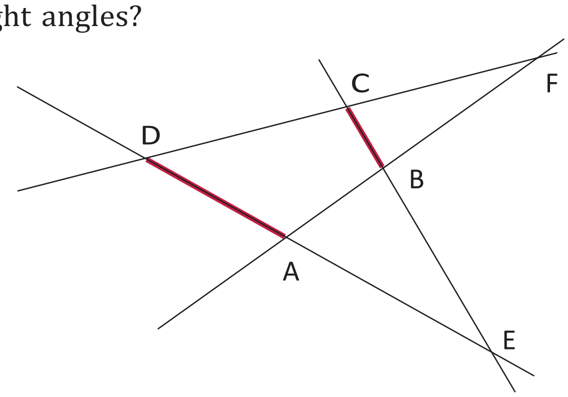
A **quadrilateral** has four straight sides and four angles.

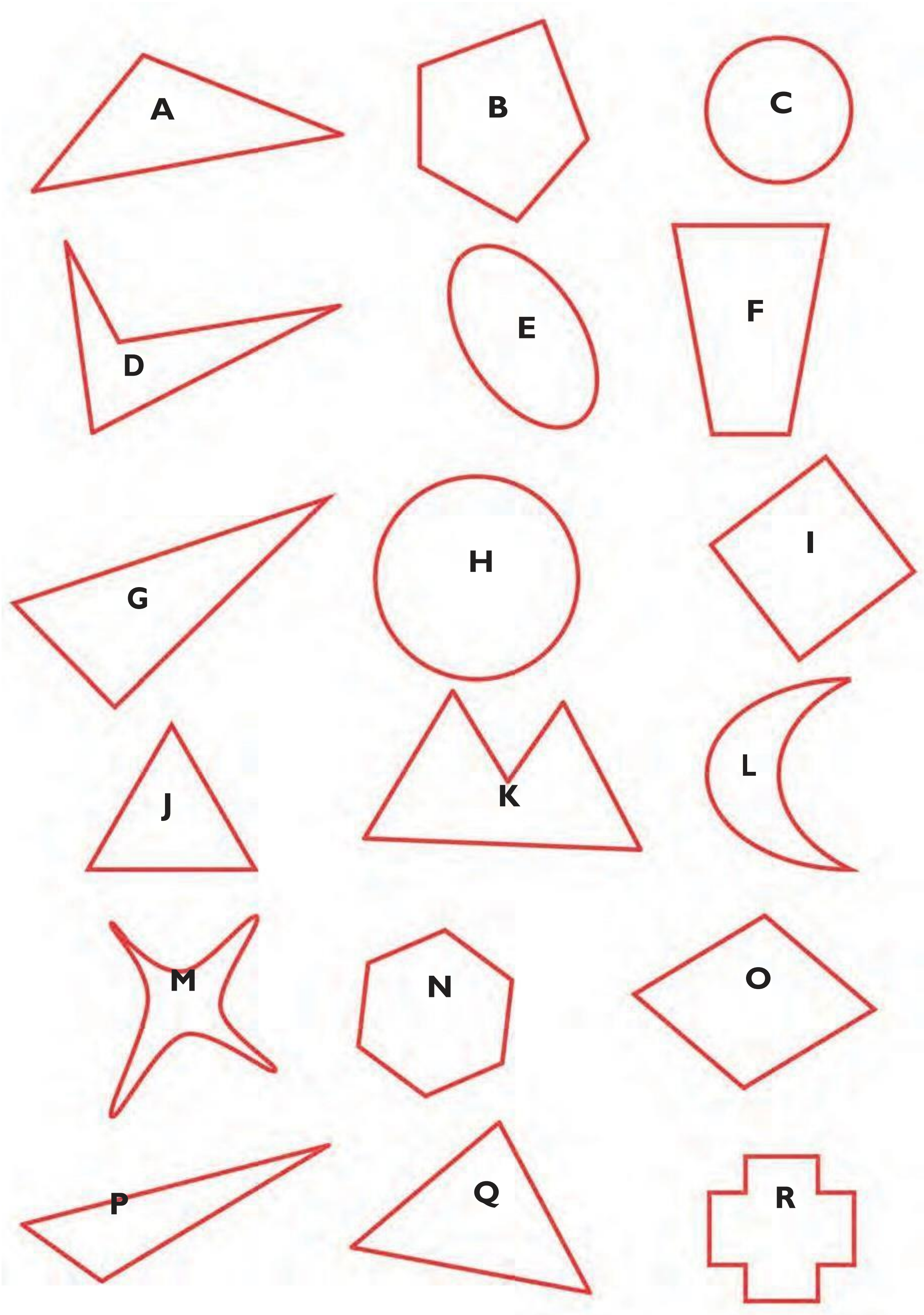
A **circle** is round and the edge is always at the same distance from the centre.

1. Which shapes on the opposite page are circles?
2. Which shapes on the opposite page are triangles?
3. Which shapes on the opposite page are quadrilaterals?

Use your ruler to do the following:

4. Make a drawing of one triangle with three acute angles, and another triangle with one obtuse angle.
5. (a) Draw a quadrilateral with two obtuse angles.
   (b) Can you draw a triangle with two obtuse angles?
6. (a) Draw a triangle with one right angle, and a triangle without any right angles.
   (b) Can you draw a triangle with two right angles?
   (c) Can you draw a quadrilateral with four right angles?
7. These four lines form quadrilateral ABCD. The two red sides, BC and AD, are called **opposite sides** of quadrilateral ABCD.
   Which other two sides of ABCD are also opposite sides?



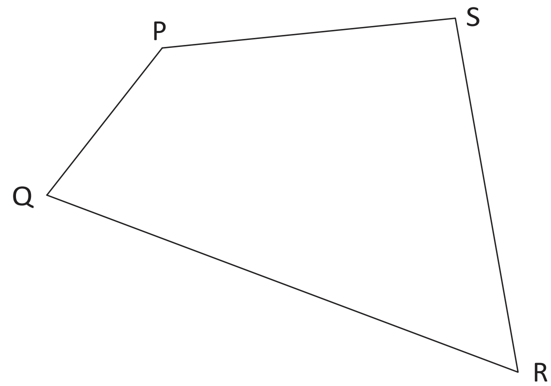

A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R

8. The lines DA and AB in the figure in question 7 are called **adjacent sides**. They meet at a point that is one of the vertices (corner points) of the quadrilateral.
   (a) Name another two adjacent sides in ABCD.
   (b) AB is adjacent to DA in the quadrilateral ABCD. Which other side of ABCD is also adjacent to DA?

9. William says:
   "Each side of a quadrilateral has two adjacent sides. Each side of a quadrilateral also has two opposite sides."
   Is William correct? Give reasons for your answer.

10. William also says:
    "In a triangle, each side is adjacent to all the other sides."
    Is this true? Give a reason for your answer.

11. In each case, say whether the two sides are opposite sides or adjacent sides of the quadrilateral PQRS.
    (a) QP and PS
    (b) QP and SR
    (c) PQ and RQ
    (d) PS and QR
    (e) SR and QR



5.2 Different types of triangles

equilateral, isosceles, scalene and right-angled triangles

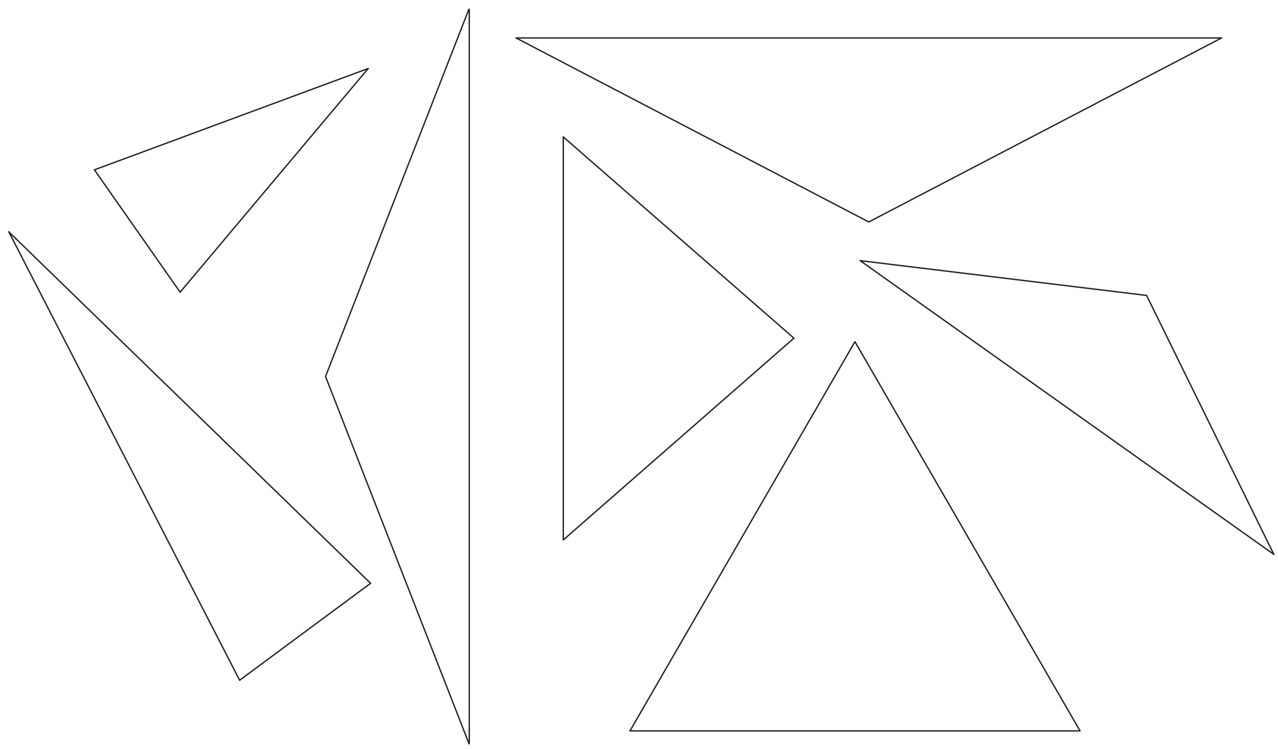
A triangle with two equal sides is called an **isosceles triangle**.

A triangle with three equal sides is called an **equilateral triangle**.

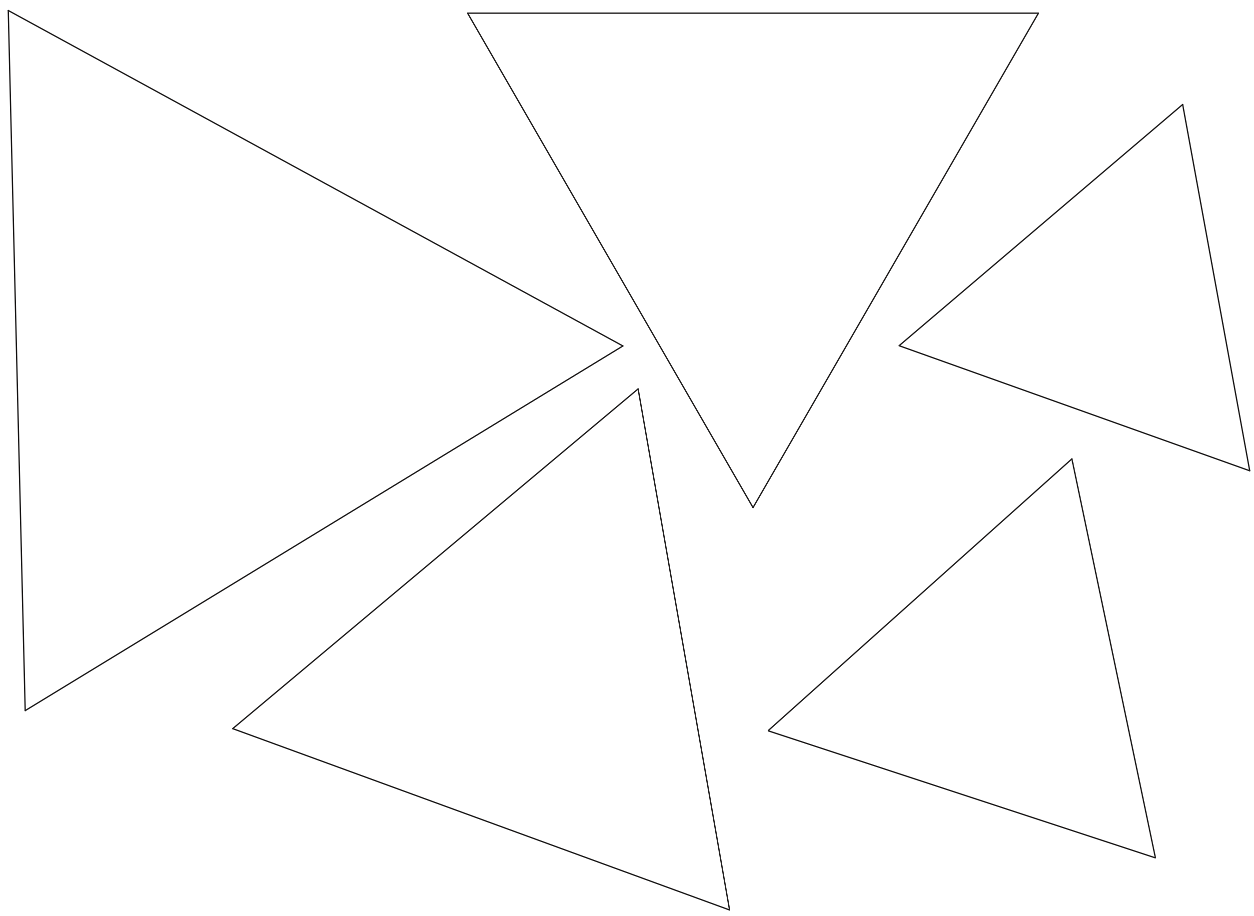
A triangle with a right angle is called a **right-angled triangle**.

A triangle with three sides with different lengths and no right angle is called a **scalene triangle**.

1. Measure every angle in each of the **isosceles triangles** given on the next page. Do you notice anything special? If you are not sure, draw more isosceles triangles in your exercise book.

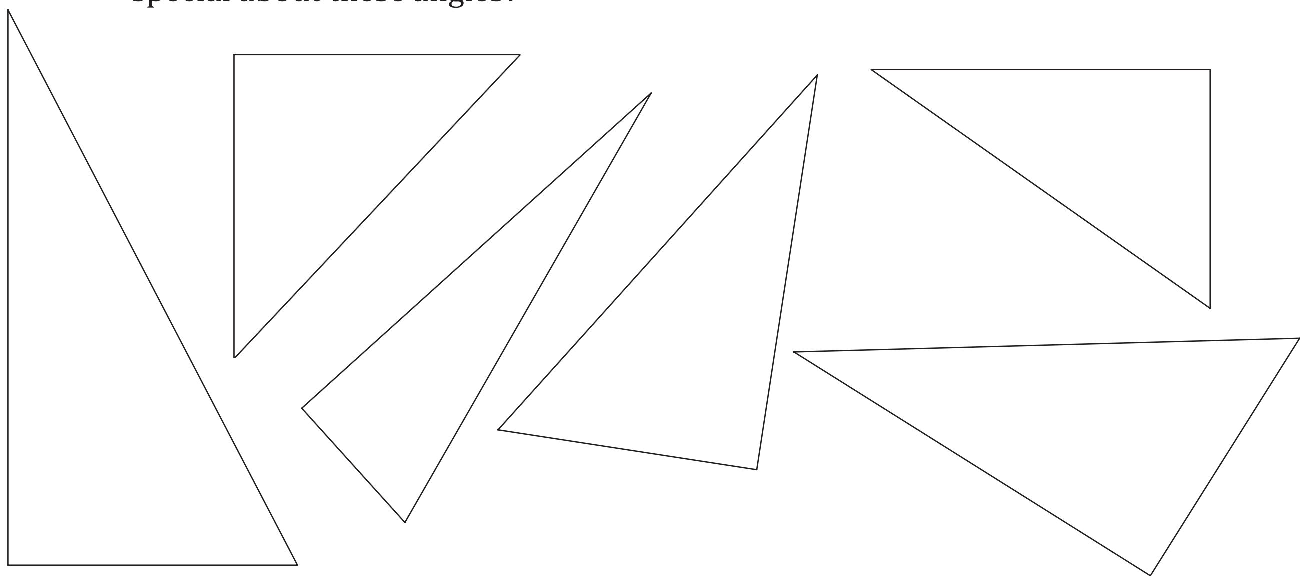


2. Measure the angles and sides of the following triangles. What is special about these triangles? In other words, what makes these triangles different to other triangles?



- These triangles are called **equilateral triangles**.

3. (a) Measure each angle in each of the following triangles. Do you notice anything special about these angles?



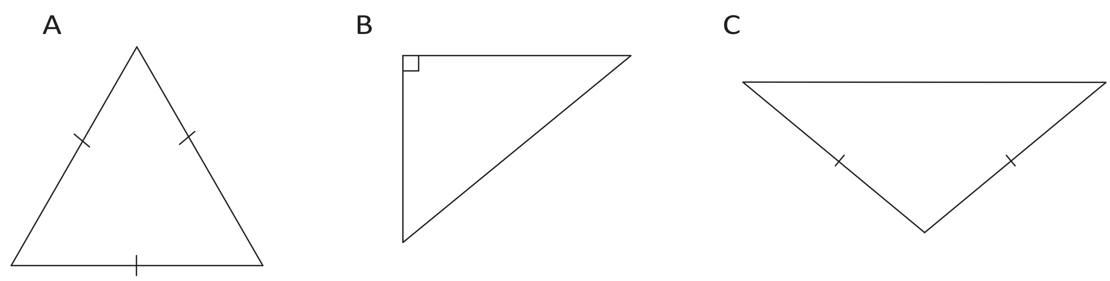
(b) Identify the longest side in each of the triangles. If you are not sure which one is the longest side, measure the sides. What do you notice about the longest side in each of these triangles?

- These triangles are called **right-angled triangles**.

comparing and describing triangles

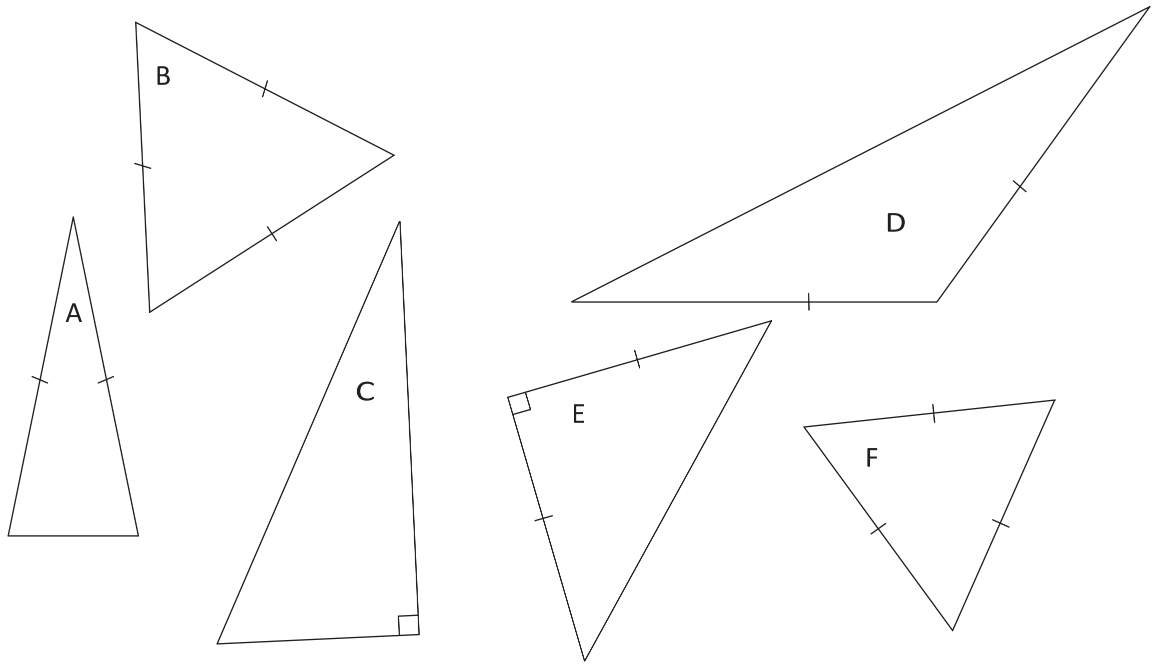
When two or more sides of a shape are equal in length, we show this using short lines on the equal sides.

1. Use the following triangles to answer the questions that follow.



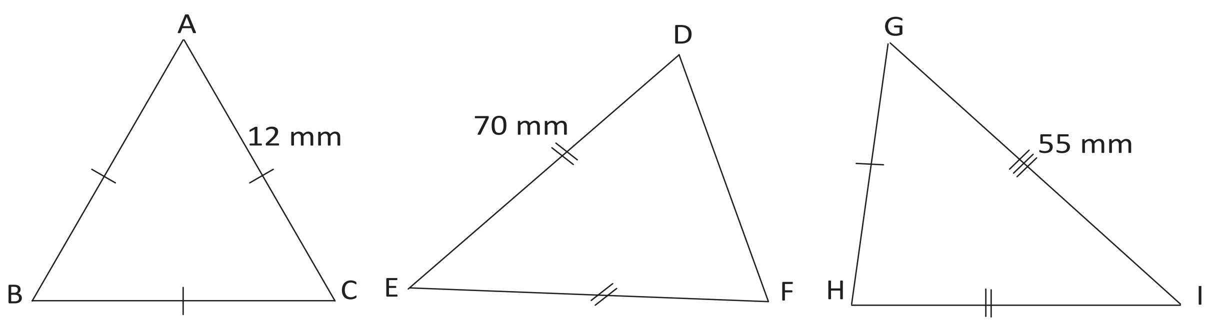
(a) Which triangle has only two sides that are equal?
What is this type of triangle called?
(b) Which triangle has all three sides equal?
What is this type of triangle called?
(c) Which triangle has an angle equal to 90°?
What is this type of triangle called?

2. In your exercise book, write down the type of each of the following triangles.



finding unknown sides in triangles

1. (a) Name each type of triangle below.

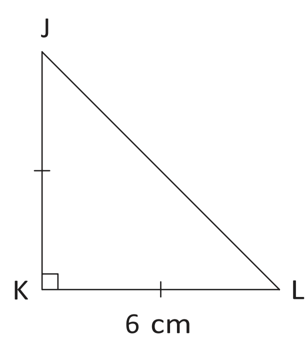


All three sides are equal *Two sides are equal* *No sides are equal*

(b) Use the given information to determine the length of the following sides:
AB BC EF

(c) Can you determine the lengths of GH and HI? Explain your answer.

2. The square in the corner of ΔJKL shows that it is a right angle. Give a reason for each of your answers below.



(a) Is this triangle scalene, isosceles, or equilateral?
(b) Name the two sides of the triangle that are equal.
(c) What is the length of JK?
(d) Name two equal angles in this triangle.ngles.
(e) What is the size of $\hat{J}$ and $\hat{L}$?

5.3 Different types of quadrilaterals

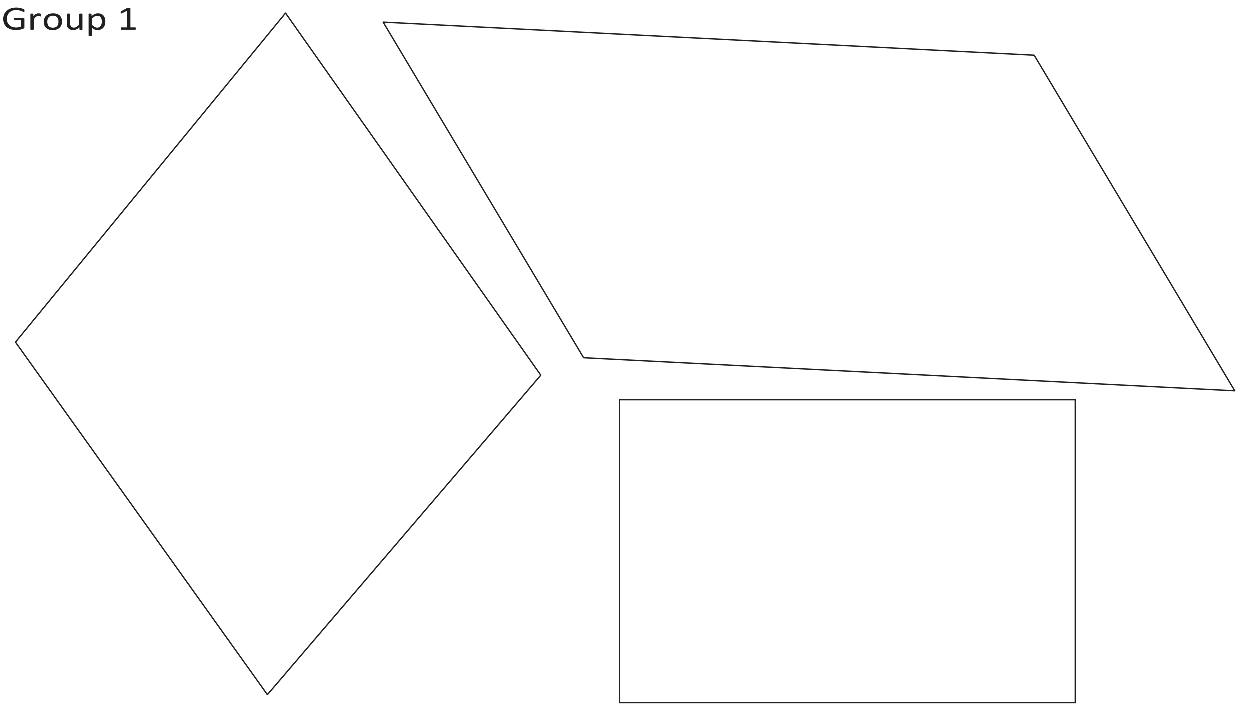
investigating quadrilaterals

1. The two pages that follow show different groups of quadrilaterals.
   (a) In which groups are both pairs of opposite sides parallel?
   (b) In which groups are only some adjacent sides equal?
   (c) In which groups are all four angles equal?
   (d) In which groups are all the sides in each quadrilateral equal?
   (e) In which groups are all four sides equal?
   (f) In which groups is each side perpendicular to the sides adjacent to it?
   (g) In which groups are opposite sides equal?
   (h) In which groups is at least one pair of adjacent sides equal?
   (i) In which groups is at least one pair of opposite sides parallel?
   (j) In which groups are all the angles right angles?

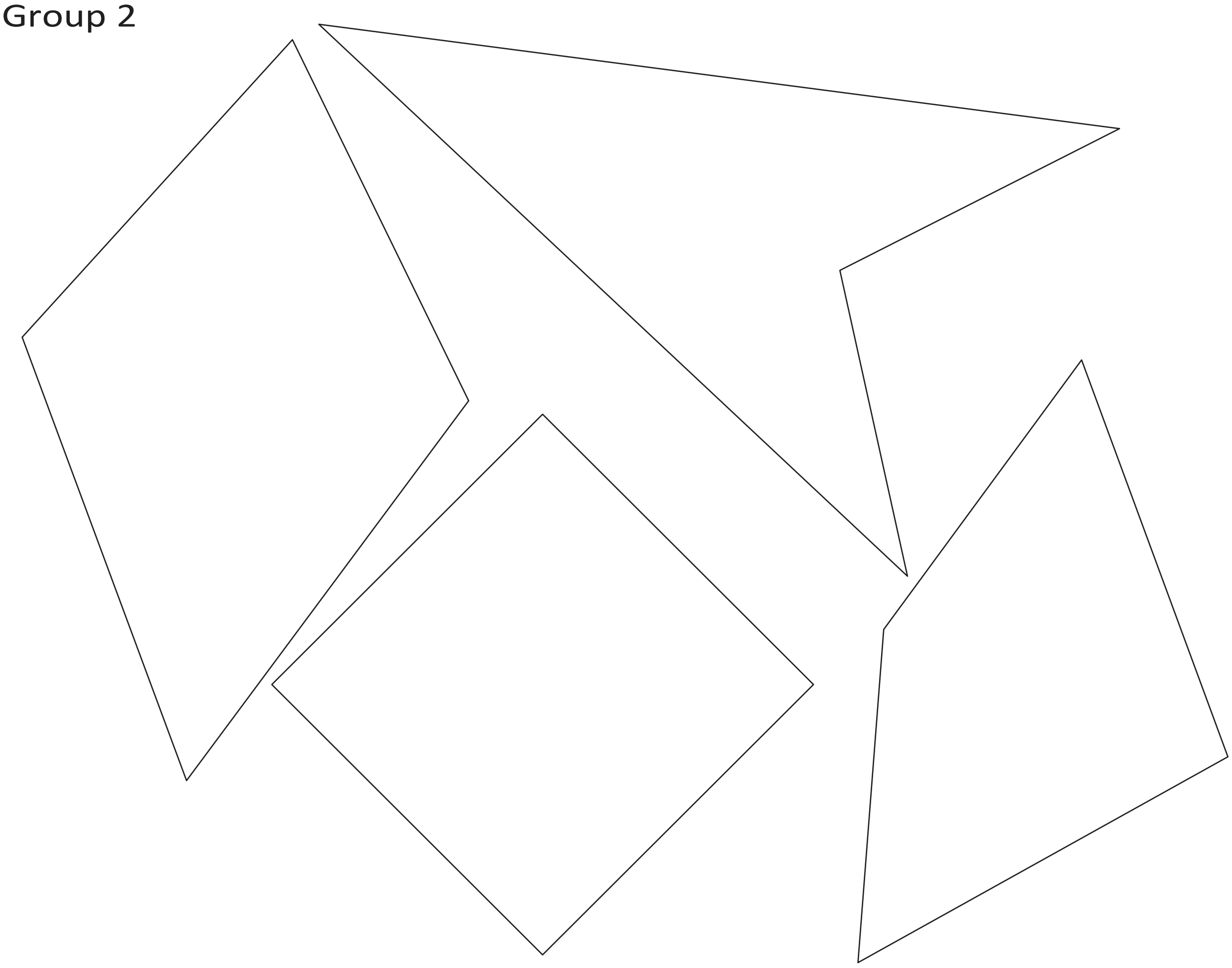
2. The figures in group 1 are called **parallelograms**.
   (a) What do you observe about the opposite sides of parallelograms?
   (b) What do you observe about the angles of parallelograms?

3. The figures in group 2 are called **kites**.
   (a) What do you observe about the sides of kites?
   (b) What else do you observe about the kites?

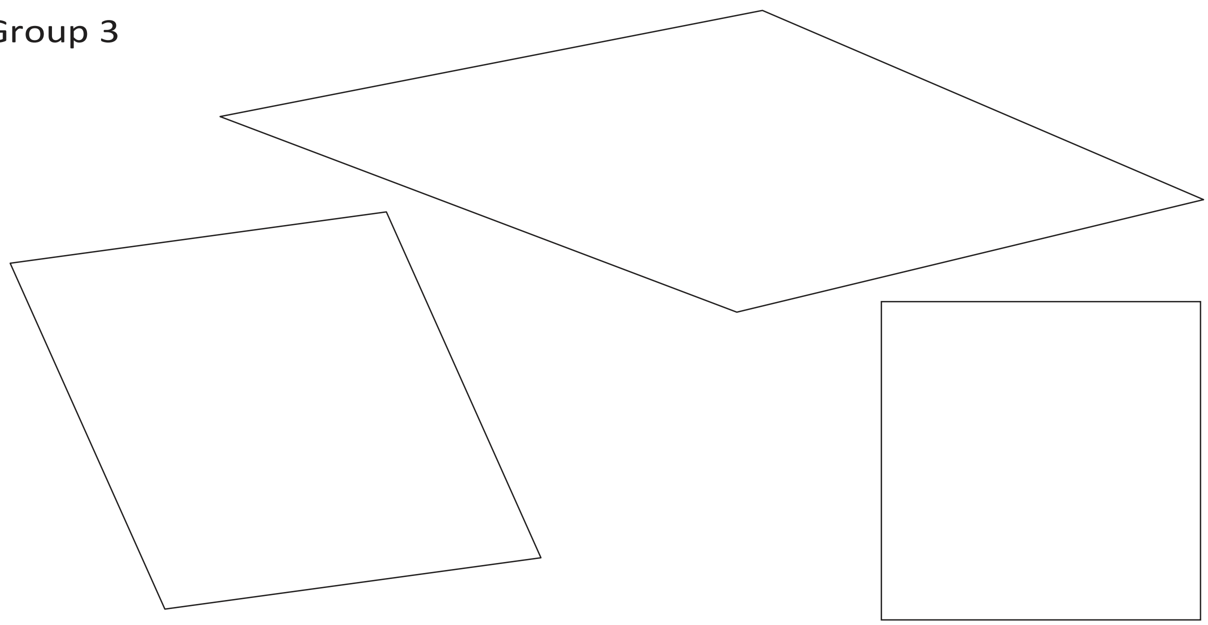
Group 1



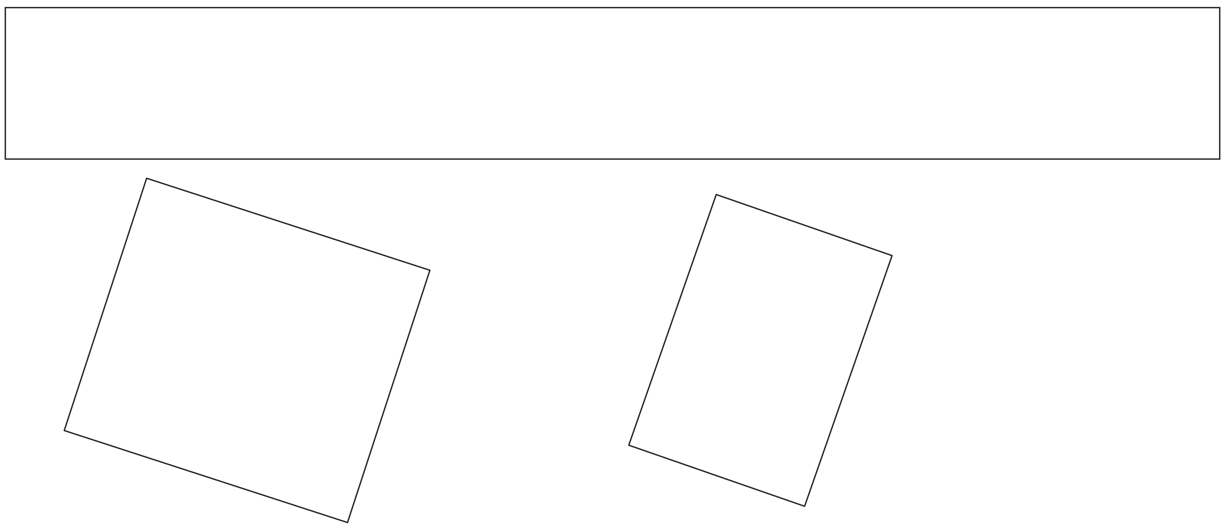
Group 2



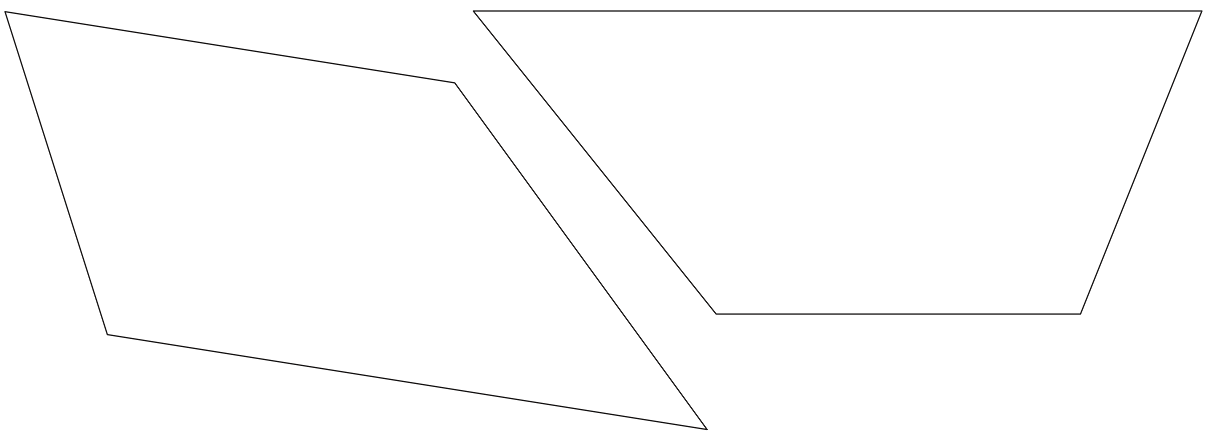
Group 3



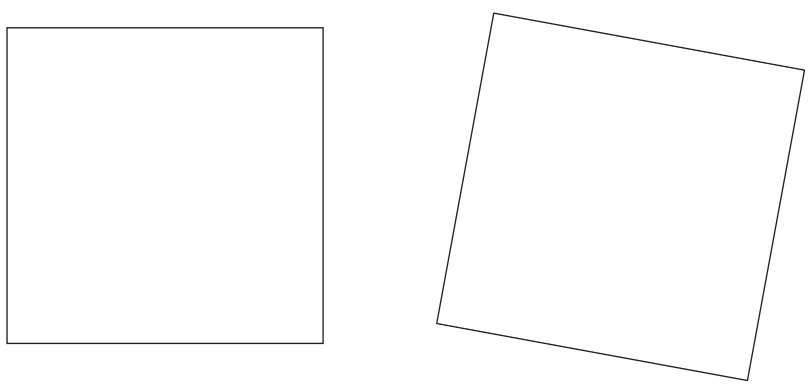
Group 4



Group 5



Group 6



4. The figures in group 3 are called **rhombi**.
   (a) What do you observe about the sides of rhombi?
   (b) What else do you observe about the rhombi?

Note:
One **rhombus**; two or more **rhombi**.

5. The figures in group 4 are called **rectangles**.
   (a) What do you observe about the opposite sides of rectangles?
   (b) What do you observe about the angles of rectangles?
   (c) What do you observe about the adjacent sides of rectangles?

6. The figures in group 5 are called **trapeziums**.
   (a) What do you observe about the opposite sides of trapeziums?

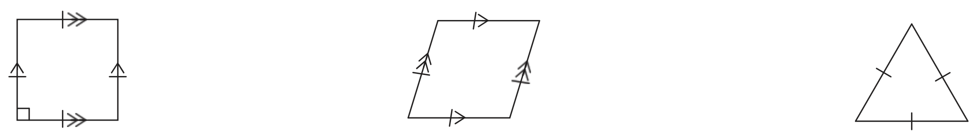
The arrows show which sides are parallel to each other.

7. The figures in group 6 are called **squares**.
   (a) What do you observe about the sides of squares?
   (b) What do you observe about the angles of squares?

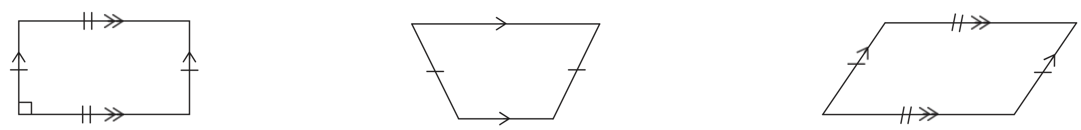
comparing and describing shapes

1. Name each shape in each group.

Group A



Group B



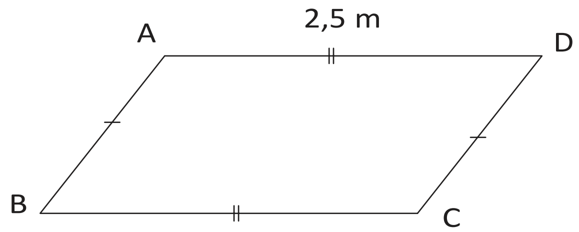
2. In what way(s) are the figures in each group the same?

3. In what way(s) does one of the figures in each group differ from the other two figures in the group?

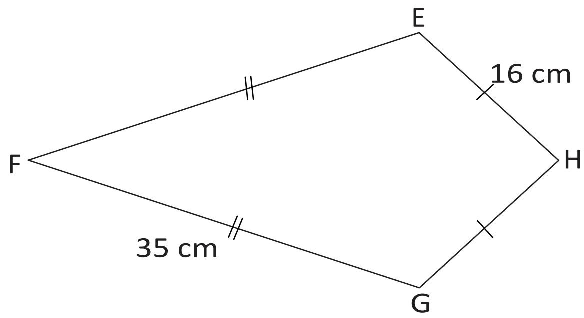
finding unknown sides in quadrilaterals

Use what you know about the sides and angles of quadrilaterals to answer the following questions. **Give reasons for your answers.**

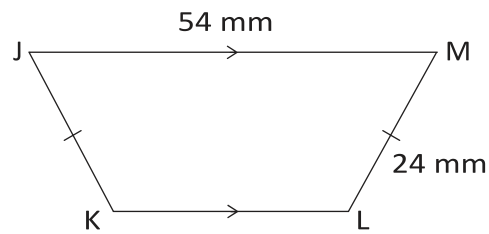
1. (a) What type of quadrilateral is ABCD?
   (b) Name a side equal to AB.
   (c) What is the length of BC?



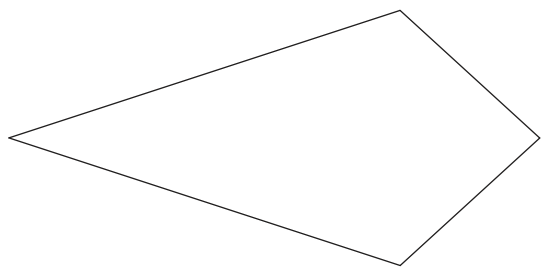
2. (a) What type of quadrilateral isEFGH?
   (b) What are the lengths of the sides EF and GH?



3. (a) What type of quadrilateral isJKLM?
   (b) What is the length of JK?

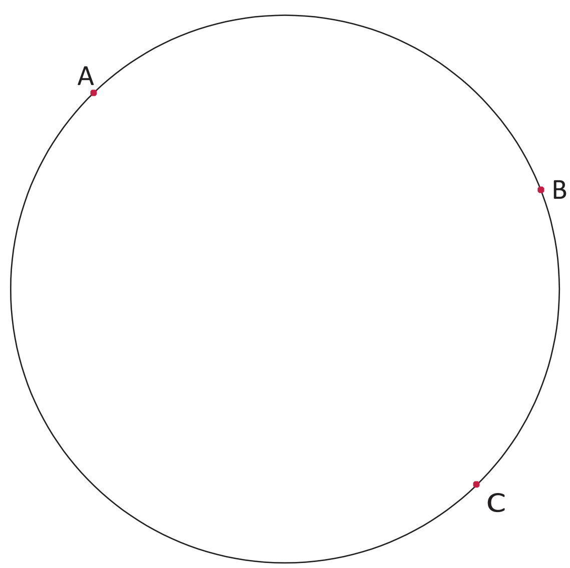


4. Figure PQRS is a kite with PQ = 4 cm and QR = 10 cm. Copy and complete the following drawing by:
   (a) labelling the vertices of the kite
   (b) showing on the drawing which sides are equal
   (c) labelling the length of each side.



5.4 Circles

1. (a) Copy the circle on the right. Make a dot in the middle of the circle. Write the letter M next to the dot. If your dot is in the middle of the circle, it is called the **midpoint** or **centre**.
   (b) Draw lines MA, MB and MC from M to the red points A, B and C.



The three red points are on the circle with midpoint M.

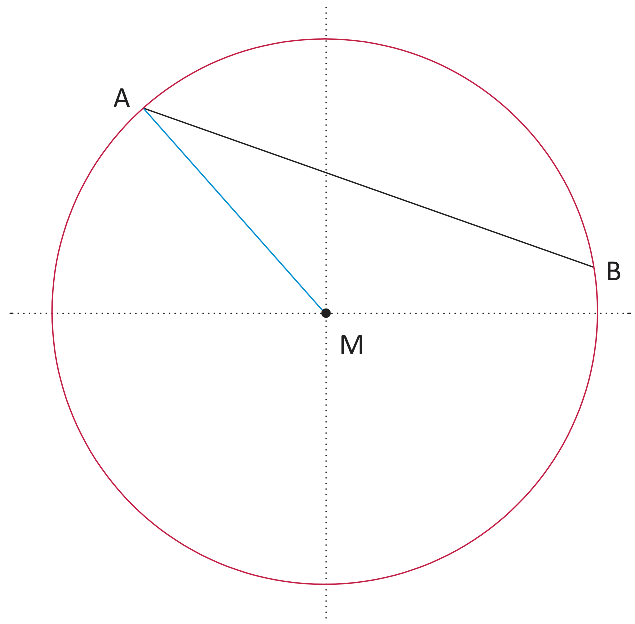
A straight line, such as AC, drawn across a circle and passing through its midpoint is called the **diameter** of the circle.

2. Measure MA, MB and MC.
   If MA, MB and MC are equal in length, you have chosen the midpoint well.
   If they are not equal, you may wish to improve your sketch of a circle and its parts.

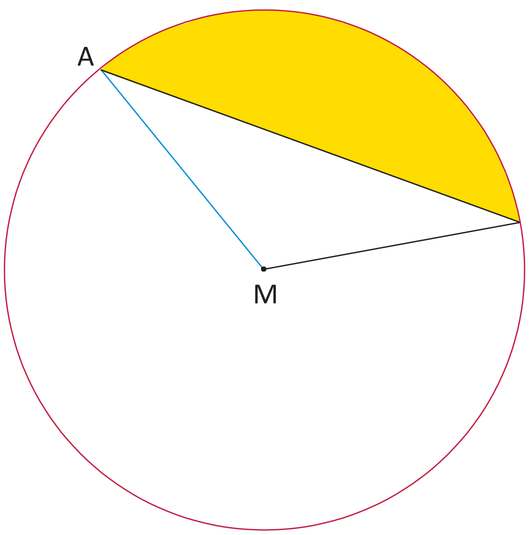
A straight line from the midpoint of a circle to a point on the circle is called a **radius** of the circle.

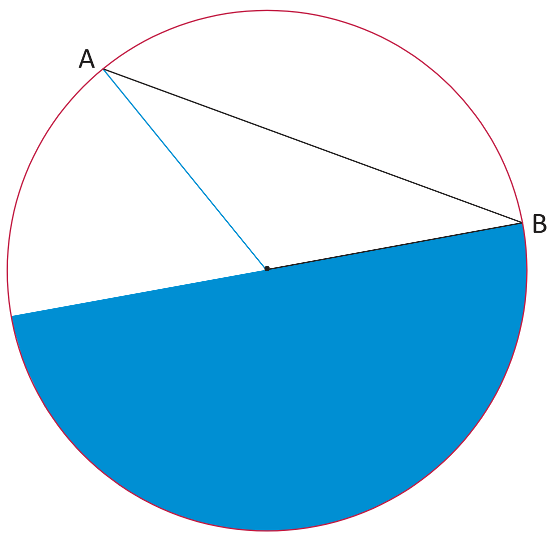
The blue line, MA, is a **radius**. Any straight line from the centre to the circle is a radius.

The black line AB joins two points on the circle. We call this line a **chord** of the circle.

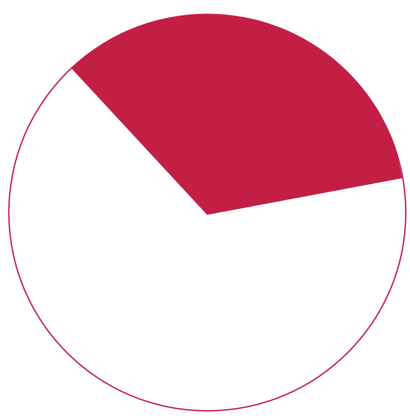


In the following two diagrams, the coloured sections are **segments** of a circle. A segment is the area between a chord and an arc.





In the circle on the right, the red section is called a **sector** of a circle. As you can see, a sector is the region between two radii and an arc.



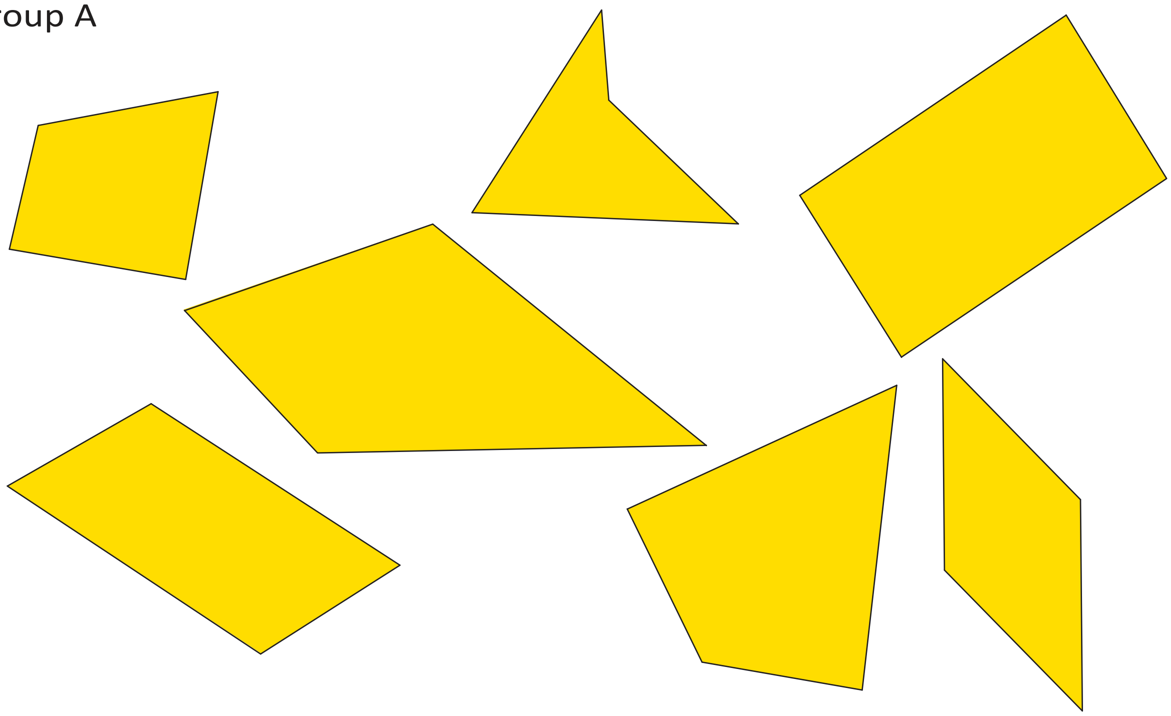
5.5 Similar and congruent shapes

Three groups of quadrilaterals are shown on this page and the next.

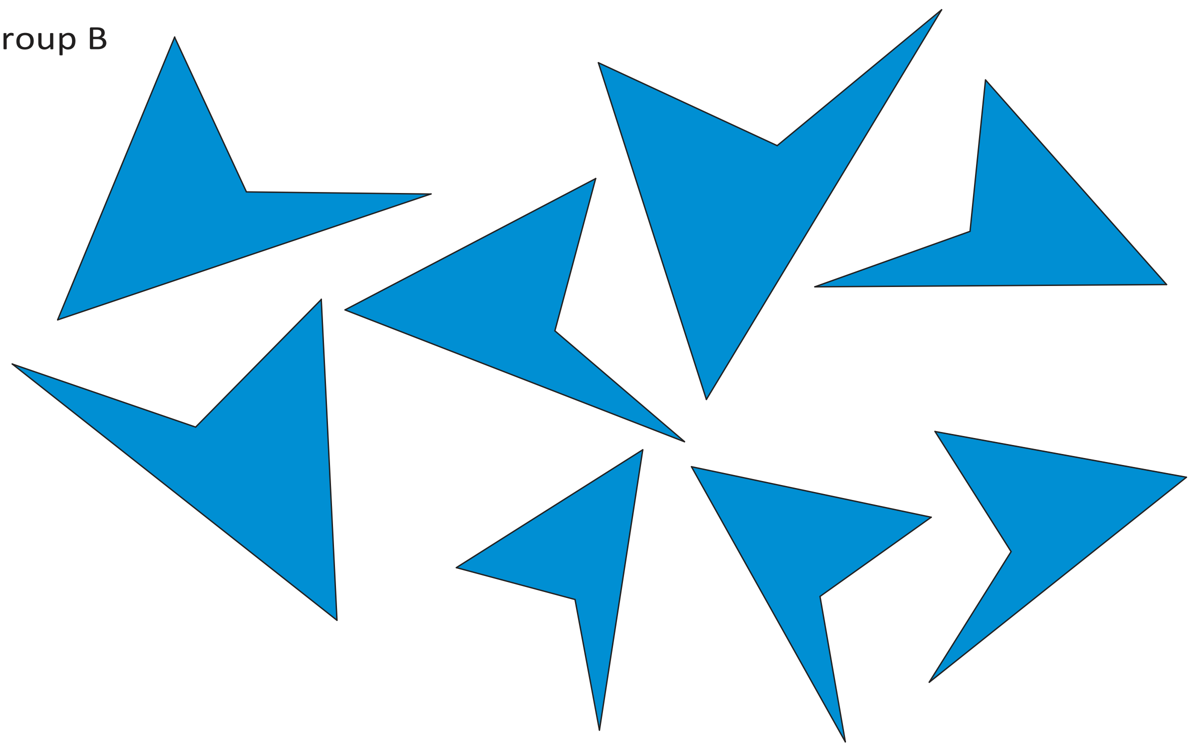
What makes each group different from the other groups, apart from the colours?

1. Group A
2. Group B
3. Group C

Group A

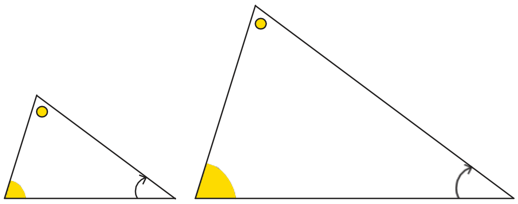


Group B



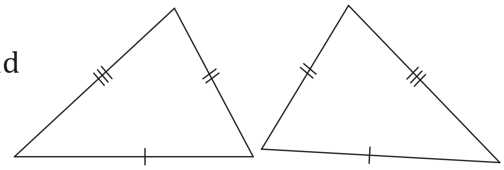
Group C

Shapes that have the same form, such as the blue shapes on the previous page, are said to be **similar** to each other. Similar shapes may differ in size, but will always have the same shape.



Example of similar shapes

Shapes that have the same form and the same size, such as the red shapes on the previous page, are said to be **congruent** to each other. These shapes are always the same size and shape.

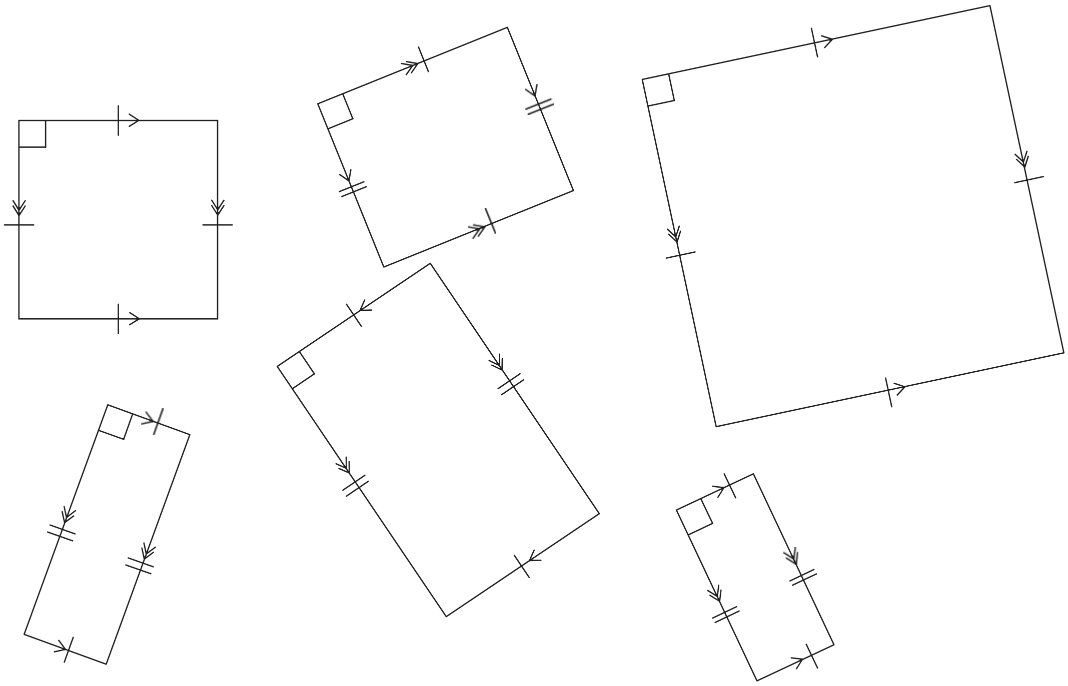


Example of congruent shapes

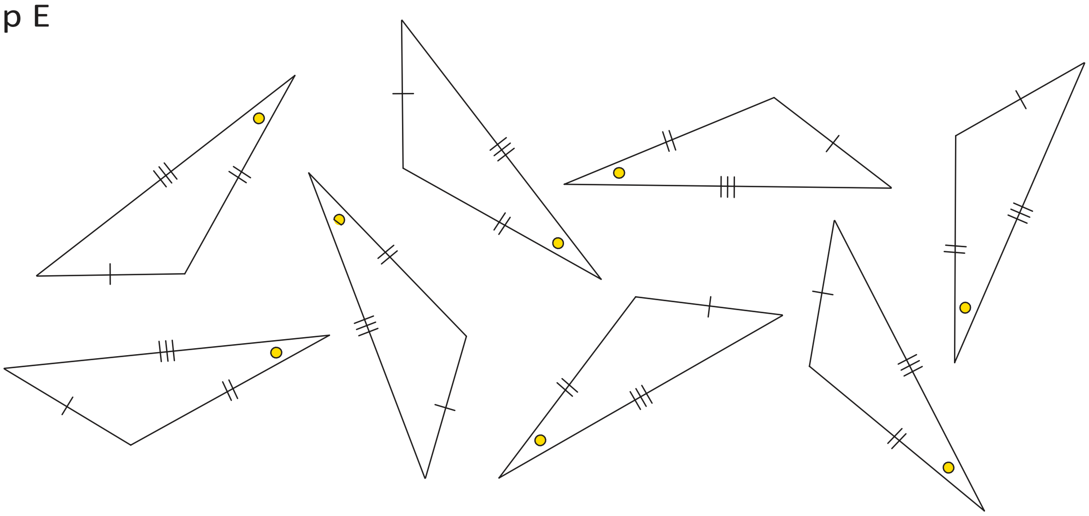
4. Are the red shapes on the previous page *similar* to each other?

5. Look at groups D, E, F, and G on this page and the next. In each case, say whether the shapes are similar and congruent, similar but not congruent, or neither similar nor congruent.
   (a) Group D
   (b) Group E
   (c) Group F
   (d) Group G

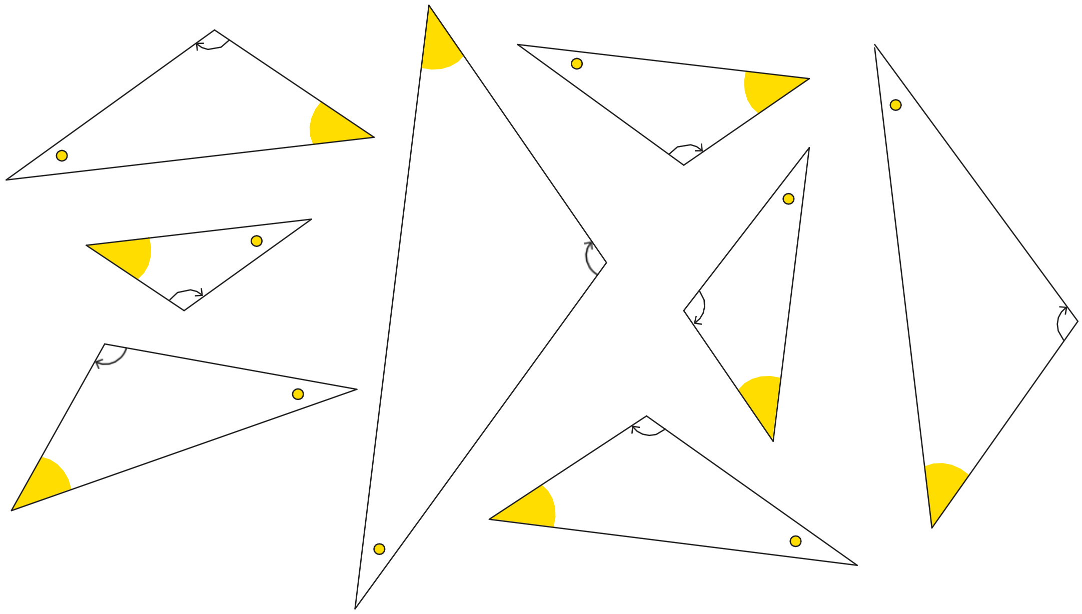
Group D



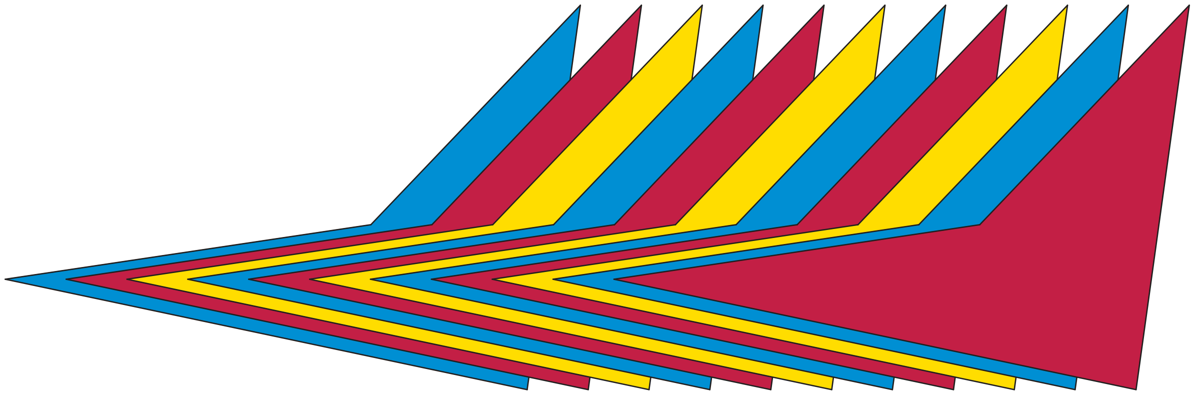
Group E



Group F



Group G



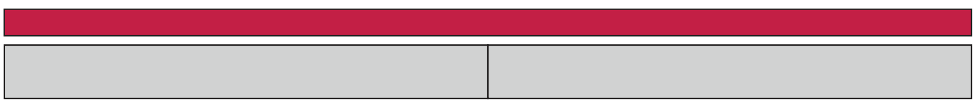
Chapter 6
Fractions

6.1 Measuring accurately with parts of a unit

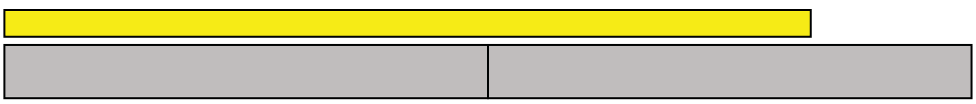
a strange measuring unit

In this activity, you will measure lengths with a unit called a *greystick*. The grey measuring stick below is exactly one greystick long. You will use this stick to measure different objects.

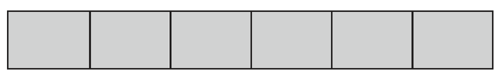
The red bar below is exactly two greysticks long.



As you can see, the yellow bar below is longer than one greystick but shorter than two greysticks.

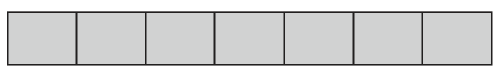


To try to measure the yellow bar accurately, we will divide one greystick into six equal parts:



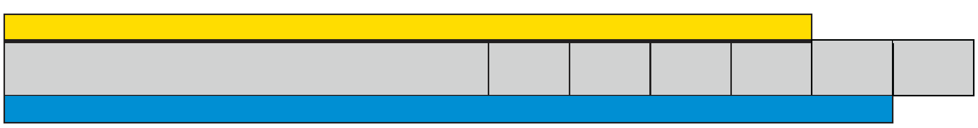
So each of these parts is **one sixth** of a greystick.

This greystick ruler is divided into seven equal parts:



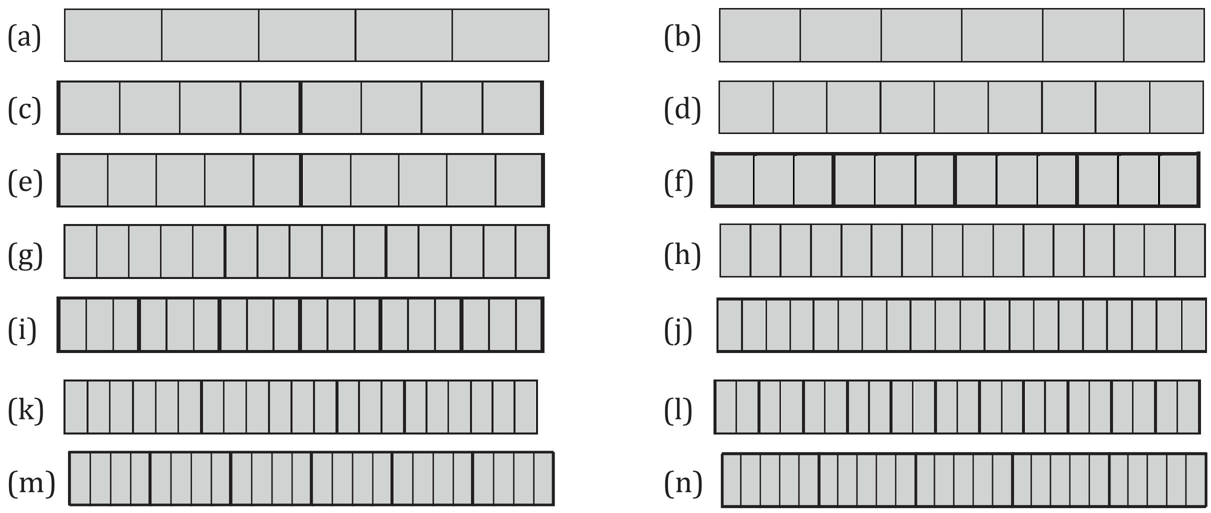
Each part is **one seventh** of a greystick.

1. Do you think one can say the yellow bar is **one and four sixths of a greystick** long?



2. Describe the length of the blue bar, on the previous page, in words.

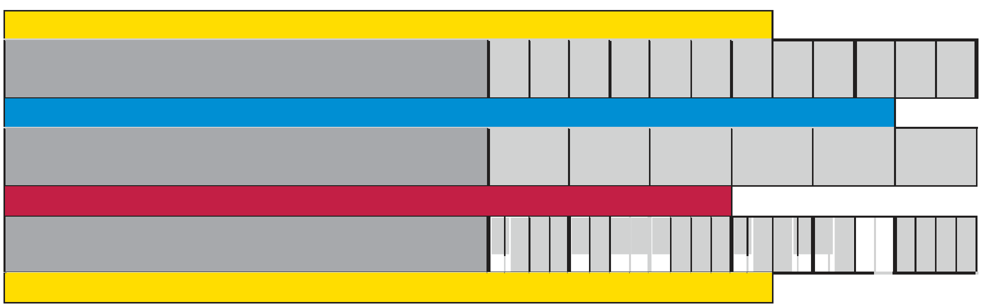
3. In each case below, say what the smaller parts of the greystick may be called. Write your answers in words.



How did you find out what to call the small parts?

Write all your answers to the following questions *in words*.

4. (a) How long is the upper yellowbar?

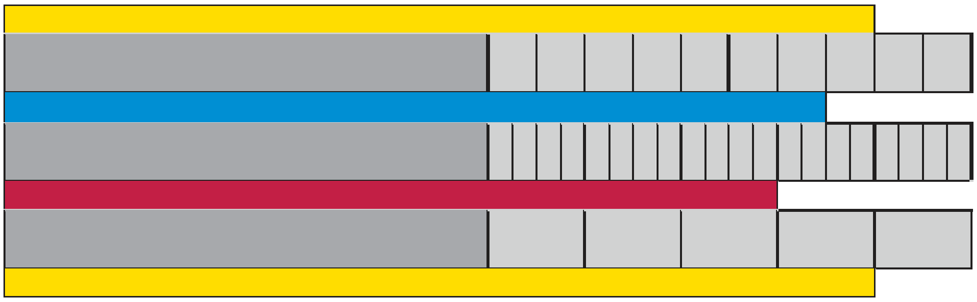


(b) How long is the lower yellow bar?

5. (a) How long is the blue bar above?
(b) How long is the red bar above?

6. (a) How many twelfths of a greystick is the same length as one sixth of a greystick?
(b) How many twenty-fourths is the same length as one sixth of a greystick?
(c) How many twenty-fourths is the same length as seven twelfths of a greystick?

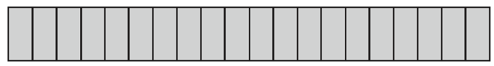
7. (a) How long is the upper yellow bar on the following page?
(b) How long is the lower yellow bar on the following page?



(c) How long is the blue bar?

(d) How long is the red bar?

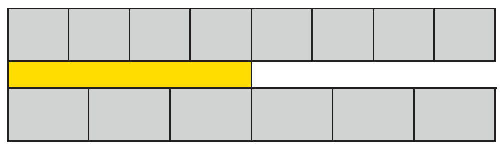
8. (a) How many fifths of a greystick is the same length as 12 twentieths of a greystick?



(b) How many fourths (or quarters) of a greystick is the same length as 15 twentieths of a greystick?

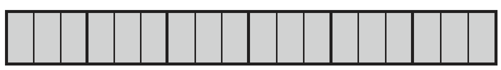
describe the same length in different ways

Two fractions may describe the same length. You can see here that three sixths of a greystick is the same as four eighths of a greystick.



When two fractions describe the same portion we say they are **equivalent**.

1. (a) What can each small part on this greystick be called?



(b) How many eighteenths is one sixth of the greystick?

(c) How many eighteenths is one third of the greystick?

(d) How many eighteenths is five sixths of the greystick?

2. (a) Write (in words) the names of four different fractions that are all equivalent to three quarters.

(b) Which equivalents for two thirds can you find on the greysticks?

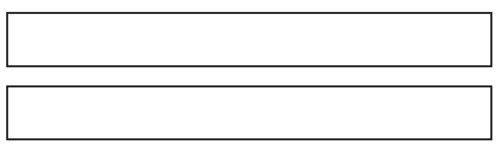
3. The information that two thirds is equivalent to four sixths, to six ninths and to eight twelfths is written in the second row of the table on the following page. Copy the table and complete the other rows of the table in the same way.

thirds	fourths	fifths	sixths	eighths	ninths	tenths	twelfths	twentieths
1								
2	–	–	4	–	6	–	8	–
–	3							
–	–	1						
–	–	2						
–	–	3						
–	–	4						

4. Copy and complete this table in the same way as the table in question 3.

fifths	tenths	fifteenths	twentieths	twenty-fifths	fiftieths	hundredths
1						
2						
3						
4						
5						
6						
7						

5. Copy the greysticks. Draw on them to show that three fifths and nine fifteenths are equivalent. Draw freehand; you need not measure and draw accurately.



6. Copy and complete these tables in the same way as the table in question 4.

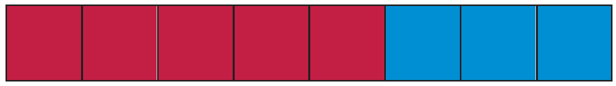
eighths	sixteenths	twenty-fourths
1		
2		
3		
4		
5		
6		
7		
8		
9		

twenty-fourths	sixths	twelfths	eighteenths
	1		
	2		
	3		
	4		
	5		
	6		
	7		
	8		
	9		

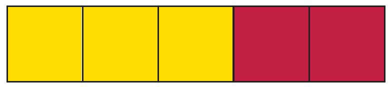
7. (a) How much is five twelfths plus three twelfths?
   (b) How much is five twelfths plus one quarter?
   (c) How much is five twelfths plus three quarters?
   (d) How much is one third plus one quarter? It may help if you work with the equivalent fractions in twelfths.

6.2 Different parts in different colours

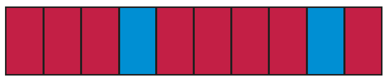
This strip is divided into eight equal parts. Five eighths of this strip is red.



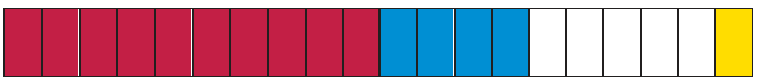
1. What part of the strip is blue?
2. What part of this strip is yellow?



3. What part of the strip is red?
4. What part of this strip is coloured blue and what part is coloured red?

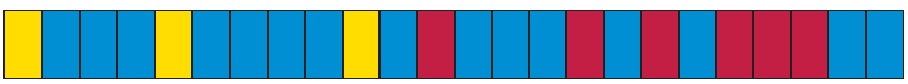


5. (a) What part of this strip is blue, what part is red and what part is white?



   (b) Express your answer differently with equivalent fractions.

6. A certain strip is not shown here. Two ninths of the strip is blue, and three ninths of the strip is green. The rest of the strip is red. What part of the strip is red?
7. What part of the strip below is yellow, what part is blue, and what part is red?



The number of parts in a fraction is called the **numerator** of the fraction. For example, the numerator in five sixths is five.

The type of part in a fraction is called the **denominator**. It is the name of the parts that are being referred to and it is determined by the size of the part. For example, sixths is the denominator in five sixths.

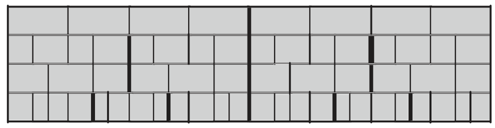
> $^5/_6$ is a short way to write five sixths.
> We may also write $\frac{5}{6}$.
> Even when we write $^5/_6$ or $\frac{5}{6}$, we still say "five sixths".
> $/6$ and $\overline{6}$ are short ways to write *sixths*.

To **enumerate** means "to find the number of".
To **denominate** means "to give a name to".

The numerator (number of parts) is written above the line of the fraction: $\frac{\textit{numerator}}{\dots}$

The denominator is indicated by a number written below the line: $\frac{\dots}{\textit{denominator}}$

8. Consider the fraction three quarters. It can be written as $\frac{3}{4}$.



(a) Multiply both the numerator and the denominator by two to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$? You may check on the diagram.

(b) Multiply both the numerator and the denominator by three to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

(c) Multiply both the numerator and the denominator by four to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

(d) Multiply both the numerator and the denominator by six to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

6.3 Combining fractions

bigger and smaller parts

Gertie was asked to solve this problem:

A team of road-builders built $\frac{8}{12}$ km of road in one week, and $\frac{10}{12}$ km in the next week. What is the total length of road that they built in the two weeks?

She thought like this to solve the problem:

*$\frac{8}{12}$ is **eight twelfths** and $\frac{10}{12}$ is **ten twelfths**, so altogether it is **eighteen twelfths**.*

I can write $\frac{18}{12}$ or "18 twelfths".

*I can also say 12 twelfths of a kilometre is 1 kilometre, **18 twelfths is 1 km and 6 twelfths of a kilometre**.*

This I can write as $1\frac{6}{12}$. It is the same as $1\frac{1}{2}$ km.

Gertie was also asked the question: How much is $4\frac{5}{9} + 2\frac{7}{9}$?

She thought like this to answer it:

$4\frac{5}{9}$ is four wholes and five ninths, and $2\frac{7}{9}$ is two wholes and seven ninths.

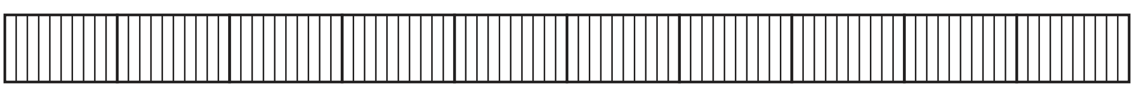
So altogether it is six wholes and 12 ninths. But 12 ninths is nine ninths (one whole) and three ninths, so I can say it is seven wholes and three ninths.

I can write $7\frac{3}{9}$.

1. Would Gertie be wrong if she said her answer was $7\frac{1}{3}$?
2. Senthereng has $4\frac{7}{12}$ bottles of cooking oil. He gives $1\frac{5}{12}$ bottles to his friend Willem. How much oil does Senthereng have left?
3. Margaret has $5\frac{5}{8}$ bottles of cooking oil. She gives $3\frac{7}{8}$ bottles to her friend Naledi. How much oil does Margaret have left?
4. Calculate each of the following:
   (a) $4\frac{2}{7} - 3\frac{6}{7}$ (b) $3\frac{6}{7} + \frac{3}{7}$ (c) $3\frac{6}{7} + 1\frac{4}{5}$
   (d) $4\frac{3}{8} - 2\frac{4}{5}$ (e) $1\frac{3}{10} - \frac{2}{3}$ (f) $3\frac{5}{10} - 1\frac{1}{2}$
   (g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$ (h) $6\frac{2}{5} + 2\frac{1}{4} - \frac{1}{2}$
   (i) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$
   (j) $2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7}$
   (k) $(4\frac{2}{7} + 1\frac{4}{7}) - 2\frac{1}{3}$ (l) $(2\frac{7}{10} + 3\frac{2}{5}) - (1\frac{2}{5} + 3\frac{7}{10})$
5. Neo's report had five chapters. The first chapter was $\frac{3}{4}$ of a page, the second chapter was $2\frac{1}{2}$ pages, the third chapter was $3\frac{3}{4}$ pages, the fourth chapter was three pages and the fifth chapter was $1\frac{1}{2}$ pages. How many pages was Neo's report in total?

6.4 Tenths and hundredths (percentages)

1. (a) 100 children each get three biscuits. How many biscuits is this in total?
   (b) 500 sweets are shared equally between 100 children. How many sweets does each child get?
2. The picture below shows a strip of licorice. The very small pieces can easily be broken off on the thin lines. How many very small pieces are shown in the picture?

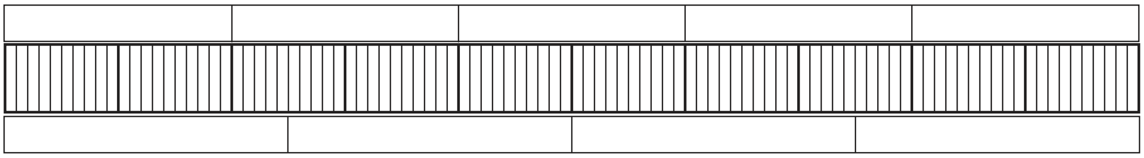


3. Gatsha runs a spaza shop. He sells strips of licorice like the above for R2 each.
   (a) What is the cost of one very small piece of licorice, when you buy from Gatsha?
   (b) Jonathan wants to buy one fifth of a strip of licorice. How much should he pay?
   (c) Batseba eats 25 very small pieces. What part of a whole strip of licorice is this?

Each small piece of the above strip is **one hundredth** of the whole strip.

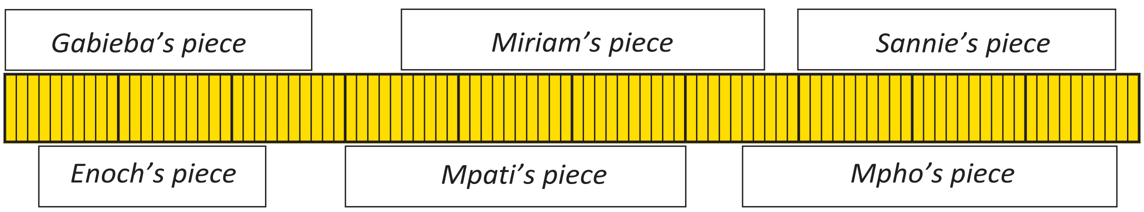
4. (a) Why can each small piece be called *one hundredth* of the whole strip?
   (b) How many hundredths is the same as one tenth of the strip?

Gatsha often sells parts of licorice strips to customers. He uses a "quarters marker" and a "fifths marker" to cut off the pieces correctly from full strips. His two markers are shown below, next to a full strip of licorice.



5. (a) How many hundredths is the same as two fifths of the whole strip?
   (b) How many tenths is the same as $\frac{2}{5}$ of the whole strip?
   (c) How many hundredths is the same as $\frac{3}{4}$ of the whole strip?
   (d) Freddie bought $\frac{60}{100}$ of a strip. How many fifths of a strip is this?
   (e) Jamey bought part of a strip for R1,60. What part of a strip did she buy?

6. Gatsha, the owner of the spaza shop, sold pieces of yellow licorice to different children. Their pieces are shown below. How much (what part of a whole strip) did each of them get?



7. The yellow licorice shown above costs R2,40 (240 cents) for a strip. How much does each of the children have to pay? Round off the amounts to the nearest cent.

8. (a) How much is $\frac{1}{100}$ of 300 cents? (b) How much is $\frac{7}{100}$ of 300 cents?
   (c) How much is $\frac{25}{100}$ of 300 cents? (d) How much is $\frac{1}{4}$ of 300 cents?
   (e) How much is $\frac{40}{100}$ of 300 cents? (f) How much is $\frac{2}{5}$ of 300 cents?

9. Explain why your answers for questions 8(e) and 8(f) are the same.

Another word for **hundredth** is **per cent**.
Instead of saying Miriam received **32 hundredths** of a licorice strip, we can say Miriam received **32 per cent** of a licorice strip. The symbol for per cent is%.

10. How much is 80% of each of the following?
    (a) R500 (b) R480 (c) R850 (d) R2 400
11. How much is 8% of each of the amounts in 10(a), (b), (c) and (d)?
12. How much is 15% of each of the amounts in 10(a), (b), (c) and (d)?
13. Building costs of houses increased by 20%. What is the new building cost for a house that previously cost R110 000 to build?
14. The value of a car decreases by 30% after one year. If the price of a new car is R125 000, what is the value of the car after one year?
15. Investigate which denominators of fractions can easily be converted to powers of 10.

6.5 Thousandths, hundredths and tenths

many equal parts

1. In a camp for refugees, 50 kg of sugar must be shared equally between 1 000 refugees. How much sugar will each refugee get? Keep in mind that 1 kg is 1 000 g. You can give your answer in grams.
2. How much is each of the following?
    (a) one tenth of R6 000
    (b) one hundredth of R6 000
    (c) one thousandth of R6 000
    (d) ten hundredths of R6 000
    (e) 100 thousandths of R6 000
    (f) seven hundredths of R6 000
    (g) 70 thousandths of R6 000
    (h) seven thousandths of R6 000
3. Calculate.
    (a) $\frac{3}{10} + \frac{5}{8}$
    (b) $3\frac{3}{10} + 2\frac{4}{5}$
    (c) $\frac{3}{10} + \frac{7}{100}$
    (d) $\frac{3}{10} + \frac{70}{100}$
    (e) $\frac{3}{10} + \frac{7}{1\,000}$
    (f) $\frac{3}{10} + \frac{70}{1\,000}$
4. Calculate.
    (a) $\frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$
    (b) $\frac{3}{10} + \frac{70}{100} + \frac{400}{1\,000}$
    (c) $\frac{6}{10} + \frac{20}{100} + \frac{700}{1\,000}$
    (d) $\frac{2}{10} + \frac{5}{100} + \frac{4}{1\,000}$
5. In each case investigate whether the statement is true or not, and give reasons for your final decision.
    (a) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{3}{100} + \frac{46}{1\,000}$
    (b) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{7}{10} + \frac{2}{100} + \frac{6}{1\,000}$
    (c) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$
    (d) $\frac{676}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$

6.6 Fraction of a fraction

form parts of parts

1. (a) How much is one fifth of R60?
   (b) How much is three fifths of R60?

2. How much is seven tenths of R80? (You may first work out how much one tenth of R80 is.)

3. In Britain the unit of currency is pound sterling, in Western Europe it is the euro, and in Botswana it is the pula.
   (a) How much is two fifths of 20 pula?
   (b) How much is two fifths of 20 euro?
   (c) How much is two fifths of 12 pula?

4. Why was it so easy to calculate two fifths of 20, but difficult to calculate two fifths of 12?

There is a way to make it easy to calculate something like three fifths of R4. You just change the rands to cents!

5. Calculate each of the following. You may change the rands to cents to make it easier.
   (a) three eighths of R2,40
   (b) seven twelfths of R6
   (c) two fifths of R21
   (d) five sixths of R3

6. You will now do some calculations about secret objects.
   (a) How much is three tenths of 40 secret objects?
   (b) How much is three eighths of 40 secret objects?

The secret objects in question 6 are fiftieths of a rand.

7. (a) How many fiftieths is three tenths of 40 fiftieths?
   (b) How many fiftieths is five eighths of 40 fiftieths?

8. (a) How many twentieths of a kilogram is the same as $\frac{3}{4}$ of a kilogram?
   (b) How much is one fifth of 15 rands?
   (c) How much is one fifth of 15 twentieths of a kilogram?
   (d) So, how much is one fifth of $\frac{3}{4}$ of a kilogram?

9. (a) How much is $\frac{1}{12}$ of 24 fortieths of some secret object?
   (b) How much is $\frac{7}{12}$ of 24 fortieths of the secret object?

10. Do you agree that the answers for the previous question are two fortieths and 14 fortieths? If you disagree, explain why you disagree.

11. (a) How much is $\frac{1}{5}$ of 80?

(b) How much is $\frac{3}{5}$ of 80?

(c) How much is $\frac{1}{40}$ of 80?

(d) How much is $\frac{24}{40}$ of 80?

(e) Explain why $\frac{3}{5}$ of 80 is the same as $\frac{24}{40}$ of 80.

12. Look again at your answers for questions 9(b) and 11(e). How much is $\frac{7}{12}$ of $\frac{3}{5}$?

Explain your answer.

The secret object in question 9 was an envelope with R160 in it.

After the work you did in questions 9, 10 and 11, you know that:

- $\frac{24}{40}$ and $\frac{3}{5}$ are just two ways to describe the same thing, and
- $\frac{7}{12}$ of $\frac{3}{5}$ is therefore the same as $\frac{7}{12}$ of $\frac{24}{40}$.

It is easy to calculate $\frac{7}{12}$ of $\frac{24}{40}$: one twelfth of 24 is 2, so seven twelfths of 24 is 14, so seven twelfths of 24 fortieths is 14 fortieths.

$\frac{3}{8}$ of $\frac{2}{3}$ can be calculated in the same way. But one eighth of $\frac{2}{3}$ is a slight problem, so it would be better to use some equivalent of $\frac{2}{3}$. The equivalent should be chosen so that it is easy to calculate one eighth of it; so it would be nice if the numerator could be eight.

$\frac{8}{12}$ is equivalent to $\frac{2}{3}$, so instead of calculating $\frac{3}{8}$ of $\frac{2}{3}$ we may calculate $\frac{3}{8}$ of $\frac{8}{12}$.

13.(a) Calculate $\frac{3}{8}$ of $\frac{8}{12}$.

(b) So, how much is $\frac{3}{8}$ of $\frac{2}{3}$?

14. In each case replace the second fraction by a suitable equivalent, and then calculate.

(a) How much is $\frac{3}{4}$ of $\frac{5}{8}$?

(b) How much is $\frac{7}{10}$ of $\frac{2}{3}$?

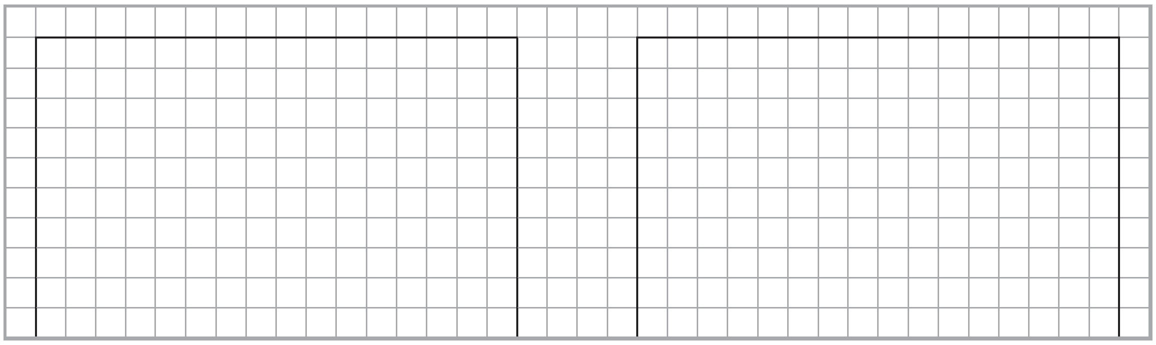
(c) How much is $\frac{2}{3}$ of $\frac{1}{2}$?

(d) How much is $\frac{3}{5}$ of $\frac{3}{5}$?

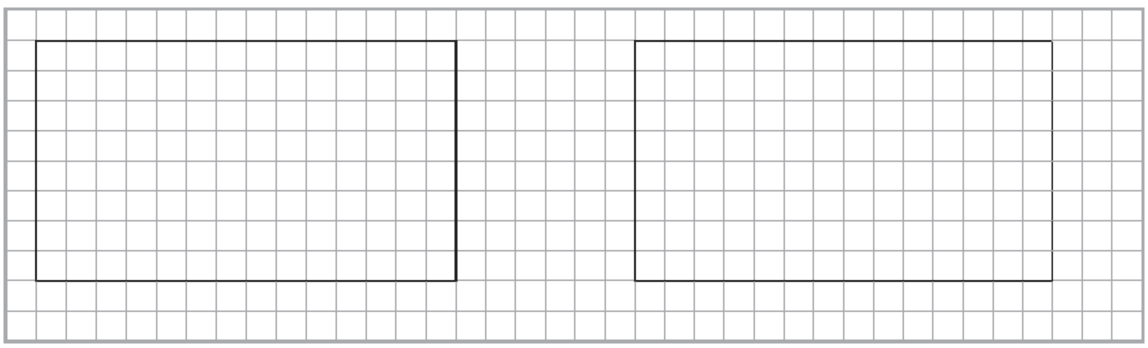
6.7 Multiplying with fractions

parts of rectangles and parts of parts

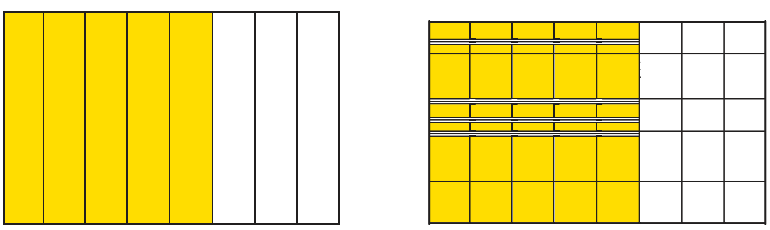
1. (a) Copy the rectangles below. Divide the rectangle on the left into eighths by drawing vertical lines. Lightly shade the left three eighths of the rectangle.
   (b) Divide the rectangle on the right into fifths drawing horizontal lines. Lightly shade the upper two fifths of the rectangle.



2. (a) Copy the rectangles below. Shade four sevenths of the rectangle on the left below.
   (b) Shade 16 twenty-eighths of the rectangle on the right below.



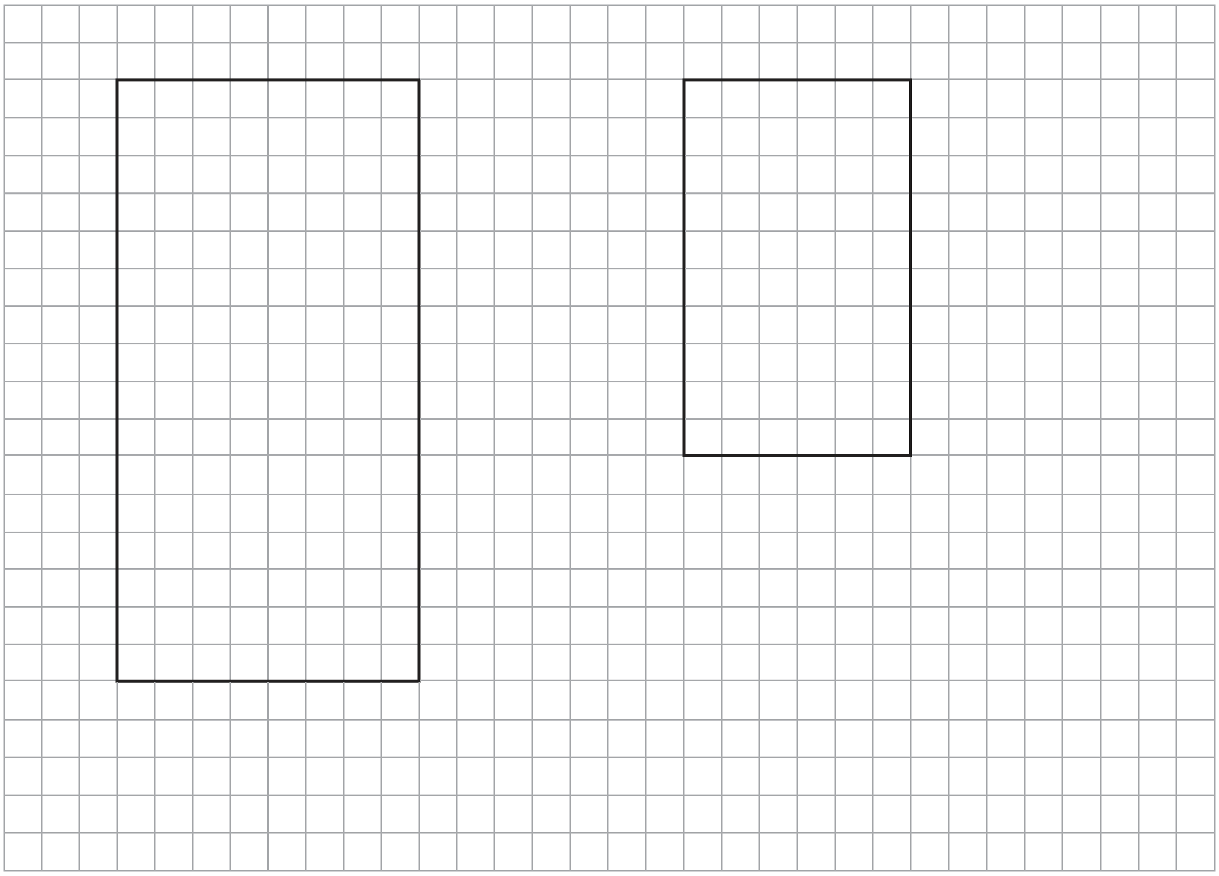
3. (a) What part of each big rectangle below is coloured yellow?



   (b) What part of the *yellow* part of the rectangle on the right is dotted?
   (c) Into how many squares is the whole rectangle on the right divided?
   (d) What part of the whole rectangle on the right is yellow *and* dotted?

4. On grid paper make diagrams to help you to figure out how much each of the following is:

(a) $\frac{3}{4}$ of $\frac{5}{8}$ (b) $\frac{2}{3}$ of $\frac{4}{5}$



Here is something you can do with the fractions $\frac{3}{4}$ and $\frac{5}{8}$:

multiply the two numerators and make this the numerator of a new fraction.
Also multiply the two denominators, and make this the denominator of a new fraction $\frac{3 \times 5}{4 \times 8} = \frac{15}{32}$.

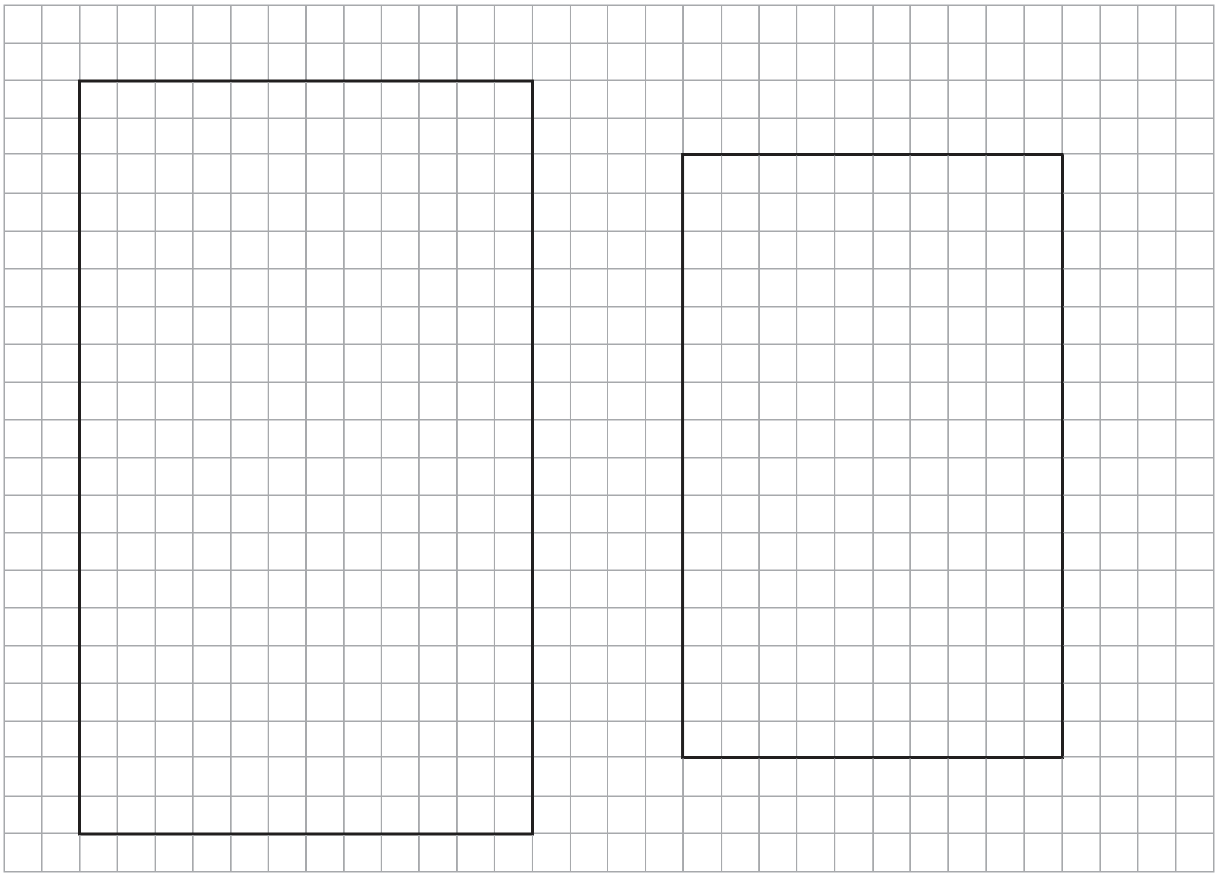
5. Compare the above with what you did in question 14(a) of section 6.6 and in question 4(a) at the top of this page. What do you notice about $\frac{3}{4}$ of $\frac{5}{8}$ and $\frac{3 \times 5}{4 \times 8}$?

6. (a) Alan has five heaps of eight apples each. How many apples is that in total?
(b) Sean has ten heaps of six quarter apples each. How many apples is that in total?

Instead of saying $\frac{5}{8}$ of R40 or $\frac{5}{8}$ of $\frac{2}{3}$ of a floor surface,
we may say $\frac{5}{8} \times$ R40 or $\frac{5}{8} \times \frac{2}{3}$ of a floor surface.

7. Use the diagrams below to figure out how much each of the following is:

(a) $\frac{3}{10} \times \frac{5}{6}$ (b) $\frac{2}{5} \times \frac{7}{8}$



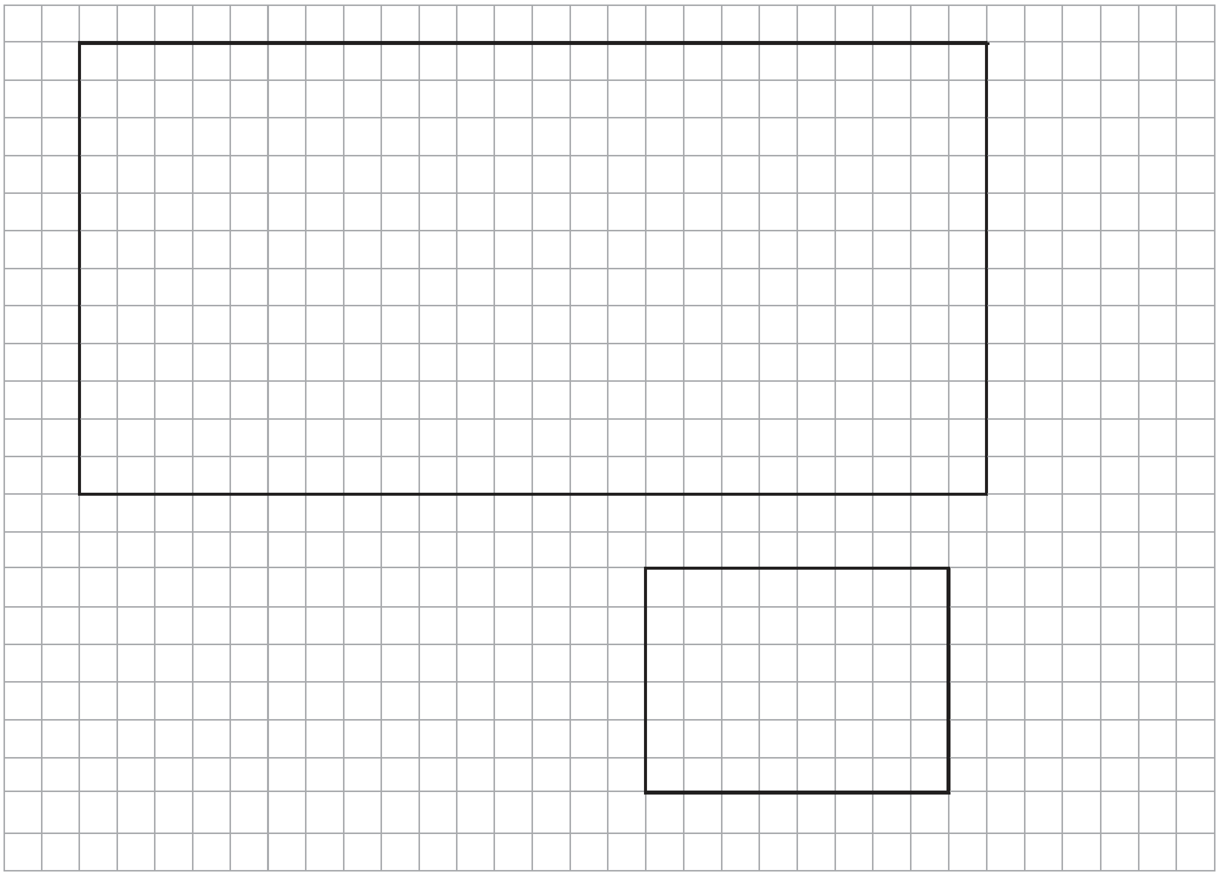
8. (a) Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for $\frac{3}{10}$ and $\frac{5}{6}$ and compare the answer to your answer for question 7(a).

(b) Do the same for $\frac{2}{5}$ and $\frac{7}{8}$.

9. Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for

(a) $\frac{5}{6}$ and $\frac{7}{12}$ (b) $\frac{3}{4}$ and $\frac{2}{3}$

10. Use the diagrams on the following page to check whether the formula $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ produces the correct answers for $\frac{5}{6} \times \frac{7}{12}$ and $\frac{3}{4} \times \frac{2}{3}$.



11. Calculate each of the following:

(a) $\frac{1}{2}$ of $\frac{1}{3}$ of R60 (b) $\frac{2}{7}$ of $\frac{2}{9}$ of R63 (c) $\frac{4}{3}$ of $\frac{2}{5}$ of R45

12.(a) John normally practises soccer for three quarters of an hour every day. Today he practised for only half his usual time. How long did he practise today?

(b) A bag of peanuts weighs $\frac{3}{8}$ of a kilogram. What does $\frac{3}{4}$ of a bag weigh?

(c) Calculate the mass of $7\frac{1}{2}$ packets of sugar if one packet has a mass of $\frac{3}{4}$ kg.

6.8 Ordering and comparing fractions

1. Order the following from the smallest to the biggest:

(a) $\frac{7}{16}$; $\frac{3}{8}$; $\frac{11}{24}$; $\frac{5}{12}$; $\frac{23}{48}$ (b) $\frac{703}{1\,000}$; $\frac{13}{20}$; $\frac{7}{10}$; 73%; $\frac{71}{100}$

2. Order the following from the biggest to the smallest:

(a) $\frac{41}{60}$; $\frac{19}{30}$; $\frac{7}{10}$; $\frac{11}{15}$; $\frac{17}{20}$ (b) $\frac{23}{24}$; $\frac{2}{3}$; $\frac{7}{8}$; $\frac{19}{20}$; $\frac{5}{6}$

3. Use the symbols = , > or < to make the following true:

(a) $\frac{7}{17}$ ☐ $\frac{21}{51}$ (b) $\frac{1}{17}$ ☐ $\frac{1}{19}$

Worksheet

1. Dothecalculationsgivenbelow.Rewriteeachquestioninthecommonfraction notation.Thenwritetheanswerinwordsandinthecommonfractionnotation.
   (a) three twentieths + five twentieths (b) five twelfths + 11 twelfths
   (c) three halves + five quarters (d) three fifths + three tenths

2. Complete the equivalent fractions.
   (a) $\frac{5}{7} = \frac{\square}{49}$ (b) $\frac{9}{11} = \frac{\square}{33}$ (c) $\frac{15}{10} = \frac{3}{\square}$
   (d) $\frac{1}{9} = \frac{4}{\square}$ (e) $\frac{45}{18} = \frac{\square}{2}$ $\frac{\ }{5} = \frac{\square}{35}$

3. Dothecalculationsgivenbelow.Rewriteeachquestioninwords.Thenwritethe answer in words and in the common fraction notation.
   (a) $\frac{3}{10} + \frac{7}{30}$ (b) $\frac{2}{5} + \frac{7}{12}$
   (c) $\frac{1}{100} + \frac{7}{10}$ (d) $\frac{3}{5} - \frac{3}{8}$
   (e) $2\frac{3}{10} + 5\frac{9}{10}$

4. Joe earns R5 000 per month. His salary increases by 12%. What is his new salary?

5. Ahmed earned R7 500 per month. At the end of a certain month, his employer raisedhis salary by 10%. However,one monthlater his employer hadto decrease his salary again by 10%. What was Ahmed's salary then?

6. Calculateeachofthefollowingandsimplifytheanswertoitslowestform:
   (a) $\frac{13}{20} - \frac{2}{5}$ (b) $3\frac{24}{100} - 1\frac{2}{10}$
   (c) $5\frac{9}{11} - 2\frac{1}{4}$ (d) $\frac{2}{3} + \frac{4}{7}$

7. Evaluate.
   (a) $\frac{1}{2} \times 9$ (b) $\frac{3}{5} \times \frac{10}{27}$ (c) $\frac{2}{3} \times 15$ (d) $\frac{2}{3} \times \frac{3}{4}$

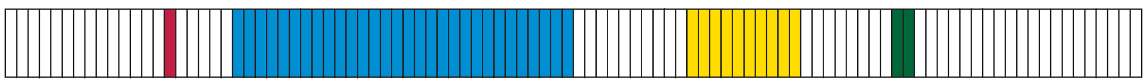
8. Calculate.
   (a) $2\frac{2}{3} \times 2\frac{2}{3}$ (b) $8\frac{2}{5} \times 3\frac{1}{3}$
   (c) $\left(\frac{1}{3} + \frac{1}{2}\right) \times \frac{6}{7}$ (d) $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}$
   (e) $\frac{5}{6} + \frac{2}{3} \times \frac{1}{5}$ (f) $\frac{3}{4} - \frac{2}{5} \times \frac{5}{6}$

Chapter 7
The decimal notation for fractions

7.1 Other symbols for tenths and hundredths

tenths and hundredths again ...

1. (a) What part of the rectangle below is coloured yellow?



   (b) What part of the rectangle is red? What part is blue? What part is green, and what part is not coloured?

0,1 is another way to write $\frac{1}{10}$ and
0,01 is another way to write $\frac{1}{100}$.
0,1 and $\frac{1}{10}$ are different notations for the same number.
$\frac{1}{10}$ is called the **(common) fraction notation**
and 0,1 is called the **decimal notation**.

2. Write the answers for 1(a) and (b) in decimal notation.

3. Three tenths and seven hundredths of a rectangle is coloured red, and two tenths and six hundredths of the rectangle is coloured brown. What part of the rectangle (how many tenths and how many hundredths) is not coloured? Write your answer in fraction notation and in decimal notation.

4. On Monday, Steve ate three tenths and seven hundredths of a strip of licorice. On Tuesday, Steve ate two tenths and five hundredths of a strip of licorice. How much licorice did he eat on Monday and Tuesday together? Write your answer in fraction notation and in decimal notation.

5. Lebogang's answer for question 4 is *five tenths and 12 hundredths*. Susan's answer is *six tenths and two hundredths*. Who is right, or are they both wrong?

The same quantity can be expressed in different ways in tenths and hundredths.

For example, *three tenths and 17 hundredths* can be expressed as *two tenths and 27 hundredths* or *four tenths and seven hundredths*.

All over the world, people have agreed to keep the number of hundredths in such statements below ten. This means that the normal way of expressing the above quantity is *four tenths and seven hundredths*.

Written in decimal notation, four tenths and seven hundredths is 0,47. This is read as *nought comma four seven* and NOT *nought comma forty-seven*.

6. What is the decimal notation for each of the following numbers?

   (a) $3\frac{7}{10}$ (b) $4\frac{19}{100}$ (c) $\frac{47}{10}$ (d) $\frac{4}{100}$

... and thousandths

0,001 is another way of writing $\frac{1}{1\ 000}$.

1. What is the decimal notation for each of the following?

   (a) $\frac{7}{1\ 000}$ (b) $\frac{9}{1\ 000}$ (c) $\frac{147}{1\ 000}$ (d) $\frac{999}{1\ 000}$

2. Write the following numbers in the decimal notation:

   (a) $2 + \frac{3}{10} + \frac{7}{100} + \frac{4}{1\ 000}$ (b) $12 + \frac{1}{10} + \frac{4}{1\ 000}$

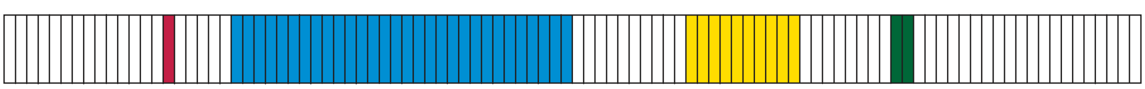
   (c) $2 + \frac{4}{1\ 000}$ (d) $67\frac{123}{1\ 000}$

   (e) $34\frac{61}{1\ 000}$ (f) $654\frac{3}{1\ 000}$

7.2 Percentages and decimal fractions

hundredths, percentages and decimals

1. The rectangle below is divided into small parts.



(a) How many of these small parts are there in the rectangle? And in one tenth of the rectangle?
(b) What part of the rectangle is blue? What part is green? What part is red?

Instead of *six hundredths*, you may say six *per cent*.
It means the same.
Ten per cent of the rectangle above is yellow.

2. Use the word "per cent" to say what part of the rectangle is green. What part is red?

3. What percentage of the rectangle is blue? What percentage is white?

We do not say: "How many per cent of the rectangle is green?" We say: "What percentage of the rectangle is green?"

The symbol % is used for "per cent".
Instead of writing "17 per cent", you may write 17%.
Per cent means *hundredths*. The symbol % is a bit like the symbol $\frac{\quad}{100}$.

4. (a) How much is 1% of R400? (In other words: How much is $\frac{1}{100}$ or 0,01 of R400?)
(b) How much is 37% of R400?
(c) How much is 37% of R700?

5. (a) 25 apples are shared equally between 100 people. What fraction of the apples does each person get? Write your answer as a common fraction and as a decimal fraction.
(b) How much is 1% (one hundredth) of 25?
(c) How much is 8% of 25?
(d) How much is 8% of 50? And how much is 0,08 of 50?

0,37 and 37% and $\frac{37}{100}$ are different symbols for the same thing: *37 hundredths.*

6. Express each of the following in three ways:
- in the *decimal notation*,
- in the *percentage notation*, and
- if possible, in the *common fraction notation, using hundredths*.

(a) three tenths
(b) seven hundredths
(c) 37 hundredths
(d) seven tenths
(e) three quarters
(f) seven eighths

7. (a) How much is three tenths of R200 and seven hundredths of R200 altogether?
(b) How much is $\frac{37}{100}$ of R200?

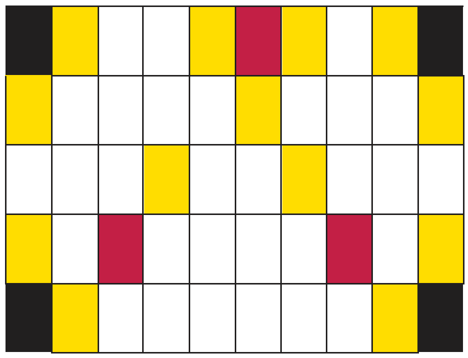
(c) How much is 0,37 of R200?

(d) And how much is 37% of R200?

8. Express each of the following in three ways:
   - in the *decimal notation*,
   - in the *percentage notation*, and
   - in the *common fraction notation*, *using hundredths*.

   (a) 20 hundredths (b) 50 hundredths
   (c) 25 hundredths (d) 75 hundredths

9. (a) Jan eats a quarter of a watermelon. What percentage of the watermelon is this?
   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk is this?
   (c) Jeminah uses 0,75 (seven tenths and five hundredths) of the paint in a tin. What percentage of the paint does she use?



10. The floor of a large room is shown alongside. What part of the floor is covered in each of the four colours? Copy the table below. Express your answer in four ways:
   (a) in the *common fraction notation*, *using hundredths*,
   (b) in the *decimal notation*,
   (c) in the *percentage notation*, and
   (d) if possible, *in the common fraction notation, as tenths* **and** *hundredths* (for example $\frac{3}{10} + \frac{4}{100}$).

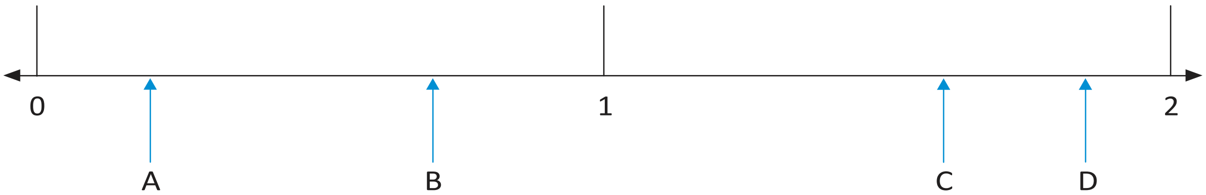
	(a)	(b)	(c)	(d)
white				
red				
yellow				
black				

7.3 Decimal measurements

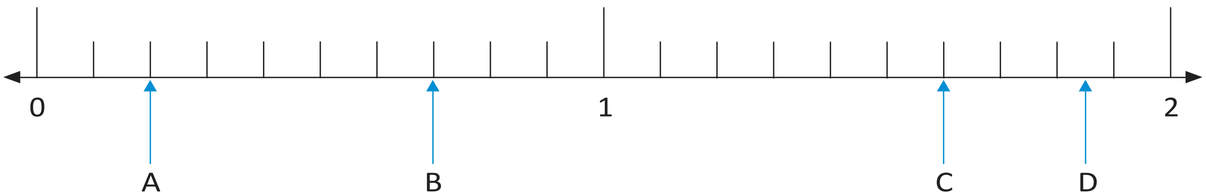
measuring on a number line

1. Read the lengths at the marked points (A to D) for the number lines on the next two pages. Give your answers as accurate as possible in decimal notation.

(a)

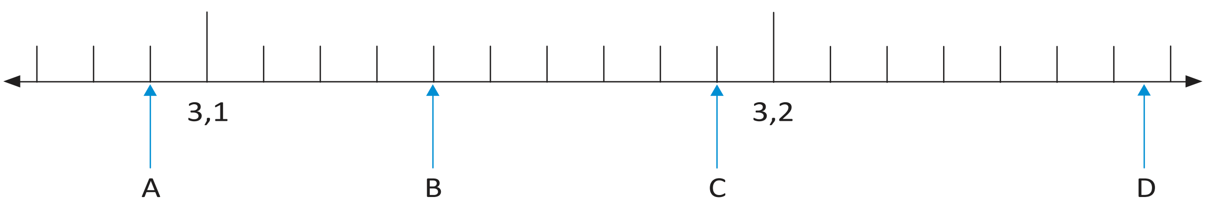


(b)

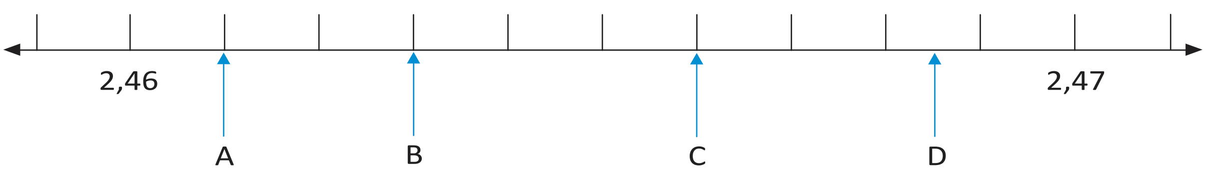


(c)

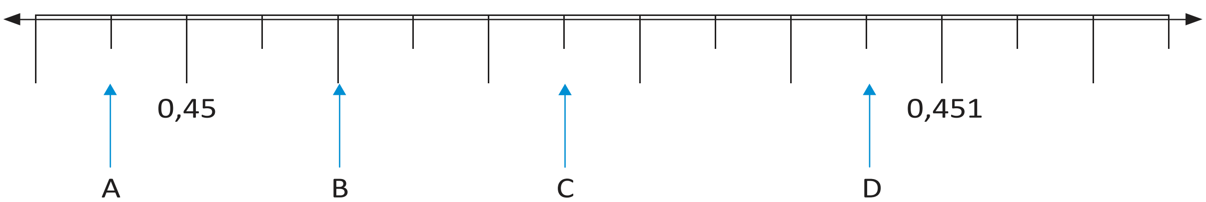
(d)



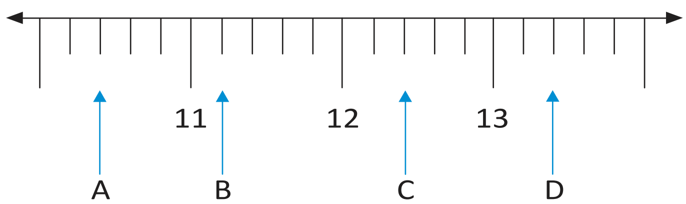
(e)



(f)

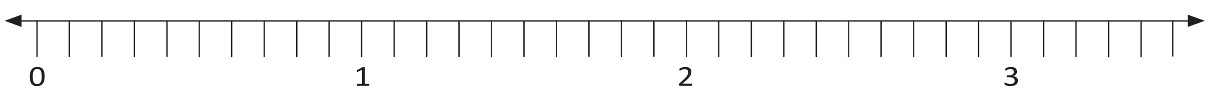


(g)



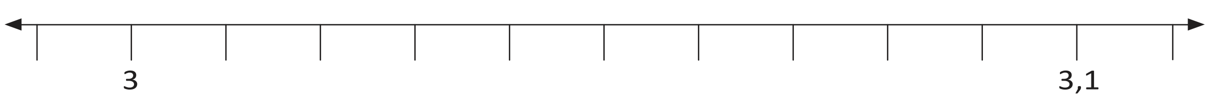
2. Copy the number line below and show the following numbers on it:

(a) 0,6 (b) 1,2 (c) 1,85 (d) 2,3
(e) 2,65 (f) 3,05 (g) 0,08



3. Copy the number line below and show the following numbers on it:

(a) 3,06 (b) 3,08 (c) 3,015
(d) 3,047 (e) 3,005



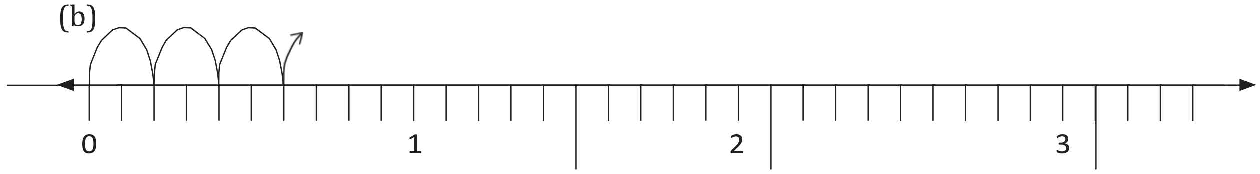
7.4 More decimal concepts

decimal jumps

Copy the number lines below. Write the next ten numbers in the number sequences and show your number sequences, as far as possible, on the number lines.

1. (a) 0,2; 0,4; 0,6; ...

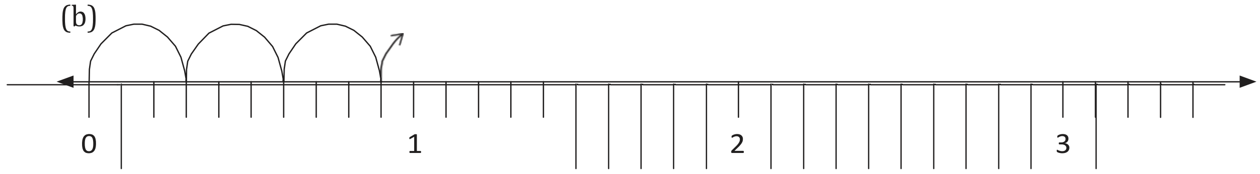
(b)



(c) How many 0,2s are there in 1?
(d) Write 0,2 as a common fraction.

2. (a) 0,3; 0,6; 0,9; ...

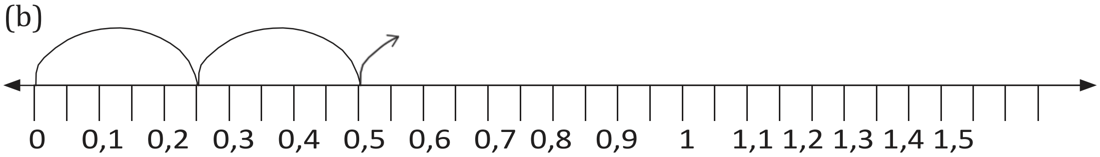
(b)



(c) How many 0,3s are there in 3?
(d) Write 0,3 as a common fraction.

3. (a) 0,25; 0,5; ...

(b)



(c) How many 0,25s are there in 1?

(d) Write 0,25 as a common fraction.

> A calculator can be programmed to do the same operation over and over again.
>
> For example, press 0,1 $\boxed{+}$ $\boxed{=}$ (do not press CLEAR or any other operation). Press the $\boxed{=}$ key repeatedly and see what happens.
>
> The calculator counts in 0,1s.

4. You can check your answers for questions 1 to 3 with a calculator. Program the calculator to help you.

5. Write the next five numbers in the number sequences:

   (a) 9,3; 9,2; 9,1; ...

   (b) 0,15; 0,14; 0,13; 0,12; ...

6. Check your answers with a calculator. Program the calculator to help you.

place value

1. Write each of the following as one number:

   (a) 2 + 0,5 + 0,07
   (b) 2 + 0,5 + 0,007
   (c) 2 + 0,05 + 0,007
   (d) 5 + 0,4 + 0,03 + 0,001
   (e) 5 + 0,04 + 0,003 + 0,1
   (f) 5 + 0,004 + 0,3 +0,01

We can write 3,784 in expanded notation as 3,784 = 3 + 0,7 + 0,08 + 0,004.
We can also name these parts as follows:

- the 3 represents the **units**
- the 7 represents the **tenths**
- the 8 represents the **hundredths**
- the 4 represents the **thousandths**

> We say: the **value** of the 7 is seven tenths but the **place value** of the 7 is tenths, because any digit **in that place** will represent the number of tenths.

For example, in 2,536 the **value** of the three is 0,03, and its **place value** is hundredths, because the value of the **place where it stands** is hundredths.

2. Now write the value (in decimal fractions) and the place value of each of the underlined digits.

(a) 2,345	(b) 4,678	(c) 1,953
(d) 34,856	(e) 564,34	(f) 0,987

7.5 Ordering and comparing decimal fractions

from biggest to smallest and smallest to biggest

1. Order the following numbers from biggest to smallest. Explain your method.

   0,8 0,05 0,5 0,15 0,465 0,55 0,75 0,4 0,62

2. Below are the results of some of the 2012 London Olympic events. Copy and complete the tables. In each case, order them from first to last place.

   (a) Women: Long jump – Final

Name	Country	Distance	Place
Anna Nazarova	RUS	6,77 m	
Brittney Reese	USA	7,12 m	
Elena Sokolova	RUS	7,07 m	
Ineta Radevica	LAT	6,88 m	
Janay DeLoach	USA	6,89 m	3rd
Lyudmila Kolchanova	RUS	6,76 m	

   (b) Women: 400 m hurdles – Final

Name	Country	Time	Place
Georganne Moline	USA	53,92 s	
Kaliese Spencer	JAM	53,66 s	4th
Lashinda Demus	USA	52,77 s	
Natalya Antyukh	RUS	52,70 s	
T’erea Brown	USA	55,07 s	
Zuzana Hejnová	CZE	53,38 s	

(c) Men: 110 m hurdles – Final

Name	Country	Time	Place
Aries Merritt	USA	12,92 s	
Hansle Parchment	JAM	13,12 s	
Jason Richardson	USA	13,04 s	
Lawrence Clarke	GBR	13,39 s	
Orlando Ortega	CUB	13,43 s	
Ryan Brathwaite	BAR	13,40 s	

(d) Men: Javelin – Final

Name	Country	Distance	Place
Andreas Thorkildsen	NOR	82,63 m	
Antti Ruuskanen	FIN	84,12 m	
Keshorn Walcott	TRI	84,58 m	
Oleksandr Pyatnytsya	UKR	84,51 m	
Tero Pitkämäki	FIN	82,80 m	
Vítezslav Veselý	CZE	83,34 m	

3. In each case, give a number that falls between the two numbers.
   (This means you may give *any* number that falls anywhere between the two numbers.)
   (a) 3,5 and 3,7 (b) 3,9 and 3,11 (c) 3,1 and 3,2

4. How many numbers are there between 3,1 and 3,2?

5. Copy and fill in <, > or=.
   (a) 0,4 ☐ 0,52 (b) 0,4 ☐ 0,32 (c) 2,61 ☐ 2,7
   (d) 2,4 ☐ 2,40 (e) 2,34 ☐ 2,567 (f) 2,34 ☐ 2,251

7.6 Rounding off

Just as whole numbers can be rounded off to the nearest 10, 100 or 1 000, decimal fractions can be rounded off to the nearest whole number or to one, two, three etc. digits after the comma. A decimal fraction is rounded off to the number whose value is closest to it. Therefore 13,24 rounded off to one decimal place is 13,2 and 13,26 rounded off to one decimal place is 13,3. A decimal ending in a 5 is an equal distance from the two numbers to which it can be rounded off. Such decimals are rounded off to the biggest number, so 13,15 rounded off to one decimal place becomes 13,2.

saying it nearly but not quite

1. Round each of the following numbers off to the nearest whole number:
   7,6 18,3 204,5 1,89 0,9 34,7 11,5 0,65
2. Round each of the following numbers off to one decimal place:
   7,68 18,93 21,47 0,643 0,938 1,44 3,81
3. Round each of the following numbers off to two decimal places:
   3,432 54,117 4,809 3,762 4,258 10,222 9,365 299,996

round off to help you calculate

1. John and three of his brothers sell an old bicycle for R44,65. How can the brothers share the money fairly?
2. A man buys 3,75 m of wood at R11,99 per metre. What does the wood cost him?
3. Estimate the answers of each of the following by rounding off the numbers:
   (a) 89,3 × 3,8 (b) 227,3 + 71,8 − 28,6

7.7 Addition and subtraction with decimal fractions

mental calculations

1. Copy and complete the number chain.

34,123	→	+ 20	→	54,123	→		→	454,123	→		→	454,023
422,011				452,011				452,021				452,023
222,011	→		→	222,211	→		→	222,231	→		→	222,232
222,489				222,482				222,422				222,222

When you add or subtract decimal fractions, you can change them to common fractions to make the calculation easier.

For example:

$0{,}4 + 0{,}5$

$= \frac{4}{10} + \frac{5}{10}$

$= \frac{9}{10}$

$= 0{,}9$

2. Calculate each of the following:

(a) 0,7 + 0,2	(b) 0,7 + 0,4	(c) 1,3 + 0,8
(d) 1,35 + 0,8	(e) 0,25 + 0,7	(f) 0,25 + 0,07
(g) 3 − 0,1	(h) 3 − 0,01	(i) 2,4 − 0,5

some real-life problems

1. The owner of an internet café looks at her bank statement at the end of the day. She finds the following amounts paid into her account: R281,45; R39,81; R104,54 and R9,80. How much money was paid into her account on that day?

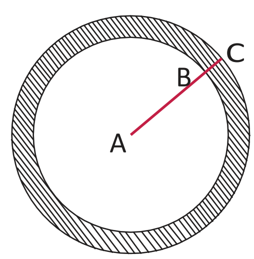
2. At the beginning of a journey the odometer in a car reads: 21 589,4. At the end of the journey the odometer reads: 21 763,7. What distance was travelled?

3. At an athletics competition, an athlete runs the 100 m race in 12,8 seconds. The announcer says that the athlete has broken the previous record by 0,4 seconds. What was the previous record?

4. In a surfing competition, five judges give each contestant a mark out of 10. The highest and the lowest marks are ignored and the other three marks are totalled. Work out each contestant's score and place the contestants in order from first to last.

A: 7,5	8	7	8,5	7,7	B: 8,5	8,5	9,1	8,9	8,7
C: 7,9	8,1	8,1	7,8	7,8	D: 8,9	8,7	9	9,3	9,1

5. A pipe is measured accurately. AC = 14,80 mm and AB = 13,97 mm. How thick is the pipe (BC)?



6. Mrs Mdlankomo buys three packets of mincemeat. The packets weigh 0,356 kg, 1,201 kg and 0,978 kg respectively. What do they weigh together?

7.8 Multiplication and decimal fractions

the power of ten

1. (a) Copy and complete the multiplication table.

×	1 000	100	10	1	0,1	0,01	0,001
6	6 000		60			0,06	
6,4		640					
0,5					0,05		
4,78	4 780		47,8				
41,2	41 200						

(b) Is it correct to say that "multiplication makes bigger"? When does multiplication make bigger?

(c) Formulate rules for multiplying with 10; 100; 1 000; 0,1; 0,01 and 0,001. Can you explain the rules?

(d) Now use your rules to calculate each of the following:

0,5 × 10   0,3 × 100   0,42 × 10   0,675 × 100

2. (a) Copy and complete the division table.

÷	0,001	0,01	0,1	1	10	100	1 000
6				6	0,6	0,06	
6,4			64	6,4			
0,5						0,005	
4,78			47,8				
41,2		4 120					

(b) Is it correct to say that "division makes smaller"? When does division make smaller?

(c) Formulate rules for dividing with 10; 100; 1 000; 0,1; 0,01 and 0,001. Can you explain the rules?

(d) Now use your rules to calculate each of the following:

0,5 ÷ 10   0,3 ÷ 100   0,42 ÷ 10

3. Copy and complete the following statements:

(a) Multiplying with 0,1 is the same as dividing by ...

(b) Dividing by 0,1 is the same as multiplying by ...

Now discuss it with a partner or explain to him or her why this is so.

4. Copy and fill in the missing numbers:

1,23456	→	× 10	→	12,3456	→		→	123,456	→		→	12 345,6
1 234,56				12 345,6				123 456				1 234 560
123,456	→		→	1,23456	→		→	0,123456	→		→	123 456

What does multiplying a decimal number with a whole number mean?

What does something like 4 × 0,5 mean?

What does something like 0,5 × 4 mean?

4 × 0,5 means four groups of $\frac{1}{2}$, which is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, which is 2.

0,5 × 4 means $\frac{1}{2}$ of 4, which is 2.

A real-life example where we would find this is:

$$\begin{aligned} 6 \times 0{,}42 \text{ kg} &= 6 \times \frac{42}{100} \\ &= (6 \times 42) \div 100 \\ &= 252 \div 100 \\ &= 2{,}52 \text{ kg} \end{aligned}$$

What really happens is that we convert 6 × 0,42 to the product of two whole numbers, do the calculation and then convert the answer to a decimal fraction again (÷ 100).

multiplying decimals with whole numbers

1. Calculate each of the following. Use fraction notation to help you.
   (a) 0,3 × 7 (b) 0,21 × 91 (c) 8 × 0,4

2. Estimate the answers to each of the following and then calculate:
   (a) 0,4 × 7 (b) 0,55 × 7
   (c) 12 × 0,12 (d) 0,601 × 2

3. Make a rule for multiplying with decimals. Explain your rule to a partner.

What does multiplying a decimal with a decimal mean?

For example, what does 0,32 × 0,87 mean?

If you buy 0,32 m of ribbon and each metre costs R0,87, you can write it as 0,32 × 0,87.

$0{,}32 \times 0{,}87 = \frac{32}{100} \times \frac{87}{100}$	[Write as common fractions]
$= \frac{32 \times 87}{10\ 000}$	[Multiplication of two fractions]
$= \frac{2\ 784}{10\ 000}$	[The product of the whole numbers 32 × 87]
$= 0{,}2784$	[Convert to a decimal by dividing the product by 10 000]

The product of two decimals is thus converted to the product of whole numbers and then converted back to a decimal.

The product of two decimals and the product of two whole numbers with the same digits differ only in terms of the place value of the products, in other words the position of the decimal comma. It can also be determined by estimating and checking.

multiplying decimals with decimals

1. Calculate each of the following. Use fraction notation to help you.
   (a) 0,6 × 0,4 (b) 0,06 × 0,4 (c) 0,06 × 0,04

Mandla uses this method to multiply decimals with decimals:

0,84 × 0,6 = (84 ÷ 100) × (6 ÷ 10)
= (84 × 6) ÷ (100 × 10)
= 504 ÷ 1 000
= 0,504

2. Calculate the following using Mandla's method:
   (a) 0,4 × 0,7 (b) 0,4 × 7 (c) 0,04 × 0,7

7.9 Division and decimal fractions

Look carefully at the following three methods of calculation:

1. 0,6 ÷ 2 = 0,3 [six tenths ÷ 2 = three tenths]

2. 12,4 ÷ 4 = 3,1 [(12 units + four tenths) ÷ 4]
   = (12 units ÷ 4) + (four tenths ÷ 4)
   = three units + one tenth
   = 3,1

3. 2,8 ÷ 5 = 28 tenths ÷ 5
= 25 tenths ÷ 5 and three tenths ÷ 5
= five tenths and (three tenths ÷ 5) [three tenths cannot be divided by 5]
= five tenths and (30 hundredths ÷ 5) [three tenths = 30 hundredths]
= five tenths and six hundredths
= 0,56

dividing decimals by whole numbers

1. Complete.

(a) 8,4 ÷ 2
= (8 ____ + 4 tenths) ÷ 2
= (8 ____ ÷ 2) + (________________)
= 4 ____ + ____ tenths
= ________

(b) 3,4 ÷ 4
= (3 units + 4 tenths) ÷ 4
= (32 ____ + 20 ________) ÷ 4
= (________ ÷ 4) + (________ ÷ 4)
= ________ + ____ hundredths
= ____

2. Calculate each of the following in the shortest possible way:
(a) 0,08 ÷ 4
(b) 14,4 ÷ 12
(c) 8,4 ÷ 7
(d) 4,5 ÷ 15
(e) 1,655 ÷ 5
(f) 0,225 ÷ 25

3. A grocer buys 15 kg of bananas for R99,90. What do the bananas cost per kilogram?

4. Given 26,8 ÷ 4 = 6,7. Write down the answers to the following without calculating:
(a) 268÷ 4
(b) 0,268 ÷ 4
(c) 26,8 ÷ 0,4

5. Given 128 ÷ 8 = 16. Write down the answers to the following without calculating:
(a) 12,8÷ 8
(b) 1,28 ÷ 8
(c) 1,28 ÷ 0,8

6. Sue pays R18,60 for 0,6 metres of material. What does one metre of material cost?

7. John buys 0,45 m of chain. The chain costs R20 per metre. What does John pay for the chain?

8. You may use a calculator for thisquestion.
Anna buys a packet of mincemeat. It weighs 0,215 kg. The price for the mincemeat is R42,95 per kilogram. What does she pay for her packet of mincemeat? (Give a sensible answer.)

Chapter 8
Relationships between variables

8.1 Constant and variable quantities

look for connections between quantities

1. (a) How many fingers does a person who is 14 years old have?
   (b) How many fingers does a person who is 41 years old have?
   (c) Does the number of fingers on a person's hand depend on their age? Explain.

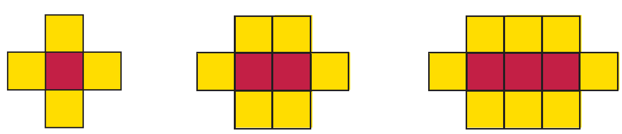
There are two quantities in the above situation: **age** and the **number of fingers** on a person's hand. The number of fingers remains the same, irrespective of a person's age. So we say the number of fingers is a **constant** quantity. However, age changes, or varies, so we say age is a **variable** quantity.

2. Now consider each situation below. For each situation, state whether one quantity influences the other. If it does, try to say *how* the one quantity will influence the other quantity. Also say whether there is a constant in the situation.

   (a) The number of calls you make and the amount of airtime left on your cell phone
   (b) The number of houses to be built and the number of bricks required
   (c) The number of learners at a school and the duration of the Mathematics period

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words, what number is linked to a specific value of the othervariable.

3. Consider the following arrangements:



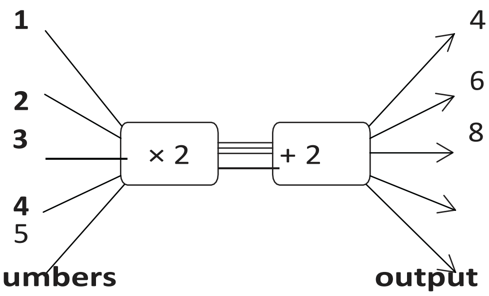
(a) How many yellow squares are there if there is only one red square?

(b) How many yellow squares are there if there are two red squares?

(c) How many yellow squares are there if there are three red squares?

(d) Copy and complete the flow diagram below by filling in the missing numbers.

Can you see the connection between the arrangements above and the flow diagram? We can also describe the relationship between the red and yellow squares in words.



input numbers
(Number of red squares)

output numbers
(Number of yellow squares)

In words:
The number of yellow squares is found by multiplying the number of red squares by 2 and then adding 2 to the answer.

(e) How many yellow squares will there be if there are 10 red squares?

(f) How many yellow squares will be there if there are 21 red squares?

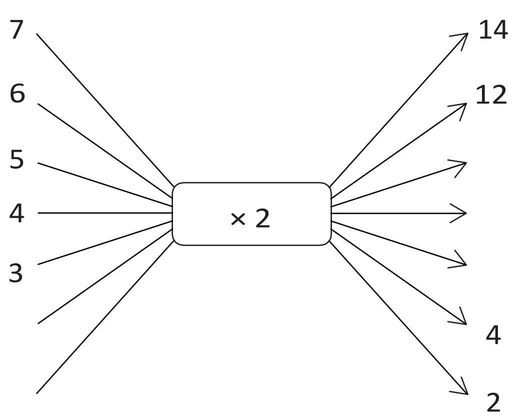
8.2 Different ways to describe relationships

complete some flow diagrams and tables of values

A relationship between two quantities can be shown with a flow diagram. In a flow diagram we cannot show all the numbers, so we show only some.

1. Copy the flow diagram below and calculate the missing input and output numbers.

(a)



Each **input number** in a flow diagram has a corresponding **output number**. The first (top) input number corresponds to the first output number. The second input number corresponds to the second output number, and so on.
We say × 2 is the **operator**.

(b) What types of numbers are given as input numbers?

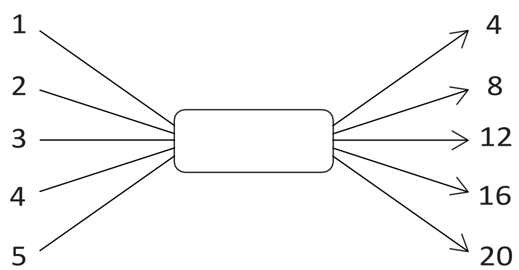
(c) In the previous flow diagram, the output number 14 corresponds to the input number 7. Copy and complete the following sentences in the same way:

In the relationship shown in the previous flow diagram, the output number ___ corresponds to the input number 5.

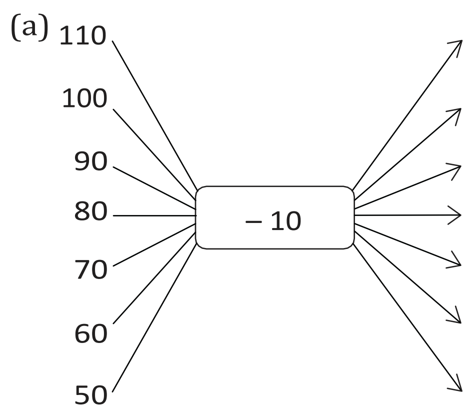
The input number ___ corresponds to the output number 6.

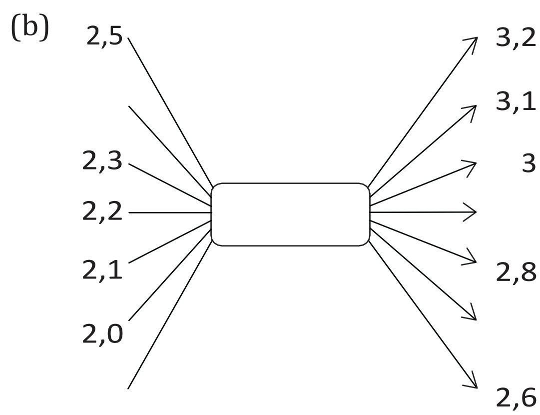
If more places are added to the flow diagram, the input number ___ will correspond to the output number 40.

2. Copy and complete this flow diagram by writing the appropriate operator, and then write the rule for completing this flow diagram in words.

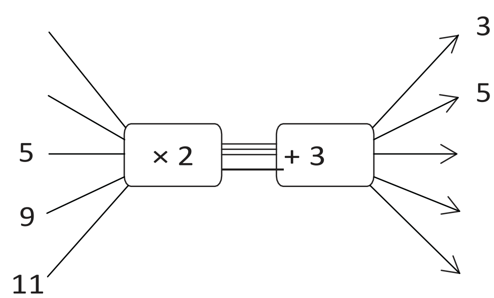


3. Copy and complete the flow diagrams below. You have to find out what the operator for (b) is and fill it in yourself.





4. Copy and complete the flow diagram:



A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

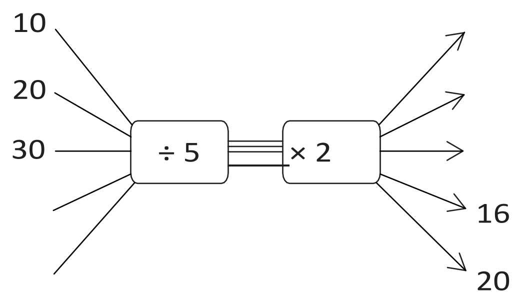
The flow diagram that you completed in question 4 shows the following information:

- Each input number is multiplied by 2 and then 3 is added to produce the output numbers.
- It shows which output number is connected to which input number.

The relationship between the input and output numbers can also be shown in a table:

Input numbers	0	1	5	9	11
Output numbers	3	5	13	21	25

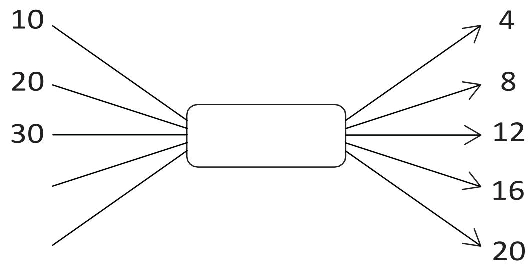
5. (a) Copy and complete the flow diagram, then describe in words how the output numbers below can be calculated.



(b) Copy and complete the table below to show which output numbers are connected to which input numbers in the above flow diagram.

Input numbers					
Output numbers					

(c) Copy the flow diagram below and fill in the appropriate operator.



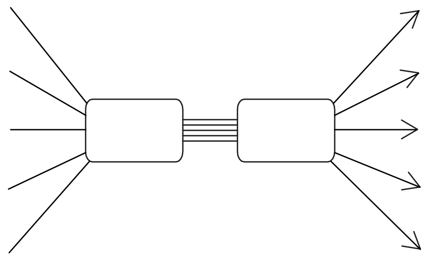
(d) The flow diagrams in question 5(a) and 5(c) have different operators, but they produce the same output values for the same input values. Explain.

6. The rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 1,8 and then add 32."

   (a) Check whether the table below was completed correctly. If you find a mistake, note what it is and correctit.

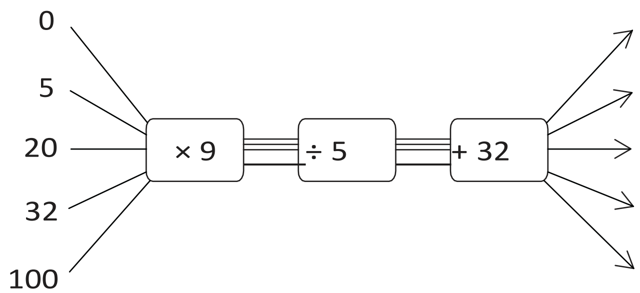
Temperature in degrees Celsius	0	5	20	32	100
Temperature in degrees Fahrenheit	32	41	68		212

   (b) Copy and complete the flow diagram to represent the information in (a).



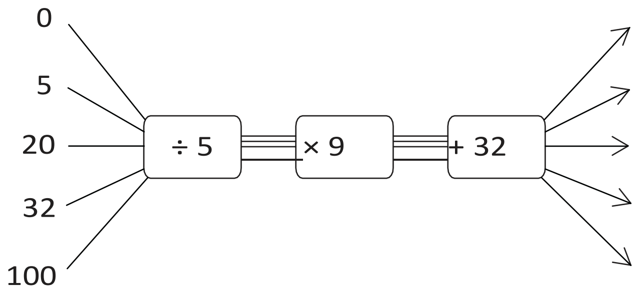
7. Another rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 9, then divide the answer by 5 and then add 32 to the answer."

   (a) Copy and complete the flow diagram below.



   (b) Why do you think the flow diagrams in questions 6(b) and 7(a) produce the same output numbers for the same input numbers, even though they have different operators?

   (c) Copy and complete the flow diagram on the next page. Does this flow diagram give the same output values as the flow diagram in question 7(a)? Explain.

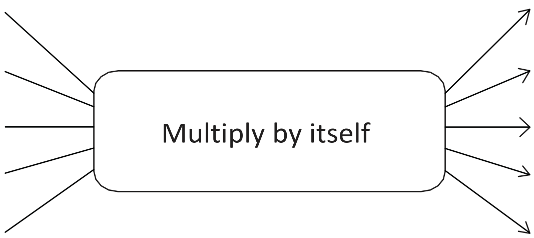


8. The rule for calculating the area of a square is: "Multiply the length of a side of the square by itself."

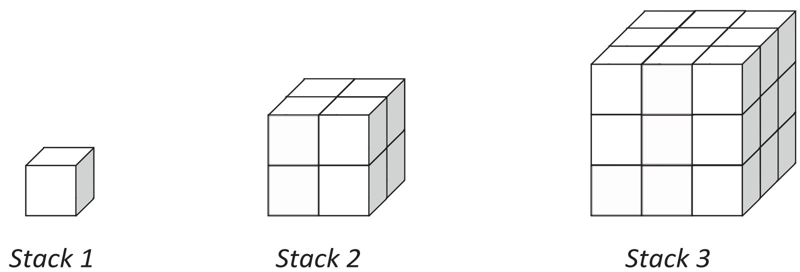
   (a) Copy and complete the table below.

Length of side	4	6		10	
Area of square			64		144

   (b) Copy and complete the flow diagram to represent the information in the table.



9. (a) The pattern below shows stacks of building blocks. The number of blocks in each stack is dependent on the number of the stack.



   Copy and complete the table below to represent the relationship between the number of blocks and the number of the stack.

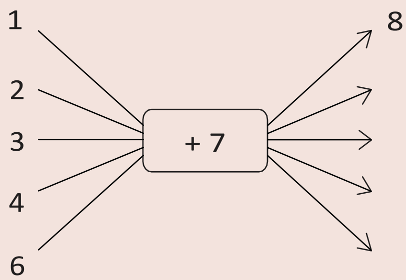
Stack number	1	2	3	4	5	6	7	8
Number of blocks	1	8						

   (b) Describe in words how the output values can be calculated.

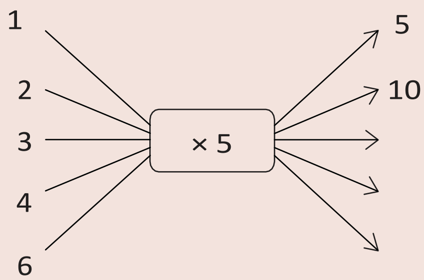
extension: linking flow diagrams, tables of values, and rules

1. Copy and complete the flow diagrams below.

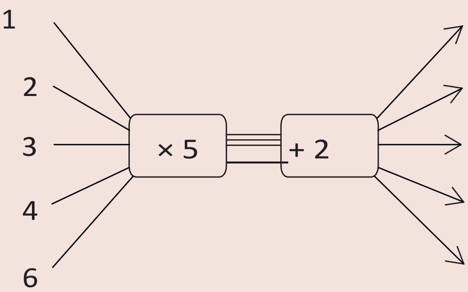
(a)



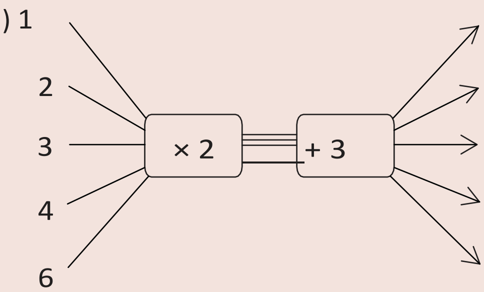
(b)



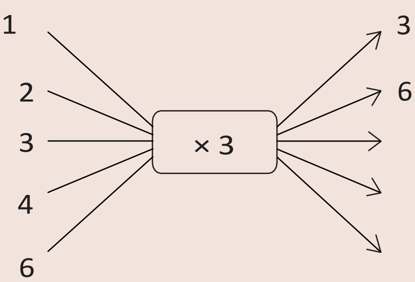
(c)



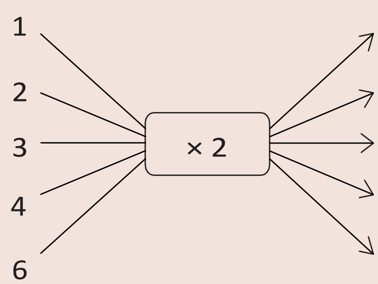
(d)



(e)



(f)



2. Calculate the differences between the consecutive output numbers and compare them to the differences between the consecutive input numbers. Consider the operator of the flow diagram. What do you notice?

3. determine the rule to calculate the missing output numbers in this relationship and copy and complete the table:

Input numbers	1	2	3	4	5	7	10
Output numbers	9	16	23				

Chapter 9
Perimeter and areaof 2D shapes

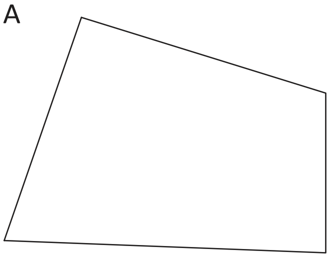
9.1 Perimeter of polygons

The **perimeter** of a shape is the total distance around the shape, or the lengths of its sides added together. Perimeter (P) is measured in units such as millimetres (mm), centimetres (cm) and metres (m).

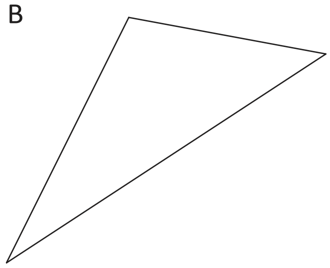
measuring perimeters

1. (a) Use a compass and/or a ruler to measure the length of each side in figures A to C. For each figure, write the measurements in millimetres.
   (b) Write down the perimeter of each figure.

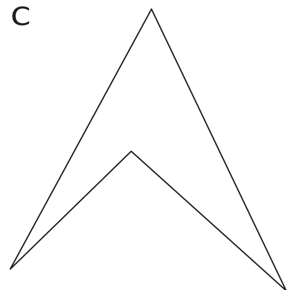
A



B

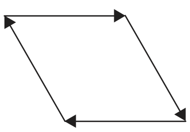


C

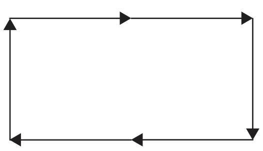


2. The following shapes consist of arrows that are equal in length.
   (a) What is the perimeter of each shape in number of arrows?
   (b) If each arrow is 30 mm long, what is the perimeter of each shape in millimetres?

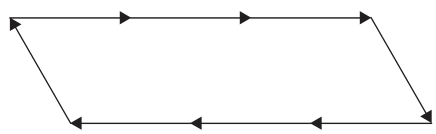
A



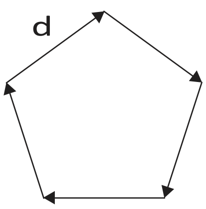
B



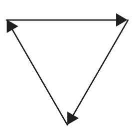
C



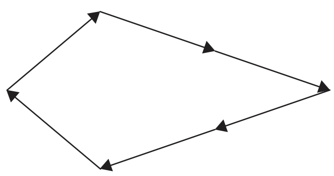
d



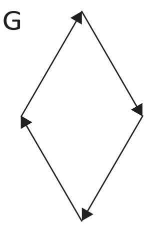
E



F

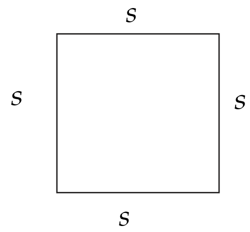


G



9.2 Perimeter formulae

If the sides of a square are all s units long:

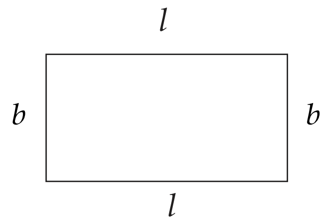


Perimeter of square $= s + s + s + s$

$= 4 \times s$

or $P = 4s$

If the length of a rectangle is l units and the breadth (width) is b units:



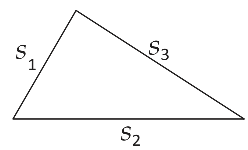
Perimeter of rectangle $= l + l + b + b$

$= 2 \times l + 2 \times b$

$= 2l + 2b$

or $P = 2(l + b)$

A triangle has three sides, so:



Perimeter of triangle $= s_1 + s_2 + s_3$

or $P = s_1 + s_2 + s_3$

applying perimeter formulae

1. Calculate the perimeter of a square if the length of one of its sides is 17,5 cm.
2. One side of an equilateral triangle is 32 cm. Calculate the triangle's perimeter.
3. Calculate the length of one side of a square if the perimeter of the square is 7,2 m. (Hint: $4s$ =? Therefore s=?)
4. Two sides of a triangle are 2,5 cm each. Calculate the length of the third side if the triangle's perimeter is 6,4 cm.
5. A rectangle is 40 cm long and 25 cm wide. Calculate its perimeter.
6. Calculate the perimeter of a rectangle that is 2,4 m wide and 4 m long.
7. The perimeter of a rectangle is 8,88 m. How long is the rectangle if it is 1,2 m wide?
8. Do the necessary calculations and then copy and complete the tables. (All the measurements refer to rectangles.)

	Length	Breadth	Perimeter
(a)	74 mm	30 mm	
(c)		1,125 cm	6,25 cm
(e)	7,5 m	3,8 m	

	Length	Breadth	Perimeter
(b)	25 mm		90 mm
(d)	5,5 cm		22 cm
(f)		2,5 m	12 m

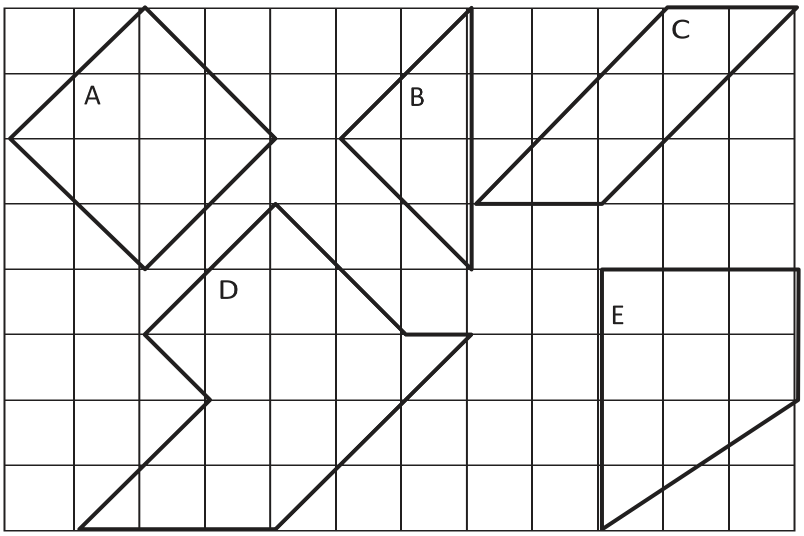
9.3 Area and square units

The **area** of a shape is the size of the flat surface surrounded by the border (perimeter) of the shape.

Usually, area (A) is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2).

square units to measure area

1. In your book, write down the area of figures A to E below by counting the square units. (Remember to add halves or smaller parts of squares.)



A is _ square units.

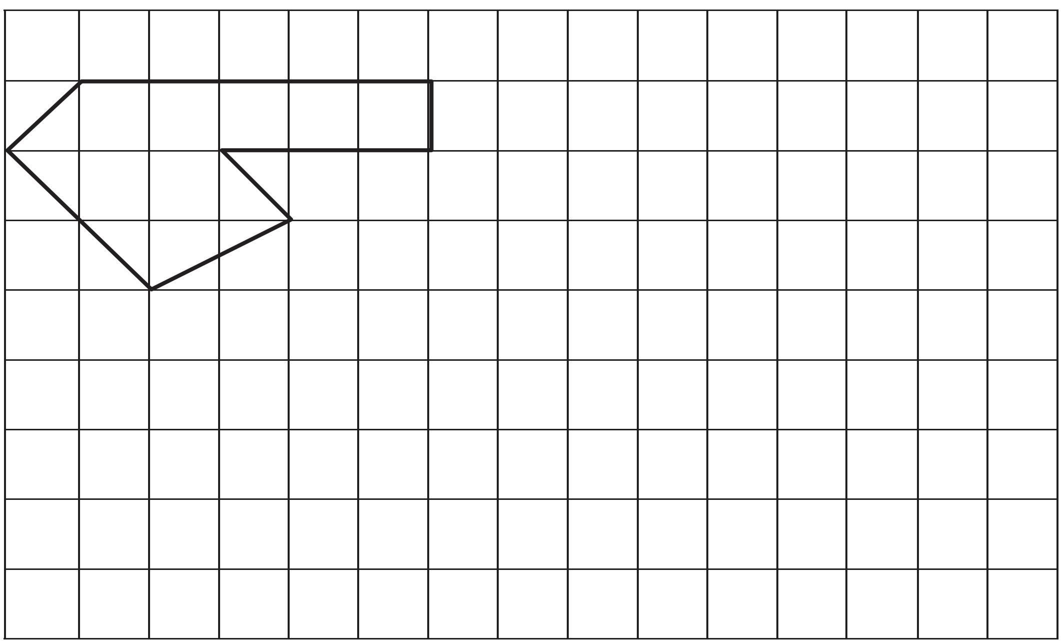
B is _ square units.

C is _ square units.

D is _ square units.

E is _ square units.

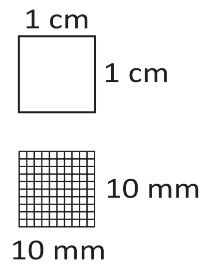
2. Each square in the grid below measures 1 cm^2 (1 cm × 1cm).



(a) What is the area of the shape drawn on the grid?

(b) Trace the grid, then draw two shapes of your own. The shapes should have the same area, but different perimeters.

conversion of units



The figure on the right shows a square with sides of 1 cm. The area of the square is one square centimetre (1 cm^2).

How many squares of 1 mm by 1 mm (1 mm^2) would fit into the 1 cm^2 square? Copy and complete: 1 cm^2 = ___ mm^2

To change cm^2 to mm^2:

1 cm^2 = 1 cm × 1 cm

= 10 mm × 10 mm

= 100 mm^2

Similarly, to change mm^2 to cm^2:

1 mm^2 = 1 mm × 1 mm

= 0,1 cm × 0,1 cm

= 0,01 cm^2

We can use the same method to convert between other square units too. Copy and complete:

From m^2 to cm^2:

1 m^2 = 1 m × 1 m

= ___ cm × ___ cm

= ___ cm^2

From cm^2 to m^2:

1 cm^2 = 1 cm × 1 cm

= 0,01 m × 0,01 m

= ___ m^2

So, to convert between m^2, cm^2 and mm^2 you do the following:

- cm^2 to mm^2 → multiply by 100
- mm^2 to cm^2 → divide by 100
- m^2 to cm^2 → multiply by 10 000
- cm^2 to m^2 → divide by 10 000

Do the necessary calculations in your exercise book. Then copy the conversions and fill in your answers.

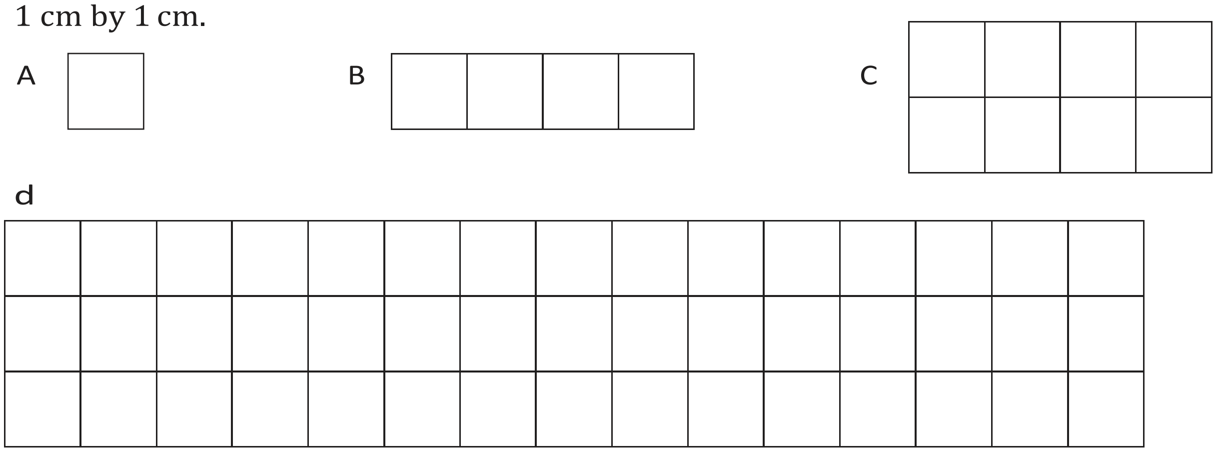
1. (a) 5 m^2 = ___ cm^2 (b) 5 cm^2 = ___ mm^2
   (c) 20 cm^2 = ___ m^2 (d) 20 mm^2 = ___ cm^2

2. (a) 25 m^2 = ___ cm^2 (b) 240 000 cm^2 = ___ m^2
   (c) 460,5 mm^2 = ___ cm^2 (d) 0,4 m^2 = ___
   (e) 12 100 cm^2 = ___ m^2 (f) 2,295 cm^2 = ___ mm^2

9.4 Area of squares and rectangles

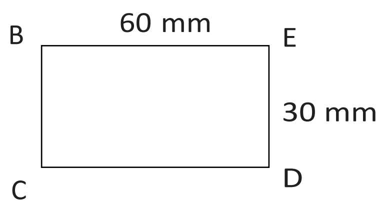
investigating the area of squares and rectangles

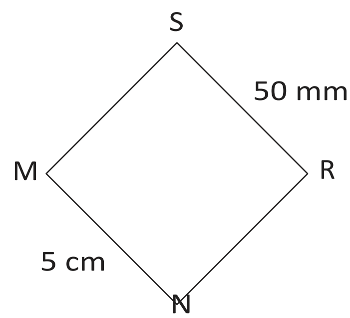
1. Each of the following four figures is divided into squares of equal size, namely 1 cm by 1 cm.



   (a) Give the area of each figure in square centimetres (cm^2).
   (b) Is there a shorter method to work out the area of each figure? Explain.

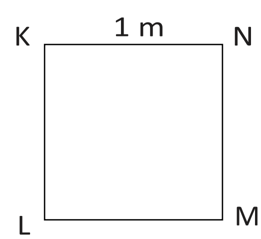
2. Figure BCDE is a rectangle and MNRS is a square.





   (a) How many cm^2 (1 cm × 1 cm) would fit into rectangle BCDE?
   (b) How many mm^2 (1 mm × 1 mm) would fit into rectangle BCDE?
   (c) What is the area of square MNRS in cm^2?
   (d) What is the area of square MNRS in mm^2?

3. Figure KLMN is a square with sides of 1 m.



   (a) How many squares with sides of 1 cm would fit along the length of the square?
   (b) How many squares with sides of 1 cm would fit along the breadth of the square?
   (c) How many squares (cm^2) would therefore fit into the whole square?
   (d) Copy and complete: $1\ m^2 =$ ___ ___ cm^2

A quick way of calculating the number of squares that would fit into a rectangle is to multiply the number of squares that would fit along its length by the number of squares that would fit along its breadth.

formulae: area of rectangles and squares

In the rectangle on the right:

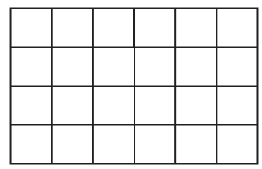
Number of squares = squares along the length × squares along the breadth

$= 6 \times 4$

$= 24$

l = 6 squares

b = 4 squares



From this we can deduce the following:

Area of rectangle = length of rectangle × breadth of rectangle

$A = l \times b$

(where A is the area in square units, l is the length and b is the breadth)

Area of square = length of side × length of side

$A = l \times l$

$= l^2$

(where A is the area in square units, and l is the length of a side)

The units of the values used in the calculations must be the same. Remember:

- 1 m = 100 cm and 1 cm = 10 mm
- 1 cm^2 = 1 cm × 1 cm = 10 mm × 10 mm = 100 mm^2
- 1 m^2 = 1 m × 1 m = 100 cm × 100 cm = 10 000cm^2
- 1 mm^2 = 1 mm × 1 mm = 0,1 cm × 0,1 cm = 0,01 cm^2
- 1 cm^2 = 1 cm × 1 cm = 0,01 m × 0,01 m = 0,0001 m^2

Examples

1. Calculate the area of a rectangle with a length of 50 mm and a breadth of 3 cm. Give the answer in cm^2.

   Solution:

   Area of rectangle = $l \times b$

   = (50 × 30) mm^2 or A = (5 × 3) cm^2

   = 1 500 mm^2 or = 15 cm^2

2. Calculate the area of a square bathroom tile with a side of 150 mm.

Solution:

Area of square tile = $l \times l$

$= (150 \times 150)\ \text{mm}^2$

$= 22\,500\ \text{mm}^2$

The area is therefore 22 500 mm^2 (or 225 cm^2).

3. Calculate the length of a rectangle if its area is 450 cm^2 and its width is 150 mm.

Solution:

Area of rectangle = $l \times b$

$450 = l \times 15$

$30 \times 15 = l \times 15$ or $450 \div 15 = l$

$30 = l$ $30 = l$

The length is therefore 30 cm (or 300 mm).

applying the formulae

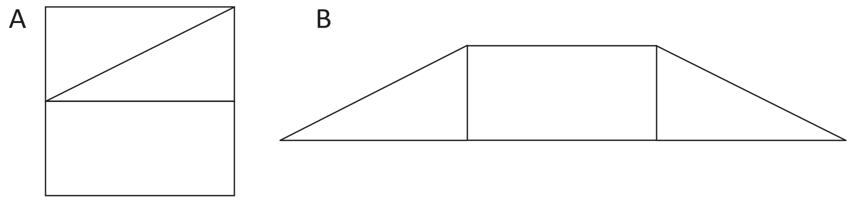
1. Calculate the area of each of the following shapes:
   (a) a rectangle with sides of 12 cm and 9 cm
   (b) a square with sides of 110 mm (answer in cm^2)
   (c) a rectangle with sides of 2,5 cm and 105 mm (answer in mm^2)
   (d) a rectangle with a length of 8 cm and a perimeter of 24 cm

2. A rugby field has a length of 100 m (goal post to goal post) and a breadth of 69 m.
   (a) What is the area of the field (excluding the area behind the goal posts)?
   (b) What would it cost to plant new grass on that area at a cost of R45/m^2?
   (c) Another unit for area is the hectare (ha). It is mainly used for measuring land. The size of 1 ha is the equivalent of 100 m × 100 m. Is a rugby field greater or smaller than 1 ha? Explain your answer.

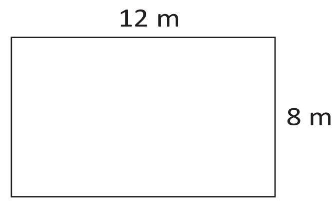
3. Do the necessary calculations and then copy and complete the table below. (All the measurements refer to rectangles.)

	Length	Breadth	Area
(a)	m	8 m	120 m^2
(b)	120 mm	mm	60 cm^2
(c)	3,5 m	4,3 m	m^2
(d)	2,3 cm	cm	2,76 cm^2
(e)	5,2 m	460 cm	m^2

4. Figure A is a square with sides of 20 mm. It is cut as shown in A and the parts are combined to form figure B. Calculate the area of figure B.

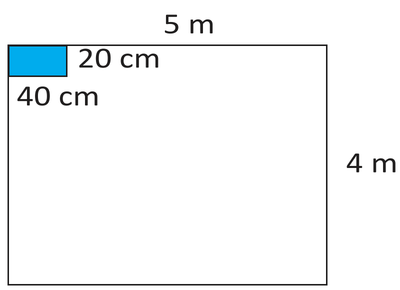


5. Margie plants a vegetable patch measuring 12 m × 8 m.



   (a) What is the area of the vegetable patch?
   (b) She plants carrots on half of the patch, and tomatoes and potatoes on a quarter of the patch each. Calculate the area covered by each type of vegetable?
   (c) How much will she pay to put fencing around the patch? The fencing costsR38/m.

6. Mr Allie has to tile a kitchen floor measuring 5 m × 4 m. The blue tiles he uses each measure 40 cm × 20 cm.



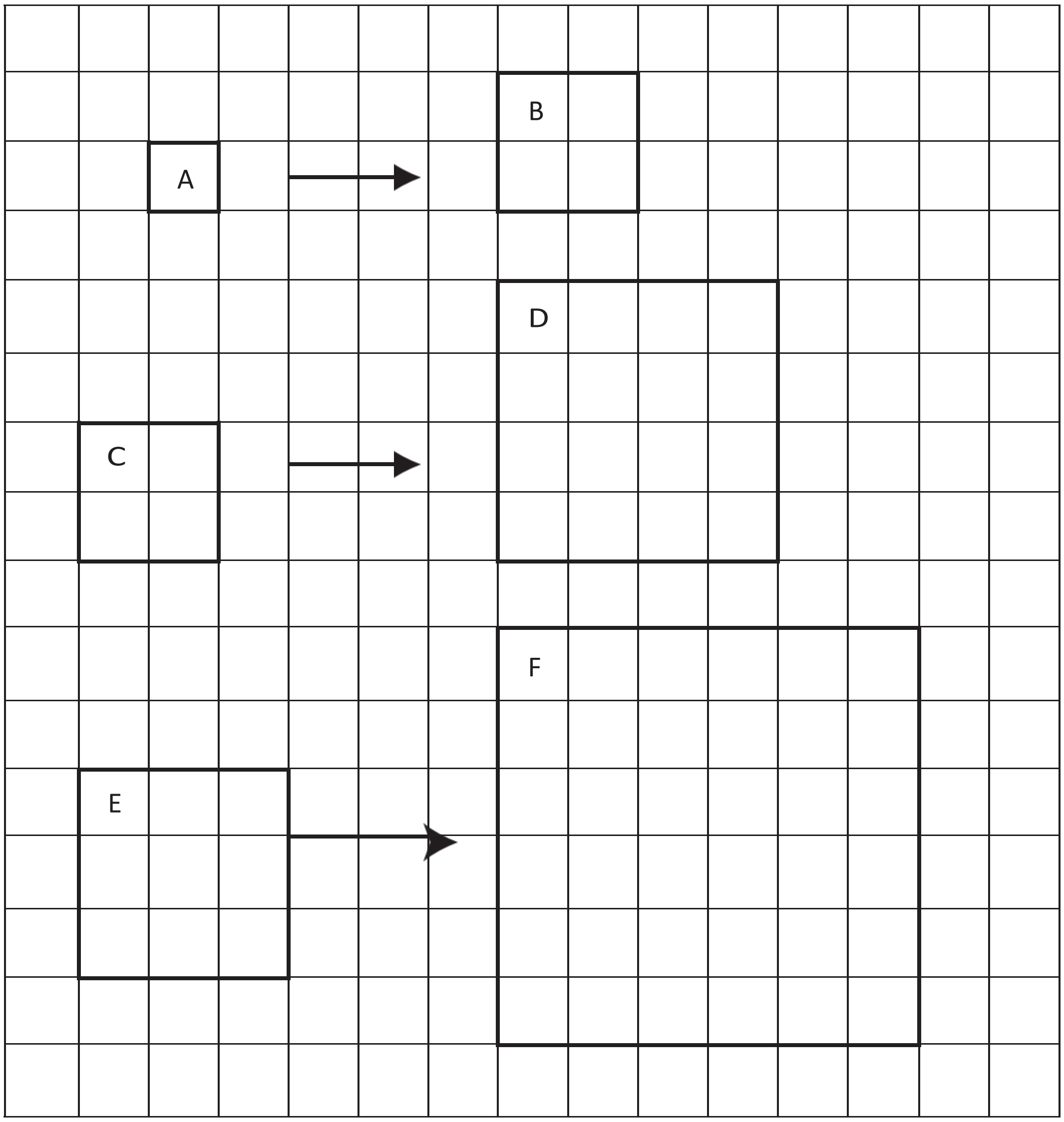
   (a) How many tiles does Mr Allie need?
   (b) The tiles are sold in boxes containing 20 tiles. How many boxes should he buy?

doubling a side and its effect on area

When a side of a square is doubled, will the area of the square also be doubled?

Copy the grid on page 141. The size of each square making up the grid is 1 cm × 1 cm.

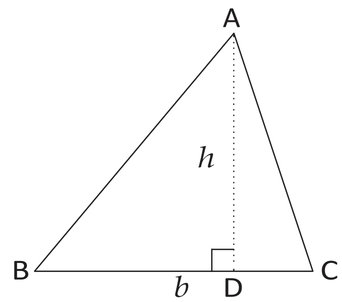
1. (a) For each square drawn on the grid, label the lengths of its sides.
   (b) Write down the area of each square. (Write the answer inside the square.)
2. Notice that the second square in each pair of squares has a side length that is double the side length of the firstsquare.
3. Compare the areas of the squares in each pair; then copy and complete the following: When the side of a square is doubled, its area ...



9.5 Area of triangles

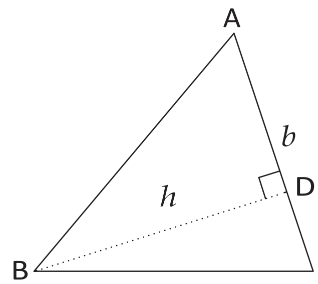
heights and bases of a triangle

The **height (h)** of a triangle is a perpendicular line segment drawn from a vertex to its opposite side. The opposite side, which forms a right angle with the height, is called the **base (b)** of the triangle. Any triangle has three heights and three bases.



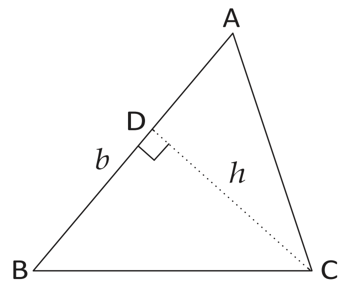
Ad = height

BC = base



Bd = height

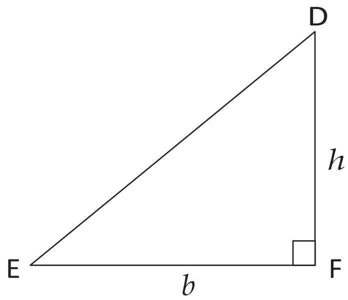
AC = base



Cd = height

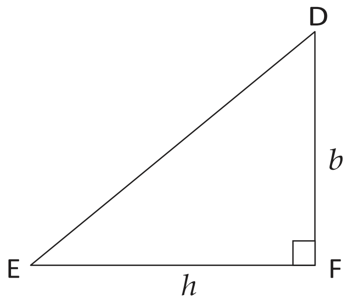
AB = base

In a right-angled triangle, two sides are already at right angles:



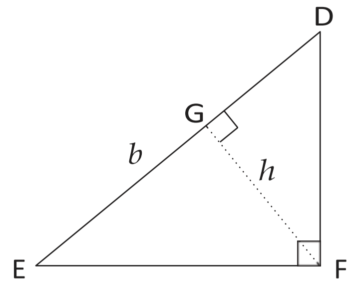
dF = height

EF = base



EF = height

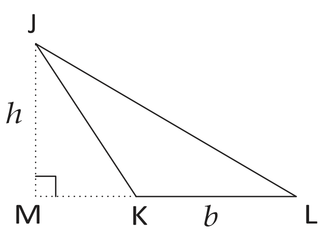
dF = base



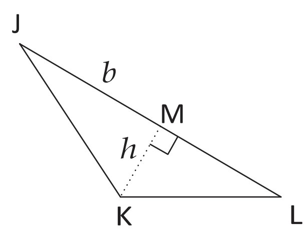
FG = height

dE = base

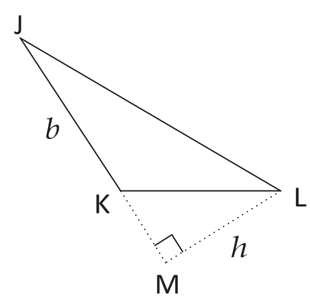
Sometimes a base must be extended outside of the triangle in order to draw the perpendicular height. This is shown in the first and third triangles below. Note that the extended part does not form part of the base's measurement:



JM = height
KL = base

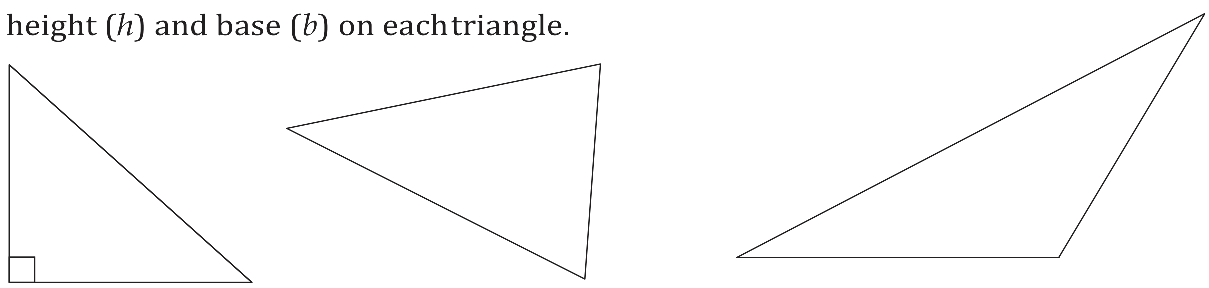


KM = height
JL = base



LM = height
JK = base

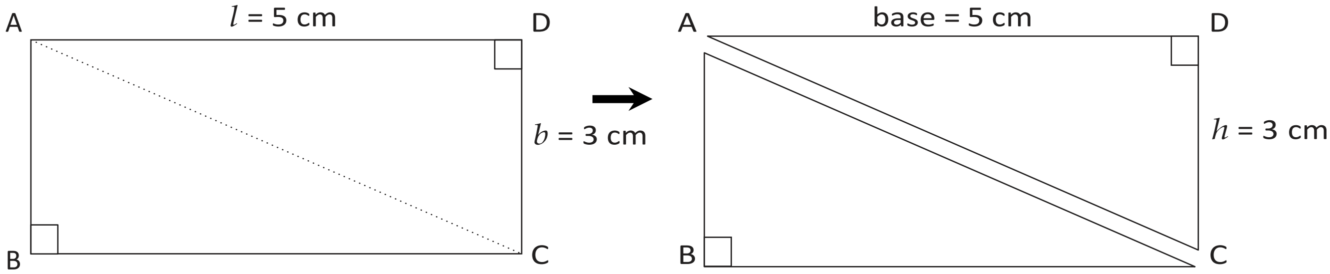
1. Copy the following triangles. Draw any height in each of the triangles. Label the height (h) and base (b) on each triangle.



2. Label another set of heights and bases on each triangle of question 1.

formula: area of a triangle

ABCD is a rectangle with length = 5 cm and breadth = 3 cm. When A and C are joined, it creates two triangles that are equal in area: ΔABC and ΔADC.



Area of rectangle = $l \times b$

Area of ΔABC (or ΔADC) = $\frac{1}{2}$ (Area of rectangle)

$= \frac{1}{2}(l \times b)$

In rectangle ABCD, AD is its length and CD is its breadth.

But look at ΔADC. Can you see that AD is a base and CD is its height?

So instead of saying:

Area of ΔADC or any other triangle = $\frac{1}{2}(l \times b)$

we say:

Area of a triangle $= \frac{1}{2}$ (base × height)

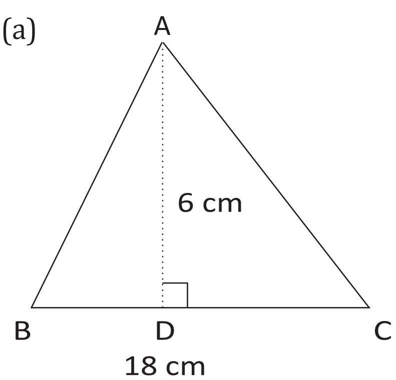
$= \frac{1}{2}(b \times h)$

In the formula for the area of a triangle, b means "base" and not "breadth", and h means perpendicular height.

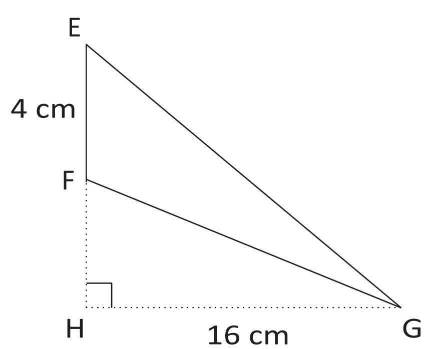
applying the area formula

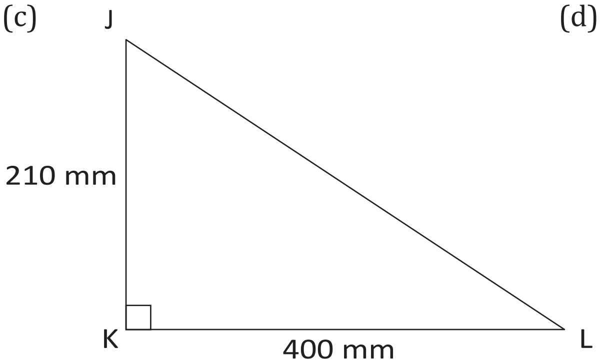
1. Use the formula to calculate the areas of the following triangles: ΔABC, ΔEFG, ΔJKL and ΔMNP.

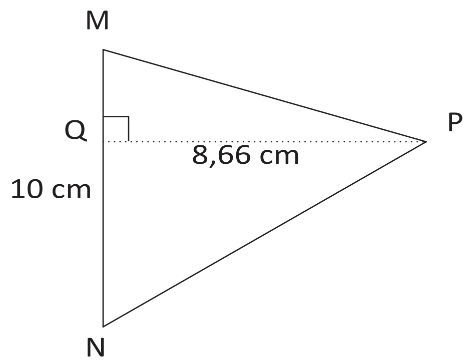
(a)



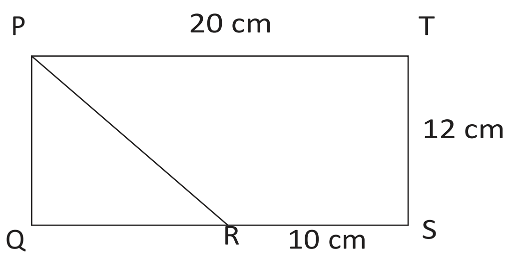
(b)

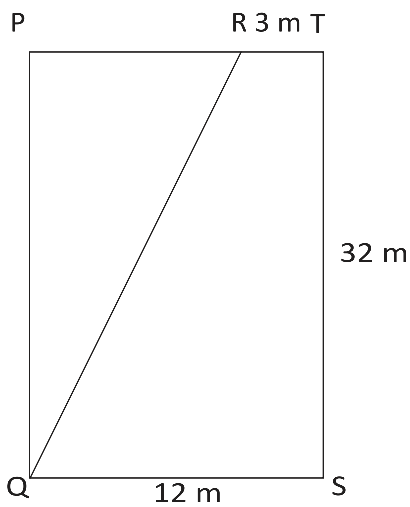


(c) 

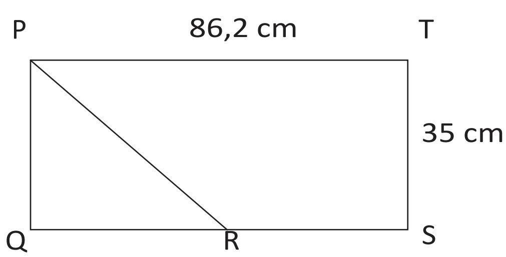
(d) 

2. PQST is a rectangle in each case below. Calculate the area of ΔPQR each time.

(a) 

(b) 

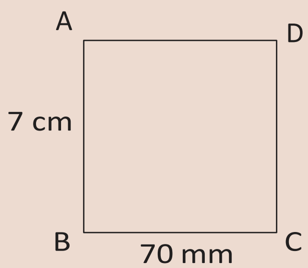
(c) R is the midpoint of QS.

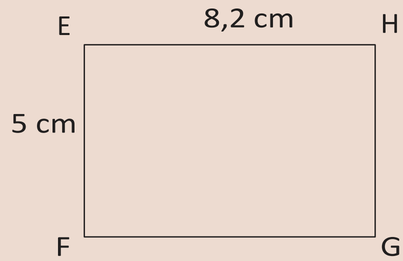


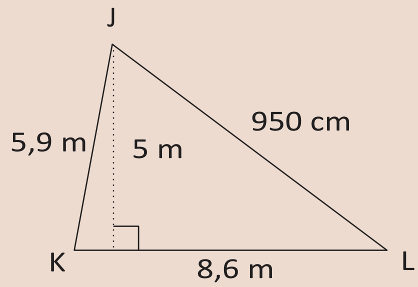
3. In ΔABC, the area is 42 m^2, and the perpendicular height is 16 m. Find the length of the base.

Worksheet

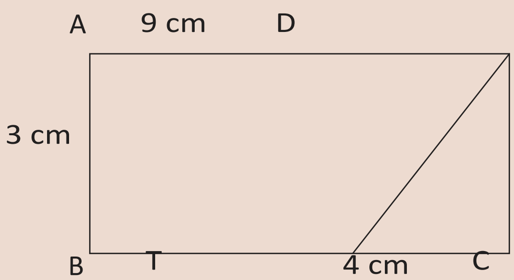
1. Calculate the perimeter (P) and area (A) of the following figures:







2. Figure ABCD is arectangle:
   AB = 3 cm, AD = 9 cm and TC = 4 cm.



   (a) Calculate the perimeter ofABCD.

   (b) Calculate the area ofABCD.

   (c) Calculate the area of ΔDTC.

   (d) Calculate the area of ABTD.

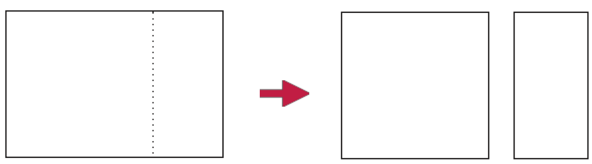
Chapter 10
Surface area and volume of 3D objects

10.1 Surface area of cubes and rectangular prisms

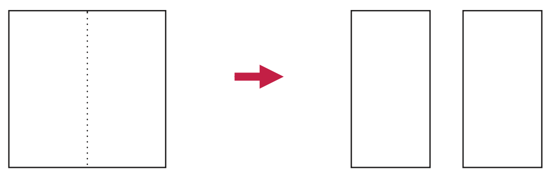
investigating surface area

1. Follow the instructions below to make a paper cube.

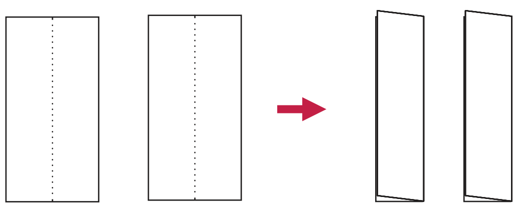
Step 1: Cut off part of an A4 sheet so that you are left with a square.



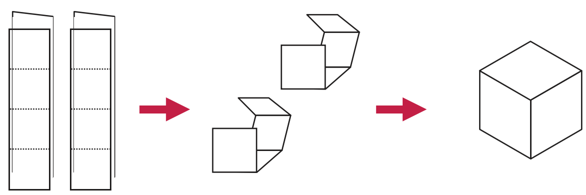
Step 2: Cut the square into two equal halves.



Step 3: Fold each half square lengthwise down the middle to form two double-layered strips.



Step 4: Fold each strip into four square sections, and put the two parts together to form a paper cube. Use sticky tape to keep it together.



2. Number each face of the cube. How many faces does the cube have?
3. Measure the side length of one face of the cube.
4. Calculate the area of one face of the cube.
5. Add up the areas of all the faces of the cube.

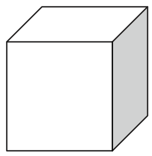
The **surface area** of an object is the sum of the areas of all its faces (or outer surfaces).

As for other areas, we measure surface area in square units, for example mm^2, cm^2 and m^2.

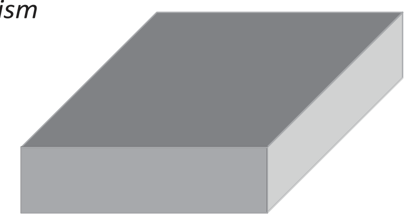
A cube has six identical square faces. A die (plural: dice) is an example of a cube.

A rectangular prism also has six faces, but its faces can be squares and/or rectangles. A matchbox is an example of a rectangular prism.

Cube



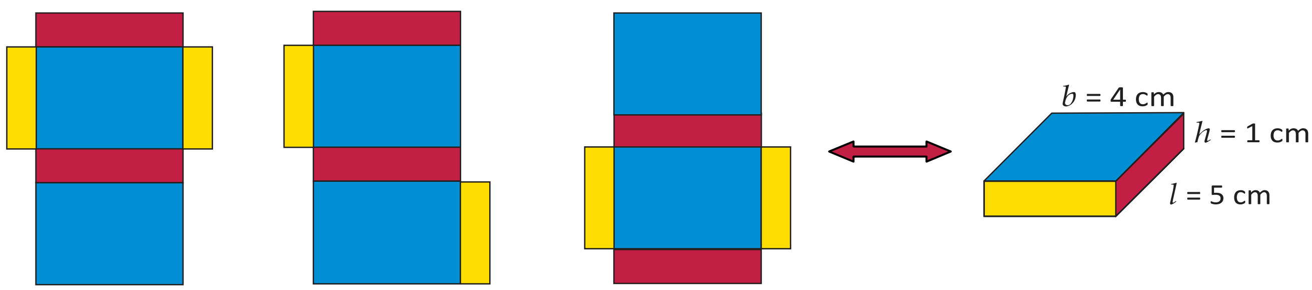
Rectangular prism

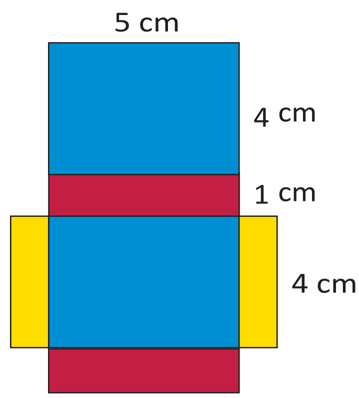


using nets of rectangular prisms and cubes

It is sometimes easier to see all the faces of a rectangular prism or cube if we look at its net. A **net** of a prism is the figure obtained when cutting the prism along some of its edges, unfolding it and laying it flat.

1. Take a sheet of paper and wrap it around a matchbox so that it covers the whole box without going over the same place twice. Cut off extra bits of paper as necessary so that you have only the paper that covers each face of the matchbox.
2. Flatten the paper and draw lines where the paper has been folded. Your sheet might look like one of the following nets (there are also other possibilities):

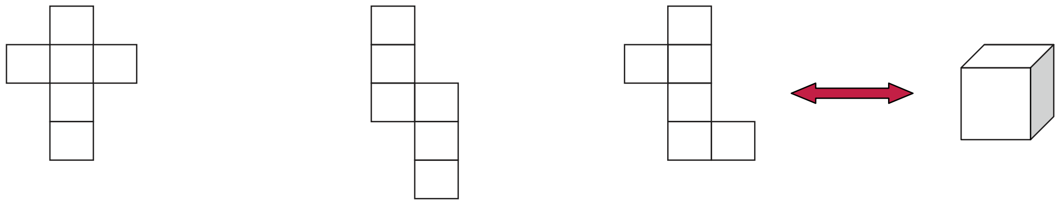




3. Notice that there are six rectangles in the net, each matching a rectangular face of the matchbox. Point to the three pairs of identical rectangles in each net.
4. Use the measurements given to work out the surface area of the prism. (Add up the areas of each face.)
5. Explain to a classmate why you think the following formula is or is not correct:

Surface area of a rectangular prism = $2(l \times b) + 2(l \times h) + 2(b \times h)$

6. Here are three different nets of the same cube.



(a) Can you picture in your mind how the squares can fold up to make a cube?

(b) If the length of an edge of the cube is 1 cm, what is the area of one of its faces? What then is the area of all its six faces?

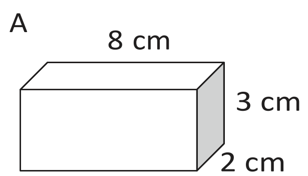
(c) Explain to a classmate why you think the following formula is or is not correct:

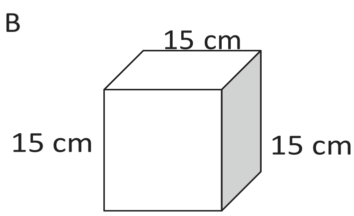
Surface area of a cube = $6(l \times l) = 6l^2$

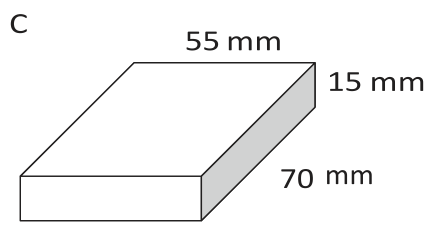
(d) If the length of an edge of the cube above is 3 cm, what is the surface area of the cube?

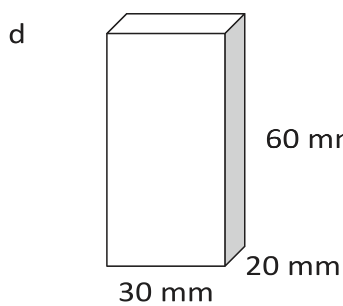
working out surface areas

1. Work out the surface areas of the following rectangular prisms and cubes.

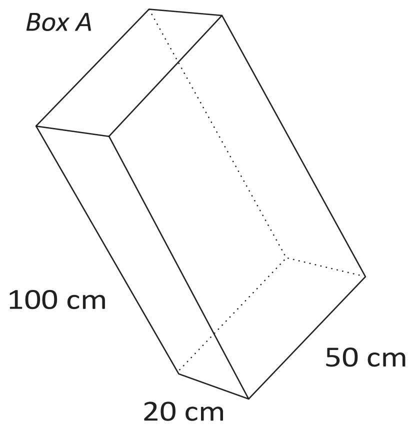


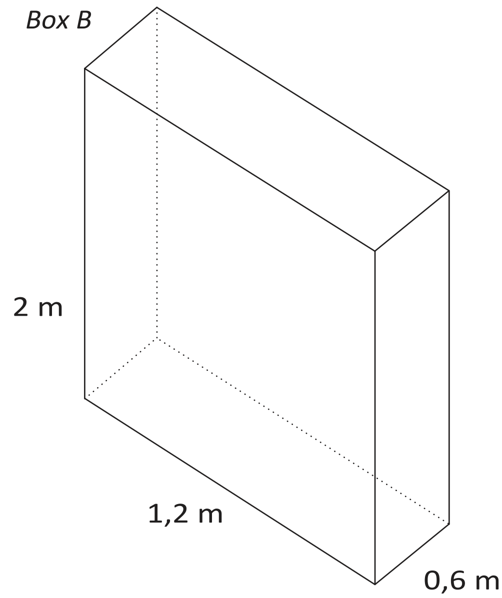




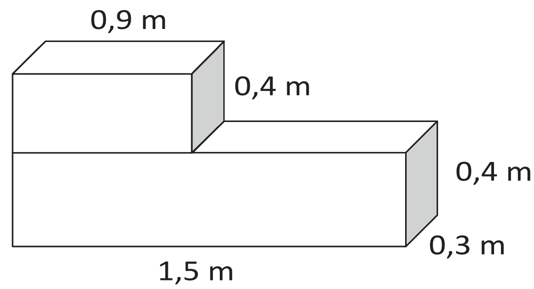


2. The two boxes on the following page are rectangular prisms. The boxes must be painted.
   (a) Calculate the total surface area of Box A and of Box B.
   (b) What will it cost to paint both boxes if the paint costs R1,34 per m^2?

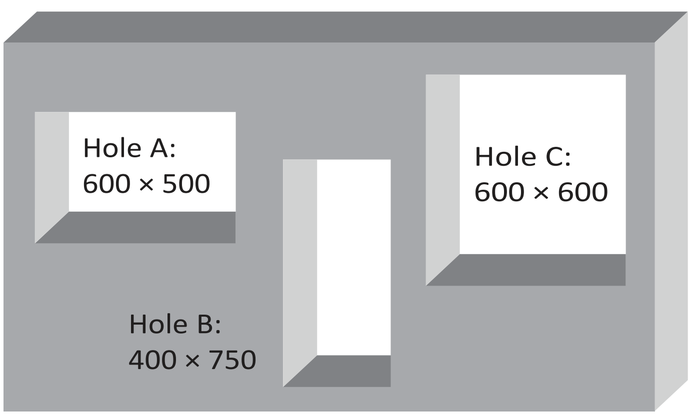




3. Two cartons, which are rectangular prisms, are glued together as shown. Calculate the surface area of this object. (Note which faces can be seen and which cannot be seen.)



4. This large plastic wall measures 3 m × 0,5 m × 1,5 m. It has to be painted for the Uyavula Literacy Project. The wall has three holes in it, labelled A, B and C, as shown. The holes go right through the wall. The measurements of the holes are in millimetres.



(a) Calculate the area of the front and back surfaces that must be painted.

(b) Calculate the area of the two side faces, as well as the top face.

(c) Calculate the total surface area of the wall, excluding the bottom and the inner surfaces where the holes are, because these will not be painted.

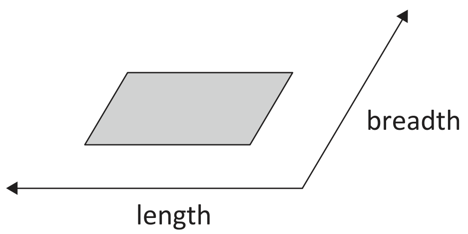
(d) What will it cost if the water-based paint costs R2,00 per m^2?

Remember from the previous chapter:
$1 \text{ cm}^2 = 100 \text{ mm}^2$
$1 \text{ m}^2 = 10\ 000 \text{ cm}^2$

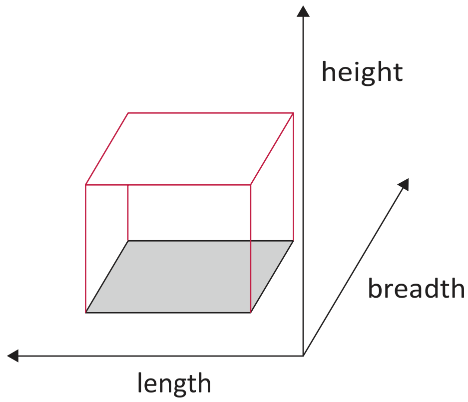
10.2 Volume of rectangular prisms and cubes

2D shapes are flat and have only two dimensions, namely length (l) and breadth (b). 3D objects have three dimensions, namely length (l), breadth (b) and height (h). You can think of a dimension as a direction in space. Look at these examples:

2D shape: rectangle



3D object: rectangular prism



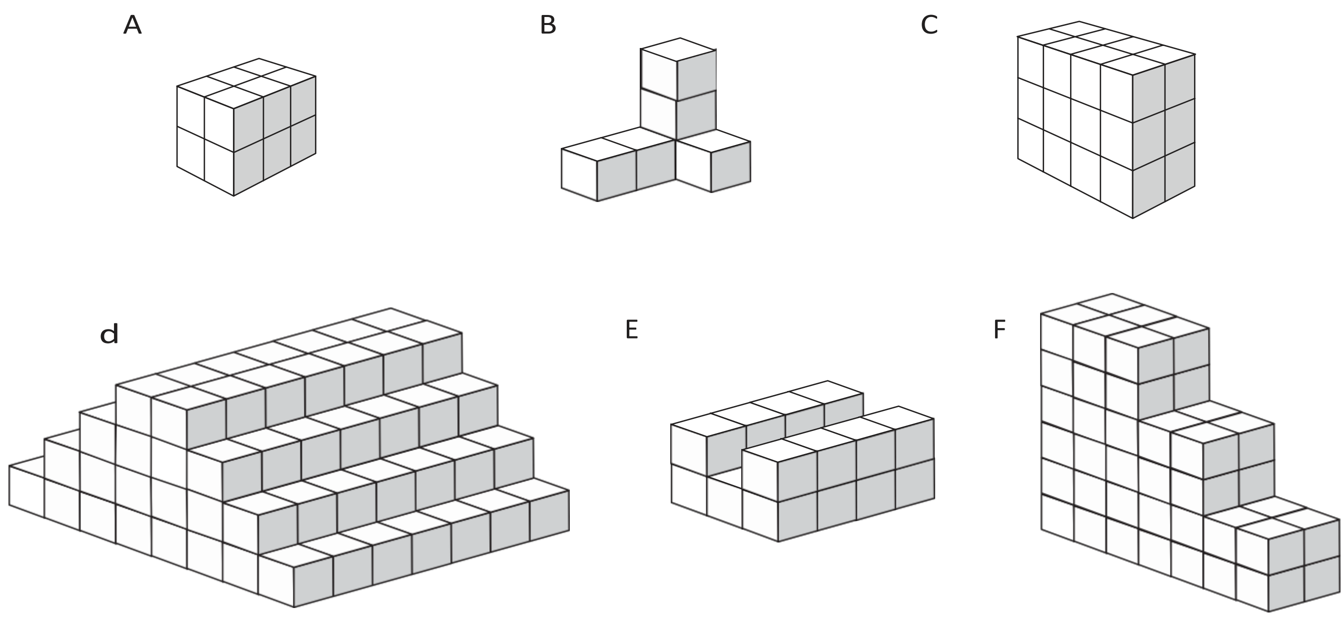
3D objects therefore take up space in a way that 2D shapes do not. We can measure the amount of space that 3D objects take up.

Every object in the real world is 3d. Even a sheet of paper is a 3d object. Its height is about 0,1 mm.

cubes to measure amount of space

We can use cubes to measure the amount of space that an object takes up.

1. Identical toy building cubes were used to make the stacks shown below.

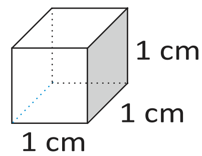


(a) Which stack takes up the least space?
(b) Which stack takes up the most space?
(c) Order the stacks from the one that takes up the least space to the one that takes up the most space. (Write the letters of the stacks.)

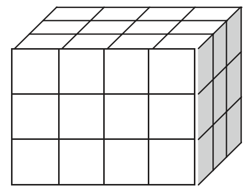
The space (in all directions) occupied by a 3D object is called its **volume**.

Cubes are the units we use to measure volume.

A cube with edges of 1 cm (that is, 1 cm × 1 cm × 1 cm) has a volume of one cubic centimetre (1 cm^3).

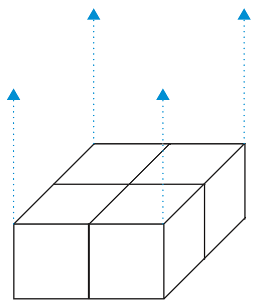


2. The figure on the right shows a rectangular prism made from 36 cubes, each with an edge length of 1 cm. The prism thus has a volume of 36 cubic centimetres (36 cm^3).

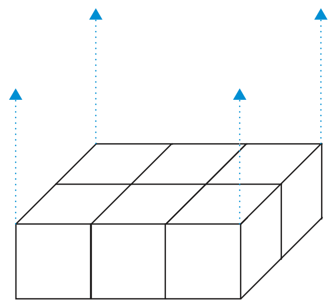


(a) The stack is taken apart and all 36 cubes are stacked again to make a new rectangular prism with a base of four cubes (see A below.) How many layers of cubes will the new prism be? What is the height of the new prism?

A



B

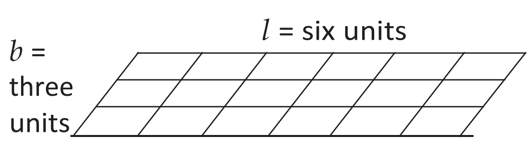


(b) Repeat (a), but this time make a prism with a base of six cubes (see B above).
(c) Which one of the rectangular prisms in questions (a) and (b) takes up the most space in all directions? (Which one has the greatest volume?)
(d) What will be the volume of the prism in question (b) if there are seven layers of cubes altogether?
(e) A prism is built with 48 cubes, each with an edge length of 1 cm. The base consists of eight layers. What is the height of the prism?

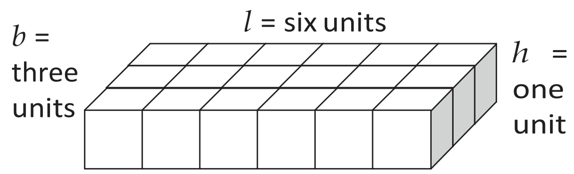
formula to calculate volume

You can think about the volume of a rectangular prism in the following way:

Step 1: Measure the area of the bottom face (also called the base) of a rectangular prism. For the prism given here: $A = l \times b = 6 \times 3 = 18$ square units.



Step 2: A layer of cubes, each one unit high, is placed on the flat base. The base now holds 18 cubes. It is 6 × 3 × 1 cubic units.

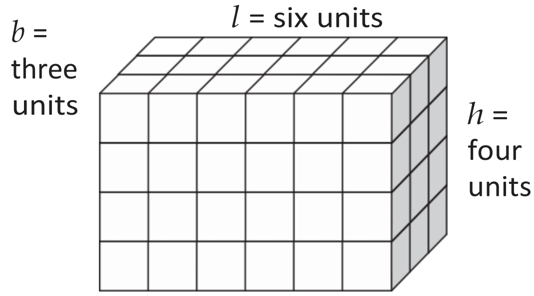


Step 3: Three more layers of cubes are added so that there are four layers altogether. The prism's height (h) is four units. The volume of the prism is:

$V = (6 \times 3) \times 4$

or V = Area of base × number of layers

$= (l \times b) \times h$



Therefore:

Volume of a rectangular prism = Area of base × height

$= l \times b \times h$

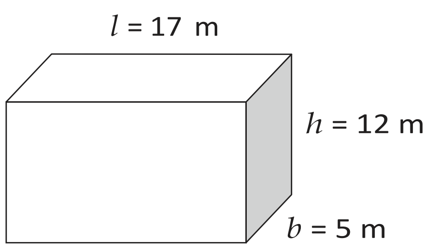
Volume of a cube $= l \times l \times l$ (edges are all the same length)

$= l^3$

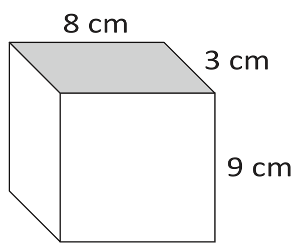
applying the formulae

1. Calculate the volume of these prisms and cubes.

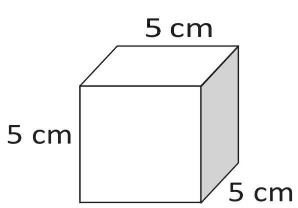
A



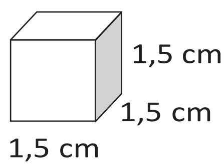
B



C



d



2. Calculate the volume of prisms with the following measurements:
   (a) l = 7 m, b = 6 m, h = 6 m
   (b) l = 55 cm, b = 10 cm, h = 20 cm
   (c) Surface of base = 48 m^2, h = 4 m
   (d) Surface of base = 16 mm^2, h = 12 mm

3. Calculate the volume of cubes with the following edge lengths:
   (a) 7 cm
   (b) 12 mm

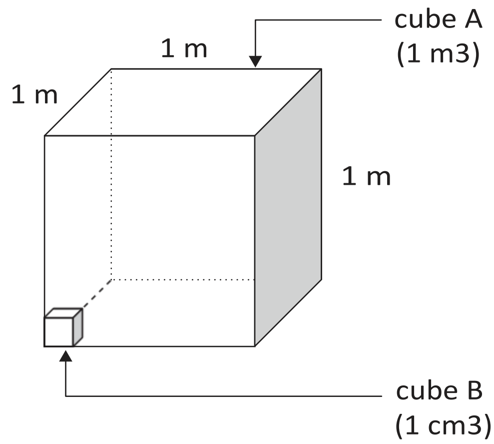
4. Calculate the volume ofthe following square-based prisms:
   (a) side of the base = 5 mm, h = 12 mm   (b) side of the base = 11 m, h = 800cm

5. The volume of a prism is 375 m^3. What is the height of the prism if its length is 8 m and its breadth is 15 m?

10.3 Converting between cubic units

cubic units to measure volume

This drawing shows a cube (A) with an edge length of 1 m. Also shown is a small cube (B) with an edge length of 1 cm.



How many small cubes can fit inside the large cube?

- 100 small cubes can fit along the length of the base of cube A (because there are 100 cm in 1 m).
- 100small cubes can fit along the breadth of the base of cube A.
- 100 small cubes can fit along the height of cube A.

Total number of 1 cm^3 cubes in 1 m^3 = 100 × 100 × 100
= 1 000 000
$\therefore$ 1 m^3 = 1 000 000 cm^3

Work out how many mm^3 are equal to 1 cm^3. Copy and complete:

1 cm^3 = 1 cm × 1 cm × 1 cm
= ___ mm × ___ mm × ___ mm
= ___ ___ mm^3

Cubic units:

1 m^3 = 1 000 000 cm^3
(multiply by 1 000 000 to change m^3 to cm^3)

1 cm^3 = 0,000001 m^3
(divide by 1 000 000 to change cm^3 to m^3)

1 cm^3 = 1 000 mm^3
(multiply by 1 000 to change cm^3 to mm^3)

1 mm^3 = 0,001 cm^3
(divide by 1 000 to change mm^3 to cm^3)

working with cubic units

1. Which unit, the cubic centimetre (cm^3) or the cubic metre (m^3), would be used to measure the volume of each of the following?
   (a) a bar of soap
   (b) a book
   (c) a wooden rafter fora roof
   (d) sand on atruck
   (e) a rectangular concrete wall
   (f) a die
   (g) water in a swimming pool
   (h) medicine in a syringe

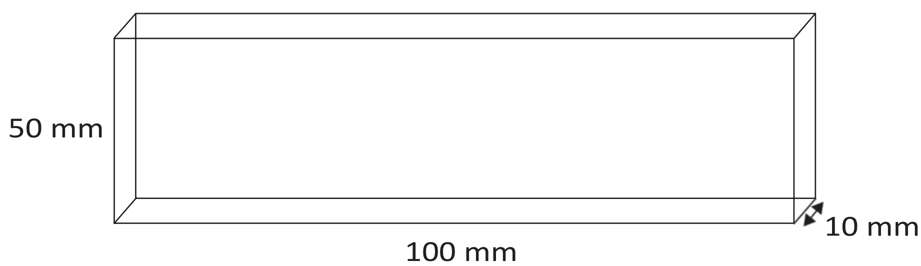
2. Write the following volumes in cm^3:
   (a) 1 000 mm^3
   (b) 3 000 mm^3
   (c) 2 500 mm^3
   (d) 4 450 mm^3
   (e) 7 824 mm^3
   (f) 50 mm^3

3. Write the following volumes in m^3:
   (a) 1 000 000 cm^3
   (b) 4 000 000 cm^3
   (c) 1 500 000 cm^3
   (d) 2 350 000 cm^3
   (e) 500 000 cm^3
   (f) 350 000 cm^3

4. Write the following volumes in cm^3:
   (a) 2 000 mm^3
   (b) 4 120 mm^3
   (c) 1,5 m^3
   (d) 34 m^3
   (e) 50000 mm^3
   (f) 2,23 m^3

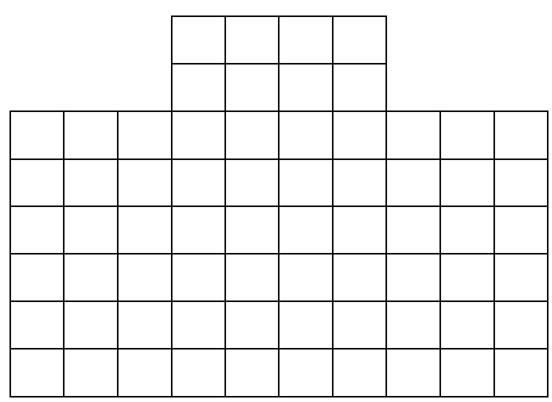
5. A rectangular hole has been dug for achildren's swimming pool. It is 7 m long, 4 m wide and 1 m deep. What is the volume of earth that has been dug out?

6. Calculate the volume of wood in the plank shown below. Answer in cm^3.



7. The drawing shows the base (viewed from below) of a stack built with 1 cm^3 cubes. The stack is 80 mm high everywhere.

   (a) What is the volume of the stack?
   (b) Copy and complete the following:
   Volume of stack = area of base ___ ___



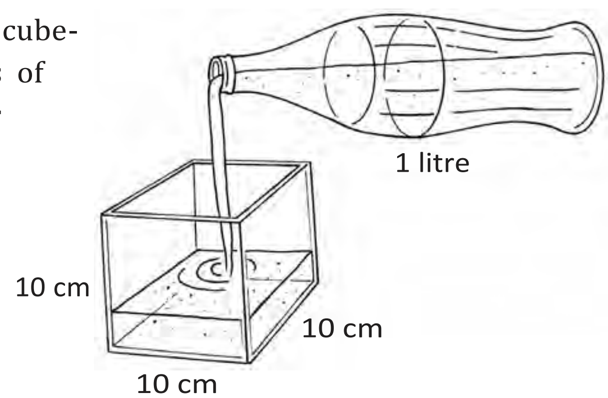
8. Calculate the volume of each of the following rectangular prisms:
   (a) length = 20 cm; breadth = 15 cm; height = 10 cm
   (b) length = 130 mm; breadth = 10 cm; height = 5 mm
   (c) length = 1 200 cm; breadth = 5,5 m; height = 3 m
   (d) length = 1,2 m; breadth = 2,25 m; height = 4 m
   (e) area of base = 300 cm^2; height = 150 mm
   (f) area of base = 12 m^2; height = 2,25 m

10.4 Volume and capacity

The space inside a container is called the internal volume, or **capacity**, of the container. Capacity is often measured in units of millilitres (ml), litres (ℓ) and kilolitres (kl). However, it can also be measured in cubic units.

equivalent units for volume and capacity

If the contents of a 1 ℓ bottle are poured into a cube-shaped container with internal measurements of 10 cm × 10 cm × 10 cm, it will fill the container exactly. Thus:



(10 cm × 10 cm × 10 cm) = 1 ℓ

or $\quad 1\,000 \text{ cm}^3 = 1\ \ell$

Since	$1\ \ell$	=	1 000 ml	
	$1\,000 \text{ cm}^3$	=	1 000 ml	[$1\ \ell = 1\,000 \text{ cm}^3$]
∴	1 cm^3	=	1 ml	[divide both sides by 1 000]

Since	1 kl	=	$1\,000\ \ell$	
		=	$1\,000 \times (1\,000 \text{ cm}^3)$	[$1\ \ell = 1\,000 \text{ cm}^3$]
		=	$1\,000\,000 \text{ cm}^3$	
		=	1 m^3	[$1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$]

This means that an object with a volume of 1 cm^3 will take up the same amount of space as 1 ml of water. Or an object with a volume of 1 m^3 will take up the space of 1 kl of water.

The following diagram shows the conversions in another way:

$$\begin{array}{ccccc} & \times 1\,000 & & \times 1\,000 & \\ 1\text{ ml} & \longrightarrow & 1\,\ell\text{ (or 1 000 ml)} & \longrightarrow & 1\text{ kl (or 1 000 }\ell) \\ 1\text{ cm}^3 & \longrightarrow & 1\,000\text{ cm}^3 & \longrightarrow & 1\text{ m}^3\text{ (or 1 000 000 cm}^3) \\ & \times 1\,000 & & \times 1\,000 & \end{array}$$

conversion is the changing of something into something else. In this case, it refers to changes between equivalent units of measurement.

From the diagram on the previous page, you can see that:

- $1\,\ell = 1\,000$ ml; 1 ml $= 0{,}001\,\ell$
- 1 kl $= 1\,000\,\ell$; $1\,\ell = 0{,}001$ kl
- 1 ml = 1 cm^3
- $1\,\ell = 1\,000$ cm^3
- 1 kl = 1 000 000 cm^3 or 1 m^3

Remember these conversions:
1 ml = 1 cm^3
1 kl = 1 m^3

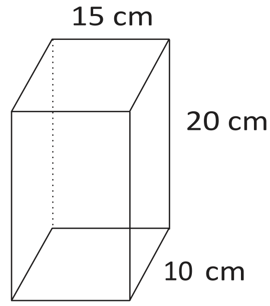
volume and capacity calculations

1. Write the following volumes in ml:
   (a) 2 000 cm^3 (b) 250 cm^3
   (c) $1\,\ell$ (d) $4\,\ell$
   (e) $2{,}5\,\ell$ (f) $6{,}85\,\ell$
   (g) $0{,}5\,\ell$ (h) 0,5 cm^3

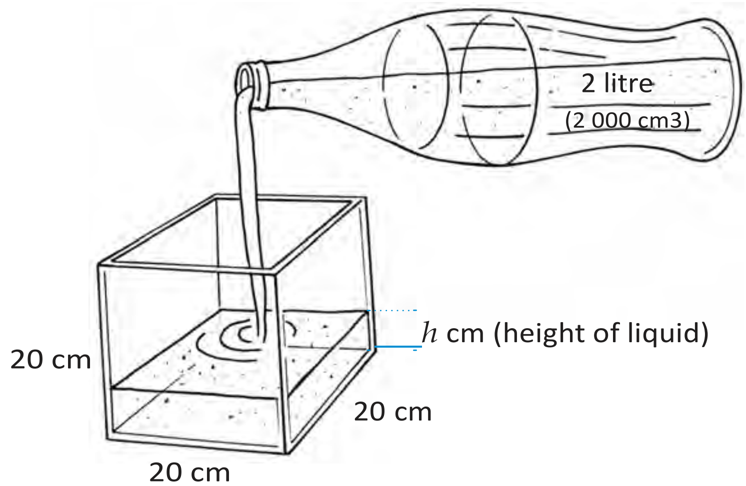
2. Write the following volumes in kl:
   (a) $2\,000\,\ell$ (b) $2\,500\,\ell$
   (c) 5 m^3 (d) 6 500 m^3
   (e) 3 000 000 cm^3 (f) 1 423 000 cm^3
   (g) $20\,\ell$ (h) $2{,}5\,\ell$

3. A glass can hold up to 250 ml of water. What is the capacity of the glass:
   (a) in ml? (b) in cm^3?

4. A vase is shaped like a rectangular prism. Its inside measurements are 15 cm × 10 cm × 20 cm. What is the capacity of the vase (in ml)?



5. A liquid is poured from a full 2 ℓ bottle into a glass tank with inside measurements of 20 cm by 20 cm by 20 cm.



   (a) What is the volume of the liquid when it is in the bottle?
   (b) What is the capacity of the bottle?
   (c) What is the volume of the liquid after it is poured into the tank?
   (d) What is the capacity of the tank?
   (e) How high does the liquid go in the tank?

In question 5 above, you should have found the following:

Volume of liquid in tank = Volume of liquid in bottle

$20 \times 20 \times h$ (liquid's height in tank) = 2 000 cm^3

$$h = \frac{2000}{(20 \times 20)}$$
$$= 5 \text{ cm}$$

Note: The capacity of the tank is 20 cm × 20 cm × 20 cm = 8 000 cm^3(8 ℓ). The volume of liquid in the bottle is 2 000 cm^3(2 ℓ).

Worksheet

1. Do the following unit conversions:

   (a) 2 348 cm^2 = ______ m^2
   (b) 5,104m^2 = ______ cm^2
   (c) 1 m^3 = ______ kl
   (d) 250 cm^3 = ______ ml = ______ ℓ
   (e) 0,5 kl = ______ ℓ = ______ ml
   (f) 6,850 ℓ = ______ ml = ______ cm^3

2. A rectangular prism measures 8 m × 4 m × 3 m. Calculate:

   (a) its surface area
   (b) its volume

3. A boy has 27 cubes, with edges of 20 mm. He uses these cubes to build one big cube.

   (a) What is the volume of the cube if he uses all 27 small cubes?
   (b) What is the edge length of the big cube?
   (c) What is the surface area of the big cube?

4. A glass tank has the following inside measurements: length = 250 mm, breadth = 120 mm and height = 100 mm. Calculate the capacity of the tank:

   (a) in cubic centimetres
   (b) in millilitres
   (c) in litres

5. Calculate the capacity of each of the following rectangular containers. The inside measurements have been given. Copy and complete the table.

	Length	Breadth	Height	Capacity
(a)	15 mm	8 mm	5 mm	cm^3
(b)	2 m	50 cm	30 cm	ℓ
(c)	3 m	2 m	1,5 m	kl

6. A water tank has a square base with internal edge lengths of 150mm. What is the height of the tank when the maximum capacity of the tank is 11 250cm^3?

Chapter 11
Numeric and geometric patterns

11.1 Number patterns in sequences

what comes next?

What may the next three numbers in each of these sequences be?

4; 8; 12; 16;20; ...   4; 8; 16; 32; 64;...

4; 8; 14; 22;32; ...   5; 7; 4; 8; 3; 9; 2;...

A set of numbers in a given order is called a **number sequence**. In some cases each number in a sequence can be formed from the previous number by performing the same or a similar action. In such a case, we can say there is a **pattern** in the sequence.

The numbers in a sequence are called the **terms** of the sequence. Terms that follow one another are said to be **consecutive**.

1. (a) Write down the next three numbers in each of these sequences:

    Sequence A: 4; 7; 10; 13; 16;...

    Sequence B: 5; 10; 20; 40; 80; ...

    Sequence C: 2; 5; 10; 17; 26; ...

   (b) Write down how you decided what the next numbers would be in each of the three sequences.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between one term and the next is constant.

A sequence can be formed by repeatedly multiplying or dividing by the same number. In this case the **ratio** between one term and the next is constant.

A sequence can also be formed in such a way that neither the difference nor the ratio between one term and the next is constant.

In sequence A of question 1 there is a **constant difference** between consecutive terms, as shown below.

Sequence A:	4	7	10	13	16
Difference:	+ 3	+ 3	+ 3	+ 3	

In sequence B of question 1 there is a **constant ratio** between consecutive terms, as shown below.

Sequence B:	5	10	20	40	80
Ratio:	× 2	× 2	× 2	× 2	

In sequence C of question 1 there is neither a constant difference nor a constant ratio between consecutive terms. There is, however, a pattern in the differences between the terms, which makes it possible to extend the sequence. Consecutive odd numbers, starting with 3, are added to form the next term.

Sequence C:	2	5	10	17	26
Difference:	+ 3	+ 5	+ 7	+ 9	

2. Write down the next five terms in each of the sequences below. In each case, describe the relationship between consecutive terms.
   (a) 100; 95; 90;85; ...
   (b) 0,3; 0,5; 0,7; 0,9; ...
   (c) 6; 18; 54; 162; ...
   (d) 1; 3; 6; 10; 15; ...
   (e) 20; 31; 42;53; ...
   (f) 10; 9,7; 9,4; 9,1;...
   (g) 18 000; 1 800; 180; 18; ...
   (h) $\frac{1}{48}$; $\frac{1}{24}$; $\frac{1}{12}$; $\frac{1}{6}$; ...
   (i) 1; 4; 9; 16; ...
   (j) 625; 125; 25;5;...

In all of the above cases it was possible to extend the sequence by repeatedly adding or subtracting a number to get the next term, or by repeatedly multiplying or dividing by a number to get the next term, or by adding different numbers according to some pattern to get the next term.

The word "recur" means "to happen again". The extension of a number sequence by repeatedly performing the same or similar action is called **recursion**. The rule that describes the relationship between consecutive terms is called a **recursive rule**.

relationships between dependent and independent variables

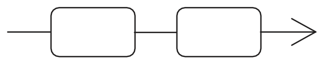
1. (a) Mr Twala pays a fee to park his car in a parking lot every day. He has to pay R3 to enter the parking lot and then a further R2 for every hour that he leaves his car there. Copy and complete the table on the following page to show how much his parking costs him per day for various numbers of hours.

Number of hours	1	2	3	4	5	6	7	8	9
Cost of parking in R	5	7	9						

(b) How did you complete this table? Describe your method.

(c) Is there another way that you could complete the table? Describe it.

(d) Thembi multiplied the number of hours by 2 and then added 3 to calculate the cost for any specific number of hours. Copy and complete the flow diagram to show Thembi's rule.



The R3 that is added is a **constant** in this situation. The number of hours and the cost are **variables**.

The rule *multiply by 2 and then add 3* describes the relationship between the two variables in this situation. The number of hours is the **independent variable**. The cost of Mr Twala's parking is the **dependent variable** because the amount he has to pay *depends on* the number of hours that he parks.

This rule describes how you can calculate the value of the *dependent* variable if the corresponding value of the *independent* variable is known. It differs from a recursive rule, which describes how you can calculate the value of the *dependent* variable that follows on a given value of the *dependent*variable.

In the case of a number sequence, the **position** (number) of the term can be taken as the independent variable, as shown for the sequence 15; 19; 23; 27; 31; . . . in this table:

Term number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31				

2. (a) Copy and complete the above table.

(b) How did you calculate term number50?

(c) Lungile reasoned like this:

I added 4 each time to complete the table. I counted backwards to see what comes before term 1. I got 11 and then I knew I had to add one 4 to 11 to get the first term.

Lungile remembered that multiplication is done before addition, unless otherwise indicated by brackets.

Complete the pattern below to show Lungile's thinking:

Term 1: $11 + 1 \times 4 = 11 + 4 = 15$ Term 2: $11 + 2 \times 4 = 11 + 8 = 19$

Term 3: ___ Term 4: ___

Term 5: ___ Term 6: ___

Term 10: ___ Term 50: ___

(d) Describe in your own words how term number 50 can be calculated.

(e) Tilly reasoned like this: *The constant difference between the terms is 4. I must add four 49 times to the first term to get the fiftieth term. So, 15 + 49 × 4 = 15 + 196 = 211.*
Complete the pattern below to demonstrate Tilly's thinking:

Term 1: 15 Term 2: 15 + 1 × 4 = 15 + 4 = 19

Term 3: 15 + 2 × 4 = 15 + 8 = 23 Term 4: ____

Term 5: ____ Term 6: ____

Term 10: ____ Term 50: ____

(f) Write the rule to calculate term number 50 in your own words.

In the example in question 2, the term number is the independent variable and the term itself is the dependent variable. So, if we know the rule that links the dependent variable and the independent variable, we can use it to determine any term for which we know the term number.

3. Write a rule to calculate the term for any term number in the sequence 15; 19; 23; 27; 31; . . . by using:
   (a) Lungile's thinking (b) Tilly's thinking

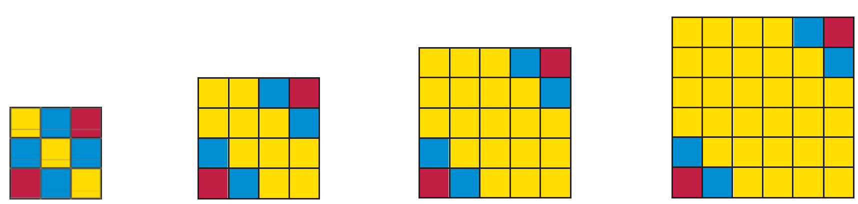
We can use n as a symbol for "any term number".
The rule to calculate the term for any term number when using Lungile's thinking will then be:
Term = $n \times 4 + 11$

   (c) Write down the rule to calculate the term for any term number in terms of n by using Tilly's thinking.

11.2 Geometric patterns

constant quantities and variable quantities

Small yellow, blue and red tiles are combined to form larger square tiles as shown below:



Tile no. 1 *Tile no. 2* *Tile no. 3* *Tile no. 4*

1. Draw tile no. 5 on grid paper. (Shade the blue and red tiles in different ways. You don't have to use colours.)

2. Copy and complete the table.

	Tile no. 1	Tile no. 2	Tile no. 3	Tile no. 4	Tile no. 5	Tile no. 10
Number of yellow tiles						
Number of red tiles						
Number of blue tiles						

3. How many red tiles are there in each bigger tile?

4. How many yellow tiles are there in each bigger tile?

5. Some of the quantities in this situation are variables and some are constants. Which are variables and which are constants?

6. Was it possible to predict the pattern on tile no. 2 by looking only at tile no. 1?

The number of red tiles is constant and the number of blue tiles is constant. It is clear that the design is such that there is always a red tile in the top right corner, and also in the bottom left corner, and that the red tiles are always "bordered" by two blue tiles each. So the number of red and blue tiles is **constant** in this situation.

The number of yellow tiles in the arrangements varies. The number of yellow tiles is a **variable** in this situation.

patterns with matches

1. A pattern with matches is shown below.

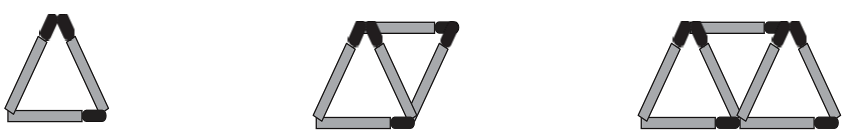


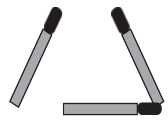
Figure 1 *Figure 2* *Figure 3*

(a) Explain how the pattern is formed.

(b) Copy and complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	3	5	7					

(c) What rule did you use to complete the table?
(d) How many matches are needed to form Figure number 9?
(e) How many matches are needed to form Figure number 17? Explain.
(f) If you used the recursive rule to complete the table, it would have taken a long time to answer question (e) because you had to add the same number many times. Try to find an easier way to answer question (e). Describe your method.
(g) Copy and complete the pattern below.
Hint: It may help to think of Figure number 1 or term 1 like this: There is one match at the beginning and two more are added every time. It helps to "see" the two matches that are added each time.



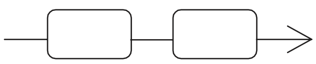
Term 1: $1 + 1 \times 2 = 3$ Term 2: $1 + 2 \times 2 = 5$

Term 3: $1 + 3 \times 2 = 7$ Term 4: ___

Term 5: ___ Term 10: ___

Term 17: ___

(h) What stays the same in the pattern in (g) and what varies?
(i) Copy the flow diagram below and use it to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure number  *Number of matches*

(j) Can you link the number of matches added each time to the number that you multiply by in the flow diagram? Explain.

2. Another pattern with matches is shown below.

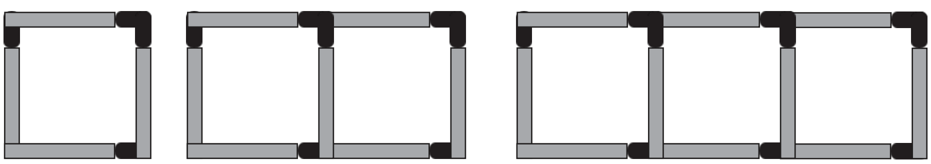


Figure 1 *Figure 2* *Figure 3*

(a) Explain how the pattern is formed.
(b) Copy and complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	4							

(c) What rule did you use to complete the table?
(d) How many matches are needed for Figure 9 (or term 9)?

(e) How many matches are needed for Figure 20 (or term 20)?

(f) What rule did you use to calculate the number of matches in question (e)?

(g) Copy and complete the pattern:

Term 1: $1 + 1 \times 3 = 4$ Term 2: $1 + 2 \times 3 = 7$

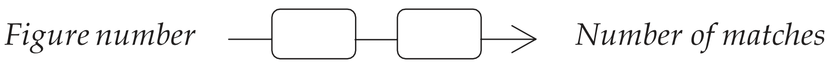
Term 3: $1 + 3 \times 3 =$ ___ Term 4: ___

Term 5: ___ Term 10: ___

Term 17: ___

(h) What stays the same in the pattern in (g) and what varies?

(i) Copy the flow diagram below and use it to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.



3. Compare the way in which the number of matches increases in question 1 to the way in which it increases in question 2. What is the same and what is different?

alphabetic patterns

Consider the figures below formed with red dots.

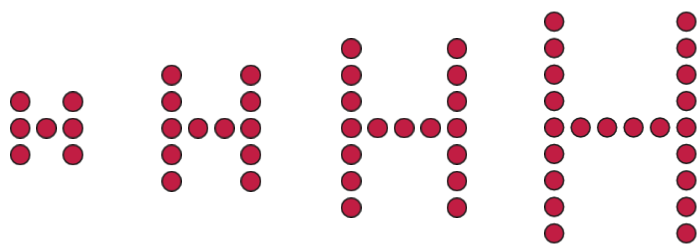


Figure 1 *Figure 2* *Figure 3* *Figure 4*

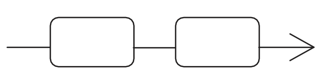
1. How many dots are used to form Figure 5?

2. Draw Figure 5.

3. Copy and complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of dots	7	12	17					

4. Copy and complete the flow diagram.



5. What rule did you use to complete the table? Describe your rule.
6. Can you think of another rule to complete the table? Describe your rule.
7. Name the dependent variable and the independent variable in this situation.

squares and cubes

1. Squares are arranged to form figures as shown below, according to a rule.

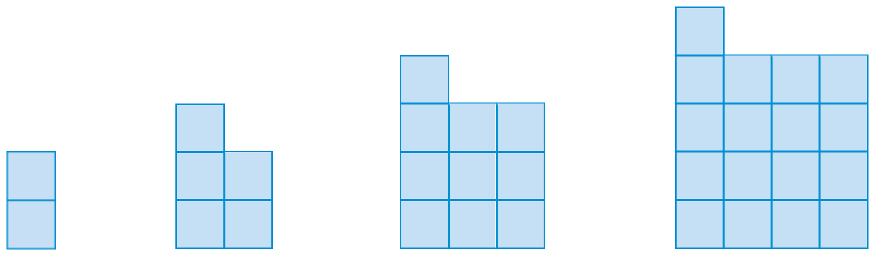


Figure 1 *Figure 2* *Figure 3* *Figure 4*

(a) Copy and complete the table. Then determine the differences between consecutive terms.

Figure number	1	2	3	4	5	6	7	8
Number of squares	2	5						

+ 3

(b) Describe the recursive rule that you can use to extend the pattern in words.

(c) Nombuso played around with the differences between consecutive terms. She noticed that the pattern (+ 3; + 5; + 7; ...) was similar to the one that you get when you calculate the differences between square numbers. This made her think that she should investigate square numbers to help her find a rule that could link the figure number and the number of squares.
Complete the following pattern along the lines of Nombuso's thinking:

Figure 1: $1 \times 1 + 1 = 1 + 1 = 2$ Figure 2: $2 \times 2 + 1 = 4 + 1 = 5$

Figure 3: ___ Figure 4: ___

Figure 5: ___ Figure 6: ___

Figure 7: ___ Figure 8: ___

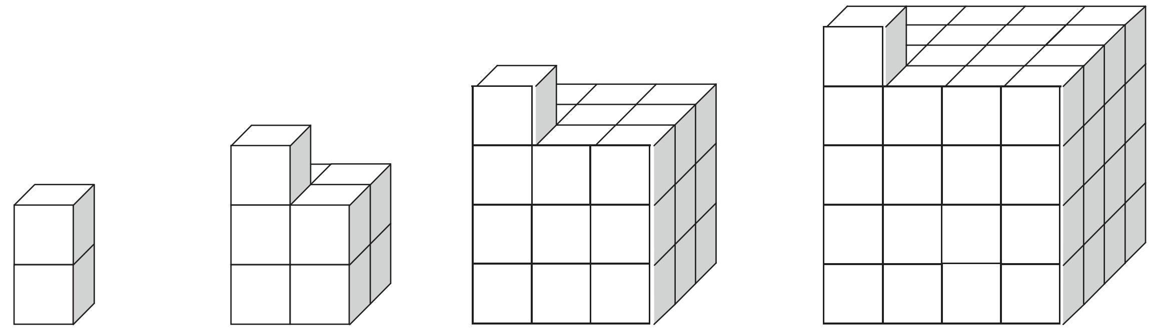
Figure 50: ___

(d) Write a rule to calculate the number of squares for any figure number.

(e) Write your rule in (d) in terms of n where n is the symbol for any figure number.

(f) Compare the sequence in this activity to the sequence in the previous activity where dots were arranged to form the letter H. Describe the way in which the dependent variable (the output number) changed in each of the sequences.

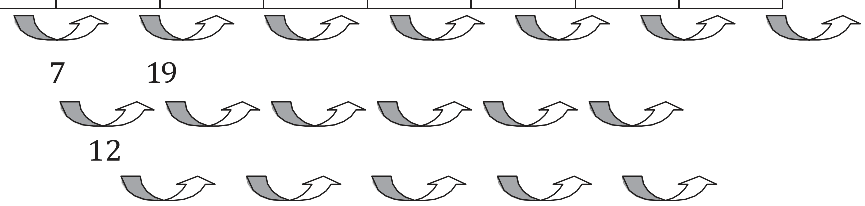
2. Identical cubes are arranged to form stacks of cubes in the following way:



Stack 1 *Stack 2* *Stack 3* *Stack 4*

(a) Copy and complete the table and the arrows. Then find the differences between consecutive terms. Do it a second and a third time. Write the differences below the arrows.

Stack number	1	2	3	4	5	6	7	8
Number of cubes	2	9	28					



(b) Describe the way in which you completed the table.

(c) David looked carefully at the structure of the stacks and did the following to link the stack number with the number of cubes in a stack. Complete the pattern.

Stack 1: $1 \times 1 \times 1 + 1 = 1 + 1 = 2$

Stack 2: $2 \times 2 \times 2 + 1 = 8 + 1 = 9$

Stack 3: $3 \times 3 \times 3 + 1 = 27 + 1 = 28$

Stack 4: $4 \times 4 \times 4 + 1 = 64 + 1 = 65$

Stack 5:

Stack 6:

Stack 7:

Stack 8:

Stack 9:

Stack 10:

(d) How many cubes will there be in stack 50?

(e) Write the rule that you used to calculate the number of cubes in stack 50 in words.

(f) Write your rule in (e) in terms of n where n is the symbol for any stack number.

3. In questions 1(a) and 2(a) you calculated the differences between the consecutive terms.

(a) What did you find when you kept on finding the differences, as suggested in question 2(a)?

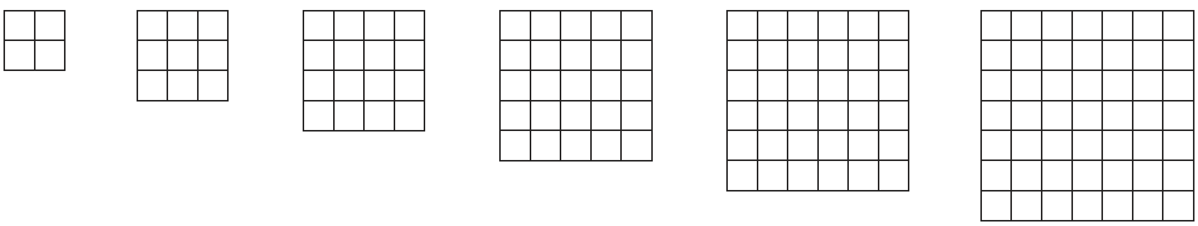
(b) Go back to question 1(a). What do you find when you keep on finding the differences between consecutive terms, like you did in question 2(a)?

my own patterns

Copy the grid, the tables and the tile template to create and describe your own geometric patterns.

Pattern A

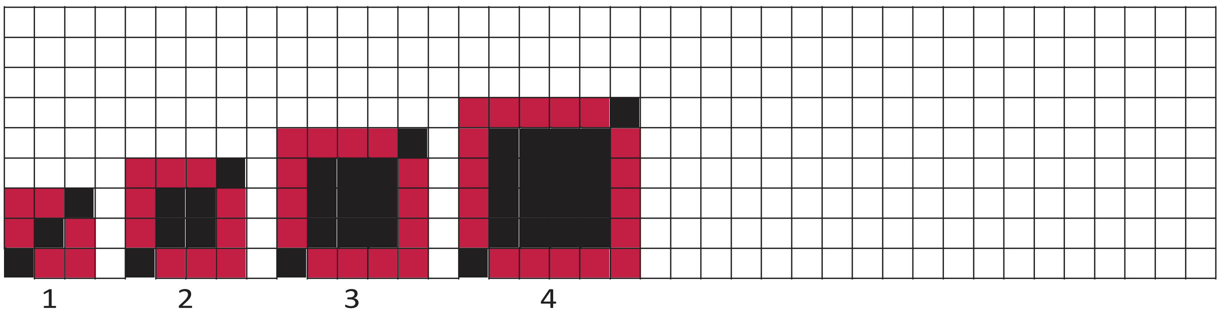
Pattern B



Chapter 12
Functions and relationships 1

12.1 From counting to calculating

1. (a) How many red squares and how many black squares are there in each of the arrangements 1, 2, 3 and 4 below? Copy and complete the table.



Arrangement number	1	2	3	4			
Number of red squares							
Number of black squares							

   (b) Imagine that arrangements 5, 6 and 7 are made according to the same pattern. How many red and how many black squares do you think there will be in each of these arrangements? Write your answers in the table you copied.
   (c) Complete arrangements 5 and 6 on grid paper, if you have not done so already.
   (d) Try to figure out how many red and how many black squares there will be in arrangements 20, 21 and 22.

2. It will be useful to have formulae to calculate the numbers of red and black squares in different arrangements like the one above.
   (a) Which of the formulae below can be used to calculate the numbers of red squares in the above arrangements? There is more than one formula that works.
   $y = 2 \times x + 4$ $\quad$ $y = 2 \times (2 \times x + 1)$ $\quad$ $y = x^2 + 2$ $\quad$ $y = 4 \times x + 2$
   (b) Natasha decided to use the formula $y = 4 \times x + 2$ to calculate the number of red squares in an arrangement. What do the symbols x and y mean in this case?
   (c) Use the formula $y = 4 \times x + 2$ to calculate the numbers of red squares in arrangements 20, 21 and 22.

(d) If your answers differ from the answers you gave in question 1(d), you have made mistakes somewhere. Find your mistakes and correct them.

(e) Copy and complete the table.

x	1	2	3	4	5	6	7	8	9
$2 \times (2 \times x + 1)$									
$2 \times x + 4$									
$4 \times x + 2$									

3. (a) Which of the formulae below can be used to calculate the numbers of black squares in the arrangements in question 1?

$z = (x + 2)^2$ $\quad$ $z = x^2 + 2$ $\quad$ $p = n^2 + 2$

(b) Copy and complete the table.

x	1	2	3	4	5	6	7	8	9
$x^2 + 2$									
$(x + 2)^2$									

4. Hilary uses x to represent the *number of squares in each side* of the arrangements.

(a) Which of these formulae can Hilary use to calculate the numbers of black squares in the arrangements in question 1?

$y = x^2 - 4 \times x + 6$ $\quad$ $y = (x - 2)^2 + 2$

(b) Which of these formulae can Hilary use to calculate the numbers of red squares?

$y = 3 \times x - 3$ $\quad$ $y = 4 \times x - 6$ $\quad$ $4 \times (x - 2) + 2$

(c) Copy and complete this table to check your answers.

x	3	4	5	6	7	8	9
$x^2 - 4 \times x + 6$							
$(x - 2)^2 + 2$							
$3 \times x - 3$							
$4 \times x - 6$							
$4 \times (x - 2) + 2$							

12.2 What to calculate and how

representing situations mathematically

1. (a) How many minutes are there in an hour?

(b) How many minutes are there in two hours?

(c) How many minutes are there in three hours?

(d) Explain how you determined the answers for questions 1(b) and (c).

The formula $m = 60 \times h$ can be used to calculate the number of minutes when the number of hours is known. The symbol h represents the number of hours and m the number of minutes.

(e) Express the formula $m = 60 \times h$ in words.

(f) Copy and complete the table.

Number of hours	1	2	3	15	24
Number of minutes	60	120			
How to calculate	60 × 1				

2. Three bus companies placed the following advertisements in a newspaper:

(a) Which of the formulae below can be used to calculate the fare for a journey with Hamba Kahle Tours?

(b) Which of the formulae can be used to calculate the fare for a journey with Saamgaan Tours?

saamgaan tours
We criss-cross every province and stop in every town and dorpie. Pay only R450 per trip plus 60c per km.

hamba kahle tours
Long distance travel is our business: R500 per trip plus 50c per km!

comfort tours
Experience what it means to travel in style. Only R480 per trip plus 55c per km.

Some formulae to calculate fares:

A. Fare = 0,50 × distance + 500
B. Fare = 50 × distance + 500
C. Fare = 0,60 × distance + 450
D. Fare = 60 × distance + 450
E. Fare = 55 × distance + 480
F. Fare = 0,55 × distance + 480

(c) Which of the above formulae can be used to calculate the fare for a journey with Comfort Tours?

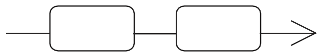
We write 50c as R0,50 or 0,50 when we do calculations.

(d) Copy and complete the table by making use of the formulae below. You may use a calculator for this question.
Fare for Hamba Kahle Tours = 0,50 × distance + 500
Fare for Saamgaan Tours = 0,60 × distance + 450
Fare for Comfort Tours = 0,55 × distance + 480

Distance in km	150	200	250	300
Hamba Kahle Tours				
Saamgaan Tours				
Comfort Tours				

(e) Which bus company is the cheapest? Explain.

(f) Copy and complete this flow diagram for the bus company that you named in question (e):



(g) Wandile wrote the formulae for calculating the fares for the different bus companies using the letter symbols x and y. Say what each letter symbol stands for in each of the following:

(i) $y = 0{,}50 \times x + 500$

(ii) $y = 0{,}60 \times x + 450$

(iii) $y = 0{,}55 \times x + 480$

(h) Which of the three bus companies would be the cheapest to use for a journey of 1 000 km?

12.3 Input and output numbers

from formulae to tables

1. For each of the tables (a) to (f) below, determine which of the following formulae could have been used to complete it:

A. $y = 5 \times x + 3$ B. $y = 3 \times x$ C. $y = 3 \times x + 2$

D. $y = 4 \times x$ E. $y = 3 \times x + 1$ F. $y = 2 \times x$

G. $y = 3 \times x + 10$ H. $y = 2 \times x - 1$ I. $y = 5 \times x$

(a)

x	1	2	3	4	5
y	13	16	19	22	25

(b)

x	1	2	3	4	5
y	8	13	18	23	28

(c)

x	1	2	3	4	5
y	4	8	12	16	20

(d)

x	1	2	3	4	5
y	5	8	11	14	17

(e)

x	1	2	3	4	5
y	5	10	15	20	25

(f)

x	1	2	3	4	5
y	1	3	5	7	9

We can complete a table of values if we are given a formula. For example, for the formula $y = 7 \times x - 3$ we can complete the table below, as shown:

For $x = 1$, $y = 7 \times 1 - 3$
$= 7 - 3$
$= 4$

For $x = 2$, $y = 7 \times 2 - 3$
$= 14 - 3$
$= 11$

For $x = 3$, $y = 7 \times 3 - 3$
$= 21 - 3$
$= 18$

x	1	2	3	4	5	6	7
y	4	11	18	25	32	39	46

2. Copy and complete the tables using the given formulae.

(a) $y = 6 \times x - 5\frac{1}{2}$

x	1	2	3	4	5	6	7
y							

(b) $y = 30 \times x + 1$

x	0,1	0,2	0,3	0,4	0,5	0,6	0,7
y							

from patterns to formulae

1. Some arrangements with black and red squares and some formulae are given below.

Formula A: $z = 2 \times n + 1$ Formula B: $z = 2 \times x - 3$

Formula C: $y = (n + 1)^2 + 2$ Formula D: $y = x^2 - (2 \times x - 3)$

(a) How many black squares will there be in the next two similar arrangements?

(b) Susan uses formulae B and D to calculate the numbers of red and black squares. What do the letter symbols z, x and y mean in Susan's work?

(c) Zain uses formulae A and C to calculate the numbers of red and black squares. What do the letter symbols z, n and y mean in Zain's work?

2. Write formulae that can be used to calculate the numbers of red and black squares in arrangements like those below. Use letter symbols of your own choice and state clearly what each of your symbols represents.

Chapter 13
Algebraic expressions 1

13.1 Describing and doing computations

different ways of describing a computation

1. The diagrams below represent arrangements of small circles. In every arrangement there are two rows of circles.

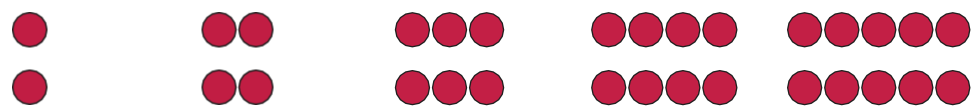


Diagram 1 *Diagram 2* *Diagram 3* *Diagram 4* *Diagram 5*

(a) The table below relates to the diagrams. Copy and complete it.

Diagram number	1	2	3	4	5
Number of circles per row					
Number of rows					
How to calculate the total number of circles per diagram (**rule**)					

In every diagram, we can identify:
- the number of rows
- the number of circles per row
- the total number of circles per arrangement.

(b) What remains the same in the diagrams above?

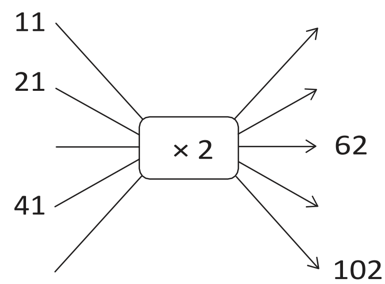
(c) What changes in the diagrams? In other words, what are the variable quantities in the situation?

(d) Copy and complete the flow diagram on the right.

(e) How many circles will diagram 11 have if the pattern is extended? Explain.

(f) What does the number 2 in the rule $2 \times n$ represent?

(g) What does the letter symbol n represent in the rule $2 \times n$?



The rule $2 \times n$ can be used to determine the total number of circles in a diagram. The number 2 in the rule $2 \times n$ remains the same all the time. We say it is a **constant**. The letter symbol n represents the number of circles per row and that is a **variable**, because it changes.

Consider the sequence 1; 3; 5; 7; 9; ...

The first odd number can be written as $2 \times \mathbf{1} - 1$.

The second odd number can be written as $2 \times \mathbf{2} - 1$.

The third odd number can be written as $2 \times \mathbf{3} - 1$.

The numbers 2 and −1 remain the same all the time; we call them **constants**. The numbers in blue change according to the position of the odd number in the sequence. We call them **variables**.

2. (a) What is the tenth odd number?
   (b) What is the thirtieth odd number?
   (c) What is the hundredth odd number?
   (d) What is the nth odd number?

3. The rule $2 \times n - 1$ can be used to determine any odd number in the sequence 1; 3; 5; 7; 9; ...

   What does the letter symbol n represent in the rule $2 \times n - 1$?

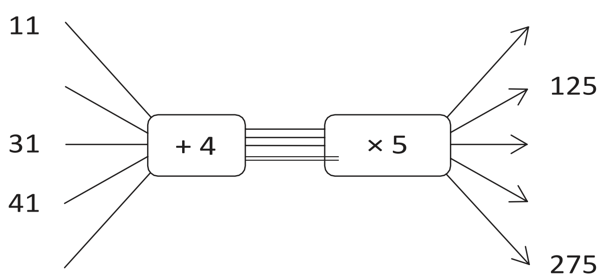
In the questions above we have used the letter symbol n to represent:

- a changing number in the rule $2 \times n$ (n represents the number of circles in a row)
- the position of the odd number in a sequence in the rule $2 \times n - 1$.

The rule $\mathbf{2 \times n}$ can be used to calculate the total number of circles in a diagram if the number of circles per row is known.

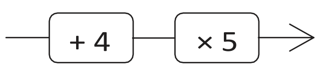
The rule $\mathbf{2 \times n - 1}$ can be used to determine any odd number in the sequence of odd numbers if its position is known.

4. (a) Copy and complete the flow diagram.



We call the numbers on the left in the flow diagram the **input numbers**.

The numbers on the right in the flow diagram, and whose values depend on the input numbers, are called the **output numbers**.

   (b) Which of the following instructions did you follow to calculate the output values of  in question 4(a)?

   Write out and place a tick mark (✓) next to the correct answer.
   A. Multiply the input number by 5 and then add 4.
   B. Add 45 to the input number.
   C. Add 4 to the input number and then multiply by 5.

5. Use 10 as the input number and calculate the output number for each of the word formulae in question 4(b).

We may write $(x + 4) \times 5$ as an abbreviation for *add 4 to the input number, then multiply by 5.*
$(x + 4) \times 5$ can be called a computational instruction or an **algebraic expression**.

The letter symbol x, or any other symbol, can be used as an abbreviation for "the input number".

In the expression $(x + 4) \times 5$, the letter symbol x can be replaced by many different input numbers. The symbol x represents a **variable quantity** or a **variable**. If, however, the expression $(x + 4) \times 5$ is equal to 35, as in the number sentence $(x + 4) \times 5 = 35$, the symbol x represents only one value, and that is 3.

In the expression $(x + 4) \times 5$, the numbers 4 and 5 are **constants**. In the number sentence $(x + 4) \times 5 = 35$, x is an **unknown value**.

6. Write the abbreviations for the following computational instructions by using x for "the input number":
   (a) Halve the input number and plus 2.
   (b) Multiply the input number by 6 and subtract 2.
   (c) Multiply the sum of the input number and 3 by 10.
   (d) Subtract 4 from the input number and multiply the answer by 7.

7. Cardo's teacher writes on the board: "Add 2 and then multiply the answer by 3." The class must use 5 as an input number and apply the computational instruction.
   (a) Cardo uses 5 as the input number and writes: $(5 + 2) \times 3$.
   Paul says $(5 + 2) \times 3$ is 7×3 which is 21. Is Paul right?
   (b) Explain your answer in (a).
   (c) Represent this flow diagram as an algebraic expression:

$$x \longrightarrow \boxed{+\ 2} \longrightarrow \boxed{\times\ 3} \longrightarrow$$

8. Express each computational instruction as a flow diagram and then write the abbreviation (algebraic expression) with x as input number:
   (a) Multiply by 4 and then subtract 8.
   (b) Subtract 8 and then multiply by 4.
   (c) Add 15 and then divide by5.
   (d) Divide by 5 and then add 15.

9. Describe each computational instruction in words:

(a) $\rightarrow \boxed{\times 4} \rightarrow \boxed{+ 7} \rightarrow$ (b) $\rightarrow \boxed{+ 7} \rightarrow \boxed{\times 4} \rightarrow$

(c) $\rightarrow \boxed{\times 9} \rightarrow \boxed{- 5} \rightarrow$ (d) $\rightarrow \boxed{- 5} \rightarrow \boxed{\times 9} \rightarrow$

10. Two algebraic expressions are given in the table. Copy the table and use the given input values (x values) to determine the corresponding output values.

x	1	2	3	4	5	6
$6 \times x + 8$	14	20	26			
$2 \times x \times (3 + 4)$						

13.2 Relationships represented in formulae

making sense of variables and constants in formulae

1. (a) Chris uses the formula $P = 2 \times l + 2 \times b$ to calculate the perimeters of rectangles of differing lengths and breadths as indicated in the table. He also calculates the area of each rectangle using the formula $A = l \times b$.
Copy and complete the table.

Rectangle	1	2	3	4
Length (l)	24	6	8	12
Breadth (b)	1	4	3	2
Perimeter $P = 2 \times l + 2 \times b$				
Area $A = l \times b$				

(b) Rita calculates the perimeter of a rectangle in a different way. She adds the value of the length of the rectangle to the value of the breadth of the rectangle and then multiplies the answer by 2. Write down the formula that Rita uses to calculate the perimeter of each rectangle. Test whether or not Rita's formula produces the same results as Chris's.

Questions 1(c) to (e) refer to the formula $P = 2 \times (l + b)$.

(c) What does the number 2 represent in the formula?
(d) What is the number 2 called?
(e) Which letter symbols represent variables in the formula $P = 2 \times (l + b)$? Explain.
(f) What can you say about the area of all of these rectangles?

2. Sindi calculates her father's age by using the formula $F = x + 37$,where x is Sindi's age. Her father passed away when Sindi was 43 years old. How old was he then?

3. Jacob wants to buy the cheapest cell phone in the market. He has already saved R45 and decides to save R5 per week until he has enough money to buy the phone. The formula $y = 45 + 5 \times w$ gives the amount of money (in rands) that Jacob has saved to buy the cell phone after w weeks.

   (a) Copy and complete the table. The first row has been done as an example.

Number of weeks (w)	How to calculate $45 + 5 \times w$	Amount saved (y)
0	$45 + 5 \times 0 = 45 + 0$	45
1		
2		
4		
5		

   (b) The cell phone that Jacob wants to buy costs R90. Will Jacob have saved enough money to be able to buy the cell phone by the eighth week? Explain.

   (c) Copy and complete the table.

	Formula: $y = 45 + 5 \times w$	Explanation
Which are constants in the formula?		
Which letter symbols represent variable quantities in the formula?		

4. Copy the table. In each of the formulae in the table, identify the symbols that represent variables and constants and fill them in.

		Symbols for variable(s)	Constant(s)
(a)	$y = 5 \times x + 7$		
(b)	$y = 100 + x$		
(c)	$y = x \div 5$		
(d)	$y = 5 \times x$		
(e)	$y = 0{,}7 \times x + 2{,}3$		

Chapter 14
Algebraic equations 1

14.1 Solving by inspection

number puzzles

Solve these number puzzles.

1. I am thinking of a certain number. If I add 3 to that number, the answer is 13. What is the number?
2. I am thinking of a certain number. If I multiply that number by 5, the answer is 30. What is the number?
3. I am thinking of a certain number. If I multiply that number by 3 and then add 4 to the result, the answer is 19.
   (a) Is the number 3? Give a reason for your answer.
   (b) Is the number 4? Give a reason for your answer.
   (c) Is the number 5? Give a reason for your answer.
   (d) Is the number 6? Give a reason for your answer.

Number puzzles like those above can be shortened by using letter symbols as placeholders for unknown numbers. In the case of question 1 we can write the following number sentence: $\boldsymbol{x + 3 = 13}$.

In the case of a number sentence such as $x + 3 = 13$ we cannot say whether it is true or false until we have determined the value of the unknown. The value of the unknown that makes the number sentence (an **equation**) true is called the **solution** of the number sentence.

For the number sentence $x + 3 = 13$, the solution is $x = 10$ because it makes the number sentence true.

A mathematical statement such as $x + 3 = 13$ that could be true or false depending on the value of x, is called an **open number sentence** or an **equation**.

To make a number sentence **true** means to find its **solution**.

the solution is there to see

The solution to the number sentence $x + 4 = 20$ can be seen at once. The value of x is 16 simply because $16 + 4 = 20$. In this case, we say we solve the number sentence **by inspection**.

Solve these number sentences (equations) by inspection.

1. (a) $x - 8 = 8$ (b) $x + 7 = 20$
   (c) $\frac{16}{x} = 8$ (d) $\frac{x}{16} = 2$
   (e) $5 \times x = 40$ (f) $8 \times x = 40$
2. (a) $84 \div x = 7$ (b) $36 \div x = 4$
   (c) $x + 56 = 100$ (d) $100 - x = 56$

14.2 Solving by the trial and improvement method

Sometimes you cannot see the solution of a number sentence (an equation) at once. Look at the following number puzzle or equation, for example:

I am thinking of a number. 6 × *the number* − 11 = 43. What is the number?

In this case, you will have to try many different possible solutions until you identify the correct one. Here we can use a method known as **trial and improvement** to determine the solution. It is shown in the table below.

Possible solution	Test	Conclusion
Try 5	$6 \times 5 - 11 = 30 - 11 = 19$	5 is too small
Try 10	$6 \times 10 - 11 = 60 - 11 = 49$	10 is too big
Try 8	$6 \times 8 - 11 = 48 - 11 = 37$	8 is too small
Try 9	$6 \times 9 - 11 = 54 - 11 = 43$	9 is the solution

Copy the tables below. Solve the following equations by means of the trial and improvement method. In each case, the solution is a number between 1 and 20.

1. $2 \times x + 13 = 37$

Possible solution	Test	Conclusion

2. $14 \times x - 21 = 77$

Possible solution	Test	Conclusion

3. $7 \times x + 8 = 71$

Possible solution	Test	Conclusion

4. $4 \times x + 7 = 31$

Possible solution	Test	Conclusion

5. $10 \times x + 11 = 141$

Possible solution	Test	Conclusion

solving by inspection or trial and improvement

Solve the following equations by inspection or by the trial and improvement method:

1. (a) $x + 5 = 2 \times x$ (b) $k \times 5 = 20 + k$
   (c) $2 \times q = 18 - q$ (d) $3 \times t = t + 22$

2. (a) $y + 6 = 4 \times y$ (b) $5 \times p = 18 + 2 \times p$
   (c) $4 \times z = 18 + z$ (d) $x \times 5 = 20$
   (e) $42 \div m = 35 - 29$ (f) $3 \times x - 2 = x + 6$

14.3 Describing problem situations with equations

from words to equations

Write an equation using a letter symbol as a placeholder for the unknown number to describe the problem in each of the situations below.

1. There are 30 learners in a class. x learners are absent and 19 are present.
2. There are 70 passengers on a bus. At a bus stop m passengers get off. There are now 23 passengers on the bus.
3. A boy buys a bicycle for R1 260 on lay-by. How many payments of R90 each must he make to pay for the bicycle? Let x be the number of payments to be made.
4. Five people share a total cost of R240 equally amongst themselves. Let c be the cost per person.
5. A school charges R100 a day for the use of its training facilities for athletes plus R30 per athlete per day for food and use of equipment. A team of athletes paid R400 for a day's practice. Let x be the number of athletes attending the training.
6. Bennie has R54 with which to buy chocolates for his friends. Each chocolate costs R6. How many chocolates can he buy for that amount? Let x be the number of chocolates that Bennie can buy.

7. Write an equation to calculate the area of a rectangle with length 2,5 cm and breadth 2 cm. Let A represent the area of the rectangle.

8. There are 38 girls in Grade 7.This is 6 more than double the number of boys.

9. Janine is 12 years old. Her father's age is 7 years plus three times Janine's age.

making sense of equations

1. Rajbansi Taxi Service charges R10 per kilometre travelled and a standard charge of R30 per trip. Consider the equation below about a taxi trip:
   $10 \times t + 30 = 80$
   (a) Explain what each number and letter symbol stands for in the equation.
   (b) Why is t multiplied by 10 in the equation?

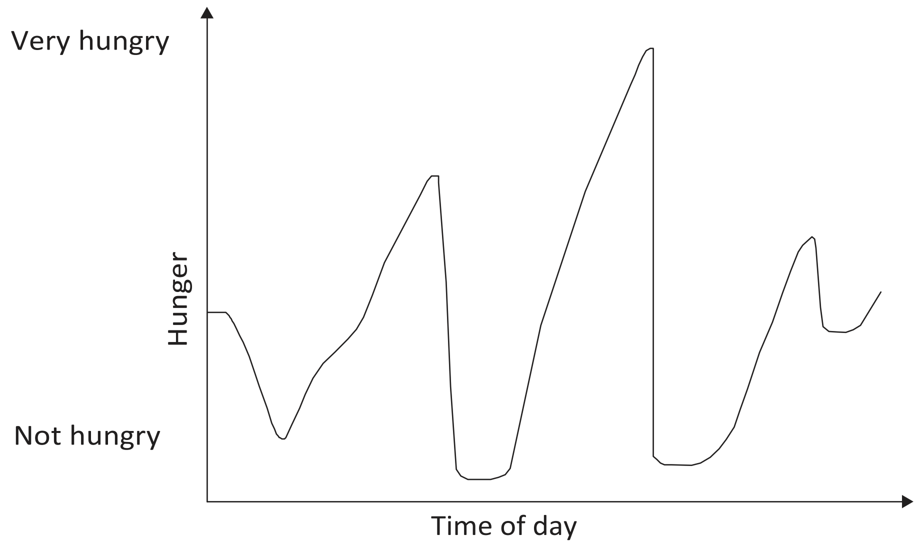
2. The cost of an adult's ticket for a music concert is four times the cost of a child's ticket. An adult's ticket costs R240. The equation below represents this problem:
   $4 \times x = 240$
   (a) What does x represent?
   (b) Why is x multiplied by 4?
   (c) Solve the equation by inspection.
   (d) How much does a child's ticket cost?

3. There are 12 eggs in a carton. Consider the equation below:
   $12 \times c = 72$
   (a) What does the letter symbol c represent in the equation?
   (b) What value of $c$makes the equation true?
   (c) What does the number 72 represent?

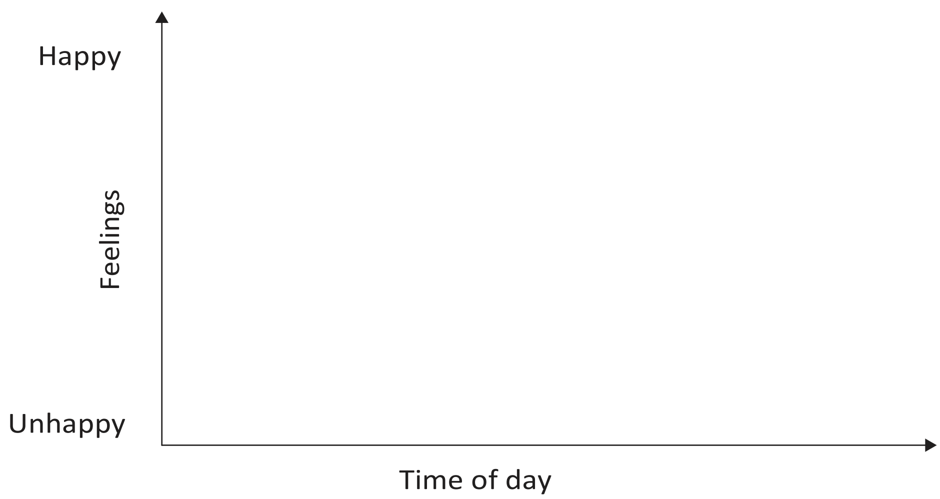
Chapter 15
Graphs

15.1 A graph can tell a story

1. Jena drew this graph to show how her feelings of hunger changed during the day. Describe in a short paragraph how her day went as far as need for food is concerned.



2. Think about a specific day and things that happened to you on that day. Use the example below to draw your own graph to show how your feelings changed during that day.



15.2 Investigating rate of change in situations

compare situations and represent them in a different way

1. Consider the situations in (a) and (b) below and copy and complete the tables to represent the relationships.

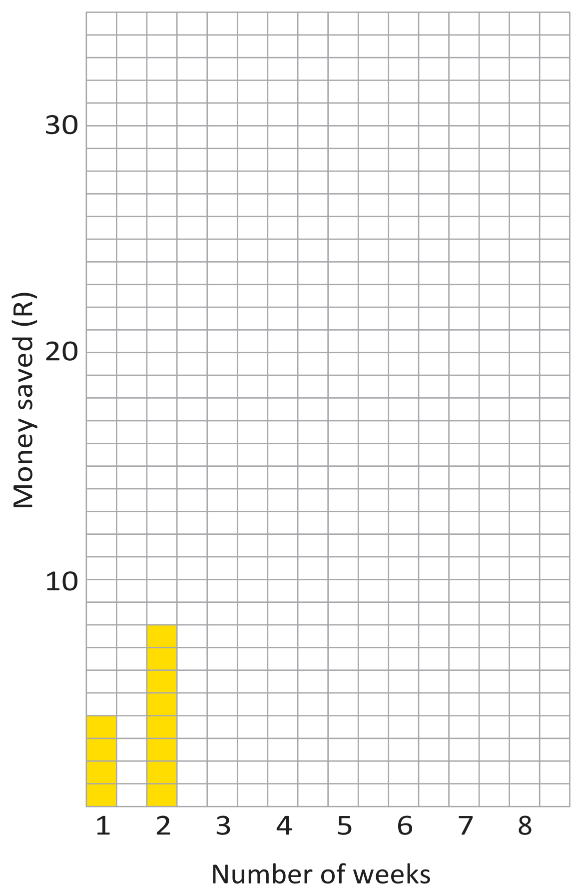
   (a) Sally is saving money to buy a CD that she really wants. She saves R4 per week.

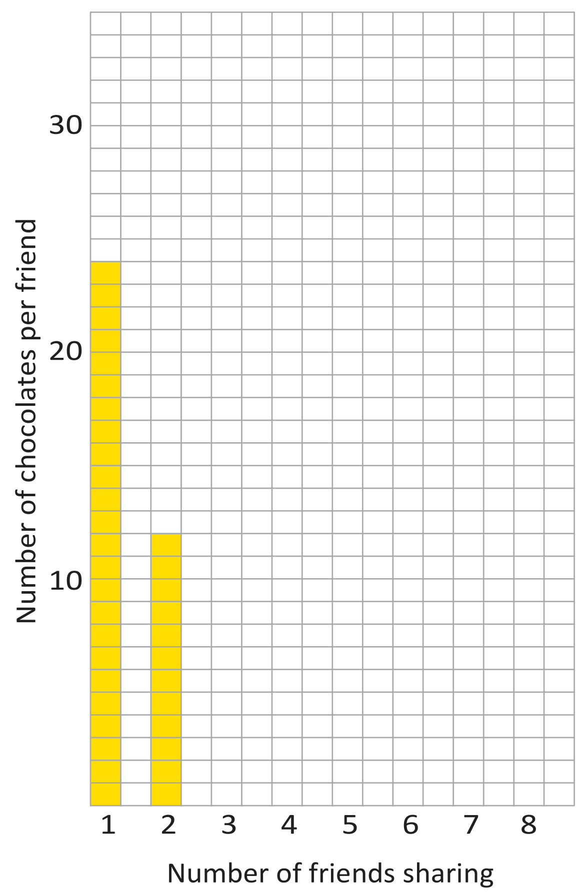
Number of weeks	1	2	3	4	5	6	7	8
Money saved in rand	4	8						

   (b) Nathi has a box of 24 chocolates. He is thinking about sharing the chocolates equally between different numbers of friends and is working out how many chocolates each friend would get.

Number of friends	1	2	3	4	5	6	7	8
Chocolates per friend	24	12						

   (c) Copy the grids below, and draw bar graphs to represent the relationships in the situations described in (a) and (b). The length of each bar should represent an output number.





2. Consider the situations in (a) and (b) below and copy and complete the tables to represent the relationships.

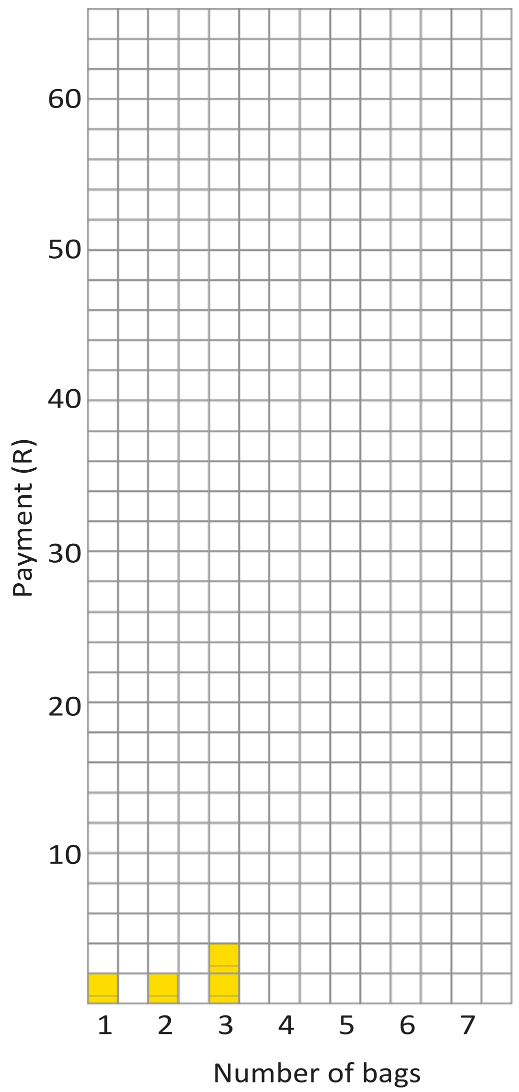
(a) Vusi collects acorns for a pig farmer who pays him per bag. Vusi thinks: "I wish Mr Bengu would agree to pay me R1 for one bag of acorns, R2 for two bags, R4 for three bags, and then keep doubling the money when another bag is added."

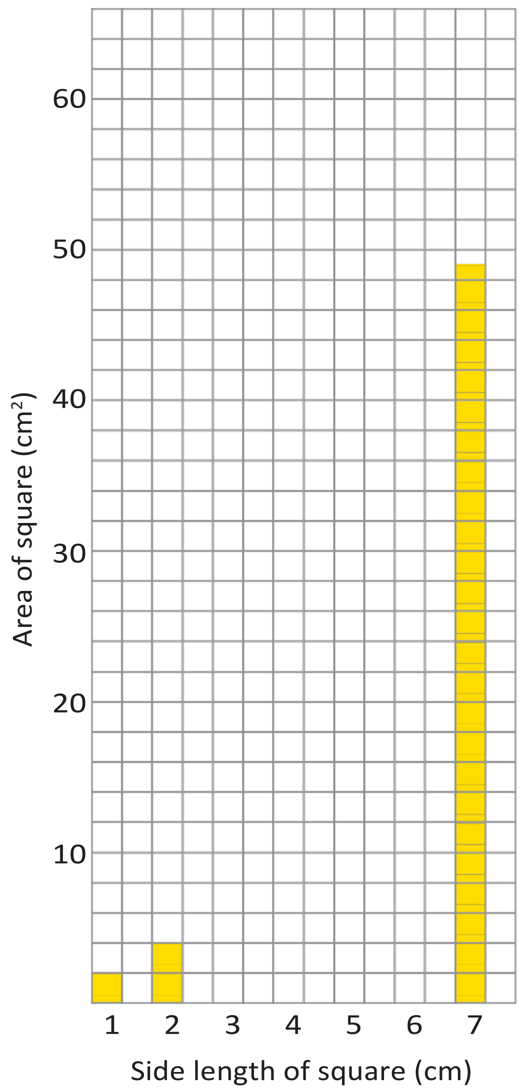
Number of bags	1	2	3	4	5	6	7
Payment (R)	1	2	4	8			

(b) Judy works out the areas of squares with different side lengths.

Side length of square (cm)	1	2	3	4	5	6	7
Area of square (cm^2)	1	4					

(c) Copy the grids below, and draw bar graphs to represent the relationships in the situations described in (a) and (b). The length of each bar should represent an output number.





3. The input numbers for the different relationships in questions 1 and 2 are the same, but the output numbers differ. Describe how the output numbers change in each of the four situations.
4. Describe briefly how the shape of the bar graphs differ.
5. Turn back to the tables of values that you made for the four relationships in questions 1 and 2. Find out how the output values changed by calculating the differences between consecutive output values, and describe the differences for each case as suggested:
   (a) Sally's savings: 4 8 12 16

   4 4

   Sally's savings grow by ______________ every week.

   (b) Chocolates per friend: 24 12 8

   12

   The number of chocolates per friend ___

   (c) Vusi's payment per bag: 1 2

   1

   The amount grows slowly at first and then ___

   (d) Areas of Judy's squares: 1 4 9

   3

   The area of the squares ___

The amount of money that Sally saves per week stays **constant**. The relationship therefore has a **constant rate of change**.

With every additional friend, the number of chocolates per friend changes. The number of chocolates changes (decreases) **rapidly at first and then it changes slower**. The **rate of change is not constant**.

The amount that Vusi would like to earn per bag of acorns grows faster and faster with each bag that he collects. The **rate of change increases**.

The area of a square also **increases faster and faster** for every centimetre added to the side length.

The **rate of change** means how fast or slow change happens per unit of time.

The **shape** of a bar graph shows the **rate of change** of the relationship. If the rate of change is constant, the shape is a **straight line**. If it is changing, the shape is a **curve**.

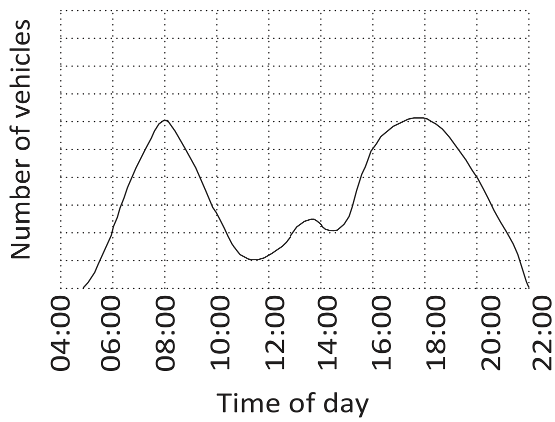
6. Refer to the bar graphs in questions 1 and 2 and link the shape of the graphs to the rate of change of the relationship.

15.3 Interpreting graphs

reading graphs

1. Look carefully at the graph.
   (a) What does this graph tell you?
   (b) Explain your answer in (a).

Traffc on the R44 (weekdays)



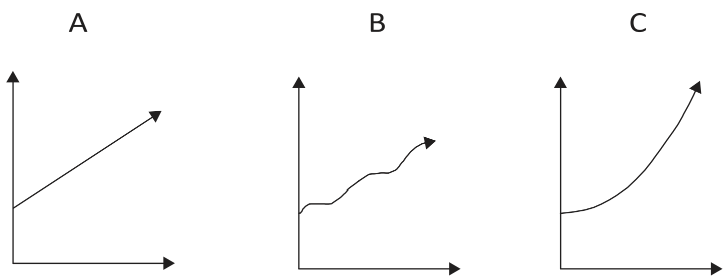
2. Mr Thatcher bought three plants in containers. The salesman at the nursery told him that one of the plants, Glamiolus, grows at a constant rate. The second plant, Bouncy Bess, grows slowly at first but then grows faster and faster. The salesman was not sure about the rate at which the third plant, Samara, grows.
   (a) What does "grows at a constant rate" mean?

   Mr Thatcher measured the three plants every week and recorded the heights in a table, given below.

   (b) Calculate the differences in height from week to week, to find the rate at which each plant grows per week.

Week	Height of A (cm)	Height of B (cm)	Height of C (cm)
1	6	8,3	10,1
2	6,3	10,2	10,6
3	6,4	12,2	11,2
4	7,2	14,1	11,9
5	7,3	16,2	12,8
6	7,4	18,3	13,9
7	9,1	20,2	15,8

   (c) Identify the plants. Which plant is plant A, which plant is plant B and which plant is plant C? Explain how you got your answers.
   (d) The three graphs on the right show the growth of the three plants. Which graph belongs to which plant? Explain.



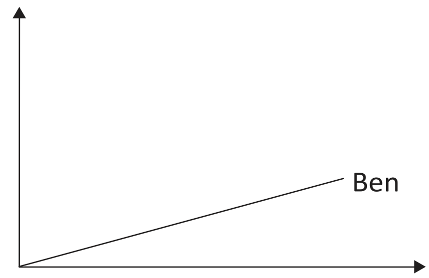
revisiting change and rate of change

In section 5.2 you compared the way in which relationships changed.

1. Consider the following situations:
   Ben saves R5 per week. Sally saves R7 per week.
   Charlie saves R5 in the first week, R6 in the second week and R7 in the third week. Every week he increases the amount that he saves by R1.

   (a) The graph shows Ben's savings. Copy the graph and draw Sally's savings and Charlie's savings. Do it on the same graph.

   (b) Describe and explain the shape of the graphs showing Sally, Ben and Charlie's savings.



The rate of change in a relationship influences the **steepness** of the graph. The higher the rate of change (i.e. the faster the output numbers change), the steeper the graph.

2. Examine the following relationships. Copy and complete the tables and calculate the differences between the output numbers indicated with arrows.

   (a) Christine wants to buy a book for her favourite teacher. The book costs R240. This is a lot of money for Christine to spend. She realises that if she asks her friend Beatrice to share the cost, she will only have to spend R120. She could even ask more classmates to join in and share the cost. Christine investigates the situation and calculates what amount everyone must pay if they share the cost equally.

Number of learners sharing the cost	1	2	3	4	6	8	10	12
Amount each learner will pay	240	120						

120

   (b) Investigate the relationship between the length of a side of a square and the perimeter of the square.

Length of a side of the square	1	2	3	4	6	7	8	9
Perimeter of the square	4	8	12					

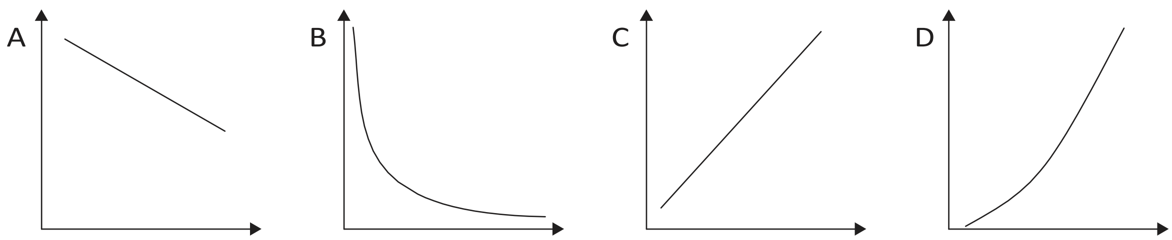
(c) Investigate the relationship between the length of a side of a square and the area of the square.

Length of a side of the square	1	2	3	4	6	7	8	9
Area of the square	1	4	9					

(d) A tall candle was lit and its length was measured and recorded every hour while it was burning.

Number of hours the candle is burning	1	2	3	4	6	7	8	9
Length of candle in centimetres	33	31	29					

3. Match each of the graphs below to one of the situations in question 2.



Write the letter of the graph next to the description of the situation:

(a) Buying a book for the teacher

(b) Length of a side of a square and the perimeter of the square

(c) Length of a side of a square and the area of the square

(d) Length of the candle and the number of hours it is burning

When we investigate the growth (or change) in a relationship, we look at the way the output numbers change.

The change can be:

- an increase or a decrease
- a constant increase, for example the perimeter of a square as the side length increases
- a constant decrease, for example the length of a burning candle
- an increase that is not constant but happens faster and faster, for example the area of a square as the side length increases
- a decrease that is not constant but happens faster and faster, for example the amount of money each friend has to pay as more and more friends share the cost.

In the case of an increase, the graph slopes like this: 

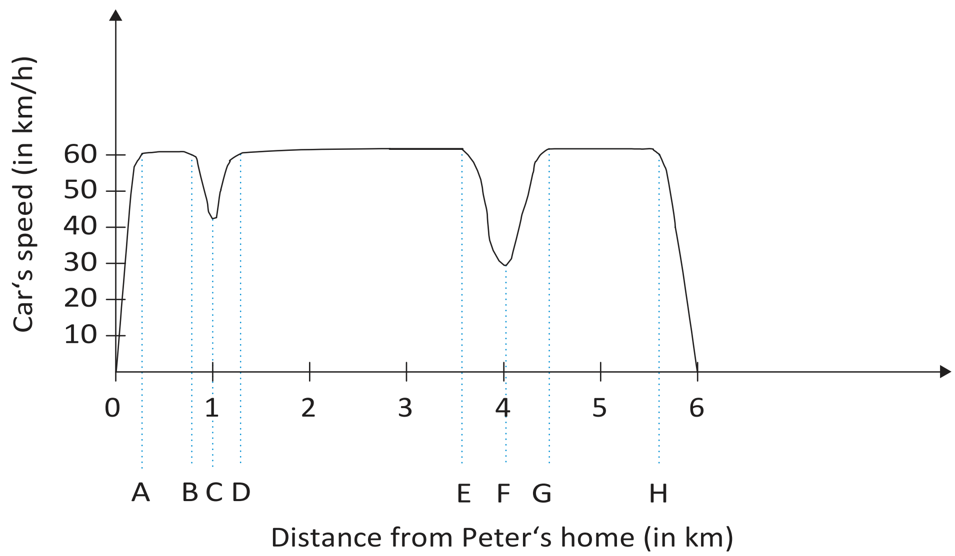
In the case of a decrease, the graph slopes like this: 

> When the increase or decrease is constant, the graph is a straight line and it is called a **linear graph**.
>
> If the increase or decrease is not constant, the graph is curved and is called a **non-linear graph**.
>
> If there is no change in the output variable, the graph is a straight horizontal line.

4. Consider the graphs in question 3 on the previous page.
   (a) Which graphs indicate a linear increase or decrease?
   (b) Which graphs indicate a decrease or increase which is not constant?

5. Peter's father drives him to school in the mornings. Below is a graph of their journey to school. Describe the story that the graph tells. What do you know about the route that they are taking?

Going to school: speed changes

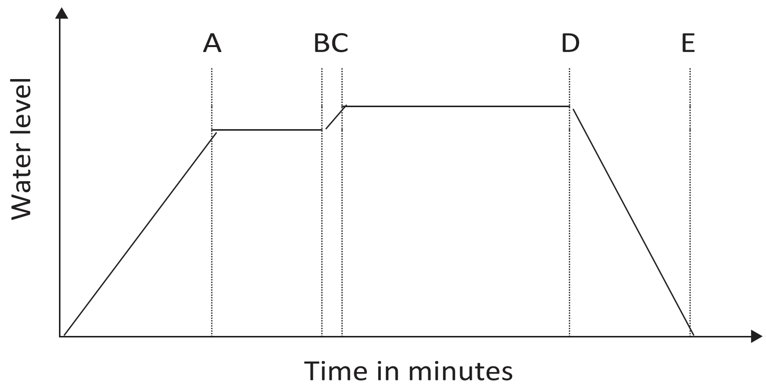


6. Consider the graph in question 5 above. Identify and indicate for the different parts of the graph listed below whether they are increasing, decreasing or constant.

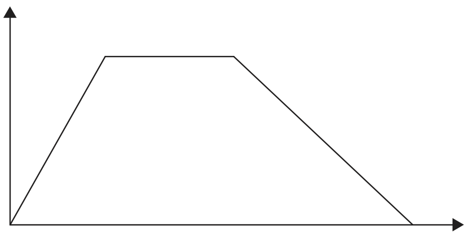
(a) 0 to A	(b) A to B	(c) B to C
(d) C to D	(e) D to E	(f) E to F
(g) F to G	(h) G to H	(i) H to 6

exploring more graphs

1. Janet takes a bath. The graph below shows the height of the water level in the bathtub as time passes. The water runs into the bath at a constant rate. Study the graph and describe what happens.



2. The axes ofthe graph below are not labelled.



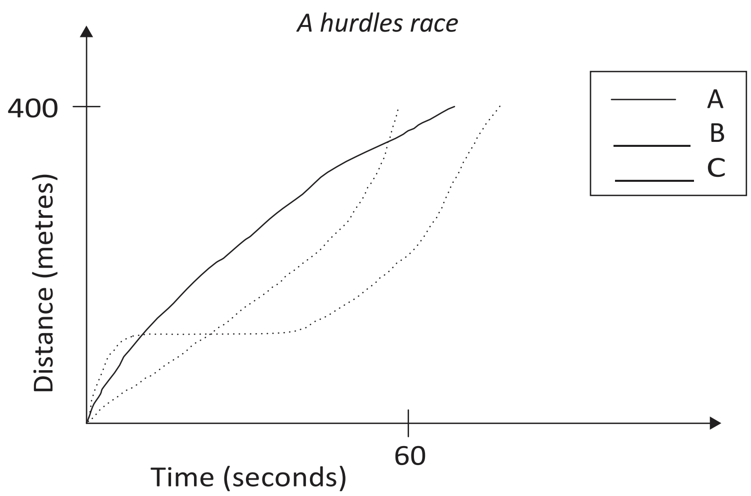
The **vertical axis** is the one that goes from bottom to top. The **horizontal axis** is the one that goes from left toright. (**axes** is the plural of **axis**.)

   (a) Which of the following sets of labels could fit the graph?
   A: **vertical axis:** time passed; **horizontal axis:** distance from home
   B: **vertical axis:** distance from home; **horizontal axis:** time passed
   C: **vertical axis:** rainfall; **horizontal axis:** temperature

   (b) Describe the story told by the graph, with the axes that you chose.

3. The graph that follows shows the distance that three athletes, A, B and C, covered in a hurdles race in a certain time.

   (a) Describe what happened during the race.

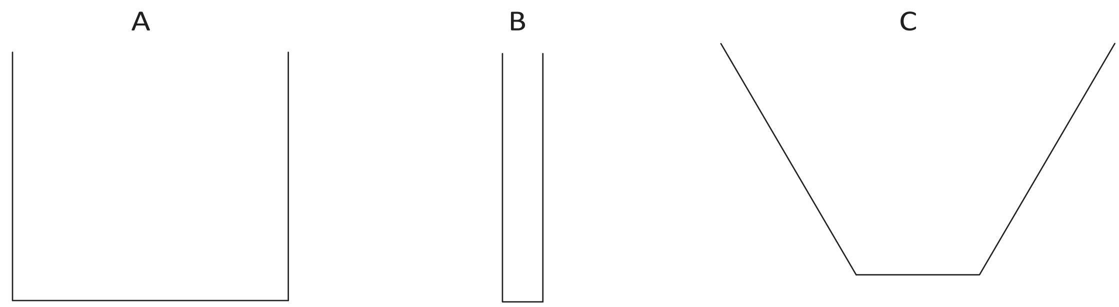


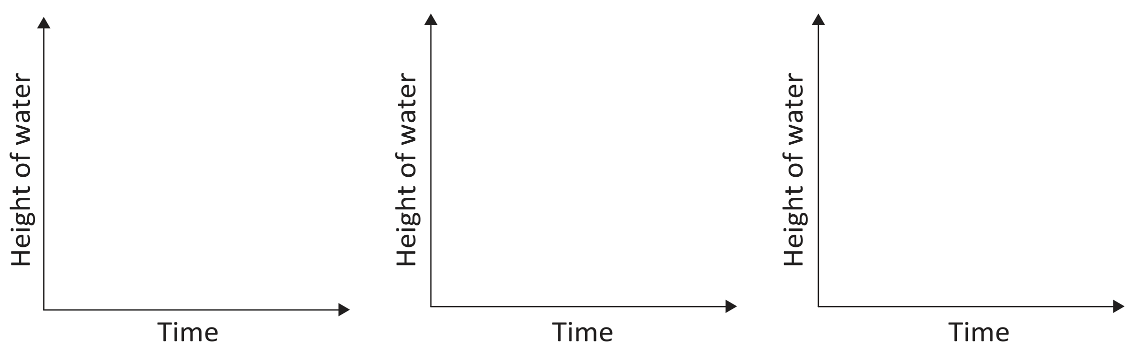
(b) How far was the race?
(c) Which of the athletes, A, B or C, won the race?
(d) Did the best athlete of the three win? Explain your answer.

4. Identify the graphs (or parts of a graph) in questions 1, 2 and 3 above that are linear and those that are non-linear.

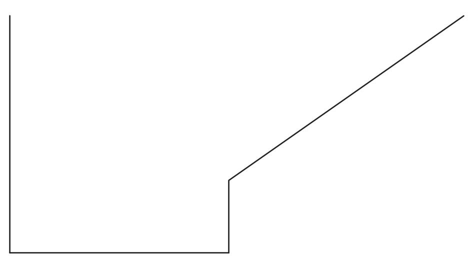
15.4 Drawing graphs

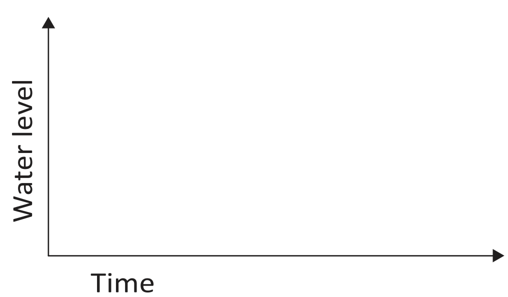
1. Water is dripping at a constant rate into three containers, A, B and C, shown below. Use the examples given below to draw graphs to show how the height of the water in each container will vary with time.



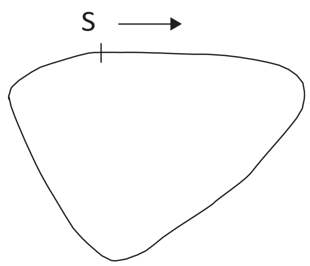


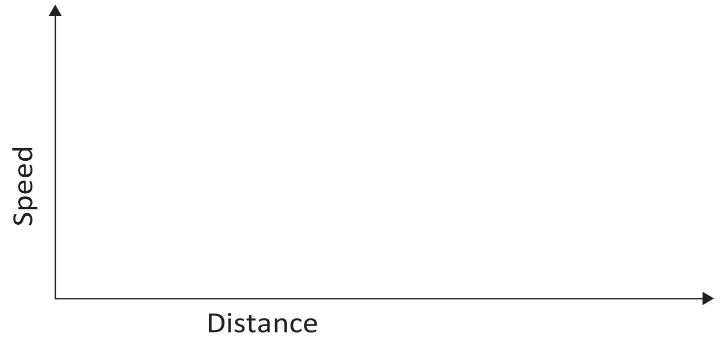
2. Use the example given below to draw a graph showing the height of the water level in the swimming pool (shown below left) if the pool is filled with a constant stream of water.


Cross-section of swimming pool



3. Use the example given below to draw a graph of the speed of a racing car as it travels once around the track shown below. S is the starting point.



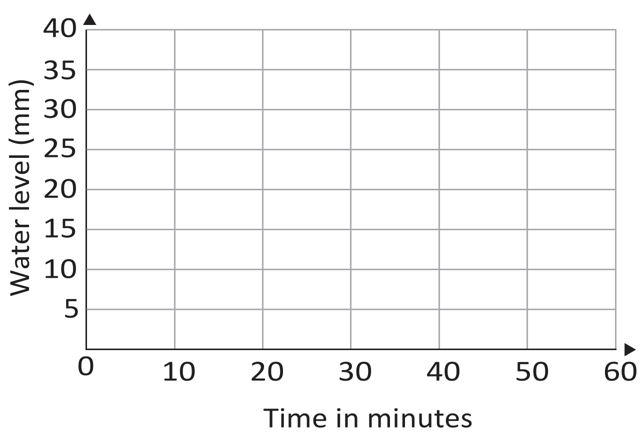


4. The Western Cape gets rain during the winter months, but in summer it is usually dry. Using the example below, draw a global graph of the average rainfall in the Western Cape during one year.

5. Use the example given below to draw a graph of the following story:

   During a rainstorm, Lydia put a measuring cup outside to measure the rainfall. After 10 minutes of hard rain the water level was 10 mm. It started to rain softer, and after 20 more minutes the water level was 15 mm.

   When Lydia went back 10 minutes later, the level was 30 mm. An hour after the storm started, the water level was still 30 mm.

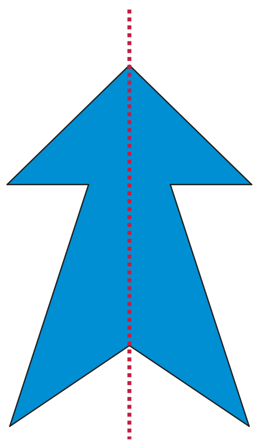


Chapter 16
Transformation geometry

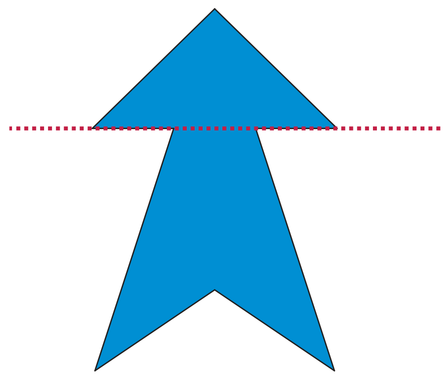
16.1 Lines of symmetry

what is the line of symmetry?

In the diagrams below, the red dotted lines divide the arrows into two parts. In which diagram does the red dotted line divide the arrow into two parts that are exactly the same?



Arrow A



Arrow B

If you were to cut out arrow A and fold it along the red dotted line, the two parts would fit perfectly on top of one another (all edges would match). The fold line is called a **line of symmetry** or an **axis of symmetry**.

A line or axis of symmetry is a line that divides a figure into two parts that have an equal number of sides, and all the corresponding sides and angles are equal. The two parts on either side of the line of symmetry are mirror images of each other. We also say the parts are **congruent**.

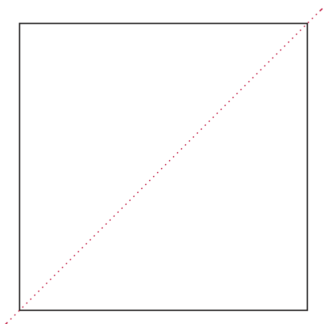
A geometric figure can have no line of symmetry, one line of symmetry, or more than one line of symmetry.

congruent figures are figures that are the same size and shape. All the sides and angles of the figures match.

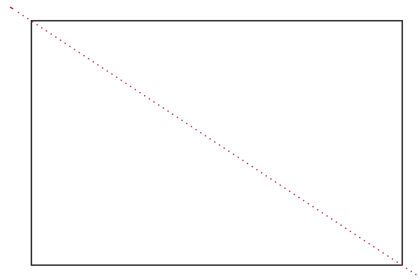
identifying lines of symmetry

1. (a) Copy each of the figures shown below. Make a tick next to each figure in which the red line is a line of symmetry.
   (b) In the figures where the red line is not a line of symmetry, draw in a line of symmetry if this is possible. If there is more than one line of symmetry, draw it in too. If a figure doesn't have any lines of symmetry, write this above the figure.

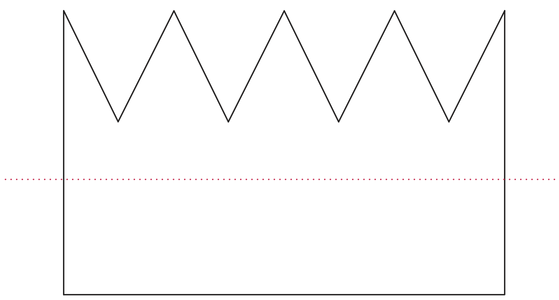
A



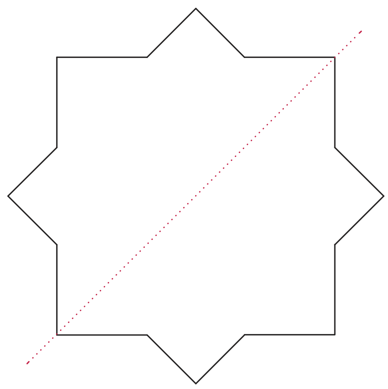
B



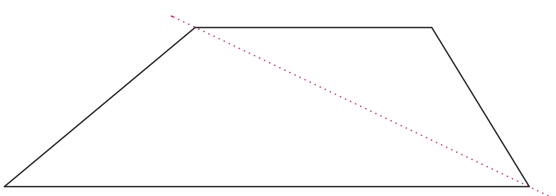
C



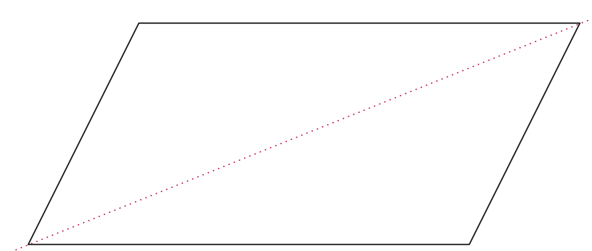
D



E

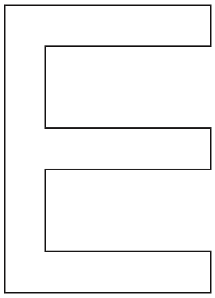


F

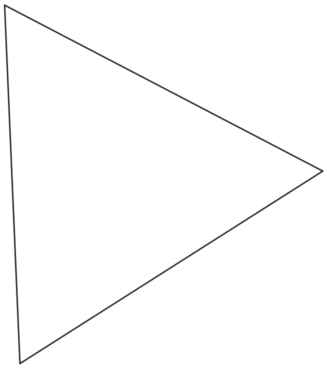


2. Copy the following geometric figures and draw the lines of symmetry. Also write down how many lines of symmetry there are in each figure.

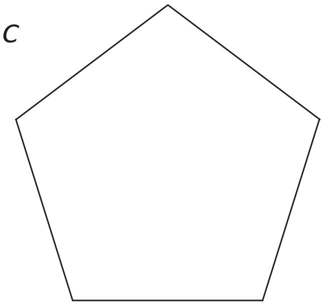
A



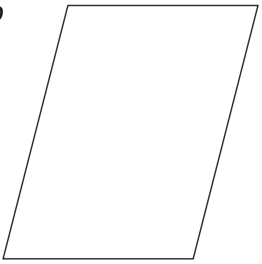
B



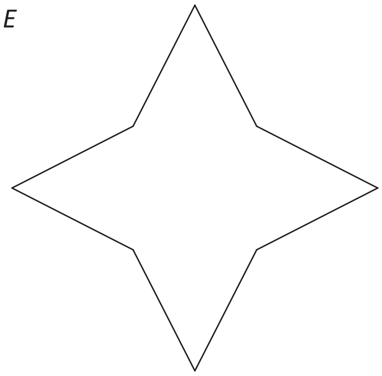
C



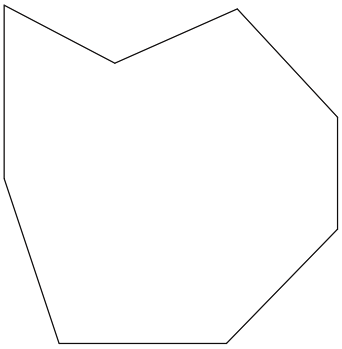
D



E

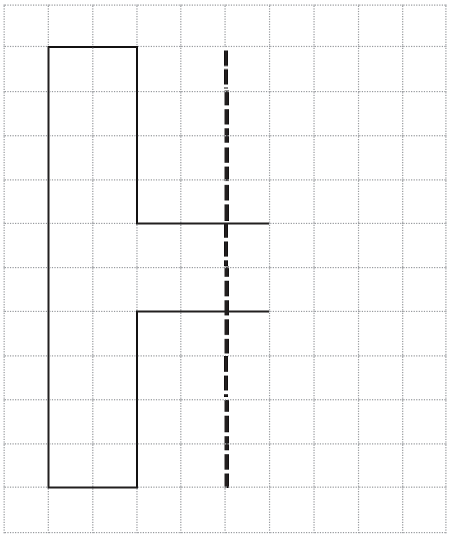


F

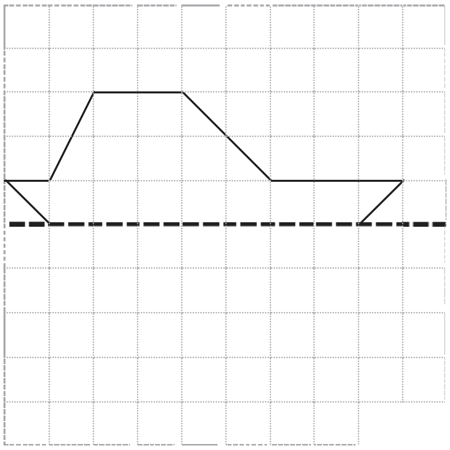


3. In each diagram, the dotted line is the axis of symmetry. Copy and complete each figure.

A



B



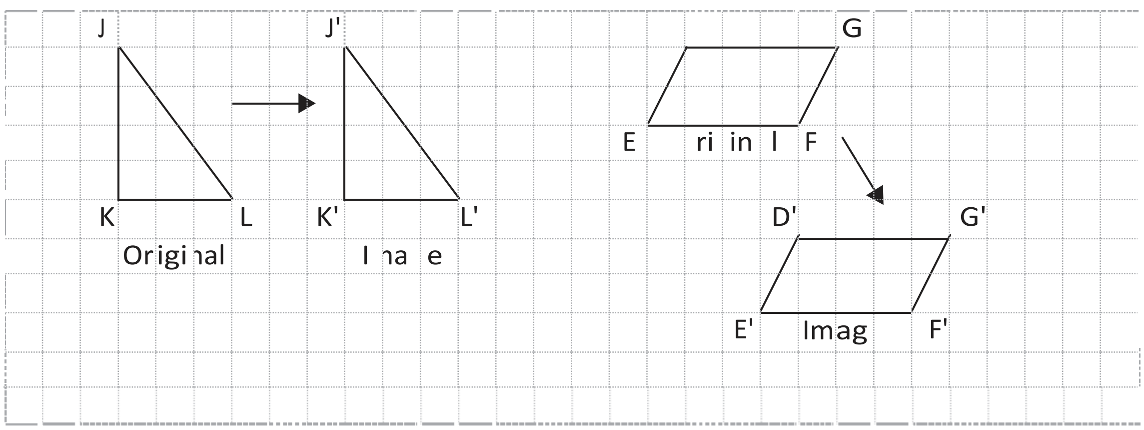
16.2 Original figures and their images

Figures can be moved around in different ways – they can be shifted, swung around and turned over. When the movement is done, the figure in its new position is called the **image** of the original figure.

> Figures can be moved in three ways: through **translation**, **reflection** and **rotation**. These transformations are often referred to as "sliding" (shifting), "flipping" (turning over) and "turning" (swinging) respectively.

16.3 Translating figures

Here are two original figures and their images after the figures were **translated**:

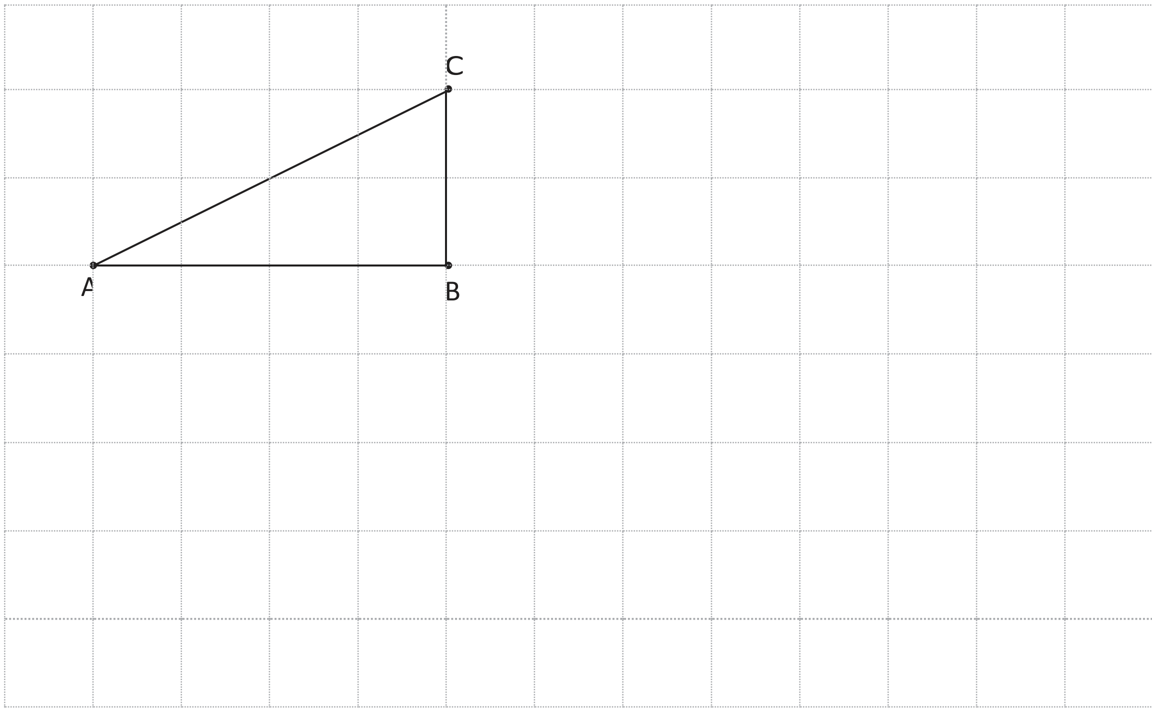


When we name the image, we use the same letters for the points that correspond to those of the original figure, but we add the prime symbol (') after each letter. The image of ΔJKL is ΔJ'K'L'. The image of parallelogram DEFG is parallelogram D'E'F'G'.

investigating the properties of translation

In a **translation**, all the points on the figure move in the same direction by the same distance. For example, look at ΔJKL above. All of its points have moved six units to the right. Also look at parallelogram DEFG above. All of its points have moved three units to the right and five units down.

1. Look at ΔABC on the following page.
   (a) Copy the triangle on the following page. Translate each of the points A, B and C five units to the right and two units down. Then join the translated points to form the image ΔA'B'C'.



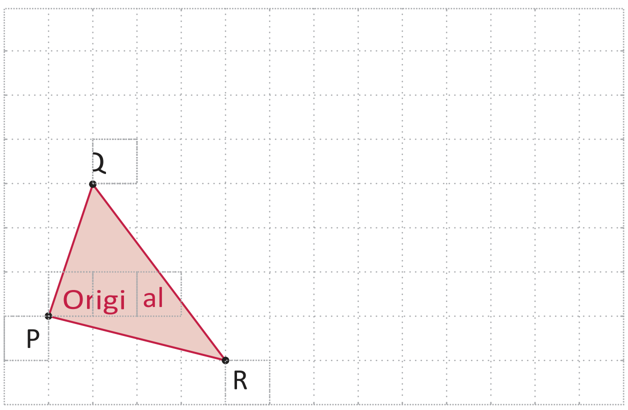
Look at the completed translation.

(b) Are the side lengths of the original triangle and those of its image the same?

(c) Is the area of the original triangle the same as the area of its image?

2. Look at ΔPQR below.

   (a) On grid paper, copy the triangle below. Translate each of the points P, Q and R four units to the right and two units up. Then join the translated points to form the image ΔP'Q'R'.



   (b) Join point P and its image, point Q and its image, and point R and its image.

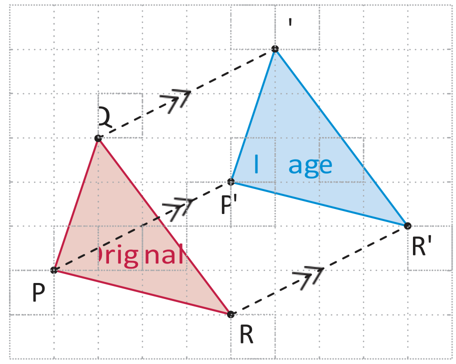
   (c) Are the line segments that join the original points to their image points equal in length?

   (d) Are the line segments that join the original points to their image points parallel?

Properties of translation

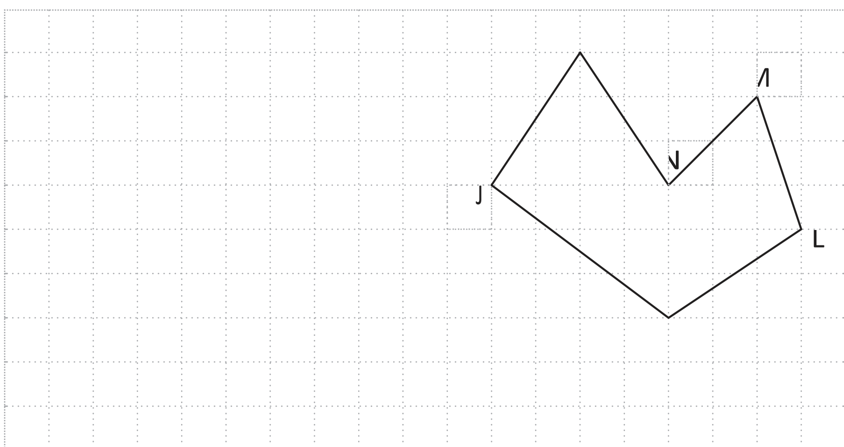
Use the diagram on the right to check if the following is true:

- The line segments that connect the vertices of the original figure to those of the image are all equal in length: PP' = RR' = QQ'
- The line segments that connect the vertices of the original figure to those of the image are all parallel to oneanother: PP' || RR' || QQ'
- When a figure is translated, its shape and size do not change. The original and its image are therefore congruent.

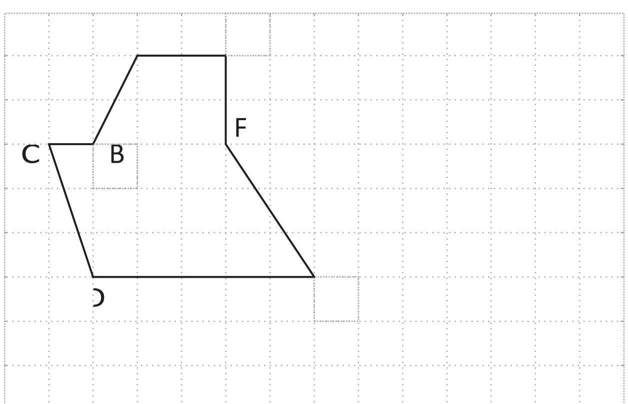


practise translating figures

1. On grid paper, copy the figure. Translate the figure eight units to the left and two units down.

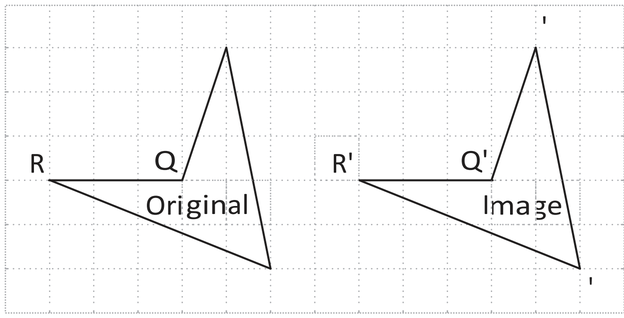


2. On grid paper, copy the figure. Translate the figure six units to the right and one unit down.

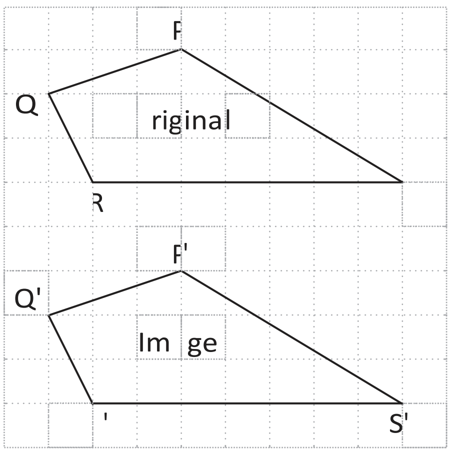


3. Describe the translation in each of the following diagrams:

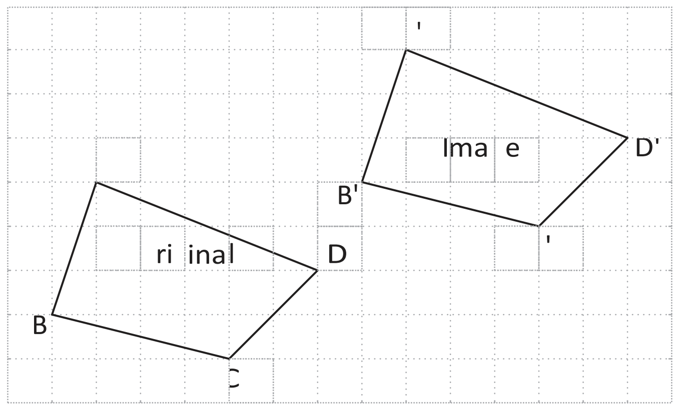
(a)



(b)



(c)



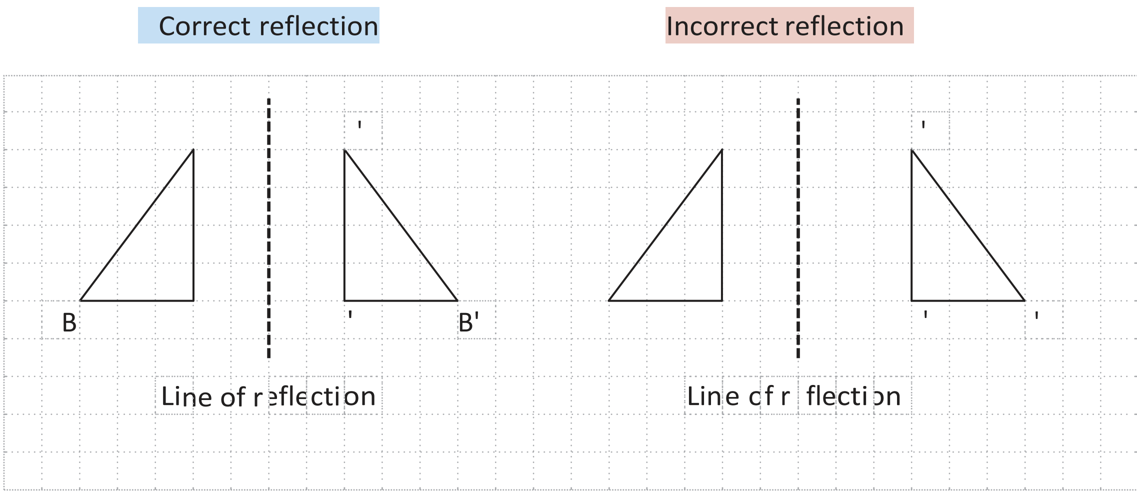
16.4 Reflecting figures

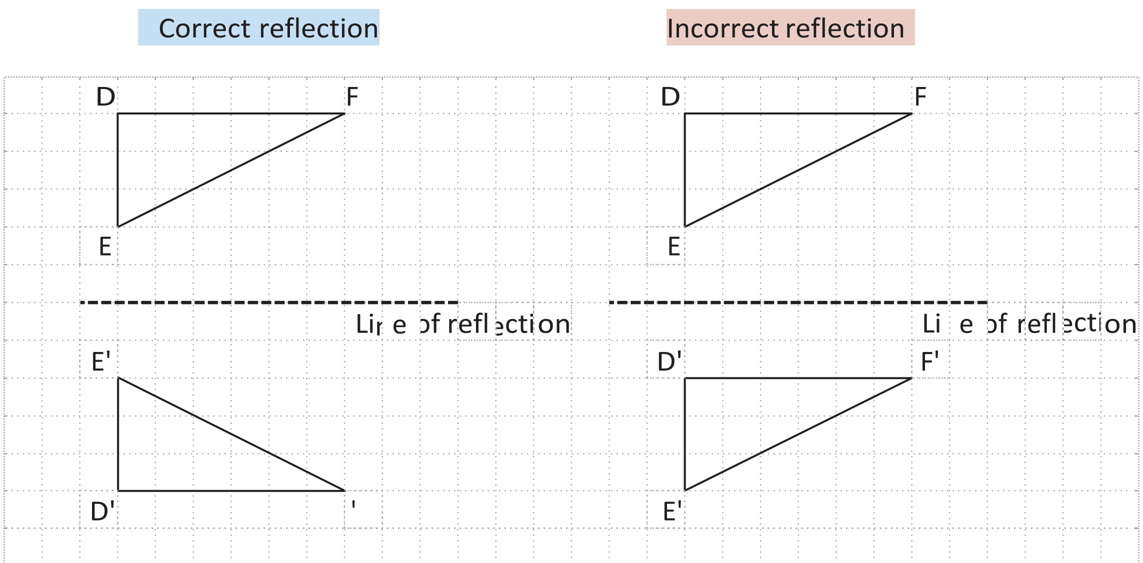
When a figure is **reflected**, it is flipped or turned over. The image that is produced is the mirror image of the original figure. The **line of reflection** is like a mirror in which the original figure is reflected.

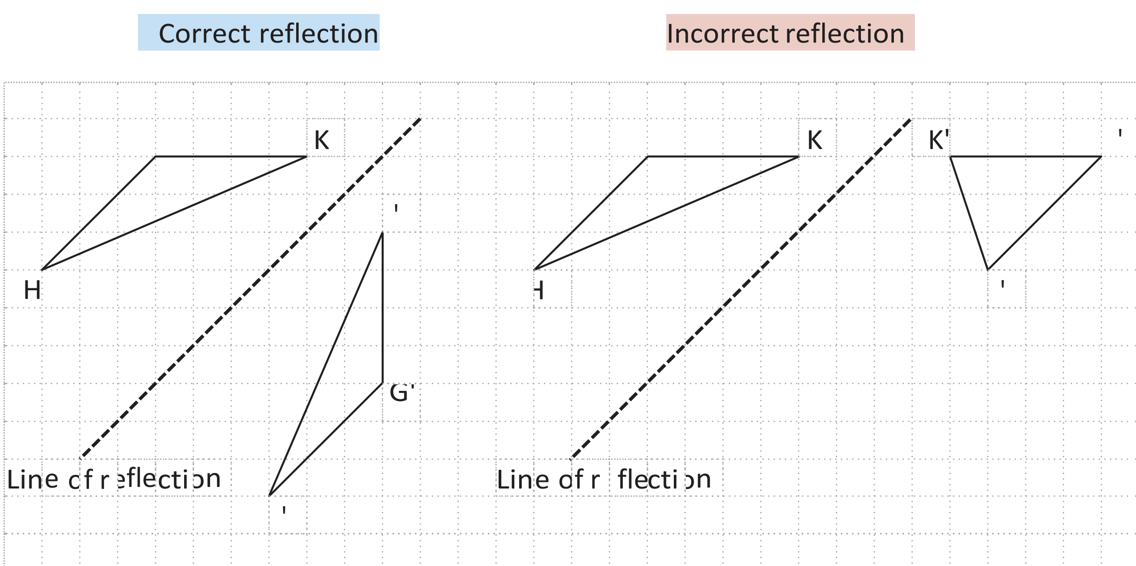
The image is produced on the opposite side of the line of reflection. Each point on the original figure and its corresponding point on the image are the same distance away from the line of reflection.

investigating the properties of reflection

The diagrams on the next page show examples of figures that have been correctly and incorrectly reflected in the lines of reflection.


Correct reflection
Incorrect reflection
B
B'
Line of reflection
Line of reflection


Correct reflection
Incorrect reflection
D
F
E
E'
D'
F'
Line of reflection
Line of reflection


Correct reflection
Incorrect reflection
K
H
G'
K'
Line of reflection
Line of reflection

1. Copy the table and write down the distance from each of the following points to the line of reflection.

Original figure	Correct reflection	Incorrect reflection
A: two units	A':	A':
B:	B':	B':
C:	C':	C':
D:	D':	D':
E:	E':	E':
F:	F':	F':
G:	G':	G':
H:	H':	H':
K:	K':	K':

2. Look at each set of *correct* reflections.
   (a) Are the side lengths of the image the same as those of the original figure?
   (b) Are the size and shape of the image the same as the size and shape of the original figure?

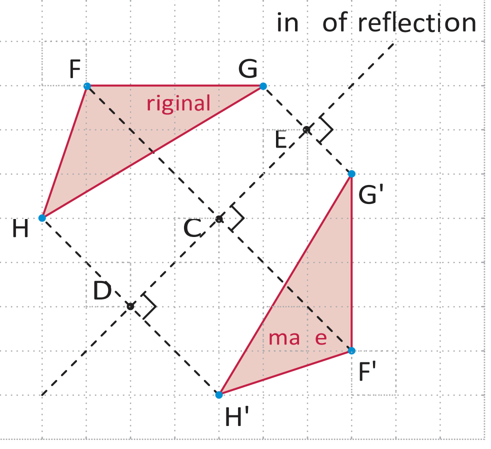
3. (a) Copy each diagram that shows the *correct* reflection, and draw a dotted line to join each point on the original figure to its corresponding reflected point (A to A', B to B', C to C' and so on).
   (b) Is the line that joins the original point to its correct reflection perpendicular to the line of reflection?

4. (a) Copy each diagram that shows the *incorrect* reflection, and draw a dotted line to join each point on the original figure to its corresponding reflected point.
   (b) Is the line that joins the original point to its incorrect reflection perpendicular to the line of reflection?

Properties of reflection

The diagram on the right shows ΔFHG and its reflection ΔF'H'G'. Notice the following properties of reflection:

- The image of ΔFHG lies on the opposite side of the line of reflection.
- The distance from the original point to the line of reflection is the same as the distance from the reflected point to the line of reflection: GE = G'E; FC = F'C and HD = H'D

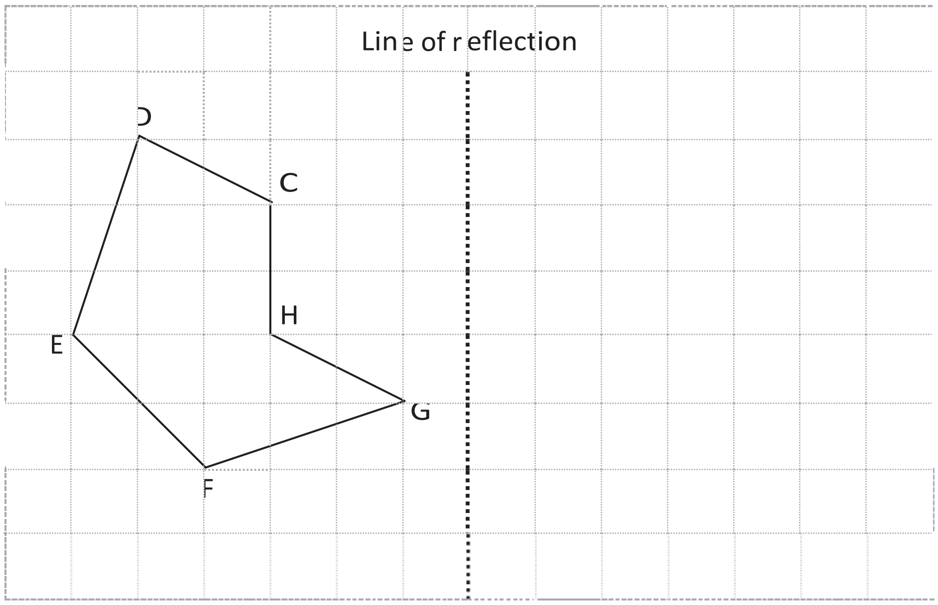


- The line that connects an original point to its image is always perpendicular (⊥) to the line of reflection: HH' ⊥ line of reflection; FF' ⊥ line of reflection, and GG' ⊥line of reflection.
- When a figure is reflected, its shape and size do not change. The original and its image are therefore congruent.

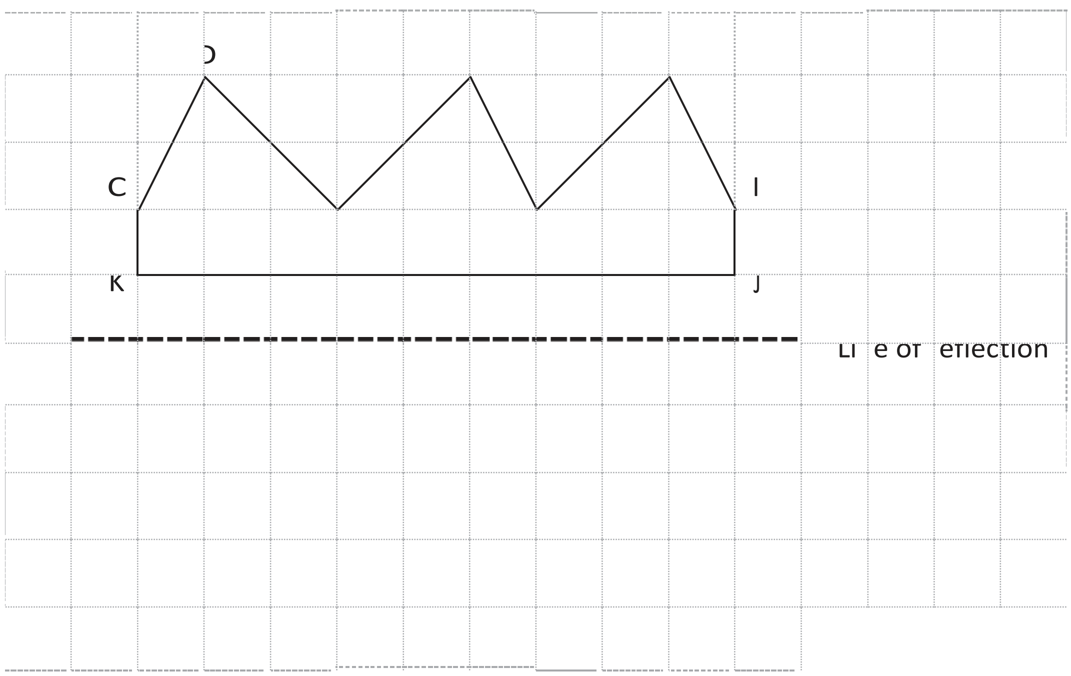
practise reflecting figures

1. Copy the figures below and reflect the figures in the given line of reflection. (*Hint*: First reflect the points; then join the reflected points.)

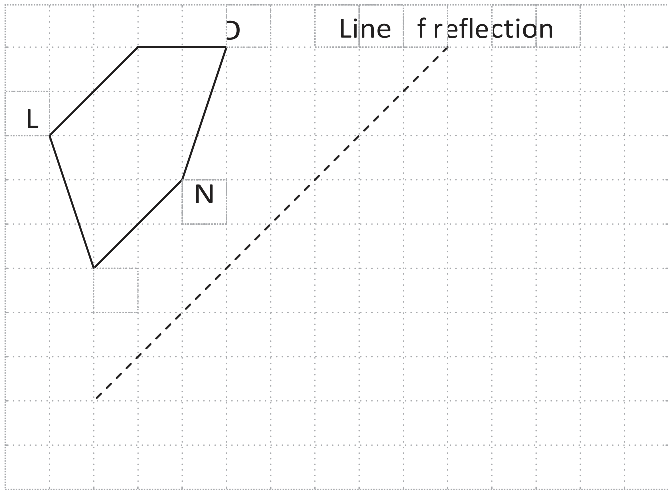
(a)



(b)

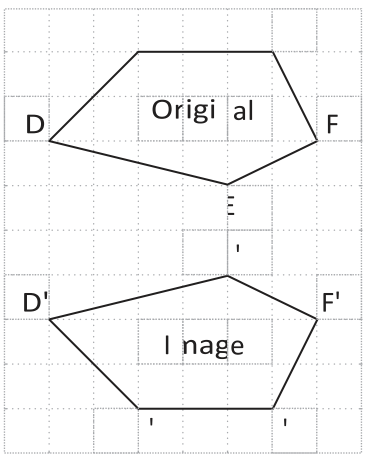


(c)

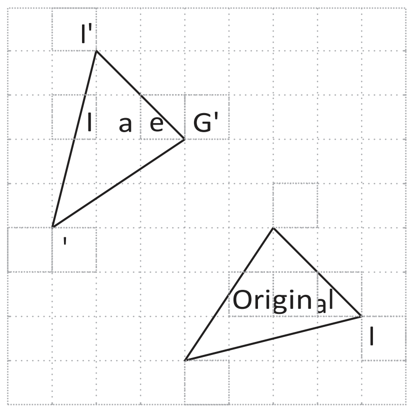


2. Copy the following and draw the line of reflection.

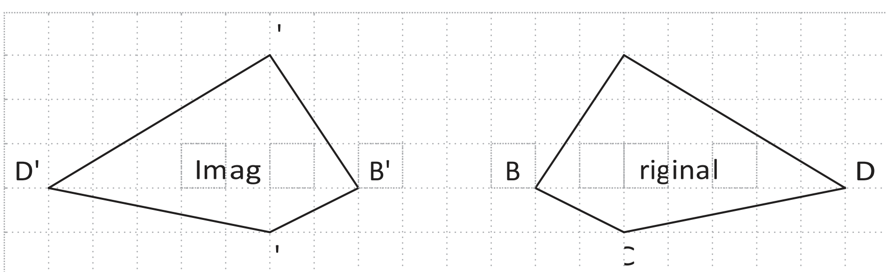
(a)



(b)

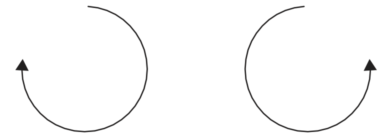


(c)



16.5 Rotating figures

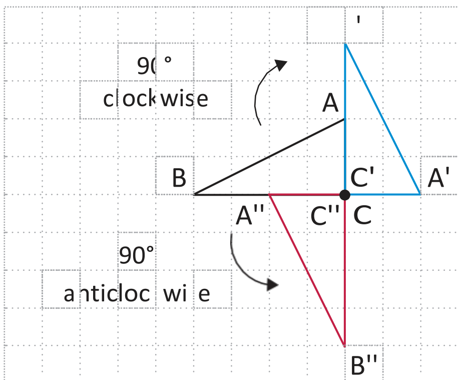
When a figure is **rotated** it is turned in a clockwise direction or in an anticlockwise direction around a particular point. This point is called the **centre of rotation** and could be inside the figure or outside of the figure.



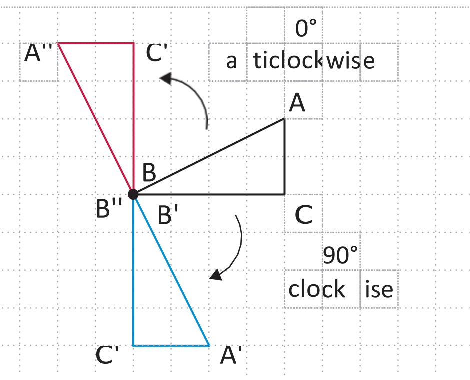
In this case, **about** means "around".

The following diagrams show ΔABC rotated 90° clockwise and 90° anticlockwise about different centres of rotation.

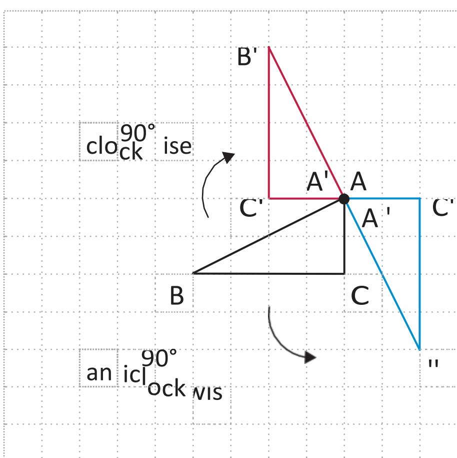
Centre of rotation is at C



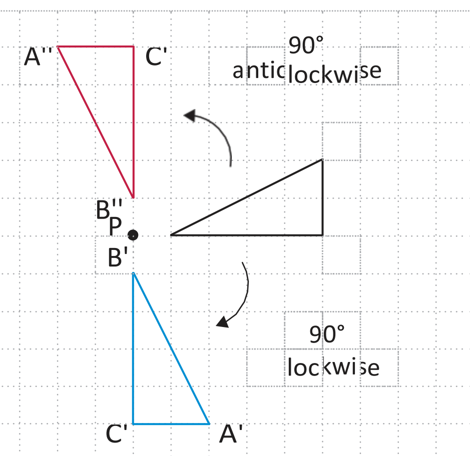
Centre of rotation is at B



Centre of rotation is at A



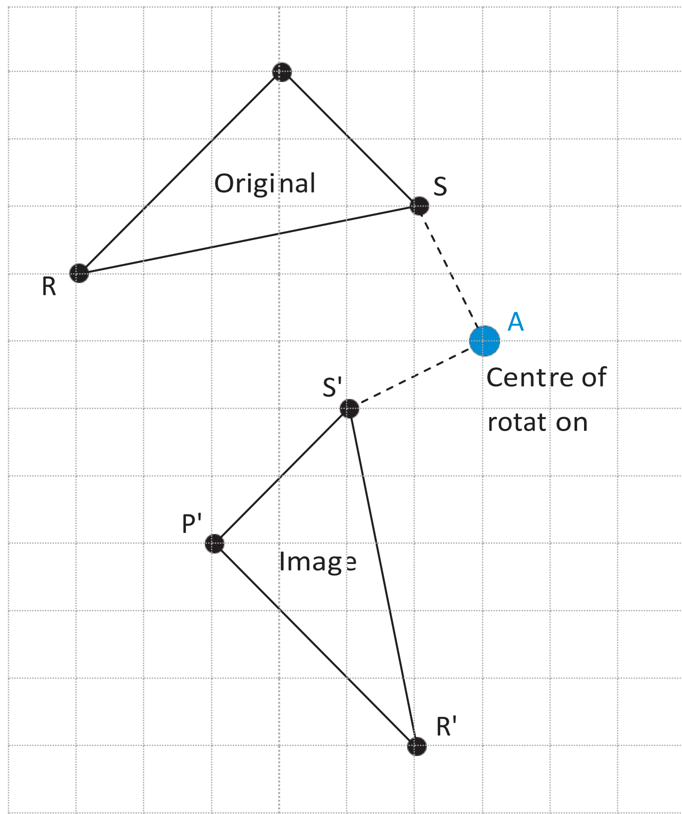
Centre of rotation is at P



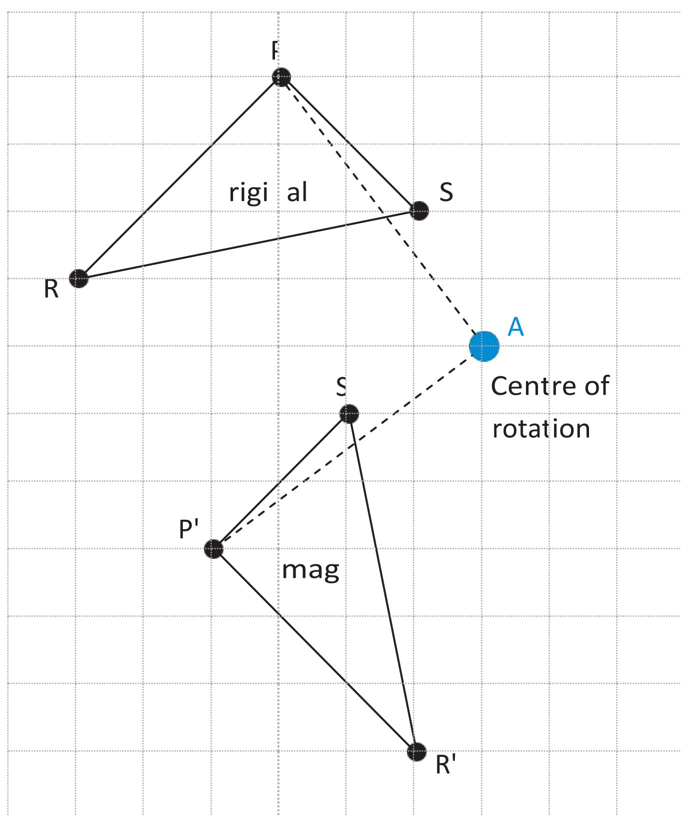
investigating the properties of rotation

In the following diagrams, the centre of rotation is point A. ΔPRS has been rotated anticlockwise through 90° about point A.

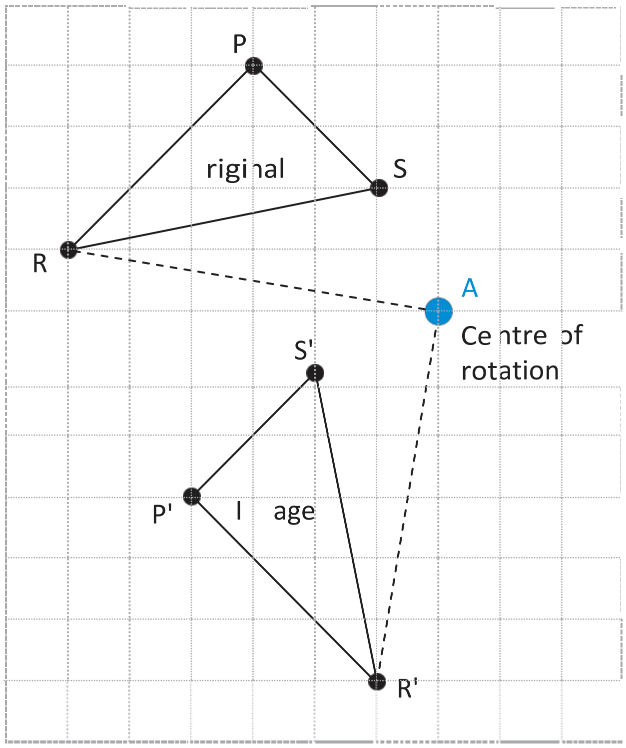
1. Lines have been drawn to join A to point S, and A to point S'.
   (a) Measure the distance from A to S.
   (b) Measure the distance from A to S'.
   (c) What do you notice about the distances in (a) and (b) above?
   (d) Measure the size of the angle SAS'. What do you notice?



2. Lines have been drawn to join A to P, and A to P'.
   (a) Measure the distance from A to P.
   (b) Measure the distance from A to P'.
   (c) What do you notice about the distances in (a) and (b) above?
   (d) Measure the size of the angle PAP'. What do you notice?



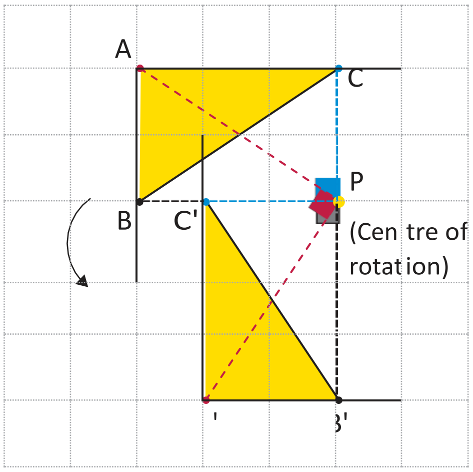
3. Lines have been drawn to join A to R, and A to R'.
   (a) Measure the distance from A toR.
   (b) Measure the distance from A toR'.
   (c) What do you notice about the distances in (a) and (b) above?
   (d) Measure the size of the angle RAR'. What do you notice?



4. In any of the diagrams in questions 1 to 3 above, measure the sides of the original triangle and the corresponding sides of the image. What do you notice?

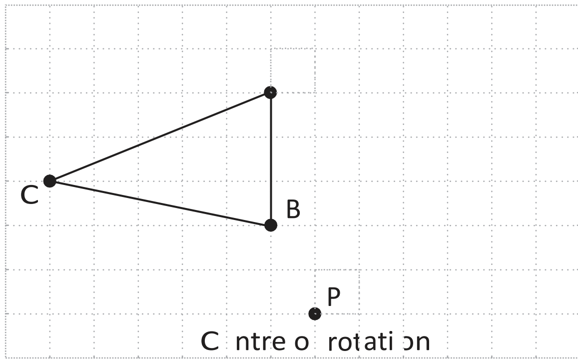
Properties of rotation

- The distance from the centre of rotation to any point on the original is equal to the distance from the centre of rotation to the corresponding point on the image. In the diagram on the right: PA = PA', PB = PB' and PC = PC'.
- The angle formed by the connecting lines between any point on the original figure, the centre of rotation and the corresponding point on the image is equal to the angle of rotation. For example, if the image is rotated through 90°, this angle will be equal to 90°. If the image is rotated through 45°, the angle will be 45°.
- When a figure is rotated, its shape and size do not change.



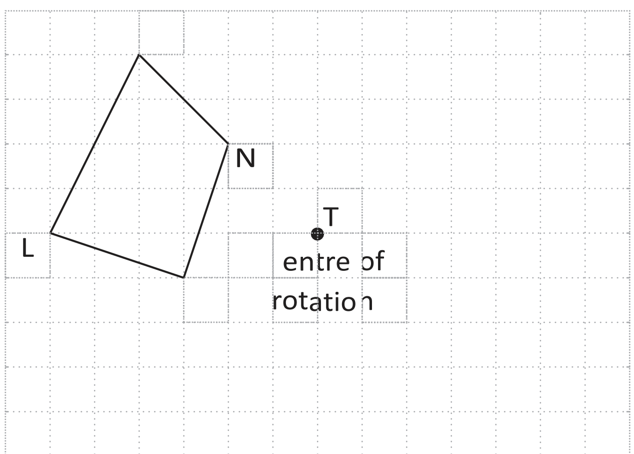
practise rotating figures

1. Rotate triangle ΔABC 90° clockwise about point P as follows:
   (a) Plot the image of each vertex on grid paper. Remember:
   - The image point must be the same distance from P as the original point.
   - The angle that is formed between the line connecting an original point to point P and the line connecting its image point to point P must be the same as the angle of rotation. In this case, it must be 90°.

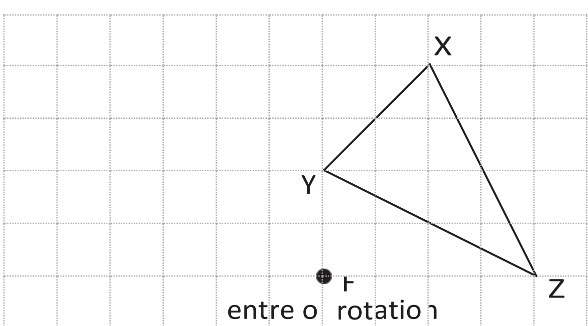


   (b) Join the image points to create ΔA'B'C'.

2. On grid paper, rotate KLMN 180° about point T.



3. On grid paper, rotate ΔXYZ 90° anticlockwise about point F.

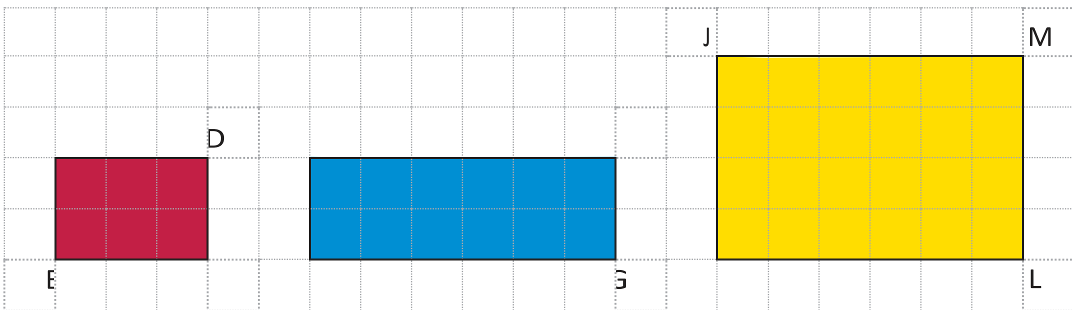


16.6 Enlarging and reducing figures

Enlarging a figure means that we make it bigger in a specific way. **Reducing** a figure means that we make it smaller in a specific way. Enlarging or reducing figures is also called **resizing**.

investigate the properties of enlargements and reductions

1. Look at the following rectangles and answer the questions below.



(a) Rectangle EFGH:

How many times is FG longer than BC?

How many times is EF longer than AB?

(b) Rectangle JKLM:

How many times is KL longer than BC?

How many times is JK longer than AB?

When the lengths of **all the sides** of a figure are **multiplied by the same number** to produce a second figure, the second figure is an **enlargement** or **reduction** of the first figure.

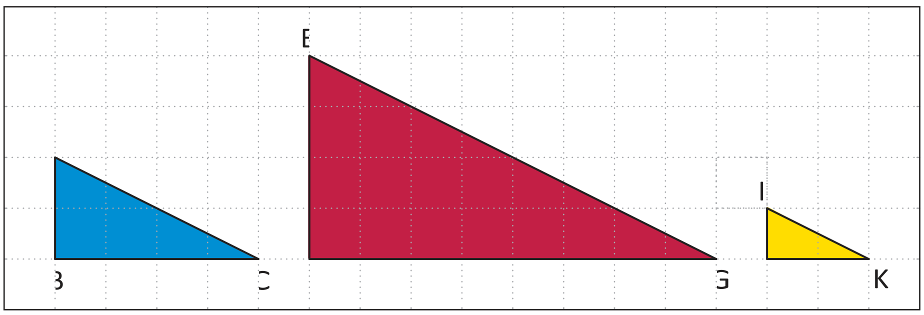
The number by which the sides are multiplied to produce an enlargement or reduction is called the **scale factor**. The scale factor in question 1(b) above is 2. We say that figure ABCD has been enlarged (or resized) by a scale factor of 2 to produce figure JKLM.

Figure EFGH is not an enlargement of figure ABCD because not *all* its sides have been increased by the *same* scale factor.

The scale factor

- When the scale factor is 1, the image is the same size as the original.
- When the scale factor is <1, the image is a reduction. For example, if the scale factor is $\frac{1}{2}$ or 0,5, each side of the image is half the length of its corresponding side in the original figure.
- When the scale factor is >1, the image is an enlargement. For example, if the scale factor is 2, each side of the image is double the length of its corresponding side in the original figure.

2. Look at the following triangles and answer the questions that follow.



(a) How many times is:
- FG longer than BC?
- EF longer than AB?
- EG longer than AC?
- JK shorter than BC?
- IJ shorter than AB?
- IK shorter than AC?

(b) Is ΔEFG an enlargement of ΔABC? Explain your answer.
(c) Is ΔIJK a reduction of ΔABC? Explain your answer.

Similar figures

When figures are enlarged or reduced, the enlarged or reduced image is **similar** to the original figure. ΔABC, ΔEFG and ΔIJK above are all similar. We also say that the lengths of their corresponding sides are **in proportion**.

If two or more figures are **similar**:
- their corresponding anglesare equal, and
- their corresponding sides are longer or shorter by the same scale factor.

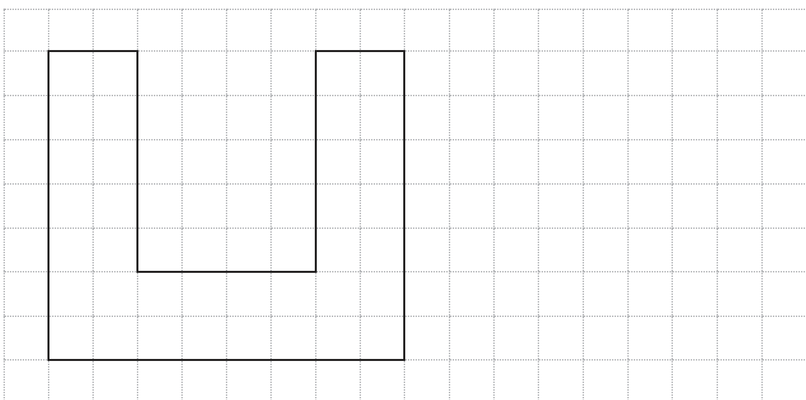
practise resizing figures

1. State whether the following scale factors will produce a larger or smaller image:

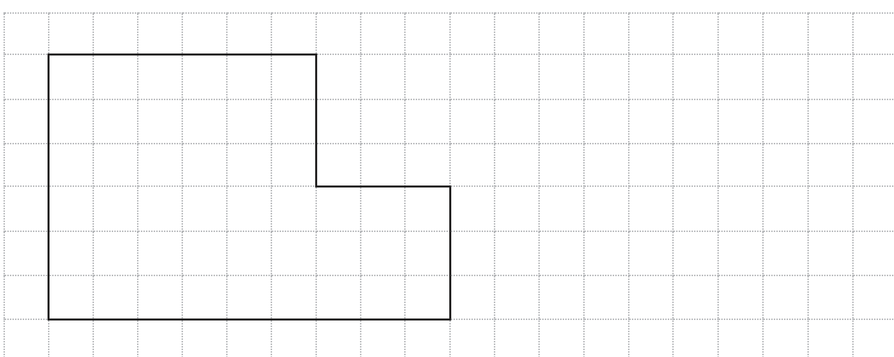
   (a) 5 (b) 0,25
   (c) 1,2 (d) $\frac{3}{8}$

2. On grid paper, enlarge the triangle below with a scale factor of 2.

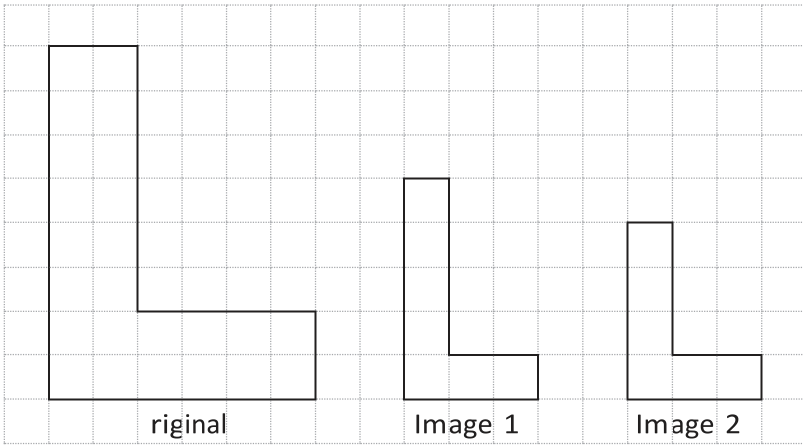
3. On grid paper, resize the following figure. Use a scale factor of 0,5.



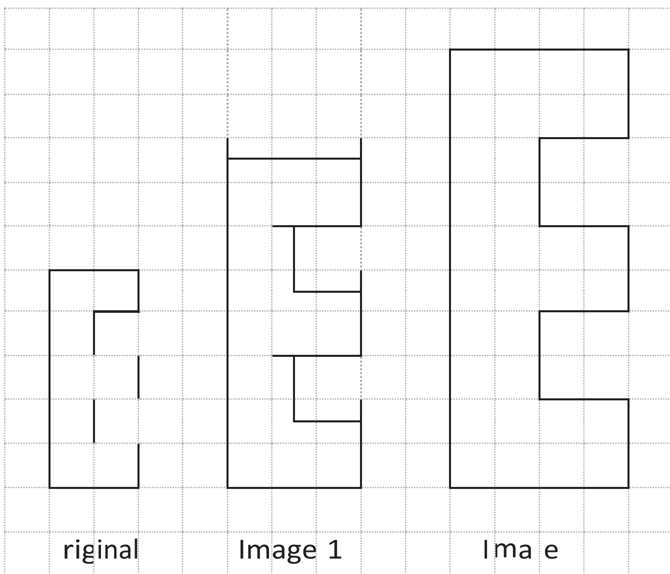
4. On grid paper, resize the figure below. Use a scale factor of $\frac{1}{3}$.



5. (a) Which image below is similar to the original?
(b) State the scale factor by which it has been resized.



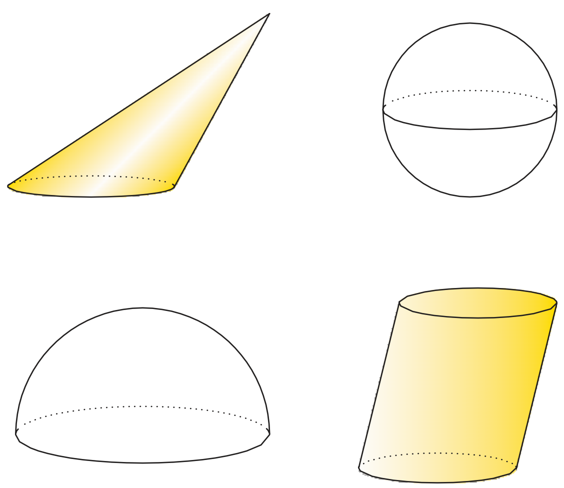
6. What scale factors were used to produce image 1 and image 2 from the original?



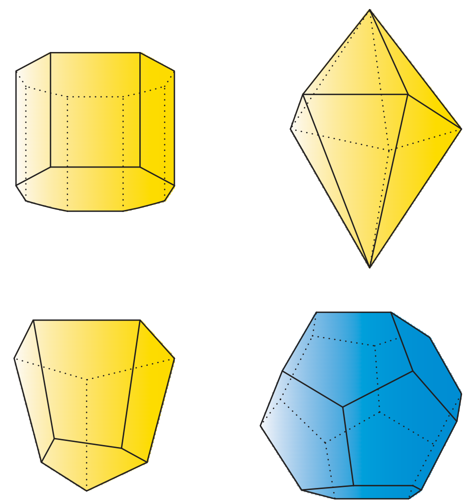
Chapter 17
Geometry of 3D objects

17.1 Classifying 3D objects

There are two main groups of objects with **three dimensions** (length, width and height), namely those with **curved surfaces** and those with **flat surfaces**. **Spheres** (balls), **cylinders** and **cones** are examples of objects with curved surfaces. Objects with only flat surfaces are called **polyhedra**.



Examples of objects with curved surfaces



Examples of objects with flat surfaces only

what is a polyhedron?

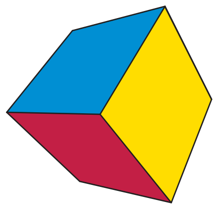
A **polyhedron** is a three-dimensional object (or 3D object) made of flat surfaces only. It has no curved surfaces. It consists of faces, edges and vertices.

A **face** is the flat surface of a 3D object.

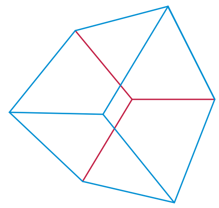
An **edge** is the segment where two faces of a polyhedron intersect.

A **vertex** is the point where the edges meet.

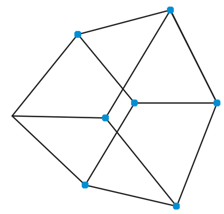
We say: one **polyhedron**; two or more **polyhedra**.
We say: one **vertex**; two or more **vertices**.



Faces

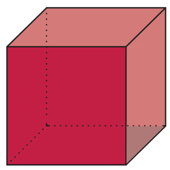


Edges

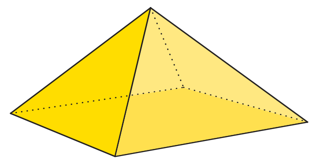


Vertices

A **cube** has six faces, 12 edges and eight vertices.

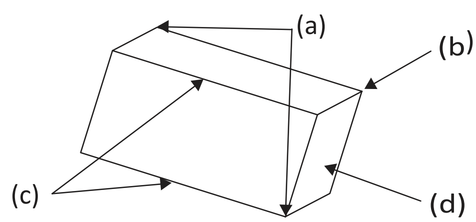


This **pyramid** has five faces, eight edges and five vertices.

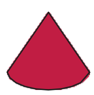
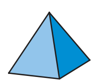
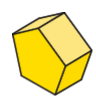
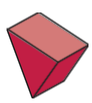
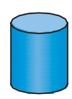
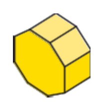


identifying and describing 3d objects

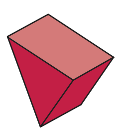
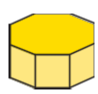
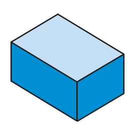
1. Identify parts (a) to (d) on the figure correctly.



2. Which of the following objects are polyhedra?

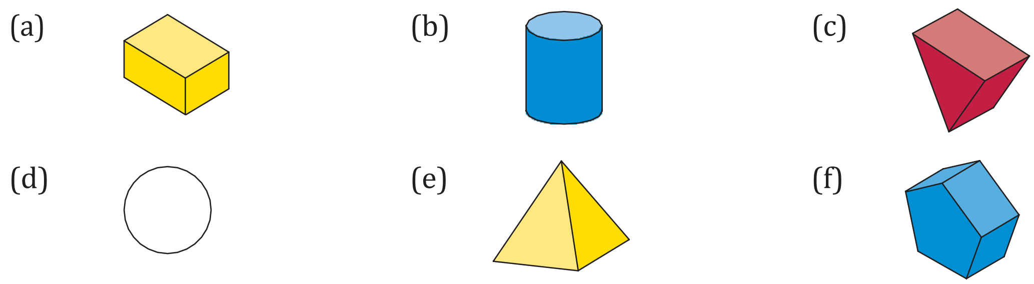
(a) 
(b) 
(c) 
(d) 
(e) 
(f) 

3. How many faces, edges and vertices do each of the following polyhedra have?

(a) 
(b) 
(c) 

4. Six learners each used play dough to make a 3D object. Use the descriptions on the next page to match each 3D object to the learner who made it.

(a) (b) (c) (d) (e) (f)



- Tumi's object has six vertices and five faces.
- Debbie's object has eight vertices and 12 edges.
- Brad made an object that has seven faces and ten vertices.
- Xola made an object with no vertices.
- Mpuka's object has eight edges and five faces.
- Maggie made an object with two circles and no vertices.

17.2 Prisms and pyramids

difference between prisms and pyramids

Prisms and **pyramids** are two special groups of polyhedra.

Prisms

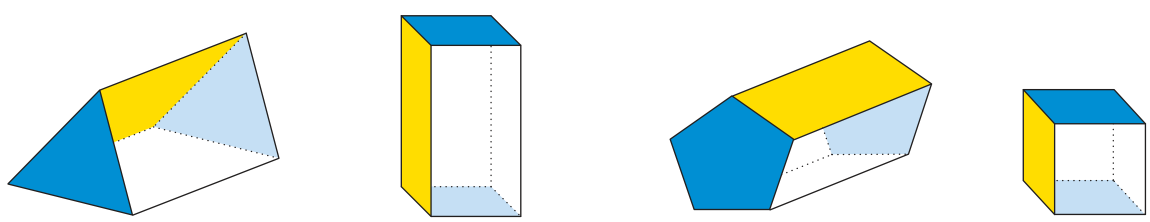
A **prism** is a polyhedron with two faces that are congruent and parallel polygons. These faces are called **bases** and they are connected by **lateral faces** that are parallelograms.

congruent means exactly the same shape and size.
lateral faces are faces that aren't bases.

In the case of **right prisms** the bases are connected by rectangles which are perpendicular to the base and the top. This means the lateral faces of a right prism make a 90° angle with the bases.

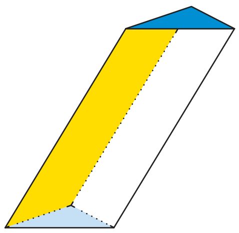
A prism is named according to the shape of its base. So a prism whose base is a triangle is called a triangular prism; a prism whose base is a rectangle or square is called a rectangular prism; and a prism with a pentagonal base is called a pentagonal prism.

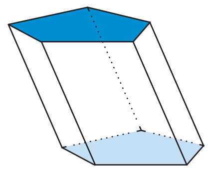
Any pair of faces in a prism that are congruent and parallel can be the bases of that prism. A cube is a special type of prism. It has six congruent faces; therefore any of its faces can be a base.



Triangular prism *Rectangular prism* *Pentagonal prism* *Cube*

These are two **oblique prisms** – the one on the left is a triangular prism and the one on the right is an oblique pentagonal prism.

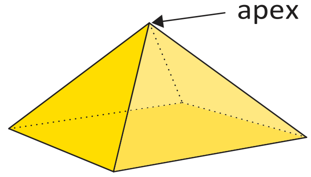




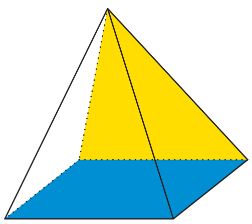
Pyramids

A **pyramid** has only one base. The lateral faces of a pyramid are always triangles. These triangles meet at the same vertex at the top. This vertex is called the **apex** of the pyramid. In a **right pyramid**, the lateral faces are isosceles triangles.

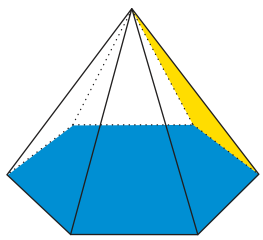
There are many types of pyramids. The type of pyramid is determined by the shape of its base. For example, a triangular-based pyramid has a triangle as its base, a square-based pyramid has a square as its base and a hexagonal-based pyramid has a hexagon as its base.



A triangular-based pyramid is also called a **triangular pyramid**; a square-based pyramid is also called a **square pyramid**; a hexagonal-based pyramid is also called a **hexagonal pyramid**, etc.

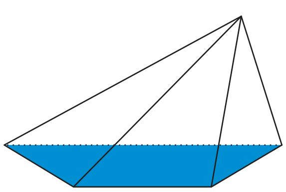


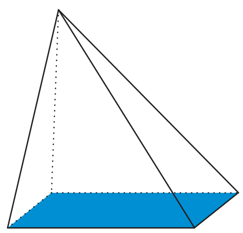
Square-based pyramid
It has five faces:
one square,
four triangles



Hexagonal-based pyramid
It has seven faces:
one hexagon,
six triangles

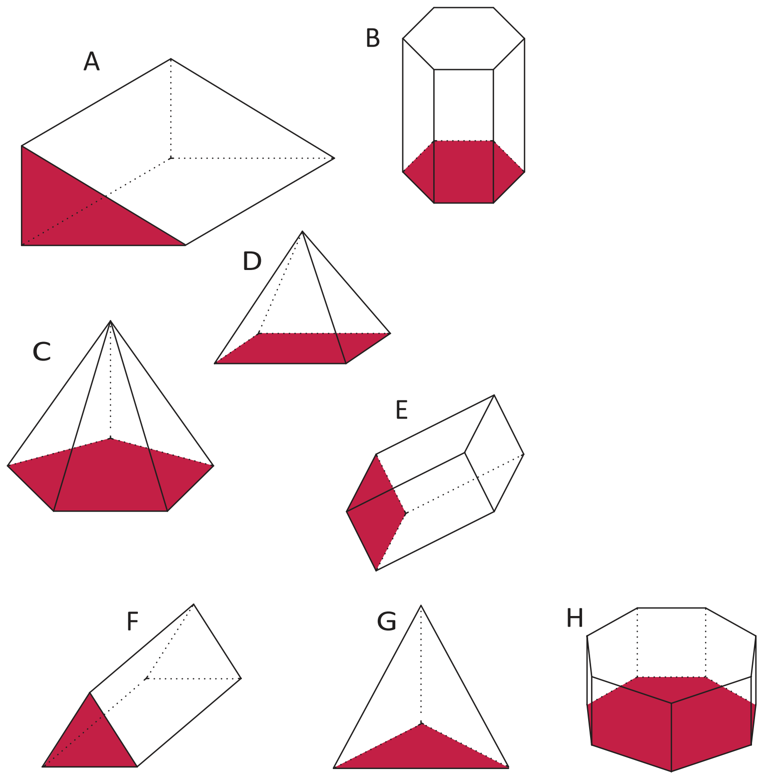
If a pyramid is not a right pyramid, it is called an **oblique pyramid**, like the two shown below. The lateral faces of an oblique pyramid are not necessarily isosceles triangles.



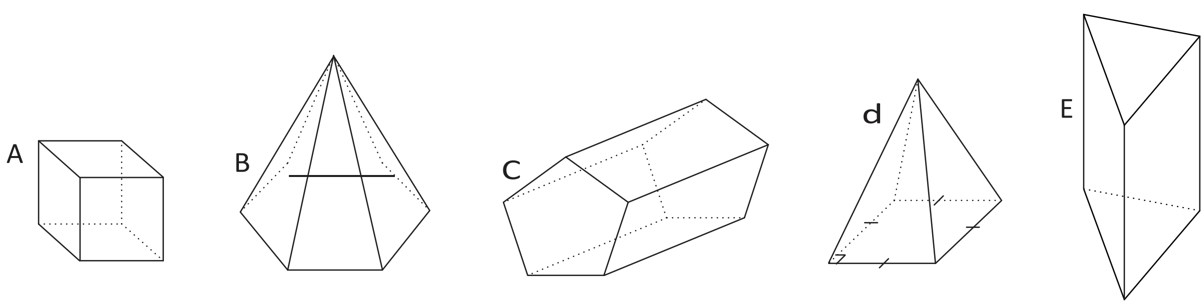


identifying prisms and pyramids

1. Copy the following figures and shade the base of each figure. Write down whether it is a prism or a pyramid. In some cases there is more than one possibility for the base.



2. Join each 3D object with its correct name. Write the letter and the 3D object's name.



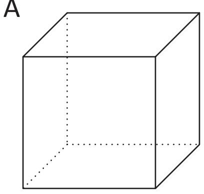
Hexagonal-based pyramid | *Triangular prism* | *Square-based pyramid* | *Cube* | *Pentagonal prism*

17.3 Describing, sorting and comparing 3D objects

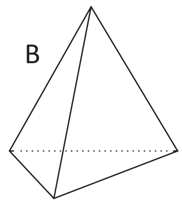
practise describing and classifying 3d objects

1. For each of the following 3D objects, name the object and describe the number of faces it has and the shapes of these faces.

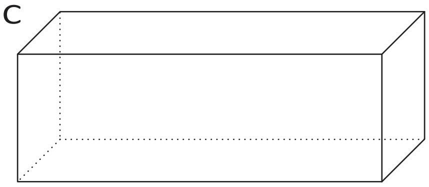
A



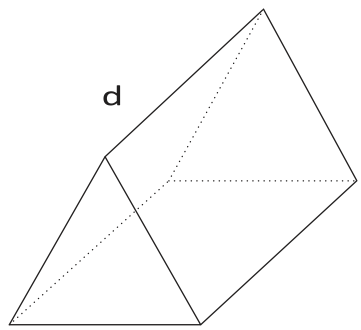
B



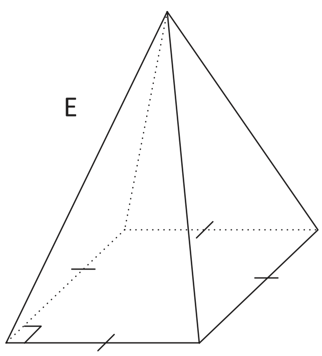
C



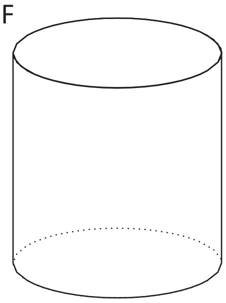
d



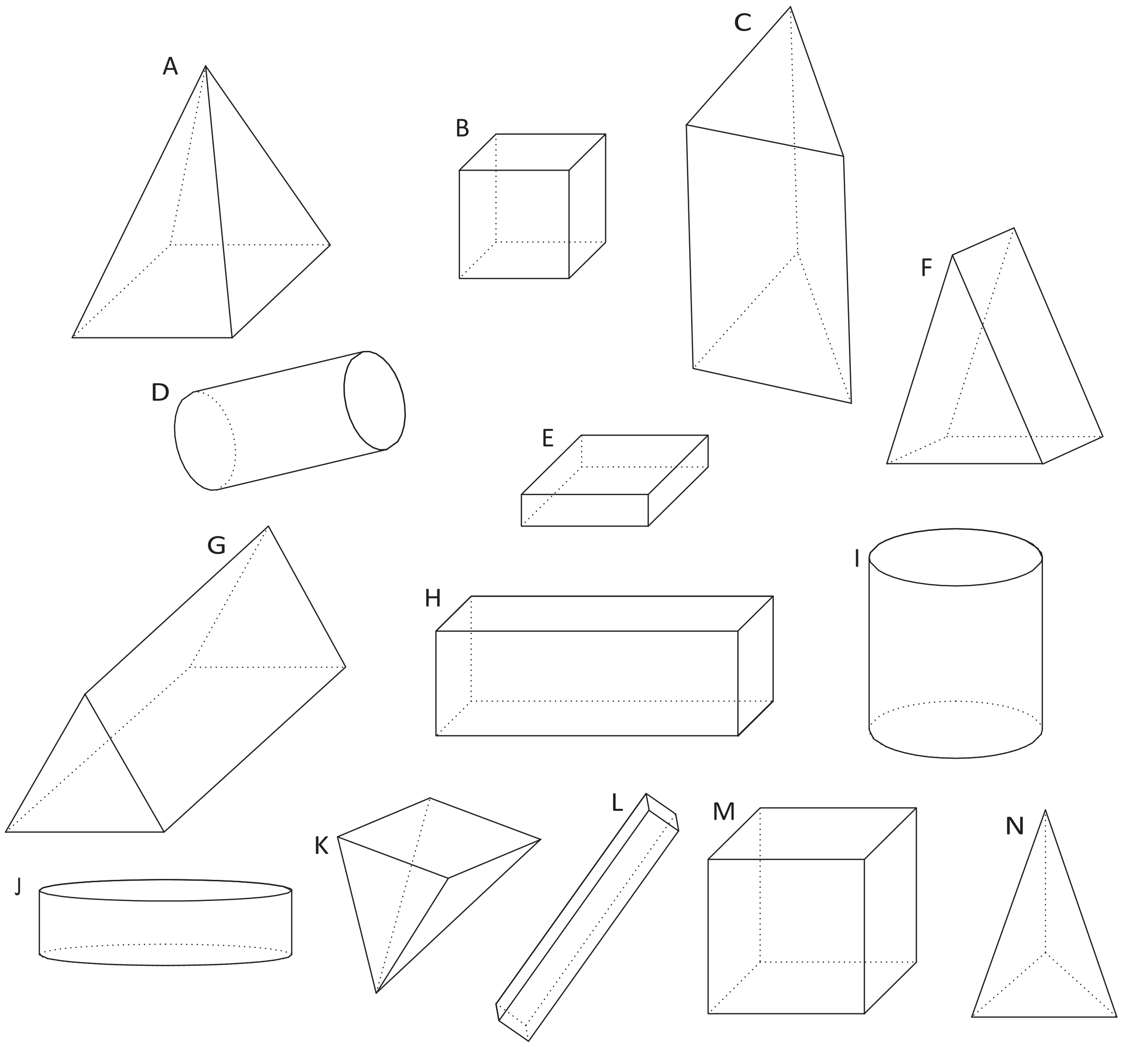
E



F



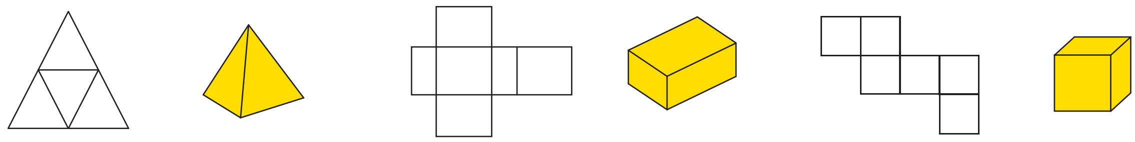
2. (a) Sort the 3D objects below into the three groups, namely prisms, pyramids and cylinders. Write down the correct letter under each group.
   (b) Further divide the prisms into three groups (cubes, rectangular prisms and triangular prisms). Write the letter under each group.



17.4 Nets of 3D objects

what is a net?

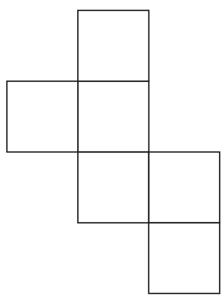
In mathematics, a **net** is a flat pattern that can be folded to form a 3D object. Different 3D objects have different nets. Sometimes the same 3D object can have different nets. Here are examples of different 3D objects and their nets.



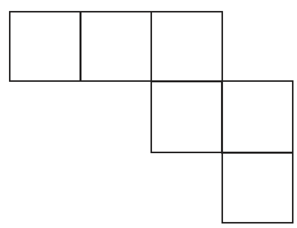
Triangular pyramid *Rectangular prism* *Cube*

In this section, you are going to focus on the net of a cube. In order for a net to form a cube, it must consist of six equal squares. But not all net patterns that consist of six squares will fold into a cube.

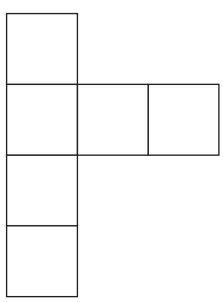
Only one of the nets below will fold into a cube. Write down which one it is.



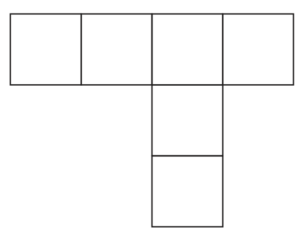
A



B



C

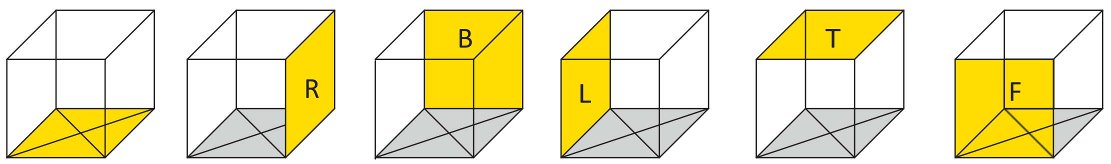


D

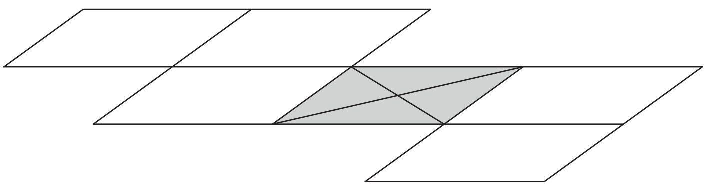
a cube or not a cube

In order to decide whether or not a net will fold into a cube, you have to imagine what will happen when you fold the net. Read through the following steps.

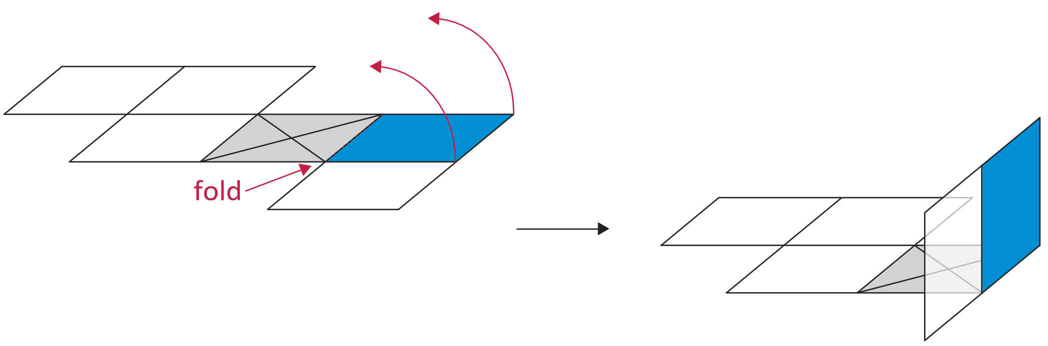
1. We can label the faces of a cube: bottom face (X), right face (R), back face (B), left face (L), top face (T) and front face (F). We will use these terms in the rest of the steps.



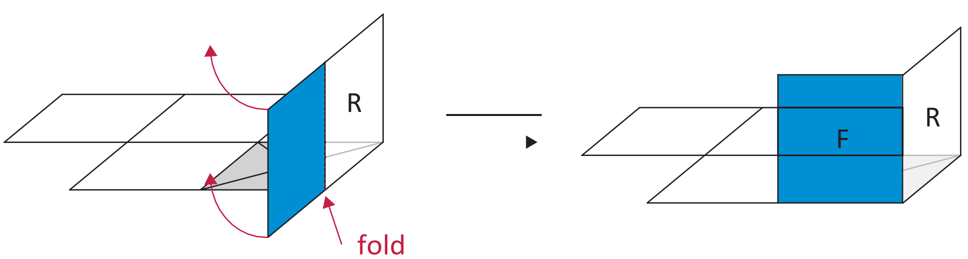
2. Start by choosing one square of the net as the bottom face (marked with an X).



3. Look at the square to the right of the X. (It is coloured blue.) If you fold the net on the red line, the blue square will be the right face of the cube.



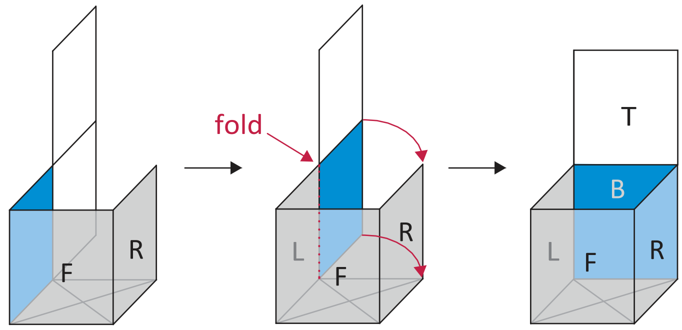
4. Look at the next square to the left of the right face. If you fold this square on the red line, it will become the front face of the cube.



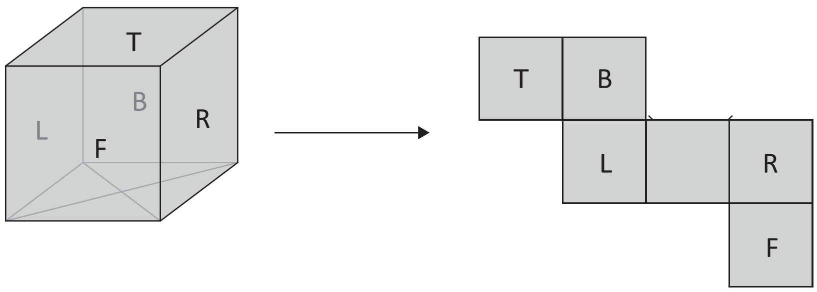
5. Look at the square to the left of the front face. It will fold to become the left face of the cube.

6. The square to the left of the left face of the cube will become the back face of the cube.

7. The last square will form the top.



8. Therefore you can label the squares on the net as follows:

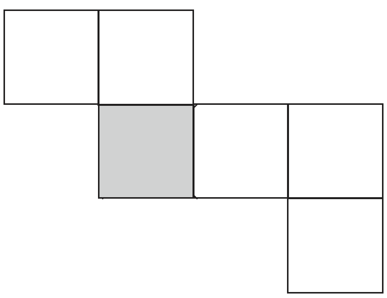


9. Since each square on the net corresponds with a face of the cube, this net can be folded into a cube.

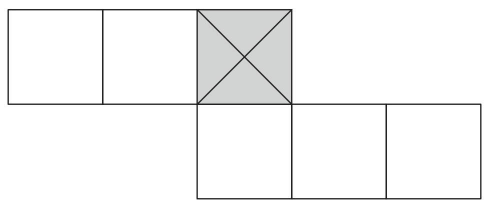
identifying the nets of a cube

1. For each of the following nets, determine whether it will fold into a cube or not by copying and labelling the squares to match the faces of a cube.

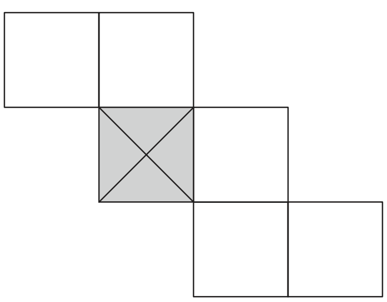
(a)



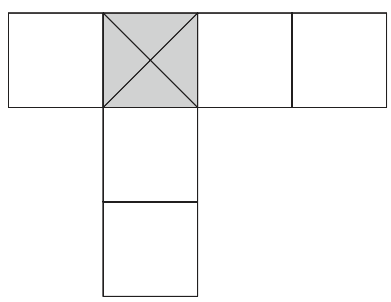
(b)



(c)



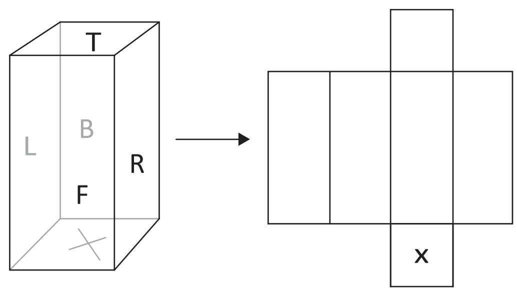
(d)



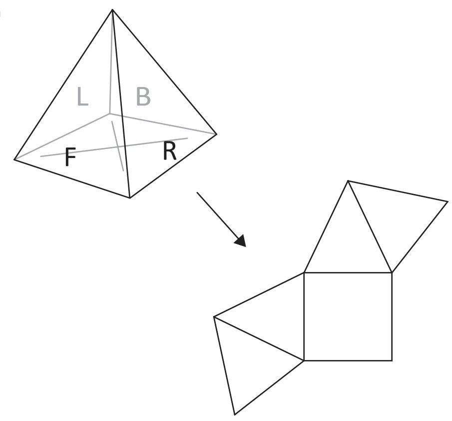
nets of other 3d objects

1. In each of the following cases, copy and label the faces on the net according to the labels on the 3D object.

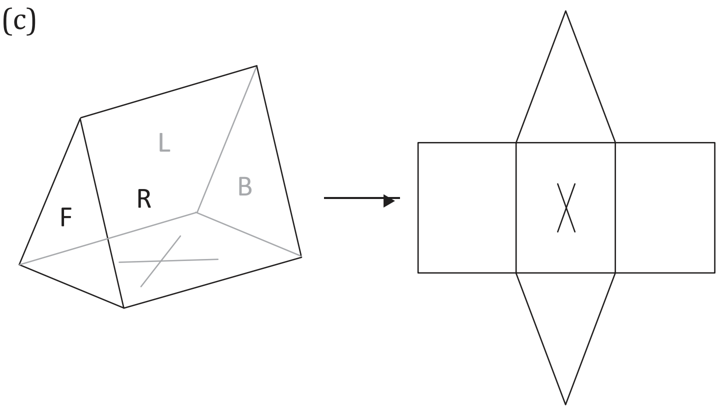
(a)



(b)

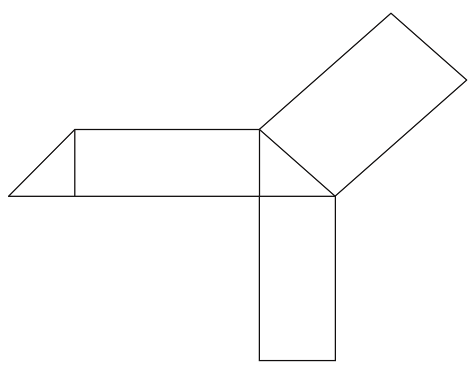


(c)

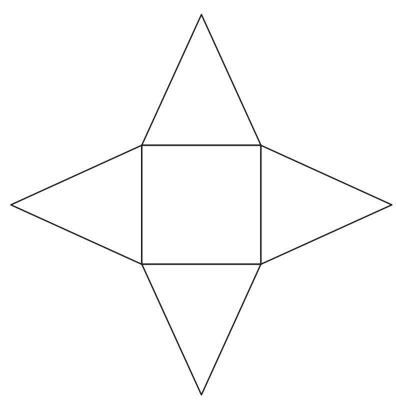


2. Decide whether the following nets will form 3D objects.

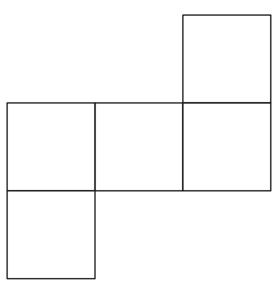
(a)



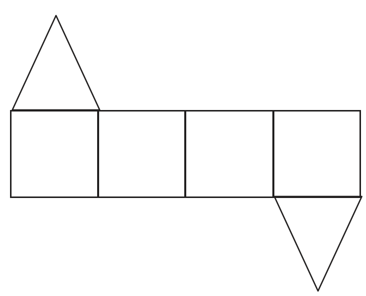
(b)



(c)

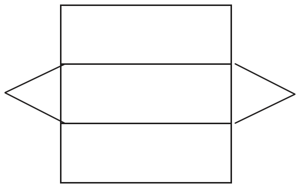


(d)

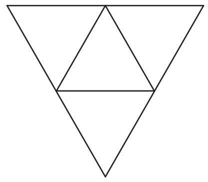


3. Use the nets to identify each of the following 3D objects.

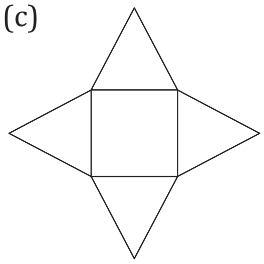
(a)



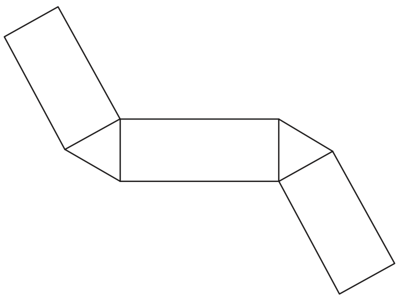
(b)



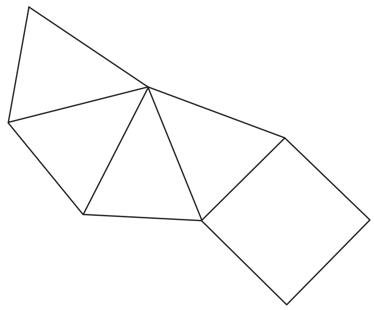
(c)



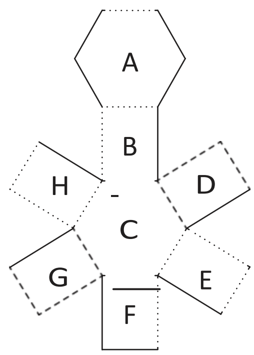
(d)



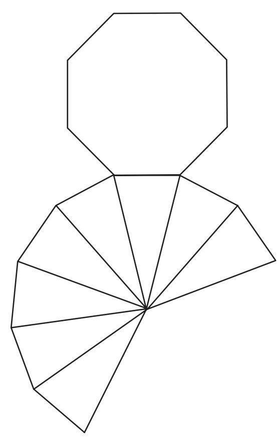
(e)



4. (a) Identify the shapes that the net on the right consists of.
   (b) How many rectangular faces does the net have?
   (c) How many other shapes does the net have? What are they?
   (d) Will this form a pyramid or a prism?
   (e) How do you know?
   (f) Name the 3D object that the net will form.



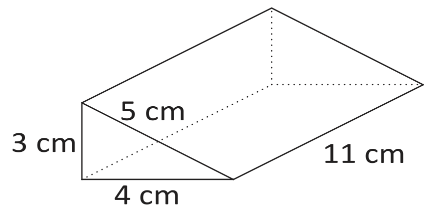
5. Answer the following questions about thisnet.
   (a) Which shapes does this net consistof?
   (b) How many triangular faces does the net have?
   (c) How many other shapes does the net have?
   (d) Will this form a pyramid or a prism?
   (e) How do you know?
   (f) Name the 3D object that the net will form.



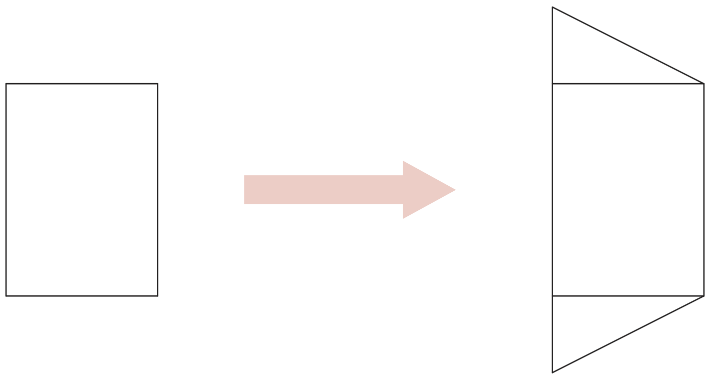
17.5 Using nets to construct cubes and prisms

how to draw a net of a prism

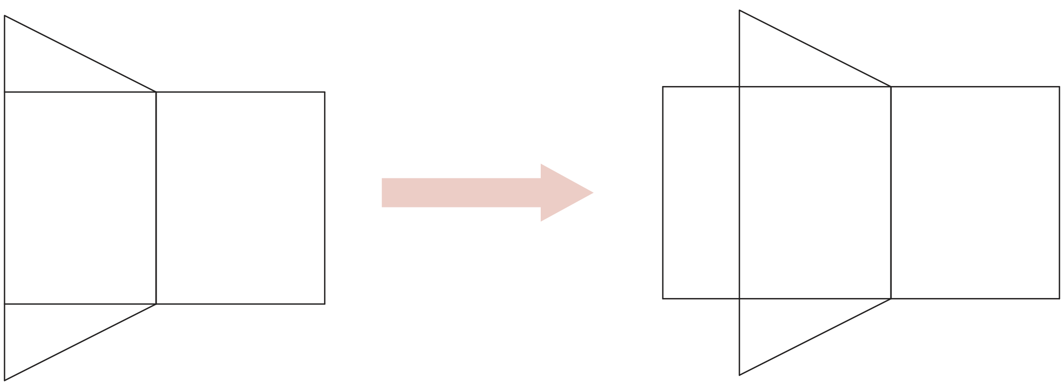
Look at the triangular prism alongside. Its faces are two right-angled triangles and three rectangles. (Since the base has three edges, there will be three rectangles.)



1. Draw a rectangle, which will be the bottom of the prism. Then add a right-angled triangle at the top and a mirror image of the triangle at the bottom of the rectangle.



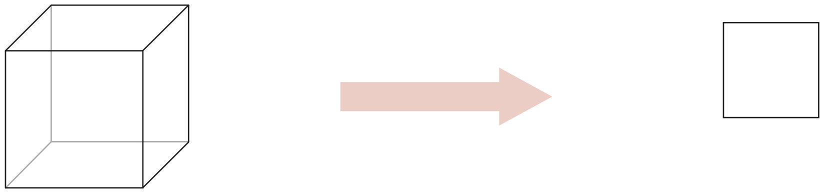
2. Draw the other two rectangles next to the centre one. The rectangle on the right must fit onto the side opposite the right angle in the triangle when it is folded, so that rectangle will be the biggest. The rectangle on the left is the smallest of the three rectangles.



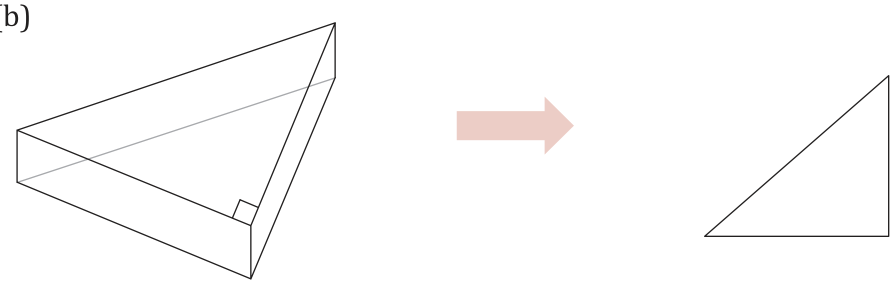
practise drawing nets and constructing 3d models

1. Copy and complete the followingnets:

   (a)

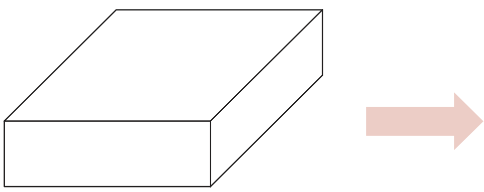


   (b)

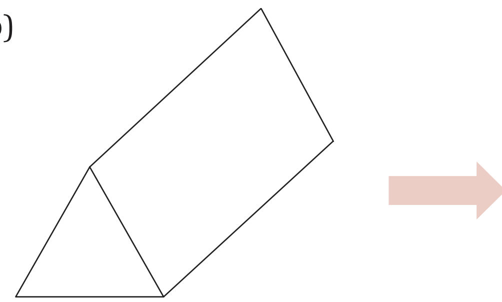


2. Draw nets for the following objects:

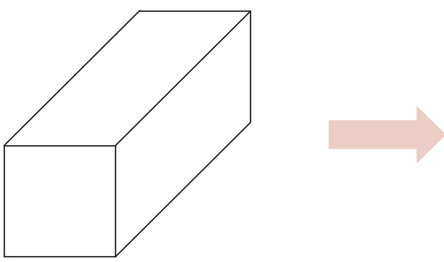
   (a)



   (b)



   (c)



3. (a) Copy each of the nets that you drew in questions 1 and 2 above onto cardboard or paper.
   (b) Cut out the nets, and fold and paste them to make each 3D object.
   (c) Write down what you found difficult in making your 3D models and how you overcame this difficulty.

Chapter 18
Integers

18.1 The need for numbers called integers

Numbers are used for many different purposes. We use numbers to say how many objects there are in a collection, for example the number of desks in a classroom. For this purpose we use the **counting numbers** 1, 2, 3, 4 . . . Numbers are also used to describe size, for example the lengths of objects. For this purpose we need more than the counting numbers, we also need **fractions.** Another purpose of numbers is to indicate position, for example the position of the right end of the red line on the pictures below.

Numbers also occur as the solutions to equations, and the natural numbers and fractions do not provide solutions for all equations. For example, there is no natural number or fraction that is the solution to the equation $10 - x = 20$. The number that provides the solution to this equation must have the property that when you subtract it, it has the same effect as when you add 10!

With a view to have numbers that can serve more purposes than counting and measuring, mathematicians have decided to also think of another kind of numbers which are called **integers**. The integers include the natural numbers, but for each natural number, for example 24, there is also another number called the **additive inverse**. For example, −24 is the additive inverse of 24. When you add a number to its additive inverse, the answer is 0. For example, 24 + (−24) = 0.

saying how cold it is

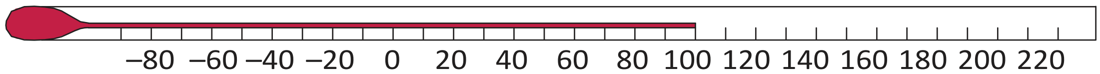
One of the uses of integers is for the measurement of temperature. If we say that the temperature is 0 when water freezes to become ice, we need numbers smaller than 0 to describe the temperature when it gets even colder than when water freezes. When water starts boiling, its temperature is 100 degrees on the scale called the Celsius scale.

Liquids expand when heated, and shrink when cooled down. So when it is warm, the liquid in a thin tube may almost fill the tube:

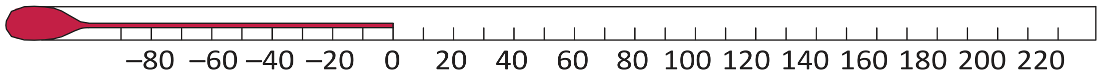
When it is cold, the column of liquid will be quite short.

This property of liquid is used to measure temperature, and an instrument like the one shown on the previous page is called a **thermometer**.

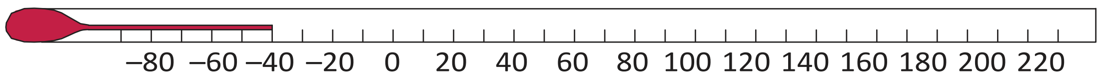
This is what a thermometer will show when it is put in water that is boiling. It shows a temperature of 100 degrees Celsius, which is written as 100 °C.



On the diagram below, you can see what a thermometer will show if it is in water that is starting to freeze. It shows a temperature of 0 °C.

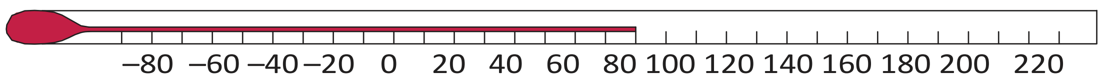


On the next diagram, you can see what a thermometer will show when the temperature is –40 °C, which is colder than any winter night you may have experienced.

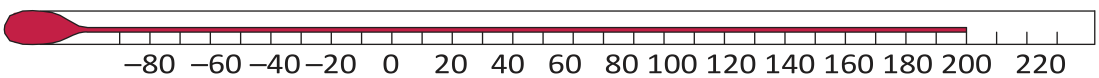


1. Write down the temperature that is shown on each of the thermometers below.

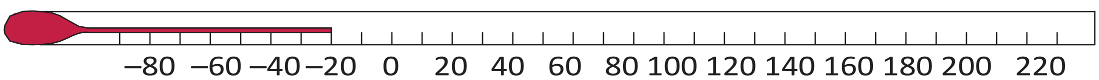
(a)



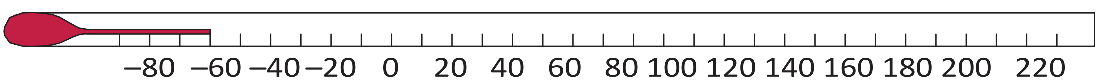
(b)



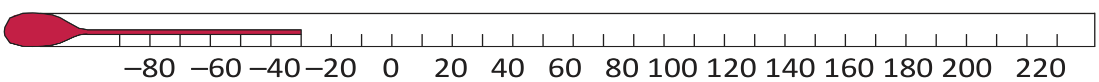
(c)



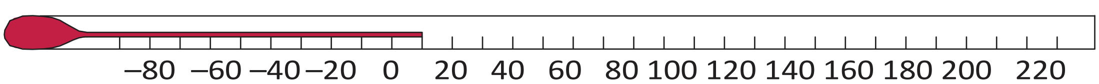
(d)



(e)



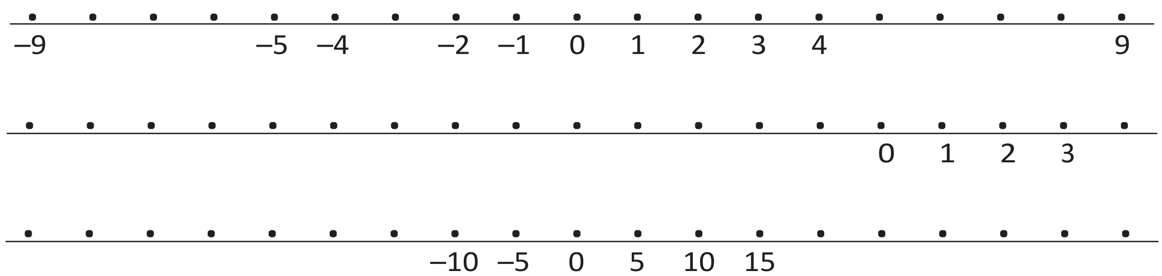
(f)



2. (a) The temperature of water in a pot is 20 °C. It is heated so that it gets 30 °C warmer. What is the temperature of the water now?
   (b) The temperature of water in a bottle is 80 °C. During the night it cools down to 30 °C. By how much has it cooled down?
   (c) In the middle of a very cold winter night the temperature outside is –20 °C. At nine o'clock in the morning it has become 30 degrees warmer. What is the temperature at nine o'clock?

3. (a) The temperature is 8 °C. What will the temperature be if it gets 10 degrees colder?
   (b) The temperature is 8 °C. What will the temperature be if it gets 20 degrees colder?
   (c) The temperature is −8 °C. What will the temperature be if it gets 10degrees warmer?
   (d) The temperature is −24 °C. What will the temperature be if it gets 10degrees warmer?

4. Some numbers are shown on the number lines below. Copy the number lines and fill in the missing numbers.



saying how much money it is

Simon is in Grade 5. He saved money in a tin. When he turned 10, his grandmother gave him R100. He also opened his savings tin on his tenth birthday and there was R260 in the tin. Simon was very happy. He said to himself: "I am very rich!"

Simon decides to buy some things that he has always wanted. This is what he decides to buy:

- a soccer ball at R160
- a pair of sunglasses at R180
- a book about animals at R90

1. How much money did Simon have in total on the day that he thought he was rich?

2. What is the total cost of the three items he wants to buy?

3. Simon decides to first buy the soccer ball only. How much money will he have after paying for the soccer ball?

4. How much money will Simon have if he buys the soccer ball and the sunglasses?

5. How much money will Simon have if he buys the soccer ball, the sunglasses and the book about animals?

Simon did these calculations while he was thinking about buying the various items:

R360 − R160 = R200
R200 − R180 = R20
R20 − R90 = (−) R70 ?

6. Fatima owns a small shop. One afternoon when she closed the shop, she had R120 cash, clients owed her R90, and she owed her suppliers R310. In Fatima's view her financial position was as follows: R120 + R90 − R310 = −R100.
   (a) On another day, Fatima ended the business day with R210 cash, clients owed her R180 and she owed her suppliers R160. What was her financial position?
   (b) On another day, Fatima ended the business day with R150 cash, clients owed her R130 and she owed her suppliers R460. What was her financial position?

About 500 years ago, some mathematicians proposed that a "negative number" may be used to describe the result in a situation like the above, where a number is subtracted from a number smaller than itself.

For example, we may say 10 − 20 = (−10)

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like health care is the profession of nurses and medical doctors.

This proposal was soon accepted by other mathematicians, and it is now used all over the world.

7. Copy the table below. Continue the lists of numbers to complete the table.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	−3	−20	150	0
9	90	6	−6	−18	125	−5
8	80	9	−9	−16	100	−10
7	70	12	−12	−14	75	−15
6	60	15	−15		50	−20
5	50					−25
4	40					
3	30					
2	20					
1	10					
0	0					
−1						

8. Calculate each of the following:
   (a) 16 − 20 (b) 16 − 30 (c) 16 − 40
   (d) 16 − 60 (e) 16 − 200 (f) 5 − 1 000

9. Jeminah has R200 in a savings account and R40 in her purse. Her brother owes her R50. How rich is she? In other words, how much money does she have?

10. Oops! Jeminah forgot that she borrowed R60 from her mother, and that she still has to pay R150 for a dress she bought last month. So how rich (or poor) is she really? In other words, how much money does she actually have?

11. In fact, Jeminah's financial situation is even worse. She has received an outstanding bill from her doctor, for R250. So how much money does she really have?

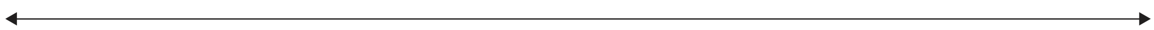
ordering and comparing integers

1. On a certain day the following minimum temperatures were provided by the weather bureau:

Bethlehem	−4 °C	Bloemfontein	−6 °C
Cape Town	7 °C	Dordrecht	−9 ºC
Durban	12 °C	Johannesburg	0 °C
Pretoria	4 °C	Queenstown	−1 °C

Arrange the temperatures from the coldest to the warmest.

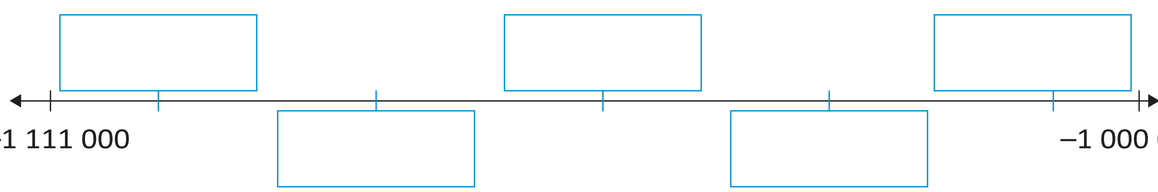
2. Copy the number line and place the following numbers on the number line as accurately as you can: 50; −2; −23; 5; −36



3. Copy the number lines. In each case, write the numbers in the boxes provided:

(a) 125 000; −178 000; −100 900; 180500

(b) −1 055 500; −1 010 100; −1 100 100; −1 032 800; −1 077 500



4. Write > or < to indicate which number is the smaller of the two.

(a) 978 543 ___ 978 534
(b) −1 043 724 ___ −1 034 724
(c) −864 026 ___ −864 169
(d) −103 232 ___ −104 326
(e) −710 742 ___ 710 741
(f) −904 700 ___ −904 704

18.2 Finding numbers that make statements true

The numbers 1, 2, 3, 4 and so on that we use for counting are called the **natural numbers**. Natural numbers are **whole numbers** – they do not contain fraction parts.

1. Is there a natural number that can be put in the brackets below to make the statement true?

   12 + (__) = 17

2. In each case below, copy the statement and insert a natural number in the space between the brackets that will make the statement true.

   (a) 15 + (__) = 21
   (b) 15 − (__) = 10
   (c) (__) + 10 = 34
   (d) (__) − 10 = 34
   (e) 3 × (__) = 18

Here is a different way to ask the same questions:
(a) What is x if $15 + x = 21$?
(b) What is x if $15 - x = 10$?
(c) What is x if $x + 10 = 34$?
(d) What is x if $x - 10 = 34$?
(e) What is x if $3 \times x = 18$?

3. (a) Can you think of a natural number that will make this statement true?

   2 × (....) = 5

   (b) Can you think of any other number that will make the statement true?

4. (a) Can you think of a natural number that will make this statement true?

   8 + (....) = 5

   (b) Can you think of any other number that will make the statement true?

We normally think of adding as making something bigger. Question 4(a) requires us to change our mind about this. We have to consider the possibility that adding a number may make something smaller.

We are looking for a number that will make the following statement true:

8 + (....) = 5

Consider this plan:
Let us agree that we will call this number *negative 3* and write it as (−3).
If we agree to this, we can say 8 + (−3) = 5.

This may seem a bit strange to you. You do not have to agree now. But even if you do not agree, let us explore how this plan may work for other numbers. What answers will a person who agrees to the plan give to the following question?

5. Calculate each of the following:

   (a) 10 + (−3) (b) 12 + (−3)
   (c) 12 + (−5) (d) 10 + (−9)
   (e) 8 + (−8) (f) 1 + (−1)

What may each of the following be equal to?
5 + (−8)
(−5) + (−8)

You might agree that:

$5 + (-5) = 0$ $\quad$ $10 + (-10) = 0$ $\quad$ and $\quad$ $20 + (-20) = 0$

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (−3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0. For example, $120 + (-120) = 0$

So, the **set of integers** consists of all the natural numbers and their additive inverses and zero.

The number zero is regarded as an integer.

6. Write the additive inverse of each of the following numbers:

(a) 24 (b) −24

(c) −103 (d) 2 348

The idea of additive inverses may be used to explain why $8 + (-5)$ is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

7. Use the idea of additive inverses to explain why each of these statements is true:

(a) $43 + (-30) = 13$ (b) $150 + (-80) = 70$

8. Calculate each of the following:

(a) $10 + 4 + (-4)$ (b) $10 + (-4) + 4$

(c) $3 + 8 + (-8)$ (d) $3 + (-8) + 8$

Natural numbers can be arranged in any order to add and subtract them. It would make things easy if we agree that this should also be the case for negative numbers.

9. Calculate each of the following:

(a) $18 + 12$ (b) $12 + 18$

(c) $2 + 4 + 6$ (d) $6 + 4 + 2$ (e) $2 + 6 + 4$

(f) $4 + 2 + 6$ (g) $4 + 6 + 2$ (h) $6 + 2 + 4$

(i) $6 + (-2) + 4$ (j) $4 + 6 + (-2)$ (k) $4 + (-2) + 6$

(l) $(-2) + 4 + 6$ (m) $6 + 4 + (-2)$ (n) $(-2) + 6 + 4$

10. Calculate each of the following:

(a) $(-5) + 10$ (b) $10 + (-5)$

(c) $(-8) + 20$ (d) $20 - 8$

(e) $30 + (-10)$ (f) $30 + (-20)$

(g) $30 + (-30)$ (h) $30 + (-40)$

(i) $10 + (-5) + (-3)$ (j) $(-5) + 7 + (-3) + 5$

(k) $(-5) + 2 + (-7) + 4$

11. In each case find the number that makes the statement true. Give your answer by writing a closed number sentence.
    (a) 20 + (an unknown number) = 50
    (b) 50 + (an unknown number) = 20
    (c) 20 + (an unknown number) = 10
    (d) (an unknown number) + (−25) = 50
    (e) (an unknown number) + (−25) = (−50)

12. Use the idea of additive inverses to explain why each of the following statements is true:
    (a) 43 + (−50) = −7 (b) 60 + (−85) = −25

Statements like these are also called number sentences.
An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**, for example: 8 − (a number) = 10.
A **closed number sentence** is where all the numbers are known, for example: 8 + 2 = 10.

statements that are true for many different numbers

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

18.3 Adding and subtracting integers

properties of integers

1. Calculate:
    (a) 80 + (−60) (b) 500 + (−200) + (−200)

2. (a) Do you agree that 20 + (−5) = 15?
    (b) What do you think 20 − (−5) should be?

3. (a) Is 100 + (−20) + (−20) = 60, or does it equal something else?
    (b) What do you think (−20) + (−20) should be equal to?

We normally think of addition and subtraction as actions that have opposite effects: what the one does is the opposite or **inverse** of what the other does.

4. Copy and complete the following as far as you can:

(a)	(b)	(c)
5 − 9 =	5 + 9 =	9 − 3 =
5 − 8 =	5 + 8 =	8 − 3 =
5 − 7 =	5 + 7 =	7 − 3 =

5 − 6 =	5 + 6 =	6 − 3 =
5 − 5 =	5 + 5 =	5 − 3 =
5 − 4 =	5 + 4 =	4 − 3 =
5 − 3 =	5 + 3 =	3 − 3 =
5 − 2 =	5 + 2 =	2 − 3 =
5 − 1 =	5 + 1 =	1 − 3 =
5 − 0 =	5 + 0 =	0 − 3 =
5 − (−1) =	5 + (−1) =	(−1) − 3 =
5 − (−2) =	5 + (−2) =	(−2) − 3 =
5 − (−3) =	5 + (−3) =	(−3) − 3 =
5 − (−4) =	5 + (−4) =	(−4) − 3 =
5 − (−5) =	5 + (−5) =	(−5) − 3 =

5. Calculate each of the following:
   (a) 20 − 20 (b) 50 − 20
   (c) (−20) − (−20) (d) (−50) − (−20)

6. In each case, suggest a number that may make the statement true. Also give an argument to support your proposal.
   (a) 20 + (a number) = 8 (b) 20 + (a number) = 28
   (c) 20 − (a number) = 28 (d) 20 − (a number) = 12

some history

The following statement is true if the number is 5:

15 − (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

But to go from 15 to 10 you have to **subtract 5**.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

"Let us just decide, and agree amongst ourselves, that the number we call *negative 5* will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5."

This means that the mathematicians agreed that 15 + (−5) is equal to 15 − 5.

Stated differently, instead of adding *negative 5* to a number, you may subtract 5.

We may agree that subtracting a negative number has the same effect as adding the additive inverse of the negative number. If we stick to this agreement, the following two calculations should have the same answer:

$10 - (-7)$ and $10 + 7$

7. Calculate.
   (a) $20 - (-10)$
   (b) $100 - (-100)$
   (c) $20 + (-10)$
   (d) $100 + (-100)$
   (e) $(-20) - (-10)$
   (f) $(-100) - (-100)$
   (g) $(-20) + (-10)$
   (h) $(-100) + (-100)$

8. Copy and complete the following as far as you can:

(a)	(b)	(c)
5 − (−9) =	(−5) + 9 =	9 − (−3) =
5 − (−8) =	(−5) + 8 =	8 − (−3) =
5 − (−7) =	(−5) + 7 =	7 − (−3) =
5 − (−6) =	(−5) + 6 =	6 − (−3) =
5 − (−5) =	(−5) + 5 =	5 − (−3) =
5 − (−4) =	(−5) + 4 =	4 − (−3) =
5 − (−3) =	(−5) + 3 =	3 − (−3) =
5 − (−2) =	(−5) + 2 =	2 − (−3) =
5 − (−1) =	(−5) + 1 =	1 − (−3) =
5 − 0 =	(−5) + 0 =	0 − (−3) =
5 − 1 =	(−5) + (−1) =	(−1) − (−3) =
5 − 2 =	(−5) + (−2) =	(−2) − (−3) =
5 − 3 =	(−5) + (−3) =	(−3) − (−3) =
5 − 4 =	(−5) + (−4) =	(−4) − (−3) =
5 − 5 =	(−5) + (−5) =	(−5) − (−3) =

9. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer.
   (a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.
   (b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.

(c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.
(d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.
(e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.
(f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.
(g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.
(h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

properties of operations

1. Calculate the following:

(a) (−3) + (−5)	(b) (−5) + (−3)
(c) 5 + (−7)	(d) (−7) + 5
(e) (−13) + 17	(f) 17 + (−13)
(g) 15 + 19	(h) 19 + 15
(i) (−21) + (−15)	(j) (−15) + (−21)

In Chapter 1 (which was about whole numbers) we said that:
addition is commutative, which means that: the numbers can be swopped around. Or, in symbols: $a + b = b + a$, where a and b are whole numbers.

2. (a) Would you say addition is also commutative when the numbers are integers?
   (b) Explain your answer.

3. Calculate the following:

(a) 9 − 5	(b) 5 − 9
(c) (−7) − 3	(d) 3 − (−7)
(e) 15 − (−12)	(f) (−12) − 15
(g) (−40) − (−23)	(h) (−23) − (−40)

4. (a) Do you think subtraction is commutative?
   (b) Explain your answer.

In Chapter 1 we also said that: when three or more whole numbers are added, the order in which you perform the calculations makes no difference. We say that: **addition is associative**.

5. Do you think addition is also associative when we work with integers? Investigate.

Chapter 19

Numeric patterns

19.1 Investigating and extending numeric patterns

patterns in two directions

1. The numbers in each row of the table form a sequence, but not all of the numbers are given.

A						4	6	8	10					
B						10	8	6	4					
C						5	8	11	14					
D						20	17	13	8					

   (a) Copy the table and fill in the missing numbers.
   (b) What is the constant difference in sequence A?
   (c) What is the constant difference in sequence C?

2. The first term of a certain sequence is 100 and the constant difference is 20.
   (a) What is the second term, and the third term, and the fourth term?
   (b) What is the tenth term in this sequence?

A constant-difference sequence is formed by adding the constant difference each time to form the next term.

3. The first term of a certain sequence is 100 and the constant difference is −20.
   (a) What is the second term, and the third term, and the fourth term?
   (b) What is the tenth term in this sequence?

4. (a) What is the constant difference in sequence B in question 1?
   (b) What is the constant difference in sequence D in question 1?

5. The sixth terms of sequences E, F and G are given in the table. Copy the table and fill in the other terms.

Term number	1	2	3	4	5	6	7	8	9
E with constant difference 10						30			
F with constant difference −5						30			
G with constant difference −10						30			

6. Investigate each of the patterns below. Find the pattern and write the next four terms in the sequence.

(a)	1	4	9	16	25	(b)	3	6	11	18	27
(c)	20	19	17	14	10	(d)	20	25	29	32	34

7. Make some numeric patterns of your own.

19.2 Making patterns from rules

1. (a) Start at 30. Add −5 and write the answer. Add −5 again and write the answer. Continue until you have a number sequence with 10 terms.
   (b) Start at −30. Add −5 and write the answer. Add −5 again and write the answer. Continue until you have a number sequence with 10 terms.
   (c) Start at −30. Add 5 and write the answer. Add 5 again and write the answer. Continue until you have a number sequence with 10 terms.

2. (a) The first term of a sequence is −10 and there is a constant difference of 5 between the terms. Write down the first ten terms of the sequence.
   (b) The first term of a sequence is −10 and there is a constant difference of −5 between the terms. Write down the first ten terms of the sequence.

3. Choose a number to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.

4. Choose a number smaller than −10 to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.

5. Choose a number to be your first term and a negative number to be a constant difference. Write the first ten terms of your sequence.

6. Choose a negative number to be your first term and another negative number to be a constant difference. Write the first ten terms of your sequence.

7. Choose a number to be your tenth term and another number to be a constant difference. Write the first ten terms of your sequence.

8. Choose a negative number to be your tenth term and another negative number to be a constant difference. Write the first ten terms of your sequence.

19.3 Making patterns from expressions

1. (a) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$2 \times x - 10$									

(b) Do the output values of $2 \times x - 10$ in the previous table form a pattern with a constant difference? If they do, what is the constant difference?

(c) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$3 \times x - 20$									

(d) What is the constant difference in (c)?

(e) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$2 - 3 \times x$									

(f) What is the constant difference in(e)?

(g) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$1 - 2 \times x$									

(h) What is the constant difference in (g)?

2. Look at the pattern: −15; −19; −23; −27; −31; . . .

In this pattern, −19 is followed by −23 and −23 is followed by −27.

(a) What number in the pattern is followed by −19?

(b) What number in the pattern is followed by −31?

(c) In the pattern, −19 follows on −15 and −23 follows on −19.
What number follows on −31?

3. A certain pattern is formed by a common difference of 6.

(a) What number follows on 23 in this pattern?

(b) What number is followed by 23 in this pattern?

(c) What number follows on 47 in thispattern?

(d) What number is followed by 47 in this pattern?

Consider the sequence: 10 6 2 −2 −6 . . . .
In this sequence, 2 follows on 6. They are called consecutive terms.

When one number follows another in a sequence
they are called **consecutive terms**.

4. Write down any two consecutive terms in the pattern formed by $2 \times x + 3$, when the input numbers are consecutive whole numbers.

5. Each pattern on the following page was formed by using one of the following expressions. Establish which pattern belongs to each expression.

(a) $2 \times x + 5$	(b) $3 \times x + 2$	(c) $4 \times x + 1$
(d) $5 \times x + 6$	(e) $6 \times x - 5$	(f) $7 \times x - 2$
(g) $1 - 4 \times x$	(h) $5 - 5 \times x$	(i) $-5 - 6 \times x$

Pattern A:	6	11	16	21	26
Pattern B:	13	17	21	25	29
Patter C:	20	23	26	29	32
Pattern D:	1	−3	−7	−11	−15
Pattern E:	31	33	35	37	39
Pattern F:	−20	−25	−30	−35	−40
Patter G:	25	31	37	43	49
Pattern H:	26	33	40	47	54
Pattern I:	−11	−17	−23	−29	−35

Sequence I in question 5 is a **decreasing** sequence; the numbers become smaller as the sequence progresses:

−11 −17 −23 −29 −35

Sequence H is an **increasing** sequence; each term is bigger than the previous term:

26 33 40 47 54

6. (a) Which sequences in question 5 are increasing sequences?
   (b) Which sequences in question 5 are decreasing sequences?
7. (a) By how much does sequence A increase from one term to the next?
   (b) By how much does sequence B increase from one term to the next?
   (c) Which of the sequences in question 5 increases by the biggest amount from one term to the next, and by how much does it increase?

Sequence G increases by 6 from term to term, and sequence E increases only by 2. We may say that sequence G **increases faster** than sequence E.

8. (a) Which of the sequences in question 5 decreases fastest?
   (b) Which of the sequences in question 5 decreases slowest?
9. (a) Write five consecutive terms of a sequence which decreases faster than sequence D in question 5.
   (b) Write five consecutive terms of a sequence which increases slower than sequence B in question 5.
10. (a) Each of the expressions below can be used to produce a sequence. Which of the expressions will produce the sequence that increases fastest?
    $3 \times x + 5$ $2 \times x + 10$ $6 \times x - 1$ $20 + 3 \times x$ $4 \times x - 9$
    (b) Think of a way in which you can test your answer, and do it.
11. In each case, state whether the sequence will be decreasing or increasing.
    $10 + 3 \times x$ $10 - 3 \times x$ $10 \times x + 3$ $3 \times x - 10$

Chapter 20
Functions and relationships 2

20.1 Relationships between variables

different ways to represent the rule for a relationship

A relationship between two variables consists of two sets of numbers as shown in the two rows of the table below. The first row contains the **input numbers** and the second row contains the **output numbers**.

x	1	2	3	4	5	6	7	8	9
y	32	39	46	53	60	67	74	81	88

For the relationship shown in the table, any output number can be calculated by multiplying the input number by 7 and adding 25 to the answer.

The way in which an output number can be calculated is called the **rule** for the relationship. The rule can be described in **words** or with a **formula**, and in some cases with a **flow diagram**.

The input numbers may also be called the values of the **input variable**, and the output numbers may also be called the values of the **output variable**.

The rule "multiply by 7 and add 25" can be represented with this flow diagram:

$\longrightarrow \boxed{\times 7} \longrightarrow \boxed{+ 25} \longrightarrow$

The same rule can also be represented with the formula below:

$y = 7 \times x + 25$

1. Calculate the value of $7 \times x + 25$ for each of the following values of x:
   (a) $x = 10$
   (b) $x = 20$
   (c) $x = 5$
   (d) $x = 15$

2. (a) What is the value of $3 \times x - 5$ if $x = 10$?
   (b) What is the value of $3 \times x - 5$ if $x = 20$?
   (c) What is the value of $3 \times x - 5$ if $x = 25$?
   (d) What is the value of $3 \times x - 5$ if $x = 100$?

3. Copy and complete the table for the values of x and $3 \times x - 5$ given in the table.

x	0	1	2	5	15		50	200	
$3 \times x - 5$						61		595	994

4. When you worked out the input number that corresponds to the output number 994 in question 3, you solved the **equation** $3 \times x - 5 = 994$.

   Write the equation that you solved when you worked out the input number that corresponds to the output number 61.

5. (a) Express each of the rules below in words.

   A $\longrightarrow \boxed{\times 5} \longrightarrow \boxed{+ 3} \longrightarrow$

   B $\longrightarrow \boxed{- 5} \longrightarrow \boxed{\times 3} \longrightarrow$

   C $\longrightarrow \boxed{\times 3} \longrightarrow \boxed{- 5} \longrightarrow$

   (b) Which of the above flow diagrams represent the same calculations as the expression $3 \times x - 5$?

Instead of $3 \times x - 5$, we may write $3x - 5$.
$3x$ means $3 \times x$.
The multiplication sign can be left out.
Instead of $3 \times (x - 5)$, we may write $3(x - 5)$.

6. (a) Which of the formulae below provide the same information as flow diagram B in question 5?

   $y = 5x - 3$ $\qquad$ $y = 3 + 5x$ $\qquad$ $y = 5(x - 3)$
   $y = 3x - 5$ $\qquad$ $y = 5x + 3$ $\qquad$ $y = 3(x - 5)$

   (b) Which of the above formulae provide the same information as flow diagram A in question 5?

formulae for tables

1. The table below shows the values of y that correspond to some of the given values of x. In this case, the output numbers form a pattern with a constant difference if the input numbers are the natural numbers.

x	1	2	3	4	5	6	7	8
y	13	21		37	45	53		

   (a) Find the output numbers that correspond to the input numbers 3, 7 and 8.

(b) Find the output numbers that correspond to the input numbers 20, 21 and 22.

(c) Which of the formulae below is the rule for the relationship between x and y in the previous table?

$y = 10x + 3$ $\quad$ $y = 8x + 5$ $\quad$ $y = 6x + 7$ $\quad$ $y = 4x + 9$ $\quad$ $y = 2x + 11$

2. Copy and complete the tables below for the formulae in question 1(c).

x	1	2	3	4	5	6	7	8
$10x + 3$								

x	1	2	3	4	5	6	7	8
$8x + 5$								

x	1	2	3	4	5	6	7	8
$6x + 7$								

x	1	2	3	4	5	6	7	8
$4x + 9$								

x	1	2	3	4	5	6	7	8
$2x + 11$								

3. In each table in question 2, the output numbers form a number pattern with a constant difference between consecutive terms. What is the constant difference in the pattern generated by each of the following expressions, when the input numbers are consecutive natural numbers? Copy and complete the table below. Also fill in the values of the expressions for $x = 0$ in the last column.

Expression	Constant difference between output numbers	Value of the expression for $x = 0$
$2x + 11$		
$4x + 9$		
$6x + 7$		
$8x + 5$		
$10x + 3$		

4. What do you think the constant differences between consecutive output numbers, and the values of the expressions for $x = 0$ may be in each of the following cases, when the input numbers are consecutive natural numbers?

Copy and complete the table.

Expression	Constant difference between output numbers	Value of the expression for $x = 0$
$5x + 7$		
$3x + 10$		
$12x + 5$		
$5x - 5$		
$(-10x) + 3$		

20.2 Integers in the rules for relationships

rules that may look strange at first

1. Copy and complete the flow diagrams.

A. $5 \rightarrow \boxed{\times 3} \rightarrow \boxed{-5} \rightarrow$

B. $5 \rightarrow \boxed{\times 3} \rightarrow \boxed{+ (-5)} \rightarrow$

C. $5 \rightarrow \boxed{-3} \rightarrow \boxed{\times 5} \rightarrow$

D. $5 \rightarrow \boxed{+ (-3)} \rightarrow \boxed{\times 5} \rightarrow$

E. $5 \rightarrow \boxed{\times 3} \rightarrow \boxed{- (-5)} \rightarrow$

F. $5 \rightarrow \boxed{\times 3} \rightarrow \boxed{+ 5} \rightarrow$

2. Describe each rule in question 1 in words, for example: "*multiply by 6 and then add* -3".

The rule *multiply by 6 and subtract the answer from 100* can be expressed with the formula $y = 100 - 6x$. This formula can also be written as $y = 100 + (-6x)$ or as $y = (-6x) + 100$.

The brackets around the $-6x$ can be left out, so the last formula above can also be written as $y = -6x + 100$.

3. Calculate y if $y = -10x + 3$, for each of the following values of x:

(a) $x = 5$ (b) $x = 10$

(c) $x = 20$ (d) $x = 1$

4. Describe each of the rules in question 1 with a formula, for example $y = 5x + 8$.

5. In each case below, predict which of the different expressions will produce the same results. You will test your predictions later, and can then mark your own answers for this question.

(a)	$20 - 5x$	$5x - 20$	$(-5x) + 20$	$20 + (-5x)$	
(b)	$20 + 5x$	$5x + 20$	$20x + 5$	$20 - (-5x)$	$5(x + 4)$
(c)	$5x - 20$	$20x - 5$	$(-20) - (-5x)$	$-((-5x) + 20)$	

6. Copy and complete the table below and then use the results to carefully check your answers to question 5.

x	0	1	5	10	100
$20 - 5x$					
$5x - 20$					
$(-5x) + 20$					
$20 + (-5x)$					
$20 + 5x$					
$5x + 20$					
$20x + 5$					
$20 - (-5x)$					
$5(x + 4)$					
$5x - 20$					
$20x - 5$					
$(-20) - (-5x)$					
$-((-5x) + 20)$					

7. In each case below, use your results in the above table or other methods to establish for which values of x the two expressions have the same value(s).

(a) $20 - 5x$ and $20 + 5x$
(b) $20 - 5x$ and $(-5x) + 20$
(c) $5x - 20$ and $(-20) - (-5x)$
(d) $5(x + 4)$ and $5x - 20$ (e)
$20 + 5x$ and $20 - (-5x)$

Chapter 21
Algebraic expressions 2

21.1 Interpret rules to calculate values of a variable

rules in verbal and symbolic form

1. Copy the table below. Do this to each of the numbers in the top row of the table, and write your answers in the bottom row: *multiply the input number by 20 and add 50 to the answer*.

x	1	2	3	4	5	6	7	8	9
y									

The sentence *multiply the input number by 20 and add 50 to the answer* is the rule that describes how the output number that corresponds to each input number in the above relationship between the variables x and y can be calculated.

The same rule can be described with the algebraic expression $20x + 50$. In this expression, the symbol x represents the input variable (the values of x). The numbers 20 and 50 are constant; they remain the same for all the different values of x.

The rule *add 50 to the input number and multiply the answer by 20* can be described with the expression $20(x + 50)$.

If there are no brackets in an expression, multiplication is done first, even if it appears later in the expression like in $30 + 5x$.

If there are brackets in an algebraic expression, the operations in brackets are to be done first.

2. Describe each of the following rulesin words.

   (a) $15x+ 30$ (b) $30 + 15x$ (c) $15(x +30)$
   (d) $15(x + 2)$ (e) $15x- 30$ (f) $15(x - 30)$ (g) $15(x - 2)$

3. What is the difference between $3(x + 5)$ and $3x+ 5$?

4. Copy and complete the table.

x	1	2	3	4	5	6	7	8	9
$15x + 30$									
$30 + 15x$									
$15(x + 30)$									
$15(x + 2)$									

5. Copy and complete the table.

x	30	40	50	60	70	80	90
$15x - 30$							
$15(x - 30)$							
$15(x - 2)$							

6. (a) Investigate which of the following rules will produce the same output numbers. You need to check for several different input numbers.

   A: Multiply the input number by 10 and then add 20.

   B: Add 20 to the input number and then multiply by 10.

   C: Add 2 to the input number and then multiply by 10.

   D: Multiply the input number by 3, add 15, add 7 times the input number, and then add 5.

x								
A								
B								
C								
D								

   (b) Describe each of the above rules with an algebraic expression.

7. (a) Which of these rules do you think will produce the same output numbers?

   A: $5x + 20$ B: $4x + 19$ C: $5(x + 20)$

   D: $20 + 5x$ E: $5(x + 4)$ F: $3x + 7 + 2x + 13$

   (b) Express each of the above rules in words.

   (c) Copy and complete this table for the rules given in question (a).

x	0	5	10	15
$5x + 20$				
$4x + 19$				
$5(x + 20)$				
$20 + 5x$				
$5(x + 4)$				
$3x + 7 + 2x + 13$				

   (d) Use your completed table to check your answer in question (a).

8. (a) Which of these rules do you think will produce the same output numbers?

A: $5x - 20$ B: $20 - 5x$ C: $5(x - 20)$

D: $3x - 18$ E: $5(x - 4)$ F: $9x + 10 - 4x - 30$

(b) Express each of the above rules in words.

(c) Copy and complete this table for the rules given in question (a).

x	20	30	40	50	60	70	80	90
$5x - 20$								
$20 - 5x$								
$5(x - 20)$								
$3x - 18$								
$5(x - 4)$								
$9x + 10 - 4x - 30$								

(d) Use your completed table to check your answer to question (a).

21.2 Slightly different kinds of rules

subtract positive and negative quantities

1. Copy and complete the table.

x	1	10	5	20	25
$10x$					
$50 - 10x$					
$20 - 10x$					
$0 - 10x$					

2. (a) Copy and complete the table.

x	0	5	10	15	20	25	30
$10x - 5$							
$5x - 10$							
$100 - 5x$							
$-100 + 5x$							
$5x - 100$							
$5 - 10x$							

(b) The values of $10x - 5$ *increase* as the values of x increase from 0 to 30. For which expressions in (a) do the values *decrease* when x is increased?

(c) Do the values of $-100 + 5x$ *increase* or *decrease* when x is increased from 0 to 30?

3. (a) The values of the expression $5x - 10$ increase when x is increased from 0 to 30. Do you think the values will increase further when x is increased beyond 30, or will they start to decrease at some stage?

   (b) Do you think the values of the expression $100 - 3x$ will increase when x is increased from 0 to 30? Explain why you think they will or will not.

The additive inverse of a number may be indicated by writing a negative sign before the number. For example, the additive inverse of 8 can be written as −8.

When a number is added to the number called its additive inverse, the answer is 0. For example, 45 + (−45) = 0 and (−12) + 12 = 0.

4. Write the additive inverse of each of the following numbers:

   20 30 −25 −20 40

5. Different values for x are given in the first row of the table below. Copy the table. Write the additive inverses of the x values in the second row, and then complete the table.

x	5	10	15	20	25	30
the additive inverse of x						
20 + (the additive inverse of x)						
20 − (the additive inverse of x)						
$20 + x$						
$20 - x$						

6. Copy and complete the table.

x	−5	−10	−15	−20	−25	−30
the additive inverse of x						
20 + (the additive inverse of x)						
20 − (the additive inverse of x)						
$20 + x$						
$20 - x$						

7. Copy and complete the table.

x	3	2	1	0	−1	−2	−3
$-x$							
$5 + (-x)$							
$5 - (-x)$							
$5 - x$							
$5 + x$							

expressions with additive inverses

1. Copy and complete the table.

x	1	5	10	20	25
$5x$					
the additive inverse of $5x$					
20 + (the additive inverse of $5x$)					
20 − (the additive inverse of $5x$)					
$3x$					
$-3x$					
$10 + (-3x)$					
$10 - 3x$					
$10 - (-3x)$					

2. Copy and complete the table below.

Note that $(-10x)$ indicates the additive inverse of $10x$.

x	1	2	3	4	−4	−3	−2
$10x - 1\ 000$							
$1\ 000 - (-10x)$							
$1\ 000 - 10x$							
$(-10x) + 1\ 000$							
$10x + 1\ 000$							
$10x + (-1\ 000)$							
$(-10x) - 1\ 000$							
$1\ 000 + (-10x)$							
$1\ 000 + 10x$							
$10x - (+1\ 000)$							

Instead of $(-10x) - 1\ 000$ we may write $-10x - 1\ 000$, in other words the brackets around the additive inverse may be left out.

Similarly, $(-10x) + 1\ 000$ may be written as $-10x + 1\ 000$.

3. Copy and complete the table.

x	1	5	10	20	25	30
$-5x + 20$						
$-5x + (-20)$						

Chapter 22
Algebraic equations 2

22.1 Describing problem situations

A **closed number sentence** is a true statement about numbers, for example $21 + 5 = 26$. All the numbers are given.

In an **open number sentence**, for example $15 + x = 21$, one or more of the numbers are unknown.

An open number sentence is also called an **equation**.

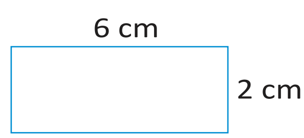
1. Jan is three years older than his sister Amanda. Amanda is 14 years old. Write a closed number sentence to show Jan's age.

2. Numbers are said to be consecutive if they follow one another. The numbers $-1, 0, 1$ are consecutive. The sum of -1, 0 and 1 is 0.
   (a) Write a closed number sentence that shows two consecutive numbers that add up to -33.
   (b) Write a closed number sentence that shows two consecutive numbers whose product is 6.

3. A cell phone costs R500 after a discount of R150 is given. Write a closed number sentence to show the original price of the cell phone.

4. When the bus leaves the terminal, it is carrying 55 people. At the first bus stop 12 people get off the bus and nine people get in. At the second bus stop, 12 people get in and nine people get off the bus. Write a closed number sentence to show the number of people that are now in thebus.

5. A rectangle is shown on theright.
   Write a closed number sentence to calculate the following:
   (a) the area of the rectangle
   (b) the perimeter of the rectangle



22.2 Analysing and interpreting equations

1. The cost of a school uniform in rand is represented by x. An alteration fee of R20 is also charged. Mr Malan paid R520 for both the school uniform and the alterations done on it.
   (a) Which of the following equations describes the above situation?
   A. $20 \times x = 520$ B. $x - 20 = 520$ C. $x + 20 = 520$ D. $20 + 20 = x$
   (b) What is the price of the uniform?

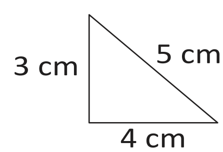
2. Five learners should each receive the same number of sweets. There are 60 sweets in total that they have to share.
   (a) Which equation describes this situation?
   A. $5 + s = 60$ B. $5s = 60$ C. $s - 5 = 60$ D. $\frac{s}{5} = 60$
   (b) How many sweets does each learner get?
   (c) What does the letter s represent in the equation you have chosen?

3. A taxi picks up n passengers at the airport and drives to the nearest hotel. When it leaves the hotel, the number of passengers in the taxi has decreased by six. There are now seven passengers in the taxi.
   (a) Which equation describes this situation?
   A. $n - 6 = 7$ B. $7 - n = 6$ C. $n + 6 = 7$ D. $n - 7 = 6$
   (b) How many passengers were in the taxi when it left the airport?

4. Write a closed number sentence for calculating the perimeter of an equilateral triangle whose sides are 5 cm long.

Remember: An equilateral triangle is a triangle in which all three sides are equal.

5. Write a closed number sentence to calculate the perimeter of the triangle shown on the right.



22.3 Solving and completing equations

solve by inspection

1. The number sentences given below are not true. Make the number sentences true by changing the numbers in blue.

(a) $13 + 7 = 22$	(b) $50 + (-50) = -100$	(c) $7 \times 8 = 54$
(d) $9 - (-3) = 6$	(e) $-5 + 12 = -7$	(f) $4 \times 6 = 28$
(g) $6 - 9 = 3$	(h) $9 - 6 = -3$	(i) $5 + (-12) = 7$
(j) $10 + (-2) = 12$	(k) $(-1) - (-1) = -2$	(l) $0 + (-2) = 0$

2. Consider the equations given below. Check whether the value given in brackets is the solution. Simply write the letter and *yes* or *no* with an explanation.
   (a) $x + 3 = 0 \quad (x = -3)$
   (b) $3 - x = 4 \quad (x = 1)$
   (c) $-5 + x + x = -11 \quad (x = -2)$
   (d) $3 - x = 4 \quad (x = -1)$

To check whether a given value is the solution or not we have to answer the following question in our minds: **does the given value make the equation true?** If it does, we say such a value is the **solution**.

3. Find the value of the unknown that makes the equation true in each case:
   (a) $x + 6 = 8$ (b) $x + 6 = 4$ (c) $x + 6 = 0$
   (d) $6 - x = 8$ (e) $6 - x = 4$ (f) $6 - x = 0$
   (g) $\frac{x}{4} = 2$ (h) $x = 4 \times 2$ (i) $\frac{x}{2} = \frac{1}{4}$

4. Three possible solutions are given in brackets below each equation, but only one is correct. Find the correct solution in each case.
   (a) $x + 27 = 27$ $\{-27; 0; 1\}$ (b) $12 = 4 - x$ $\{8; 16; -8\}$ (c) $x + 3 = 0$ $\{-3; 0; 3\}$
   (d) $5 - x = 10$ $\{-5; 0; 5\}$ (e) $5 + x = 10$ $\{-5; 0; 5\}$ (f) $-5 + x = 10$ $\{-5; -15; 15\}$
   (g) $-5 - x = 10$ $\{-5; -15; 15\}$ (h) $-5 - x = 0$ $\{-5; -15; 15\}$ (i) $5 - x = -10$ $\{-5; -15; 15\}$
   (j) $x = \frac{10}{10}$ $\{0; 1; 100\}$ (k) $10x = 0$ $\{0; 1; \frac{1}{10}\}$ (l) $\frac{x}{10} = 0$ $\{0; 1; 10\}$

5. What value for x would make each equation below true?
   (a) Let x = ... then $x + 3 = 10$ (b) Let x = ... then $x + 3 = -4$
   (c) $x + x + x = -6$ is true for x = ... (d) $x + x + x + x = -8$ is true for x = ...

6. Copy the tables below. In each case, fill in the table until you can see for what value of x the equation given above the table is true. You may add more x values of your own choice. To save time and work, you may skip columns that you think will not help you to find the solution.
   (a) $37 - 4x = 5$

x	1	10	5	6	7				
$37 - 4x$									

   (b) $50 - 7x = 22$

x	1	10	5	6					
$50 - 7x$									

(c) $100 - 3x = 49$

x	10	20	25	15	16				
$100 - 3x$									

solve by trial andimprovement

We can think of an equation as a question asking for a value that we can assign to the **unknown** to make the equation true.

Consider the equation $82 + m = 23$. We need to assign values to m until we find a value that makes the equation true, as shown in the table below.

	Equation	True/False
Let $m = -50$	$82 + (-50) = 82 - 50 = 32$	False
Let $m = -30$	$82 + (-30) = 82 - 30 = 52$	False
Let $m = -60$	$82 + (-60) = 82 - 60 = 22$	False
Let $m = -59$	$82 + (-59) = 82 - 59 = 23$	True

So $m = -59$ because $82 + (-59) = 82 - 59 = 23$

1. Determine the value of t that makes the equation $28 - t = 82$ true by making use of the trial and improvement method. Copy and complete the table.

	Equation	True/False

2. Consider the equation $w + 32 = -68$. Use the trial and improvement method to find the solution of the equation. Copy and complete the table.

	Equation	True/False

3. The equation $200 - 5t = 110$is given. What value of t makes the equation true? Copy the table below and use it to determine the solution.

	Equation	True/False

4. What value of p makes the equation $18p = 90$ true? Copy and complete the table.

	Equation	True/False

5. What value of x makes the equation $88 - 6x = 46$ true? Copy and complete the table.

	Equation	True/False

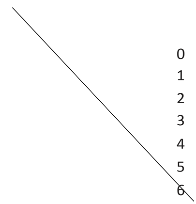
22.4 Identifying variables and constants

1. The mass of an empty truck is 2 680 kg. The truck is used to transport cement. Each pocket of cement has a mass of 90 kg. The combined mass of the truck and the cement can be calculated by means of the formula: $y = 90 \times x + 2\,680$.

   Use the terms **variable** or **constant** to describe the meaning of each symbol used in the formula. Explain your answer.

   (a) y (b) 90 (c) x (d) 2 680

2. A steel spring is suspended from a stand. Mass pieces of equal mass are hooked onto the bottom end of the spring. The length of the spring is measured with one mass piece hooked, two mass pieces hooked, three mass pieces hooked and so on. The results are shown in the table below.



Number of mass pieces	1	2	3	4	5	7	10
Length of spring in cm	48	56	64	72	80	96	120

   The formula $y = 8x + 40$ is used to predict the length of the spring for the various number of mass pieces hooked.

   Use the terms **variable** or **constant** to describe each symbol used in the formula. Explain your answer.

   (a) y (b) 8 (c) x (d) 40

22.5 Numerical values of expressions

substituting numbers into expressions

1. (a) Copy the table below. Calculate the values of each expression for the given values of x, and write your answers in the table.

x	0	2	5	10	20	50	100
$100 - 9x$							
$100 - 8x$							
$100 - 7x$							
$100 - 6x$							
$100 - 5x$							
$100 - 4x$							
$100 - 3x$							

(b) Which sequence in the above table decreases fastest, and which sequence decreases slowest?

2. (a) Copy and complete the table.

x	1	2	3	4	5	6	7
$2x + 3$							
$3x - 3$							
$3x - 2$							
$3x - 1$							

(b) For which value of x is $2x + 3$ equal to $3x - 1$?
(c) For which values of x is $2x + 3$ smaller than $3x - 1$?
(d) Do you think $2x + 3$ is smaller than $3x - 1$ for all values of x greater than 4? You may try a few numbers to help you think about this.
(e) Which sequence increases fastest, the sequence generated by $2x + 3$ or the sequence generated by $3x - 3$?

Chapter 23
Collect, organise and summarise data

23.1 Collecting data

Think of something that you really want to know about your own community or about children your age in other schools. For example, "How many Grade 7 learners in South Africa have access to a computer?" What would *you* find interesting to know about?

When you start the cycle of data handling, you start with at least one question. But there can of course be many more questions.

Once you have a research idea in mind, you can start planning how you will collect the data. When you collect data, you need to consider:

- what question you are asking
- where you will find the data to answer the question (for example, from people such as your peers, family or the wider community; or from published sources such as newspapers, books or magazines)
- how you will collect the data (for example, by using questionnaires or conducting interviews)
- who you will collect the data from (the entire population or a sample).

populations and samples: from whom to collect data

In data handling, **population** refers to the whole group you are asking the question about.
Sample refers to a small number of the group that you think will represent the whole group.

Here is an example: Thandeka wants to know about the home languages of all Grade 7s across the whole of South Africa. All Grade 7s in all of South Africa would be the population of that data. But it is not possible to reach every single Grade 7 learner in South Africa, so Thandeka could choose a sample of Grade 7 learners. For example, she could choose to collect data from her own Grade 7 class and from two other Grade 7 classes from two other schools.

But if Thandeka chose her own Grade 7 class and only two other classes from other schools, her sample would not really give information about learners across the whole of South Africa, because the learners in all three of the schools could be from the same language group.

So, how can you try to make sure that a sample gives information about the whole population? In other words, how can you make sure that your sample is **representative** of the population?

1. *Choose a big enough sample*. Generally, the bigger the sample is the more likely it is to represent the characteristics of the population.
2. *Ensure that you do not take a sample from only one of the groups within the population.* For example, if you want to find out if people like watching soccer, you cannot survey people at a Chiefs versus Pirates match. The majority of these people will almost certainly be there because they love watching soccer!

Example

Ganief wishes to find out if learners at his school like the style and colour of their school uniform and surveys ten learners in Grade 7.There are 2 000 learners at the school.

Give two reasons to explain why the sample chosen is not likely to be representative of the population.

Answer

1. The sample is too small.
2. He is only getting the views of Grade 7s, not of the learners in any of the other grades (who might have very different views).

thinking about populations and samples

1. Here are some research questions. Copy the statements below and use a **P** to show which statement describes the population and an **S** to show which statement describes a sample of the population.

   (a) What percentage of plants in the vegetable patch is affected by disease?

   ☐ All the plants in the vegetable patch

   ☐ Every fourth or fifth plant in the vegetable patch

   (b) How often do teenagers recycle plastic?

   ☐ Every teenager in South Africa

   ☐ About 40 teenagers in the community

   (c) How many hours of sleep do 10-year-olds in my community get per night?

   ☐ All 10-year-olds in the community

   ☐ About ten 10-year-olds in the community

2. You want to know the most popular colour of the learners in your school.
   (a) Write down the population of your data collection.
   (b) Write down what sample you would use.

3. Census@School took place in 2001 and 2009. These were surveys that Statistics South Africa did to show learners how information about people is collected and analysed. The Census@School wanted to know personal, community and household information about learners from Grades 3 to 12. This is how they chose their sample:

   - *A sample of 2 500 schools was selected from the Department of Basic Education's database of approximately 26 000 registered schools.*
   - *The schools were divided into groups depending on their province, school type (primary: Grades 3 to 7 only; intermediate: Grades 5 to 9 only; secondary: Grades 8 to 12 only; combined: Grades 3 to 12), and education district.*
   - *A sample of schools was selected from each of these groups.*
   - *Approximately 790 000 learners participated in the Census@School 2009.*

   This information was included in their final report.

   (a) What percentage was the sample of all the schools in the country?
   (b) Why do you think they separated the schools into groups first?
   (c) Do you think the information that they obtained from this survey would be interesting to you? Explain.

4. Unathi goes to River View Girls' Primary School. She wants to find out whether 13-year-olds in her town prefer rugby or netball. She surveys ten learners from each of the three Grade 7 classes at her school. Is the sample chosen likely to be representative of the population (13-year-olds in her town)? Explain your answer.

constructing questionnaires: how to collect data

A **questionnaire** is a sheet with questions used to collect data from people. Each **respondent** in the sample completes a questionnaire. The questions on the sheet can be structured differently, for example:

- The questions may require "yes" or "no" answers.
- A selection of answers (multiple-choice answers) may be provided for respondents to choose from.
- The respondents may enter their own views or information on the questionnaire.

A **respondent** is a person who fills in a questionnaire or from whom you collect data.

The type of responses you need (for example a simple "yes" or "no" or more detailed information) depends on the data you intend to collect.

Look at the examples below. Notice how each question is worded to be as clear as possible and to allow the data to be collected easily. (The questions in examples 4 and 5 were used by Census@School in their 2009 questionnaire.)

Example 1

do you help with chores at home?

☐ Yes ☐ No

Example 2

Which of these chores do you help with?

☐ cleaning dishes ☐ washing clothes
☐ sweeping/vacuuming ☐ making beds

Example 3

How old are you?

☐ 5–8 years ☐ 9–11 years
☐ 12–15 years ☐ 16–19 years

Note in example 3 how all the ages from 5 to 19 are covered, but without any overlaps.

Example 4

Tick the box if you have:

1	Running water inside your home	6	A cell phone
2	Electricity inside your home	7	Access to a computer
3	A radio at home	8	Access to the internet
4	A TV at home	9	Access to a library
5	A telephone at home		

Example 5

6 How tall are you without your shoes on? Answer to the nearest centimetre.
☐☐☐ centimetres

7 What is the length of your right foot, without a shoe? Answer to the nearest centimetre.
☐☐☐ centimetres

8 What is your arm span? (Open arms wide, measure the distance across your back from the tip of your right hand middle finger to the tip of your left hand middle finger.) Answer to the nearest centimetre.
☐☐☐ centimetres

making questionnaires

1. (a) Refue wants to find out how much pocket money learners in her class receive each month. She draws up the following multiple-choice question:

How much pocket money do you get?

☐ 0–10 ☐ 10–20 ☐ 20–30 ☐ 30–40

Explain why this question is not clear. Give at least three reasons.

(b) Draw up the multiple-choice question so that it will allow Refue to collect the data that she needs.

2. You want to find out which sports learners at your school play.
   (a) Describe the population of your data.
   (b) Describe the sample you will use.
3. Make a question with yes/no or multiple-choice responses to help you collect the data you need.
4. Collect your data from your population or the sample you chose. Keep your data for the next chapter.

23.2 Organising data

To organise data that we have collected, we can use tally marks and tables, dot plots, and stem-and-leaf displays. We can also group the data when there are many data values. The ways that we organise the data depends on the type of data we collected.

different types of data

Look at the five examples of questions for questionnaires on page 261.

1. Which of the examples will give you data that looks like this?

Yes	1 235 learners
No	1 265 learners

2. Which of the examples might give you data that looks like this?
   132 cm; 141 cm; 160 cm; 132 cm; 154 cm; 145 cm; 147 cm; 129 cm; 121 cm; 143 cm; 135 cm; 154 cm; 156 cm; 133 cm; 156 cm; 123 cm; 137 cm etc.
3. What could the data for example 4 look like? Copy and fill in this table to give a possible example for 30 learners. Use numbers that you have made up.

	Number of learners

4. Which of the examples might give you a data set that looks like this?

5–8 years	15
9–11 years	45
12–15 years	32
16–19 years	28

The type of data in questions 1 and 3 is called **categorical data**. This is often described by words. The categories don't have to be given in order.

The type of data in questions 2 and 4 is called **numerical data**. Numerical data can be whole numbers only, or it can include fractions.

For both of these kinds of data, your results give you a list of responses. You will soon learn how to organise these responses.

5. Classify the following data sets as categorical or numerical.
   (a) the number of pages in books
   (b) the length of learners' arm spans
   (c) learners' favourite soccer teams
   (d) the time it takes 13-year-olds to run 1,5 km
   (e) the cost of different types ofcell phones
   (f) colours of new cars manufactured

organising categorical data

Thandeka asked the following question: "Which of South Africa's official languages are the home languages of the learners in my class?"

Thandeka drew up a table with each learner's name. She then asked each learner what his or her home language was, and wrote it down as follows:

Name	Language	Name	Language	Name	Language
Nonkhanyiso	isiXhosa	Marike	Afrikaans	Herbert	Sepedi
Anna	Afrikaans	Jennifer	Sepedi	Thabo	isiXhosa
Mpho	Ndebele	Nomonde	isiXhosa	Nomi	isiXhosa
Nontobeko	isiZulu	Thandeka	Sepedi	Manare	Sepedi
Jonathan	English	Siza	isiZulu	Unathi	Sesotho
Sibongile	isiZulu	Prince	Sesotho	Gabriel	Ndebele
Dumisani	isiZulu	Duma	isiZulu	Marlene	Afrikaans
Matshediso	Sesotho	Thandile	Sepedi	Simon	Sesotho

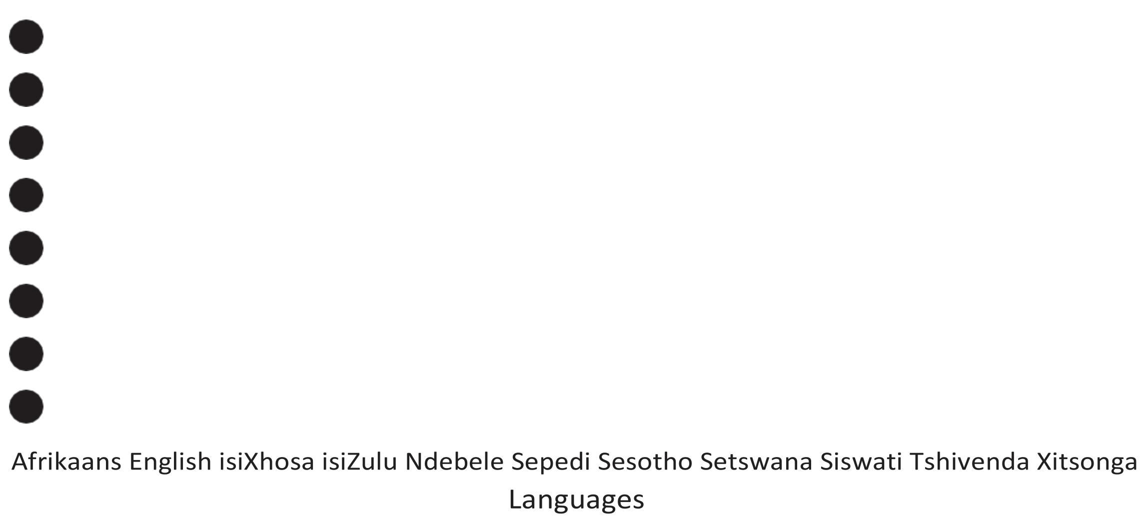
Name	Language	Name	Language	Name	Language
Chokocha	Sepedi	Nicholas	Sesotho	Miriam	Setswana
Khanyisile	isiXhosa	Jabulani	isiZulu	Sibusiso	isiZulu
Ramphamba	Tshivenda	Nomhle	isiXhosa	Mishack	isiZulu
Portia	isiZulu	Frederik	Afrikaans	Peter	Setswana
Erik	Afrikaans	Lola	Afrikaans	Maya	Afrikaans
Jan	Afrikaans	Zinzi	isiXhosa	Thobile	Sesotho
Palesa	isiZulu	Jacob	Setswana		

We don't need the learners' names in the data. This data could be written as a list of the languages, like this:

isiXhosa, Afrikaans, Sepedi, Afrikaans, Sepedi, isiXhosa, Ndebele, isiXhosa, isiXhosa, isiZulu, Sepedi, Sepedi, English, isiZulu, Sesotho, isiZulu, Sesotho, Ndebele, isiZulu, isiZulu, Afrikaans, Sesotho, Sepedi, Sesotho, Sepedi, Sesotho, Setswana, isiXhosa, isiZulu, isiZulu, Tshivenda, isiXhosa, isiZulu, isiZulu, Afrikaans, Setswana, Afrikaans, Afrikaans, Afrikaans, Afrikaans, isiXhosa, Sesotho, isiZulu, Setswana

Now work with this data set to see what story it is telling you. What do you notice about the data?

1. What do you need to find out from this list of languages?
2. Does it matter what order you write the languages in? Why or why not?
3. (a) Copy Thandeka's graph below. Place a dot above each language to show every learner who speaks that language. The languages are in alphabetical order. Try to space out the dots evenly. The dots for Afrikaans have been drawn for you. A graph like this is called a **dotplot**.



(b) Which languages have the same numbers of learners?
(c) List the languages in order from the language spoken by the most learners to the language spoken by the fewest learners.

You can also record results in a **tally table**. To do this, you draw a single line (|) for each item you count. This line is called a **tally mark**.

You group tally marks in groups of five. The fifth tally mark is always drawn horizontally to show that the group of five is complete. Then you start a new group. This makes it easy to quickly count how many tally marks there are in a particular category.

Examples of tally marks:
A count of three = |||
A count of four = ||||
A count of five = ~~||||~~
A count of seven = ~~||||~~ ||

4. (a) Copy and complete the table.

Home language of learners in the Grade 7 class

Language	Number of speakers of each home language	Total
Afrikaans	~~||||~~ |||	8
English			1
isiXhosa		
isiZulu		
Ndebele		
Sepedi		
Sesotho		
Setswana		
Siswati		
Tshivenda		
Xitsonga		
Total (whole class)		

(b) How many learners altogether were asked about their home language?
(c) Which home language occurs most often in this class?
(d) Which languages are not spoken as a home language by any of the learners in this class?
(e) Write a short paragraph to describe the home languages in Thandeka's class.

Dot plots and tally tables are used for **numerical data** too. You can write data values on prepared tally tables or dot plots as you record them. This sorts the data at the same time as it is recorded.

introducing stem-and-leaf displays

A **stem-and-leaf display** (also called a stem-and-leaf plot) is a way of listing numerical data using two columns divided by a vertical line. Each number is split across thecolumns.

For example, if the numbers in a set of data consist of digits for tens and units (such as 23, 25 and 34), the column on the right (the leaf column) shows the units digits of the numbers, and the column on the left (the stem column) shows the tens digits of the numbers.

Numerical data is data that consists of numbers.

Example 1

Show the following data set as a stem-and-leaf display:

13, 56, 20, 35, 47, 53, 12, 51, 53, 49, 34, 53

First, we order the values in the data set from smallest to biggest:

12, 13, 20, 34, 35, 47, 49, 51, 53, 53, 53, 56

In this example, the tens digits range from one to five, so we list these in the stem column. Then we fill in the units digits in theleaf column.

The stem-and-leaf display of the above data set looks like this:

Key: 1 | 2 means 12

Stems	Leaves
1	2, 3
2	0
3	4, 5
4	7, 9
5	1, 3, 3, 3, 6

Values with the same stem are written in the same row. Numbers with the same tens value are separated by a space or a comma. The first row shows the numbers 12 and 13, the second row shows 20, and the last row shows 51, 53, 53, 53 and 56.

In this example, the stem column showsthe tens digit of eachvalue.

Here the leaf column shows the units digit of each value.

Example 2

The stem-and-leaf display on the following page shows the units digits as the leaves, and both the hundreds and tens digits as the stems:

10	2, 5	Key: 10 | 2 means 102
11	0, 6	
12	1, 4, 4	
13		
14	7, 9	
15		
16	1, 3, 8	

The values shown are: 102, 105, 110, 116, 121, 124, 124, 147, 149, 161, 163, 168.

Note that if there is a 0 in the leaf column it means the unit digit is a 0, as in 110 above. When there is nothing written in the leaf column next to a stem, it means that there aren't any numbers with that particular stem. In the case of stem 13 above, for example, it means there are no values between 129 and 140.

When you draw stem-and-leaf displays, it is important that the numbers line up vertically so that you can compare the leaves. Draw lines to help you. (Or use grid paper, if you have some.)

dot plots and stem-and-leaf displays

1. Look at the following stem-and-leaf display and answer the questions below.

13	1, 9	Key: 13 | 1 means 131
14	0	
15		
16	2, 3, 5, 5, 5	
17	6, 8, 8	
18		
19	4, 6, 7	

   (a) Write down the values in the data set shown by the stem-and-leaf display.
   (b) Do most of the values fall in the 160s or 170s?
   (c) Which value occurs the most times?
   (d) Copy the above stem-and-leaf display and add the following values: 143, 167 and 199.
   (e) There are no values in the 150s. Can we add 15 | 0 to the stem-and-leaf display to show that there are no values in the 150s? Explain your answer.

2. (a) Arrange the values in the following data set in order from smallest to largest: 378, 360, 390, 378, 378, 400, 379, 382, 354, 394, 399, 395, 378, 361, 375
   (b) Organise the data set as a stem-and-leaf display. Remember to add the key.
   (c) Which value occurs most often?

3. (a) The data sets below show the sales of two new makes of cars (Jupiter and Mercury) over 24 months. Copy the number lines and draw a dot plot for each set on the number lines.

Mercury: 23, 27, 30, 27, 32, 31, 32, 32, 35, 33, 28, 39, 32, 29, 35, 36, 33, 25, 35, 37, 26, 28, 36, 30

Jupiter: 31, 44, 30, 36, 37, 34, 43, 38, 37, 35, 36, 34, 31, 32, 40, 36, 31, 44, 26, 30, 37, 43, 42, 33

Mercury

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Number of cars sold

Jupiter

20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45

Number of cars sold

(b) If you look at the dots for Mercury and the dots for Jupiter, what can you see about the sales of the two cars? What does this mean?

something to thinkabout

What kind of graph does the stem-and-leaf display look like if you turn it by 90 degrees?

1	2, 5
2	0, 6
3	1, 4, 4, 5, 5, 9
4	
5	2, 7, 8, 9
6	1, 3, 6, 7, 7
7	1, 3, 8

→

		9				
		5			7	
		5		9	7	
		4		8	6	8
5	6	4		7	3	3
2	0	1		2	1	1
1	2	3	4	5	6	7

grouping data into intervals

When a data set contains many data items, we sometimes group the data items to help us organise the data. For example, the following data set shows the number of milk bottles collected by 24 learners for recycling:

9, 10, 13, 23, 24, 26, 26, 27, 30, 31, 34, 40, 42, 49, 50, 53, 61, 64, 67, 67, 68, 69, 91, 94

We can group the data into categories called **class intervals**, such as 0–9, 10–19, 20–29, and so on. We can then count how many times a value occurs in each interval. The number of times a value occurs in an interval is called its **frequency**.

This table shows the grouped data and the frequency of the values in each interval.

Interval	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	1	2	5	3	3	2	6	0	0	2

The table shows that one learner collected 0–9 bottles, two learners collected 10–19 bottles, five learners collected 20–29 bottles, and so on. We can clearly see that most learners (six) collected 60–69 bottles.

working with grouped data

1. Anita collected data from a sample of Grade 7 learners about how far they live from the nearest grocery store. Below are the results. The values are in kilometres, correct to one decimal figure.

0,1	0,1	0,2	0,2	0,2	0,2	0,3	0,3	0,3	0,4	0,4	0,5	0,5	0,5	0,6
0,6	0,7	0,7	0,7	0,8	0,8	0,8	0,9	0,9	0,9	1	1	1	1,5	1,5
2	2	2	2	2,5	2,5	3	3	3	3,5	3,5	4	4	4,	4,5
5	5	6	6	7	7	8	8	9	10	10	15	20	23	30

(a) Copy and complete the table alongside to indicate how many of the values appear in each of the given intervals.

(b) How far do most of the learners live from the nearest grocery store?

Interval	Frequency
Less than 1,0 km	
1,0–5,9 km	
6,0–9,9 km	
10 km or further	

2. Here are the heights of 50 Grade 7 boys at a school (in centimetres):

165	148	150	160	165	150	156	155	164	162
160	158	138	158	140	146	160	148	152	139
165	148	152	139	165	148	160	163	178	138
142	179	156	160	160	171	140	160	164	135
159	143	167	138	163	164	155	160	167	165

(a) Draw a stem-and-leaf display to show this set of data. Remember to include the key.

(b) Write a short paragraph to describe the data set.

(c) Copy and complete the frequency table below for the grouped data from your stem-and-leaf display in question (a).

Class interval (cm)	Frequency
130–139	
140–149	
150–159	
160–169	
170–179	
Total	

23.3 Summarising data

When you have collected data, you often need to tell someone what you have found out. People want to know what your conclusions are, without looking at all of the data you have collected.

It is often useful to summarise a set of numerical data by using *one* value. For example, which value best summarises or describes the following data set?

0 1 1 5 8 8 9 9 10 10 10 11 11

Statisticians use any of three values that show the most central values in the set, or the value around which the other values tend to cluster. These values are called the **measures of central tendency** or **summary statistics**.

Statisticians are mathematicians who specialise in collecting, organising and analysing data.

- The **mode** is the value that occurs most frequently in the data set. In the example on the previous page, the mode is 10 because it occurs the most times (three times).
- The **median** is the value exactly in the middle of the data set when the data values are arranged in order from smallest to largest. For the data set on the previous page, the median is 9 because there are six values to the right of the first 9 and six values to the left of it.
- The **mean (average)** is the total (sum) of the values divided by the number of values in the data set. So:

$$\text{Mean} = \frac{\text{Total of values}}{\text{Number of values}} = \frac{93}{13} = 7{,}15$$

A data set can have more than one mode.

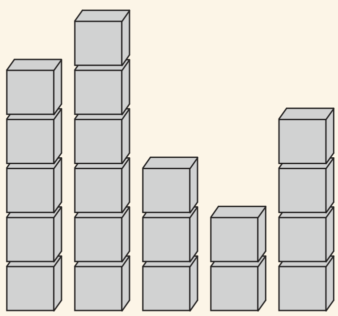
If the data set consists of an even number of items, the median = sum of the two middle values divided by 2.

In the data set on the previous page, either 10 (mode), 9 (median) or 7,15 (mean) could be used to represent the entire data set.

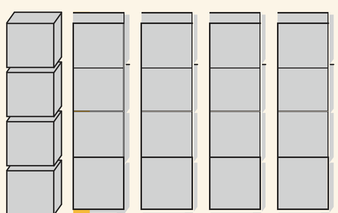
understanding the mean

This activity will help you to understand how the mean represents the whole set of data.

Make piles of blocks of different heights:



Then move blocks from the higher piles to the lower ones to make all the piles equal:



You have just found the mean: Each pile now has four blocks in it. But how do you do this if you only have the numbers 5, 6, 3, 2 and 4 to work with? You add them up and then divide the answer by the total number of values (numbers):

$$5 + 6 + 3 + 2 + 4 = 20 \qquad 20 \div 5 = 4$$

What this means is that you are finding a single number that you can use in place of all the different numbers and still get the same total.

It is also useful to know how big the spread of the data is.

The **range** of a data set is the difference between the highest value and the lowest value. For example, for the data set on the previous page, the range is:

$11 - 0 = 11$

The bigger the range, the more the data is spread out. The smaller the range, the more the data is clustered around similar values.

determining the mode, median, mean and range

1. The following data set shows the shoe sizes of a sample of learners at a school:
   1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6
   (a) What is the mode of the data set?
   (b) What is the median of the data set?
   (c) What is the mean? (Round off to the nearest whole number.)
   (d) What is the range of the data set?

2. The following data set shows the number of siblings (that is, brothers and sisters) that the learners in a sample of Grade 7 learners have:
   0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 5
   (a) How many learners are in the sample?
   (b) What is the mode of the data set?
   (c) What is the median of the data set?
   (d) What is the mean? (Round off to the nearest whole number.)
   (e) What is the range of the data set?

3. The following data set shows the number of hours worked in a week by a sample of parents at School A:
   15, 16, 20, 25, 25, 30, 40, 40, 40, 40, 40, 42, 45, 45, 48, 48
   (a) How many parents are in the sample?
   (b) What is the mode of the data set?
   (c) What is the median of the data set?
   (d) What is the mean? (Round off to one decimal place.)
   (e) What is the range of the data set?

Remember, if the number of items in a data set is even, the median = the sum of the two middle numbers divided by 2.

4. The following data set shows the number of hours worked in a week by a sample of parents at School B:
   25, 30, 35, 35, 35, 40, 40, 40, 40, 40, 42, 45, 45, 45, 48, 50
   (a) How many parents are in the sample?

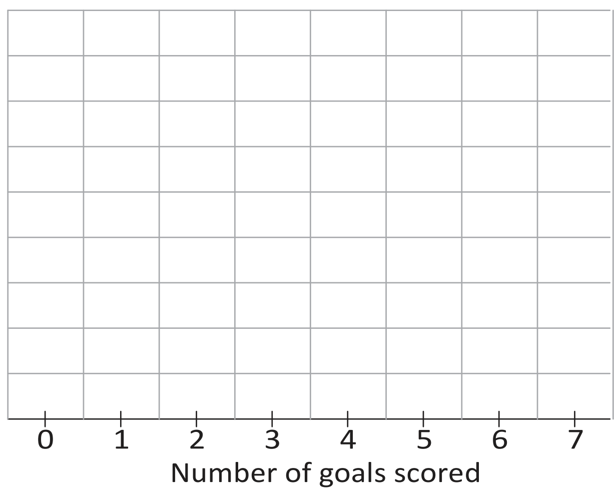
(b) What is the mode of the data set?
(c) What is the median of the data set?
(d) What is the mean? (Round off to one decimal place.)
(e) What is the range of the data set?

5. The following is a list of test scores of learners in a Grade 7 class:
40, 42, 44, 13, 10, 23, 68, 31, 69, 91, 30, 49, 50, 53, 67, 94, 61, 64, 67, 34
(a) Arrange the scores from the lowest to the highest.
(b) How many learners are in the population?
(c) What is the mode of the data set?
(d) What is the median of the data set?
(e) What is the mean?
(f) What is the range of the data set?

6. A hockey player recorded the number of goals she scored in her last 30 matches:

1	1	3	2	0	0	4	2	2	4	3	1	0	1	0
2	1	5	1	3	7	2	2	2	4	3	1	1	0	3

(a) Copy the graph below. Draw a dot plot on the number line to organise these data values.



Now use the dot plot to answer these questions.

(b) Which of the values are quite different to the other values?
(c) Which number of goals has she scored the highest number of times?
(d) Which numbers of goals did she score in the two groups with five matches each?
(e) Use the dot plot to find the mode of the data.
(f) Use the dot plot to find the median.
(g) What is the mean of the goals?

Chapter 24

Represent data

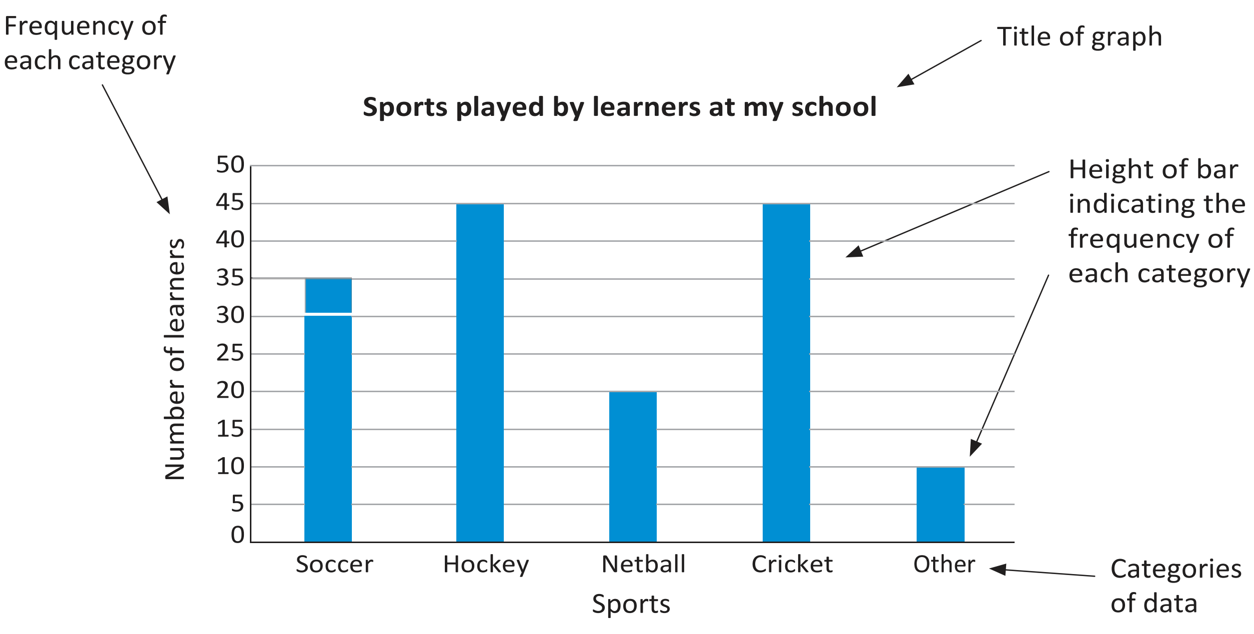
Now that we have collected and organised a set of data, we want to show the results in a useful way.

Remember when you drew dot plots in the previous chapter, you could see which categories or measurements occurred many times and which occurred only a few times. There are a few different graphs that show the important things about the data in such a way that you can see them easily. You need to be able to draw these graphs.

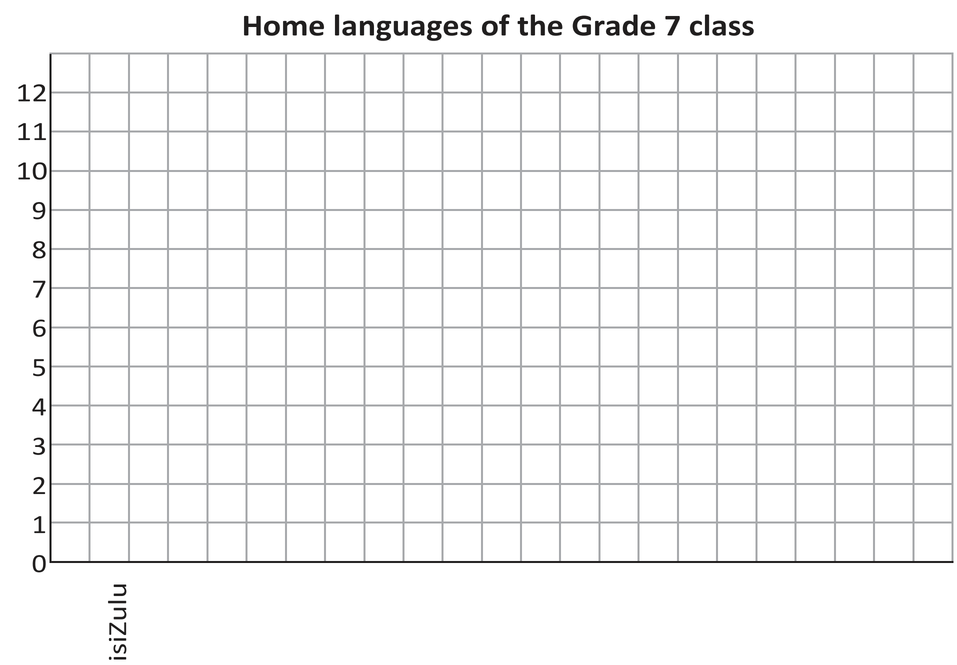
24.1 Bar graphs and double bar graphs

drawing a bar graph

A **bar graph** shows categories (or classes) of data along the horizontal axis, and the frequency of each category along the vertical axis. (Sometimes the axes are swopped around.) Here is an example of a bar graph.

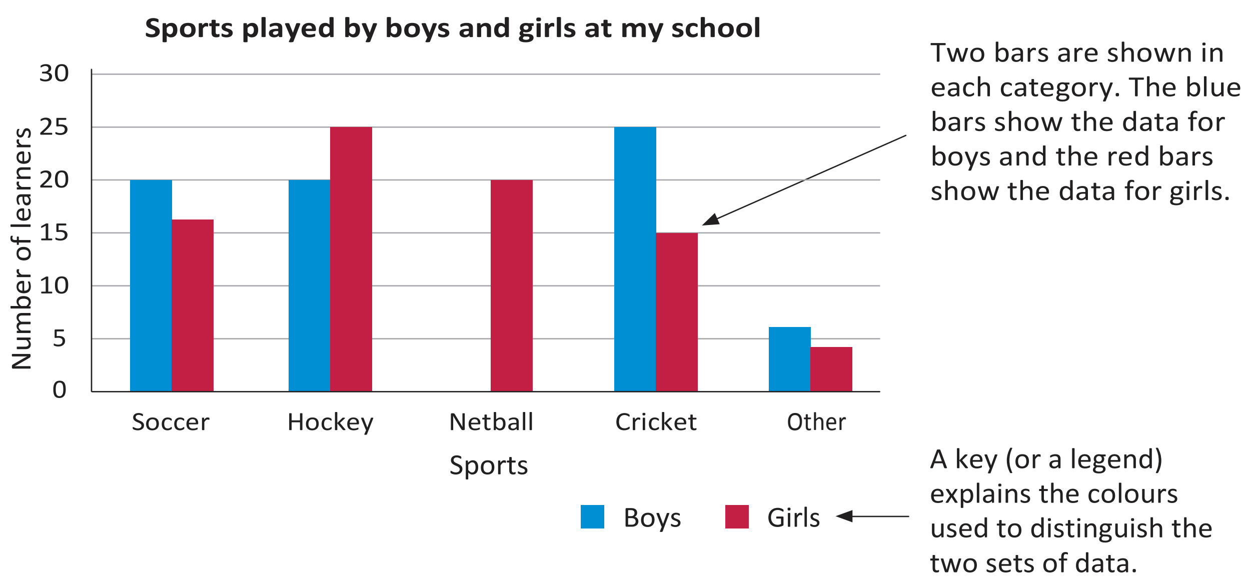


Go back to section 23.2 of Chapter 23, where you drew a dot plot and made a tally table of Thandeka's data about languages spoken in her class. Copy the grid below and use Thandeka's data to draw a bar graph. Draw the bars to the correct height by looking at the numbers on the vertical axis.

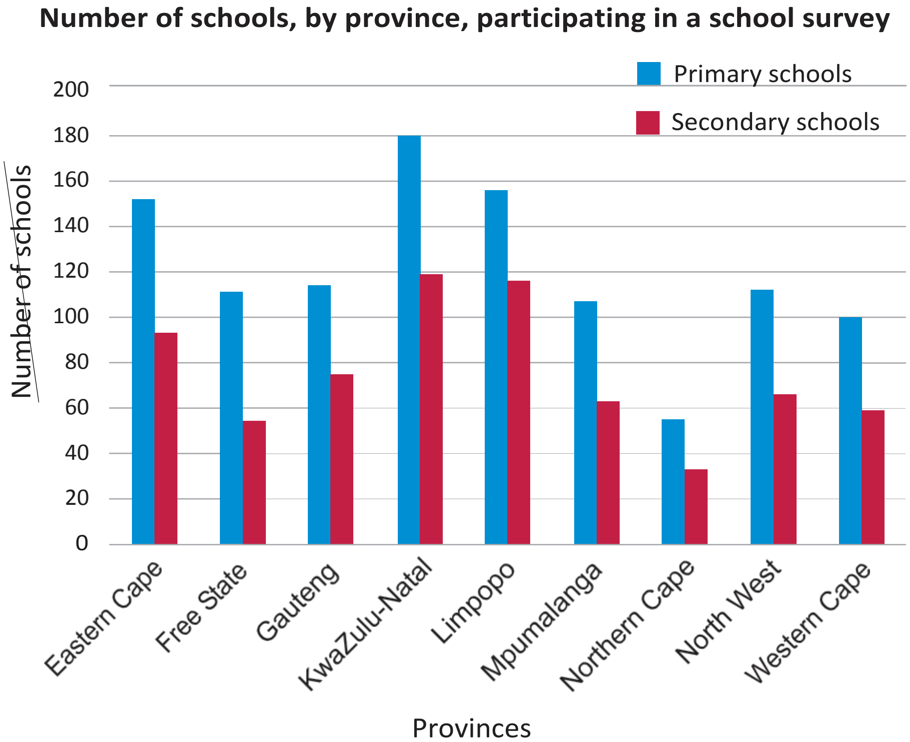


using double bar graphs

A **double bar graph** shows two sets of data for each category (or class). For example, the double bar graph below shows data collected from girls for each category, and data collected from boys for each category.



1. Look at the data below and answer the questions that follow.



(a) Did more primary schools or more secondary schools participate in the survey?
(b) Which province had fewer than 50 secondary schools participating in the survey?
(c) Which provinces had more than 150 of its primary schools participating in the survey?

2. Use grid paper, as shown in the example on page 277, to draw a double bar graph to show the following data.

Facilities available at schools in Province A and Province B

Facility	Percentage of schools in Province A	Percentage of schools in Province B
Electricity	73	50
Running water	68	45
Computers	60	20
Internet	30	10

24.2 Histograms

a situation where data has to be organised

1. Mr Makae wants to buy an orange farm. Three farms are available, each with an orchard of orange trees, and the three farms cost about the same. There are 40 orange trees on each farm. The total mass of oranges (in kilograms) harvested from each tree on each farm over the last three years is given below. Which farm should he buy?

Farm A:

426	628	467	413	862	585	652	600	734	611
741	605	536	643	833	438	613	704	623	719
719	701	501	768	642	444	751	579	695	726
616	619	441	703	902	947	785	952	725	721

Farm B:

822	736	773	674	884	463	644	433	688	487
884	530	448	410	982	638	492	638	725	621
743	661	744	530	560	745	455	943	760	734
888	457	621	969	507	500	542	831	576	801

Farm C:

438	530	743	947	450	777	859	748	473	724
750	852	428	464	725	554	758	997	467	743
722	438	779	690	785	543	752	898	474	483
460	772	544	756	491	576	482	744	701	803

2. How can the data about the orange trees on the three farms be organised so that the farmer has a clear picture of the difference between the orchards on the three farms? For now, just write down how you think the data may be organised. You will organise the data later when you do the questions that follow.

3. Copy and complete these tally and frequency tables for the data about the masses of oranges harvested on the three orange farms.

Masses of oranges harvested from different trees on Farm A

<table>
<tr><th>Mass of oranges harvested from each tree. These are called class intervals.</th><th>Number of trees that produced masses in the interval</th><th>Total</th></tr>
<tr><td>400 kg or more but less than 500 kg</td><td>卌 |</td><td></td></tr>
<tr><td>500 kg or more but less than 600 kg</td><td>||||</td><td></td></tr>
<tr><td>600 kg or more but less than 700 kg</td><td>卌 卌 ||</td><td></td></tr>
<tr><td>700 kg or more but less than 800 kg</td><td>卌 卌 |||</td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Masses of oranges harvested from different trees on Farm B

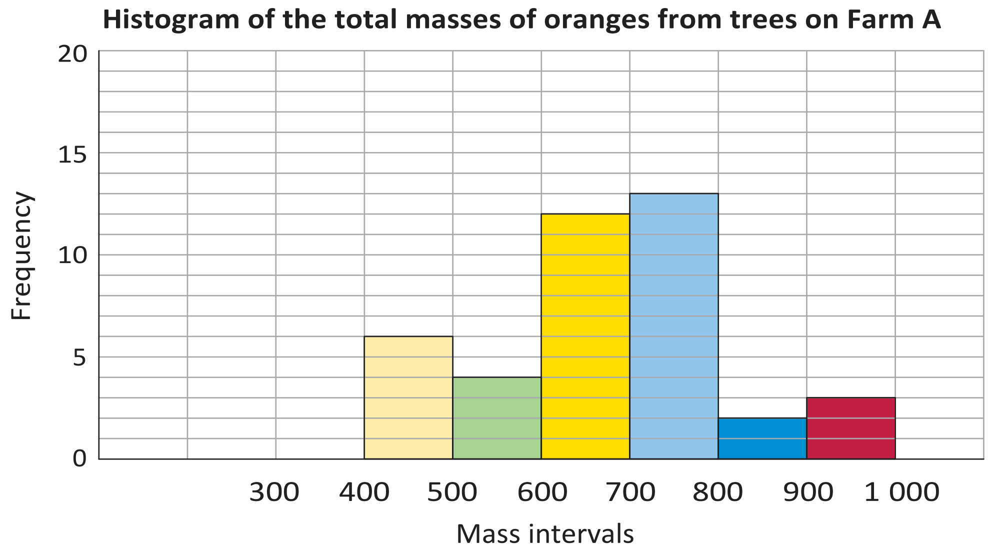
<table>
<tr><th>Class interval</th><th>Number of trees that produced masses in the interval</th><th>Total</th></tr>
<tr><td>400 kg or more but less than 500 kg</td><td>卌 |||</td><td>8</td></tr>
<tr><td>500 kg or more but less than 600 kg</td><td>卌 ||</td><td>7</td></tr>
<tr><td>600 kg or more but less than 700 kg</td><td>卌 |||</td><td></td></tr>
<tr><td>700 kg or more but less than 800 kg</td><td>卌 |||</td><td></td></tr>
<tr><td>800 kg or more but less than 900 kg</td><td></td><td></td></tr>
<tr><td>900 kg or more but less than 1 000 kg</td><td></td><td></td></tr>
</table>

Masses of oranges harvested from different trees on Farm C

<table>
<tr><th>Class interval</th><th>Number of trees that produced masses in the interval</th><th>Total</th></tr>
<tr><td>400 kg or more but less than 500 kg</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>700 kg or more but less than 800 kg</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

On the next page, you will learn how to draw graphs of the data for the three farms.

The data for Farm A is represented on the following graph.



This type of graph is called a **histogram**.

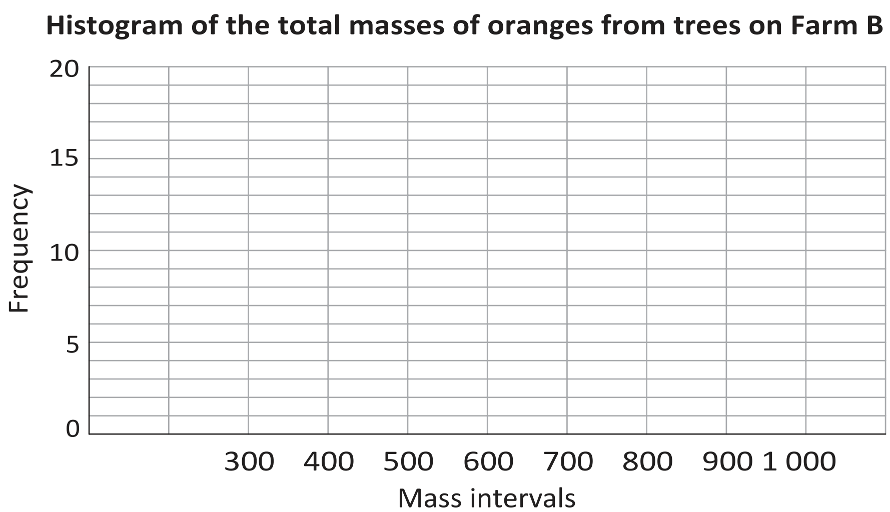
(The columns in a histogram are normally not coloured differently, or even coloured at all. In this histogram the columns are coloured only because some questions are asked about them in question 4 below.)

The numbers 400 on the left and 500 on the right of the light yellow column indicate that masses of 400 kg or more but less than 500 kg are counted in that interval.

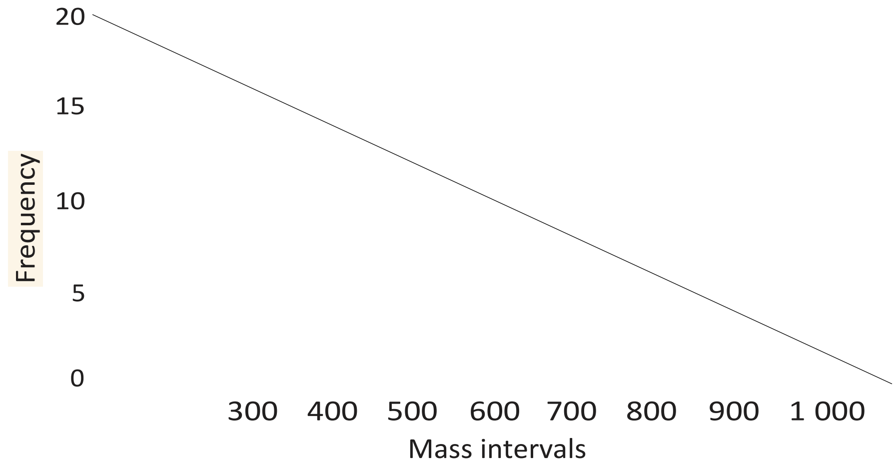
The height of each column represents the number of masses (the frequency) that fall in that interval.

4. (a) A total of 536 kg of oranges was harvested from one of the trees on Farm A over a period of the three years. In which column on the above histogram is this tree represented? Explain your answer.
   (b) Which masses are represented in the red column?
   (c) Which class interval is represented by the light blue column on the above histogram?
   (d) How many masses are represented by the green column?
   (e) Which column represents the highest frequency?

5. Copy and complete the following histograms.



Histogram of the total masses of oranges from trees on Farm C



The different class intervals are **consecutive** and cannot have values that overlap. For example, we can group heights into class intervals of 10 cm, as shown below:

Height (m)	Heights that fall in the class interval	Frequency
1,20–1,30	1,20; 1,25; 1,29	3
1,30–1,40	1,30; 1,31; 1,35; 1,39	4
1,40–1,50	1,40; 1,46; 1,48; 1,48; 1,49	5
1,50–1,60	1,53; 1,53; 1,57; 1,58; 1,59; 1,59	6

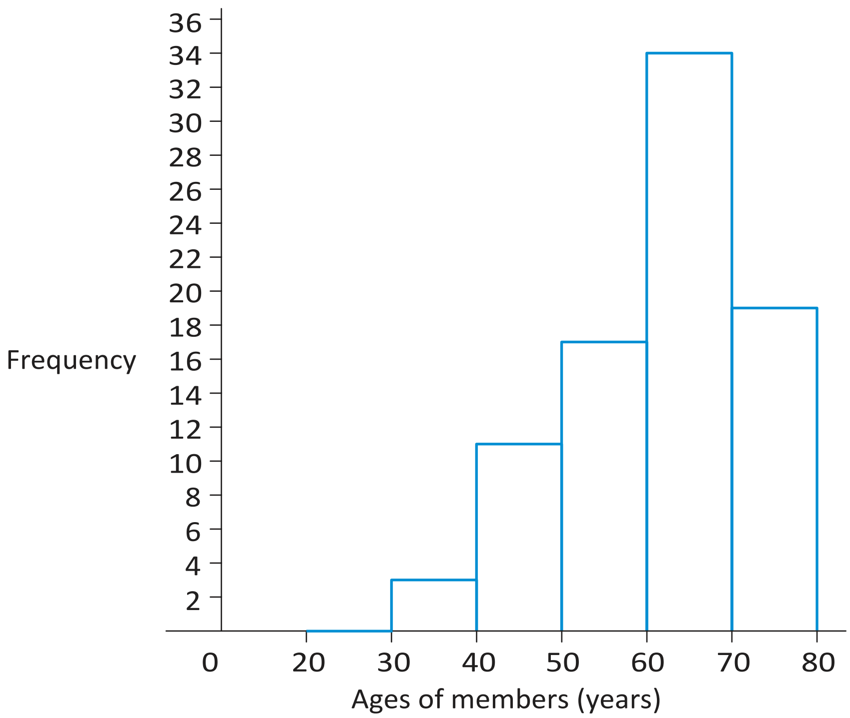
We follow the convention that the top value (also called the **upper boundary**) of each class interval is not included in the interval.

So the height of 1,20 m falls into the 1,20–1,30 m interval, but the height 1,30 m falls into the 1,30–1,40 m interval.

interpreting a histogram

Study the histogram (on page 281) showing the numbers of members, in different age groups, of a sports club. Then answer the questions that follow.

1. Complete a frequency table for the information.
2. How many of the members are in their fifties?
3. How many members does the club have?
4. When you drew a bar graph, it did not matter what order the bars were in. Does the order of the columns on the histogram matter? Explain.

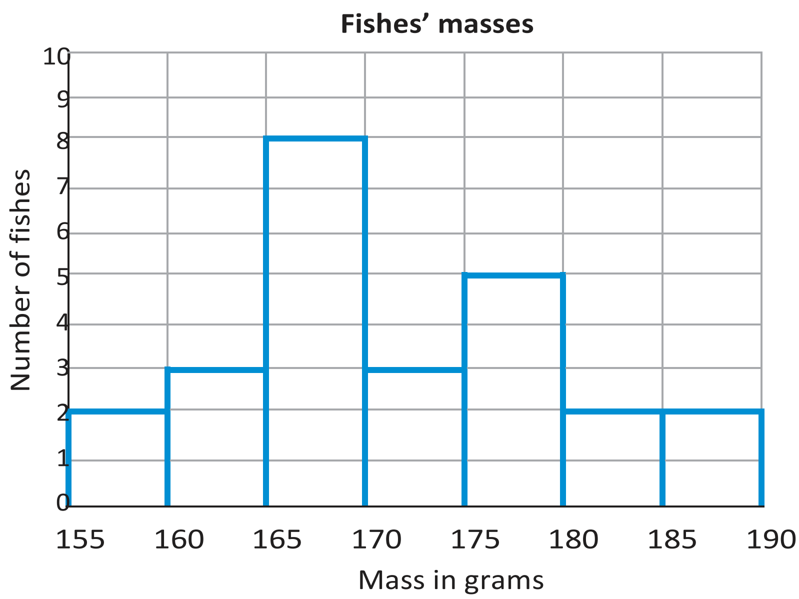


Notice that you cannot see the individual data values in a histogram – they have been “lost”. For example, below you can see a stem-and-leaf display and a histogram of the same data set.

Fishes’ masses in grams

15	7, 8
16	2, 3, 3
16	5, 5, 6, 7, 8, 8, 9, 9
17	1, 2, 3
17	6, 7, 7, 8, 8
18	0, 3
18	6, 7

Key: 15|7 means 157



A histogram usually has many more data values than a stem-and-leaf display – too many to show in a stem-and-leaf display. It would, for example, be difficult to put the 84 values for the members of the sports club onto a stem-and-leaf display.

drawing more histograms

1. The table shows how long it takes learners from a Grade 7 class at Western Primary to travel to school each day. In question (d) you will use a histogram to represent the data in the table.

Time (minutes)	Frequency
0–10	7
10–20	18
20–30	11
30–40	3

   (a) How many learners were asked about their travelling hours?

   (b) Look at the grid provided in question (d). What do you have to consider in order to help you decide on a scale division for the vertical axis?

   (c) What scale will you use on the horizontal axis? Explain your answer.

   (d) Copy the grid and draw a histogram of the data.

2. The table shows how much money different vendors earn selling their goods every week.

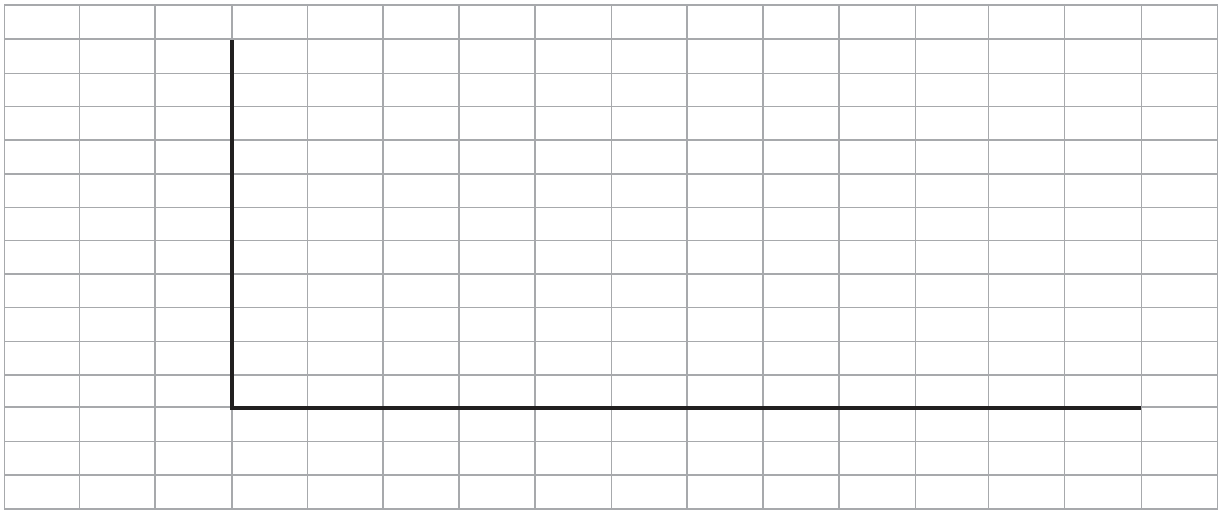
Money (R)	Frequency
0–100	6
100–200	9
200–300	11
300–400	7
400–500	5

   (a) How many vendors were asked about their earnings?

   (b) Copy the grid on the following page. Decide on a scale for the vertical axis of a histogram and indicate it on the axis.

   (c) Decide on a scale for the horizontal axis and indicate it on the axis.

(d) Complete the histogram showing the data.



3. In a Natural Sciences class, learners planted beans and measured the heights of the bean plants after two months. Here is the data they collected (in centimetres):

34 65 72 42 37 29 78 43 79 91 43 45 28 42 79

34 92 87 40 43 43 78 82 47 85 43 32 86 76

(a) Copy and complete this frequency table:

Height of bean plants (cm)	Tally	Frequency
20–30		
30–40		
40–50		
50–60		
60–70		
70–80		
80–90		
90–100		
Total		

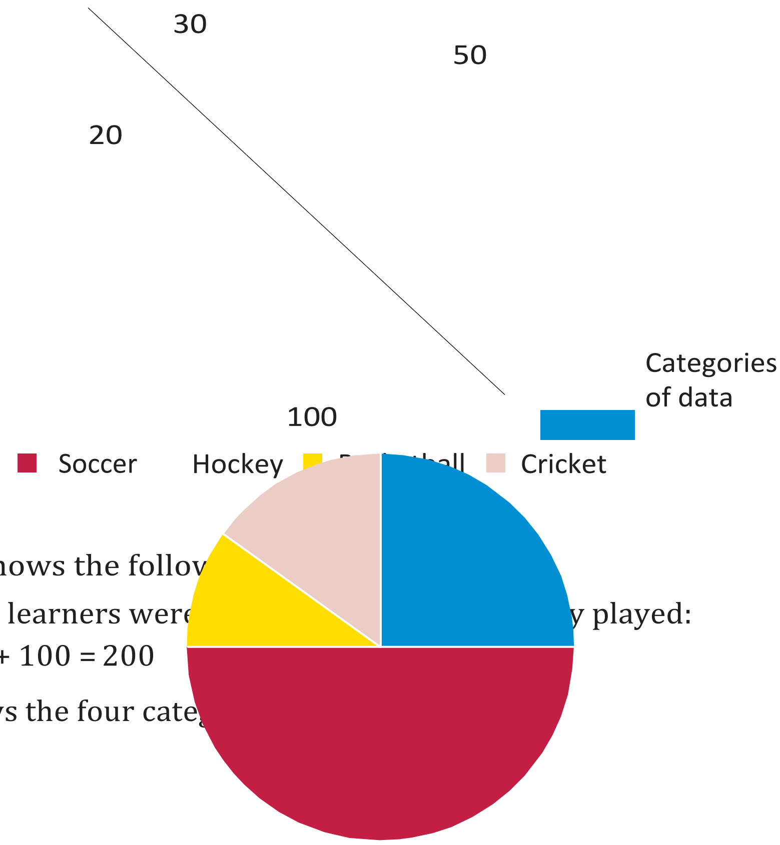
(b) Draw a histogram of this data.

24.3 Pie charts

A **pie chart** consists of a circle divided into slices (**sectors**), where the slices show how the different categories of data make up the whole set of data. Bigger categories of data have bigger slices of the circle.

Look at the example of a pie chart below.

Number of learners playing sports



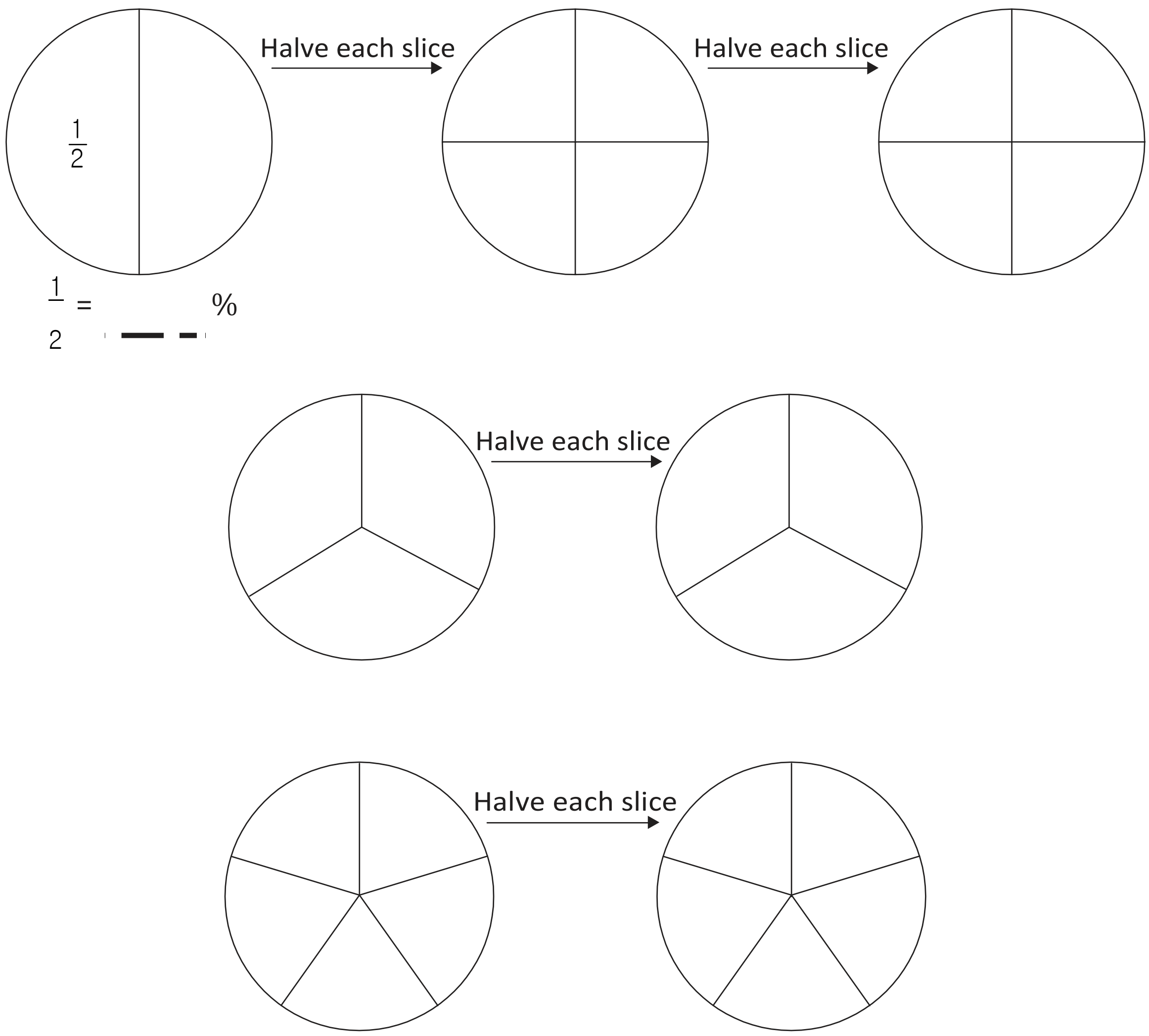
The pie chart shows the follov

- A total of 200 learners were … y played:
  20 + 30 + 50 + 100 = 200
- The key shows the four cate
  - soccer
  - hockey
  - basketball
  - cricket.
- 100 of the 200 learners play hockey. This is the largest category, and gets the biggest slice (half of the whole).
- 20 of the 200 learners play basketball. This is the smallest category, and gets the smallest slice (one tenth of the whole).

You will learn how to draw accurate pie charts in later grades. In this grade, you will estimate the portions of a pie chart that each category of data requires.

estimating sizes of slices in a pie chart

1. (a) Copy the following diagrams and add the slices indicated. Write down the fraction of a whole that each slice in the following diagrams shows.



   (b) For each diagram in question 1(a), write down what percentage each fraction is equal to.

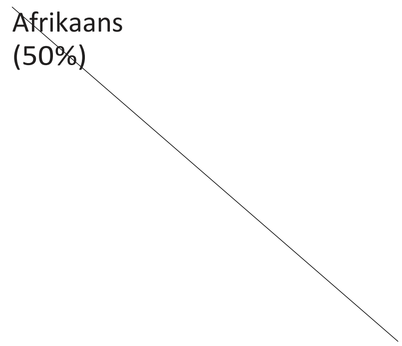
You can use the diagrams above to estimate the sizes of slices when drawing your own pie charts.

2. Copy the pie charts on the following page. Use the data in each of the following tables to complete the pie charts. You must:
   - label the major sector
   - divide the other sector into the parts that represent the other languages
   - label each sector.

(a)

Province: Western Cape

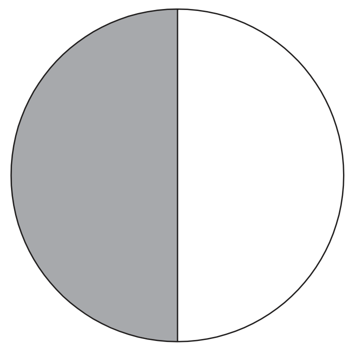
Major languages	Frequency (in %)
Afrikaans	50%
English	20%
isiXhosa	25%
Other	5%



(b)

Province: KwaZulu-Natal

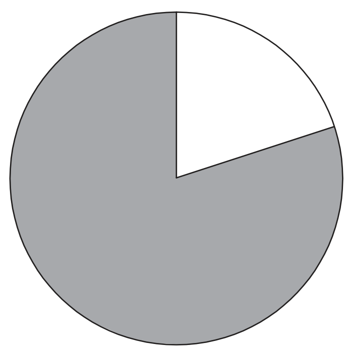
Major languages	Frequency (in %)
English	15%
isiZulu	80%
Other	5%



(c)

Province: Limpopo

Major languages	Frequency (in %)
Sepedi	50%
Tshivenda	15%
Xitsonga	20%
Other	15%

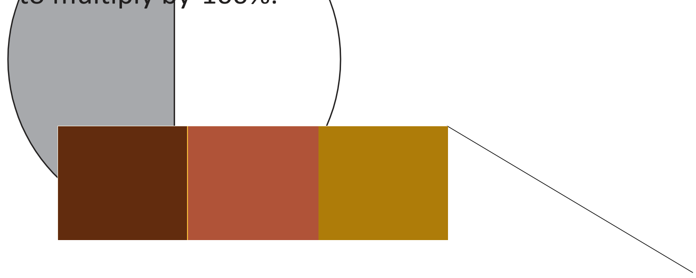


representing data as fractions and percentages in pie charts

To represent data in a pie chart, you need to know how to convert (change) the frequencies of the different categories into a fraction or percentage of the total.

1. The learners in Class A were asked how many languages they could speak. The table shows the data that was collected. Copy the table.
   (a) Complete the "Fraction" column by determining what fraction of the whole each category is.
   (b) Complete the "Percentage" column by converting the fraction to a percentage.

Remember, to convert a common fraction to a percentage you have to multiply by 100%.



Number of languages spoken by learners in Class A

Languages	Frequency	Fraction	Percentage
One language	10	$\frac{10}{40} = \frac{1}{4}$	25%
Two languages	20		
Three languages	6		
Four languages	2		
More than four languages	2		
Total	**40**	$\mathbf{\frac{40}{40}}$	**100%**

(c) Draw a pie chart of the data in your completed table. Use a circular object to draw the circle. Then estimate the sizes of the various slices of the pie chart.

2. The learners in Class B were asked how many languages they could speak. The table shows the data that was collected. Copy the table.

    (a) Complete the "Fraction" column by determining what fraction of the whole each category is.

    (b) Complete the "Percentage" column by converting the fraction to a percentage.

Number of languages spoken by learners in Class B

Languages	Frequency	Fraction	Percentage
One language	12	$\frac{12}{60} = \frac{1}{5}$	20%
Two languages	30		
Three languages	12		
Four languages	3		
More than four languages	3		
Total	**60**	$\mathbf{\frac{60}{60}}$	**100%**

(c) Draw a pie chart to represent the data in your completed table.

Chapter 25
Interpret, analyseand report on data

25.1 Interpreting and reporting on data

critically reading and reporting on data

1. Read the following paragraph and answer the questions that follow.

   In 2009, a sample of 2 500 schools from about 26 000 schools across South Africa took part in a survey to provide data about learners and schools. The sample included schools from each province as follows: 415 schools from the Eastern Cape, 238 from the Free State, 265 from Gauteng, 386 from KwaZulu-Natal, 326 from Limpopo, 248 from Mpumalanga, 129 from the Northern Cape, 275 from North West and 218 from the Western Cape.

   Adapted from: *Census @ School Results 2009*, Statistics South Africa

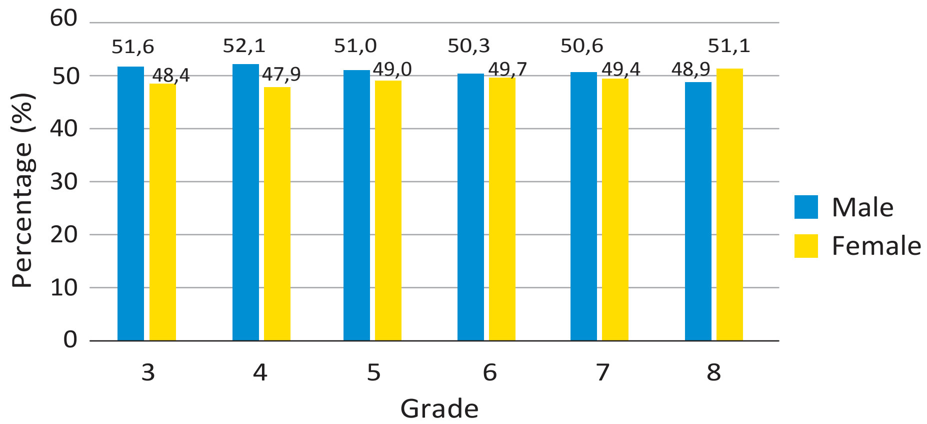
   (a) What was the population of the survey?
   (b) What was the sample of the survey?
   (c) Which province were most of the schools from?
   (d) Which province were the fewest schoolsfrom?
   (e) Copy the table below. Complete the first two columns of the table by listing the provinces in order from the province that had the most schools to the province that had the fewest schools participating in the survey.

Province	Number of schools	Percentage of all schools

   (f) Complete the last column by working out the percentage of the whole that the schools in each province make up. You may use your calculator for this question. (Round off to one decimalplace.)
   (g) Write three to five lines as a summary report of the data described in the paragraph above. The summary should give an idea of the highest and lowest data items, as this indicates the range of the data.

2. The graph that follows shows the percentage ofmale and female learners at schools in Grades 3 to 8 in 2009.

Percentage of male and female learners in Grades 3 to 8



(Source: *Census @ School Results 2009*, Statistics South Africa)

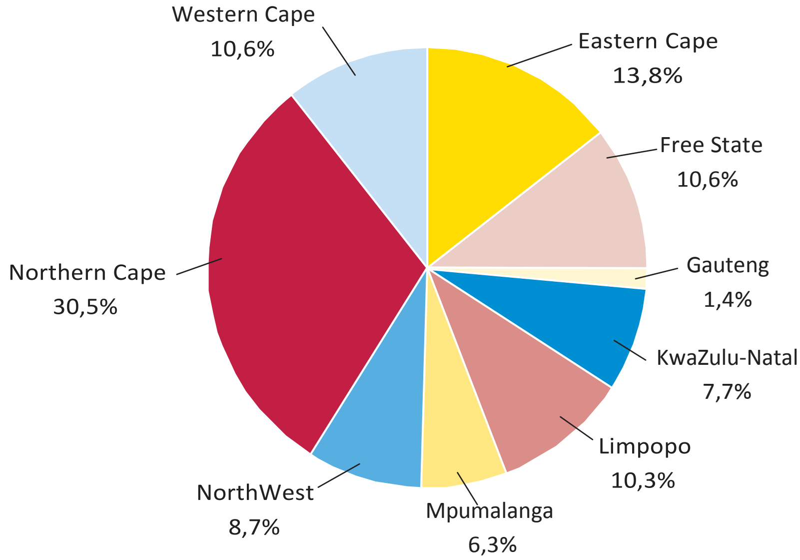
(a) Which grade has the highest percentage of females?
(b) Which grade has the lowest percentage of females?
(c) Which grade has the highest percentage of males?
(d) Which grade has the lowest percentage of males?
(e) If 150 000 Grade 6 learners took part in the survey, how many girls and how many boys were there in Grade 6? You may use your calculator.
(f) Copy and complete the following summary report:

> The graph shows that the number of male learners seems to (decrease/increase) the higher the grade. For example, in Grade 3, ___ % learners weremale compared to ___ % in Grade 8. The number of female learners seems to (decrease/increase) the higher the grade. For example, in Grade 3, ___ % learners were female compared to ___ % in Grade 8.

(g) Based on the graph, would you expect there to be more or fewer males in Grade 10? Explain your answer.
(h) Based on the graph, would you expect there to be more or fewer females in Grade 10? Explain your answer.

3. The pie chart on the following page shows the land area of each province in 2011.
   (a) Which province has the largest land area?
   (b) Which province has the smallest land area?
   (c) Which three provinces have more or less the same land area?
   (d) How much bigger is the Northern Cape than Gauteng? (Use a calculator.)
   (e) Are we able to tell from the pie chart which province has the largest population? Explain your answer.
   (f) If the total land area of South Africa is 1 200 000 km^2, how manysquare kilometres are the largest and the smallest provinces?

(g) Write a short paragraph to summarise the data shown in the pie chart.



(Source: *Census 2011: Census in brief*, Statistics South Africa)

25.2 Identifying bias and misleading data

Sometimes the ways in which data is presented could be intentionally or unintentionally **biased** or misleading. As you work through the following activities, think carefully about:

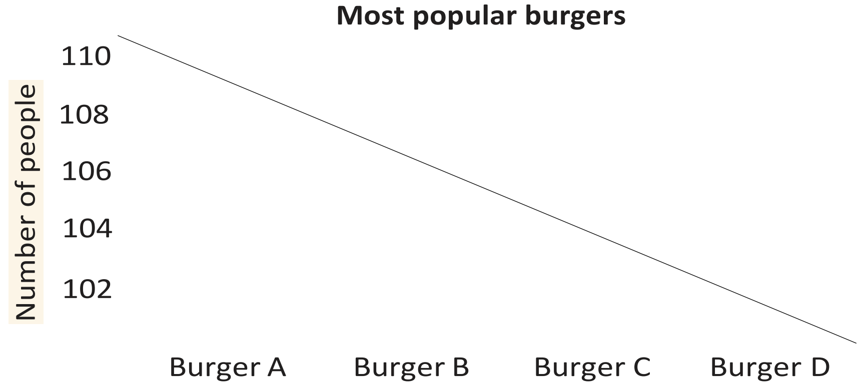
- data that is not necessarily shown by the graph
- when, how and where the data was collected
- which scales are used on the graphs
- which summary statistics (mean, median and mode) are used to summarise the data.

bias means that a person prefers a certain idea and possibly does not give equal chance to a different idea.

critically analysing data

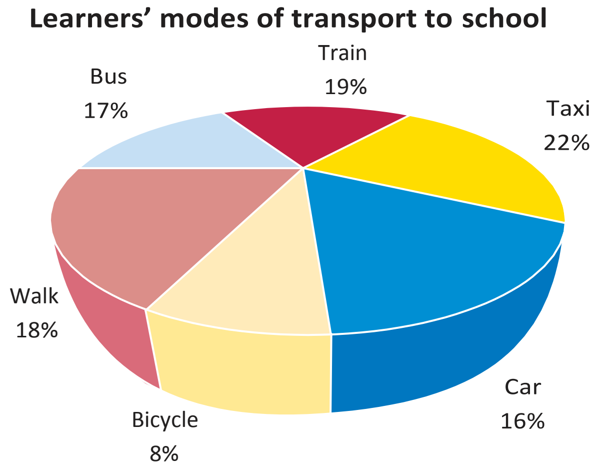
1. Look at the bar graph below and answer the questions that follow.

Most popular burgers

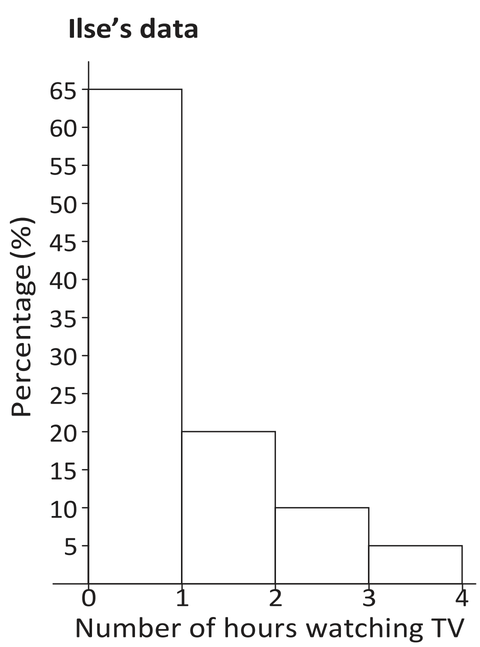


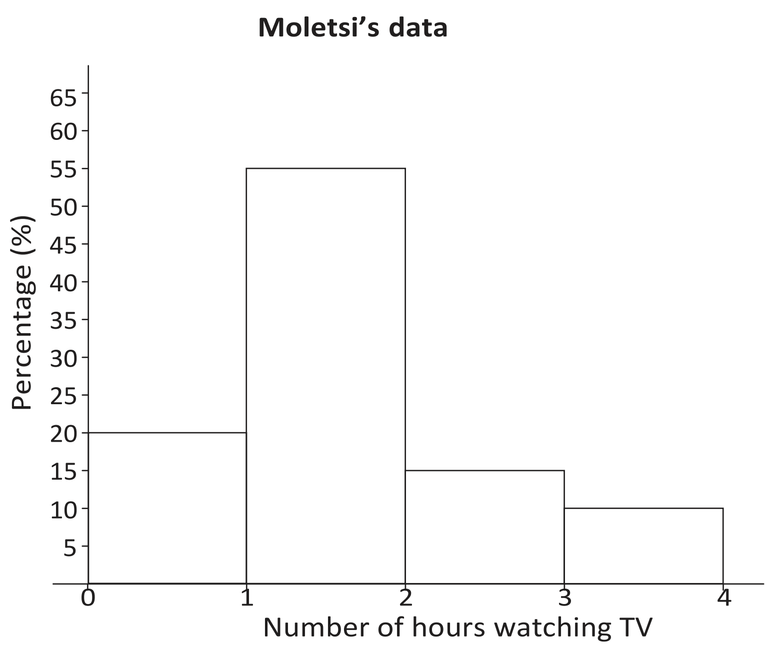
(a) Which burger is the most popular?
(b) The heights of the bars indicate that burger A is liked by five times as many people as burger B. Is this true? Look at the vertical scale.
(c) Redraw the bar graph, but show the full vertical scale.

2. Look at the pie chart.
   (a) What is the second most common mode of transport that learners use?
   (b) Which mode of transport is the least common one?
   (c) Is the pie chart misleading in any way? Explain.



3. Ilse and Moletsi wanted to find out more about the number of hours people spend watching TV on a particular public holiday. Ilse did her survey on the public holiday from 13:00 to 15:00. She visited a supermarket and asked adult respondents to complete her questionnaire. Moletsi did his survey on the same day from 17:00 to 19:00. He went from door to door in his neighbourhood and asked the children to complete his questionnaire.

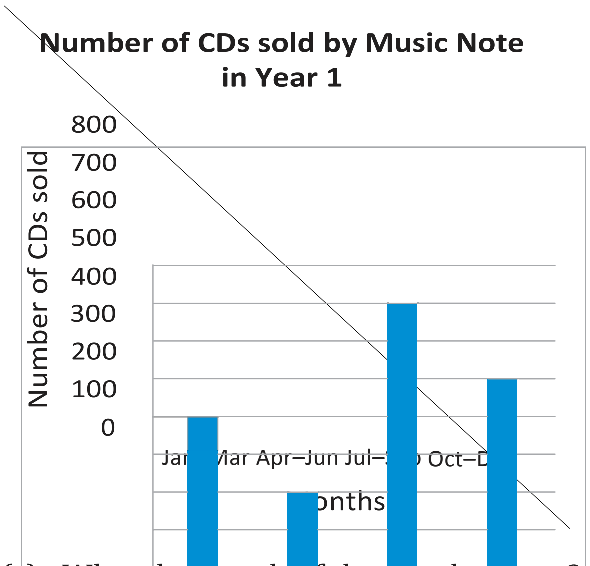


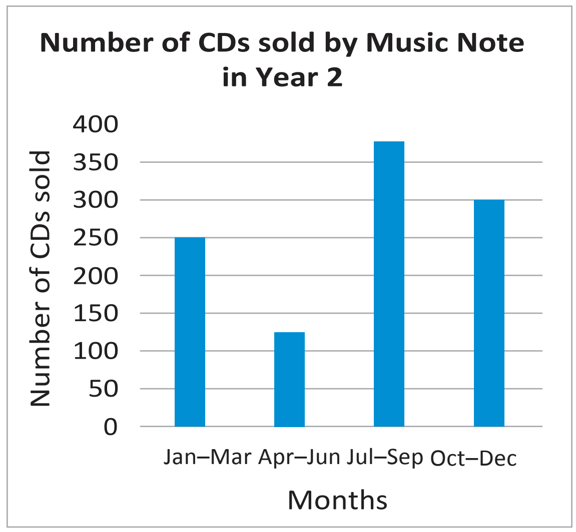


(a) According to Ilse's data, how long did most people spend watching TV on the public holiday?

(b) According to Moletsi's data, how long did most people spend watching TV on the public holiday?
(c) Write a paragraph to summarise and compare Ilse's data and Moletsi's data.
(d) How could the time when the data was collected have affected the data?
(e) How could the place where the data was collected have affected the data?
(f) How could the people from whom data was collected have affected the data?

4. Look at the following graphs and answer the questions that follow.





(a) What does each of the graphs show?
(b) How many CDs were sold in July to September of Year 1?
(c) How many CDs were sold in July to September of Year 2?
(d) The heights of the bars indicate that Music Note sold more CDs in October to December of Year 2 than in the same months of Year 1. Is this the case?
(e) How many CDs were sold altogether in Year 1?
(f) How many CDs were sold altogether in Year 2?
(g) Explain why the heights of the bars seem to indicate that Music Note sold more or less the same number of CDs in both years, which is not true.

5. The following table shows the Mathematics marks of Class A and Class B.

Class A	94, 42, 23, 67, 67, 68, 13, 53, 44, 34, 64, 69, 50, 31, 91, 40, 10, 30, 49, 61
Class B	74, 26, 65, 45, 71, 77, 58, 35, 39, 45, 68, 45, 57, 62, 29, 55, 23, 56, 38, 36, 50, 64, 58, 32, 42

(a) Find the range of each set ofdata.
(b) What can you say about the two classes by looking at the range of marks?

(c) Copy and complete the table by calculating the mean (average) Mathematics mark for each class. You may use your calculator.

Class	Total marks	Number of marks	Mean
Class A			
Class B			

(d) Compare the two sets ofdata using the means.

(e) Copy and complete the table by finding the median for each class.

Class	Marks from highest to lowest	Middle position	Median
Class A			
Class B			

(f) Compare the two sets of data using the medians.

(g) Copy and complete the table by finding the mode for each class.

Class	Highest frequency	Mode
Class A		
Class B		

(h) Compare the two sets of data usingthe mode.

(i) Which of the following do you think best represents each set of data: mean, median or mode? Explain your answer.

Chapter 26
Probability

26.1 Possible and actual outcomes, and frequencies

what can you expect?

You will soon do an experiment. To do the experiment you need a bag like a plastic shopping bag or a brown paper bag. You also need three objects of the same size and shape, like three buttons, bottle tops or small square pieces of cardboard. The three objects must look different, for example they should have different colours such as yellow, red and blue. If you use cardboard squares, you can write "yellow", "red" and "blue" on them.

1. (a) Put your three objects in your bag. You will later draw one object out of the bag, without looking inside. Can you say whether the object that you will draw will be the yellow one, the blue one or the red one?
   (b) Discuss this with two classmates.

2. (a) Now draw an object out of the bag, write down its colour, and put it back.
   (b) You will soon do this 12 times. Can you say how many times you will draw each of the three colours? If you think you can, write down your prediction.
   (c) Compare your predictions with two classmates.
   (d) Can you think of any reason why you may draw blue more often than red or yellow, when you do the experiment described in (b)?

3. (a) Draw an object out of the bag, write down its colour, and put it back. Do this 12 times and write down the colour of the object each time.
   (b) Write your results in a table like the one below.

Outcome	Yellow	Red	Blue
Number of times obtained			

What you did in question 3 is called a **probability experiment**. Each time you drew an object out of the bag, you performed a **trial**.

Each time you performed a trial, three different things could have happened. These are called the **possible outcomes**.

Each time you performed a trial, one of the possible outcomes actually occurred. This is called the **actual outcome**.

The number of times that a specific outcome occurred during an experiment is called the **actual frequency** of that outcome.

4. (a) What were the possible outcomes in the experiment that you did in question 3?
   (b) How many trials did you perform in the experiment?
   (c) What was the actual outcome in the third trial that you performed?
   (d) What was the actual frequency of drawing a blue object during the 12 trials in the experiment that you did?

26.2 Relative frequencies

Thomas also did the experiment in question 3 on page 294 but he performed more trials and his results were as follows:

Outcome	Yellow	Red	Blue
Number of times obtained	5	7	8

1. (a) How many trials did Thomas perform in total?
   (b) What fraction of the trials produced yellow as an outcome?
   (c) What fraction of the trials produced red as an outcome?
   (d) What fraction of the trials produced blue as an outcome?

The fraction of the trials in an experiment that produce a specific outcome is called the **relative frequency** of that outcome.

$$\text{Relative frequency of an outcome} = \frac{\text{number of times the outcome occurred}}{\text{total number of trials}}$$

A relative frequency can be expressed as a common fraction, as a decimal or as a percentage. The relative frequencies in the results of the experiment Thomas did (question 1) were one quarter for yellow, seven twentieths for red and two fifths for blue. Expressed as percentages, the relative frequencies were 25%, 35% and 40%. The **range** of Thomas's relative frequencies, expressed as percentages, is 15% (40% – 25%).

2. (a) Use your calculator to calculate the relative frequencies that you obtained for the three different outcomes in the experiment you did in question 3 on page 294. Express them both as fractions and percentages.
   (b) Calculate the range of the relative frequencies of the three outcomes for the results of the experiment you did in question 3.
   (c) You will soon repeat the experiment with three possible outcomes and 12 trials that you did. Do you think the results will be the same as the first time you did the experiment?

3. (a) Join with three or four classmates to work as a team, and discuss question 2(c).
   (b) Assign the "names" A, B, C, D and E (if there are five of you) to the team members and copy and complete the table below for the experiment you did in question 3 on page 294. Give the relative frequencies as percentages. Note that to calculate the relative frequencies for the totals as percentages, you have to use your calculators.

	Actual frequencies			Relative frequencies%			Range
	Yellow	Red	Blue	Yellow	Red	Blue	
Experiment 1 by A							
Experiment 1 by B							
Experiment 1 by C							
Experiment 1 by D							
Experiment 1 by E							
Totals for experiment 1							

   (c) Which of the ranges is the smallest?

26.3 More trials and relative frequencies

what happens when you conduct many trials?

1. Join up with your teammates of the previous activity. Each of you will soon repeat the experiment you did previously. You will put a yellow object, a red object and a blue object in a bag, draw one object and note the colour. You will do this 12 times. This will be experiment 2.
   (a) Do you expect that the results will, in some ways, be the same as for experiment 1 in the previous section? Do not talk to your teammates yet. Form your own opinion, and also consider *why* you think the results will be different or the same.
   (b) Share your ideas with your teammates.

You will soon repeat the experiment and write the results in the rows for "experiment 2" on the table on the next page. You will repeat it once more and write the results in the rows for "experiment 3". If you have time left, you may repeat it once more as "experiment 4".

2. (a) Look at the table on the next page. Certain rows are for the outcomes that you and your teammates obtain. The shaded rows are for adding different sets of outcomes together. Think about what may happen and predict in what rows the ranges will be smaller than in other rows, and in what row the range will be the smallest of all. Copy the table.

(b) Share your ideas with your teammates.

3. (a) Copy the totals for "experiment 1" into the first row of the table. Do the experiment described in question 1 and enter the results in the rows for "experiment 2". Calculate the relative frequencies and the range.
   (b) Add in the results of your teammates, add up the totals and calculate the relative frequencies and the range of thetotals.

4. Repeat question 3, and enter the results in the rows for "experiment 3".

		Actual frequencies			Relative frequencies%			Range
		Yellow	Red	Blue	Yellow	Red	Blue	
1	Totals for experiment 1							
2	Experiment 2 by A							
3	Experiment 2 by B							
4	Experiment 2 by C							
5	Experiment 2 by D							
6	Experiment 2 by E							
7	Totals for experiment 2							
8	Totals for experiments 1 and 2 combined							
9	Experiment 3 by A							
10	Experiment 3 by B							
11	Experiment 3 by C							
12	Experiment 3 by D							
13	Experiment 3 by E							
14	Totals for experiment 3							
15	Totals for experiments 1, 2 and 3 combined							
16	Experiment 4 by A							
17	Experiment 4 by B							
18	Experiment 4 by C							
19	Experiment 4 by D							
20	Experiment 4 by E							
21	Totals for experiment 4							
22	Totals for experiments 1, 2, 3 and 4 combined							