MATHEMATICS

Grade 8

CAPS

Learner Book

2017 Edition

Developed and funded as an ongoing project by the Sasol Inzalo Foundation in partnership with the UkuqondaInstitute.

Published by The Ukuqonda Institute
9 Neale Street, Rietondale 0084
Registered as a Title 21 company, registration number 2006/026363/08
Public Benefit Organisation, PBO Nr. 930035134
Website: http://www.ukuqonda.org.za

This edition published in 2017

ISBN: *************

This book was developed with the participation of the Department of Basic Education of South Africa with funding from the Sasol Inzalo Foundation.

Contributors:
Herholdt Bezuidenhout, Lucinda Cruickshank, Marthinus de Jager, Gudrun Elliott, Andrew Hofmeyr, Piet Human, Louise Keegan, Erna Lampen, Nathi Makae, Enoch Masemola, Alwyn Olivier, Cerenus Pfeiffer, Rika Potgieter, Johan Pretorius, Renate Röhrs, Paul van Koersveld, Therine van Niekerk, Dirk Wessels

Illustrations and computer graphics:
Leonora van Staden, Lisa Steyn Illustration
Zhandré Stark, Lebone Publishing Services

Computer graphics for chapter frontispieces: Piet Human

Cover illustration: Leonora van Staden

Text design: Mike Schramm

Layout and typesetting: Lebone Publishing Services

Printed by: [printer name and address]

Contents

Term 1

Term 2

Term 3

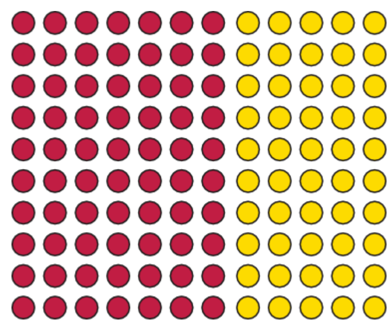
Term 4

Chapter 1
Whole numbers

1.1 Properties of whole numbers

the commutative property of addition and multiplication

1. Which of the following calculations would you choose to calculate the number of yellow beads in this pattern? Do not do any calculations now, just make a choice.



   (a) 7 + 7 + 7 + 7 + 7
   (b) 10 + 10 + 10 + 10 + 10 + 10 + 10
   (c) 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5
   (d) 5 + 5 + 5 + 5 + 5 + 5 + 5
   (e) 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7
   (f) 10 + 10 + 10 + 10 + 10

2. (a) How many red beads are there in the pattern? How many yellow beads are there?
   (b) How many beads are there in the pattern in total?

3. (a) Which expression describes what you did to calculate the total number of beads: 70 + 50 *or* 50 + 70?
   (b) Does it make a difference?
   (c) Which expression describes what you did to calculate the number of red beads: 7 × 10 *or* 10 × 7?
   (d) Does it make a difference?

We say: **addition and multiplication are commutative**. The numbers can be swopped around as their order does not change the answer. This does **not** work for subtraction and division, however.

4. Calculate each of the following:

(a) 5 × 8	(b) 10× 8	(c) 12× 8	(d) 8 × 12
(e) 6 × 8	(f) 3 × 7	(g) 6 × 7	(h) 7 × 6

the associative property of addition and multiplication

Lebogang and Nathi both have to calculate 25 × 24.

Lebogang calculates 25 × 4 and then multiplies by 6.
Nathi calculates 25 × 6 and then multiplies by 4.

1. Will they get the same answer or not?

> If three or more numbers have to be multiplied, it does not matter which two of the numbers are multiplied first.
> This is called the **associative property of multiplication**.
> We also say **multiplication is associative**.

2. Do the following calculations. **Do not use a calculator now**.
   (a) 4 + 7 + 5 + 6 (b) 7 + 6 + 5 + 4
   (c) 6 + 5 + 7 + 4 (d) 7 + 5 + 4 + 6

3. (a) Is additionassociative?
   (b) Illustrate your answer with an example.

4. Find the value of each expression by working in the easiest possible way:
   (a) 2 × 17× 5 (b) 4 × 7 × 5
   (c) 75 + 37+ 25 (d) 60 + 87 + 40 + 13

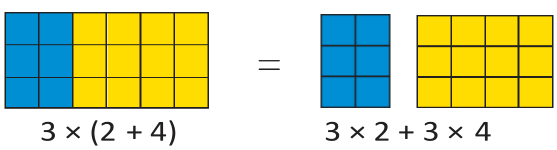
5. What must you add to each of the following numbers to get 100?
   82 44 56 78 24 89 77

6. What must you multiply each of these numbers by to get 1 000?
   250 125 25 500 200 50

7. Calculate each of the following. Note that you can make the work very easy by deciding how to group the operations.
   (a) 82 + 54 + 18 + 46 + 237 (b) 24 + 89 + 44 + 76 + 56 + 11
   (c) 25 × (86 × 4) (d) 32 × 125

more conventions and the distributive property

The distributive property is a useful property because it allows us to do this:



Both answers are 18. We used brackets in the first example to show that the addition operation must be done first. Otherwise, we would have done the multiplication first. For example, the expression 3 × 2 + 4 means "multiply 3 by 2; then add 4". It does **not** mean "add 2 and 4; then multiply by 3".

The expression 4 + 3 × 2 also means "multiply 3 by 2; then add 4".

> If you wish to specify that addition or subtraction should be **done first**, that part of the expression should be enclosed **in brackets**.

The distributive property can be used to break up a difficult multiplication into smaller parts. For example, it can be used to make it easier to calculate 6 × 204:

6 × 204 can be rewritten as 6 × (200 + 4) (Remember the brackets!)
= 6 × 200 + 6 × 4
= 1 200 + 24
= 1 224

Multiplication can also be distributed over subtraction, for example to calculate 7 × 96:

7 × 96 = 7 × (100 − 4)
= 7 × 100 − 7 × 4
= 700 − 28
= 672

1. Here are some calculations with answers. Rewrite them with brackets to make all the answers correct.
   (a) 8 + 6 × 5= 70 (b) 8 + 6 × 5 = 38 (c) 5 + 8 × 6 − 2 = 52
   (d) 5 + 8 × 6 − 2 = 76 (e) 5 + 8 × 6 − 2 = 51 (f) 5 + 8 × 6 − 2 = 37

2. Calculate the following:
   (a) 100 × (10 + 7) (b) 100 × 10 + 100 × 7
   (c) 100 × (10 − 7) (d) 100 × 10 − 100 × 7

3. Copy and complete the table:

×	8	5	4	9	7	3	6	2	10	11	12
7											
3				27				6			
9											
5											
8											
6											
4					28						
2											
10	80									110	
12											
11											

4. Use the various mathematical conventions for numerical expressions to make the following calculations easier. Show how you work them out.
   (a) 18 × 50 (b) 125× 28 (c) 39 × 220
   (d) 443 + 2 100 + 557 (e) 318 + 650+ 322 (f) 522 + 3 003 + 78

Two more properties of numbers are:
- **The additive property of 0:** when we add zero to any number, the answer is that number.
- **The multiplicative property of 1:** when we multiplyanynumberby1,theansweristhatnumber.

1.2 Calculations with whole numbers

estimating, approximating and rounding

1. Copy and complete the following statements by giving answers to these questions, without doing any calculations with the given numbers.
   (a) Is 8 × 117 more than 2 000 or less than 2 000? __ __ __than 2 000
   (b) Is 27 × 88 more than 3 000 or less than 3 000? __ __ __than 3 000
   (c) Is 18 × 117 more than 3 000 or less than 3 000? __ __ __than 3 000
   (d) Is 47 × 79 more than 3 000 or less than 3 000? __ __ __than 3 000

What you have done when you tried to give answers to questions 1(a) to (d), is called **estimation**. To estimate is to try to get close to an answer without actually doing the calculations with the given numbers.

An estimate may also be called an **approximation**.

2. Look at question 1 again.
   (a) The numbers 1 000, 2 000, 3 000, 4 000, 5 000, 6 000, 7 000, 8 000, 9 000 and 10 000 are all multiples of a thousand. In each case, write down the multiple of 1 000 that you think is closest to the answer. The numbers you write down are called **estimates**.
   (b) In some cases you may achieve a **better estimate** by adding 500 to your estimate, or subtracting 500 from it. If so, you may add or subtract 500.
   (c) If you wish, you may write what you believe is an even better estimate by adding or subtracting some hundreds.

3. (a) Use a calculator to find the exact answers for the calculations in question 1. Calculate the **error** in your last approximation of each of the answers in question 1.
   (b) What was your smallest error?

The difference between an estimate and the actual answer is called the **error**.

4. Think again about what you did in question 2. In 2(a) you tried to approximate the answers to the nearest **1 000**. In 2(c) you tried to approximate the answers to the nearest **100**. Describe what you tried to achieve in question 2(b).

5. Estimate the answers for each of the following products and sums. Try to approximate the answers for the **products** to the nearest thousand, and for the **sums** to the nearest hundred.

(a) 84 × 178	(b) 677 + 638
(c) 124 × 93	(d) 885 + 473
(e) 79 × 84	(f) 921 + 367
(g) 56 × 348	(h) 764 + 829

6. Use a calculator to find the exact answers for the calculations in question 5. Calculate the error in each of your approximations. Use the second line in each question to do this.

Calculating with "easy" numbers that are close to given numbers is a good way to obtain approximate answers, for example:

- To approximate 764 + 829, you may calculate 800 + 800 to get the approximate answer 1 600; with an error of 7.
- To approximate 84 × 178, you may calculate 80 × 200 to get the approximate answer 16 000; with an error of 1 048.

7. Calculate with "easy" numbers close to the given numbers to produce approximate answers for each product below. **Do not use a calculator.** When you have made your approximations, use a calculator to work out the precise answers.

(a) 78× 46	(b) 67 × 88
(c) 34 × 276	(d) 78 × 178

rounding off and compensating

The word **compensate** means to do things that will remove damage.

1. (a) Approximate the answer for 386 + 3 435, by rounding both numbers off to the nearest hundred, and then adding the rounded numbers.
   (b) Because you rounded 386 up to 400, you introduced an error of 14 in your approximate answer. What error did you introduce by rounding 3 435 down to 3 400?
   (c) What combined (total) error did you introduce by rounding both numbers off before calculating?
   (d) Use your knowledge of the total error to correct your approximate answer, so that you have the correct answer for 386 + 3 435.

In question 1, you used **rounding off** and **compensating** to find the correct answer for 386 + 3 435. By rounding the numbers off you introduced errors. You then compensated for the errors by making adjustments to your answer.

2. Round off and compensate to calculate each of the following accurately:
   (a) 473 + 638 (b) 677 + 921

Subtraction can also be done in this way. For example, to work out R5 362 – R2 687, you may round R2 687 up to R3 000. You may do this in the following ways:

- Rounding R2 687 up to R3 000 can be done in two steps: 2 687 + 13 = 2 700, and 2 700 + 300 = 3 000. In total, 313 is added.
- You can now add 313 to 5 362 too: R5 362 + 313 = 5 675.
- Instead of calculating R5 362 – R2 687, which is a bit difficult, you may calculate R5 675 – R3 000. This is easy: R5 675 – R3 000 = R2 675.

This means that R5 362 – R2 687 = R2 675, because
R5 362 – R2 687 = (R5 362 + R313) – (R2 687 + R313).

adding numbers in parts written in columns

Numbers can be added by thinking of their **parts** as we say the numbers. For example, we say 4 994 as *four thousand nine hundred and ninety-four*. This can be written in expanded notation as 4 000 + 900 + 90 + 4.

Similarly, we can think of 31 837 as 30 000 + 1 000 + 800 + 30 + 7.

31837 + 4 994 can be calculated by working with the various kinds of parts separately. To make this easy, you can write the numbers below each other so that the units are below the units, the tens below the tens and so on, as shown on the right.

31
837

We write only this:	In your mind you can see this:				
31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4

The numbers in each column can be added to get a new set of numbers.

31 837	30 000	1 000	800	30	7
4 994		4 000	900	90	4
11					11
120				120	
1 700			1 700		
5 000		5 000			
30 000	30 000				
36 831	It is easy to add the new set of numbers to get the answer.				

The work may start with the 10 000s or any other parts. Starting with the units, as shown on page 6, makes it possible to do more of the work mentally, and write less, as shown below.

31 837 4 994 36 831	To achieve this, only the units digit 1 of the 11 is written in the first step. The 10 of the 11 is remembered and added to the 30 and 90 of the tens column, to get 130.

We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 130: only the digit "3" is written (in the tens column, so it means 30), and the 100 is carried to the next step.

1. Calculate each of the following without using a calculator:

   (a) 4 638 + 2 667 (b) 748 + 7246

2. Impilo Enterprises plans a new computerised training facility in their existing building. The training manager has to keep the total expenditure budget under R1 million. This is what she has written so far:

Architects and builders	R 102 700
Painting and carpeting	R 42 600
Security doors and blinds	R 52 000
Data projector	R 4 800
25 new secretary chairs	R 50 400
24 desks for workstations	R123 000
1 desk for presenter	R 28 000
25 new computers	R300 000
12 colour laser printers	R 38 980

Work out the total cost of all the items for which the training manager has budgeted.

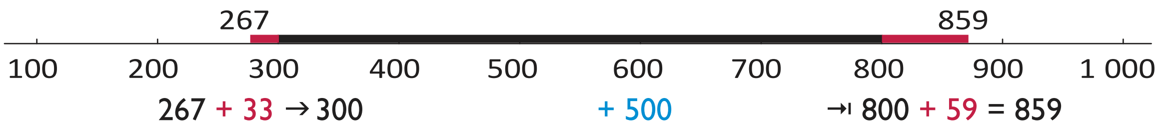
3. Calculate each of the following, without using a calculator:

   (a) 7 828 + 6 284 (b) 7 826 + 888 + 367
   (c) 657 + 32 890 + 6 542 (d) 6 666 + 3 333 + 1

methods of subtraction

There are many ways to find the difference between two numbers. For example, to find the difference between 267 and 859, you may think of the numbers as they would appear on a number line, for example:

We may think of the distance between 267 and 859 as three steps: from 267 to 300, from 300 to 800, and from 800 to 859. How big are each of these three steps?



This number line shows that 859 – 267 is 33 + 500 + 59.

1. Calculate 33 + 500 + 59 to find the answer for 859 – 267.
2. Calculate each of the following. You may think of working out the distance between the two numbers as shown above, or use any other method you prefer.
   Do not use a calculator now.
   (a) 823 – 456
   (b) 1 714–829
   (c) 3 045 – 2 572
   (d) 5 131–367

Like addition, subtraction can also be done by working with the different parts in which we say numbers. For example, 8 764 – 2 352 can be calculated as follows:

8 thousand – 2 thousand = 6 thousand
7 hundred – 3 hundred = 4 hundred
6 tens – 5 tens = 1 ten
4 units – 2 units = 2 units
So, 8 764 – 2 352 = 6 412

Subtraction by parts is more difficult in some cases, for example 6 213 – 2 758:

6 000 – 2 000 = 4 000. This step is easy, but the following steps cause problems:

200 – 700 =?
10 – 50 =?
3 – 8 =?

One way to overcome these problems is to work with negative numbers:
200 – 700 = (–500)
10 – 50 = (–40)
3 – 8 = (–5)
4 000 – 500 → 3 500 – 45 =

Fortunately, the parts and sequence of work may be rearranged to overcome these problems, as shown below:

Instead of		we may do		
3 – 8 =	?	13 – 8 =	5	"borrow" 10 from below
10 – 50 =	?	100 – 50 =	50	"borrow" 100 from below
200 – 700 =	?	1 100 – 700 =	400	"borrow" 1 000 from below
6 000 – 2 000 =	?	5 000 – 2 000 =	3 000	

This reasoning can also be set out in columns:

Instead of				we may do				but write only this			
6 000	200	10	3	5 000	1 100	100	13	6	2	1	3
2 000	700	50	8	2 000	700	50	8	2	7	5	8
				3 000	400	50	5	3	4	5	5

3. (a) Complete the above calculations and find the answer for 6 213 – 2 758.
   (b) Use the borrowing technique to calculate 823 – 376 and 6 431 – 4 968.
4. Check your answers in question 3(b) by doing addition.

With some practice, you can learn to subtract using borrowing without writing all the steps. When calculating 6 213 – 2 758, use the column method, as shown on the right.

```
  6 213
  2 758
      5
     50
    400
  3 000
  3 455
```

If you work more mentally, you will write even less, as shown below:

```
6 213
2 758
3 455
```

Do not use a calculator when you do question 5, because the purpose of this work is for you to understand the methods of subtraction. What you learn here will help you to understand **algebra**better at a later stage.

5. Calculate each of the following:
   (a) 7 342 – 3 877 (b) 8 653 – 1 856 (c) 5 671 – 4 528

You may use a calculator to do questions 6 and 7.

6. Estimate the difference between the two car prices in each case, to the nearest R1 000 or closer. Then calculate the difference.
   (a) R102 365 and R98 128 (b) R63 378 and R96 889
7. First estimate the answers to the nearest 100 000, 10 000 or 1 000. Then calculate.
   (a) 238 769 – 141 453 (b) 856 333 – 739 878 (c) 65 244 – 39 427

a method of multiplication

You can calculate 7 × 4 598 in parts, as shown here:

$$7 \times 4\,000 = 28\,000$$
$$7 \times 500 = 3\,500$$
$$7 \times 90 = 630$$
$$7 \times 8 = 56$$

The four partial products can now be added to get the answer, which is 32 186. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown on the right.

```
    4 5 9 8
×         7
        5 6
      6 3 0
    3 5 0 0
  2 8 0 0 0
  3 2 1 8 6
```

When working from right to left in the columns, you can "carry" parts of the partial answers to the next column. Instead of writing 56, only the 6 of the product 7 × 8 is written down. The 50 is kept in mind, and added to the 630 obtained when 7 × 90 is calculated in the next step.

$$\begin{array}{r} 4\ 5\ 9\ 8 \\ \underline{\quad\quad\quad 7} \\ 3\ 2\ 1\ 8\ 6 \end{array}$$

1. Calculate each of the following. Do not use a calculator now.
   (a) 27 × 649 (b) 75 × 1 756 (c) 348 × 93
2. Use your calculator to check your answers for question 1. Redo the questions for which you had the wrong answers.
3. Calculate each of the following. Do not use a calculator now.
   (a) 67 × 276 (b) 84 × 178
4. Use a calculator to check your answers for question 3. Redo the questions for which you had the wrong answers.

long division

1. The municipal head gardener wants to buy young trees to plant along the main street of the town. The young trees cost R27 each, and an amount of R9 400 has been budgeted for trees. He needs 324 trees. Do you think he has enough money?
2. (a) How much will 300 trees cost?
   (b) How much money will be left if 300 trees are bought?
   (c) How much money will be left if 20 more trees are bought?

The municipal gardener wants to work out exactly how many trees, at R27 each, he can buy with the budgeted amount of R9 400. His thinking and writing are described below.

Step 1:

What he writes:	What he thinks:
27 | 9 400	*I want to find out how many chunks of 27 there are in 9 400.*

Step 2:

What he writes:	What he thinks:
300	*I think there are at least 300 chunks of 27 in 9 400.*
27 | 9 400	
<u>8 100</u>	*300 × 27 = 8 100. I need to know how much is left over.*
1 300	*I want to find out how many chunks of 27 there are in 1 300.*

Step 3: (He has to rub out the one "0" of the 300 on top, to make space.)

What he writes:	What he thinks:
340	*I think there are at least 40 chunks of 27 in 1 300.*
27 | 9 400	
<u>8 100</u>	
1 300	
<u>1 080</u>	*40 × 27 = 1 080. I need to know how much is left over.*
220	*I want to find out how many chunks of 27 there are in 220. Perhaps I can buy some extra trees.*

Step 4: (He rubs out another "0".)

What he writes: What he thinks:

```
      348
27 | 9 400
     8 100
     1 300
     1 080
       220
       216
         4
```

I think there are at least 8 chunks of 27 in 220.

8 × 27 = 216

So, I can buy 348 young trees and will have R4 left.

Do not use a calculator to do questions 3 and 4. The purpose of this work is for you to develop a good understanding of how division can be done. Check all your answers by doing multiplication.

3. (a) Graham bought 64 goats, all at the same price. He paid R5 440 in total. What was the price for each goat? You can start by working out how much he would have paid if he paid R10 per goat. You can start with a bigger step if you wish.
   (b) Mary has R2 850 and she wants to buy candles for her sister's wedding reception. The candles cost R48 each. How many candles can she buy?

4. Calculate each of the following. Do not use a calculator.
   (a) 7 234 ÷ 48 (b) 3 267 ÷ 24
   (c) 9 500 ÷ 364 (d) 8 347 ÷ 24

1.3 Multiples, factors and prime factors

multiples and factors

1. The numbers 6; 12; 18; 24; ... are **multiples** of 6.
   The numbers 7; 14; 21; 28; ... are **multiples** of 7.

If n is a natural number, $6n$ represents the multiples of 6.

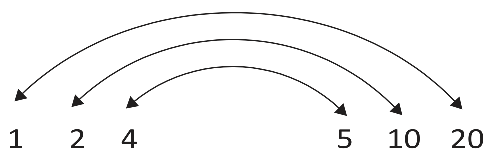
   (a) What is the hundredth number in each sequence above?
   (b) Is 198 a number in the first sequence?
   (c) Is 175 a number in the second sequence?

Of which numbers is 20 a multiple?

20 = 1 × 20 = 2 × 10 = 4 × 5 = 5 × 4 = 10 × 2 = 20 × 1

20 is a multiple of 1; 2; 4; 5; 10 and 20, and all of these numbers are factors of 20.

Factors come in pairs. The following pairs are factors of 20:



2. A rectangle has an area of 30 cm. What are the possible lengths of the sides of the rectangle in centimetres if the lengths of the sides are natural numbers?

3. Are 4, 8, 12 and 16 factors of 48? Simon says that all multiples of 4 smaller than 48 are factors of 48. Is he right?

4. We have defined factors in terms of the product of *two* numbers. What happens if we have a product of *three* or more numbers, for example: 210 = 2 × 3 × 5 × 7?
   (a) Explain why 2, 3, 5 and 7 are factors of 210.
   (b) Are 2 × 3, 3 × 5, 5 × 7, 2 × 5 and 2 × 7 factors of 210?
   (c) Are 2 × 3 × 5, 3 × 5 × 7 and 2 × 5 × 7 factors of 210?

5. Is 20 a factor of 60? What factors of 20 are also factors of 60?

prime numbers and composite numbers

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.
   (a) 66 (b) 67
   (c) 68 (d) 69
   (e) 70 (f) 71
   (g) 72 (h) 73

The number 36 can be formed as 2 × 2 × 3 × 3. Because 2 and 3 are used twice, they are called **repeated factors** of 36.

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*?

A number that cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*, is called a **prime number**.

3. Which of the numbers in question 1 are prime numbers?

Composite numbers are natural numbers with more than two different factors. The sequence of composite numbers is 4; 6; 8; 9; 10; 12; ...

4. Are the statements below true or false? If you answer “false”, explain why.
   (a) All prime numbers are odd numbers.
   (b) All composite numbers are evennumbers.
   (c) 1 is a prime number.
   (d) If a natural number is not prime, then it is composite.
   (e) 2 is a composite number.
   (f) 785 is a prime number.
   (g) A prime number can only end in 1, 3, 7 or 9.
   (h) Every composite number is divisible by at least one prime number.

5. We can find out if a given number is prime by systematically checking whether the primes 2; 3; 5; 7; 11; 13; ... are factors of the given number or not.

   To find possible factors of 131, we need to consider only the primes 2; 3; 5; 7 and 11. Why not 13; 17; 19; ...?

6. Determine whether the following numbers are prime or composite. If the number is composite, write down at least two factors of the number (besides 1 and the number itself).

   (a) 221 (b) 713

prime factorisation

Every composite number can be expressed as the product of prime factors and this can happen in only one way.

To find all the factors of a number you can write the number as the product of prime factors; first by writing it as the product of two convenient (composite) factors and then by splitting these factors into smaller factors until all factors are prime. Then you take all the possible combinations of the products of the prime factors.

Example: Find the factors of 84.

Write 84 as the product of prime factors by starting with different known factors:

$84 = 4 \times 21$ or $84 = 7 \times 12$ or $84 = 2 \times 42$

$= 2 \times 2 \times 3 \times 7$ $= 7 \times 3 \times 4$ $= 2 \times 6 \times 7$

$= 7 \times 3 \times 2 \times 2$ $= 2 \times 2 \times 3 \times 7$

A more systematic way of finding the prime factors of a number would be to start with the prime numbers and try the consecutive prime numbers 2; 3; 5; 7; ... as possible factors. The work may be set out as shown below.

2	1 430
5	715
11	143
13	13
	1

$1\ 430 = 2 \times 5 \times 11 \times 13$

3	2 457
3	819
3	273
7	91
13	13
	1

$2\ 457 = 3 \times 3 \times 3 \times 7 \times 13$

We can use exponents to write the products of prime factors more compactly as products of powers of prime factors.

$$2\ 457 = 3 \times 3 \times 3 \times 7 \times 13 = 3^3 \times 7 \times 13$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

$$1\ 500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 = 2^2 \times 3 \times 5^3$$

1. Express the following numbers as the product of powers of primes:

   (a) 792 (b) 444

2. Find the prime factors of the following numbers:

   28 32 124 36 42 345 182

common multiples and factors

1. Is 4 × 5 a multiple of 4? Is 4 × 5 a multiple of 5?
2. Comment on the following statement:
   The product of numbers is a multiple of each of the numbers in the product.

We use **common multiples** when fractions with different denominators are added. To add $\frac{2}{3} + \frac{3}{4}$, the common denominator is 3 × 4, so the sum becomes $\frac{8}{12} + \frac{9}{12}$.
In the same way, we could use 6 × 8 = 48 as a common denominator to add $\frac{1}{6} + \frac{3}{8}$, but 24 is the **lowest common multiple** (**LCM**) of 6 and 8.

Prime factorisation makes it easy to find the **LCM** or highest common factor.

When we simplify a fraction, we divide the same number into the numerator and the denominator. For the simplest fraction, use the **highest common factor** (**HCF**) to divide into both the numerator and denominator.

The HCF is divided into the numerator and the denominator to write the fraction in its **simplest form**.

So $\frac{36}{144} = \frac{2 \times 2 \times 3 \times 3}{2 \times 2 \times 2 \times 2 \times 3 \times 3} = \frac{1}{4}$

Use prime factorisation to determine the LCM and HCF of 32, 48 and 84 in a systematic way:

$$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$
$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$$

The LCM is a **multiple** and therefore, all of the factors of all the numbers must divide into it.

All of the factors that are present in the three numbers must also be factors of the LCM, even if it is a factor of only one of the numbers. But because it has to be the lowest common multiple, there are no unnecessary factors in the LCM.

The highest power of each factor is in the LCM, because all of the other factors can divide into it. In 32, 48 and 84, the highest power of 2 is 2^5, the highest power of 3 is 3 and the highest power of 7 is 7.

LCM = $2^5 \times 3 \times 7 = 672$

The HCF is a common factor. Therefore, for a factor to be in the HCF, it must be a factor of *all* of the numbers. The only number that appears as a factor of all three numbers is 2. The lowest power of 2 is 2^2; therefore, the HCF is 2^2.

3. Determine the LCM and the HCF of the numbers in each case:
   (a) 24; 28; 42
   (b) 17; 21;35
   (c) 75; 120; 200
   (d) 18; 30; 45

investigate prime numbers

You may use a calculator for this investigation.

1. Find all the prime numbers between 110 and 130.
2. Find all the prime numbers between 210 and 230.
3. Find the biggest prime number smaller than 1 000.

1.4 Solving problems

rate and ratio

You may use a calculator for the work in this section.

1. Tree plantations in the Western Cape are to be cut down in favour of natural vegetation. There are roughly 3 000 000 trees on plantations in the area and it is possible to cut them down at a **rate** of 15 000 trees per day with the labour available. How many working days will it take before all the trees will be cut down?

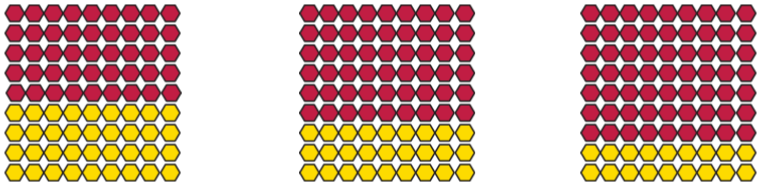
Instead of saying "... per day", people often say "at a **rate** of ... per day". Speed is a way in which to describe the rate of movement.

The word **per** is often used to describe a rate and can mean *for every*, *for*, *in each*, *in*, *out of*, or *every*.

2. A car travels a distance of 180 km in two hours on a straight road. How many kilometres can it travel in three hours at the same speed?

3. Thobeka wants to order a book that costs \$56,67. The rand-dollar exchange rateis R13,79 to a dollar. What is the price of the book in rands?

4. In pattern A below, there are five red beads for every four yellowbeads.



Pattern A *Pattern B* *Pattern C*

Describe patterns B and C in the same way.

5. Copy and complete the following table to show how many screws are produced by two machines in different periods oftime:

Number of hours	1	2	3	5	8
Number of screws at machine A	1 800				
Number of screws at machine B	2 700				

(a) How much faster is machine B than machine A?
(b) How many screws will machine B produce in the same time that it takes machine A to make 100 screws?

The patterns in question 4 can be described like this: In pattern A, the **ratio** of yellow beads to red beads is 4 to 5. This is written as 4 : 5. In pattern B, the ratio between yellow beads and red beads is 3: 6. In pattern C the ratio is 2 : 7.

In question 5, machine A produces two screws for every three screws that machine B produces. This can be described by saying that the ratio between the production speeds of machines A and B is 2 : 3.

6. Nathi, Paul and Tim worked in Mr Setati's garden. Nathi worked for five hours, Paul for four hours and Tim for three hours. Mr Setati gave the boys R600 for their work. How should they divide the R600 among the three of them?

We use **ratios** to show how many times more, or less, one quantity is than another.

A **ratio** is a comparison of two (or more) quantities. The number of hours that Nathi, Paul and Tim worked are in the ratio 5: 4 : 3. To be fair, the money should also be shared in that ratio. Therefore, Nathi should receive five parts, Paul four parts and Tim three parts of the money. There were 12 parts, which means Nathi should receive $\frac{5}{12}$ of the total amount, Paul should get $\frac{4}{12}$ and Tim should get $\frac{3}{12}$.

7. Ntabi uses three packets of jelly to make a pudding for eight people. How many packets of jelly does she need to make a pudding for 16 people? And for 12 people?

8. Which rectangle is more like a square: a 3 × 5 rectangle or a 6 × 8 rectangle? Explain.

To increase 40 in the ratio 2 : 3 means that the 40 represents two parts and must be increased so that the new number represents three parts. If 40 represents two parts, 20 represents one part. The increased number will therefore be 20 × 3 = 60.

Remember that if you multiply by 1, the number does not change. If you multiply by a number greater than 1, the number increases. If you multiply by a number smaller than 1, the number decreases.

9. (a) Increase 56 in the ratio 2 : 3.
   (b) Decrease 72 in the ratio 4 : 3.

10. (a) Divide 840 in the ratio 3 : 4.
   (b) Divide 360 in the ratio 1 : 2 : 3.

11. Look at the following data about the performance of different athletes during a walking event. Investigate the data to find out who walks the fastest and who walks the slowest. Arrange the athletes from the fastest walker to the slowest walker.
   (a) First make estimates to do the investigation.
   (b) Then use your calculator to do the investigation.

Athlete	A	B	C	D	E	F
Distance walked in metres	2 480	4 283	3 729	6 209	3 112	5 638
Time taken in minutes	17	43	28	53	24	45

profit, loss, discount and interest

1. (a) How much is one eighth of R800?
   (b) How much is one hundredth of R800?
   (c) How much is seven hundredths of R800?

Rashid is a furniture dealer. He buys a couch for R2 420. He displays the couch in his showroom with the price marked as R3 200. Rashid offers a discount of R320 to customers who pay cash.

The amount for which a dealer buys an article from a producer or manufacturer is called the **cost price**. The price marked on the article is called the **marked price** and the price of the article after discount is the **selling price**.

2. (a) What is the cost price of the couch in Rashid's furniture shop?
   (b) What is the marked price?
   (c) What is the selling price for a customer who pays cash?
   (d) How much is ten hundredths of R3 200?

The **discount** on an article is always less than the marked price of the article. In fact, it is only a fraction of the marked price. The discount of R320 that Rashid offers on the couch is ten hundredths of the marked price.

Another word for hundredths is **percentage**, and the symbol for percentage is %. We can therefore say that Rashid offers a discount of 10%.

A percentage is a number of hundredths.

18% is 18 hundredths and 25% is 25 hundredths.

% is a symbol for hundredths. 8% means eight hundredths and 15% means 15 hundredths. The symbol % is just a variation of the $\frac{}{100}$ that is used in the common fraction notation for hundredths.

For example: 8% is $\frac{8}{100}$.

A discount of 6% on an article can be calculated in two steps:

Step 1: Calculate one hundredth of the marked price (divide by 100).

Step 2: Calculate six hundredths of the marked price (multiply by six).

3. Calculate a discount of 6% on each of the following marked prices of articles:
   (a) R3 600 (b) R9 360

4. (a) How much is one hundredth of R700?
   (b) A customer pays cash for a coat marked at R700.
   He is given a R63 discount. How many hundredths of R700 is this?
   (c) What is the percentage discount?

5. A client buys a blouse marked at R300 and she is given R36 discount for paying cash. Work as in question 4 to determine what percentage discount she was given.

You may use a calculator to do questions 6, 7 and 8.

6. A dealer buys an article for R7 500 and makes the price 30% higher. The article is sold at a 20% discount.
   (a) What is the selling price of the article?
   (b) What is the dealer's percentage profit?

When a person borrows money from a bank or some other institution, he or she normally has to pay for the use of the money. This is called **interest**.

7. Sam borrows R7 000 from a bank at 14% interest for one year. How much does he have to pay back to the bank at the end of the period?

8. Jabu invests R5 600 for one year at 8% interest.
   (a) What will the value of his investment be at the end of that year?
   (b) At the end of the year, Jabu does not withdraw the investment or the interest earned; instead, he reinvests it for another year. How much will it be worth at the end of the second year?
   (c) What will the value of Jabu's investment be after five years?

Chapter 2
Integers

2.1 What is beyond 0?

why people decided to have negative numbers

On the right, you can see how Jimmy prefers to work when doing calculations, such as 542 + 253.

He tries to calculate 542 − 253 in a similar way:

500 − 200 = 300

40 − 50 = ?

Jimmy clearly has a problem. He reasons as follows:

I can subtract 40 from 40; that gives 0. But then there is still 10 that I have to subtract.

He decides to deal with the 10 that he still has to subtract later, and continues:

500 − 200 = 300

40 − 50 = 0, *but there is still 10 that I have to subtract.*

2 − 3 = 0, *but there is still 1 that I have to subtract.*

500 + 200 = 700
40 + 50 = 90
2 + 3 = 5
700 + 90 + 5 = 795

1. (a) What must Jimmy still subtract, and what will his final answer be?
   (b) When Jimmy did another subtraction problem, he ended up with this writing at one stage:
   600 and (−)50 and (−)7
   What do you think Jimmy's final answer for this subtraction problem is?

About 500 years ago, some **mathematicians** proposed that a "negative number" may be used to describe the result in a situation, such as in Jimmy's subtraction problem above, where a number is subtracted from a number smaller than itself.

For example, we may say 10 − 20 = (−10).

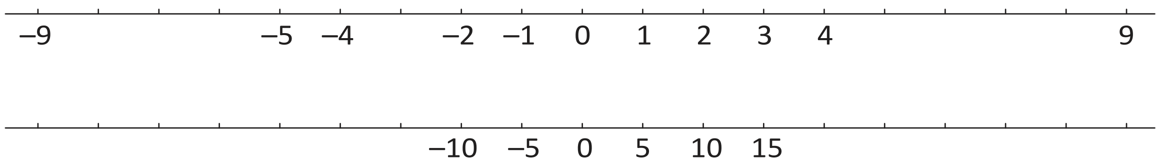
This proposal was soon accepted by other mathematicians, and it is now used all over the world.

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like healthcare is the profession of nurses and medical doctors.

2. Calculate each of the following:

(a) 16 − 20 (b) 16 − 30
(c) 16 − 40 (d) 16 −60
(e) 16 − 200 (f) 5 − 1 000

3. Some numbers are shown on the number lines below. Copy the number lines and numbers shown, and fill in the missing numbers.



The following statement is true if the number is 5:

15 − (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

The numbers 1; 2; 3; 4 etc. are called the **natural numbers**. The natural numbers, 0 and the negative whole numbers together, are called the **integers**.

But to go from 15 to 10 you have to subtract 5.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

Let us decide, and agree amongst ourselves, that the number we call ***negative 5*** *will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5.*

This means that the mathematicians agreed that 15 + (−5) is equal to 15 − 5.

Stated differently, instead of adding *negative 5* to a number, you may subtract 5.

Adding a negative number has the same effect as subtracting a natural number.

For example: 20 + (−15) = 20 − 15 = 5

4. Calculate each of the following:

(a) 500 + (−300) (b) 100 + (−20) + (−40)
(c) 500 + (−200) + (−100) (d) 100 + (−60)

5. Make a suggestion of what the answer for (−20) + (−40) should be. Give reasons for your suggestion.

6. Copy and continue the lists of numbers to complete the following table:

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	−3	−20	150	0
9	90	6	−6	−18	125	−5
8	80	9	−9	−16	100	−10
7	70	12	−12	−14	75	−15
6	60	15	−15		50	−20
5	50					−25
4	40					
3	30					
2	20					
1	10					
0	0					
−1	−10					

The following statement is true if the number is 5:

15 + (a certain number) = 20

What properties should a number have so that it makes the following statement true?

15 − (a certain number) = 20

To go from 15 to 20 you have to add 5. The number we need to make the sentence *15 − (a certain number)* = 20 true, must have the following property:

If you **subtract** this number, it should have the **same effect** as to **add 5**.

Let us agree that 15 − (−5) is equal to 15 + 5.

Stated differently, instead of subtracting *negative 5* from a number, you may add 5.

Subtracting a negative number has the same effect as adding a natural number.
For example: 20 − (−15) = 20 + 15 = 35

7. Calculate each of the following:

(a) 30 − (−10) (b) 30 + 10 (c) 30 + (−10) (d) 30 − 10
(e) 30 − (−30) (f) 30 + 30 (g) 30+ (−30) (h) 30 − 30

You probably agree that:

5 + (−5) = 0 10 + (−10) = 0 and 20 + (−20) = 0

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (−3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0 (the additive property of 0). For example, 120 + (−120) = 0.

What may each of the following be equal to?
(−8) + 5
(−5) + (−8)

8. Write the additive inverse of each of the following numbers:

(a) 24 (b) −24
(c) −103 (d) 2 348

The idea of additive inverses may be used to explain why 8 + (−5) is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

9. Use the idea of additive inverses to explain why each of these statements is true:

(a) 43 + (−30)= 13 (b) 150 + (−80) = 70

statements that are true for many different numbers

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed?

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed?

2.2 Adding and subtracting with integers

adding can make less and subtraction can make more

1. Calculate each of thefollowing:

(a) 10 + 4+ (−4) (b) 10 + (−4) + 4
(c) 3 + 8+ (−8) (d) 3 + (−8) + 8

Natural numbers can be arranged in any order to add and subtract them. This is also the case for integers.

The numbers 1; 2; 3; 4; ... that we use to count, are called **natural numbers**.

2. Calculate each of the following:

(a) 18 + 12 (b) 12 + 18 (c) 2 + 4 + 6
(d) 6 + 4 + 2 (e) 2 + 6 + 4 (f) 4 + 2 + 6
(g) 4 + 6 + 2 (h) 6 + 2 + 4 (i) 6 + (−2) + 4
(j) 4 + 6 + (−2) (k) 4 + (−2) + 6 (l) (−2) + 4 + 6
(m) 6 + 4 + (−2) (n) (−2) + 6 + 4 (o) (−6) + 4 + 2

3. Calculate each of the following:

(a) (−5) + 10 (b) 10 + (−5) (c) (−8) + 20
(d) 20 − 8 (e) 30 + (−10) (f) 30 + (−20)
(g) 30 + (−30) (h) 10 + (−5) + (−3)
(i) (−5) + 7 + (−3) + 5 (j) (−5) + 2 + (−7) + 4

4. In each case, find the number that makes the statement true. Give your answer by writing a closed number sentence.

(a) 20 + (an unknown number) = 50
(b) 50 + (an unknown number) = 20
(c) 20 + (an unknown number) = 10
(d) (an unknown number) + (−25) = 50
(e) (an unknown number) + (−25) = −50

Statements like these are also called number sentences.

An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**:

8 − (a number) = 10

A **closed number sentence** is where all the numbers are known:

8 + 2 = 10

5. Use the idea of additive inverses to explain why each of the following statements is true:

(a) 43 + (−50) = −7 (b) 60 + (−85) = −25

6. Copy and complete the following table as far as you can:

(a)	(b)	(c)
5 − 8 =	5 + 8 =	8 − 3 =
5 − 7 =	5 + 7 =	7 − 3 =
5 − 6 =	5 + 6 =	6 − 3 =
5 − 5 =	5 + 5 =	5 − 3 =
5 − 4 =	5 + 4 =	4 − 3 =
5 − 3 =	5 + 3 =	3 − 3 =
5 − 2 =	5 + 2 =	2 − 3 =
5 − 1 =	5 + 1 =	1 − 3 =
5 − 0 =	5 + 0 =	0 − 3 =
5 − (−1) =	5 + (−1) =	(−1) − 3 =
5 − (−2) =	5 + (−2) =	(−2) − 3 =
5 − (−3) =	5 + (−3) =	(−3) − 3 =
5 − (−4) =	5 + (−4) =	(−4) − 3 =
5 − (−5) =	5 + (−5) =	(−5) − 3 =
5 − (−6) =	5 + (−6) =	(−6) − 3 =

7. Calculate each of the following:

(a) 80 + (−60) (b) 500 + (−200) + (−200)

8. (a) Is 100 + (−20) + (−20) = 60, or does it equal to something else?
   (b) What do you think (−20) + (−20) is equal to?

9. Calculate each of the following:
   (a) 20 − 20 (b) 50 − 20 (c) (−20) − (−20) (d) (−50) − (−20)

10. Calculate each of the following:
   (a) 20 − (−10) (b) 100 − (−100) (c) 20 + (−10) (d) 100 + (−100)
   (e) (−20) − (−10) (f) (−100) − (−100) (g) (−20) + (−10) (h) (−100) + (−100)

11. Copy and complete the following table as far as you can:

(a)	(b)	(c)
5 − (−8) =	(−5) + 8 =	8 − (−3) =
5 − (−7) =	(−5) + 7 =	7 − (−3) =
5 − (−6) =	(−5) + 6 =	6 − (−3) =
5 − (−5) =	(−5) + 5 =	5 − (−3) =
5 − (−4) =	(−5) + 4 =	4 − (−3) =
5 − (−3) =	(−5) + 3 =	3 − (−3) =
5 − (−2) =	(−5) + 2 =	2 − (−3) =
5 − (−1) =	(−5) + 1 =	1 − (−3) =
5 − 0 =	(−5) + 0 =	0 − (−3) =
5 − 1 =	(−5) + (−1) =	(−1) − (−3) =
5 − 2 =	(−5) + (−2) =	(−2) − (−3) =
5 − 3 =	(−5) + (−3) =	(−3) − (−3) =
5 − 4 =	(−5) + (−4) =	(−4) − (−3) =
5 − 5 =	(−5) + (−5) =	(−5) − (−3) =

12. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer:
   (a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.
   (b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.
   (c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.
   (d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.

(e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.
(f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.
(g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.
(h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

comparing integers and solving problems

1. Rewrite and include <, > or = to make the relationship between the numbers true:

   (a) −103 ☐ −99 (b) −699 ☐ −701

   (c) 30 ☐ −30 (d) 10 − 7 ☐ −(10 − 7)

   (e) −121 ☐ −200 (f) 12 − 5 ☐ −(12 + 5)

   (g) −199 ☐ −110

2. At 5 a.m. in Bloemfontein the temperature was −5 °C. At 1 p.m., it was 19 °C. By how many degrees did the temperature rise?

3. A diver swims 150 m below the surface of the sea. She moves 75 m towards the surface. How far below the surface is she now?

4. One trench in the ocean is 800 m deep and another is 2 200 m deep. What is the difference in their depths?

5. An island has a mountain which is 1 200 m high. The surrounding ocean has a depth of 860 m. What is the difference in height?

6. On a winter's day in Upington the temperature rose by 19 °C. If the minimum temperature was −4 °C, what was the maximum temperature?

2.3 Multiplying and dividing with integers

multiplication with integers

1. Calculate each of the following:
   (a) −5 + −5 + −5 + −5 + −5 + −5 + −5 + −5 + −5 + −5
   (b) −10 + −10 + −10 + −10 + −10
   (c) −6 + −6 + −6 + −6 + −6 + −6 + −6 + −6
   (d) −8 + −8 + −8 + −8 + −8 + −8
   (e) −20 + −20 + −20 + −20 + −20 + −20 + −20

2. In each case, show whether you agree (✓) or disagree (✗) with the givenstatement:
   (a) $10 \times (-5) = 50$
   (b) $8 \times (-6) = (-8) \times 6$
   (c) $(-5) \times 10 = 5 \times (-10)$
   (d) $6 \times (-8) = -48$
   (e) $(-5) \times 10 = 10 \times (-5)$
   (f) $8 \times (-6) = 48$
   (g) $4 \times 12 = -48$
   (h) $(-4) \times 12 = -48$

Multiplication of integers is commutative:

$(-20) \times 5 = 5 \times (-20)$

3. Is addition of integers commutative? Demonstrate your answer with three different examples.

4. Calculate each of the following:
   (a) $20 \times (-10)$
   (b) $(-5) \times 4$
   (c) $(-20) \times 10$
   (d) $4 \times (-25)$
   (e) $29 \times (-20)$
   (f) $(-29) \times (-2)$

5. Calculate each of the following:
   (a) $10 \times 50 + 10 \times (-30)$
   (b) $50 + (-30)$
   (c) $10 \times (50 + (-30))$
   (d) $(-50) + (-30)$
   (e) $10 \times (-50) + 10 \times (-30)$
   (f) $10 \times ((-50) + (-30))$

The product of two positive numbers is a positive number, for example: $5 \times 6 = 30$.

The product of a positive number and a negative number is a negative number, for example: $5 \times (-6) = -30$.

The product of a negative number and a positive number is a negative number, for example: $(-5) \times 6 = -30$.

6. (a) Four numerical expressions are given below. Which expressions do you expect to have the same answers? Do not do the calculations.

   $14 \times (23 + 58)$ $\quad$ $23 \times (14 + 58)$ $\quad$ $14 \times 23 + 14 \times 58$ $\quad$ $14 \times 23 + 58$

   (b) What property of operations is demonstrated by the fact that two of the above expressions have the same value?

7. Consider your answers for question 6.
   (a) Does multiplication distribute over addition in the case of integers?
   (b) Illustrate your answer with two examples.

8. Three numerical expressions are given below. Which expressions do you expect to have the same answers? Do not do the calculations.

   $10 \times ((-50) - (-30))$ $\quad$ $10 \times (-50) - (-30)$ $\quad$ $10 \times (-50) - 10 \times (-30)$

9. Do the three sets of calculations given in question 8.

Your work in questions 5, 8 and 9 demonstrates that multiplication with a positive number distributes over addition and subtraction of integers. For example:

$$10 \times \mathbf{(}5 + (-3)\mathbf{)} = 10 \times 2 = \mathbf{20} \text{ and } 10 \times 5 + 10 \times (-3) = 50 + (-30) = \mathbf{20}$$

$$10 \times \mathbf{(}5 - (-3)\mathbf{)} = 10 \times 8 = \mathbf{80} \text{ and } 10 \times 5 - 10 \times (-3) = 50 - (-30) = \mathbf{80}$$

10.Calculate: $(-10) \times \mathbf{(}5 + (-3)\mathbf{)}$

Now consider the question of whether or not multiplication with a negative number distributes over addition and subtraction of integers. For example, would $(-10) \times 5 + (-10) \times (-3)$ also have the answer -20, as does $(-10) \times \mathbf{(}5 + (-3)\mathbf{)}$?

11. What must $(-10) \times (-3)$ be equal to, if we want $(-10) \times 5 + (-10) \times (-3)$ to be equal to -20?

In order to ensure that multiplication distributes over addition and subtraction in the system of integers, we have to agree that

(a negative number) × (a negative number) is a positive number.

For example: $(-10) \times (-3) = 30$.

12. Calculate each of the following:

(a) $(-10) \times (-5)$

(b) $(-10) \times 5$

(c) 10×5

(d) $10 \times (-5)$

(e) $(-20) \times (-10) + (-20) \times (-6)$

(f) $(-20) \times \mathbf{(}(-10) + (-6)\mathbf{)}$

(g) $(-20) \times (-10) - (-20) \times (-6)$

(h) $(-20) \times \mathbf{(}(-10) - (-6)\mathbf{)}$

Here is a summary of the **properties of integers** that make it possible to do calculations with integers:

- When a number is added to its additive inverse, the result is 0, for example $(+12) + (-12) = 0$.
- Adding an integer has the same effect as subtracting its additive inverse. For example, $3 + (-10)$ can be calculated by doing $3 - 10$, and the answer is -7.
- Subtracting an integer has the same effect as adding its additive inverse. For example, $3 - (-10)$ can be calculated by doing $3 + 10$, and the answer is 13.
- The product of a positive and a negative integer is negative, for example $(-15) \times 6 = -90$.
- The product of a negative and a negative integer is positive, for example $(-15) \times (-6) = 90$.

division with integers

1. (a) Calculate 25×8.
   (b) How much is $200 \div 25$?
   (c) How much is $200 \div 8$?

Division is the inverse of multiplication. Therefore, if two numbers and the value of their product are known, the answers to two division problems are also known.

2. Calculate each of the following:
   (a) $25 \times (-8)$ (b) $(-125) \times 8$

3. Use the work you have done for question 2 to write the answers for the following division questions:
   (a) $(-1\,000) \div (-125)$ (b) $(-1\,000) \div 8$
   (c) $(-200) \div 25$ (d) $(-200) \div 8$

4. Can you also work out the answers for the following division questions by using the work you have done for question2?
   (a) $1\,000 \div (-125)$ (b) $(-1\,000) \div (-8)$
   (c) $(-100) \div (-25)$ (d) $100 \div (-25)$

When two numbers are multiplied, for example $30 \times 4 = 120$, the word "product" can be used in various ways to describe the situation:

- An expression that specifies multiplication only, such as 30×4, is called a **product** or a product expression.
- The answer obtained is also called the product of the two numbers. For example, 120 is called the **product of 30 and 4**.

An expression that specifies division only, such as $30 \div 5$, is called a **quotient** or a **quotient expression.** The answer obtained is also called the quotient of the two numbers. For example, 6 is called the **quotient of 30 and 5**.

5. In each case, state whether you agree or disagree with the statement, and give an example to illustrate your answer:
   (a) The quotient of a positive and a negative integer is negative.
   (b) The quotient of a positive and a positive integer is negative.
   (c) The quotient of a negative and a negative integer is negative.
   (d) The quotient of a negative and a negative integer is positive.

6. Do the necessary calculations to enable you to provide the values of the quotients:
   (a) $(-500) \div (-20)$ (b) $(-144) \div 6$ (c) $1\,440 \div (-60)$
   (d) $(-1\,440) \div (-6)$ (e) $-14\,400 \div 600$ (f) $500 \div (-20)$

the associative properties of operations with integers

Multiplication of whole numbers is **associative**. This means that in a product with several factors, the factors can be placed in any sequence, and the calculations can be performed in any sequence. For example, the following sequences of calculations will all produce the same answer:

A. 2 × 3, the answer of 2 × 3 multiplied by 5, the new answer multiplied by 10
B. 2 × 5, the answer of 2 × 5 multiplied by 10, the new answer multiplied by 3
C. 10 × 5, the answer of 10 × 5 multiplied by 3, the new answer multiplied by 2
D. 3 × 5, the answer of 3 × 5 multiplied by 2, the new answer multiplied by 10

1. Do the four sets of calculations given in A to D to check whether or not they really produce the same answers.
2. (a) If the numbers 3 and 10 in the calculation sequences A, B, C and D are replaced with −3 and −10, do you think the four answers will still be the same?
   (b) Investigate to check your expectation.

Multiplication with integers is associative.

The calculation sequence A can be represented in symbols in only two ways:

- 2 × 3 × 5 × 10: The convention to work from left to right unless otherwise indicated with brackets, ensures that this representation corresponds to A.
- 5 × (2 × 3) × 10, where brackets are used to indicate that 2 × 3 should be calculated first. When brackets are used, there are different possibilities to describe the same sequence.

3. Express the calculation sequences B, C and D given above symbolically, without using brackets.
4. Investigate, in the same way that you did for multiplication in question 2, whether or not addition with integers is associative. Use sequences of four integers.
5. (a) Calculate: 80 − 30 + 40 − 20
   (b) Calculate: 80 + (−30) + 40 + (−20)
   (c) Calculate: 30 − 80 + 20 − 40
   (d) Calculate: (−30) + 80 + (− 20) + 40
   (e) Calculate: 20 + 30 − 40 − 80

mixed calculations with integers

1. Calculate each of the following:
   (a) −3 × 4 + (−7) × 9
   (b) −20(−4 − 7)
   (c) 20 × (−5) − 30 × 7
   (d) −9(20 − 15)
   (e) −8 × (−6) − 8 × 3
   (f) (−26 − 13) ÷ (−3)
   (g) −15 × (−2) + (−15) ÷ (−3)
   (h) −15(2 − 3)
   (i) (−5 + −3) × 7
   (j) −5 × (−3 + 7) + 20 ÷ (−4)
2. Calculate each of the following:
   (a) 20 × (−15 + 6) − 5 × (−2 − 8) − 3 × (−3 − 8)
   (b) 40 × (7 + 12 − 9) + 25 ÷ (−5) − 5 ÷ 5

(c) $-50(20 - 25) + 30(-10 + 7) - 20(-16 + 12)$
(d) $-5 \times (-3 + 12 - 9)$
(e) $-4 \times (30 - 50) + 7 \times (40 - 70) - 10 \times (60 - 100)$
(f) $-3 \times (-14 + 6) \times (-13 + 7) \times (-20 + 5)$
(g) $20 \times (-5) + 10 \times (-3) + (-5) \times (-6) - (3 \times 5)$
(h) $-5(-20 - 5) + 10(-7 - 3) - 20(-15 - 5) + 30(-40 - 35)$
(i) $(-50 + 15 - 75) \div (-11) + (6 - 30 + 12) \div (-6)$

2.4 Squares, cubes and roots with integers

squares and cubes of integers

1. Calculate each of the following:
   (a) 20×20 (b) $20 \times (-20)$
2. Write the answers for each of the following:
   (a) $(-20) \times 20$ (b) $(-20) \times (-20)$
3. Copy and complete the following table:

x	1	−1	2	−2	5	−5	10	−10
x^2 which is $x \times x$								
x^3								

4. In each case, state for which values of x, in the table in question 3, the given statement is true.
   (a) x^3 is a negative number (b) x^2 is a negative number
   (c) $x^2 > x^3$ (d) $x^2 < x^3$
5. Copy and complete the following table:

x	3	−3	4	−4	6	−6	7	−7
x^2								
x^3								

6. Ben thinks of a number. He adds 5 to it and his answer is 12.
   (a) What number did he think of?
   (b) Is there another number that would also give 12 when 5 is added to it?
7. Lebo also thinks of a number. She multiplies the number by itself and gets 25.
   (a) What number did she think of?
   (b) Is there more than one number that will give 25 when multiplied by itself?

8. Mary thinks of a number and calculates (the number) × (the number) × (the number). Her answer is 27. What number did Mary think of?

> 10^2 is 100 and $(-10)^2$ is also 100.
> Both 10 and (−10) are called **square roots** of 100. 10 may be called the **positive square root** of 100, and (−10) may be called the **negative square root** of 100.

9. Write the positive square root and the negative square root of each number:
   (a) 64 (b) 9

10. Copy and complete the following table:

Number	1	4	9	16	25	36	49	64
Positive square root			3					8
Negative square root			−3					−8

11. Copy and complete the following tables:

(a)

x	1	2	3	4	5	6	7	8
x^3								

(b)

x	−1	−2	−3	−4	−5	−6	−7	−8
x^3								

> 3^3 is 27 and $(-5)^3$ is −125.
> 3 is called the **cube root** of 27, because $3^3 = 27$.
> −5 is called the cube root of −125 because $(-5)^3 = -125$.

12. Copy and complete the following table:

Number	−1	8	−27	−64	−125	−216	1 000
Cube root			−3				10

The symbol $\sqrt{\ }$ is used to indicate "root".

$\sqrt[3]{-125}$ represents the **cube root** of −125. That means $\sqrt[3]{-125} = -5$.

$\sqrt[2]{36}$ represents the **positive square root** of 36, and $-\sqrt[2]{36}$ represents the **negative square root**. The "2" that indicates "square" is normally omitted, so $\sqrt{36} = 6$ and $-\sqrt{36} = -6$.

13. Copy and complete the following table:

$\sqrt[3]{-8}$	$\sqrt{121}$	$\sqrt[3]{-64}$	$-\sqrt{64}$	$\sqrt{64}$	$\sqrt[3]{-1}$	-1	$\sqrt[3]{-216}$

Worksheet

1. Use the numbers −8, −5 and −3 to demonstrate each of the following:
   (a) Multiplication with integers distributes over addition.
   (b) Multiplication with integers distributes over subtraction.
   (c) Multiplication with integers isassociative.
   (d) Addition with integers isassociative.

2. Calculateeachofthefollowingwithoutusingacalculator:
   (a) $5 \times (-2)^3$ (b) $3 \times (-5)^2$
   (c) $2 \times (-5)^3$ (d) $10 \times (-3)^2$

3. Useacalculator to workouteachofthe following:
   (a) $24 \times (-53) + (-27) \times (-34) - (-55) \times 76$
   (b) $64 \times (27 - 85) - 29 \times (-47 + 12)$

4. Useacalculator to workouteachofthe following:
   (a) $-24 \times 53 + 27 \times 34 + 55 \times 76$
   (b) $64 \times (-58) + 29 \times (47 - 12)$

If you do not get the same answers in questions 3 and 4, you have made mistakes.

Chapter 3
Exponents

3.1 Revision

exponential notation

1. Calculate each of the following:
   (a) $2 \times 2 \times 2$
   (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   (c) $3 \times 3 \times 3$
   (d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

Instead of writing $3 \times 3 \times 3 \times 3 \times 3 \times 3$, we can write 3^6.
We read this as "3 to the power of 6". The number 3 is the **base** and 6 is the **exponent**.
When we write $3 \times 3 \times 3 \times 3 \times 3 \times 3$ as 3^6, we are using **exponential notation**.

2. Write each of the following in exponential form:
   (a) $2 \times 2 \times 2$
   (b) $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   (c) $3 \times 3 \times 3$
   (d) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

3. Calculate each of the following:
   (a) 5^2
   (b) 2^5
   (c) 10^2
   (d) 15^2
   (e) 3^4
   (f) 4^3
   (g) 2^3
   (h) 3^2

squares

To square a number is to multiply it by itself.
The square of 8 is 64 because 8×8 equals 64.
We write 8×8 as 8^2 in exponential form.
We read 8^2 as **eight squared**.

1. Copy and complete the following table:

	Number	Square the number	Exponential form	Square
(a)	1			
(b)	2			
(c)	3			

	Number	Square the number	Exponential form	Square
(d)	4			
(e)	5			
(f)	6			
(g)	7			
(h)	8	8×8	8^2	64
(i)	9			
(j)	10			
(k)	11			
(l)	12			

2. Calculate each of the following:

   (a) $3^2 \times 4^2$ (b) $2^2 \times 3^2$ (c) $2^2 \times 5^2$ (d) $2^2 \times 4^2$

3. Copy and complete the following statements to make them true:

   (a) $3^2 \times 4^2 = \_\_^2$ (b) $2^2 \times 3^2 = \_\_^2$

   (c) $2^2 \times 5^2 = \_\_^2$ (d) $2^2 \times 4^2 = \_\_^2$

cubes

To cube a number is to multiply it by itself and then by itself again. The cube of 3 is 27 because $3 \times 3 \times 3$ equals 27.

We write $3 \times 3 \times 3$ as 3^3 in exponential form.

We read 3^3 as **three cubed**.

1. Copy and complete the following table:

	Number	Cube the number	Exponential form	Cube
(a)	1			
(b)	2			
(c)	3	$3 \times 3 \times 3$	3^3	27
(d)	4			
(e)	5			
(f)	6			
(g)	7			

	Number	Cube the number	Exponential form	Cube
(h)	8			
(i)	9			
(j)	10			

2. Calculate each of the following:

   (a) $2^3 \times 3^3$ (b) $2^3 \times 5^3$ (c) $2^3 \times 4^3$ (d) $1^3 \times 9^3$

3. Which of the following statements are true? If a statement is false, rewrite it as a true statement.

   (a) $2^3 \times 3^3 = 6^3$ (b) $2^3 \times 5^3 = 7^3$ (c) $2^3 \times 4^3 = 8^3$ (d) $1^3 \times 9^3 = 10^3$

square and cube roots

Tofind the square root of a number we ask the question: Which number was multiplied by itself to get a square?

The square root of 16 is 4 because 4 × 4 = 16.

The question: **Which number was multiplied by itself to get 16?** is written mathematically as $\sqrt{16}$.

The answer to this question is written as $\sqrt{16} = 4$.

1. Copy and complete the following table:

	Number	Square of the number	Square root of the square of the number	Reason
(a)	1			
(b)	2			
(c)	3			
(d)	4	16	4	4 × 4 = 16
(e)	5			
(f)	6			
(g)	7			
(h)	8			
(i)	9			
(j)	10			
(k)	11			
(l)	12			

2. Calculate the following. Justify your answer.

(a) $\sqrt{144}$ (b) $\sqrt{100}$ (c) $\sqrt{81}$ (d) $\sqrt{64}$

To find the cube root of a number we ask the question: Which number was multiplied by itself and again by itself to get a cube?

The cube root of 64 is 4 because 4 × 4 × 4 = 64.

Thequestion:**Whichnumberwasmultipliedby itself and again by itself (or cubed) to get 64?** is written mathematically as $\sqrt[3]{64}$. The answer to this question iswritten as $\sqrt[3]{64} = 4$.

3. Copy and complete the following table:

	Number	Cube of the number	Cube root of the cube of the number	Reason
(a)	1			
(b)	2			
(c)	3			
(d)	4	64	4	4 × 4 × 4 = 64
(e)	5			
(f)	6			
(g)	7			
(h)	8			
(i)	9			
(j)	10			

4. Calculate the following and give reasons for your answers:

(a) $\sqrt[3]{216}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{125}$

(d) $\sqrt[3]{27}$ (e) $\sqrt[3]{64}$ (f) $\sqrt[3]{1\,000}$

3.2 Working with integers

representing integers in exponential form

1. Calculate the following, without using a calculator:

(a) −2 × −2 × −2 (b) −2 × −2 × −2 × −2

(c) −5 × −5 (d) −5 × −5 × −5

(e) −1 × −1 × −1 × −1 (f) −1 × −1 × −1

2. Calculate each of the following:

(a) -2^2 (b) $(-2)^2$

(c) $(-5)^3$ (d) -5^3

3. Use your calculator to work out the answers to question 2.

(a) Are your answers to question 2(a) and (b) different or the same as those of the calculator?

(b) If your answers are different to those of the calculator, try to explain how the calculator did the calculations differently from you.

> The calculator "understands" $\mathbf{-5^2}$ and $\mathbf{(-5)^2}$ as two different numbers.
>
> It understands $\mathbf{-5^2}$ as $\mathbf{-5 \times 5 = -25}$ and $\mathbf{(-5)^2}$ as $\mathbf{-5 \times -5 = 25}$.

4. Write the following in exponential form:

(a) $-2 \times -2 \times -2$ (b) $-2 \times -2 \times -2 \times -2$

(c) -5×-5 (d) $-5 \times -5 \times -5$

(e) $-1 \times -1 \times -1 \times -1$ (f) $-1 \times -1 \times -1$

5. Calculate each of the following:

(a) $(-3)^2$ (b) $(-3)^3$

(c) $(-2)^4$ (d) $(-2)^6$

(e) $(-2)^5$ (f) $(-3)^4$

6. Say whether the sign of the answer is negative or positive. Explain why.

(a) $(-3)^6$ (b) $(-5)^{11}$

(c) $(-4)^{20}$ (d) $(-7)^5$

7. Say whether the following statements are true or false. If a statement is false, rewrite it as a correct statement.

(a) $(-3)^2 = -9$ (b) $-3^2 = 9$

(c) $(-5^2) = -5^2$ (d) $(-1)^3 = -1^3$

(e) $(-6)^3 = -18$ (f) $(-2)^6 = 2^6$

3.3 Laws of exponents

product of powers

1. A product of 2s is given below. Describe it using exponential notation, that is, write it as a power of 2.

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

2. Express each of the following as a product of the powers of 2, as indicated by the brackets.
   (a) $(2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2)$
   (b) $(2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$
   (c) $(2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$
   (d) $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
   (e) $(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2)$

3. Copy and complete the following statements so that they are true. You may want to refer to your answers to question 2(a) to (e) to help you.
   (a) $2^3 \times$ ___ $= 2^{12}$
   (b) $2^5 \times$ ___ $\times 2^2 = 2^{12}$
   (c) $2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 \times 2^2 =$ ___
   (d) $2^8 \times$ ___ $= 2^{12}$
   (e) $2^3 \times 2^3 \times 2^3 \times$ ___ $= 2^{12}$
   (f) $2^6 \times$ ___ $= 2^{12}$
   (g) $2^2 \times 2^{10} =$ ___

Suppose we are asked to simplify: $3^2 \times 3^4$.

The solution is: $3^2 \times 3^4 = 9 \times 81$

$= 729$

$= 3^6$

The base (3) is a repeated factor. The exponents (2 and 4) tell us the number of times each factor is repeated.

We can explain this solution in the following manner:

$$3^2 \times 3^4 = \underbrace{3 \times 3}_{\text{2 factors}} \times \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 factors}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\text{6 factors}} = 3^6$$

4. Copy and complete the following table:

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$2^7 \times 2^3$			
(b)	$5^2 \times 5^4$			
(c)	$4^1 \times 4^5$			
(d)	$6^3 \times 6^2$			
(e)	$2^8 \times 2^2$			
(f)	$5^3 \times 5^3$			
(g)	$4^2 \times 4^4$			
(h)	$2^1 \times 2^9$			

When you multiply two or more powers that have the same base, the answer has the same base, but its exponent is equal to the sum of the exponents of the numbers you are multiplying.

We can express this symbolically as $a^m \times a^n = a^{m+n}$, where m and n are natural numbers and a is not zero.

5. What is wrong with these statements? Correct each one.
   (a) $2^3 \times 2^4 = 2^{12}$
   (b) $10 \times 10^2 \times 10^3 = 10^{1 \times 2 \times 3} = 10^6$
   (c) $3^2 \times 3^3 = 3^6$
   (d) $5^3 \times 5^2 = 15 \times 10$

6. Express each of the following numbers as a single power of 10.

   Example: 1 000 000 as a power of 10 is 10^6.

   (a) 100
   (b) 1 000
   (c) 10 000
   (d) $10^2 \times 10^3 \times 10^4$
   (e) 100 × 1 000 × 10 000
   (f) 1 000 000 000

7. Write each of the following products in exponential form:
   (a) $x \times x \times x \times x \times x \times x \times x \times x \times x$
   (b) $(x \times x) \times (x \times x \times x) \times (x \times x \times x \times x)$
   (c) $(x \times x \times x \times x) \times (x \times x) \times (x \times x) \times x$
   (d) $(x \times x \times x \times x \times x \times x) \times (x \times x \times x)$
   (e) $(x \times x \times x) \times (y \times y \times y)$
   (f) $(a \times a) \times (b \times b)$

8. Copy and complete the following table:

	Product of powers	Repeated factor	Total number of times the factor is repeated	Simplified form
(a)	$x^7 \times x^3$			
(b)	$x^2 \times x^4$			
(c)	$x^1 \times x^5$			
(d)	$x^3 \times x^2$			
(e)	$x^8 \times x^2$			
(f)	$x^3 \times x^3$			
(g)	$x^1 \times x^9$			

raising a power to a power

1. Copy and complete the table of powers of 2:

x	1	2	3	4	5	6	7	8	9	10	11
2^x	2	4									
	2^1	2^2	2^3								

x	12	13	14	15	16	17	18
2^x							

2. Copy and complete the table of powers of 3:

x	1	2	3	4	5	6	7	8	9
3^x									
	3^1	3^2	3^3						

x	10	11	12	13	14
3^x					

3. Copy and complete the table. You can read the values from the tables you made in questions 1 and 2.

Product of powers	Repeated factor	Power of power notation	Total number of repetitions	Simplified form	Value
$2^4 \times 2^4 \times 2^4$	2	$(2^4)^3$	12	2^{12}	4 096
$3^2 \times 3^2 \times 3^2 \times 3^2$					
$2^3 \times 2^3 \times 2^3 \times 2^3 \times 2^3$					
$3^4 \times 3^4 \times 3^4$					
$2^6 \times 2^6 \times 2^6$					

4. Copy and complete by using your table of powers of 2 to find the answers for the following:
   (a) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ = ___ = ___ ___
   (b) $(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$ = ___ = ___ ___
   (c) 16^3 = ___ = ___ = ___ ___

5. Use your table of powers of 2 to find the answers for the following:
   (a) Is $16^3 = 2^{12}$?
   (b) Is $2^4 \times 2^4 \times 2^4 = 2^{12}$?
   (c) Is $2^4 \times 2^3 = 2^{12}$?
   (d) Is $(2^4)^3 = 2^4 \times 2^4 \times 2^4$?
   (e) Is $(2^4)^3 = 2^{12}$?
   (f) Is $(2^4)^3 = 2^{4+3}$?
   (g) Is $(2^4)^3 = 2^{4 \times 3}$?
   (h) Is $(2^2)^5 = 2^{2+5}$?

6. (a) Express 8^5 as a power of 2. It may help to first express 8 as a power of 2.
   (b) Can $(2^3) \times (2^3) \times (2^3) \times (2^3) \times (2^3)$ be expressed as $(2^3)^5$?
   (c) Is $(2^3)^5 = 2^{3+5}$ or is $(2^3)^5 = 2^{3 \times 5}$?

7. (a) Express 4^3 as a power of 2.
(b) Calculate $2^2 \times 2^2 \times 2^2$ and express your answer as a single power of 2.
(c) Can $(2^2) \times (2^2) \times (2^2)$ be expressed as $(2^2)^3$?
(d) Is $(2^2)^3 = 2^{2+3}$ or is $(2^2)^3 = 2^{2 \times 3}$?

8. Simplify the following:
Example: $(10^2)^2 = 10^2 \times 10^2 = 10^{2+2} = 10^4 = 10\ 000$

(a) $(3^3)^2$ (b) $(4^3)^2$ (c) $(2^4)^2$ (d) $(9^2)^2$
(e) $(3^3)^3$ (f) $(4^3)^3$ (g) $(5^4)^3$ (h) $(9^2)^3$

$(a^m)^n = a^{m \times n}$, where m and n are natural numbers and a is not equal to zero.

9. Simplify:
(a) $(5^4)^{10}$ (b) $(10^4)^5$ (c) $(6^4)^4$ (d) $(5^4)^{10}$

10. Write 5^{12} as a power of powers of 5 in two different ways.

To simplify $(x^2)^5$ we can write it out as a product of powers or we can use a shortcut.

$(x^2)^5 = x^2 \times x^2 \times x^2 \times x^2 \times x^2$
$= \underbrace{x \times x} \times \underbrace{x \times x} \times \underbrace{x \times x} \times \underbrace{x \times x} \times \underbrace{x \times x} = x^{10}$

2 × 5 factors = 10 factors

11. Copy and complete the following table:

	Expression	Write as a product of the powers and then simplify	Use the rule $(a^m)^n$ to simplify
(a)	$(a^4)^5$	$a^4 \times a^4 \times a^4 \times a^4 \times a^4$ $= a^{4+4+4+4+4} = a^{20}$	$(a^4)^5 = a^{4 \times 5} = a^{20}$
(b)	$(b^{10})^5$		
(c)	$(x^7)^3$		
(d)		$s^6 \times s^6 \times s^6 \times s^6$ $= s^{6+6+6+6}$ $= s^{24}$	
(e)			$y^{3 \times 7} = y^{21}$

power of a product

1. Copy and complete the table. You may use your calculator when you are not sure of a value.

	x	1	2	3	4	5
(a)	2^x	$2^1 = 2$				
(b)	3^x		$3^2 = 9$			
(c)	6^x			$6^3 = 216$		

2. Use the table in question 1 to answer the questions below. Are these statements true or false? If a statement is false, rewrite it as a correct statement.
   (a) $6^2 = 2^2 \times 3^2$ (b) $6^3 = 2^3 \times 3^3$
   (c) $6^5 = 2^5 \times 3^5$ (d) $6^8 = 2^4 \times 3^4$

3. Copy and complete the following table:

	Expression	The bases of the expression are factors of ...	Equivalent expression
(a)	$2^6 \times 5^6$	10	10^6
(b)	$3^2 \times 4^2$		
(c)	$4^2 \times 2^2$		
(d)			56^5
(e)			30^3
(f)	$3^5 \times x^5$	$3x$	$(3x)^5$
(g)	$7^2 \times z^2$		
(h)	$4^3 \times y^3$		
(i)			$(2m)^6$
(j)			$(2m)^3$
(k)	$2^{10} \times y^{10}$		$(2y)^{10}$

12^2 can be written in terms of its factors as $(2 \times 6)^2$ or as $(3 \times 4)^2$.

We already know that $12^2 = 144$.

What this tells us is that both $(2 \times 6)^2$ and $(3 \times 4)^2$ also equal to 144.

We write $12^2 = (2 \times 6)^2$ or $12^2 = (3 \times 4)^2$

$= 2^2 \times 6^2$ $= 3^2 \times 4^2$

$= 4 \times 36$ $= 9 \times 16$

$= 144$ $= 144$

A product raised to a power is the product of the factors each raised to the same power.

Using symbols, we write $(a \times b)^m = a^m \times b^m$, where m is a natural number and a and b are not equal to zero.

4. Write each of the following expressions as an expression with one base.

Example: $3^{10} \times 2^{10} = (3 \times 2)^{10} = 6^{10}$

(a) $3^2 \times 5^2$ (b) $5^3 \times 2^3$ (c) $7^4 \times 4^4$

(d) $2^3 \times 6^3$ (e) $4^4 \times 2^4$ (f) $5^2 \times 7^2$

5. Write the following as a product of powers.

Example: $(3x)^3 = 3^3 \times x^3 = 27x^3$

(a) 6^3 (b) 15^2 (c) 21^4 (d) 6^5

(e) 18^2 (f) $(st)^7$ (g) $(ab)^3$ (h) $(2x)^2$

(i) $(3y)^5$ (j) $(3c)^2$ (k) $(gh)^4$ (l) $(4x)^3$

6. Simplify the following expressions.

Example: $3^2 \times m^2 = 9 \times m^2 = 9m^2$

(a) $3^5 \times b^5$ (b) $2^6 \times y^6$ (c) $x^2 \times y^2$

(d) $10^4 \times x^4$ (e) $3^3 \times x^3$ (f) $5^2 \times t^2$

(g) $6^3 \times m^7$ (h) $12^2 \times a^2$ (i) $n^3 \times p^9$

a quotient of powers

Consider the following table:

x	1	2	3	4	5	6
2^x	2	4	8	16	32	64
3^x	3	9	27	81	243	729
5^x	5	25	125	625	3 125	15 625

Answer questions 1 to 4 by referring to the table when needed.

1. Give the value of each of the following:

(a) 3^4 (b) 2^5 (c) 5^6

2. (a) Calculate $3^6 \div 3^3$. (Read the values of 3^6 and 3^3 from the table and then divide. You may use a calculator where necessary.)

(b) Calculate 3^{6-3}.

(c) Is $3^6 \div 3^3$ equal to 3^3? Explain.

3. (a) Calculate the value of 2^{6-2}.
   (b) Calculate the value of $2^6 \div 2^2$.
   (c) Calculate the value of $2^{6 \div 2}$.
   (d) Read the value of 2^3 from the table.
   (e) Read the value of 2^4 from the table.
   (f) Which of the statements below is true? Give an explanation for your answer.

   A. $2^6 \div 2^2 = 2^{6-2} = 2^4$ B. $2^6 \div 2^2 = 2^{6 \div 2} = 2^3$

To calculate 4^{5-3}, we first do the calculation in the exponent, that is, we subtract 3 from 5. Then we can calculate 4^2 as $4 \times 4 = 16$.

4. Say which of the statements below are true and which are false. Rewrite false statements as correct statements.
   (a) $5^6 \div 5^4 = 5^{6 \div 4}$ (b) $3^{4-1} = 3^4 \div 3$
   (c) $5^6 \div 5 = 5^{6-1}$ (d) $2^5 \div 2^3 = 2^2$

$a^m \div a^n = a^{m-n}$
Here, m and n are natural numbers, and m is a number greater than n and a is not zero.

5. Simplify the following. Do not use a calculator.
   Example: $3^{17} \div 3^{12} = 3^{17-12} = 3^5 = 243$
   (a) $2^{12} \div 2^{10}$ (b) $6^{17} \div 6^{14}$ (c) $10^{20} \div 10^{14}$ (d) $5^{11} \div 5^8$

6. Simplify the following:
   (a) $x^{12} \div x^{10}$ (b) $y^{17} \div y^{14}$ (c) $t^{20} \div t^{14}$ (d) $n^{11} \div n^8$

the power of zero

1. Simplify the following:
   (a) $2^{12} \div 2^{12}$ (b) $6^{17} \div 6^{17}$ (c) $6^{14} \div 6^{14}$ (d) $2^{10} \div 2^{10}$

We define $a^0 = 1$.
Any number raised to the power of zero is always equal to 1.

2. Simplify the following:
   (a) 100^0 (b) x^0 (c) $(100x)^0$ (d) $(5x^3)^0$

3.4 Calculations

mixed operations

Simplify the following:

1. $3^3 + \sqrt[3]{-27} \times 2$ 2. $5 \times (2 + 3)^2 + (-1)^0$

3. $3^2 \times 2^3 + 5 \times \sqrt{100}$

4. $\frac{\sqrt[3]{1\,000}}{\sqrt{100}} + (4 - 1)^2$

5. $\sqrt{16} \times \sqrt{16} + \sqrt[3]{216} + 3^2 \times 10$

6. $4^3 \div 2^3 + \sqrt{144}$

3.5 Squares, cubes and roots of rational numbers

squaring a fraction

Squaring or cubing a fraction or a decimal fraction is no different from squaring or cubing an integer.

1. Copy and complete the following table:

	Fraction	Square the fraction	Value of the square of the fraction
(a)	$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
(b)	$\frac{2}{3}$		
(c)	$\frac{3}{4}$		
(d)	$\frac{2}{5}$		
(e)	$\frac{3}{5}$		
(f)	$\frac{2}{6}$		
(g)	$\frac{3}{7}$		
(h)	$\frac{11}{12}$		

2. Calculate each of the following:

(a) $\left(\frac{3}{2}\right)^2$ (b) $\left(\frac{4}{5}\right)^2$ (c) $\left(\frac{7}{8}\right)^2$

3. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to calculate $(0{,}6)^2$.

(b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0{,}8)^2$.

finding the square root of a fraction

1. Copy and complete the following table:

	Fraction	Writing the fraction as a product of factors	Square root
(a)	$\frac{81}{121}$		
(b)	$\frac{64}{81}$		
(c)	$\frac{49}{169}$		
(d)	$\frac{100}{225}$		

2. Determine the following:

(a) $\sqrt{\frac{25}{16}}$ (b) $\sqrt{\frac{81}{144}}$ (c) $\sqrt{\frac{400}{900}}$ (d) $\sqrt{\frac{36}{81}}$

3. (a) Use the fact that 0,01 can be written as $\frac{1}{100}$ to calculate $\sqrt{0,01}$.

(b) Use the fact that 0,49 can be written as $\frac{49}{100}$ to calculate $\sqrt{0,49}$.

4. Calculate each of the following:

(a) $\sqrt{0,09}$ (b) $\sqrt{0,64}$ (c) $\sqrt{1,44}$

cubing a fraction

One half cubed is equal to one eighth.

We write this as $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

1. Calculate each of the following:

(a) $\left(\frac{2}{3}\right)^3$ (b) $\left(\frac{5}{10}\right)^3$ (c) $\left(\frac{5}{6}\right)^3$ (d) $\left(\frac{4}{5}\right)^3$

2. (a) Use the fact that 0,6 can be written as $\frac{6}{10}$ to find $(0,6)^3$.

(b) Use the fact that 0,8 can be written as $\frac{8}{10}$ to calculate $(0,8)^3$.

(c) Use the fact that 0,7 can be written as $\frac{7}{10}$ to calculate $(0,7)^3$.

3.6 Scientific notation

very large numbers

1. Express each of the following as a single number. Do not use a calculator.
   Example: 7,56 × 100 can be written as 756.

(a) 3,45 × 100	(b) 3,45 × 10	(c) 3,45 × 1 000
(d) $2,34 \times 10^2$	(e) 2,34 × 10	(f) $2,34 \times 10^3$
(g) $10^4 \times 10^2$	(h) $10^0 \times 10^6$	(i) $3,4 \times 10^5$

We can write 136 000 000 as $1,36 \times 10^8$.
$1,36 \times 10^8$ is called the **scientific notation** for 136 000 000.
In scientific notation, a number is expressed in two parts: a number between 1 and 10 multiplied by a power of 10. The exponent must always be an integer.

2. Write the following numbers in scientific notation:

(a) 367 000 000	(b) 21 900 000
(c) 600 000 000 000	(d) 178

3. Write each ofthe following numbers in the ordinary way.
   For example: $3,4 \times 10^5$ written in the ordinary way is 340 000.

   (a) $1,24 \times 10^8$ (b) $9,2074 \times 10^4$ (c) $1,04 \times 10^6$ (d) $2,05 \times 10^3$

4. The universe is 15000 000 000 years old. Express the age of the universe in scientific notation.

5. The average distance from the earth to the sun is 149 600 000 km. Express this distance in scientific notation.

Because it is easier to multiply powers of ten without a calculator, **scientific notation** makes it possible to do calculations in your head.

6. Explain why the number 24×10^3 is not in scientific notation.

7. Calculate the following. Do not use a calculator.
   Example: $3\ 000\ 000 \times 90\ 000\ 000 = 3 \times 10^6 \times 9 \times 10^7 = 3 \times 9 \times 10^{6+7}$
   $= 27 \times 10^{13} = 270\ 000\ 000\ 000\ 000$

(a) 13 000 × 150 000	(b) 200 × 6 000 000
(c) 120 000 × 120000 000	(d) 2,5 × 40 000 000

8. Copy the statements and use > or < to compare these numbers:

(a) $1,3 \times 10^9$ ☐ $2,4 \times 10^7$	(b) $6,9 \times 10^2$ ☐ $4,5 \times 10^3$
(c) $7,3 \times 10^4$ ☐ $7,3 \times 10^2$	(d) $3,9 \times 10^6$ ☐ $3,7 \times 10^7$

Worksheet

1. Calculate:

   (a) 11^2 (b) $3^2 \times 4^2$

   (c) 6^3 (d) $\sqrt{121}$

   (e) $(-3)^2$ (f) $\sqrt[3]{125}$

2. Simplify:

   (a) $3^4 \times m^6$ (b) $b^2 \times n^6$

   (c) $y^{12} \div y^5$ (d) $(10^2)^3$

   (e) $(2w^2)^3$ (f) $(3d^5)(2d)^3$

3. Calculate:

   (a) $\left(\frac{2}{5}\right)^2$ (b) $\sqrt{\frac{9}{25}}$

   (c) $(6^4y^2)^0$ (d) $(0{,}7)^2$

4. Simplify:

   (a) $(2^2 + 4)^2 + \frac{6^2}{3^2}$ (b) $\sqrt[3]{-125} - 5 \times 3^2$

5. Write3×10^9intheordinaryway.

6. The first birds appeared on earth about 208 000 000 years ago. Write this number in scientific notation.

Chapter 4
Numeric and geometric patterns

4.1 The term-term relationship in a sequence

going from one term to the next

Write down the next three numbers in each of the sequences below. Also explain in writing, in each case, how you figured out what the numbers should be.

A list of numbers which form a pattern is called a **sequence**. Each number in a sequence is called a **term** of the sequence. The first number is the first term of the sequence.

1. Sequence A: 2; 5; 8; 11; 14; 17; 20;23; __
2. Sequence B: 4; 5; 8; 13; 20; 29; 40; __
3. Sequence C: 1; 2; 4; 8; 16; 32; 64; __
4. Sequence D: 3; 5; 7; 9; 11; 13; 15; 17;19; __
5. Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; __
6. Sequence F: 2; 6; 18; 54; 162; 486; __
7. Sequence G: 1; 5; 9; 13; 17; 21; 25; 29;33; __
8. Sequence H: 2; 4; 8; 16; 32; 64; __

Numbers that follow one another are said to be **consecutive**.

adding or subtracting the same number

1. Which sequences above are of the same kind as sequence A? Explain your answer.

Amanda explains how she figured out how to continue sequence A:

> *I looked at the first two numbers in the sequence and saw that I needed 3 to go from 2 to 5. I looked further and saw that I also needed 3 to go from 5 to 8. I tested that and it worked for all the next numbers.*
>
> *This gave me a rule I could use to extend the sequence: add 3 to each number to find the next number in the pattern.*

Tamara says you can also find the pattern by working backwards and subtracting 3 each time:

14 − 3 = 11; 11 − 3 = 8; 8 − 3 = 5; 5 − 3 = 2

When the **differences** between consecutive terms of a sequence are the same, we say the difference is **constant**.

2. Provide a rule to describe the relationship between the numbers in the sequence. Use this rule to calculate the missing numbers in each sequence.

   (a) 1; 8; 15; ___; ___; ___; ___; ___; ...

   (b) 10 020; ___; ___; ___; 9 980; 9 970; ___; ___; 9 940; 9 930; ...

   (c) 1,5; 3,0; 4,5; ___; ___; ___; ___; ___; ...

   (d) 2,2; 4,0; 5,8; ___; ___; ___; ___; ___; ...

   (e) $45\frac{3}{4}$; $46\frac{1}{2}$; $47\frac{1}{4}$; 48; ___; ___; ___; ___; ___; ...

   (f) ___; 100,49; 100,38; 100,27; ___; ___; 99,94; 99,83; 99,72; ...

3. Copy and complete the following table:

Input number	1	2	3	4	5		12		n
Input number + 7	8			11		15		30	

multiplying or dividing with the same number

Take another look at sequence F on page 49: 2; 6; 18; 54; 162; 486; ...

Piet explains that he figured out how to continue the sequence F:

I looked at the first two terms in the sequence and wrote 2 ×? = 6.
When I multiplied the first number by 3, I got the second number: 2 × 3 = 6.
I then checked to see if I could find the next number if I multiplied 6 by 3: 6 × 3 = 18.
I continued checking in this way: 18 × 3 = 54; 54 × 3 = 162 and so on.
This gave me a rule I can use to extend the sequence and my rule was: multiply each number by 3 to calculate the next number in the sequence.

Zinhle says you can also find the pattern by working backwards and dividing by 3 each time:

54 ÷ 3 = 18; 18 ÷ 3 = 6; 6 ÷ 3 = 2

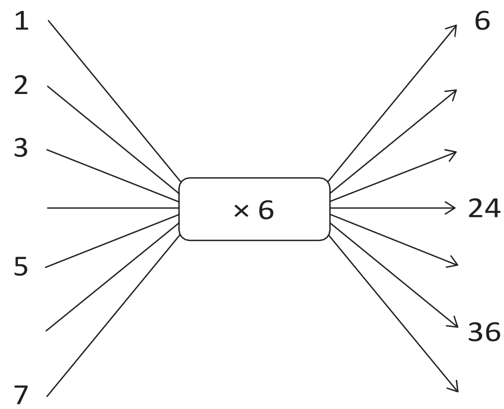
The number that we multiply with to get the next term in the sequence is called a **ratio**. If the number we multiply with remains the same throughout the sequence, we say it is a **constant ratio**.

1. Check whether Piet's reasoning works for sequence H on page 49: 2; 4; 8; 16; 32; 64; ...
2. Describe, in words, the rule for finding the next number in the sequences on the next page. Write down the next five terms of each sequence.

(a) 1; 10; 100; 1 000; ___ (b) 16; 8; 4; 2; (c) 7; −21; 63; −189; ___

(d) 3; 12, 48; ___ (e) 2 187; −729; 243; −81; ___

3. (a) Copy and fill in the missing output and input numbers:



What is the term-to-term rule for the output numbers here, + 6 or × 6?

(b) Copy and complete the following table:

Input numbers	1	2	3	4	5		12	x
Output numbers	6			24		36		

neither adding nor multiplying by the same number

1. Consider sequences A to H again and answer the questions that follow:
   - Sequence A: 2; 5; 8; 11; 14; 17; 20; 23; ...
   - Sequence B: 4; 5; 8; 13; 20; 29; 40; ...
   - Sequence C: 1; 2; 4; 8; 16; 32; 64; ...
   - Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19;...
   - Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...
   - Sequence F: 2; 6; 18; 54; 162; 486; ...
   - Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...
   - Sequence H: 2; 4; 8; 16; 32; 64; ....

   (a) Which other sequence(s) is/are of the same kind as sequence B? Explain.

   (b) In what way are sequences B and E different from the other sequences?

There are sequences where there is neither a constant difference nor a constant ratio between consecutive terms and yet a pattern still exists, as in the case of sequences B and E.

2. Consider the sequence: 10; 17; 26; 37; 50;...

   (a) Write down the next five numbers in the sequence.

   (b) Eric observed that he can calculate the next term in the sequence as follows: 10 + **7** = 17; 17 + **9** = 26; 26 + **11** = 37. Use Eric's method to check whether your numbers in question 2(a) above are correct.

3. Which of the statements below can Eric use to describe the relationship between the numbers in the sequence in question 2? Test the rule for the first three terms of the sequence and then simply write "yes" or "no" next to each statement.

   (a) Increase the difference between consecutive terms by two each time.
   (b) Increase the difference between consecutive terms by one each time.
   (c) Add two more than you added to get the previous term.

4. Provide a rule to describe the relationship between the numbers in the sequences below. Use your rule to provide the next five numbers in each sequence.

   (a) 1; 4; 9; 16; 25; ___ (b) 2; 13; 26; 41; 58; ___

   (c) 4; 14; 29; 49; 74; ___ (d) 5; 6; 8; 11; 15;20; ___

4.2 The position-term relationship in a sequence

using position to make predictions

1. Take another look at sequences A to H. Which sequence(s) are of the same kind as sequence A? Explain.

   Sequence A: 2; 5; 8; 11; 14; 17; 20;23;...
   Sequence B: 4; 5; 8; 13; 20; 29; 40; ...
   Sequence C: 1; 2; 4; 8; 16; 32; 64; ...
   Sequence D: 3; 5; 7; 9; 11;13; 15; 17; 19;...
   Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40; ...
   Sequence F: 2; 6; 18; 54; 162; 486; ...
   Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33; ...
   Sequence H: 2; 4; 8; 16; 32; 64; ...

Sizwe has been thinking about Amanda and Tamara's explanations of how they worked out the rule for sequence A and has drawn up a table. He agrees with them but says that there is another rule that will also work. He explains:

My table shows the terms in the sequence and the difference between consecutive terms:

	1st term	2nd term	3rd term	4th term						
A:	5	8	11	14						
differences	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	+ 3	

Sizwe reasons that the following rule will also work:

Multiply the position of the number by 3 and add 2 to the answer.

I can write this rule as a number sentence: Position of the number × 3 + 2

I use my number sentence to check: ***1*** × 3 + 2 = 5; ***2*** × 3 + 2 = 8; ***3*** × 3 + 2 = 11

2. (a) What do the numbers in bold in Sizwe's number sentence stand for?
   (b) What does the number 3 in Sizwe's number sentence stand for?

3. Consider the sequence 5; 8; 11; 14;...
   Apply Sizwe's rule to the sequence and determine:
   (a) term number 7 of the sequence
   (b) term number 10 of the sequence
   (c) the hundredth term of the sequence

4. Consider the sequence: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...
   (a) Use Sizwe's explanation to find a rule for this sequence.
   (b) Determine the 28th term of the sequence.

more predictions

Copy and complete the following tables by calculating the missing terms:

1.

Position in sequence	1	2	3	4	10	54
Term	4	7	10	13		

2.

Position in sequence	1	2	3	4	8	16
Term	4	9	14	19		

3.

Position in sequence	1	2	3	4	7	30
Term	3	15	27			

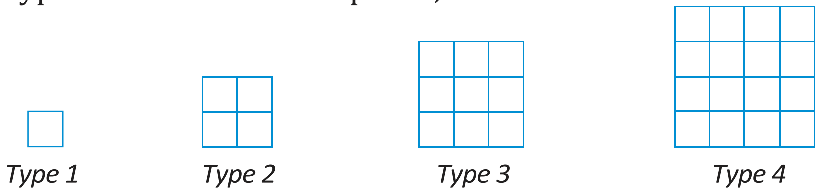
4. Copy the following table. Use the rule **position in the sequence × (position in the sequence + 1)** to complete it.

Position in sequence	1	2	3	4	5	6
Term	2					

4.3 Investigating and extending geometric patterns

square numbers

A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.

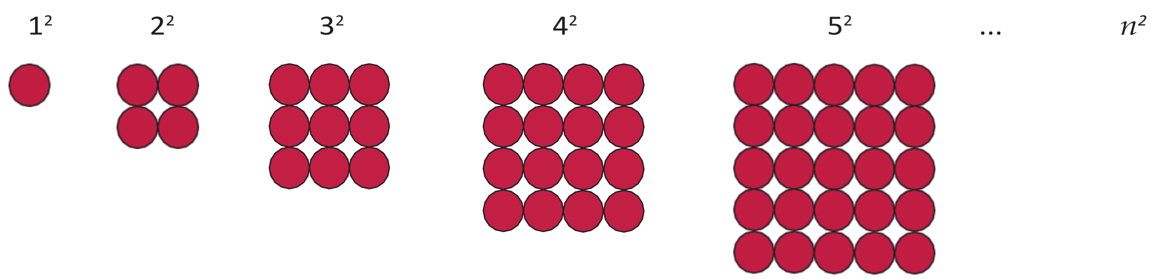


1. How many windowpanes will there be in type 5?
2. How many windowpanes will there be in type 6?
3. How many windowpanes will there be in type 7?
4. How many windowpanes will there be in type 12? Explain.
5. Copy and complete the table. Show your calculations.

Frame type	1	2	3	4	15	20
Number of windowpanes	1	4	9	16		

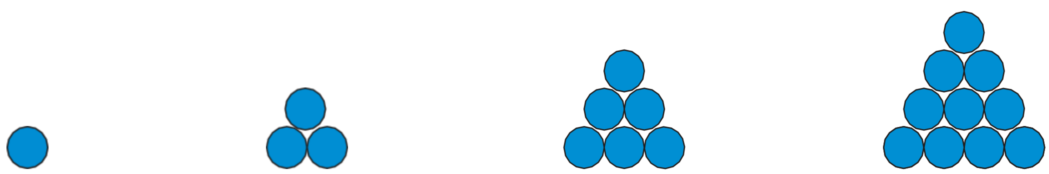
In algebra we think of a square as a number that is obtained by multiplying a number by itself. So, 1 is also a square because $1 \times 1 = 1$.

The symbol n is used below to represent the *position number* in the expression that gives the rule (n^2) when generalising.



triangular numbers

Therese uses circles to form a pattern of triangular shapes:



Picture 1 *Picture 2* *Picture 3* *Picture 4*

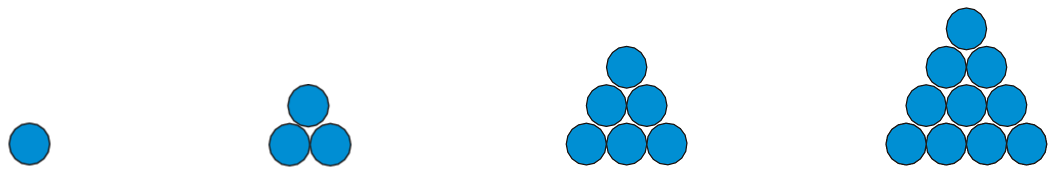
1. If the pattern is continued, how many circles must Therese have:
   (a) in the bottom row of picture 5?
   (b) in the second row from the bottom of picture 5?
   (c) in the third row from the bottom of picture 5?
   (d) in the second row from the top of picture 5?
   (e) in the top row of picture 5?
   (f) in total in picture 5? Show your calculations.
2. How many circles does Therese need to form triangle picture 7? Show the calculation.
3. How many circles does Therese need to form triangle picture 8?

4. Copy and complete the following table. Show all your work.

Picture number	1	2	3	4	5	6	12	15
Number of circles	1	3	6	10				

More than 2 500 years ago, Greek mathematicians already knew that the numbers 3, 6, 10, 15 and so on could form a triangular pattern. They represented these numbers with dots which they arranged in such a way that they formed equilateral triangles, hence the name **triangular numbers**. Algebraically, we think of them as sums of consecutive natural numbers starting with 1.

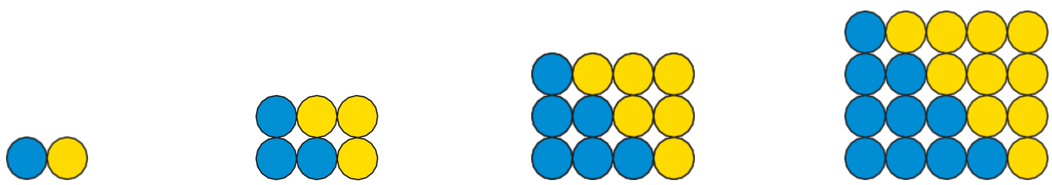
Let us revisit the activity on triangular numbers that we did on the previous page.



Picture 1 *Picture 2* *Picture 3* *Picture 4*

So far, we have determined the number of circles in the pattern by adding consecutive natural numbers. If we were asked to determine the number of circles in picture 200, for example, it would take us a very long time to do so. We need to find a quicker method of finding any triangular number in the sequence.

Consider the arrangement below.



Picture 1 *Picture 2* *Picture 3* *Picture 4*

We have added the yellow circles to the original blue circles and then rearranged the circles in such a way that they are in a rectangular form.

5. Picture 2 is three circles long and two circles wide. Copy and complete the following sentences:

   (a) Picture 3 is ___ circles long and ___ circles wide.

   (b) Picture 1 is ___ circles long and ___ circle wide.

   (c) Picture 4 is ___ circles long and ___ circles wide.

   (d) Picture 5 is ___ circles long and ___ circles wide.

6. How many circles will there be in a picture that is:
   (a) ten circles long and nine circles wide?
   (b) seven circles long and six circles wide?
   (c) six circles long and five circles wide?
   (d) 20 circles long and 19 circles wide?

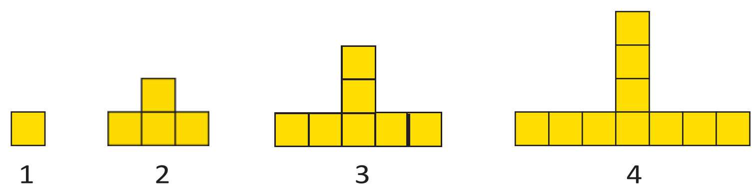
Suppose we want to use a quicker method to determine the number of circles in picture 15. We know that picture 15 is 16 circles long and 15 circles wide. This gives a total of $15 \times 16 = 240$ circles. But we must compensate for the fact that the yellow circles were originally not there by halving the total number of circles. In other words, the original figure has $240 \div 2 = 120$ circles.

7. Use the above reasoning to calculate the number of circles in:
   (a) picture 20 (b) picture 35

4.4 Describing patterns in different ways

t-shaped numbers

The pattern below is made from squares.



1. (a) How many squares will there be in pattern 5?
   (b) How many squares will there be in pattern 15?
   (c) Copy and complete the following table:

Pattern number	1	2	3	4	5	6	20
Number of squares	1	4	7	10			

You can use the following three plans (or methods) to calculate the number of squares for pattern 20. Study each onecarefully.

Plan A:

To get from one square to four squares, you have to add three squares. To get from four squares to seven squares, you have to add three squares. To get from seven squares to ten squares, you have to add three squares. Continue to add three squares for each pattern until pattern 20.

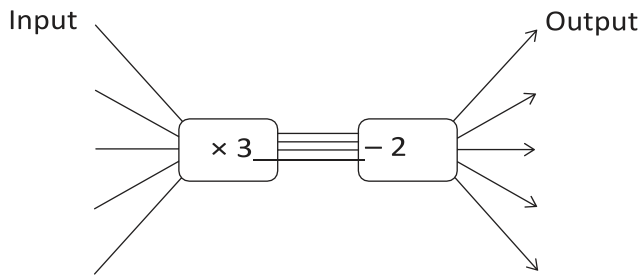
Plan B:

Multiply the pattern number by three and subtract two. Pattern 20 will therefore have $20 \times 3 - 2$ squares.

Plan C:

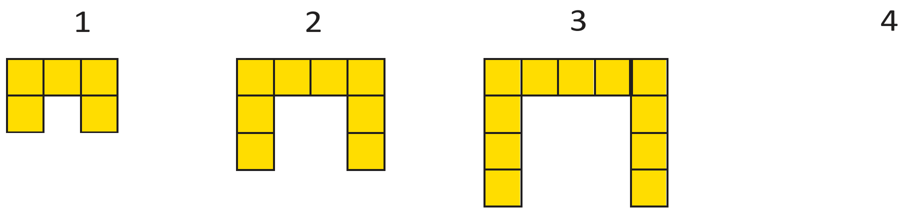
The number of squares in pattern 5 is 13. Pattern 20 will therefore have 13 × 4 = 52 squares because 20 = 5 × 4.

2. (a) Which method or plan (A, B or C) will give the right answer? Explain why.
   (b) Which of the above plans did you use? Explain why?
   (c) Can this flow diagram be used to calculate the number of squares?

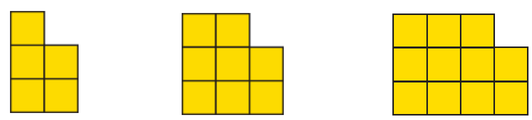


… and some other shapes

1. Three figures are given below. Draw the next figure in the tile pattern.

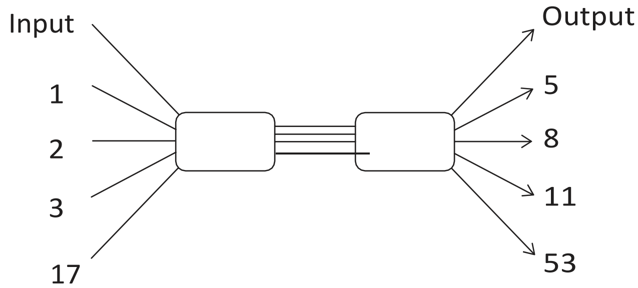


2. (a) If the pattern is continued, how many tiles will there be in the 17th figure? Answer this question by analysing what happens.
   (b) Thato decides that it easier for him to see the pattern when the tiles are rearranged as shown on the right:



3 × 1 + 2   3 × 2 + 2   3 × 3 + 2

   Use Thato's method to determine the number of tiles in the 23rd figure.
   (c) Copy and complete the following flow diagram by writing the appropriate operators so that it can be used to calculate the number of tiles in any figure of the pattern.



   (d) How many tiles will there be in the 50th figure if the pattern is continued?

Worksheet

1. Writedownthenextfourtermsineachsequence.Alsoexplain,ineachcase,how you figured out what the terms are.

   (a) 2; 4; 8; 14; 22; 32; 44; __

   (b) 2; 6; 18; 54; 162; __

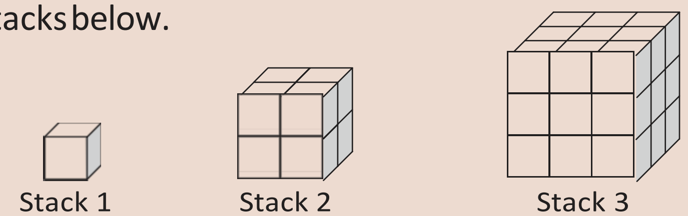
   (c) 1; 7; 13; 19; 25; __

2. (a) Copy and complete the following table by calculating the missing terms:

Position in sequence	1	2	3	4	5	7	10
Term	3	10	17				

   (b) Write the rule to calculate the term from the position in the sequence in words.

3. Consider the stacks below.



   (a) How many cubes will there be in stack 5?

   (b) Copy and complete the following table:

Stack number	1	2	3	4	5	6	10
Numberofcubes	1	8	27				

   (c) Write down the rule to calculate the number of cubes for any stack number.

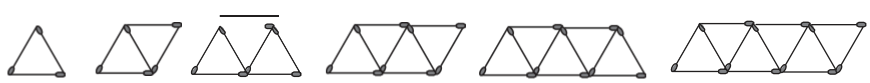
Chapter 5
Functions and relationships

5.1 Constant and variable quantities

looking for connections between quantities

Consider the following seven situations. There are two quantities in each situation. For each quantity, state whether it is constant (always the same number) or whether it changes. Also state, in each case, whether one quantity has an influence on the other. If it has, try to say how the one quantity will influence the other quantity.

1. Your age and the number of fingers on your hands
2. The number of calls you make and the airtime left on your cell phone
3. The length of your arm and your ability to finish Mathematics tests quickly
4. The number of identical houses to be built and the number of bricks required
5. The number of learners at a school and the length of the school day
6. The number of learners at a school and the number of classrooms needed
7. The number of matches in each arrangement, and the number of triangles in the arrangement:

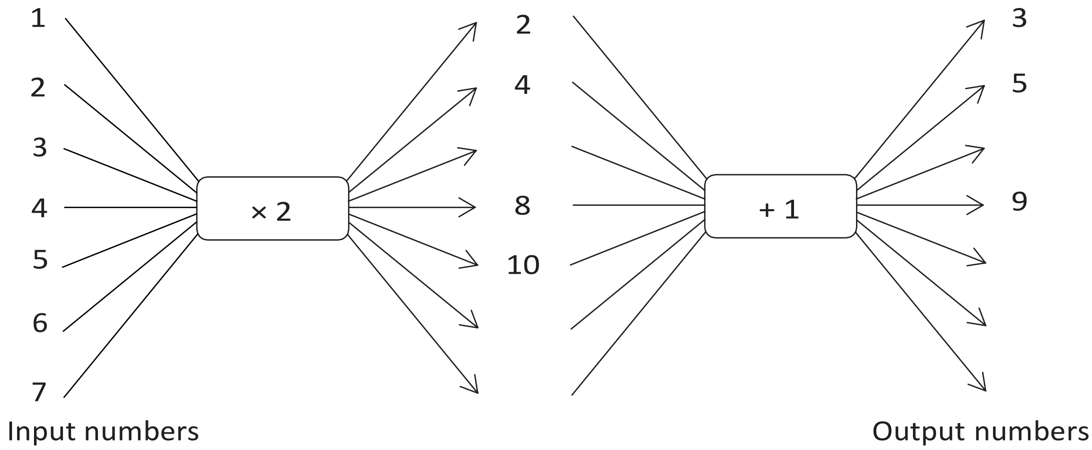


If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words what number, is linked to a specific value of the other quantity.

A quantity that changes is called a **variable quantity**, or just a **variable**.

8. (a) Look at the match arrangements in question 7. If you know that there are three triangles in an arrangement, can you say with certainty how many matches there are in that specific arrangement?
   (b) How many matches are there in the arrangement with ten triangles?
   (c) Is there another possible answer for question (b)?

9. Copy and complete the following flow diagrams by filling in all the missing numbers. Do you see any connections between the situation in question 7 on page 59 and this flow diagram? If so, describe the connections.

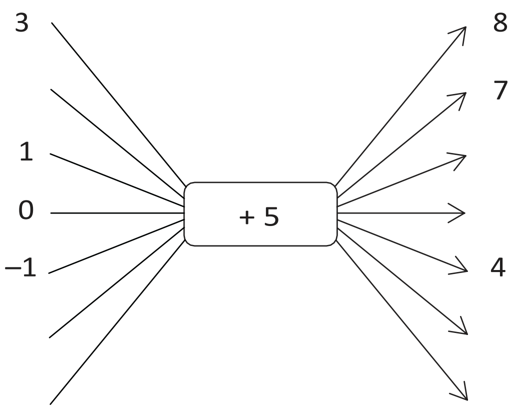


completing some flow diagrams

A relationship between two quantities can be shown with a flow diagram, such as those below. Unfortunately, only some of the numbers can be shown on a flow diagram.

1. Copy the following flow diagram. Calculate the output numbers. Some input numbers are missing. Choose and insert your own input numbers.

(a)



Each input number in a flow diagram has a corresponding **output number**. The first (top) **input number** corresponds to the first output number. The second input number corresponds to the second output number and so on. We call + 5 the **operator**.

(b) What type of numbers are the given input numbers?

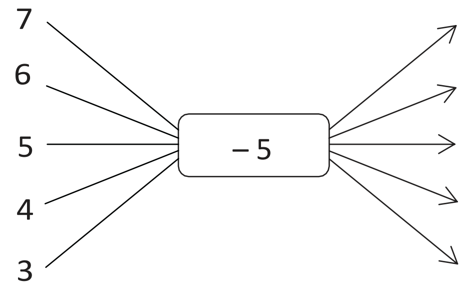
(c) In the above flow diagram, the output number 8 corresponds to the input number 3. Copy and complete the following sentences:

In the relationship shown in the above flow diagram, the output number ___ *corresponds to the input number* −1.

The input number ___ *corresponds to the output number 7.*

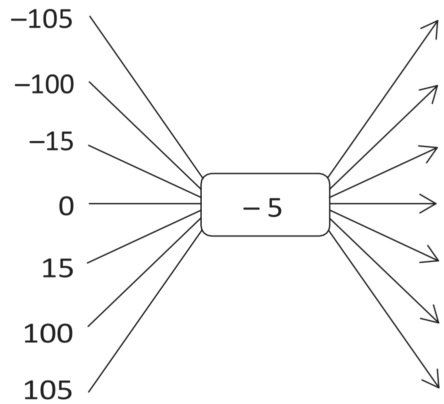
If more places are added to the flow diagram, the input number ___ *will correspond to the output number 31.*

2. (a) Copy and complete this flow diagram.
   (b) Compare this flow diagram to the flow diagram in question 1. What link do you find between the two?

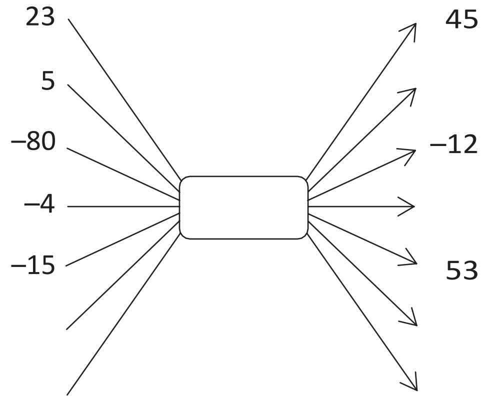


3. Copy and complete the following flow diagrams. You have to find out what the **operator** for (b) is, and fill it in yourself.

(a)

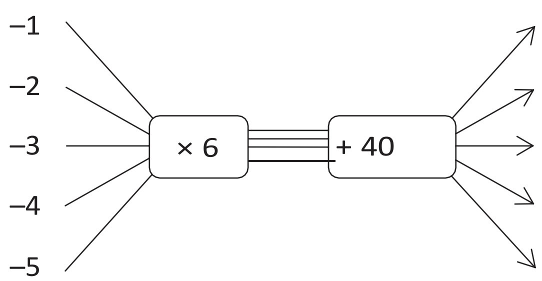


(b)



   (c) What number can you add in (a), instead of subtracting 5, that will produce the same output numbers?
   (d) What number can you subtract in (b), instead of adding a number, that will produce the same output numbers?

4. Copy and complete the following flow diagram:



A completed flow diagram shows two kinds of information:

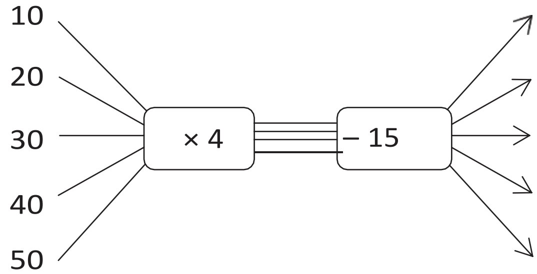
- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

The flow diagram that you completed in question 4 shows the following information:

- Each input number is multiplied by 6, then 40 is added to produce the output numbers.
- The input and output numbers are connected, as shown in the table on page 62.

Input numbers	−1	−2	−3	−4	−5
Output numbers	34	28	22	16	10

5. (a) Describe, in words, how the following output numbers can be calculated:

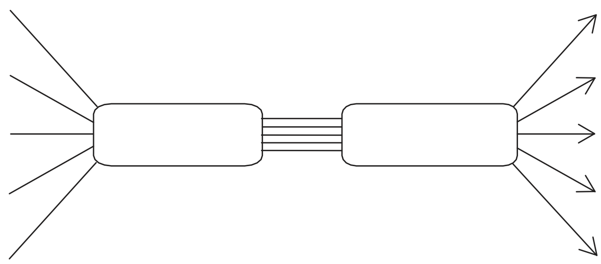


(b) Copy the following table and use it to show which output numbers are connected to which input numbers in the above flow diagram.

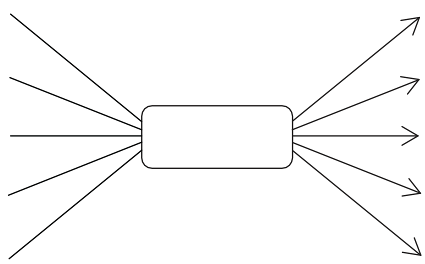
6. The following information is available about the floor space and cost of houses in a new development. The cost of an empty stand is R180 000.

Floor space in square metres (m^2)	90	120	150	180	210
Cost of house and stand	540 000	660 000	780 000	900 000	1 020 000

(a) Represent the above information in the following flow diagram:



(b) Show what the houses only will cost, if you get the stand for free.

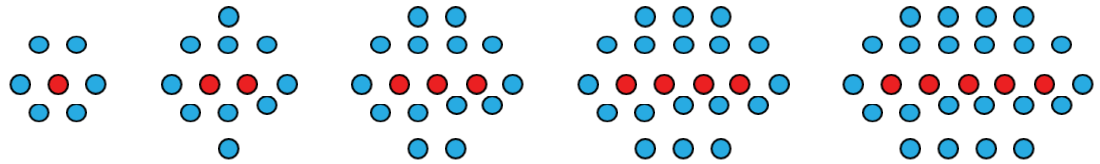


(c) Try to figure out what the cost of a house and stand will be, if there are exactly one hundred 1 m by 1 m sections of floor space in the house.

5.2 Different ways to describe relationships

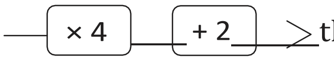
a relationship between red dots and blue dots

Here is an example of a relationship between two quantities:



In each arrangement there are some red dots and some blue dots.

1. How many blue dots are there if there is one red dot?
2. How many blue dots are there if there are two red dots?
3. How many blue dots are there if there are three red dots?
4. How many blue dots are there if there are four red dots?
5. How many blue dots are there if there are five reddots?
6. How many blue dots are there if there are six red dots?
7. How many blue dots are there if there are seven red dots?
8. How many blue dots are there if there are ten red dots?
9. How many blue dots are there if there are 20 red dots?
10. How many blue dots are there if there are 100 red dots?
11. Which of the following descriptions correctly describe the relationship between the number of blue dots and the number of red dots in the above arrangements? Test each description thoroughly for all the above arrangements.

    (a) The number of red dots  the number of blue dots

    (b) To calculate the number of blue dots you multiply the number of red dots by 2, add 1 and multiply the answer by 2

    (c) The number of blue dots = 2 × the number of red dots + 4

something to think about

Are there different possibilities for the number of blue dots if there are three red dots in the arrangement?

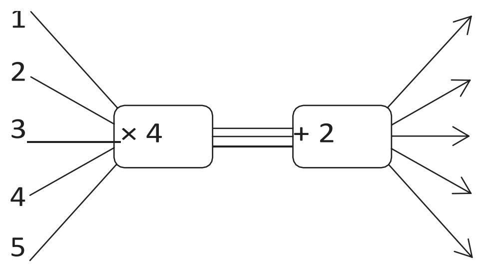
Are there different possibilities for the number of blue dots if there are two red dots in the arrangement?

Are there different possibilities for the number of blue dots if there are 20 red dots in the arrangement?

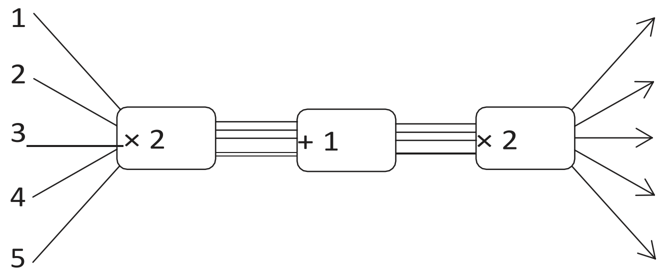
(d)

Number of red dots	1	2	3	4	5	6
Number of blue dots	6	10	14	18	22	26

(e)



(f)



(g) The number of blue dots = 4 × the number of red dots + 2

(h) The number of blue dots = 2 × (2 × the number of red dots + 1)
(Remember that the calculations inside the brackets are done first.)

The descriptions in (c), (g) and (h) above are called **word formulae**.

translating between different languages of description

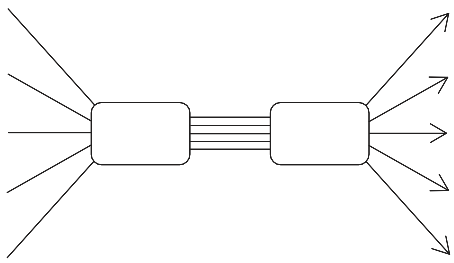
A relationship between two quantities can be described in different ways, including:

- a table of values of the two quantities
- a flow diagram
- a word formula
- a symbol formula (or symbolic formula).

You will learn about symbolic formulae in Section 5.3.

1. The relationship between two quantities is described as follows:
   The second quantity is always three times the first quantity plus 8.
   The first quantity varies between 1 and 5, and it is always a whole number.

(a) Describe this relationship using the flow diagram.



(b) Describe the relationship between the two quantities using this table:

(c) Describe the relationship between the two quantities using a word formula.

2. The relationship between two quantities is described as follows:

The input numbers are the first five odd numbers.

value of the one quantity → + 5 → × 3 → *the corresponding value of the other quantity*

(a) Describe this relationship using a table.

(b) Describe the relationship using a word formula.

5.3 Algebraic symbols for variables and relationships

describing procedures in different ways

1. In each case, do four things:
   - Complete the table.
   - Describe the relationship with a word formula.
   - Describe the input numbers in words.
   - Describe the output numbers in words.

(a) input number → × 10 → + 15 → output number

Input number	5	10	15	20	25	30
Output number						

output number =

(b) input number → + 15 → × 10 → output number

Input number	5	10	15	20	25	30
Output number						

(c) input number → × 2 → + 3 → × 5 → output number

Input number	5	10	15	20	25	30
Output number						

Formulae with symbols

Instead of writing "input number" and "output number" in formulae, you may just write a single letter symbol as an abbreviation.

Many years ago, mathematicians adopted the convention of using the letter symbol x as an abbreviation for the "input number", and the letter symbol y as an abbreviation for the "output number".

Letter symbols other than x and y are also used to indicate variable quantities.

The word formula you wrote for question 1(a) can be written more shortly as:

$$y = 10 \times x + 15$$

Mathematicians have also agreed that one may leave the ×-sign (multiplication sign) out when writing **symbolic formulae.**

Note that it is not at all wrong to use the multiplication sign in symbolic formulae.

So, instead of $y = 10 \times x + 15$, we may write $y = 10x + 15$.

2. Rewrite your word formulae in questions 1(b) and 1(c) as symbolic formulae.

3. Write a word formula for each of the following relationships:

   (a) $y = 7x + 10$ (b) $y = 7(x + 10)$ (c) $y = 7(2x + 10)$

writing symbolic formulae

Describe each of the following relationships with a symbolic formula:

1. To calculate the output number, the input number is multiplied by 4 and 7 is subtracted from the answer.

2. To calculate the output number, 7 is subtracted from the input number and the answer is multiplied by 5.

3. To calculate the output number, 7 is subtracted from the input number, the answer is multiplied by 5 and 3 is added to this answer.

Chapter 6
Algebraic expressions 1

6.1 Algebraic language

words, diagrams and symbols

1. Copy and complete the following table:

	Words	Flow diagram	Expression
	Multiply a number by two and add six to the answer.	→ [× 2] → [+ 6] →	$2 \times x + 6$
(a)	Add three to a number and then multiply the answer by two.		
(b)		→ [× 5] → [− 1] →	
(c)			$7 + 4 \times x$
(d)			$10 - 5 \times x$

An **algebraic expression** indicates **a sequence of calculations** that can also be described in words or by means of a flow diagram.

The flow diagram illustrates the **order** in which the calculations must be done.

In algebraic language, the **multiplication sign is usually omitted**. So, we write $2x$ instead of $2 \times x$. We also write $x \times 2$ as $2x$.

2. Write the following expressions in "normal" algebraic language:
   (a) $-2 \times a + b$
   (b) $a2$

looking different but yet the same

1. Copy and complete the table by calculating the numerical values of the expressions for the values of x. Some answers for $x = 1$ have been done for you as an example.

	x	1	3	7	10
(a)	$2x + 3x$	$2 \times 1 + 3 \times 1$ $2 + 3 = 5$			
(b)	$5x$				
(c)	$2x + 3$				
(d)	$5x^2$	$5 \times (1)^2$ $5 \times 1 = 5$			

2. Do the expressions $2x + 3x$ and $5x$ in question 1 above, produce different answers or the same answer for:
   (a) $x = 3$? (b) $x = 10$?
3. Do the expressions $2x + 3$ and $5x$ produce different answers or the same answer for:
   (a) $x = 3$? (b) $x = 10$?
4. Although they may look different, write down all the algebraic expressions in question 1 that have the same numerical value for the same value(s) of x. Justify your answer.

One of the things we do in algebra is to **evaluate** expressions. When we evaluate an expression we choose or are given a value of the variable in the expression. Now that we have an actual value, we can carry out the operations in the expression using this value, as in the examples given in the table.

Algebraic expressions that have the same numerical value for the same value of x but look different, are called **equivalent expressions**.

5. Say whether the following statements are true or false. Explain your answer in each case.
   (a) The expressions $2x + 3x$ and $5x$ are equivalent.
   (b) The expressions $2x + 3$ and $5x$ are equivalent.
6. Consider the expressions $3x + 2z + y$ and $6xyz$.
   (a) What is the value of $3x + 2z + y$ for $x = 4$, $y = 7$ and $z = 10$?
   (b) What is the value of $6xyz$ for $x = 4$, $y = 7$ and $z = 10$?
   (c) Are the expressions $3x + 2z + y$ and $6xyz$ equivalent? Explain.

Remember that $6xyz$ is the same as $6 \times x \times y \times z$.

To show that the two expressions in question 5(a) are equivalent, we write $2x + 3x = 5x$.

We can explain this as:

$2x + 3x = (x + x) + (x + x + x) = 5x$

We say the expression $2x + 3x$ **simplifies** to $5x$.

The term $3x$ is a product. The number 3 is called the **coefficient** of x.

7. In each case below, write down an expression equivalent to the one given:
   (a) $3x + 3x$ (b) $3x + 8x + 2x$
   (c) $8b + 2b + 2b$ (d) $7m + 2m + 10m$
   (e) $3x^2 + 3x^2$ (f) $3x^2 + 8x^2 + 2x^2$

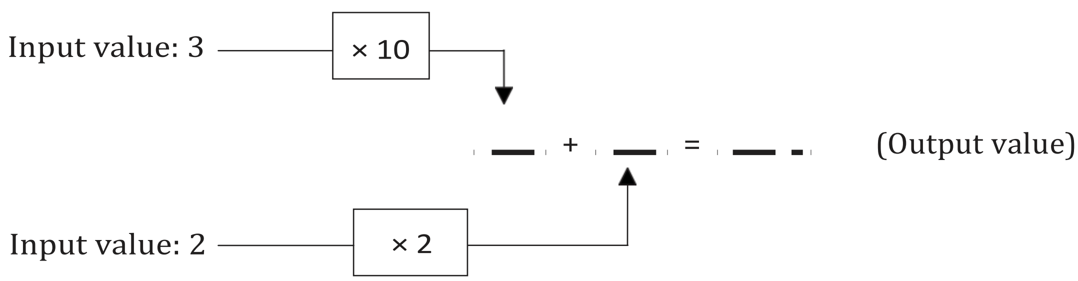
8. What is the coefficient of x^2 for the expression equivalent to $3x^2 + 8x^2 + 2x^2$?

In an expression that can be written as a sum, the different parts of the expression are called the **terms of the expression**. For example, $3x$, $2z$ and y are the terms of the expression $3x + 2z + y$.

An expression can have **like terms** or **unlike terms**, or both.

Like terms are terms that have the **same variable(s) raised to the same power**. The terms $2x$ and $3x$ are examples of like terms.

9. (a) Copy the diagram. Calculate the numerical value of $10x + 2y$ for $x = 3$ and $y = 2$ by completing the empty spaces in the diagram.



   (b) What is the output value for the expression $12xy$ for $x = 3$ and $y = 2$?
   (c) Are the expressions $10x + 2y$ and $12xy$ equivalent? Explain.
   (d) Are the terms $10x$ and $2y$ like or unlike terms? Explain.

10. (a) Which of the following algebraic expressions do you think will give the same results?
   A. $6x + 4x$ B. $10x$ C. $10x^2$ D. $9x + x$
   (b) Test the algebraic expressions you have identified for the following values of x:
   $x = 10$ $x = 17$ $x = 54$
   (c) Are the terms $6x$ and $4x$ like or unlike terms? Explain.
   (d) Are the terms $10x$ and $10x^2$ like or unlike terms? Explain.

11. Ashraf and Hendrik have a disagreement about whether the terms $7x^2y^3$ and $301y^3x^2$ are like terms or not. Hendrik thinks they are not, because in the first term the x^2 comes before the y^3, whereas in the second term the y^3 comes before the x^2.

    Explain to Hendrik why his argument is not correct.

12. Explain why the terms $5abc$, $10acb$ and $15cba$ are like terms.

6.2 Add and subtract like terms

rearrange terms and then combine like terms

1. Copy and complete the table by evaluating the expressions for the given values of x:

x	1	2	10
$30x + 80$	$30 \times 1 + 80$ $= 30 + 80 = 110$		
$5x + 20$			
$30x + 80 + 5x + 20$			
$35x + 100$			
$135x$			

2. Write down all the expressions in the table that are equivalent.

3. Tim thinks that the expressions $135x$ and $35x + 100$ are equivalent because for $x = 1$, they both have the same numerical value 135. Explain to Tim why the two expressions are not equivalent.

We have already come across the commutative and associative properties of operations. We will now use these properties to help us form equivalent algebraic expressions.

Commutative property

The order in which we add or multiply numbers does not change the answer: $a + b = b + a$ and $ab = ba$

Associative property

The way in which we group three or more numbers when adding or multiplying does not change the answer: $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$

We can find an equivalent expression by **rearranging** and **combining like terms**, as shown below:

$30x + 80 + 5x + 20$
Hence $30x + (80 + 5x) + 20$
Hence $30x + (5x + 80) + 20$
$= (30x + 5x) + (80 + 20)$
$= 35x + 100$

Like terms are combined to form a single term.

The terms 80 and 20 are called **constants**. The numbers 30 and 5 are called **coefficients**.

Brackets are used in the expression on the left to show how the like terms have been rearranged.

The terms $30x$ and $5x$ are combined to get the new term $35x$, and the terms 80 and 20 are combined to form the new term: 100. We say that the **expression** $30x + 80 + 5x + 20$ is **simplified** to a new expression $35x + 100$.

4. Simplify the following expressions:
   (a) $13x + 7 + 6x - 2$ (b) $21x - 8 + 7x + 15$ (c) $18c - 12d + 5c - 7c$
   (d) $3abc + 4 + 7abc - 6$ (e) $12x^2 + 2x - 2x^2 + 8x$ (f) $7m^3 + 7m^2 + 9m^3 + 1$

When you are not sure about whether or not you correctly simplified an expression, it is always advisable to check your work by evaluating the original expression and the simplified expression for some values.

When we use a value of the variable in the expression, we call it a **substitution**.

5. Make a simpler expression that is equivalent to the given expression. Test your answer for three different values of x, and redo your work until you get it right.
   (a) Simplify $(15x + 7y) + (25x + 3 + 2y)$ (b) Simplify $12mn + 8mn$

In questions 6 to 8 below, write down the letter representing the correct answer. Explain why you think your answer is correct.

6. The sum of $5x^2 + x + 7$ and $x - 9$ is:
   A. $x^2 - 2$ B. $5x^2 + 2x + 16$ C. $5x^2 + 16$ D. $5x^2 + 2x - 2$
7. The sum of $6x^2 - x + 4$ and $x^2 - 5$ is equivalent to:
   A. $7x^2 - x + 9$ B. $7x^2 - x - 1$ C. $6x^4 - x - 9$ D. $7x^4 - x - 1$
8. The sum of $5x^2 + 2x + 4$ and $3x^2 - 5x - 1$ can be expressed as:
   A. $8x^2 + 3x + 3$ B. $8x^2 + 3x - 3$ C. $8x^2 - 3x + 3$ D. $8x^2 - 3x - 3$

Combining like terms is a useful algebraic habit. It allows us to replace an expression with another expression that may be convenient to work with.

Do the questions on page 72 to get a sense of what we are talking about.

convenient replacements

1. Consider the expression $x + x + x + x + x + x + x + x + x + x$. What is the value of the expression in each of the following cases?
   (a) $x = 2$ (b) $x = 50$
2. Consider the expression $x + x + x + z + z + y$. What is the value of the expression in each of the following cases?
   (a) $x = 4, y = 7, z = 10$ (b) $x = 0, y = 8, z = 22$
3. Suppose you have to evaluate $3x + 7x$ for $x = 20$. Will calculating 10×20 give the correct answer? Explain.

Suppose we evaluate the expression $3x + 7x$ for $x = 20$ without first combining the like terms. We will have to do **three** calculations, namely 3×20, then 7×20 and then find the sum of the two: $3 \times 20 + 7 \times 20 = 60 + 140 = 200$.

But if we first combine the like terms $3x$ and $7x$ into one term $10x$, we only have to do **one calculation**: $10 \times 20 = 200$. This is one way of thinking about the convenience or usefulness of simplifying an algebraic expression.

4. The expression $5x + 3x$ is given and you are required to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.
5. Suppose you have to evaluate $7x + 5$ for $x = 10$. Will calculating 12×10 give the correct answer? Explain.
6. The expression $5x + 3$ is given and you have to evaluate it for $x = 8$. Will calculating 8×8 give the correct answer? Explain.

Samantha was asked to evaluate the expression $12x^2 + 2x - 2x^2 + 8x$ for $x = 12$. She thought to herself that just substituting the value of x directly into the terms would require a lot of work. She first combined the like terms as shown below:

$12x^2 - 2x^2 + 2x + 8x$
$= 10x^2 + 10x$

The terms $+ 2x$ and $- 2x^2$ change positions by the commutative property of operations.

Then for $x = 10$, Samantha found the value of $10x^2 + 10x$ by calculating:

$10 \times 10^2 + 10 \times 10$
$= 1\ 000 + 100$
$= 1\ 100$

Use Samatha's way of thinking for questions 7 to 9.

7. What is the value of $12x + 25x + 75x + 8x$ when $x = 6$?
8. Evaluate $3x^2 + 7 + 2x^2 + 3$ for $x = 5$.
9. When Zama was asked to evaluate the expression $2n - 1 + 6n$ for $n = 4$, she wrote down the following:
   $2n - 1 + 6n = n + 6n = 6n^2$
   Hence for $n = 4$: $6 \times (4)^2 = 6 \times 8 = 48$. Explain where Zama went wrong and why.

Worksheet

1. Copy and complete the following table:

	Words	Flow diagram	Expression
(a)	Multiply a number by three and add two to the answer	→ [× 3] → [+ 2] →	
(b)			$9x - 6$
(c)		→ [× 7] → [− 3] →	$7x - 3$

2. Which of the following pairs consist of like terms? Explain.
   A. $3y; -7y$ B. $14e^2; 5e$ C. $3y^2z; 17y^2z$ D. $-bcd; 5bd$

3. Write the following in the "normal" algebraic way:
   (a) $c2 + d3$ (b) $7 \times d \times e \times f$

4. Consider the expression $12x^2 - 5x + 3$.
   (a) What is the number 12 called? (b) Write down the coefficient of x.
   (c) What name is given to the number 3?

5. Explain why the terms $5pqr$, $-10prq$ and $15qrp$ are like terms.

6. If $y = 7$, what is the value of each of the following?
   (a) $y + 8$ (b) $9y$ (c) $7 - y$

7. Simplify the followingexpressions:
   (a) $18c + 12d + 5c - 7c$ (b) $3def + 4 + 7def - 6$

8. Evaluate the following expressions for $y = 3, z = -1$:
   (a) $2y^2 + 3z$ (b) $(2y)^2 + 3z$

9. Write each of the following algebraic expressions in the simplest form:
   (a) $5y + 15y$ (b) $5c + 6c - 3c + 2c$ (c) $4b + 3 + 16b - 5$
   (d) $7m + 3n + 2 - 6m$ (e) $5h^2 + 17 - 2h^2 + 3$ (f) $7e^2f + 3ef + 2 + 4ef$

10. Evaluate each of the following expressions:
    (a) $3y + 3y + 3y + 3y + 3y + 3y$ for $y = 18$
    (b) $13y + 14 - 3y + 6$ for $y = 200$
    (c) $20 - y^2 + 101y^2 + 80$ for $y = 1$
    (d) $12y^2 + 3yz + 18y^2 + 2yz$ for $y = 3$ and $z = 2$

Chapter 7
Algebraic equations 1

7.1 Setting up equations

An **equation** is a mathematical sentence that is true for some numbers but false for other numbers. The following are examples of equations:

$x + 3 = 11$ and $2x = 8$

$x + 3 = 11$ is true if $x = 8$, but false if $x = 3$.

When we look for a number or numbers that make an equation true, we say that we are **solving the equation**. For example, $x = 4$ is the **solution** of $2x = 8$ because it makes $2x = 8$ true. (Check: $2 \times 4 = 8$)

looking for numbers to make statements true

1. Are the following statements true or false? Justify your answer.
   (a) $x - 3 = 0$, if $x = -3$ (b) $x^3 = 8$, if $x = -2$ (c) $3x = -6$, if $x = -3$
   (d) $3x = 1$, if $x = 1$ (e) $6x + 5 = 47$, if $x = 7$

2. Find the original number. Show your reasoning.
   (a) A number multiplied by 10 is 80.
   (b) Add 83 to a number and the answer is 100.
   (c) Divide a number by 5 and the answer is 4.
   (d) Multiply a number by 4 and the answer is 20.
   (e) Twice a number is 100. (f) A number raised to the power 5 is 32.
   (g) A number raised to the power 4 is −81. (h) Fifteen times a number is 90.
   (i) 93 added to a number is −3. (j) Half a number is 15.

3. Write the equations below in words using "a number" in place of the letter symbol x. Then write what you think "the number" is in each case.
   Example: $4 + x = 23$. *Four plus a number equals twenty-three. The number is 19.*
   (a) $8x = 72$ (b) $\frac{2x}{5} = 2$ (c) $2x + 5 = 21$
   (d) $12 + 9x = 30$ (e) $30 - 2x = 40$ (f) $5x + 4 = 3x + 10$

7.2 Solving equations by inspection

the answer is in plain sight

1. Seven equations are given in the following table. Use the table to find out for which of the given values of x it will be true that the left-hand side of the equation is equal to the right-hand side.

You can read the solutions of an equation from a table.

x	−3	−2	−1	0	1	2	3	4
$2x + 3$	−3	−1	1	3	5	7	9	11
$x + 4$	1	2	3	4	5	6	7	8
$9 - x$	12	11	10	9	8	7	6	5
$3x - 2$	−11	−8	−5	−2	1	4	7	10
$10x - 7$	−37	−27	−17	−7	3	13	23	33
$5x + 3$	−12	−7	−2	3	8	13	18	23
$10 - 3x$	19	16	13	10	7	4	1	−2

(a) $2x + 3 = 5x + 3$ (b) $5x + 3 = 9 - x$ (c) $2x + 3 = x + 4$
(d) $10x - 7 = 5x + 3$ (e) $3x - 2 = x + 4$ (f) $9 - x = 2x + 3$
(g) $10 - 3x = 3x - 2$

Two or more equations can have the same solution. For example, $5x = 10$ and $x + 2 = 4$ have the same solution; $x = 2$ is the solution for both equations.

Two equations are called **equivalent** if they have the same solution.

2. Which of the equations in question 1 have the same solutions? Explain.

3. Copy and complete the following table. Then answer the questions that follow.

You can also do a search by narrowing down the possible solution to an equation.

x	0	5	10	15	20	25	30	35	40
$2x + 3$									
$3x - 10$									

(a) Can you find a solution for $2x + 3 = 3x - 10$ in the table?
(b) What happens to the values of $2x + 3$ and $3x - 10$ as x increases? Do they become bigger or smaller?
(c) Is there a point where the value of $3x - 10$ becomes bigger or smaller than the value of $2x + 3$ as the value of x increases? If so, between which x-values does this happen?

This point where the two expressions are equal is called the **break-even point**.

(d) Now that you narrowed down where the possible solution can be, try other possible values for x until you find out for what value of x the statement $2x + 3 = 3x - 10$ is true.

"Searching" for the solution of an equation by using tables or by narrowing down to the possible solution is called **solution by inspection**.

7.3 More examples

looking for and checking solutions

1. What is the solution for the equations shown below?

   (a) $x - 3 = 4$ (b) $x + 2 = 9$ (c) $3x = 21$ (d) $3x + 1 = 22$

When a certain number is the solution of an equation we say that the number **satisfies** the equation.
For example, $x = 4$ satisfies the equation $3x = 12$ because $3 \times 4 = 12$.

2. Choose the number in brackets that satisfies the equation. Explain your choice.

(a) $12x = 84$	{5; 7; 10; 12}
(b) $\frac{84}{x} = 12$	{−7; 0; 7; 10}
(c) $48 = 8k + 8$	{−5; 0; 5; 10}
(d) $19 - 8m = 3$	{−2; −1; 0; 1; 2}
(e) $20 = 6y - 4$	{3; 4; 5; 6}
(f) $x^3 = -64$	{−8; −4; 4; 8}
(g) $5^x = 125$	{−3; −1; 1; 3}
(h) $2^x = 8$	{1; 2; 3; 4}
(i) $x^2 = 9$	{1; 2; 3; 4}

3. What makes the following equations true? Check your answers.

   (a) $m + 8 = 100$ (b) $80 = x + 60$ (c) $26 - k = 0$ (d) $105 \times y = 0$
   (e) $k \times 10 = 10$ (f) $5x = 100$ (g) $\frac{15}{t} = 5$ (h) $3 = \frac{t}{5}$

4. Solve the following equations by inspection. Check your answers.

   (a) $12x + 14 = 50$ (b) $100 = 15m + 25$
   (c) $\frac{100}{x} = 20$ (d) $7m + 5 = 40$
   (e) $2x + 8 = 10$ (f) $3x + 10 = 31$
   (g) $-1 + 2x = -11$ (h) $2 + \frac{x}{7} = 5$
   (i) $100 = 64 + 9x$ (j) $\frac{2x}{6} = 4$

Chapter 8
Algebraic expressions 2

8.1 Expanding algebraic expressions

multiply often or multiply once: it is your choice

1. (a) Calculate 5 × 13 and 5 × 87 and add the two answers.
   (b) Add 13 and 87, and then multiply the answer by 5.
   (c) If you do not get the same answer for questions 1(a) and 1(b), you have made a mistake. Redo your work until you get it right.

If you work correctly, you get the same answer for questions 1(a) and 1(b); this is an example of a certain property of addition and multiplication called the **distributive property**. You use this property each time you multiply a number in parts. For example, you may calculate 3 × 24 by calculating 3 × 20 and 3 × 4, and then add the two answers:

$$3 \times 24 = 3 \times 20 + 3 \times 4$$

What you saw in question 1 was that 5 × 100 = 5 × 13 + 5 × 87. This can also be expressed by writing 5(13 + 87).

The word *distribute* means "to spread out". The distributive properties may be described as follows:
$a(b + c) = ab + ac$ and
$a(b - c) = ab - ac$,
where a, b and c can be any numbers.

2. (a) Calculate 10 × 56. (b) Calculate 10 × 16 + 10 × 40.

3. Write down any two numbers smaller than 100. Let us call them x and y.
   (a) Add your two numbers and multiply the answer by 6.
   (b) Calculate $6 \times x$ and $6 \times y$ and add the two answers.
   (c) If you do not get the same answers for (a) and (b), you have made a mistake somewhere. Correct your work.

4. Copy and complete the followingtable:

(a)

x	1	2	3	4	5
$3(x + 2)$					
$3x + 6$					
$3x + 2$					
$3(x - 2)$					
$3x - 6$					
$3x - 2$					

(b) If you do not get the same answers for the expressions $3(x + 2)$ and $3x + 6$, and for $3(x - 2)$ and $3x - 6$, you have made a mistake somewhere. Correct your work.

In algebra we normally write $3(x + 2)$ instead of $3 \times (x + 2)$. The expression $3 \times (x + 2)$ does not mean that you should first multiply by 3 when you evaluate the expression for a certain value of x. The brackets tell you that the first thing you should do is add the value(s) of x to 2 and then multiply the answer by 3.

However, instead of first adding the values within the brackets and then multiplying the answer by 3, we may just do the calculation $3 \times x + 3 \times 2 = 3x + 6$ as shown in the table.

(c) Which expressions amongst those given in the table on page 77 are equivalent? Explain.

(d) For what value(s) of x is $3(x + 2) = 3x + 2$?

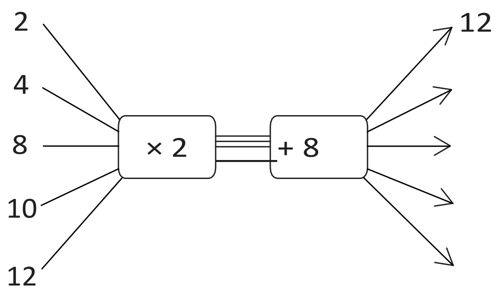
(e) Try to find a value of x such that $3(x + 2) \neq 3x + 6$.

If multiplication is the last step in evaluating an algebraic expression, then the expression is called a **product expression** or, briefly, a **product**. The way you evaluated the expression $3(x + 2)$ in the table is an example of a product expression.

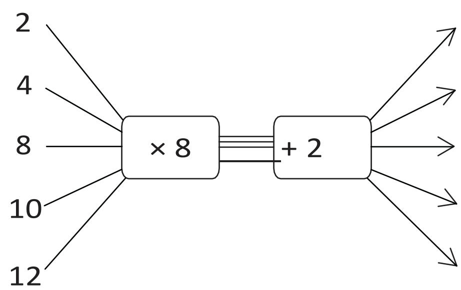
5. (a) Determine the value of $5x + 15$ if $x = 6$.
   (b) Determine the value of $5(x + 3)$ if $x = 6$.
   (c) Can we use the expression $5x + 15$ to calculate the value of $5(x + 3)$ for any values of x? Explain.

6. Copy and complete the following flow diagrams:

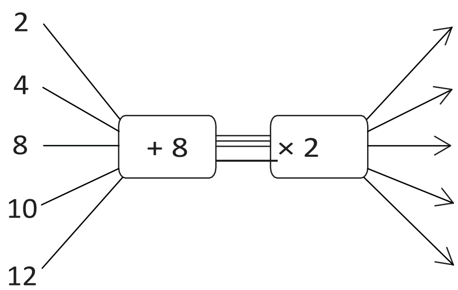
(a)



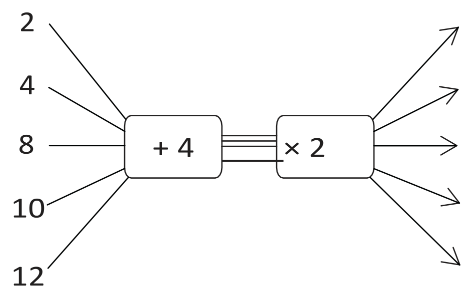
(b)



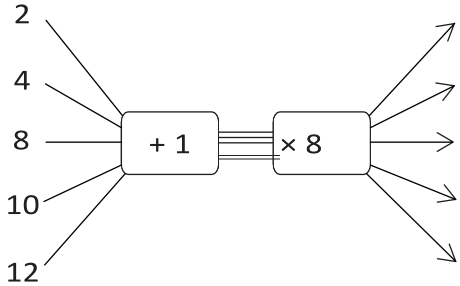
(c)



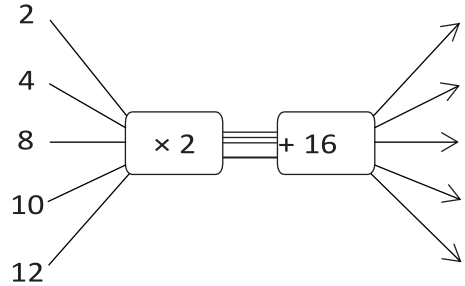
(d)



(e)



(f)



7. (a) Which of the flow diagrams in question 6(a) to (f) produce the same output numbers?
   (b) Write an algebraic expression for each of the flow diagrams in question 6.

product expressions and sum expressions

1. Copy and complete thefollowing:
   (a) (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)

   = ___ × (___)

   (b) (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6) + (3 + 6)

   = (3 + 3 + ___ ___) + (___ ___ ___)

   = (_ × _) + (_ × _)

2. Copy and complete the following:
   (a) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$

   = ___(___)

   (b) $(3x + 6) + (3x + 6) + (3x + 6) + (3x + 6) + (3x + 6)$

   = $(3x + 3x +$ ___ ___ ___) + (___ ___ ___)

   = (_ × _) (_ × _) _

3. In each case, write an expression without brackets that will give the same results as the given expression.

(a) $3(x+7)$	(b) $10(2x+1)$
(c) $x(4x+6)$	(d) $3(2p+q)$
(e) $t(t+9)$	(f) $x(y+z)$
(g) $2b(b+a-4)$	(h) $k^2(k-m)$

The process of writing product expressions as sum expressions is called **expansion**. It is sometimes also referred to as **multiplication of algebraic expressions**.

4. (a) Copy and complete the following table for the given values of x, y and z.

	$3(x + 2y + 4z)$	$3x + 6y + 12z$	$3x + 2y + 4z$
$x = 1$ $y = 2$ $z = 3$			
$x = 10$ $y = 20$ $z = 30$			
$x = 23$ $y = 60$ $z = 100$			
$x = 14$ $y = 0$ $z = 1$			
$x = 5$ $y = 9$ $z = 32$			

(b) Which sum expression and product expression are equivalent?

5. For each expression, write an equivalent expression without brackets.

(a) $2(x^2 + x + 1)$

(b) $p(q + r + s)$

(c) $-3(x + 2y + 3z)$

(d) $x(2x^2 + x + 7)$

(e) $6x(8 - 2x)$

(f) $12x(4 - x)$

(g) $3x(8x - 5) - 4x(6x - 5)$

(h) $10x(3x(8x - 5) - 4x(6x - 5))$

8.2 Simplifying algebraic expressions

expand, rearrange and then combine like terms

1. Write the shortest possible equivalent expression without brackets.

(a) $x + 2(x + 3)$

(b) $5(4x + 3) + 5x$

(c) $5(x + 5) + 3(2x + 1)$

(d) $(5 + x)^2$

(e) $-3(x^2 + 2x - 3) + 3(x^2 + 4x)$

(f) $x(x - 1) + x + 2$

When you are not sure if you have simplified an expression correctly, you should always check your work by evaluating the original expression and the simplified expression for some values of the variables.

2. (a) Evaluate $x(x + 2) + 5x^2 - 2x$ for $x = 10$.
   (b) Evaluate $6x^2$ for $x = 10$.
   (c) Can we use the expression $6x^2$ to calculate the values of the expression $x(x + 2) + 5x^2 - 2x$ for any given value of x? Explain.

This is how a sum expression for $x(x + 2) + 5x^2 - 2x$ can be made:

$x(x + 2) + 5x^2 - 2x = x \times x + x \times 2 + 5x^2 - 2x$

$= x^2 + 2x + 5x^2 - 2x$

$= x^2 + 5x^2 + 2x - 2x$ [Rearrange and combine like terms.]

$= 6x^2 + 0$

$= 6x^2$

3. Evaluate the following expressions for $x = -5$:
   (a) $x + 2(x+ 3)$
   (b) $5(4x + 3) + 5x$
   (c) $5(x + 5) + 3(2x+ 1)$
   (d) $(5 + x)^2$
   (e) $-3(x^2 + 2x - 3) + 3(x^2+ 4x)$
   (f) $x(x - 1) + x + 2$

4. Copy and complete the following table for the given values of x, y and z.

x	100	80	10	20	30
y	50	40	5	5	20
z	20	30	2	15	10
$x + (y - z)$					
$x - (y - z)$					
$x - y - z$					
$x - (y + z)$					
$x + y - z$					
$x - y + z$					

5. Say whether the following statements are true or false. Refer to the table in question 4. For any values of x, y and z:
   (a) $x + (y - z) = x + y - z$
   (b) $x - (y - z) = x - y - z$

6. Write the expressions without brackets. Do not simplify.
   (a) $3x - (2y+ z)$
   (b) $-x + 3(y - 2z)$

We can simplify algebraic expressions by using properties of operations as shown:

$(5x + 3) - 2(x + 1)$

Hence $5x + 3 - 2x - 2$

Hence $5x - 2x + 3 - 2$

Hence $3x + 1$

$x - (y + z) = x - y - z$

Addition is both associative and commutative.

7. Write an equivalent expression without brackets for each of the following expressions and then simplify:

(a) $22x + (13x - 5)$ (b) $22x - (13x - 5)$

(c) $22x - (13x + 5)$ (d) $4x - (15 - 6x)$

8. Simplify:

(a) $2(x^2 + 1) - x - 2$ (b) $-3(x^2 + 2x - 3) + 3x^2$

Some of the techniques we have used so far to form equivalent expressions include:

- removebrackets
- rearrangeterms
- combine like terms.

8.3 Simplifying quotient expressions

from quotient expressions to sum expressions

1. Copy and complete the table for the given values of x.

x	1	7	−3	−10
$7x^2 + 5x$				
$\frac{7x^2 + 5x}{x}$				
$7x + 5$				
$7x + 5x$				
$7x^2 + 5$				

2. (a) What is the value of $7x + 5$ for $x = 0$?
(b) What is the value of $\frac{7x + 5x}{x}$ for $x = 0$?

(c) Which of the two expressions, $7x + 5$ or $\frac{7x^2 + 5x}{x}$, requires fewer calculations? Explain.

(d) Are the expressions $7x + 5$ and $\frac{7x^2 + 5x}{x}$ equivalent, $x = 0$ excluded? Explain.

(e) Are there any other expressions that are equivalent to $\frac{7x^2 + 5x}{x}$ from those given in the table? Explain.

If division is the last step in evaluating an algebraic expression, then the expression is called a **quotient expression** or an **algebraic fraction**.

The expression $\frac{7x^2 + 5x}{x}$ is an example of a quotient expression or algebraic fraction.

3. Copy and complete the following table for the given values of x:

x	5	10	–5	–10
$10x - 5x^2$	$50 - 125$ $= -75$			
$5x$	5×5 $= 25$			
$\frac{10x - 5x^2}{5x}$	$\frac{50 - 125}{25}$ $= \frac{-75}{25}$ $= -3$			
$2 - x$	$2 - 5$ $= -3$			

(a) What is the value of $2 - x$ for $x = 0$?

(b) What is the value of $\frac{10x - 5x^2}{5x}$ for $x = 0$?

(c) Are the expressions $2 - x$ and $\frac{10x - 5x^2}{5x}$ equivalent, $x = 0$ excluded? Explain.

(d) Which of the two expressions $2 - x$ or $\frac{10x - 5x^2}{5x}$ requires fewer calculations? Explain.

We have found that quotient expressions, such as $\frac{10x - 5x^2}{5x}$ can sometimes be manipulated to give equivalent expressions, such as $2 - x$.

The value of this is that these equivalent expressions require fewer calculations.

The expressions $\frac{10x - 5x^2}{5x}$ and $2 - x$ are not quite equivalent because for $x = 0$, the value of $2 - x$ can be calculated, while the first expression has no value.

However, we can say that the two expressions are equivalent if they have the same values for all values of x admissible for both expressions.

How is it possible that $\frac{7x^2+5x}{x} = 7x + 5$ and $\frac{10x-5x^2}{5x} = 2 - x$ for all admissible values of x? We say $x = 0$ is not an admissible value of x because division by 0 is not allowed.

One of the methods for finding equivalent expressions for algebraic fractions is by means of division:

$$\frac{7x^2+5x}{x} = \frac{1}{x}(7x^2 + 5x) \quad \text{[just as } \frac{3}{5} = 3 \times \frac{1}{5}\text{]}$$

$$= \left(\frac{1}{x} \times 7x^2\right) + \left(\frac{1}{x} \times 5x\right) \quad \text{[distributive property]}$$

$$= \frac{7x^2}{x} + \frac{5x}{x}$$

$$= 7x + 5 \quad \text{[provided } x \neq 0\text{]}$$

4. Use the method shown on the previous page to simplify each fraction below:

(a) $\frac{8x + 10z + 6}{2}$

(b) $\frac{20x^2 + 16x}{4}$

(c) $\frac{9x^2y + xy}{xy}$

(d) $\frac{21ab - 14a^2}{7a}$

Simplifying a quotient expression can sometimes lead to a result which still contains quotients, as you can see in the following example:

$$\frac{5x^2 + 3x}{x^2}$$

$$= \frac{5x^2}{x^2} + \frac{3x}{x^2}$$

$$= 5 + \frac{3}{x}$$

5. (a) Evaluate $\frac{5x^2+3x}{x^2}$ for $x = -1$.

(b) For the expression $\frac{5x^2 + 3x}{x^2}$ to be equivalent to $5 + \frac{3}{x}$, which value of x must be excluded? Why?

6. Simplify the following expressions:

(a) $\frac{8x^2 + 2x + 4}{2x}$ (b) $\frac{4n + 1}{n}$

7. Evaluate:

(a) $\frac{8x^2 + 2x + 4}{2x}$ for $x = 2$ (b) $\frac{4n + 1}{n}$ for $n = 4$

8. Simplify:

(a) $\frac{6x^4 - 12x^3 + 2}{2x}$ (b) $\frac{-6n^4 - 4n}{6n}$

9. When Natasha and Lebogang were asked to evaluate the expression $\frac{x^2 + 2x + 1}{x}$ for $x = 10$, they did it in different ways.

Natasha's calculation:

$10 + 2 + \frac{1}{10}$

$= 12\frac{1}{10}$

Lebogang's calculation:

$\frac{100 + 20 + 1}{10}$

$= \frac{121}{10}$

$= 12\frac{1}{10}$

Explain how each of them thought about evaluating the given expression.

8.4 Squares, cubes and roots of expressions

simplifying squares and cubes

Study the following example:

$(3x)^2 = 3x \times 3x$ — Meaning of squaring

$= 3 \times x \times 3 \times x$

$= 3 \times 3 \times x \times x$ — Multiplication is commutative: $a \times b = b \times a$

$= 9x^2$ — We say that $(3x)^2$ simplifies to $9x^2$

1. Simplify the following expressions:

(a) $(2x)^2$ (b) $(2x^2)^2$ (c) $(-3y)^2$

2. Simplify the following expressions:

(a) $25x - 16x$ (b) $4y + y + 3y$ (c) $a + 17a - 3a$

3. Simplify:

(a) $(25x - 16x)^2$ (b) $(4y + y + 3y)^2$ (c) $(a + 17a - 3a)^2$

Study the following example:

$(3x)^3 = 3x \times 3x \times 3x$ — Meaning of cubing

$= 3 \times x \times 3 \times x \times 3 \times x$

$= 3 \times 3 \times 3 \times x \times x \times x$ — Multiplication is commutative: $a \times b = b \times a$

$= 27x^3$ — We say that $(3x)^3$ simplifies to $27x^3$.

4. Simplify:
   (a) $(2x)^3$ (b) $(-x)^3$ (c) $(5a)^3$
   (d) $(7y^2)^3$ (e) $(-3m)^3$ (f) $(2x^3)^3$

5. Simplify:
   (a) $5a - 2a$ (b) $7x + 3x$ (c) $4b + b$

6. Simplify:
   (a) $(5a - 2a)^3$ (b) $(7x + 3x)^3$ (c) $(4b + b)^3$
   (d) $(13x - 6x)^3$ (e) $(17x + 3x)^3$ (f) $(20y - 14y)^3$

Always remember to test if the simplified expression is equivalent to the given expression for at least three different values of the givenvariable.

square and cube roots of expressions

1. Thabang and his friend Vuyiswa were asked to simplify $\sqrt{2a^2 \times 2a^2}$.
   Thabang reasoned as follows:
   To find the square root of a number is the same as asking yourself the question: "Which number was multiplied by itself?" The number that is multiplied by itself is $2a^2$ and therefore $\sqrt{2a^2 \times 2a^2} = 2a^2$.
   Vuyiswa reasoned as follows:
   I should first simplify $2a^2 \times 2a^2$ to get $4a^4$ and then calculate $\sqrt{4a^4} = 2a^2$.
   Which of the two methods do you prefer? Explain why.

2. Say whether each of the following is true or false. Give a reason for your answer.
   (a) $\sqrt{6x \times 6x} = 6x$ (b) $\sqrt{5x^2 \times 5x^2} = 5x^2$

3. Simplify:
   (a) $y^6 \times y^6$ (b) $125x^2 + 44x^2$

4. Simplify:
   (a) $\sqrt{y^{12}}$ (b) $\sqrt{125x^2 + 44x^2}$
   (c) $\sqrt{25a^2 - 16a^2}$ (d) $\sqrt{121y^2}$

(e) $\sqrt{16a^2 + 9a^2}$

(f) $\sqrt{25a^2 - 9a^2}$

5. What does it mean to find the cube root of $8x^3$ written as $\sqrt[3]{8x^3}$?

6. Simplify:

(a) $2a \times 2a \times 2a$

(b) $10b^3 \times 10b^3 \times 10b^3$

(c) $3x^3 \times 3x^3 \times 3x^3$

(d) $-3x^3 \times -3x^3 \times -3x^3$

7. Determine the following:

(a) $\sqrt[3]{1000b^9}$

(b) $\sqrt[3]{2a \times 2a \times 2a}$

(c) $\sqrt[3]{27x^3}$

(d) $\sqrt[3]{-27x^3}$

8. Simplify each of the following expressions:

(a) $6x^3 + 2x^3$

(b) $-m^3 - 3m^3 - 4m^3$

9. Determine the following:

(a) $\sqrt[3]{6x^3 + 2x^3}$

(b) $\sqrt[3]{-8m^3}$

(c) $\sqrt[3]{125y^3}$

(d) $\sqrt[3]{93a^3 + 123a^3}$

worksheet

1. Simplify:

(a) $2(3b + 1) + 4$

(b) $6 - (2 + 5e)$

(c) $18mn + 22mn + 70mn$

(d) $4pqr + 3 + 9pqr$

2. Evaluate each of the following expressions for $m = 10$:

(a) $3m^2 + m + 10$

(b) $5(m^2 - 5) + m^2 + 25$

3. (a) Simplify: $\frac{4b + 6}{2}$

(b) Evaluate the expression $\frac{4b + 6}{2}$ for $b = 100$.

4. Simplify:

(a) $(4g)^2$

(b) $(6y)^3$

(c) $(7s + 3s)^2$

5. Determinethefollowing:

(a) $\sqrt{121b^2}$

(b) $\sqrt[3]{64y^3}$

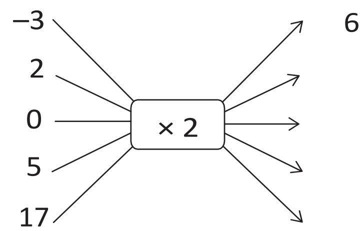
(c) $\sqrt{63d^2 + 18d^2}$

Chapter 9

Algebraic equations 2

9.1 Thinking forwards and backwards

doing and undoing what has been done



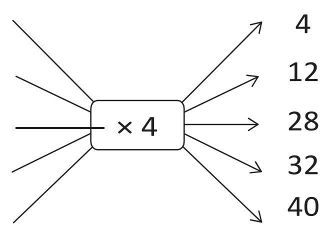
1. Copy and complete the flow diagram on the right by finding the output values.

2. Copy and complete the following table:

x	-3	-2	0	5	17
$2x$					

3. Evaluate $4x$ if:

   (a) $x = -7$ (b) $x = 10$ (c) $x = 0$



4. (a) Copy and complete the flow diagram on the right by finding the input values.

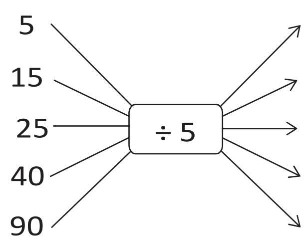
   (b) Puleng put another integer into the flow diagram and got -68 as an answer. Which integer did she put in? Show your calculation.

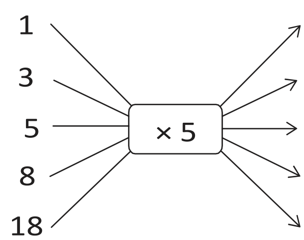
   (c) Explain how you worked to find the input numbers when you did question 4(a).

5. (a) Copy and complete the following table:

x					
$5x$	5	15	25	40	90

   (b) Copy and complete the following flow diagrams:





   (c) Explain how you completed the table.

One of the things we do in algebra is to **evaluate** expressions. When we evaluate expressions we replace a variable in the expression with an **input number** to obtain the value of the expression called the **output number**. We think of this process as a **doing process**.

However, in other cases we may need to undo what was done. When we know what output number was obtained but do not know what input number was used, we have to **undo** what was done in evaluating the expression. In such a case we say we are **solving an equation**.

6. Look again at questions 1 to 5. For each question, say whether the question required a doing or an undoing process. Give an explanation for your answer (for example: input to output).

7. (a) Copy and complete the following flowdiagrams:

   1 → [+ 6] →   7 → [− 6] →

   (b) What do you observe?

8. (a) Copy and complete the following flow diagrams:

   2 → [× 5] →   10 → [÷ 5] →

   (b) What do you observe?

9. (a) Copy and complete the following flow diagrams:

   20 → [× 5] → [+ 5] →   105 → [− 5] → [÷ 5] →

   (b) What do you observe?

10. (a) Copy and complete the following flow diagram:

   64 → [÷ 8] → [+ 12] →

   (b) What calculations will you do to determine what the input number was when the output number is 20?

Solve the following problems by undoing what was done to get the answer:

11. When a certain number is multiplied by 10 the answer is 150. What is the number?

12. When a certain number is divided by 5 the answer is 1. What is the number?

13. When 23 is added to a certain number the answer is 107.What is the original number?

14. When a certain number is multiplied by 5 and 2 is subtracted from the answer, the final answer is 13. What is the original number?

> Moving from the output value to the input value is called **solving the equation for the unknown**.

9.2 Solving equations using the additive and multiplicative inverses

finding the unknown

Consider the equation $3x + 2 = 23$.

We can represent the equation $3x + 2 = 23$ in a flow diagram, where x represents an unknown number:

x —— [× 3] —— [+ 2] ——→ 23

When you reverse the process in the flow diagram, you start with the output number 23, then subtract 2 and then divide the answer by 3:

23 —— [− 2] —— [÷ 3] ——→

We can write all of the above reverse process as follows:

Subtract 2 from both sides of the equation:

$$3x + 2 - 2 = 23 - 2$$

$$3x = 21$$

Divide both sides by 3:

$$\frac{3x}{3} = \frac{21}{3}$$

$$x = 7$$

We say $x = 7$ is the solution of $3x + 2 = 23$, because $3 \times 7 + 2 = 23$. We say that $x = 7$ makes the equation $3x + 2 = 23$ true.

> The numbers + 2 and − 2 are **additive inverses** of each other. When we add a number and its additive inverse we always get 0.
> The numbers 3 and $\frac{1}{3}$ are **multiplicative inverses** of each other. When we multiply a number and its multiplicative inverse we always get 1, so $3 \times \frac{1}{3} = 1$.

The additive and multiplicative inverses help us to isolate the unknown value or the input value.

Solve the equations below by using the additive and multiplicative inverses. Check your answers.

1. $x + 10 = 0$ 2. $49x + 2 = 100$

3. $2x = 1$ 4. $20 = 11 - 9x$

In some cases you need to collect like terms before you can solve the equations using additive and multiplicative inverses, as in the example below:

Example: Solve for x: $7x + 3x = 10$

$$10x = 10$$

$$\frac{10x}{10} = \frac{10}{10}$$

$$x = 1$$

5. $4x + 6x = 20$ 6. $5x = 40 + 3x$

7. $3x + 1 - x = 0$ 8. $x + 20 + 4x = -55$

Also remember:

- **the multiplicative property of 1:** the product of any number and 1 is that number.
- **the additive property of 0:** the sum of any number and 0 is that number.

$7x$ and $3x$ are like terms and can be replaced with one equivalent expression: $(7 + 3)x = 10x$.

9.3 Solving equations involving powers

Solving an exponential equation is the same as asking the question: **To what exponent must the base be raised in order to make the equation true?**

1. Copy and complete the following table:

x	1	3	5	7
2^x				

2. Copy and complete the following table:

x		2		5
3^x	1		27	

Karina solved the equation $3^x = 27$ as follows:

$3^x = 27$

Hence $3^x = 3^3$

Hence $x = 3$

The number 27 can be expressed as 3^3 because $3^3 = 27$.

3. Now use Karina's method and solve for x in each of the following:

(a) $2^x = 32$ (b) $4^x = 16$ (c) $6^x = 216$ (d) $5^{x+1} = 125$

Chapter 10
Construction of geometric figures

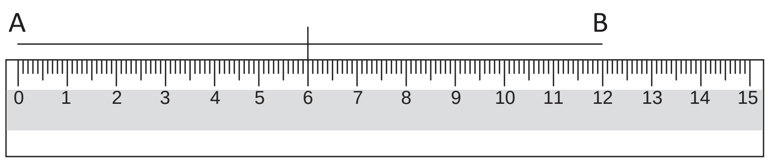
10.1 Bisecting lines

When we construct, or draw geometric figures, we often need to bisect lines or angles. To **bisect** means to cut something into two equal parts. There are different ways to bisect a line segment.

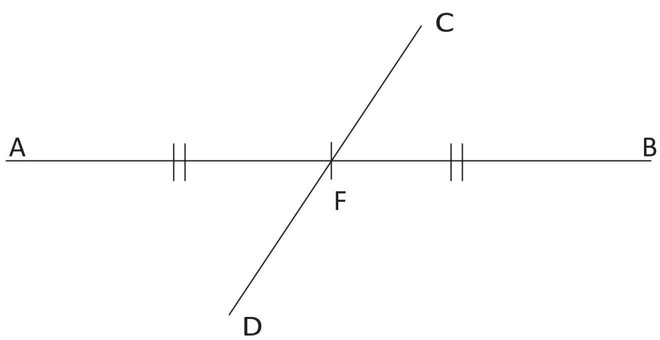
bisecting a line segment with a ruler

1. Read through the following steps:

Step 1: Draw line segment AB and determine its midpoint.



Step 2: Draw any line segment through the midpoint.



The small marks on AF and FB show that AF and FB are equal.

CD is called a **bisector** because it bisects AB. AF = FB.

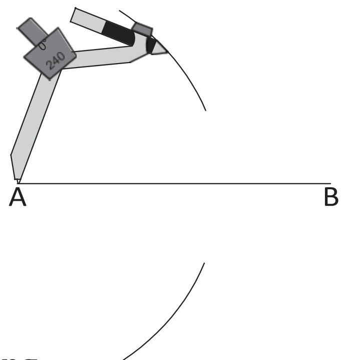
2. Use a ruler to draw and bisect the following line segments:
AB = 6 cm and XY = 7 cm.

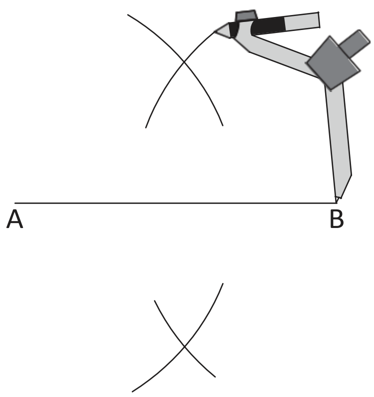
In Grade 6, you learnt how to use a compass to draw circles and parts of circles called arcs. We can use arcs to bisect a line segment.

bisecting a line segment with a compass and ruler

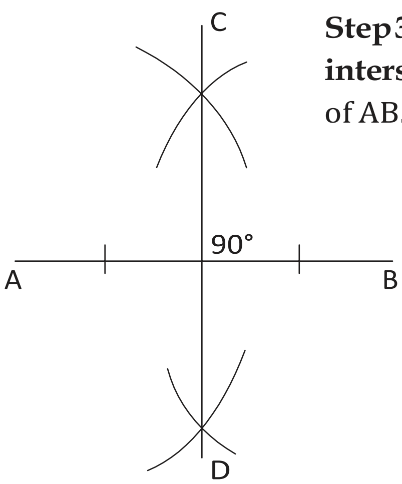
1. Read through the following steps:

Step 1: Place the compass on one endpoint of the line segment (point A). Draw an arc above and below the line. (Notice that all the points on the arc above and below the line are the same distance from point A.)





Step 2: Without changing the compass width, place the compass on point B. Draw an arc above and below the line so that the arcs cross the first two. (The two points where the arcs cross are the same distance away from point A and from point B.)



Step 3: Use a ruler to join the points where the arcs **intersect**. This line segment (CD) is the bisector of AB.

> **intersect** means to cross or meet.
> A **perpendicular** is a line that meets another line at an angle of 90°.

Notice that CD is also **perpendicular** to AB. Therefore, CD is a **perpendicular bisector**.

2. Use a compass and a ruler to practise drawing perpendicular bisectors on line segments.

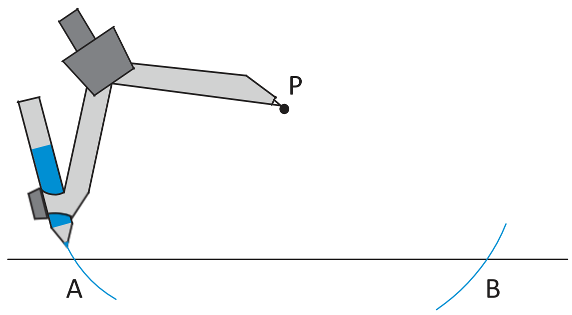
> **try this!**
> Use only a protractor and ruler to draw a perpendicular bisector on a line segment. (Remember that we use a protractor to measure angles.)

10.2 Constructing perpendicular lines

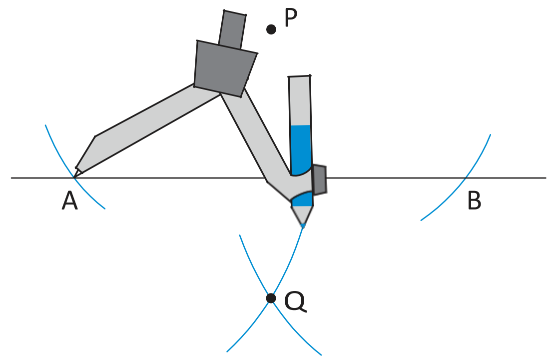
a perpendicular line from a given point

1. Read through the followingsteps:

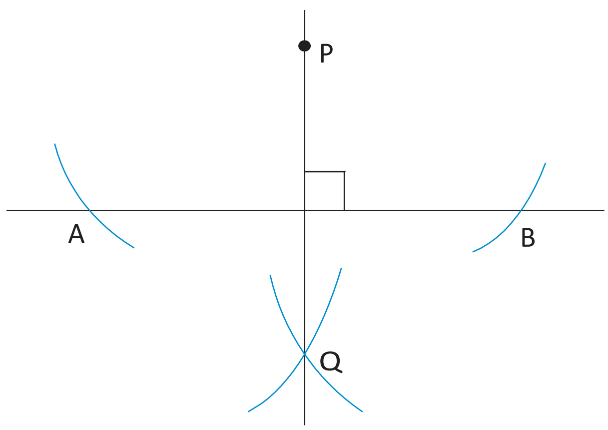
Step 1: Place your compass on the given point (point P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.



Step 2: From each arc on the line, draw another arc on the opposite side of the line from the given point (P). The two new arcs will intersect.

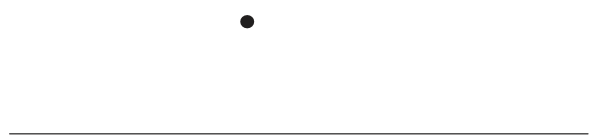


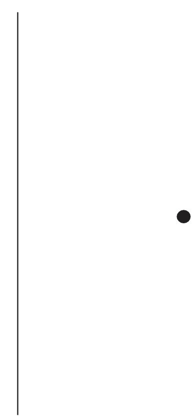
Step 3: Use your ruler to join the given point (P) to the point where the arcs intersect (Q).



PQ is perpendicular to AB.
We also write it like this: PQ $\perp$ AB.

2. Use your compass and ruler to draw a perpendicular line from each given point to the line segment:

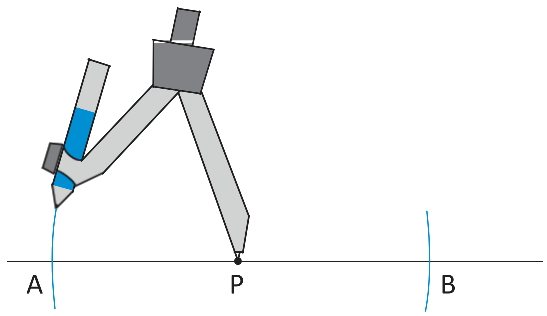




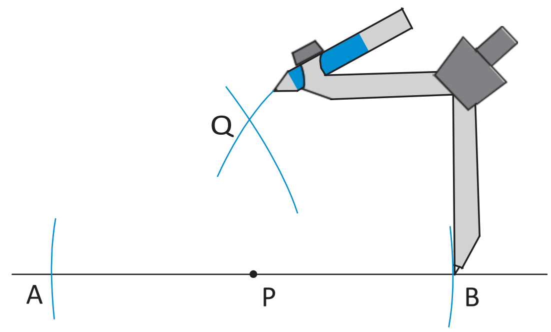
a perpendicular line at a given point on a line

1. Read through the following steps:

Step 1: Place your compass on the given point (P). Draw an arc across the line on each side of the given point. Do not adjust the compass width when drawing the second arc.

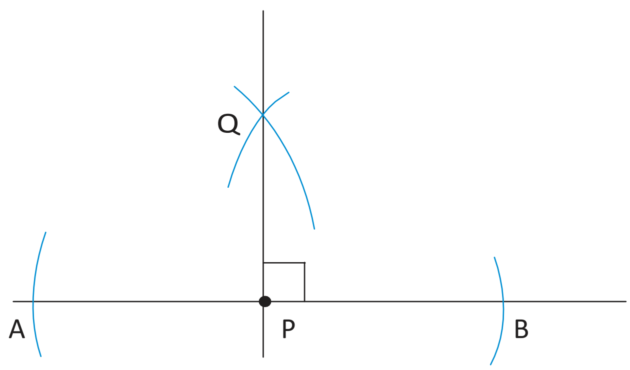


Step 2: Open your compass so that it is wider than the distance from one of the arcs to point P. Place the compass on each arc and draw an arc above or below the point P. The two new arcs will intersect.

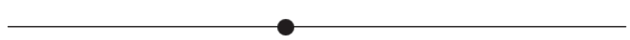


Step 3: Use your ruler to join the given point (P) and the point where the arcs intersect (Q).

PQ ⊥ AB



2. Copy the following diagrams. Use your compass and ruler to draw a perpendicular at the given point on each line:



10.3 Bisecting angles

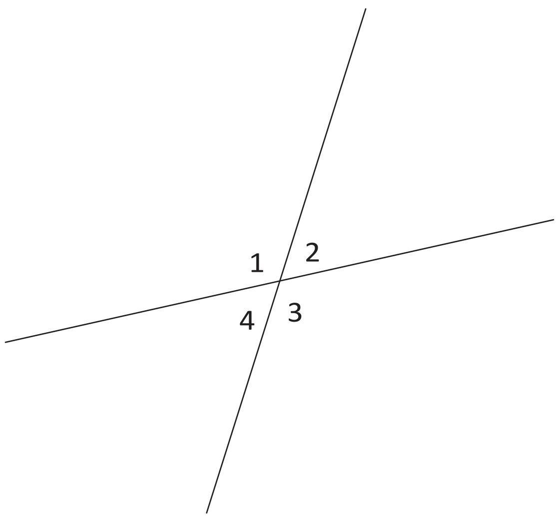
Angles are formed when any two lines meet. We use degrees (°) to measure angles.

measuring and classifying angles

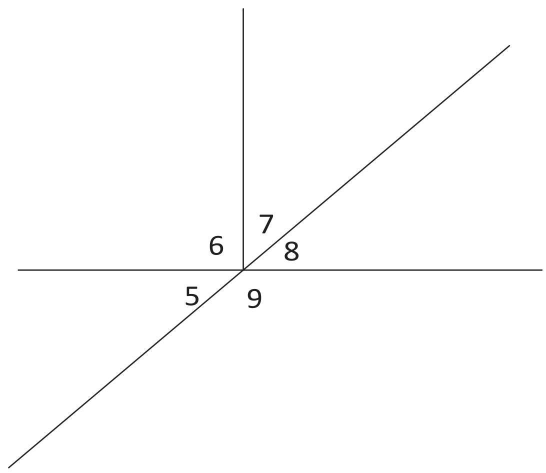
In the figures below, each angle has a number from 1 to 9.

1. Use a protractor to measure the sizes of all the angles in each figure. Write your answers for each figure.

(a)



(b)



2. Copy the statements below and use your answers to write the angle sizes:

$\hat{1} = \_\_\_^\circ$

$\hat{1} + \hat{2} = \_\_\_^\circ$

$\hat{1} + \hat{4} = \_\_\_^\circ$

$\hat{2} + \hat{3} = \_\_\_^\circ$

$\hat{3} + \hat{4} = \_\_\_^\circ$

$\hat{1} + \hat{2} + \hat{4} = \_\_\_^\circ$

$\hat{1} + \hat{2} + \hat{3} + \hat{4} = \_\_\_^\circ$

$\hat{6} = \_\_\_^\circ$

$\hat{7} + \hat{8} = \_\_\_^\circ$

$\hat{6} + \hat{7} + \hat{8} = \_\_\_^\circ$

$\hat{5} + \hat{6} + \hat{7} = \_\_\_^\circ$

$\hat{6} + \hat{5} = \_\_\_^\circ$

$\hat{5} + \hat{6} + \hat{7} + \hat{8} = \_\_\_^\circ$

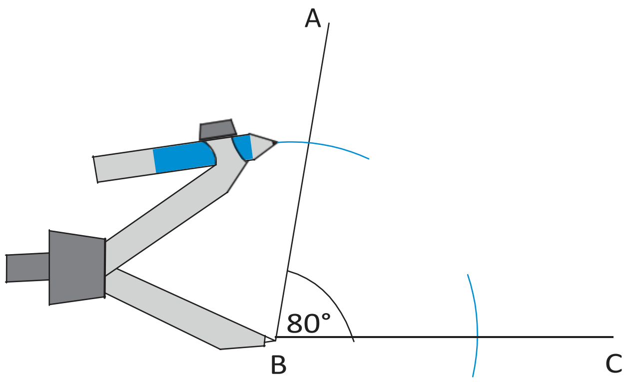
$\hat{5} + \hat{6} + \hat{7} + \hat{8} + \hat{9} = \_\_\_^\circ$

3. Next to each of your answers above, write down what type of angle it is, namely acute, obtuse, right, straight, reflex or revolution.

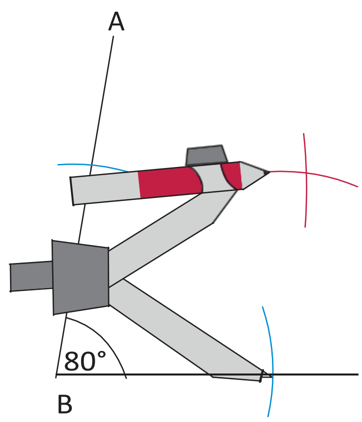
bisecting angles without a protractor

1. Read through the following steps:

Step 1: Place the compass on the vertex of the angle (point B). Draw an arc across each arm of the angle.

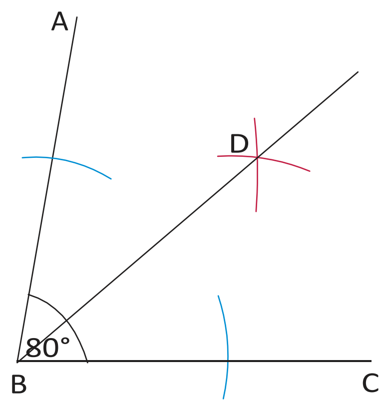


Step 2: Place the compass on the point where one arc crosses an arm and draw an arc inside the angle. Without changing the compass width, repeat for the other arm so that the two arcs cross.



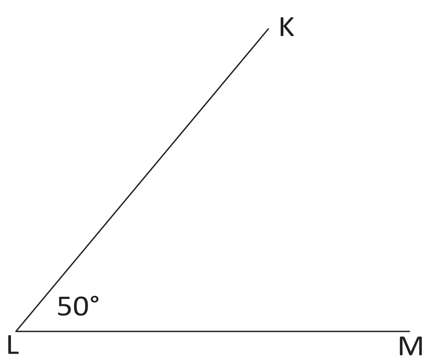
Step 3: Use a rulerto join the vertex to the point where the arcs intersect (D).

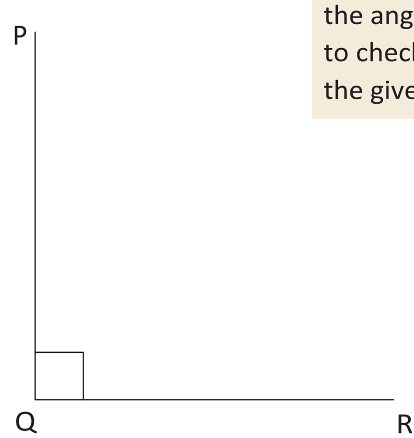
DB is the bisector of $A\hat{B}C$.



2. Copy the following angles and use your compass and ruler to bisect the angles.

You could measure each of the angles with a protractor to check if you have bisected the given angle correctly.



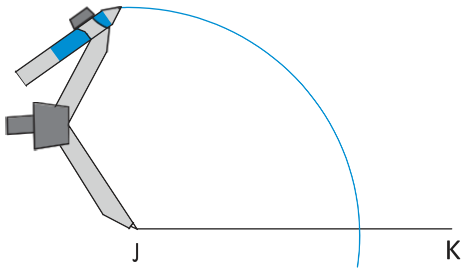


10.4 Constructing special angles without a protractor

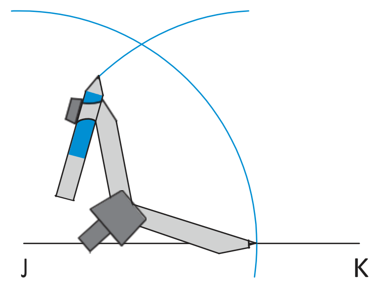
constructing angles of 60°, 30° and 120°

1. Read through the following steps:

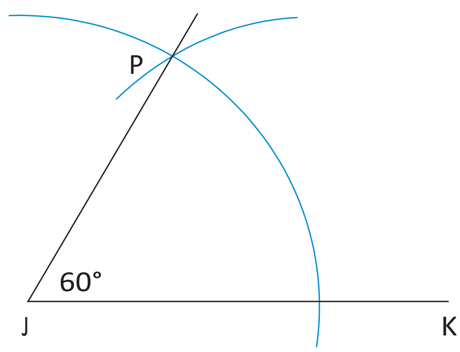
Step 1: Draw a line segment (JK). With the compass on point J, draw an arc across JK and up over above point J.



Step 2: Without changing the compass width, move the compass to the point where the arc crosses JK, and draw an arc that crosses the first one.



Step 3: Join point J to the point where the two arcs meet (point P). $P\hat{J}K = 60°$



2. (a) Copy the drawing at the top of page 99. Construct an angle of 60° at point B.
   (b) Bisect the angle you constructed.
   (c) Do you notice that the bisected angle consists of two 30° angles?
   (d) Extend line segment BC to A. Then measure the angle adjacent to the 60° angle. What is its size?
   (e) What do the 60° angle and its adjacent angle add up to?

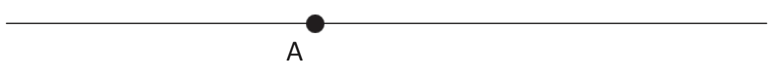
When you learn more about the properties of triangles later, you will understand why the method above creates a 60° angle. Or can you already work this out now? (Hint: What do you know about equilateral triangles?)

adjacent means "next to".

constructing angles of 90° and 45°

1. Construct an angle of 90° at point A. Go back to Section 10.2 if you need help.
2. Bisect the 90° angle to create an angle of 45°. Go back to Section 10.3 if you need help.

challenge
Try to construct the following angles without using a protractor: 150°, 210° and 135°.



10.5 Constructing triangles

In this section, you will learn how to construct triangles. You will need a pencil, a protractor, a ruler and a compass.

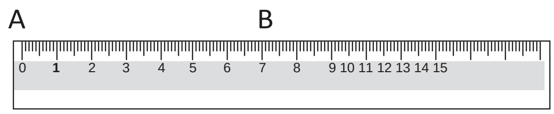
A triangle has three sides and three angles. We can construct a triangle when we know some of its measurements, that is, its sides, its angles, or some of its sides and angles.

constructing triangles

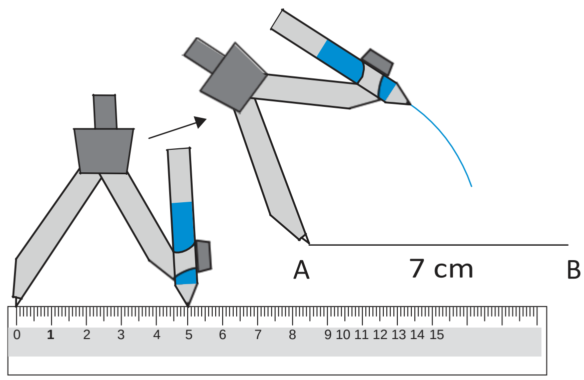
Constructing triangles when three sides are given

1. Read through the following steps. They describe how to construct ΔABC with side lengths of 3 cm, 5 cm and 7 cm.

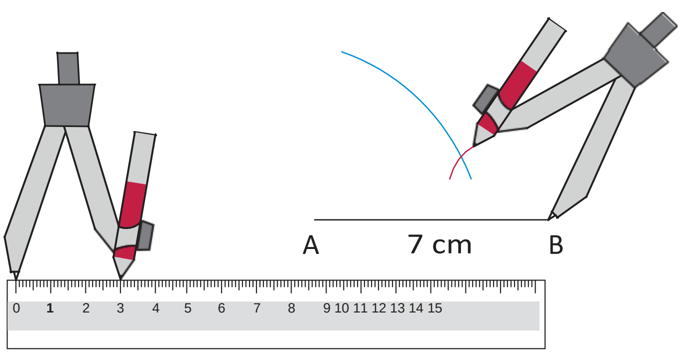
Step 1: Draw one side of the triangle using a ruler. It is often easier to start with the longest side.



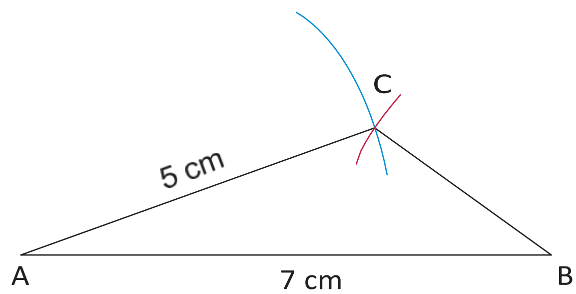
Step 2: Set the compass width to 5 cm. Draw an arc 5 cm away from point A. The third vertex of the triangle will be somewhere along this arc.



Step 3: Set the compass width to 3 cm. Draw an arc from point B. Note where this arc crosses the first arc. This will be the third vertex of the triangle.



Step 4: Use your ruler to join points A and B to the point where the arcs intersect (C).

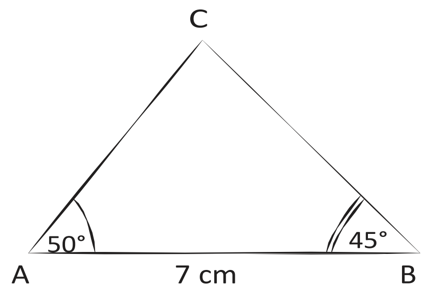


2. Follow the steps above to construct the following triangles:
   (a) ΔABC with sides 6 cm, 7 cm and 4 cm
   (b) ΔKLM with sides 10 cm, 5 cm and 8 cm
   (c) ΔPQR with sides 5 cm, 9 cm and 11 cm

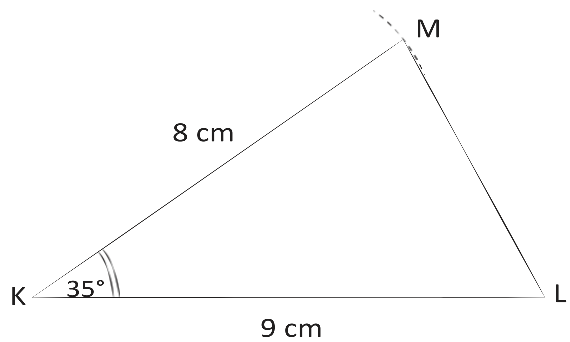
Constructing triangles when certain angles and sides are given

3. Use the rough sketches in (a) to (c) below to construct accurate triangles, using a ruler, compass and protractor. Make sure that:
   - the dotted lines show where you have to use a compass to measure the length of a side
   - you use a protractor to measure the size of the given angles.

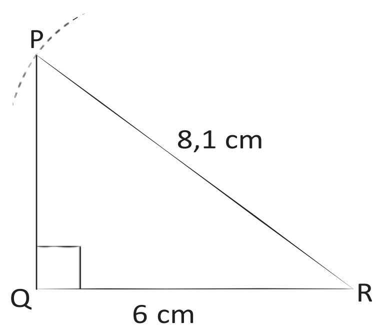
   (a) Construct ΔABC, with **two angles and one side given**.



   (b) Construct a ΔKLM, with **two sides and an angle given**.



   (c) Construct a right-angled ΔPQR, with the **hypotenuse and one other side given**.



4. Measure the missing angles and sides of each triangle in 3(a) to (c) on the previous page. Write the measurements next to your completed constructions.
5. Compare each of your constructed triangles in question 3(a) to (c) with a classmate's triangles. Are the triangles exactly the same?

If triangles are exactly the same, we say they are **congruent**.

challenge

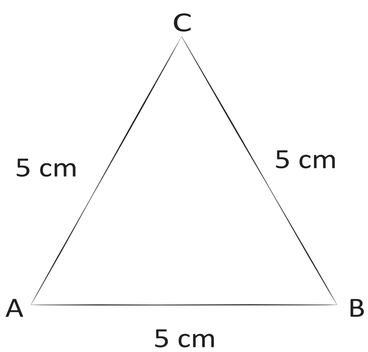
1. Construct these triangles:
   (a) ΔSTU, with three angles given: $\hat{S} = 45°$, $\hat{T} = 70°$ and $\hat{U} = 65°$.
   (b) ΔXYZ, with two sides and the angle opposite one of the sides given: $\hat{X} = 50°$, XY = 8 cm and XZ = 7 cm.
2. Can you find more than one solution for each triangle above? Explain your findings to a classmate.

10.6 Properties of triangles

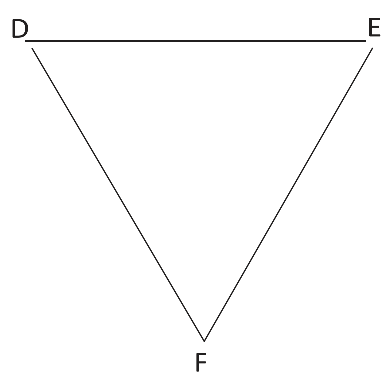
The angles of a triangle can be the same size or different sizes. The sides of a triangle can be the same length or differentlengths.

properties of equilateral triangles

1. (a) Construct ΔABC.
   (b) Measure and label the sizes of all its sides and angles.



2. Measure and write down the sizes of the sides and angles of ΔDEF on the right.



3. Both triangles in questions 1 and 2 are called **equilateral triangles**. Discuss with a classmate if the following is true for an equilateral triangle:
   - All the sides are equal.
   - All the angles are equal to 60°.

properties of isosceles triangles

1. (a) Construct ΔDEF with EF = 7 cm, $\hat{E}$ = 50° and $\hat{F}$ = 50°.

   Also construct ΔJKL with JK = 6 cm, KL = 6 cm and $\hat{J}$ = 70°.

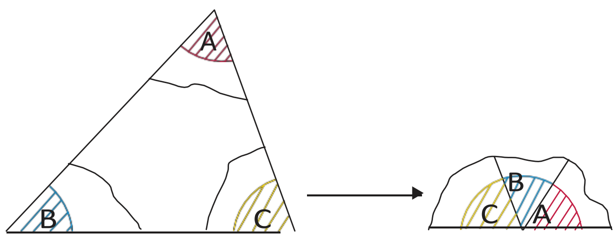
   (b) Measure and label all the sides and angles of each triangle.

2. Both triangles above are called **isosceles triangles**. Discuss with a classmate whether the following is true for an isosceles triangle:
   - Only two sides are equal.
   - Only two angles are equal.
   - The two equal angles are opposite the two equal sides.

the sum of the angles in a triangle

1. Look at your constructed triangles ΔABC, ΔDEF and ΔJKL above and on the previous page. What is the sum of the three angles each time?

2. Did you find that the sum of the interior angles of each triangle is 180°? Do the following to check if this is true for other triangles.



   (a) On a clean sheet of paper, construct any triangle. Label the angles A, B and C and cut out the triangle.

   (b) Neatly tear the angles off the triangle and fit them next to one another.

   (c) Notice that $\hat{A}$, $\hat{B}$ and $\hat{C}$ form a straight angle. Complete: $\hat{A} + \hat{B} + \hat{C}$ = ___°

We can conclude that the interior angles of a triangle always add up to 180°.

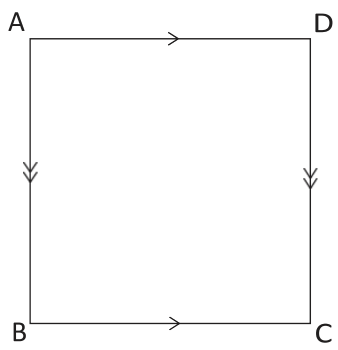
10.7 Properties of quadrilaterals

A quadrilateral is any closed shape with four straight sides. We classify quadrilaterals according to their sides and angles. We note which sides are parallel, perpendicular or equal. We also note which angles are equal.

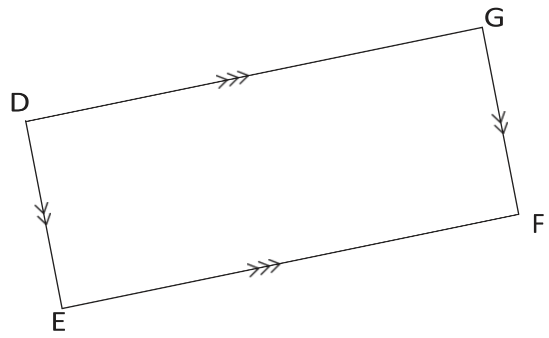
properties of quadrilaterals

1. Measure and write down the sizes of all the angles and the lengths of all the sides of each of the following quadrilaterals:

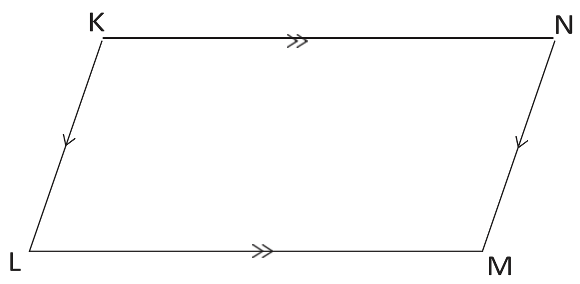
Square



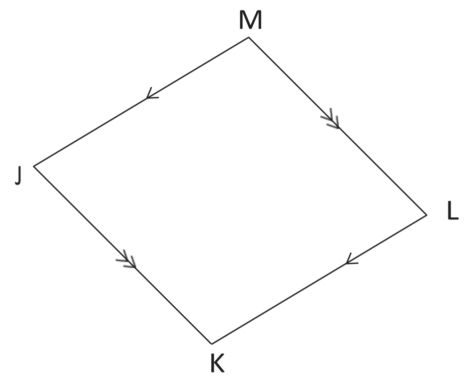
Rectangle



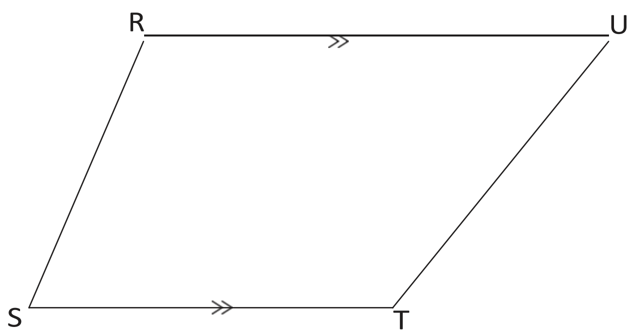
Parallelogram



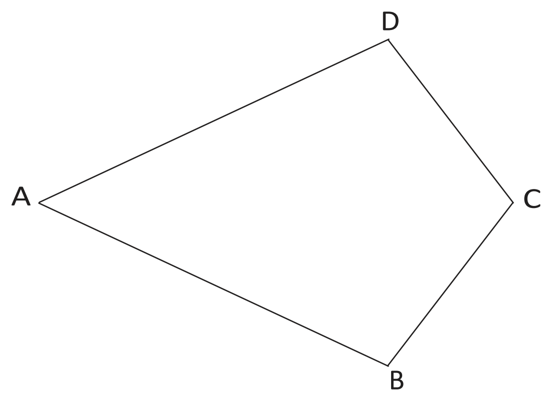
Rhombus



Trapezium



Kite



2. Use your answers in question 1. Copy the following table and place a ✓ in the correct box to show which property is correct for each shape.

Properties	Parallelogram	Rectangle	Rhombus	Square	Kite	Trapezium
Only one pair of sides are parallel						
Opposite sides are parallel						
Opposite sides are equal						
All sides are equal						
Two pairs of adjacent sides are equal						
Opposite angles are equal						
All angles are equal						

sum of the angles in a quadrilateral

1. Add up the four angles of each quadrilateral on the previous page. What do you notice about the sum of the angles of each quadrilateral?
2. Did you find that the sum of the interior angles of each quadrilateral equals 360°? Do the following to check if this is true for other quadrilaterals:
   (a) On a clean sheet of paper, use a ruler to construct any quadrilateral.
   (b) Label the angles A, B, C and D. Cut out the quadrilateral.
   (c) Neatly tear the angles off the quadrilateral and fit them next to one another.
   (d) What do you notice?

We can conclude that the interior angles of a quadrilateral always add up to 360°.

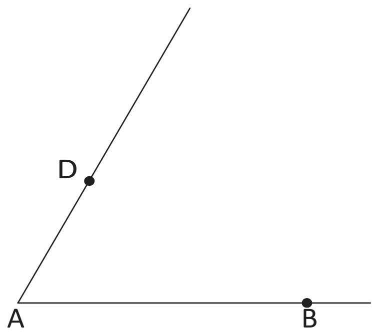
10.8 Constructing quadrilaterals

You learnt how to construct perpendicular lines in Section 10.2. If you know how to construct parallel lines, you should be able to construct any quadrilateral accurately.

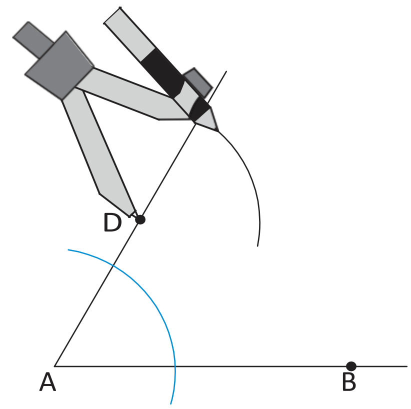
constructing parallel lines to draw quadrilaterals

1. Read through the following steps:

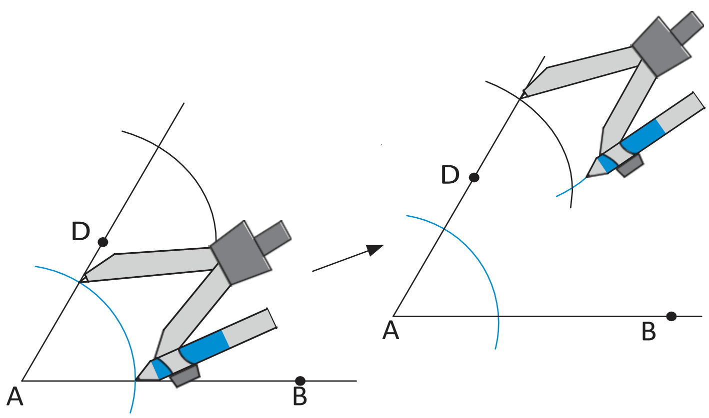
Step 1: From line segment AB, mark a point D. This point D will be on the line that will be parallel to AB. Draw a line from A through D.



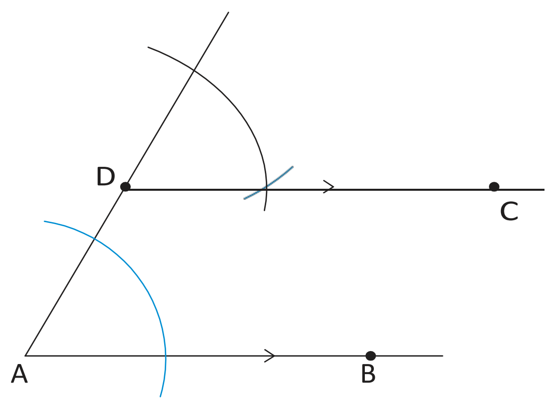
Step 2: Draw an arc from A that crosses AD and AB. Keep the same compass width and draw an arc from point D as shown.



Step 3: Set the compass width to the distance between the two points where the first arc crosses AD and AB. From the point where the second arc crosses AD, draw a third arc to cross the second arc.



Step 4: Draw a line from D through the point where the two arcs meet. DC is parallel to AB.



2. Practise drawing a parallelogram, square and rhombus.
3. Use a protractor to draw quadrilaterals with at least one set of parallel lines.

Worksheet

1. Do the followingconstruction:

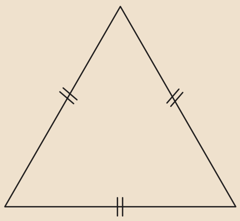
   (a) Use a compass and ruler to construct equilateral ΔABC with sides of 9 cm.

   (b) Without using a protractor, bisect $\hat{B}$. Let the bisector intersect AC at point D.

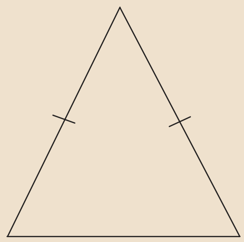
   (c) Use a protractor to measure $A\hat{D}B$. Write the measurement on the drawing.

2. Name the following types of triangles and quadrilaterals:

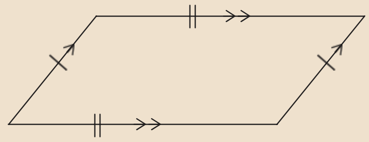
A



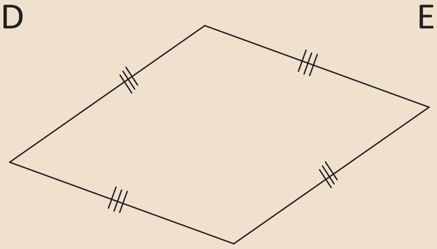
B



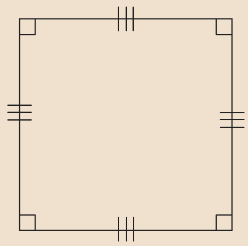
C



D



E



F

3. Which of the following quadrilaterals matches each description below? (There may be more than one answer for each.)

parallelogram; rectangle; rhombus; square; kite; trapezium

   (a) All sides are equal and all angles are equal.

   (b) Two pairs of adjacent sides are equal.

   (c) Onepairofsidesisparallel.

   (d) Opposite sides are parallel.

   (e) Opposite sides are parallel and all angles are equal.

   (f) All sides areequal.

Chapter 11
Geometry of 2D shapes

11.1 Types of triangles

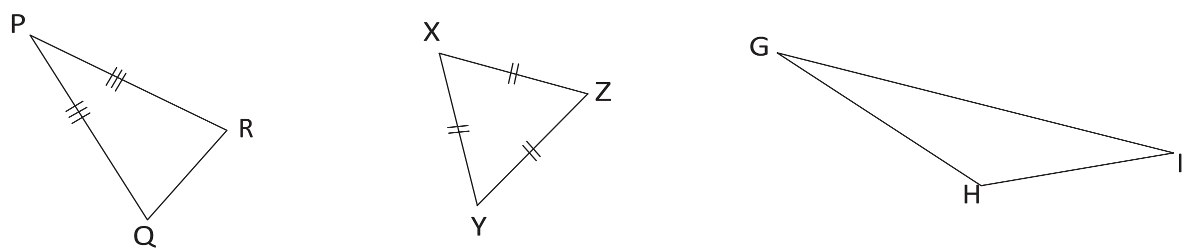
By now, you know that a triangle is a closed 2D shape with three straight sides. We can classify or name different types of triangles according to the lengths of their sides and according to the sizes of theirangles.

naming triangles according to their sides

1. Copy the table and match the name of each type of triangle with its correct description:

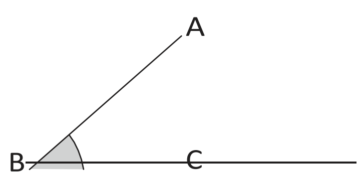
Name of triangle	Description of triangle
Isosceles triangle	All the sides of a triangle are equal.
Scalene triangle	None of the sides of a triangle are equal.
Equilateral triangle	Two sides of a triangle are equal.

2. Name each type of triangle by looking at its sides:

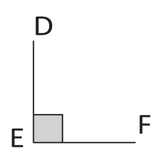


naming triangles according to their angles

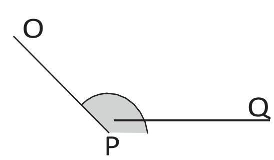
Remember the following types of angles:



Acute angle
(< 90°)

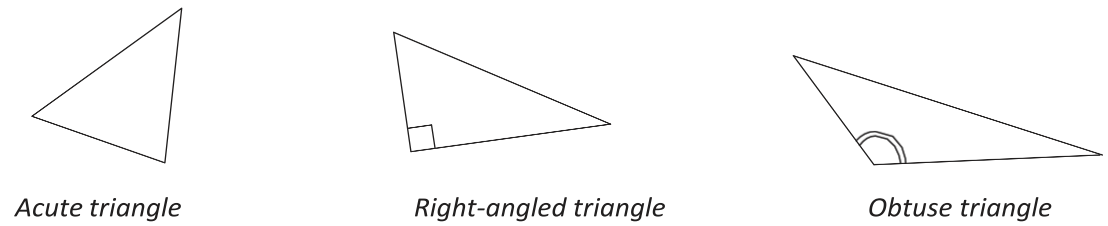


Right angle
(= 90°)



Obtuse angle
(between 90° and 180°)

Study the following triangles and then answer the questions:



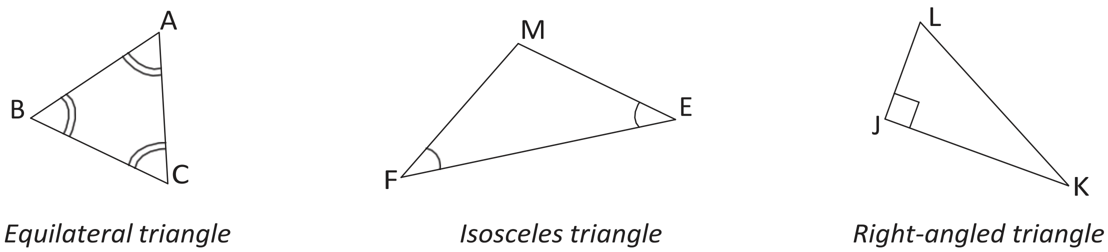
1. Are all the angles of a triangle always equal?
2. When a triangle has an obtuse angle, what kind of triangle is it called?
3. When a triangle has only acute angles, what kind of triangle is it called?
4. What size must one of the angles of a triangle be for it to be called a right-angled triangle?

investigating the angles and sides of triangles

1. (a) What is the sum of the interior angles of a triangle?
   (b) Can a triangle have two right angles? Explain your answer.
   (c) Can a triangle have more than one obtuse angle? Explain your answer.

If you cannot work out the answers in question 1(b) and (c), try to construct the triangles to find the answers.

2. Look at the triangles below. The arcs show which angles are equal.



   (a) ΔABC is an equilateral triangle. What do you notice about its angles?
   (b) ΔFEM is an isosceles triangle. What do you notice about its angles?
   (c) ΔJKL is a right-angled triangle. Is its longest side opposite the 90° angle?
   (d) Construct any three right-angled triangles on a sheet of paper. Is the longest side always opposite the 90° angle?

Properties of triangles:

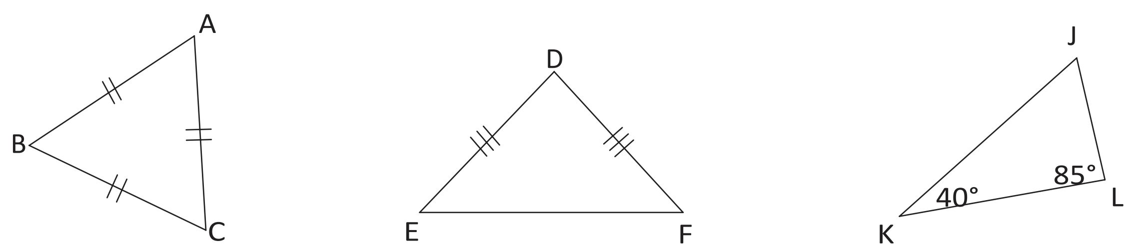
- The **sum of the interior angles** of a triangle is 180°.
- An **equilateral triangle** has all sides equal and each interior angle is equal to 60°.
- An **isosceles triangle** has two equal sides and the angles opposite the equal sides are equal.
- A **scalene triangle** has no sides equal.
- A **right-angled triangle** has a right angle (90°).
- An **obtuse triangle** has one obtuse angle (between 90° and 180°).
- An **acute triangle** has three acute angles (< 90°).

interior angles are the angles inside a closed shape; not the angles outside of it.

11.2 Unknown angles and sides of triangles

You can use what you know about triangles to obtain other information. When you work out new information, you must always give reasons for the statements you make.

Look at the following examples of working out unknown angles and sides when certain information is given. The reason for each statement is written in square brackets.



$\hat{A} = \hat{B} = \hat{C} = 60°$	[Angles in an equilateral Δ = 60°]
DE = DF	[Given]
$\hat{E} = \hat{F}$	[Angles opposite the equal sides of an isosceles Δ are equal]
$\hat{J} = 55°$	[The sum of the interior angles of a Δ = 180°; so $\hat{J} = 180° - 40° - 85°$]

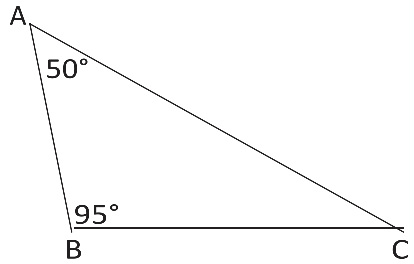
You can shorten the following reasons in the ways shown:

- Sum of interior angles (∠s) of a triangle (Δ) = 180°: **Interior ∠s of Δ**
- Isosceles triangle has two sides and two angles equal: **Isosceles Δ**
- Equilateral triangle has three sides and three angles equal: **Equilateral Δ**
- Angles forming a straight line = 180°: **Straightline**

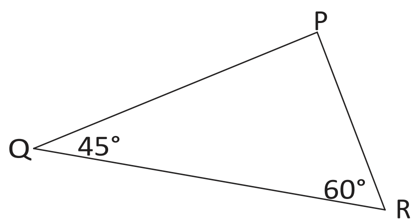
working out unknown angles and sides

Find the sizes of unknown angles and sides in the following triangles. Always give reasons for every statement.

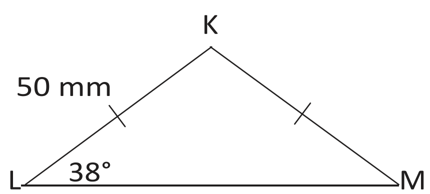
1. Find Ĉ.



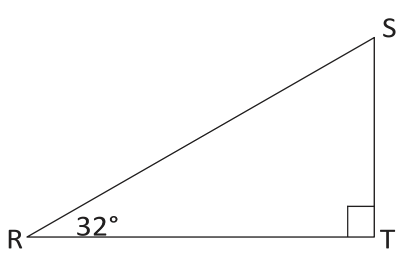
2. Find P̂.



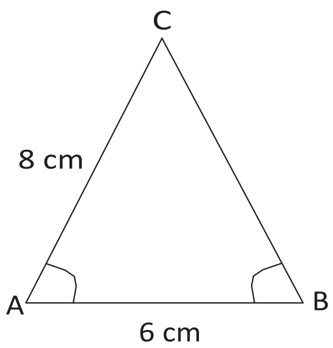
3. (a) Find KM.
   (b) Find K̂.



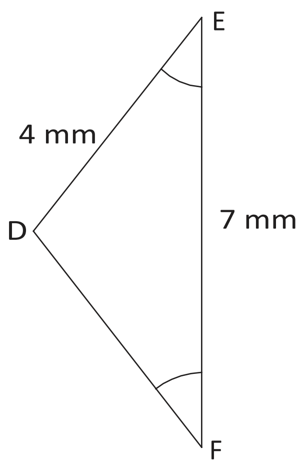
4. What is the size of Ŝ?



5. (a) Find CB.
   (b) Find $\hat{C}$ if $\hat{A}$ = 50°.

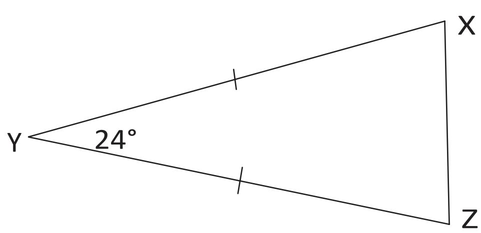


6. (a) Find DF.
   (b) Find $\hat{E}$ if $\hat{D}$ = 100°.

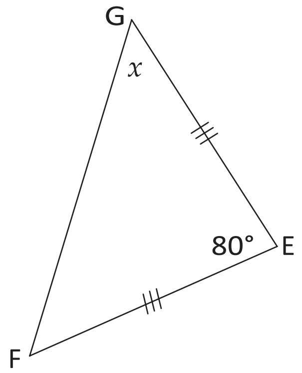


working out more unknown angles and sides

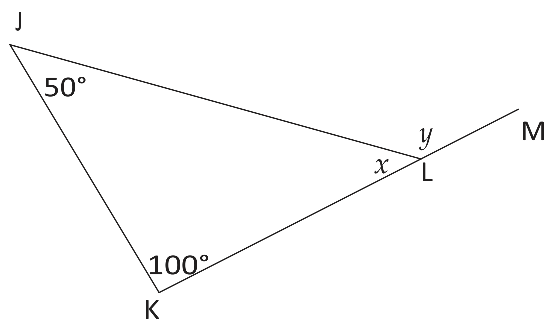
1. Calculate the size of $\hat{X}$ and $\hat{Z}$.



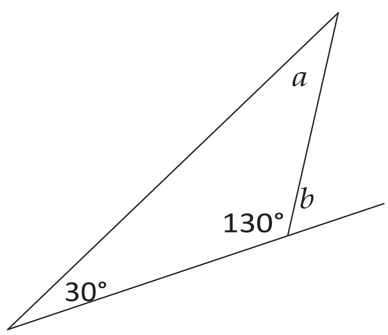
2. Calculate the size of x.



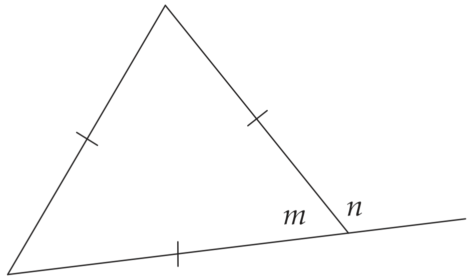
3. KLM is a straight line. Calculate the size of x and y.



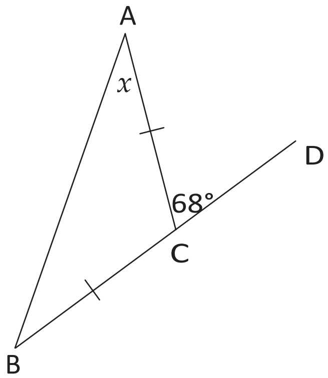
4. Angle b and an angle with a size of 130° form a straight angle. Calculate the size of a and b.



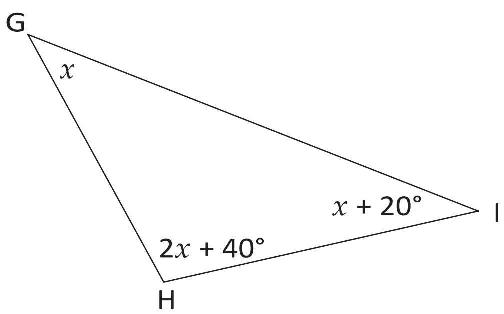
5. Angle m and n form a straight angle. Calculate the size of m and n.



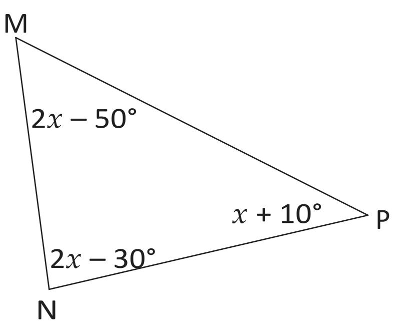
6. BCD is a straight line segment. Calculate the size of x.



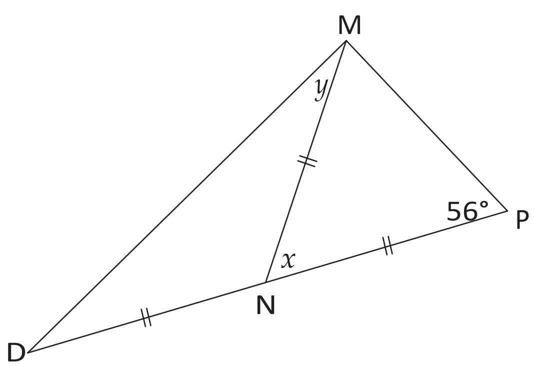
7. Calculate the size of x and then the size of $\hat{H}$.



8. Calculate the size of $\hat{N}$.

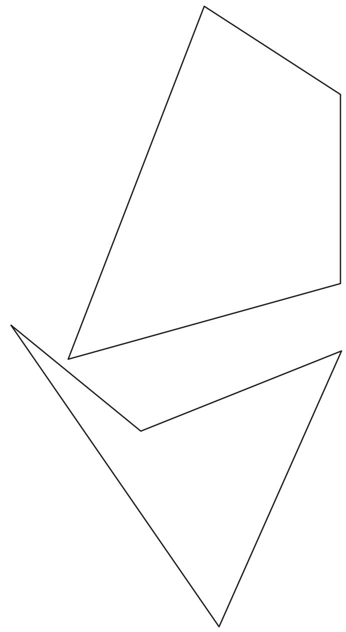


9. DNP is a straight line. Calculate the size of x and y.



11.3 Types of quadrilaterals and their properties

A quadrilateral is a figure with four straight sides which meet at four vertices. In many quadrilaterals all the sides are of different lengths and all the angles are of different sizes.

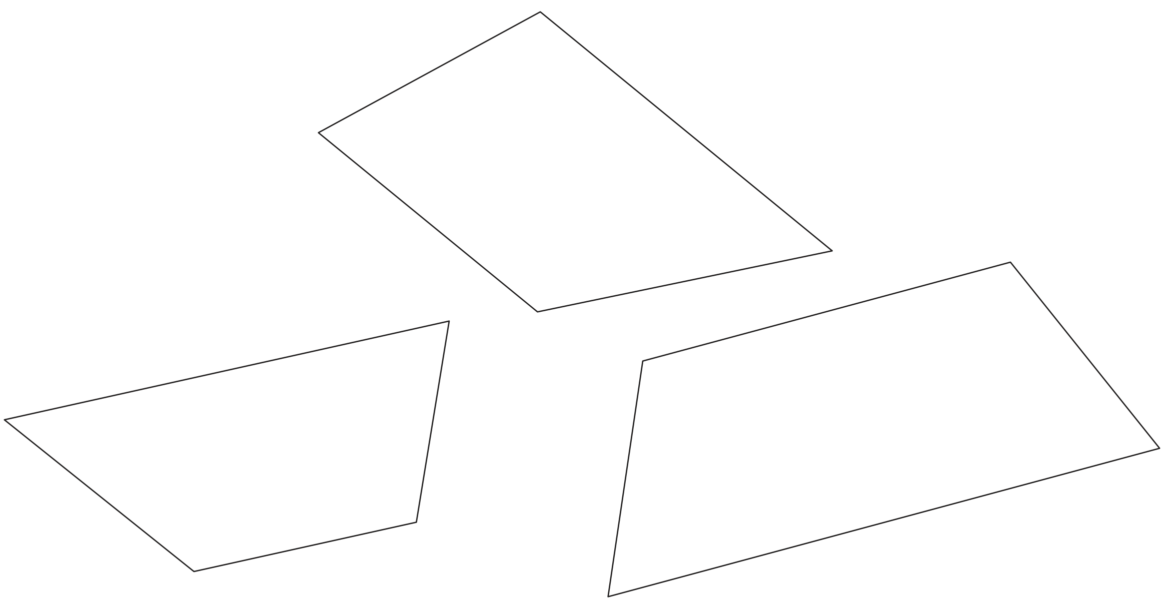


> You have previously worked with these types of quadrilaterals, in which some sides have the same lengths, and some angles may be of the same size:
> parallelograms
> rectangles
> kites
> rhombuses
> squares
> trapeziums

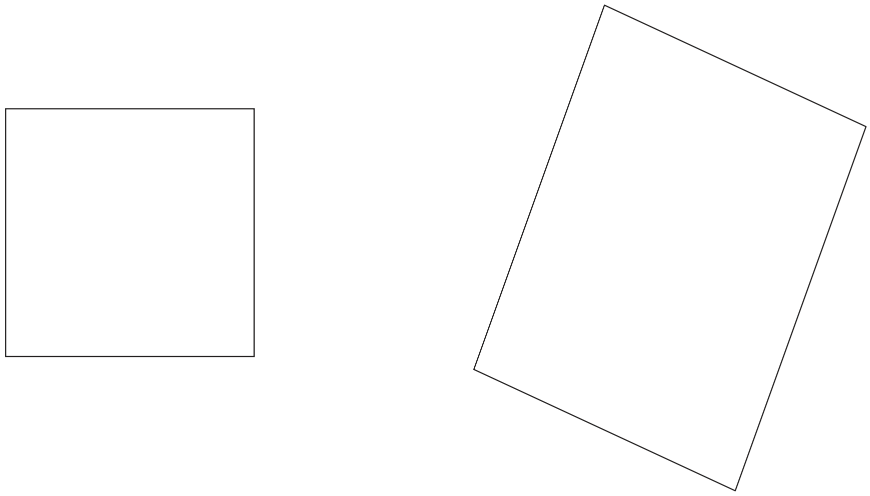
the properties of different types of quadrilaterals

1. In each of the following questions, different examples of a certain type of quadrilateral are given. In each case identify which kind of quadrilateral it is. Describe the properties of each type by making statements about the lengths and directions of the sides and the sizes of the angles of each type. You may have to take some measurements to be able to do this.

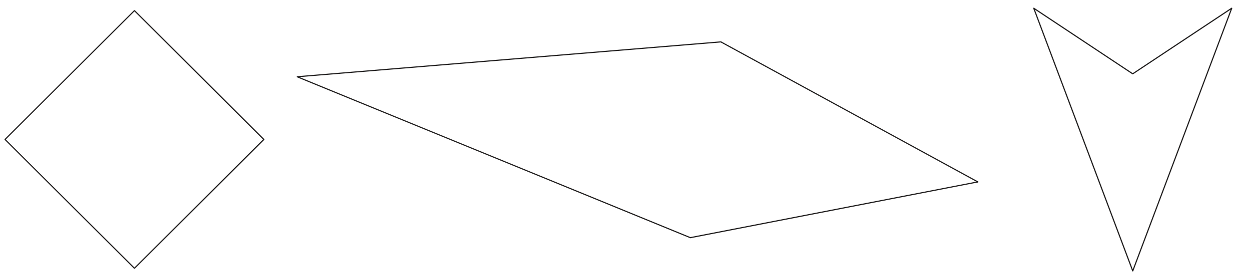
(a)



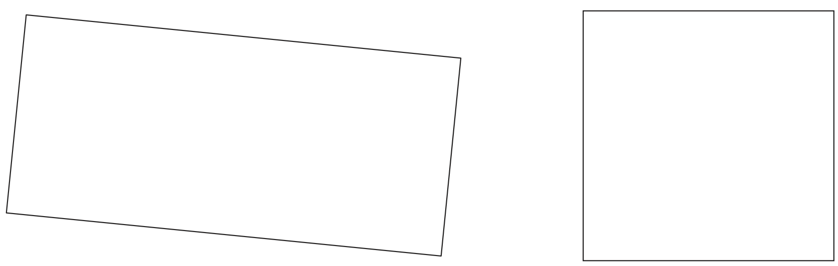
(b)



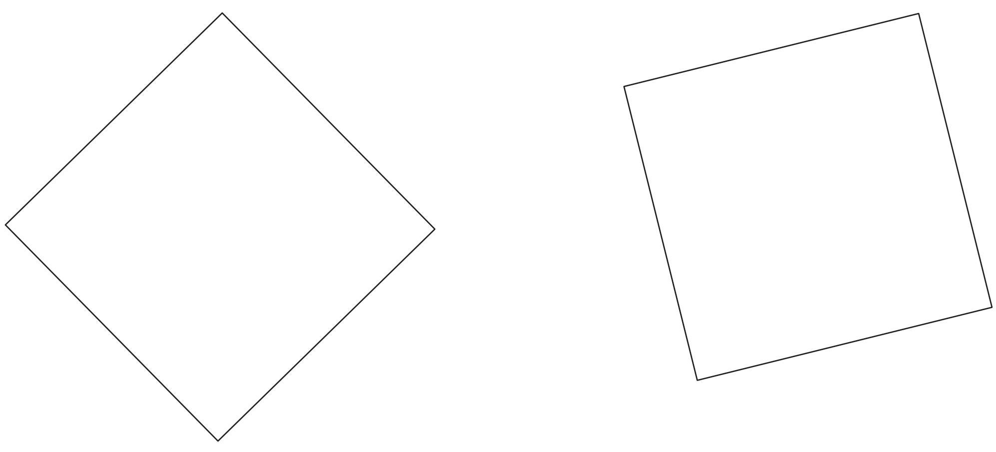
(c)



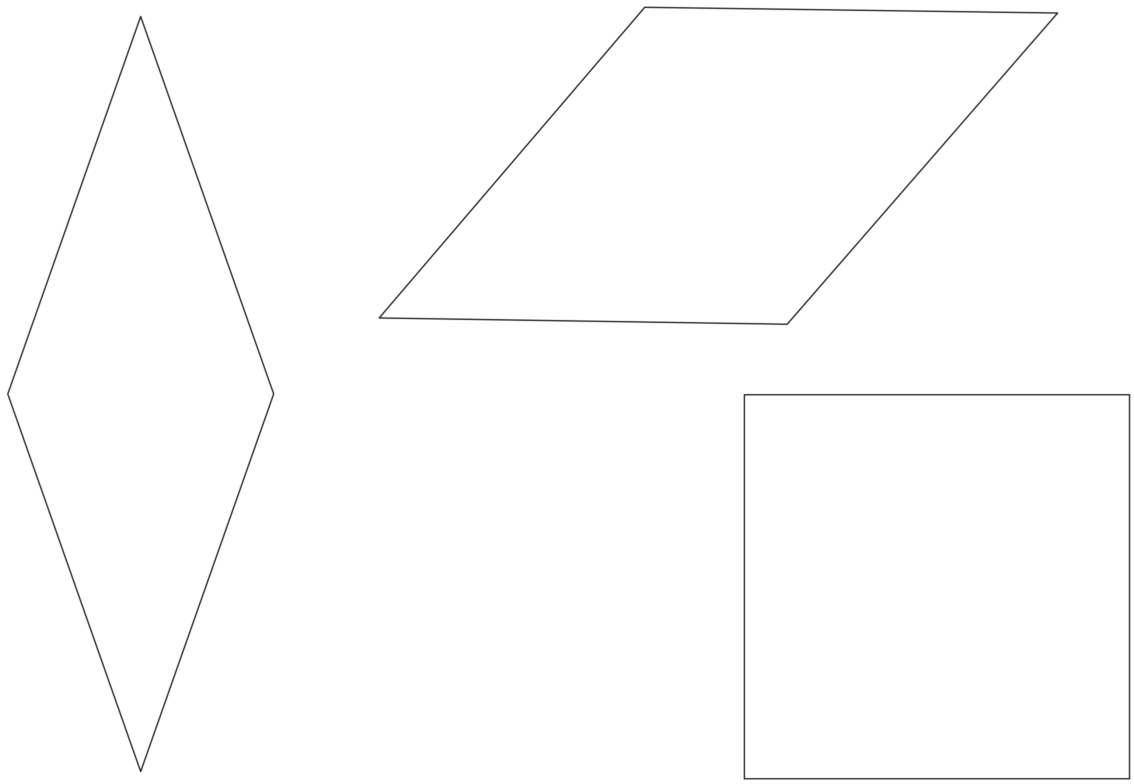
(d)



(e)



(f)



2. Use your completed lists and the drawings in question 1 to determine if the following statements are true or false:

(a) A rectangle is a parallelogram.
(b) A square is a parallelogram.
(c) A rhombus is a parallelogram.
(d) A kite is a parallelogram.
(e) A trapezium is a parallelogram.
(f) A square is a rhombus.
(g) A square is a rectangle.
(h) A square is akite.
(i) A rhombus is a kite.
(j) A rectangle is a rhombus.
(k) A rectangle is a square.

If a quadrilateral has *all* the properties of another quadrilateral, you can define it in terms of the other quadrilateral, as you have found above.

A **convention** is something (such as a definition or method) that most people agree on, accept and follow.

3. Here are some conventional definitions of quadrilaterals:
   - A **parallelogram** is a quadrilateral with two opposite sides parallel.
   - A **rectangle** is a parallelogram that has all four angles equal to 90°.
   - A **rhombus** is a parallelogram with all four sides equal.
   - A **square** is a rectangle with all four sides equal.
   - A **trapezium** is a quadrilateral with one pair of opposite sides parallel.
   - A **kite** is a quadrilateral with two pairs of adjacent sides equal.

Write down other definitions that work for the following quadrilaterals:

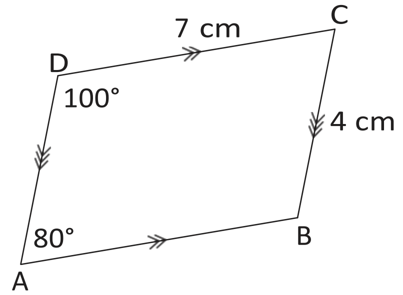
(a) Rectangle (b) Square (c) Rhombus
(d) Kite (e) Trapezium

11.4 Unknown angles and sides of quadrilaterals

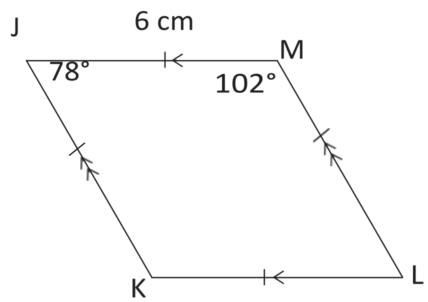
finding unknown angles and sides

Find the length of all the **unknown sides** and **angles** in the following quadrilaterals. Give reasons to justify your statements. (Also recall that the sum of the angles of a quadrilateral is 360°.)

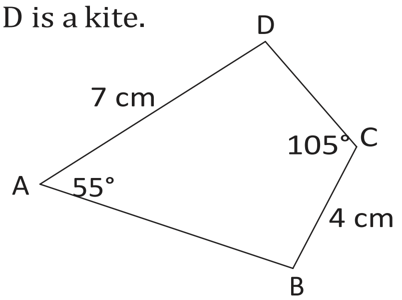
1.



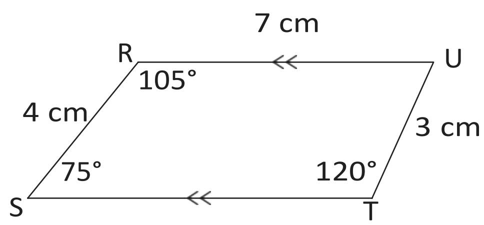
2.



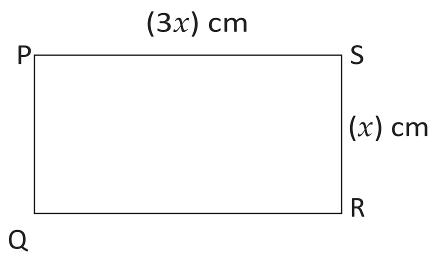
3. ABCD is a kite.



4. The perimeter of RSTU is 23 cm.



5. PQRS is a rectangle and has a perimeter of 40 cm.



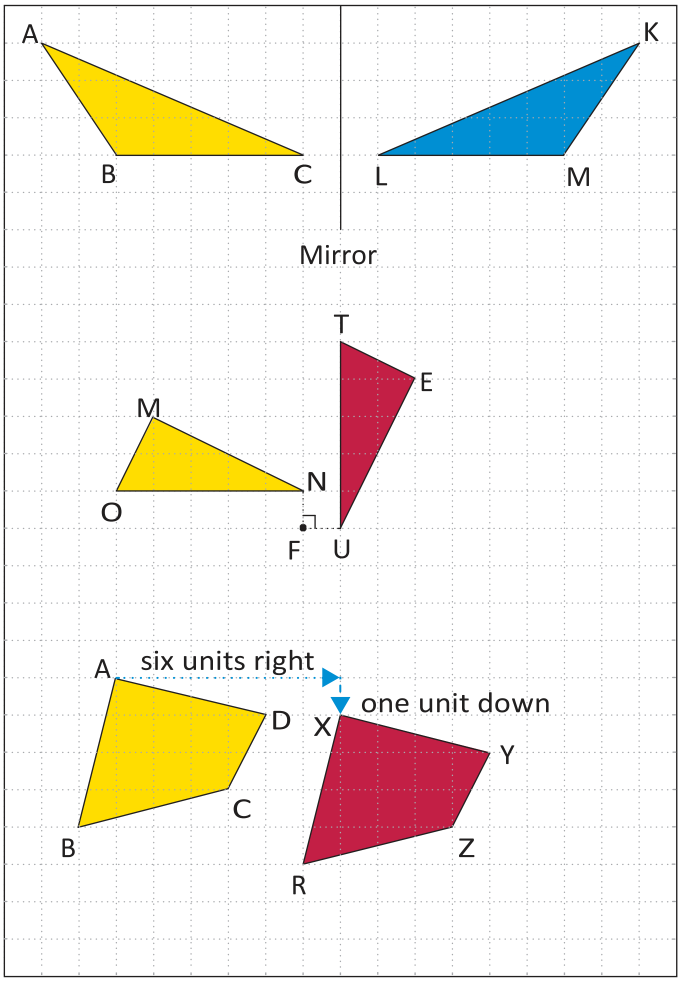
11.5 Congruency

what is congruency?

1. ΔABC is reflected in the vertical line (mirror) to give ΔKLM.
   Are the sizes and shapes of the two triangles exactly the same?

2. ΔMON is rotated 90° around point F to give you ΔTUE.
   Are the sizes and shapes of ΔMON and ΔTUE exactly the same?

3. Quadrilateral ABCD is translated six units to the right and one unit down to give quadrilateral XRZY.
   Are ABCD and XRZY exactly the same?



In the previous activity, each of the figures was transformed (reflected, rotated or translated) to produce a second figure. The second figure in each pair has **the same angles**, **side lengths**, **size and area** as the first figure. The second figure is therefore an **accurate copy** of the first figure.

When one figure is an image of another figure, we say that the two figures are **congruent**.

The symbol for congruency is: ≡

The word **congruent** comes from the Latinword*congruere*, which means "to agree". Figures are congruent if they match up perfectly when laid on top of each other.

Notation of congruent figures

When we name shapes that are congruent, we name them so that the matching, or corresponding, angles are in the same order. For example, in ∆**ABC** and ∆**KLM** on the previous page:

$\hat{A}$ is congruent to (matches and is equal to) $\hat{K}$

$\hat{B}$ is congruent to $\hat{M}$.

$\hat{C}$ is congruent to $\hat{L}$.

We therefore use this notation: ∆**ABC** ≡ ∆**KML**.

Similarly, for the other pairs of figures on the previous page:

∆**MON** ≡ ∆**TUE** and **ABCD** ≡ **XRZY**.

We cannot assume that, when the angles of polygons are equal, the polygons are congruent. You will learn about the conditions of congruence in Grade 9.

The notation of congruent figures also shows which sides of the two figures correspond and are equal. For example, ∆ABC ≡ ∆KML shows that:

AB = KM, BC = ML and AC = KL.

The incorrect notation ∆ABC ≡ ∆KLM would show the following incorrect information:

$\hat{B} = \hat{L}$, $\hat{C} = \hat{M}$, AB = KL and AC = KM.

identifying congruent angles and sides

Copy the following table and write down which angles and sides are equal between each pair of congruent figures:

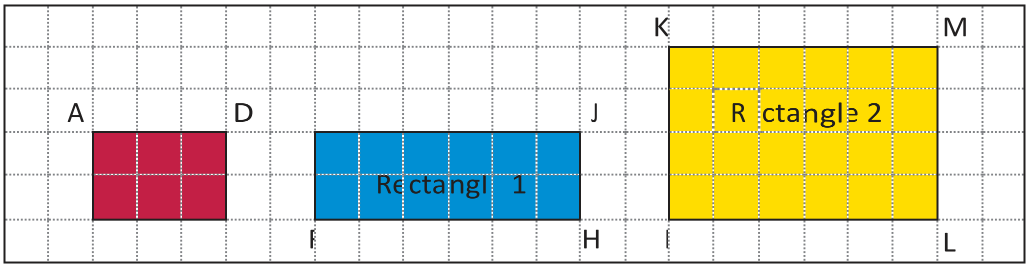
1. ∆PQR ≡∆UCT	2. ∆KLM ≡ ∆UWC
3. ∆GHI ≡ ∆QRT	4. ∆KJL ≡ ∆POQ

11.6 Similarity

In Grade 7, you learnt that two figures are **similar** when they have the **same shape** (their angles are equal) but they may be **different sizes**. The sides of one figure are proportionally longer or shorter than the sides of the other figure; that is, the length of each side is multiplied or divided by the same number. We say that one figure is an enlargement or a reduction of the other figure.

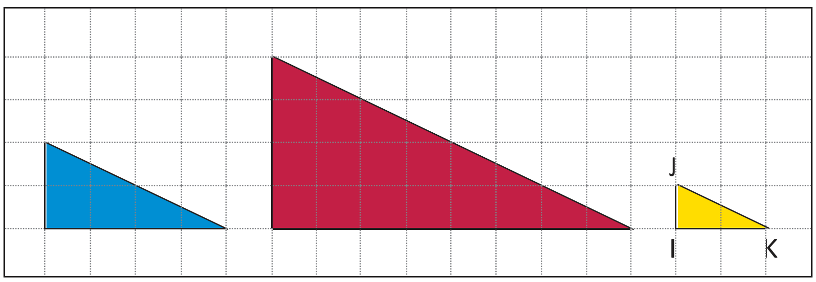
checking for similarity

1. Look at the rectangles below and answer the questions that follow:



   (a) Look at rectangle 1 and ABCD. How many times is FH longer than BC? How many times is GF longer than AB?
   (b) Look at rectangle 2 and ABCD. How many times is IL longer than BC? How many times is LM longer than CD?
   (c) Is rectangle 1 or rectangle 2 an enlargement of rectangle ABCD? Explain your answer.

2. Look at the triangles below and answer the questions that follow:



   (a) How many times is:
      - FG longer than BC?
      - HG longer than AC?
      - JI shorter than AB?
      - HF longer than AB?
      - IK shorter than BC?
      - JK shorter than AC?
   (b) Is ΔHFG an enlargement of ΔABC? Explain your answer.
   (c) Is ΔJIK a reduction of ΔABC? Explain your answer.

In the previous activity, rectangle KILM is an enlargement of rectangle ABCD. Therefore, ABCD is similar to KILM. The symbol for "is similar to" is: ///. So we write: ABCD /// KILM.

The triangles on the previous page are also similar. ΔHFG is an enlargement of ΔABC, and ΔJIK is a reduction of ΔABC.

In ΔABC and ΔHFG, $\hat{A} = \hat{H}$, $\hat{B} = \hat{F}$ and $\hat{C} = \hat{G}$. We write it like this: ΔABC /// ΔHFG.

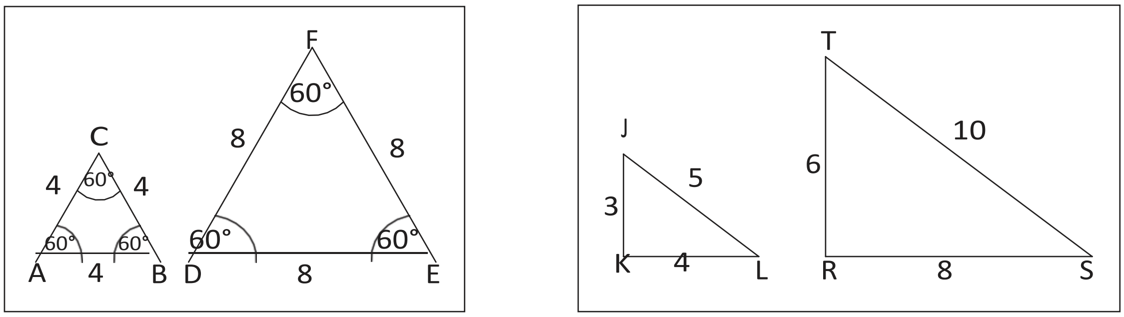
In the same way, ΔABC /// ΔJIK.

When you enlarge or reduce a polygon, you need to enlarge or reduce all its sides proportionally, or by the same ratio. This means that you multiply or divide each length by the same number.

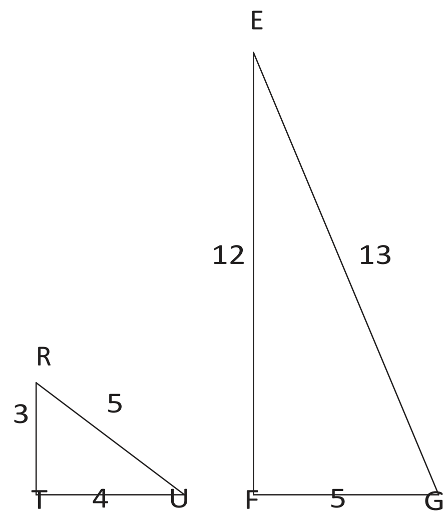
Similar figures are figures that have the same angles (same shape) but are not necessarily the same size.

using properties of similar and congruent figures

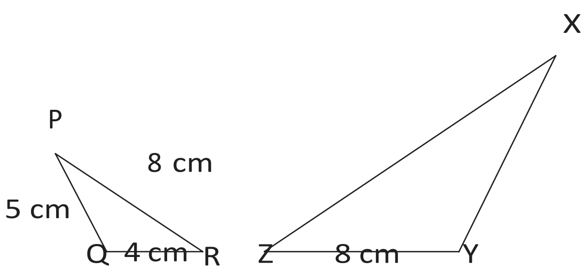
1. Are the triangles in each pair similar? Give a reason for each answer.



2. Is ΔRTU ///ΔEFG? Give a reason for your answer.



3. ΔPQR ///ΔXYZ. Determine the length of XZ and XY.



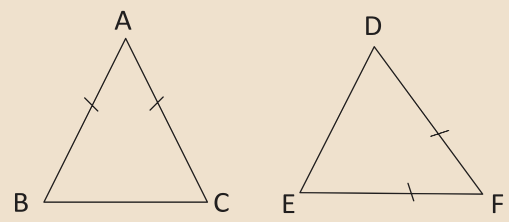
4. Are the following statements true or false? Explain your answers.
   (a) Figures that are congruent are similar.
   (b) Figuresthat aresimilar arecongruent.
   (c) All rectangles are similar.
   (d) All squares are similar.

Worksheet

1. Study the triangles below and answer the following questions:

   (a) Choose the correct answer and write it down.
   ΔABC is:

   ☐ acute and equilateral
   ☐ obtuse and scalene
   ☐ acute and isosceles
   ☐ right-angledandisosceles



   (b) If AB = 40 mm, what is the length of AC?

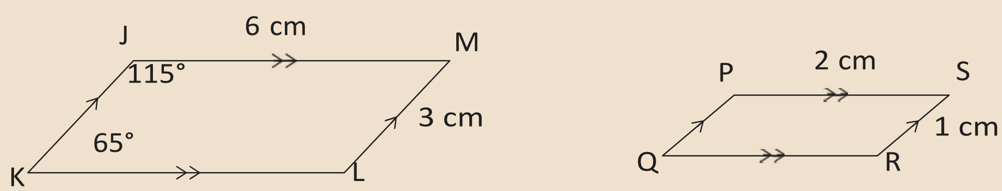
   (c) If $\hat{B}$ = 80°, what is the size of $\hat{C}$ and of $\hat{A}$?

   (d) ΔABC ≡ ΔFDE. Name all the sides in the two triangles that are equal to AB.

   (e) Name the side that is equal to DE.

   (f) If $\hat{F}$ is 40°, what is the size of $\hat{B}$?

2. Look at figures JKLM and PQRS. (Give reasons for your answers below.)



   (a) What type of quadrilateral is JKLM?

   (b) Is JKLM///PQRS?

   (c) What is the size of $\hat{L}$?

   (d) What is the size of $\hat{S}$?

   (e) What is the length of KL?

Chapter 12

Geometry of straight lines

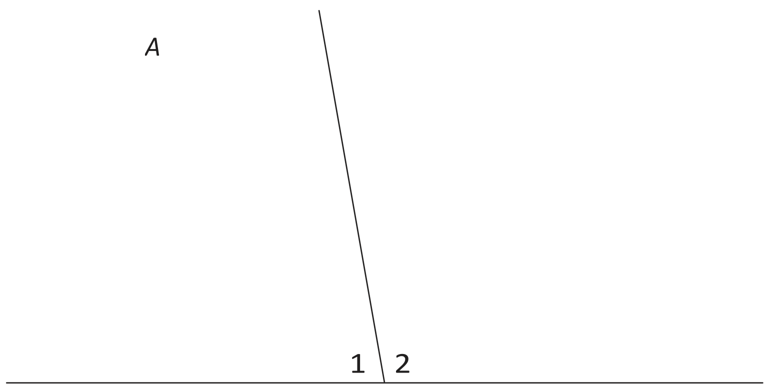
12.1 Angles on a straight line

sum of angles on a straight line

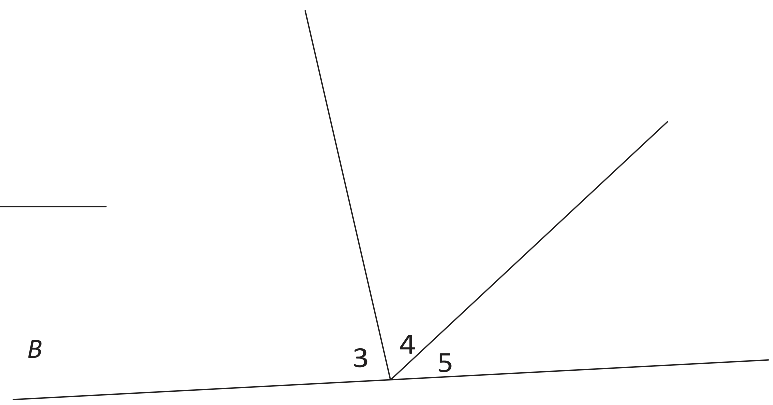
In the figures below, each angle is given a label from 1 to 5.

1. Use a protractor to measure the sizes of all the angles in each figure. Write down your answers.

A



B

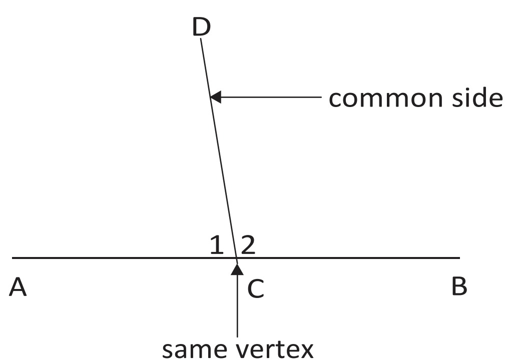


2. Use your answers to determine the following sums:

   (a) $\hat{1} + \hat{2}$ (b) $\hat{3} + \hat{4} + \hat{5}$

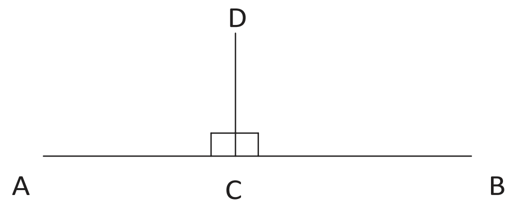
The sum of angles that are formed on a straight line is equal to 180°. (We can shorten this property as: ∠s on a straight line.)

Two angles whose sizes add up to 180° are also called **supplementary** angles, for example $\hat{1} + \hat{2}$. Angles that share a vertex and a common side are said to be **adjacent.** So $\hat{1} + \hat{2}$ are therefore also called **supplementary adjacent angles**.



When two lines are perpendicular, their adjacent supplementary angles are each equal to 90°.

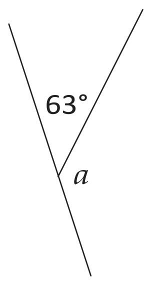
In the drawing alongside, D$\hat{C}$A and D$\hat{C}$B are adjacent supplementary angles because they are next to each other (adjacent) and they add up to 180° (supplementary).



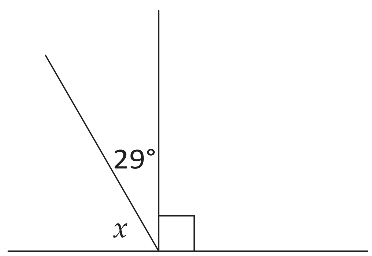
finding unknown angles on straight lines

Work out the sizes of the unknown angles below. Build an equation each time as you solve these geometric problems. Always give a reason for every statement you make.

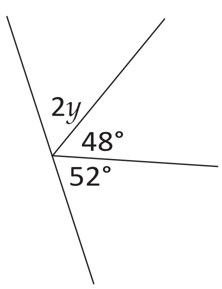
1. Calculate the size of a.



2. Calculate the size of x.

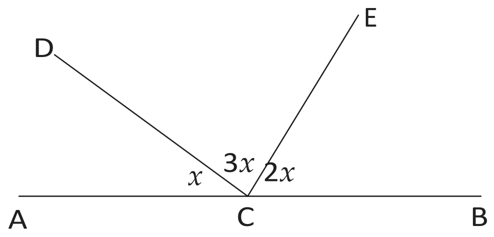


3. Calculate the size of y.

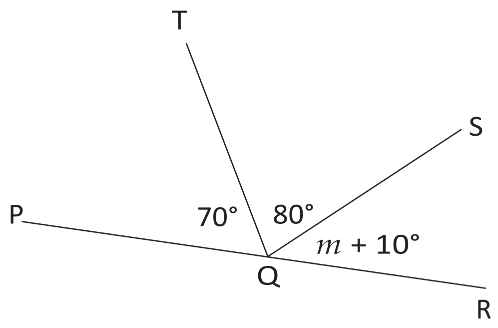


finding more unknown angles on straight lines

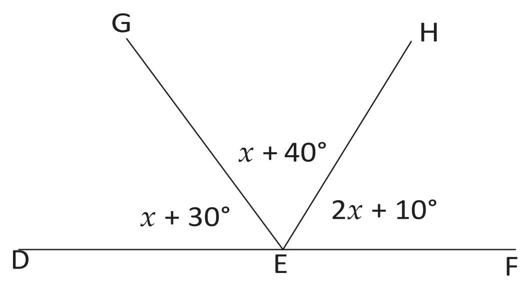
1. Calculate the size of:
   (a) x
   (b) $E\hat{C}B$



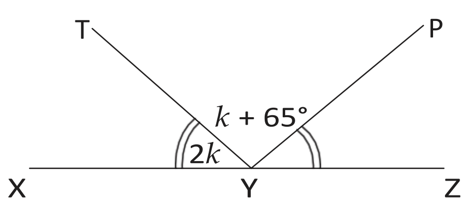
2. Calculate the size of:
   (a) m
   (b) $S\hat{Q}R$



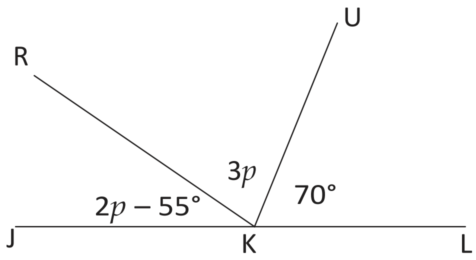
3. Calculate the size of:
   (a) x
   (b) $H\hat{E}F$



4. Calculate the size of:
   (a) k
   (b) $T\hat{Y}P$



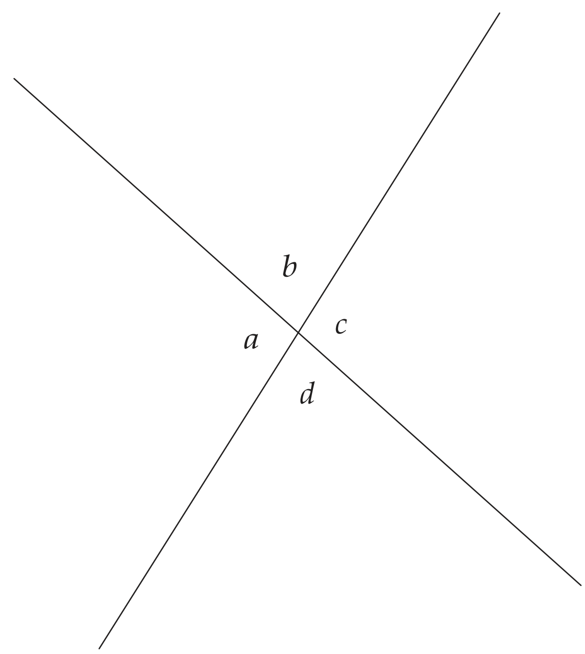
5. Calculate the size of:
   (a) p
   (b) $J\hat{K}R$



12.2 Vertically opposite angles

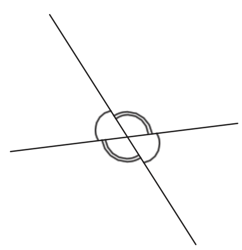
what are vertically opposite angles?

1. Use a protractor to measure thesizes of all the angles in the figure. Write down your answers.



2. Notice which angles are equal and how these equal angles are formed.

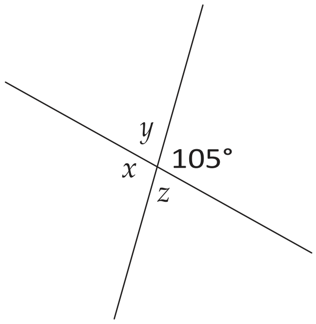
Vertically opposite angles (**vert. opp. ∠s**) are the angles opposite each other when two lines intersect.
Vertically opposite angles are **always equal**.



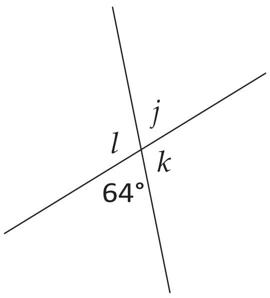
finding unknown angles

Calculate the sizes of the unknown angles in the following figures. Always give a reason for every statement you make.

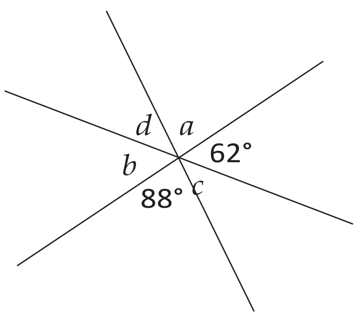
1. Calculate x, y and z.



2. Calculate j, k and l.



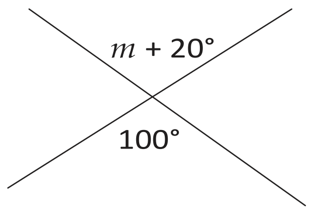
3. Calculate a, b, c and d.



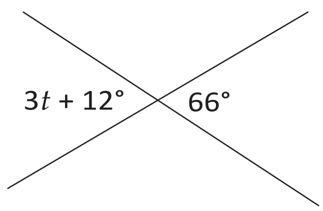
equations using vertically opposite angles

Vertically opposite angles are always equal. We can use this property to build an equation. Then we solve the equation to find the value of the unknown variable.

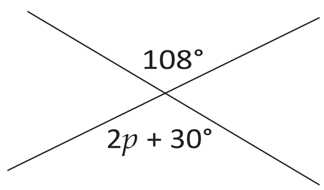
1. Calculate the value of m.



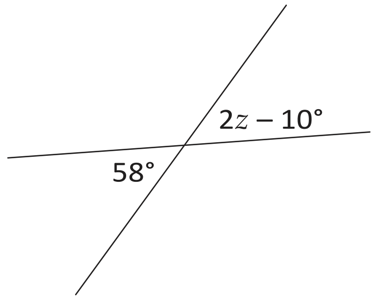
2. Calculate the value of t.



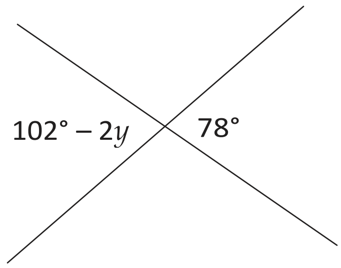
3. Calculate the value of p.



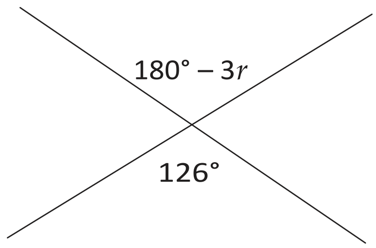
4. Calculate the value of z.



5. Calculate the value of y.



6. Calculate the value of r.

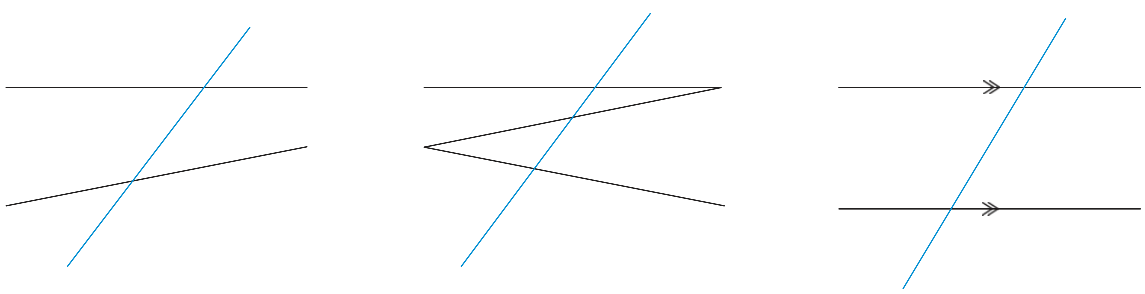


12.3 Lines intersected by a transversal

pairs of angles formed by a transversal

A **transversal** is a line that crosses at least two other lines.

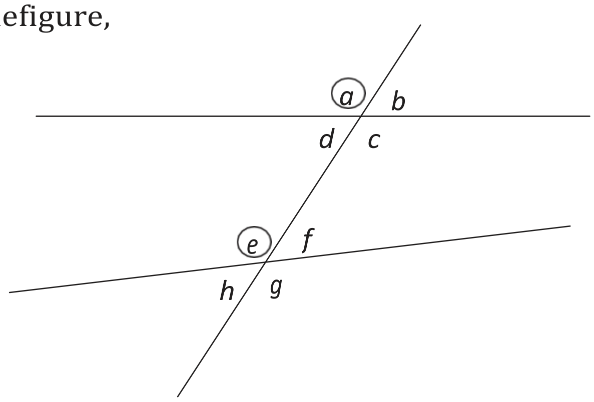
The blue line is the transversal.



When a transversal intersects two lines, we can compare the sets of angles on the two lines by looking at their positions.

The angles that lie on the same side of the transversal and are in matching positions are called **corresponding angles (corr. ∠s)**. In the figure, these are corresponding angles:

- a and e
- b and f
- d and h
- c and g.



1. In the figure, a and e are both left of the transversal and above a line.

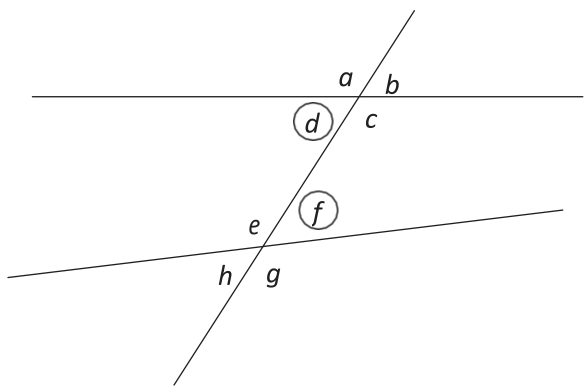
   Write down the location of the following corresponding angles. The first one has been done for you.

   b and f: Right of the transversal and above the lines.

   d and h c and g

Alternate angles (alt. ∠s) lie on opposite sides of the transversal, but are not adjacent or vertically opposite. When the alternate angles lie between the two lines, they are called **alternate interior angles**. In the figure, these are alternate interior angles:

- d and f
- c and e.



When the alternate angles lie outside of the two lines, they are called **alternate exterior angles**. In the figure, these are alternate exterior angles:

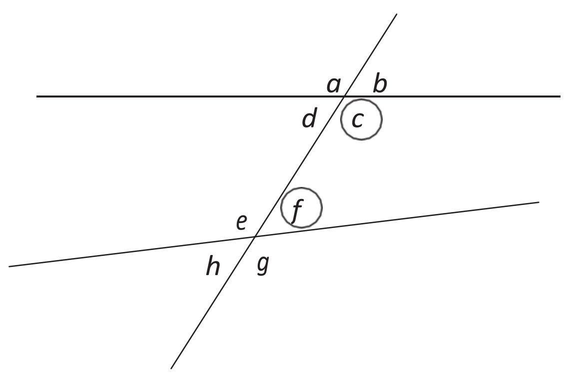
- a and g
- b and h.

2. Write down the location of the following alternate angles:

   d and f c and e a and g b and h

Co-interior angles (co-int. ∠s) lie on the same side of the transversal and between the two lines. In the figure, these are co-interior angles:

- c and f
- d and e.

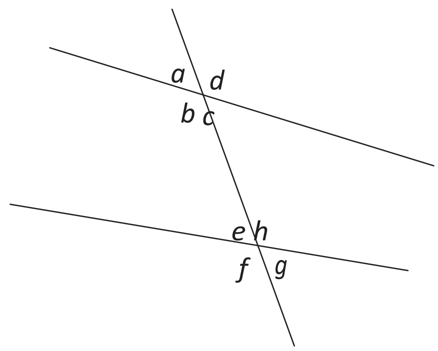


3. Write down the location of the following co-interior angles:

   d and e c and f

identifying types of angles

Two lines are intersected by a transversal, as shown below.



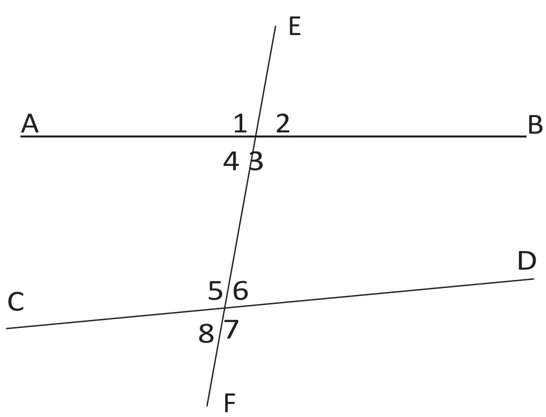
Write down the following pairs of angles:

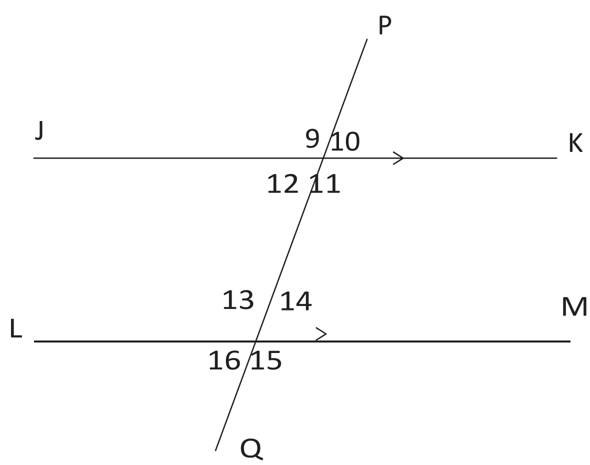
1. two pairs of corresponding angles
2. two pairs of alternate interior angles
3. two pairs of alternate exterior angles
4. two pairs of co-interior angles
5. two pairs of vertically opposite angles

12.4 Parallel lines intersected by a transversal

investigating angle sizes

In the figure below on the left-hand side, EF is a transversal to AB and CD. In the figure below on the right-hand side, PQ is a transversal to parallel lines JK and LM.





1. Use a protractor to measure the sizes of all the angles in each figure. Write down the measurements of each angle.

2. Copy the table and use your measurements to complete it.

Angles	When two lines are not parallel	When two lines are parallel
Corr. ∠s	$\hat{1}$ = ____ ; $\hat{5}$ = ____ $\hat{4}$ = ____ ; $\hat{8}$ = ____ $\hat{2}$ = ____ ; $\hat{6}$ = ____ $\hat{3}$ = ____ ; $\hat{7}$ = ____	$\hat{9}$ = ____ ; $\hat{13}$ = ____ $\hat{12}$ = ____ ; $\hat{16}$ = ____ $\hat{10}$ = ____ ; $\hat{14}$ = ____ $\hat{11}$ = ____ ; $\hat{15}$ = ____
Alt. int. ∠s	$\hat{4}$ = ____ ; $\hat{6}$ = ____ $\hat{3}$ = ____ ; $\hat{5}$ = ____	$\hat{12}$ = ____ ; $\hat{14}$ = ____ $\hat{11}$ = ____ ; $\hat{13}$ = ____
Alt. ext. ∠s	$\hat{1}$ = ____ ; $\hat{7}$ = ____ $\hat{2}$ = ____ ; $\hat{8}$ = ____	$\hat{9}$ = ____ ; $\hat{15}$ = ____ $\hat{10}$ = ____ ; $\hat{16}$ = ____
Co-int. ∠s	$\hat{4} + \hat{5}$ = ____ $\hat{3} + \hat{6}$ = ____	$\hat{12} + \hat{13}$ = ____ $\hat{11} + \hat{14}$ = ____

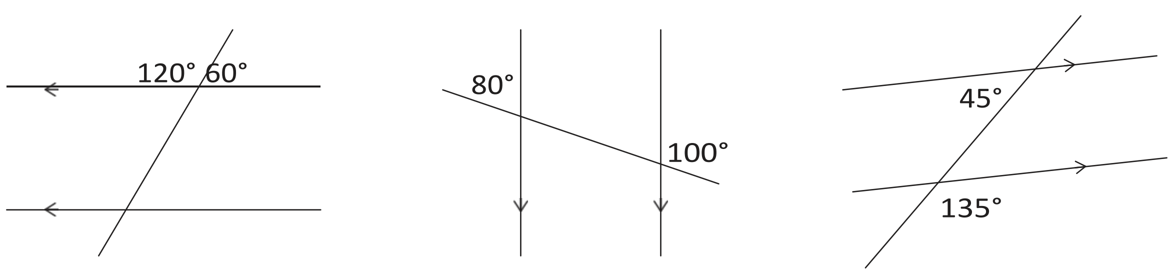
3. Look at your completed table in question 2. What do you notice about the angles formed when a transversal intersects parallel lines?

When lines are parallel:

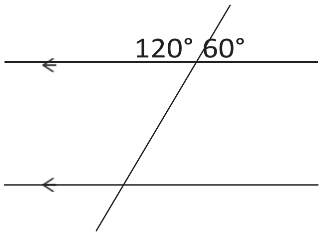
- corresponding angles are equal
- alternate interior angles are equal
- alternate exterior angles are equal
- co-interior angles add up to 180°.

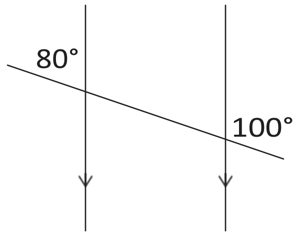
identifying angles on parallel lines

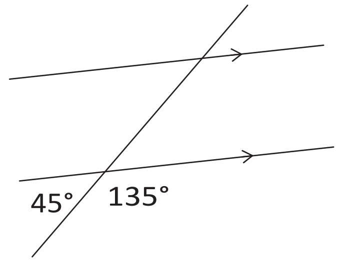
1. Copy these drawings and fill in the corresponding angles to those given:



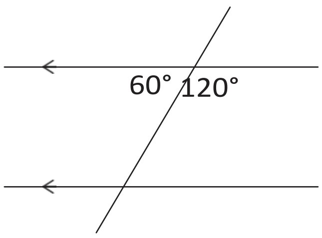
2. Copy the following drawings and fill in the alternate exterior angles:

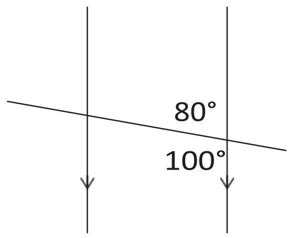


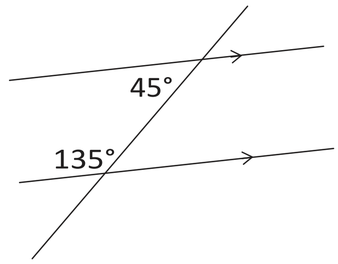




3. (a) Copy the drawings and fill in the alternate interior angles.
   (b) Circle the two pairs of co-interior angles in each figure.

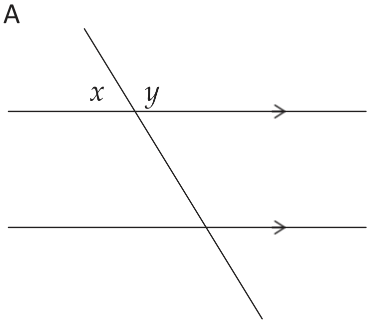




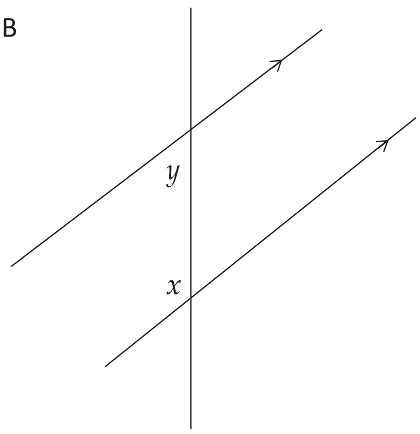


4. (a) Copy the drawings below. Without measuring, fill in all the angles in the following figures that are equal to x and y.
   (b) Explain your reasons for each x and y that you filled in to your partner.

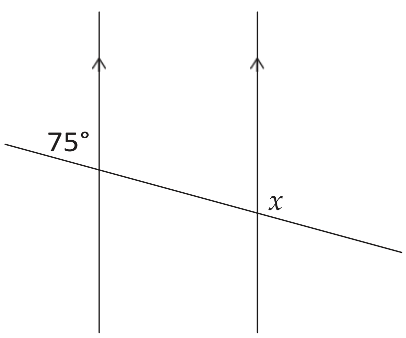
A

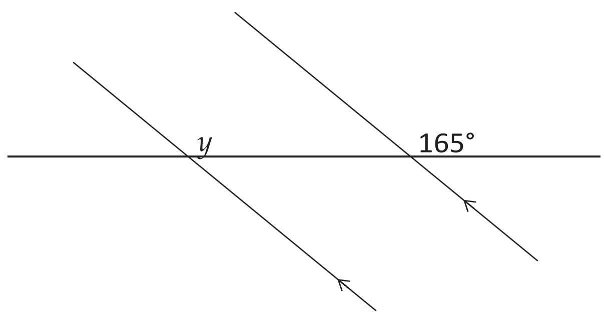


B



5. Give the value of x and y below:



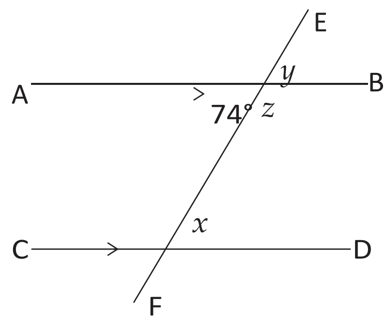


12.5 Finding unknown angles on parallel lines

working out unknownangles

Work out the sizes of the unknown angles. Give reasons for your answers. (The first one has been done as an example.)

1. Find the sizes of x, y and z.



$x = 74°$ [alt. ∠ with given 74°; AB // CD]

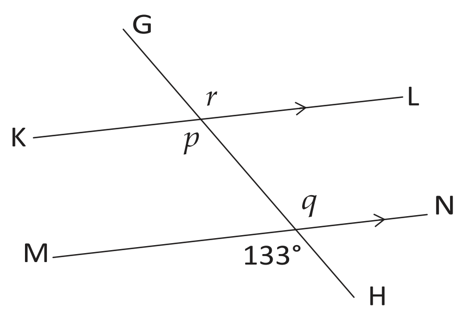
$y = 74°$ [corr. ∠ with x; AB // CD]

or $y = 74°$ [vert. opp. ∠ with given 74°]

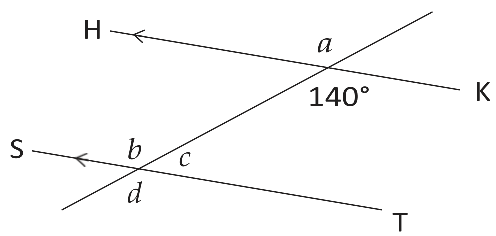
$z = 106°$ [co-int. ∠ with x; AB // CD]

or $z = 106°$ [∠s on a straight line]

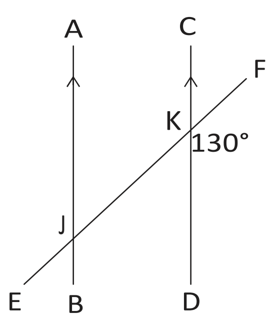
2. Work out the sizes of p, q and r.



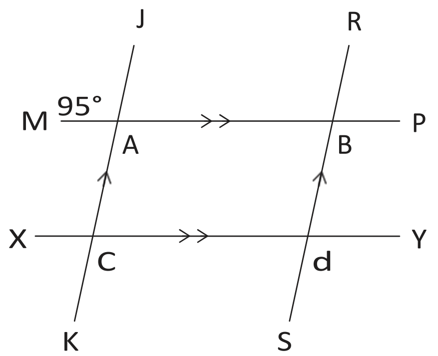
3. Find the sizes of a, b, c and d.



4. Find the sizes of all the angles in this figure.

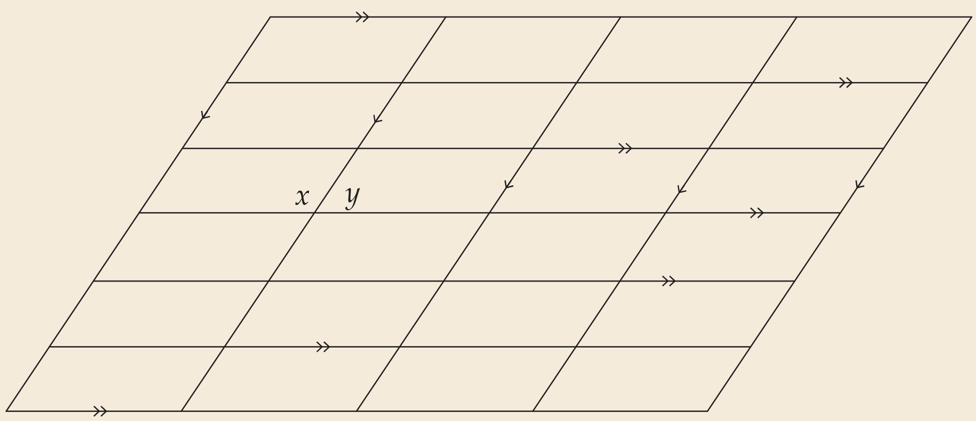


5. Find the sizes of all the angles. (Can you see two transversals and two sets of parallel lines?)

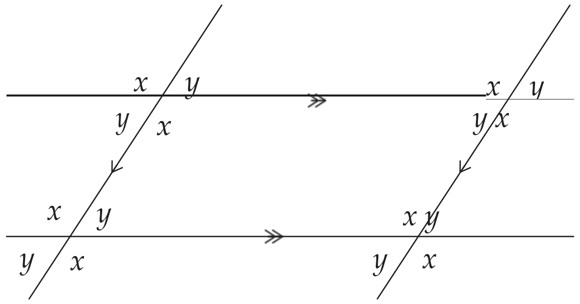


extension

Two angles in the following diagram are given as x and y. Copy the diagram and fill in all the angles that are equal to x and y.



sum of the angles in a quadrilateral



The diagram on the right is a section of the previous diagram.

1. What kind of quadrilateral is in the diagram? Give a reason for your answer.
2. Look at the top left intersection. Complete the following equation: Angles around a point = 360°

   $\therefore \quad x + y +$ ___ + ___ = 360°

3. Look at the interior angles of the quadrilateral on page 136. Copy and complete the following equations:

Sum of angles in the quadrilateral = $x + y +$ ___ + ___

From question 2: $x + y +$ ___ + ___ = 360°

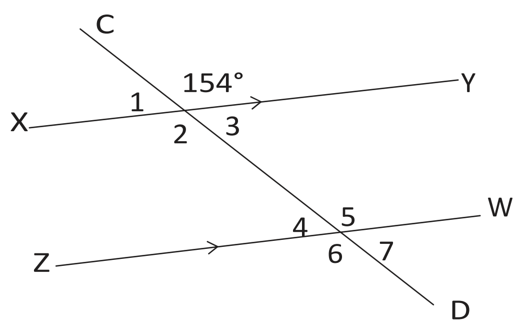
$\therefore$ Sum of angles in a quadrilateral = ___°

Can you think of another way to use the diagram on page 136 to work out the sum of the angles in a quadrilateral?

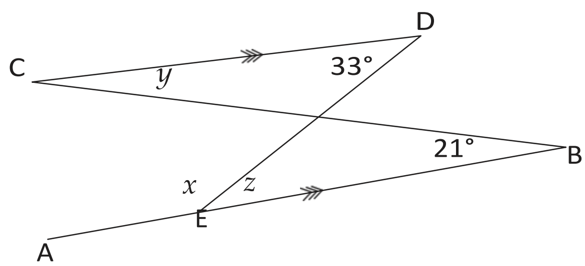
12.6 Solving more geometric problems

angle relationships on parallel lines

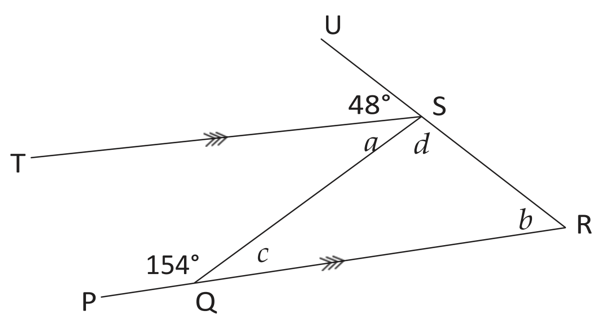
1. Calculate the sizes of $\hat{1}$ to $\hat{7}$.



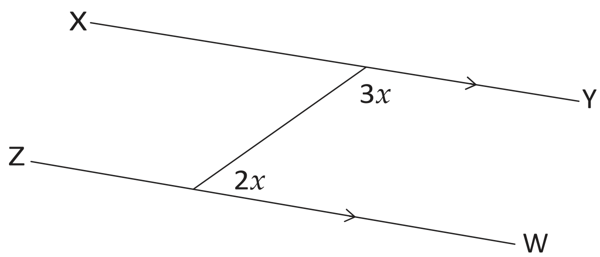
2. Calculate the sizes of x, y and z.



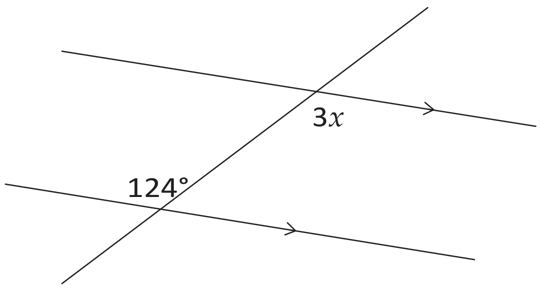
3. Calculate the sizes of a, b, c and d.



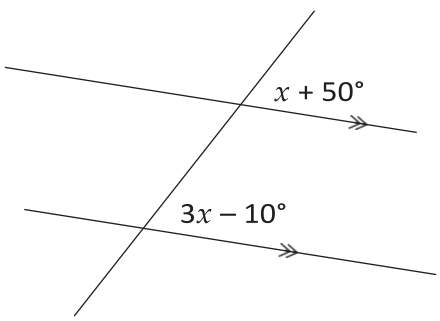
4. Calculate the size of x.



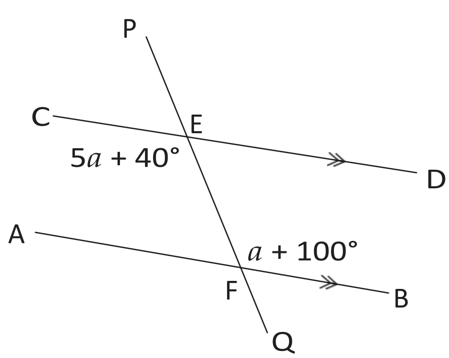
5. Calculate the size of x.



6. Calculate the size of x.

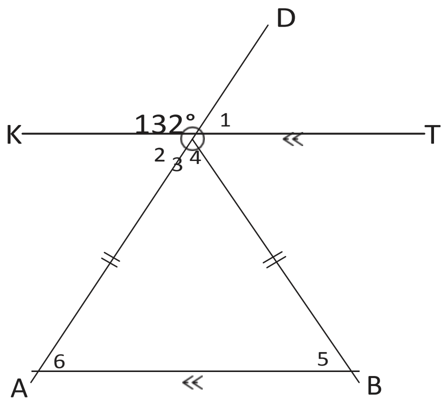


7. Calculate the sizes of a and $C\hat{E}P$.

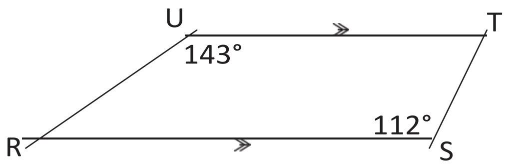


including properties of triangles and quadrilaterals

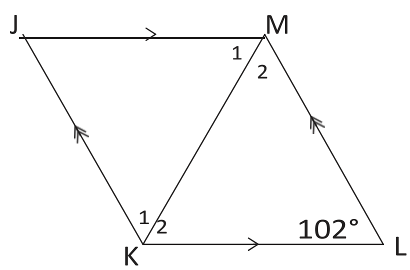
1. Calculate the sizes of $\hat{1}$ to $\hat{6}$.



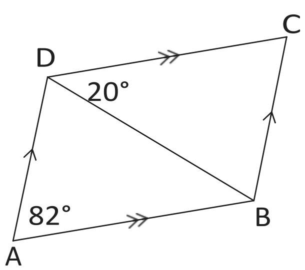
2. RSTU is a trapezium. Calculate the sizes of $\hat{T}$ and $\hat{R}$.



3. JKLM is a rhombus. Calculate the sizes of $J\hat{M}L$, $\hat{M}_2$ and $\hat{K}_1$.

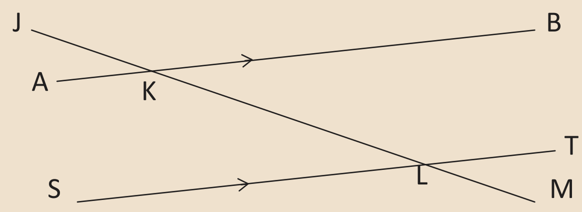


4. ABCD is a parallelogram. Calculate the sizes of $A\hat{D}B$, $A\hat{B}D$, $\hat{C}$ and $D\hat{B}C$.



Worksheet

1. Look at the drawing below. Name the items listed alongside it.



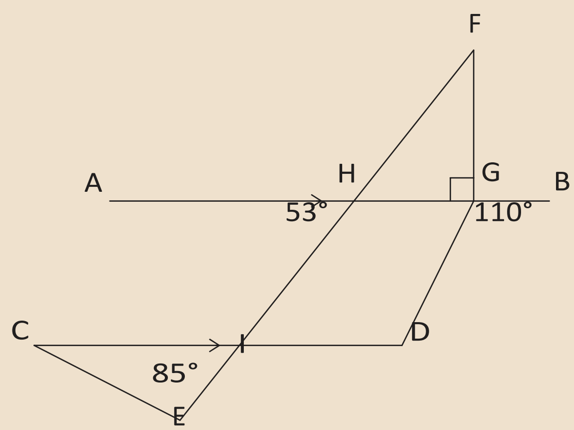
(a) a pair of vertically opposite angles

(b) a pair of corresponding angles

(c) a pair of alternate interior angles

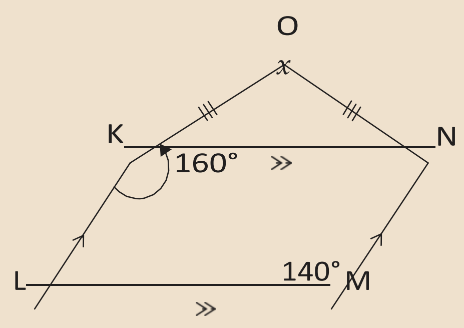
(d) a pair of co-interior angles

2. In the diagram, AB // CD. Calculate the sizes of $F\hat{H}G$, $\hat{F}$, $\hat{C}$ and $\hat{D}$. Give reasons for your answers.



3. In the diagram, OK = ON, KN // LM, KL // MN and $L\hat{K}O = 160°$.

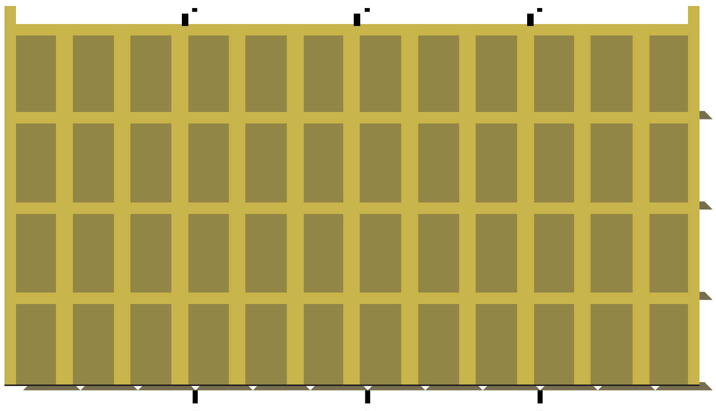
   Calculate the value of x. Give reasons for your answers.



Chapter 13
Common fractions

13.1 Equivalent fractions

sharing chocolate in different ways



1. (a) John eats three quarters of a chocolate slab like the one above. How many small pieces of chocolate is that?
   (b) How many small pieces are there in the whole slab of chocolate?
   (c) Ratti eats six eighths of a chocolate slab like the one above. Who eats more, Ratti or John, or do they eat the same amount of chocolate? Explain your answer.
2. A slab of chocolate like the above one has to be shared fairly between 16 people. That means each person should get one sixteenth of the slab. How many small pieces of chocolate should each person get?
3. What fraction of the whole slab is one of the small pieces?
4. (a) Is it true that each person in question 2 should get one sixteenth of the slab?
   (b) Is it true that each person in question 2 should get three forty-eighths of the slab?
   (c) Is one sixteenth of the slab of chocolate precisely the same amount of chocolate as three forty-eighths of the slab?
5. How many forty-eighths of a slab will each person get in each of the following cases, if the slab is equally shared among the number of people indicated?
   (a) between two people
   (b) between three people
   (c) betweenfour people
   (d) between six people

(e) between eight people

(f) between 12 people

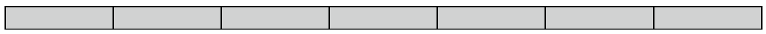
(g) between 16 people

(h) between 24 people

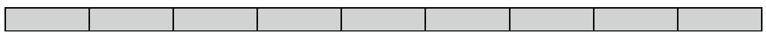
6. In each case below, state what the smaller parts of the grey strip may be called.

(a) 

(b) 

(c) 

(d) 

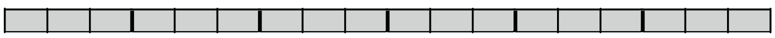
(e) 

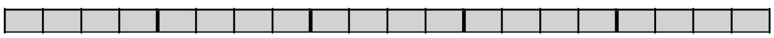
(f) 

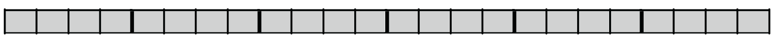
(g) 

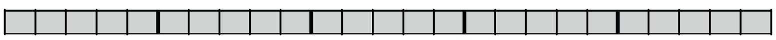
(h) 

(i) 

(j) 

(k) 

(l) 

(m) 

7. (a) A whole slab of chocolate is divided equally between a number of people and each person gets one eighth of the slab. How many people are there?
   (b) How many people are there if each person gets one twelfth of the slab?
   (c) How many people are there if each person gets one sixteenth of the slab?

8. If each small piece is one forty-eighth of a slab of chocolate, how many pieces are there in each of the following?

(a) one twelfth of a slab

(b) one eighth of a slab

(c) one third of a slab

(d) one twenty-fourth of a slab

(e) one sixth of a slab

(f) one sixteenth of a slab

9. If each small piece is one forty-eighth of a slab of chocolate, how many pieces are there in each of the following?

(a) five twelfths of a slab

(b) three eighths of a slab

(c) two thirds of a slab

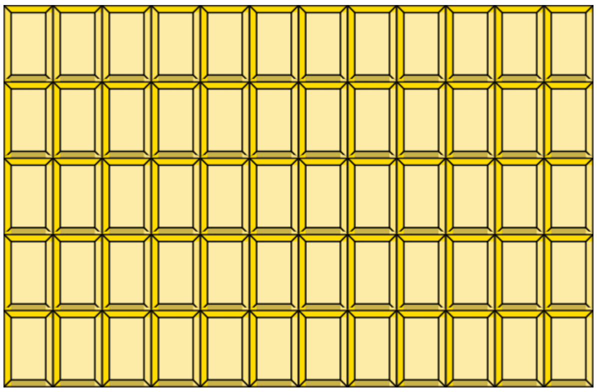
(d) 17 twenty-fourths of a slab

(e) five sixths of a slab

(f) 13 sixteenths of a slab

10. In each of the following say which fraction of the slab gives you more chocolate, or whether the two quantities are the same. How do you know this?
    (a) five sixths of a slab or 13 sixteenths of a slab
    (b) five twelfths of a slab or three eighths of a slab
    (c) two thirds of a slab or 17 twenty-fourths of a slab
11. (a) How many forty-eighths of a slab is one third of a slab and one eighth of a slab together?
    (b) How much of a slab is one sixth of a slab and three eighths of a slab together?
    (c) How much chocolate is five sixths of a slab and seven eighths of a slab together?
12. (a) How many eighths of a slab is 18 forty-eighths of a slab? How did you work this out?
    (b) How many sixths of a slab is 32 forty-eighths of a slab? How did you work this out?

Now, here is a different slab of chocolate:



13. What fraction of the whole slab is each one of the small pieces?
14. How many sixtieths of the yellow 60-piece slab is each of the following?
    (a) one fifth of the slab
    (b) one twelfth of the slab
15. To answer question 14, you may just have counted the small pieces on the diagram. What calculations could you have done to find the answers for question 14?
16. How many sixtieths of the yellow 60-piece slab is each of the following?
    (a) one twentieth of the slab
    (b) one sixth of the slab
    (c) nine twentieths of the slab
17. In each case below, state which is more chocolate, or whether the two fractions of the slab are the same amount of chocolate. How do you know?
    (a) 14 twentieths or seven tenths
    (b) 13 twentieths or nine fifteenths
    (c) three fifths or seven twelfths

18. In each case below, work out how much of a slab is made up of the two parts together:
    (a) 14twentieths and seven tenths. At the end, give your answer as a number of tenths.
    (b) 13 twentieths and nine fifteenths. Give your final answer as wholes and quarters.
    (c) three fifths and seven twelfths

using fraction notation

Instead of writing five forty-eighths, we may write $\frac{5}{48}$.

This is called the **common fraction notation**.

The number 48 below the line is called the **denominator** and it shows that the whole was divided into 48 equal pieces, so each piece is one forty-eighth of the whole. The denominator shows the **unit** in which the number is expressed.

The number 5 above the line is called the **numerator** and it indicates the **number** of pieces.

A number that is made up of a whole number and a fraction, like two and three fifths, can be written as a **mixed number**: $2\frac{3}{5}$.

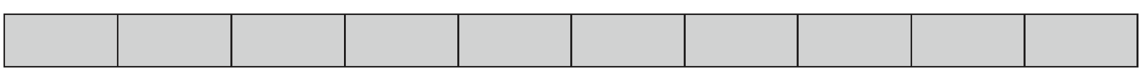
1. Write each of the following numbers in fraction notation:
    (a) seven twentieths   (b) three and five eighths
    (c) two and seven ninths   (d) one and seventenths

2. Write each of the following numbers in words:
    (a) $\frac{23}{100}$ (b) $3\frac{5}{30}$ (c) $2\frac{5}{18}$ (d) $\frac{17}{25}$

3. (a) The strip below is divided into five equal parts.
    What part of the whole strip is each of the five parts?

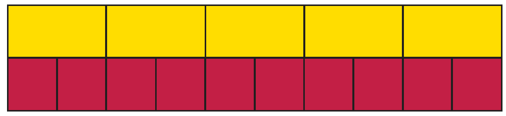
    (b) If you divide each fifth into six smaller equal parts, how many smaller parts will there be altogether?
    (c) What fraction of the whole strip is each of these smaller parts?

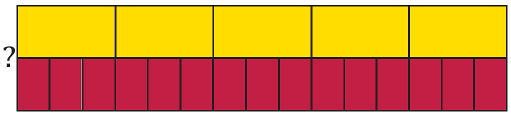
4. (a) The strip below is divided into ten equal parts.
    What part of the whole strip is each of the ten parts?



    (b) If you divide each tenth into four smaller equal parts, how many smaller parts will there be altogether?
    (c) What fraction of the whole strip is each of these smaller parts?

(d) If you divide each tenth into five smaller equal parts, how many smaller parts will there be altogether?
(e) What fraction of the whole strip is each of these smaller parts?
(f) If you divide each tenth into ten smaller equal parts, how many smaller parts will there be altogether?
(g) What fraction of the whole strip is each of these smaller parts?



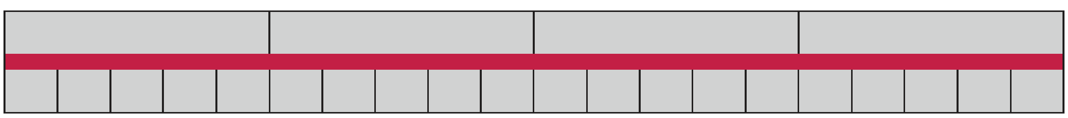


5. (a) How many tenths make up one fifth? You may use the diagram on the right to figure this out.
(b) How many fifteenths are there in one fifth?
(c) How many fifteenths are there in three fifths?
(d) How many twentieths are there in one fifth? If you need help with this, draw a diagram like those in question 5(a) and (b) to help you. Your diagram need not be accurate.
(e) How many twentieths are there in one quarter?
(f) How many twentieths are there in three quarters?
(g) How many twentieths do you think will make up one tenth? If you need help, make marks on the diagram in question 5(a) to help you.

Your answers for question 5 can also be written in fraction notation. For example, your answer for question 5(c) can be written as $\frac{3}{5} = \frac{9}{15}$.

6. Write each of your other answers for question 5 in fraction notation.

7. In this question write the fractions in words. Decide whether each statement is true or false and give reasons for your answers.
(a) $\frac{15}{20}$ of the red strip below is longer than $\frac{3}{4}$ of the strip

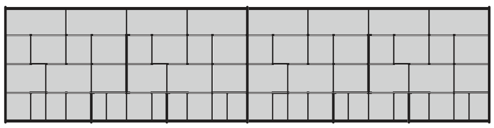


(b) $\frac{9}{15}$ is a bigger number than $\frac{3}{5}$
(c) $\frac{2}{3}$ is a smaller number than $\frac{7}{12}$

The same number can be expressed in different units. For example, the number $\frac{3}{4}$ can be expressed in eighths as $\frac{6}{8}$, in twentieths as $\frac{15}{20}$, in sixtieths as $\frac{45}{60}$ and in many other units. $\frac{3}{4}$, $\frac{6}{8}$, $\frac{15}{20}$ and $\frac{45}{60}$ are all different ways of expressing the same number. Hence they are called **equivalent fractions**.

Equivalent fractions let us write the same number in different ways, for example: $\frac{3}{4} = \frac{6}{8} = \frac{15}{20} = \frac{45}{60}$

8. Write your answers in words and in fraction notation, and explain your answers.
   (a) Express $\frac{3}{8}$ in sixteenths and in fortieths.
   (b) Express $\frac{3}{5}$ in tenths, twentieths, fortieths and hundredths.
   (c) Express $\frac{7}{10}$ in fortieths, fiftieths and hundredths.
9. Consider the fraction three quarters. It can be written as $\frac{3}{4}$.
   (a) Multiply both the numerator and the denominator by 2 to form a "new" fraction. Is the "new" fraction equivalent to $\frac{3}{4}$? You can check your answer on this diagram.



   (b) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 3 to form a "new" fraction. Is the "new" fraction equivalent to $\frac{3}{4}$?
   (c) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 4 to form a "new" fraction. Is the new fraction equivalent to $\frac{3}{4}$?
   (d) Multiply both the numerator and the denominator of $\frac{3}{4}$ by 6 to form a "new" fraction. Is the new fraction equivalent to $\frac{3}{4}$?

$\frac{15}{20}$ is equivalent to $\frac{3}{4}$ because there are five twentieths in one quarter, and so there are 15 twentieths in three quarters. $\frac{9}{16}$ is not equivalent to $\frac{3}{4}$ because there are four sixteenths in one quarter. Therefore, three quarters is 12 sixteenths; not nine sixteenths.

10. Decide whether the two given numbers are equal or not. Explain your answer. If they are not equal, state which one is bigger and explain why you say so. You may first write the fractions in words if that helps you.
   (a) $\frac{5}{8}$ and $\frac{3}{5}$ (Hint: express both numbers in fortieths)
   (b) $\frac{7}{10}$ and $\frac{5}{8}$
   (c) $\frac{4}{5}$ and $\frac{7}{8}$

13.2 Adding and subtracting fractions

To add or subtract fractions, all the fractions must be expressed in the same unit.

1. Calculate each of the fractions on page 147. The work that you did in question 10 above may help you.

(a) $\frac{5}{8} + \frac{3}{5}$

(b) $\frac{7}{10} + \frac{5}{8}$

(c) $\frac{7}{10} + \frac{3}{8}$

(d) $\frac{5}{8} - \frac{3}{5}$

(e) $\frac{7}{10} - \frac{3}{8}$

(f) $6 \times \frac{5}{8}$ (which is $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$)

(g) $8 \times \frac{7}{10}$

To compare, add or subtract fractions, for example $\frac{5}{8}$ and $\frac{3}{5}$, find a fraction unit in which both fractions can be expressed so that you can compare them. We call this a **common denominator**. The "product" of the two denominators is helpful to find such a unit. In this case, 5 × 8 = 40. Since one eighth is five fortieths, $\frac{5}{8}$ is 25 fortieths or $\frac{25}{40}$. Since one fifth is eight fortieths, $\frac{3}{5}$ is 24 fortieths or $\frac{24}{40}$. So, $\frac{5}{8}$ is bigger than $\frac{3}{5}$.

2. In each question, explain why the two given numbers are equal or why they are not equal. If they are not equal, state which one is bigger and explain why you say so. You may first write the fractions in words if that will help you.

(a) $\frac{5}{8}$ and $\frac{2}{3}$ (b) $\frac{5}{6}$ and $\frac{7}{8}$ (c) $\frac{3}{4}$ and $\frac{4}{5}$

(d) $\frac{5}{12}$ and $\frac{2}{3}$ (e) $\frac{7}{12}$ and $\frac{3}{8}$ (f) $\frac{9}{20}$ and $\frac{4}{15}$

(g) $\frac{3}{10}$ and $\frac{1}{4}$ (h) $\frac{7}{10}$ and $\frac{5}{8}$ (i) $\frac{9}{13}$ and $\frac{11}{17}$

3. Add the two fractions given in each part of question 2. Show how you work it out.

(a) $\frac{5}{8} + \frac{2}{3}$ (b) $\frac{5}{6} + \frac{7}{8}$ (c) $\frac{3}{4} + \frac{4}{5}$

(d) $\frac{5}{12} + \frac{2}{3}$ (e) $\frac{7}{12} + \frac{3}{8}$ (f) $\frac{9}{20} + \frac{4}{15}$

(g) $\frac{3}{10} + \frac{1}{4}$ (h) $\frac{7}{10} + \frac{5}{8}$ (i) $\frac{9}{13} + \frac{11}{17}$

4. Now subtract the smaller number from the bigger number in each part of question 2.

(a) $\frac{2}{3} - \frac{5}{8}$
(b) $\frac{7}{8} - \frac{5}{6}$
(c) $\frac{4}{5} - \frac{3}{4}$
(d) $\frac{2}{3} - \frac{5}{12}$
(e) $\frac{7}{12} - \frac{3}{8}$
(f) $\frac{9}{20} - \frac{4}{15}$
(g) $\frac{3}{10} - \frac{1}{4}$
(h) $\frac{7}{10} - \frac{5}{8}$
(i) $\frac{9}{13} - \frac{11}{17}$

5. Calculate each of the following:

(a) $3\frac{2}{3} - 1\frac{5}{6}$

(b) $5\frac{6}{7} + \frac{3}{8}$

(c) $12\frac{5}{8} + 7\frac{4}{9}$

(d) $4\frac{5}{12} - 2\frac{3}{10}$

(e) $1\frac{3}{10} - \frac{2}{3}$

(f) $2\frac{7}{15} - 1\frac{3}{8}$

(g) $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$

(h) $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$

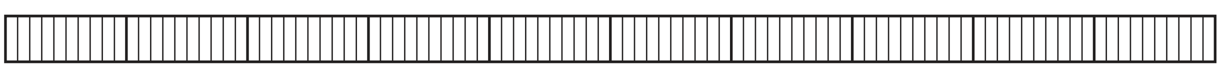
(i) $\frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8} + \frac{7}{8}$

(j) $2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12} + 2\frac{4}{12}$

13.3 Tenths, hundredths and thousandths

a useful family of fraction units

1. (a) Shade three tenths of the following strip:

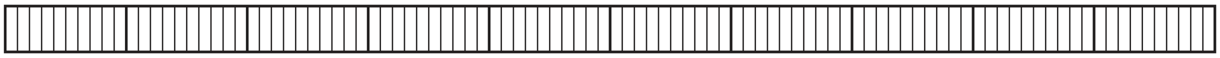


(b) Into how many smaller parts is each tenth of the above strip divided?
(c) How many of these smaller parts are there in the whole strip?
(d) What is each of these smaller parts called?

(e) How many hundredths make up two fifths of the strip?

(f) How many hundredths make up one quarter of the strip?

(g) Shade 37 hundredths of the following strip:



2. Express each of the following numbers as a number of hundredths, and write your answers in fraction notation:

(a) four fifths (b) one twentieth

(c) seven twentieths (d) one twenty-fifth

(e) 17 twenty-fifths (f) seven fiftieths

Because one twentieth is five hundredths, seven twentieths is 35 hundredths. This can also be expressed in fraction notation: $\frac{35}{100} = \frac{7}{20}$.

$\frac{7}{20}$ is called the **simplest form** of $\frac{35}{100}$ because $\frac{35}{100}$ cannot be expressed with a smaller numerator than 7.

3. Express each of the following fractions in its simplest form:

(a) $\frac{75}{100}$ (b) $\frac{60}{100}$

(c) $\frac{65}{100}$ (d) $\frac{90}{100}$

4. Calculate each of the following, and express your answer in its simplest form:

(a) $\frac{3}{25} + \frac{4}{20}$ (b) $\frac{6}{25} + \frac{6}{20}$ (c) $\frac{7}{100} + \frac{9}{200}$

5. (a) How much is $\frac{1}{100}$ of R400?

(b) How much is $\frac{7}{100}$ of R250?

(c) How much is $\frac{25}{100}$ of R600?

(d) How much is $\frac{1}{4}$ of R600?

(e) How much is $\frac{40}{100}$ of R700?

(f) How much is $\frac{2}{5}$ of R700?

Instead of writing $\frac{40}{100}$ of R700, we may write $\frac{40}{100} \times$ R700.

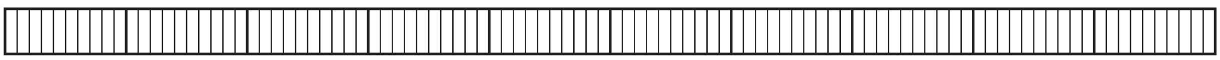
6. Explain why your answers for question 5(e) and (f) are the same.

Another word for *hundredth* is *per cent*.
Instead of saying:
*Miriam received **32 hundredths** of the prize money*,
we can say:
*Miriam received **32 per cent** of the prize money.*
The symbol for per cent is% (percentage).

7. How much is 80% of each of the following?
   (a) R900
   (b) R650
   (c) R250
   (d) R3 400

8. How much is 8% of each of the amounts in 7(a), (b), (c) and (d)?

9. How much is 15% of each of the amounts in 7(a), (b), (c) and (d)?



The above strip is divided into hundredths. Imagine that each of the hundredths is divided into ten equal parts (they will be almost impossible to see).

10.(a) How many of these very small parts will there be in the whole strip?
   (b) What could each of these very small parts be called?

11. How much is each of the following?
   (a) one tenth of R6 000
   (b) one hundredth of R6 000
   (c) one thousandth of R6 000
   (d) ten hundredths of R6 000
   (e) 100 thousandths of R6 000
   (f) seven hundredths of R6 000
   (g) 70 thousandths of R6 000
   (h) one ten thousandth of R6 000

12. Calculate each of the following:
   (a) $\frac{3}{10} + \frac{5}{8}$
   (b) $3\frac{3}{10} + 2\frac{4}{5}$
   (c) $\frac{3}{10} + \frac{7}{100}$
   (d) $\frac{3}{10} + \frac{70}{100}$
   (e) $\frac{3}{10} + \frac{7}{1\,000}$
   (f) $\frac{3}{10} + \frac{70}{1\,000}$

13. Calculate each of the following:
   (a) $\frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$
   (b) $\frac{3}{10} + \frac{70}{100} + \frac{400}{1\,000}$
   (c) $\frac{6}{10} + \frac{20}{100} + \frac{700}{1\,000}$
   (d) $\frac{2}{10} + \frac{5}{100} + \frac{4}{1\,000}$

14. In each case investigate whether the statement is true or not, and give reasons for your decision.

(a) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{3}{100} + \frac{46}{1\,000}$

(b) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{7}{10} + \frac{2}{100} + \frac{46}{1\,000}$

(c) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$

(d) $\frac{676}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$

13.4 Fraction of a fraction

calculate parts of wholes and parts of parts

To calculate $\frac{7}{20}$ (seven twentieths) of R500 you can first calculate one twentieth, and then multiply by 7:

one twentieth of R500 is R500 ÷ 20 = R25, so $\frac{7}{20}$ of R500 is 7 × R25 = R175.

This means that to calculate $\frac{7}{20}$ of R500, you work out (500 ÷ 20) × 7. You divide by the denominator and then multiply by the numerator.

$\frac{7}{20}$ of 500 is the same as $\frac{7}{20} \times 500$.

1. Calculate each of the following:
   (a) $\frac{9}{25}$ of R500
   (b) $\frac{9}{20}$ of R500
   (c) $\frac{9}{125}$ of R500
2. A small choir of eight members won the second prize in a competition and they received two fifths of the total prize money. They shared the money equally between themselves. The total prize money was R1 000. How much prize money did each member of the choir get?
3. (a) How much is $\frac{7}{8}$ of 400?
   (b) How much is $\frac{2}{5}$ of your answer for (a)?
   (c) How much is $\frac{7}{20}$ of 400?

4. Here is Nathi's answer to question 2 on page 151:

One fifth of R1 000 is R200, so two fifths is R400. So the choir team received R400 in total. Each member received one eighth of the R400, which is R400 ÷ 8 = R50.

(a) Compare your own answer to Nathi's answer. If they are different, work them out again and find out who is right.

(b) Do you agree that $\frac{1}{20}$ of R1 000 is R50?

(c) Explain why the answer for question 2 is the same as $\frac{1}{20}$ of R1 000.

5. Copy the table below and write your answers in it. For each of the following numbers, 80, 180, 260, 360 and 2 400, work out:

(a) How much is $\frac{2}{5}$ of each of the numbers?

(b) How much is $\frac{3}{4}$ of each of your answers for (a)?

(c) How much is $\frac{6}{20}$ of each of the numbers?

Number	80	180	260	360	2 400
$\frac{2}{5}$ of the number					
$\frac{3}{4}$ of the answer					
$\frac{6}{20}$ of the number					

6. Use your answers for question 5 to answer the following questions:

(a) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R80?

(b) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R180?

(c) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R260?

(d) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R360?

(e) How much is $\frac{3}{4}$ of $\frac{2}{5}$ of R2 400?

7. To calculate $\frac{3}{4}$ of $\frac{2}{5}$ of a number you did this: the number ÷ 4 × 3 ÷ 5 × 2.

(a) Investigate whether the number × 3 × 2 ÷ 5 ÷ 4 will give the same results as the number ÷ 4 × 3 ÷ 5 × 2, for the numbers in question 5 or any other numbers you may choose.

(b) Investigate whether or not the number × 6 ÷ 20 will give the same results as the number × 3 × 2 ÷ 5 ÷ 4.

(c) Investigate whether or not the number × 3 ÷ 10 will give the same results as the number × 6 ÷ 20.

> Instead of $\frac{3}{4}$ of $\frac{2}{5}$ we may write $\frac{3}{4} \times \frac{2}{5}$.
>
> $\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5}$

To multiply by a mixed number like $2\frac{7}{8}$, it is good practice to express the whole number part in the same fraction units as the fraction part, for example:
two wholes is 16 eighths, so $2\frac{7}{8}$ is $\frac{16}{8} + \frac{7}{8} = \frac{23}{8}$.

8. Calculate each of the following:

(a) $\frac{3}{10} \times \frac{12}{25}$

(b) $\frac{5}{18} \times \frac{4}{35}$

(c) $\left(\frac{1}{3} + \frac{1}{2}\right) \times \frac{6}{7}$

(d) $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}$

(e) $2\frac{3}{5} \times \frac{5}{6}$

(f) $2\frac{3}{4} \times 3\frac{2}{5}$

(g) $2\frac{2}{3} \times 2\frac{2}{3}$

(h) $8\frac{2}{5} \times 3\frac{1}{3}$

(i) $\frac{6}{7} \times \left(\frac{1}{3} + \frac{1}{2}\right)$

(j) $\frac{6}{7} \times \frac{1}{3} + \frac{6}{7} \times \frac{1}{2}$

(k) $\frac{6}{7} \times \left(\frac{1}{2} - \frac{1}{3}\right)$

(l) $\frac{6}{7} \times \frac{1}{2} - \frac{6}{7} \times \frac{1}{3}$

(m) $\left(\frac{5}{6} + \frac{2}{3}\right) \times \frac{1}{5}$

(n) $\frac{5}{6} \times \frac{1}{5} + \frac{2}{3} \times \frac{1}{5}$

(o) $\frac{3}{4} - \frac{2}{5} \times \frac{5}{6}$

(p) $\frac{7}{8} \times \left(\frac{4}{7} + \frac{2}{5}\right)$

squares and cubes and roots of fractions

1. Calculate each of the following:

(a) $\frac{3}{10} \times \frac{3}{10}$

(b) $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}$

(c) $\left(\frac{3}{5}\right)^2$

(d) $\left(\frac{5}{9}\right)^2$

(e) $\left(\frac{3}{5}\right)^3$

(f) $\left(\frac{1}{4}\right)^2$

(g) $\left(\frac{1}{4}\right)^3$ (h) $\left(\frac{4}{7}\right)^2$

(i) $\left(\frac{5}{8}\right)^3$ (j) $\left(\frac{5}{8}\right)^2$

(k) $\left(\frac{5}{12}\right)^3$ (l) $\left(\frac{5}{12}\right)^2$

2. What number multiplied by itself will give $\frac{9}{16}$?

This number is called the square root of $\frac{9}{16}$. It can be written as $\sqrt{\frac{9}{16}}$.

3. Find each of the following. In some cases, your answers to question 1 will help you.

(a) $\sqrt{\frac{4}{9}}$ (b) $\sqrt[3]{\frac{27}{64}}$

(c) $\sqrt{\frac{25}{81}}$ (d) $\sqrt[3]{\frac{125}{343}}$

(e) $\sqrt{\frac{25}{36}}$ (f) $\sqrt[3]{\frac{125}{216}}$

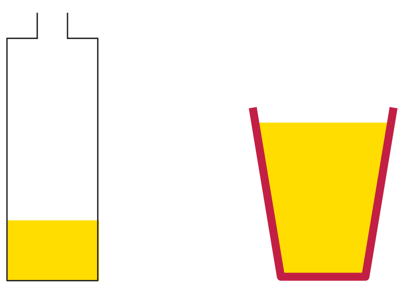
(g) $\sqrt{\frac{9}{100}}$ (h) $\sqrt[3]{\frac{27}{1\,000}}$

13.5 Division by a fraction

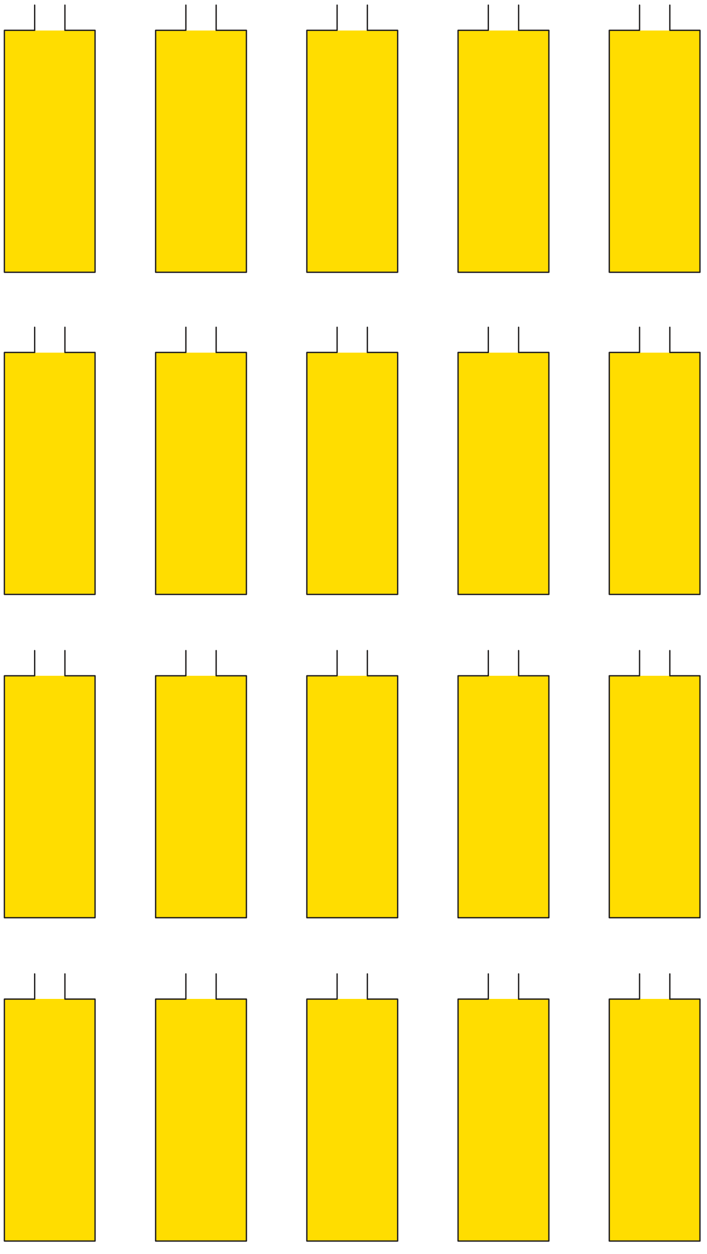
serving juice

Jamie pours juice from bottles into glasses.

He uses three quarters of a bottle of juice to fill one glass.



1. How many bottles will Jamie need to fill ten glasses?
2. How many bottles will Jamie need to fill 30 glasses?
3. How many bottles will Jamie need to fill 100 glasses?
4. How many bottles will Jamie need to fill 180 glasses?
5. How many bottles will Jamie need to fill 37 glasses?
6. How many glasses can Jamie fill from 20 full bottles of juice?



7. How many glasses can Jamie fill from 36 full bottles of juice?

On another day Jamie uses different size glasses. He needs five eighths of a bottle of juice to fill one of these glasses.

8. How many bottles of juice does Jamie need to fill 50 of these glasses?
9. How many of these glasses can Jamie fill from 25 full bottles of juice?

Jamie changes glasses again. For the new glasses, he needs $\frac{7}{10}$ of a full bottle of juice to fill one glass.

10. How many bottles of juice does Jamie need to fill 44 of these glasses?
11. How many of these glasses can Jamie fill from 25 full bottles of juice?
12. How many glasses can Jamie fill from 36 full bottles of juice if he needs three quarters of a bottle to fill one glass?

doing the juice calculations more quickly

1. Ria has R850 and chickens cost R67 each. What operation does she need to do to work out how many chickens she can buy?
2. Jamie has 16 bottles of juice and needs three quarters of a bottle to fill one glass.
   (a) How many quarters of a bottle of juice are there in 16 full bottles?
   (b) How many glasses can he fill with these quarters?

In question 2 you have worked out how many glasses, each taking $\frac{3}{4}$ of a bottle, can be filled from 16 bottles. You did this by first working out the total number of quarters in 16 bottles, and then dividing by 3 to find out how many glasses can be filled.
Do questions 3 and 4 in the same way.

3. Jamie has 20 bottles of juice and needs five eighths of a bottle to fill one glass. To work out how many glasses he can fill, he needs to work out 20 divided by $\frac{5}{8}$. Work in the same way you did for question 2 to find out.
4. Jamie has 25 bottles of juice and needs $\frac{3}{5}$ of a bottle to fill one glass. How many glasses can he fill?

In questions 2, 3 and 4 you have actually done the following calculations:

In question 2 you have calculated $16 \div \frac{3}{4}$, by doing $16 \times 4 \div 3$.

In question 3 you have calculated $20 \div \frac{5}{8}$, by doing $20 \times 8 \div 5$.

In question 4 you have calculated $25 \div \frac{3}{5}$, by doing $25 \times 5 \div 3$.

To divide by a fraction, you multiply by the denominator and divide by the numerator.

5. Calculate each of the following:
   (a) $9 \div \frac{2}{3}$
   (b) $12 \div \frac{3}{8}$
   (c) $15 \div \frac{7}{10}$
   (d) $2 \div \frac{3}{20}$
   (e) $20 \div \frac{7}{12}$
   (f) $120 \div 3\frac{3}{5}$
6. Calculate each of the following:
   (a) $9 \times \frac{3}{2}$
   (b) $12 \times \frac{8}{3}$
   (c) $15 \times \frac{10}{7}$
   (d) $2 \times \frac{20}{3}$
   (e) $20 \times \frac{12}{7}$
   (f) $120 \times \frac{5}{18}$
7. What do you notice about your answers for questions 5 and 6?

To divide by a fraction, we may turn the fraction around and multiply! For example, $15 \div \frac{7}{10} = 15 \times \frac{10}{7}$.

$\frac{10}{7}$ is the **reciprocal** (also called the multiplicative inverse) of $\frac{7}{10}$.

Division is the inverse of multiplication.

The method of dividing by multiplying by the reciprocal also works when a fraction is divided by a fraction. For example $\frac{5}{18} \div \frac{7}{10}$ can be calculated by doing $\frac{5}{18} \times \frac{10}{7}$.

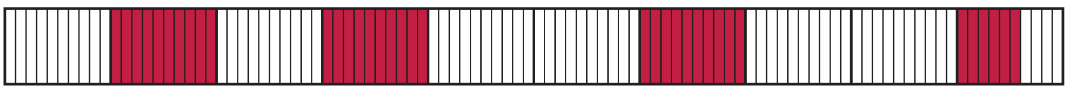
8. Calculate each of the following:
   (a) $\frac{7}{10} \div \frac{3}{20}$
   (b) $\frac{9}{10} \div \frac{3}{18}$
   (c) $\frac{17}{20} \div \frac{2}{7}$
   (d) $2\frac{7}{10} \div \frac{3}{5}$
   (e) $4\frac{7}{8} \div \frac{2}{3}$
   (f) $5\frac{7}{8} \div 2\frac{3}{5}$

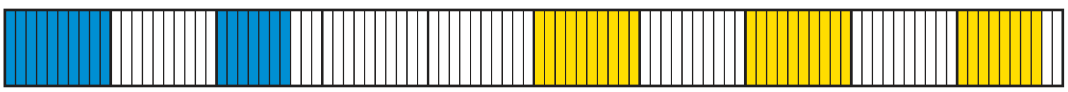
Chapter 14
Fractions in decimal notation

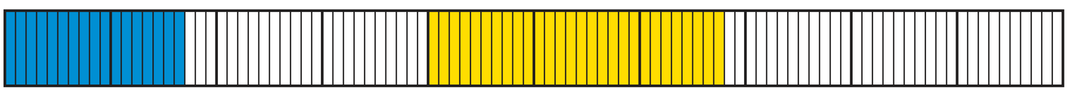
14.1 Equivalent forms

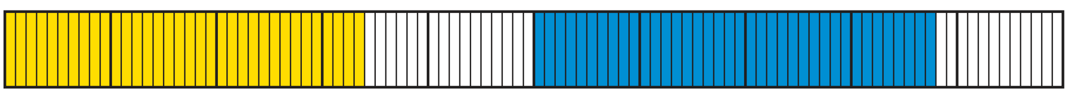
fractions in decimal notation

1. What fraction of each rectangle is coloured in? Copy the table and write your answers in it.

(a) 

(b) 

(c) 

(d) 

Coloured in	Fraction notation	Decimal notation
(a) Red		
(b) Blue		
Yellow		
(c) Blue		
Yellow		
(d) Yellow		
Blue		

2. Now find out what fraction in each rectangle in question 1 is not coloured in. Copy the table and fill in your answers.

Not coloured in	Fraction notation	Decimal notation
(a)		
(b)		
(c)		
(d)		

Decimal fractions and common fractions are simply different ways of expressing the same number. We call them different **notations**.

To write a **common fraction as a decimal fraction**, we must first express the common fraction with a power of ten (10, 100, 1 000, etc.) as the denominator.

For example: $\frac{9}{20} = \frac{9}{20} \times \frac{5}{5} = \frac{45}{100} = 0{,}45$

If you have a calculator, you can also divide the numerator by the denominator to get the decimal form of a fraction, for example: $\frac{9}{20} = 9 \div 20 = 0{,}45$.

To write a **decimal fraction as a common fraction**, we must first express it as a common fraction with a power of ten as the denominator and then simplify if necessary.

For example: $0{,}65 = \frac{65}{100} = \frac{65 \div 5}{100 \div 5} = \frac{13}{20}$

3. Give the decimal form of each of the following numbers:

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{7}{5}$ $\frac{7}{2}$ $\frac{65}{100}$

4. Write the following as decimal fractions:

(a) $2 \times 10 + 1 \times 1 + \frac{3}{10}$

(b) $3 \times 1 + 6 \times \frac{1}{100}$

(c) three hundredths

(d) $7 \times \frac{1}{1\,000}$

5. Write each of the following numbers as fractions in their simplest form:

0,2 0,85 0,07 12,04 40,006

6. Write each of the following in the decimal notation:
   (a) five + 12 tenths
   (b) two + three tenths + 17 hundredths
   (c) 13 hundredths + 15 thousandths
   (d) seven hundredths + 154 hundredths

hundredths, percentages and decimals

It is often difficult to compare fractions with different denominators. Fractions with the same denominator are easier to compare. For this and other reasons, fractions are often expressed as hundredths. A fraction expressed as hundredths is called a **percentage**.

Instead of six hundredths we can say 6 per cent, $\frac{6}{100}$ or 0,06.

6 per cent, $\frac{6}{100}$ and 0,06 are just three different ways of writing the same number.

The symbol % is used for per cent.
Instead of writing "17 per cent", we may write 17%.

1. Write each of the following in three ways: in decimal notation, in percentage notation and in common fraction notation. Leave your answers in hundredths.
   (a) 80 hundredths
   (b) five hundredths
   (c) 60 hundredths
   (d) 35 hundredths

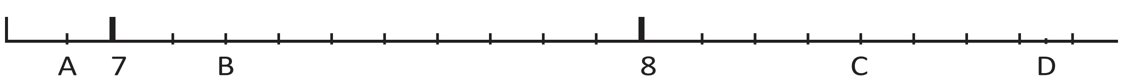
2. Copy and complete the following table:

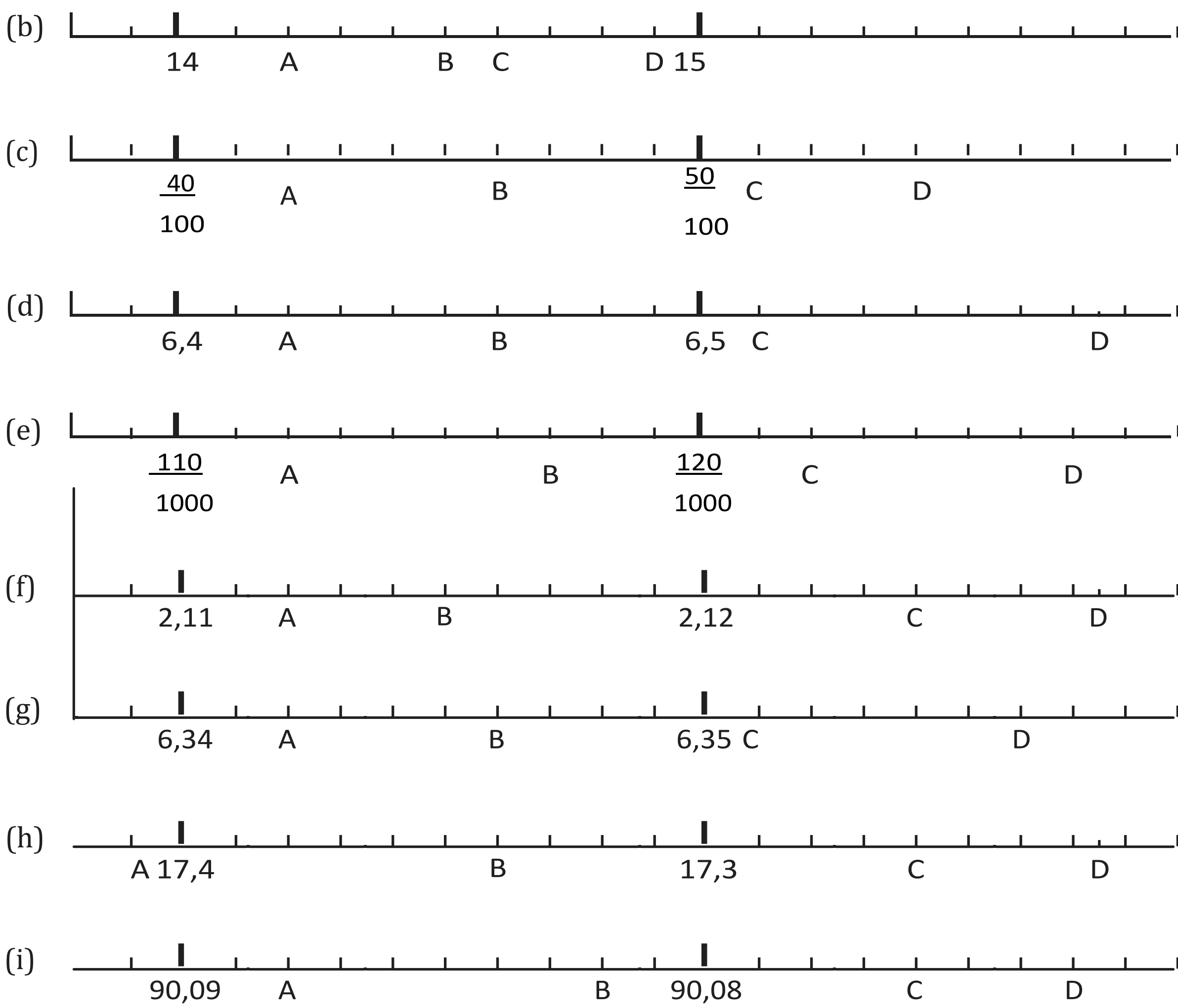
Common fraction	Decimal fraction	Percentage
	0,3	
$\frac{1}{4}$		
		15%
$\frac{1}{8}$		
	0,55	
		1%

14.2 Ordering and comparing decimal fractions

bigger, smaller or the same?

1. Copy the number lines. Write the values of the marked points (A to D) in as accurately as possible in **decimal notation**. Write the values **beneath** the letters A to D.

   (a)




2. Order the following numbers from biggest to smallest. Explain your thinking.

   5 267   1 263   1 300   12 689   635   1 267   125   126   12

3. Order the following numbers from biggest to smallest. Explain your method.

   0,8   0,05   0,901   0,15   0,465   0,55   0,75   0,4   0,62
   0,901   0,8   0,75   0,62   0,55   0,465   0,4   0,15   0,05

4. Write down three different numbers that are bigger than the first number and smaller than the second number:

   (a) 5 and 5,1   (b) 5,1 and 5,11   (c) 5,11 and 5,12
   (d) 5,111 and 5,116   (e) 0 and 0,001   (f) $\frac{1}{2}$ and 1

5. Write down the bigger of the two numbers:

   (a) 2,399 and 2,6   (b) 5,604 and 5,64   (c) 0,11 and 0,087
   (d) $\frac{3}{4}$ and 50%   (e) $\frac{75}{100}$ and $\frac{50}{100}$ (f) 0,125 and 0,25

6. The table gives information about two world-champion heavyweight boxers. If they fight against one another, who would you expect to have the advantage, and why?

	Wladimir Klitschko	Alexander Povetkin
Height (m)	1,98	1,88
Weight (kg)	112	103,3
Reach (m)	2,03	1,91

7. Copy and fill in <, > or=:

   (a) 3,09 □ 3,9 (b) 3,9 □ 3,90 (c) 2,31 □ 3,30

   (d) 3,197 □ 3,2 (e) 4,876 □ 5,987 (f) 123,321 □ 123,3

8. How many numbers are there between 3,1 and 3,2?

14.3 Rounding off decimal fractions

Decimal fractions can be rounded in the same way as whole numbers. They can be rounded to the nearest whole number or to one, two, three, etc. figures after the comma.

If the last digit of the number is 5 or bigger, it is rounded **up** to the next number. For example: 13,5 rounded to the nearest whole number is 14; 13,526 rounded to two figures after the comma is 13,53. If the last digit is 4 or less, it is rounded **down** to the previous number. For example: 13,4 rounded to the nearest whole number is 13.

let's round off

1. Round each of the following numbers off to the nearest whole number:

   29,34 3,65 14,452 3,299 39,1 564,85 1,768

2. Round each of the following numbers off to one decimal place:

   19,47 421,34 489,99 24,37 6,77

3. Round each of the following numbers off to two decimal places:

   8,345 6,632 5,555 34,239 21,899

4. Mr Peters buys a radio for R206,50. The shop allows him to pay it off over six months. How must he pay back the money?

5. (a) Mrs Smith buys a box of 10 kg grapes at the market for R24,77. She must divide it between herself and two friends. How much does each person get?

   (b) How much must each person pay Mrs Smith for the grapes?

6. Estimate the answers for each of the following by rounding off the numbers:
   (a) 1,43 × 1,62 (b) 3,89 × 4,21

14.4 Calculations with decimal fractions

To **add** and **subtract** decimal fractions:

- tenths may be added to tenths
- tenths may be subtracted from tenths
- hundredths may be added to hundredths
- hundredths may be subtracted from hundredths, etc.

let's docalculations!

1. Four consecutive stages in a cycling race are 21,4 km, 14,7 km, 31 km and 18,6 km long. How long is the whole race?

2. Calculate each of the following:
   (a) 16,52 + 2,35 (b) 16,52 + 9,38 (c) 16,52 + 9,78
   (d) 30,08 + 2,9 (e) 0,042 + 0,103 (f) 9,99 + 0,99

3. Calculate each of the following:
   (a) 45,67 − 23,25 (b) 45,67 − 23,80 (c) 187,6 − 98,45
   (d) 1,009 − 0,998 (e) 0,9 − 0,045 (f) 65,7 − 37,6

4. The following set of measurements (in cm) was recorded during an experiment:
   56,8; 55,4; 78,9; 57,8; 34,2; 67,6; 45,5; 34,5; 64,5; 88
   (a) Find the sum of the measurements and round it off to the nearest whole number.
   (b) First round off each measurement to the nearest whole number and then find the sum.
   (c) Which of your answers in 4(a) and (b) is closest to the actual sum? Explain why.

5. By how much is 0,7 greater than 0,07?

6. The difference between two numbers is 0,75. The bigger number is 18,4. What is the other number?

To **multiply** fractions written as decimals, convert the fractions to whole numbers by multiplying by powers of 10 (for example, 0,3 × 10 = 3), do your calculations with the whole numbers, and then convert back to decimals again.

For example: $13{,}1 \times 1{,}01$

$13{,}1 \mathbf{\times 10} \times 1{,}01 \mathbf{\times 100} = 131 \times 101 = 13\,231$; $13\,231 \mathbf{\div 10 \div 100} = 13{,}231$

When you do **division** you can first multiply the number and the divisor by the same number to make the working easier.

For example: $21{,}7 \div 0{,}7 = (21{,}7 \mathbf{\times 10}) \div (0{,}7 \mathbf{\times 10}) = 217 \div 7 = 31$

7. Calculate each of the following. You may use fraction notation if you wish.
   (a) $0{,}12 \times 0{,}3$ (b) $0{,}12 \times 0{,}03$ (c) $1{,}2 \times 0{,}3$
   (d) $350 \times 0{,}043$ (e) $0{,}035 \times 0{,}043$ (f) $0{,}13 \times 0{,}16$
   (g) $1{,}3 \times 1{,}6$ (h) $0{,}13 \times 1{,}6$

8. $30{,}5 \times 1{,}3 = 39{,}65$. Use this answer to work out each of the following:
   (a) $3{,}05 \times 1{,}3$ (b) $305 \times 1{,}3$ (c) $0{,}305 \times 0{,}13$
   (d) 305×13 (e) $39{,}65 \div 30{,}5$ (f) $39{,}65 \div 0{,}305$
   (g) $39{,}65 \div 0{,}13$ (h) $3{,}965 \div 130$

9. $3{,}5 \times 4{,}3 = 15{,}05$. Use this answer to work out each of the following:
   (a) $3{,}5 \times 43$ (b) $0{,}35 \times 43$ (c) $3{,}5 \times 0{,}043$
   (d) $0{,}35 \times 0{,}43$ (e) $15{,}05 \div 0{,}35$ (f) $15{,}05 \div 0{,}043$

10. Calculate each of the following. You may convert to whole numbers to make it easier.
   (a) $62{,}5 \div 2{,}5$ (b) $6{,}25 \div 2{,}5$
   (c) $6{,}25 \div 0{,}25$ (d) $0{,}625 \div 2{,}5$

14.5 Solving problems

1. (a) Divide R44,45 between seven people so that each one receives the same amount.
   (b) John saves R15,25 every week. He now has R106,75 saved up. For how many weeks has he been saving?

2. (a) Calculate $14{,}5 \div 6$, correct to two decimal places.
   (b) Calculate $7{,}41 \div 5$, correct to one decimal place.

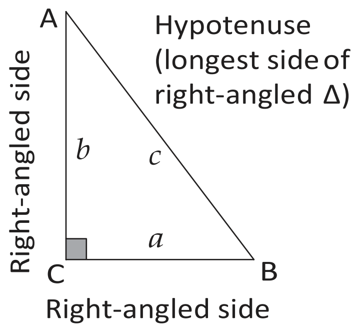
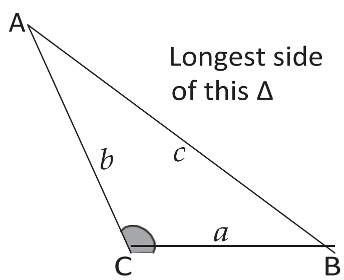
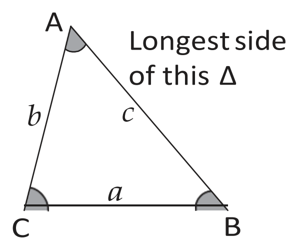
3. Determine the value of x. (Give answers rounded to two decimal places.)
   (a) $7{,}1 \div x = 4{,}2$ (b) $x \div 0{,}7 = 6{,}2$ (c) $12 \div x = 6{,}4$
   (d) $x \div 3{,}5 = 7$ (e) $2{,}3 \times x = 6$ (f) $0{,}023 \times x = 8$

4. (a) 1 ℓ of water weighs almost 0,995 kg. What will 50 ℓ of water weigh? What will 0,5 ℓ of water weigh?
   (b) Mince meat costs R36,65 per kilogram (kg). What will 3,125 kg of mince meat cost? What will 0,782 kg of mince meat cost?

Chapter 15
The theorem of Pythagoras

15.1 The lengths of sides of right-angled triangles

what do you remember about triangles?

Right-angled triangle (Δ): one angle is 90°.	**Obtuse-angled triangle (Δ): one angle is obtuse (between 90° and 180°).**	**Acute-angled triangle (Δ): all angles are acute (less than 90°).**
		

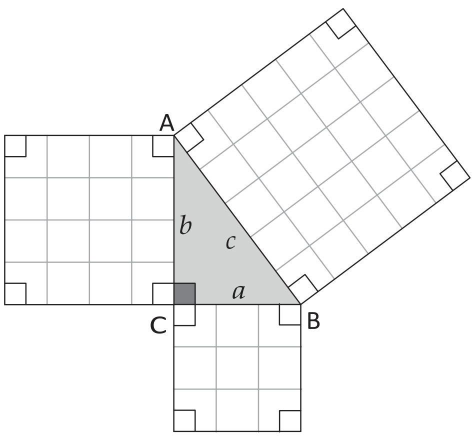
If the vertices of a triangle are labelled A, B and C, the sides opposite these vertices are often labelled as a, b and c, as shown in the above diagrams.

We use the word **hypotenuse** to indicate the side opposite the 90° angle of a right-angled triangle. The hypotenuse is always the longest side of a right-angled triangle. A triangle with no right angle, therefore, does not have a hypotenuse.

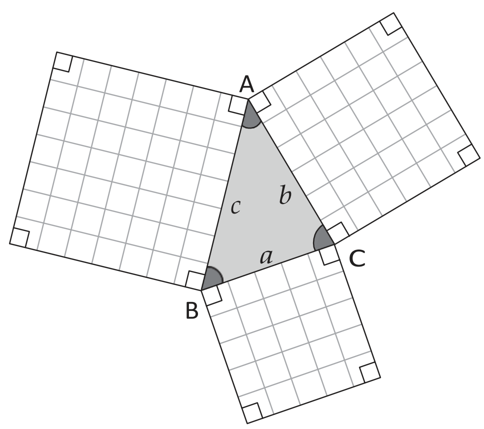
investigating the relationship between the lengths of sides

1. Study the figures below. Each triangle in the following four figures has a square drawn on each of its sides. So, in figure (a), a = three units, b = four units and c = five units long.

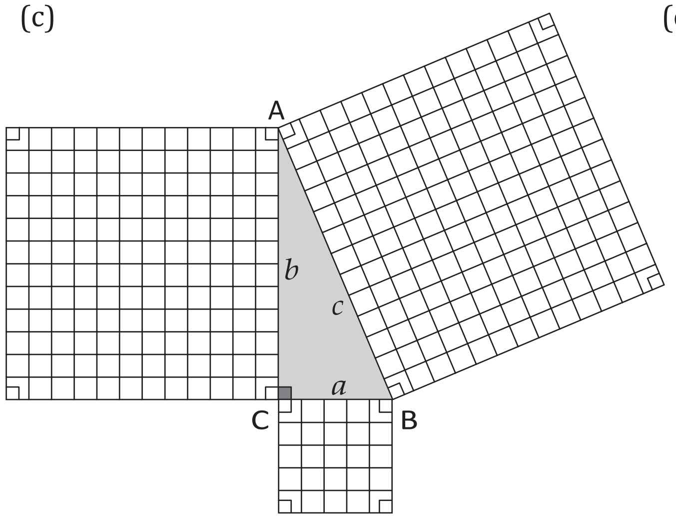
(a)



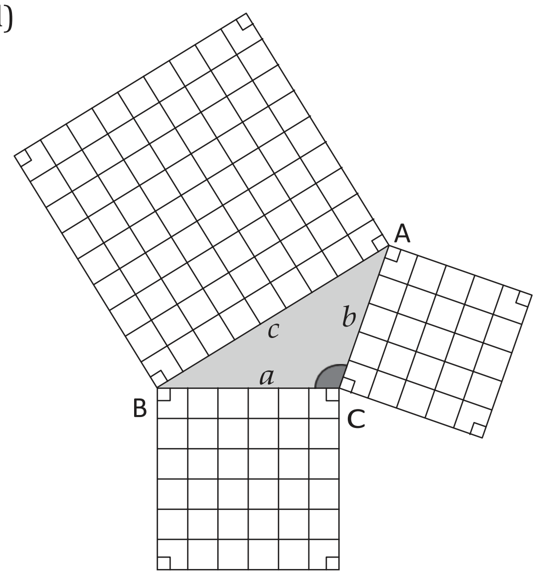
(b)



(c)



(d)



2. Copy the following table. Refer to the four figures from question 1(a) to (d) to complete the table.

Figure	Type of triangle	Length of side ***a***	Length of side ***b***	Length of side ***c***	a^2	b^2	c^2
(a)							
(b)							
(c)							
(d)							

3. Look at your completed table and then copy the statements below and insert =, > or <.

$a^2 + b^2$ ☐ c^2 when ΔABC is an acute-angled triangle.

$a^2 + b^2$ ☐ c^2 when ΔABC is an obtuse-angled triangle.

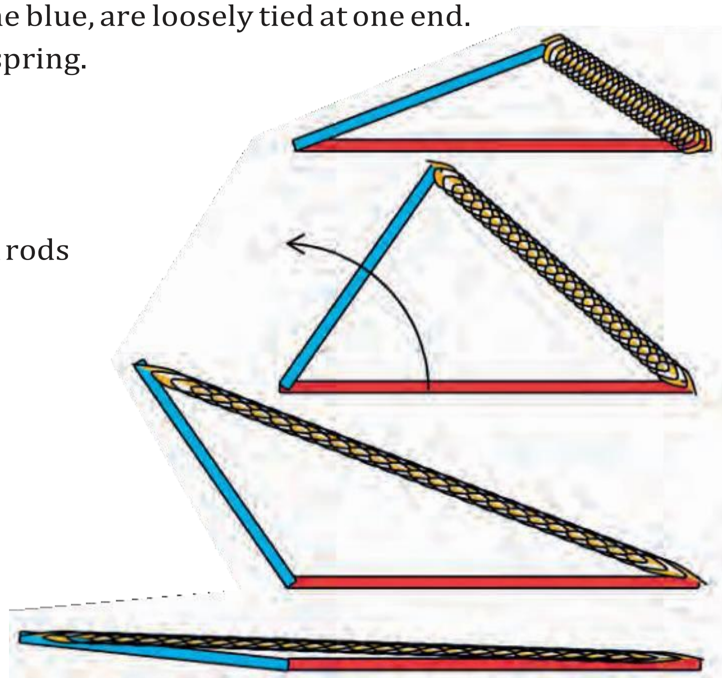
$a^2 + b^2$ ☐ c^2 when ΔABC is a right-angled triangle.

4. Which of the following statements are correct?
   A. In any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.
   B. If a triangle is acute-angled, then the square of the length of the longest side is equal to the sum of the squares of the lengths of the other two sides.
   C. If a triangle is right-angled, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.
   D. In any obtuse-angled triangle, the area of the square on the longest side is equal to the sum of the area of the squares on the other two sides.

5. The following table gives the side lengths *a*, *b* and *c* of ten triangles. Copy and complete the table to decide what type of triangle each triangle is (acute-angled, obtuse-angled or right-angled).

a	b	c	$a^2 + b^2$	c^2	Fill in =, < or >	Type of triangle
7	8	10	$7^2 + 8^2 = 113$	$10^2 = 100$	$a^2 + b^2 > c^2$	Acute-angled
4	5	8	$4^2 + 5^2 = 41$	$8^2 = 64$	$a^2 + b^2 < c^2$	Obtuse-angled
6	8	10	$6^2 + 8^2 = 100$		$a^2 + b^2 = c^2$	Right-angled
8	13	17			$a^2 + b^2 \quad c^2$	
3	4	5			$a^2 + b^2 \quad c^2$	
5	6	7			$a^2 + b^2 \quad c^2$	
5	12	13			$a^2 + b^2 \quad c^2$	
15	8	17			$a^2 + b^2 \quad c^2$	
11	60	61			$a^2 + b^2 \quad c^2$	
12	35	37			$a^2 + b^2 \quad c^2$	

6. Two pieces of wood, one red and one blue, are loosely tied at one end. The two free ends are linked by a spring.



The angle between the two wooden rods can be changed.

Describe how this angle affects the length of the spring.

15.2 Working with the theorem of Pythagoras

The special relationship between the lengths of the sides of a right-angled triangle is known as the **theorem of Pythagoras**. It can be stated in terms of area as follows:

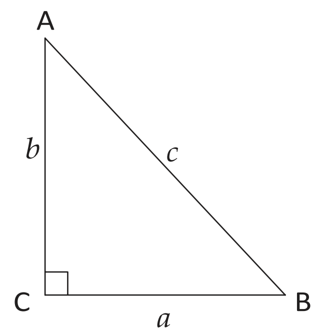
> If a triangle has a right angle, then the area of the square with a side on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

The reference to area can be left out.

> If a triangle is a right-angled triangle, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

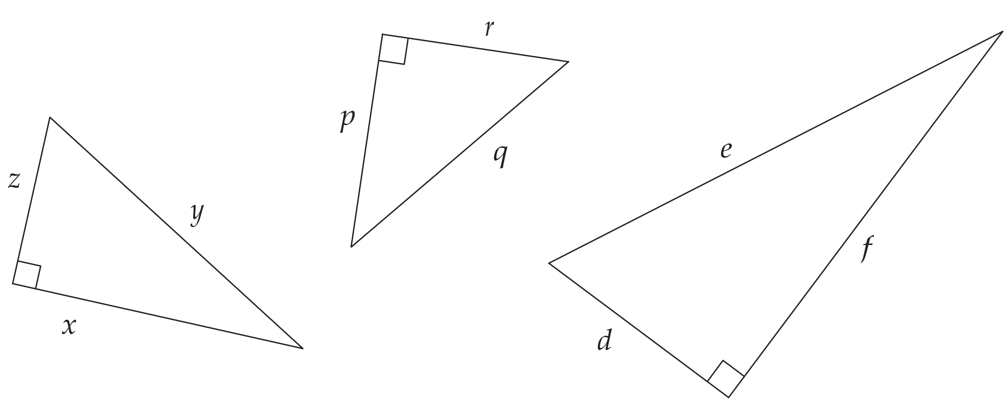
We can express the relationship between the lengths of the sides of the triangle by means of the equation $c^2 = a^2 + b^2$; where c represents the length of the hypotenuse and a and b represent the lengths of the other two sides.

a note about pythagoras
Pythagoras lived in about 500 BCE. The theorem is named after Pythagoras because he may have been the first person to prove it. However, the theorem was known and used in other parts of the world such as Egypt 1 200 years before Pythagoras was born.



working with the formula

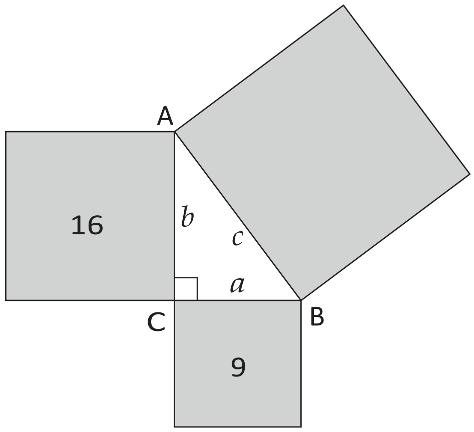
1. Write a Pythagorean equation for each of the following triangles. Explain what each letter symbol represents.



2. Study the following worked example:

Example

Consider the triangle below. Side a is three units long and side b is four units long. What is the length of side c?



If side a is three units long, and side b is four units long, then, according to Pythagoras'theorem:

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c^2 = 9 + 16$

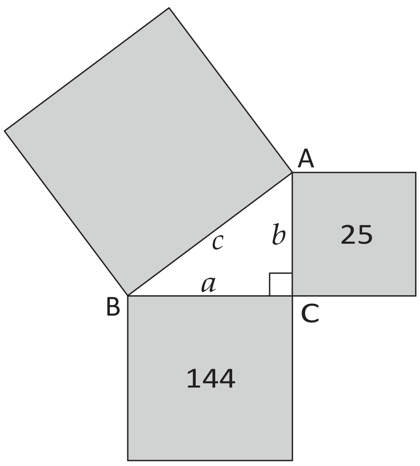
$c^2 = 25$

$\sqrt{c^2} = \sqrt{25}$

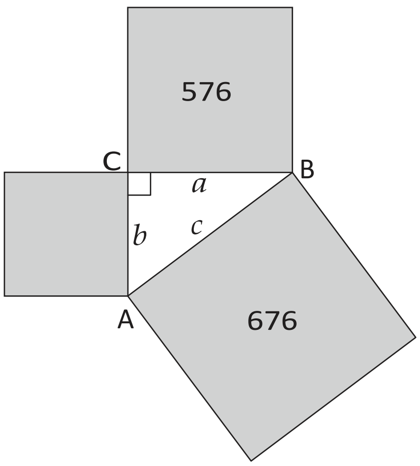
c = five units

3. The areas of some of the following squares are given. Calculate the areas of each of the squares that are not given and the lengths of all the sides.

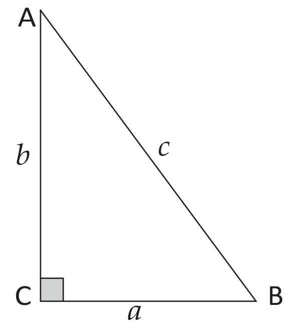
(a)



(b)



4. The following table gives information about the sides of five right-angled triangles. The letter symbol c represents the length of the hypotenuse in all cases. Use Pythagoras' theorem to complete the table, leaving answers in surd form if necessary.



a	b	c	a^2	b^2	$a^2 + b^2$	c^2
7	24					
16		34				
10				576		
			16	49		
	1		1			

15.3 Finding the missing sides in right-angled triangles

We can use the theorem of Pythagoras to calculate the length of the third side of a right-angled triangle if we know the lengths of the other two sides.

Example 1

A right-angled triangle has side a = six units and side b = eight units. Calculate the length of side c.

$$c^2 = a^2 + b^2$$
$$= 6^2 + 8^2$$
$$= 36 + 64$$
$$= 100$$
$$\sqrt{c^2} = \sqrt{100}$$
$$c = 10$$
$$\therefore c = 10 \text{ units}$$

Example 2

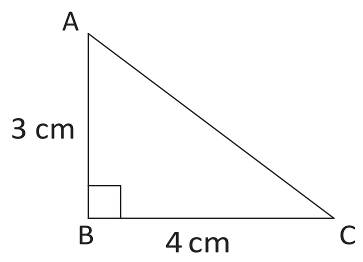
A right-angled triangle has side a = five units and side b = three units. Calculate the length of side c.

$$c^2 = a^2 + b^2$$
$$= 5^2 + 3^2$$
$$= 25 + 9$$
$$= 34$$
$$\sqrt{c^2} = \sqrt{34}$$
$$c = \sqrt{34} \text{ (leave in surd form)}$$
$$\therefore c = \sqrt{34} \text{ units}$$

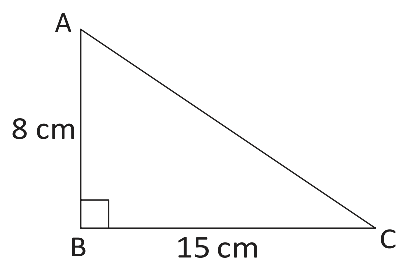
calculating the length of the hypotenuse

Use the formula for the theorem of Pythagoras to calculate the length of the hypotenuse. Leave answers in surd form if necessary.

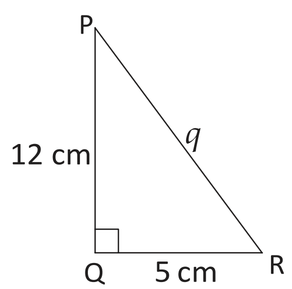
1.



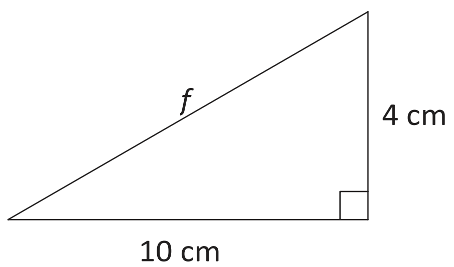
2.



3.



4.

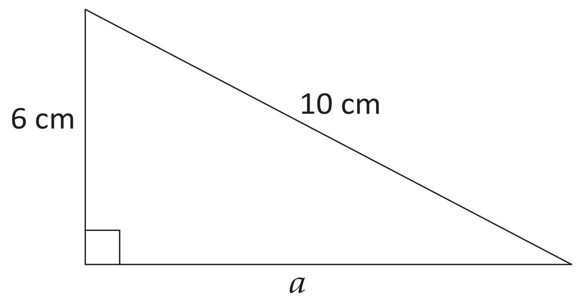


5. A right-angled triangle with hypotenuse c and sides of the following lengths:
a = 9 cm and b = 40 cm.

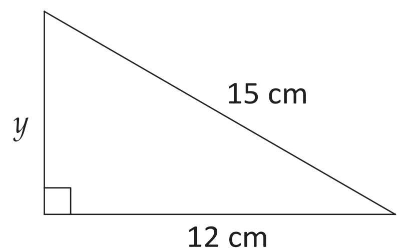
calculating the length of any side in a right-angled triangle

Calculate the missing sides in the following triangles. Do not use a calculator and leave the answers in the simplest surd form where necessary.

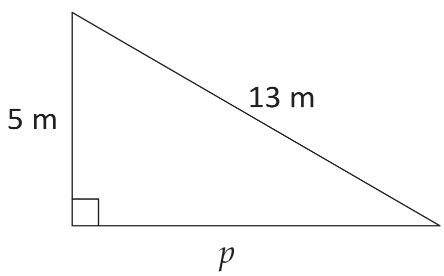
1.



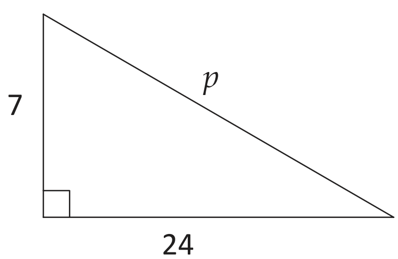
2.



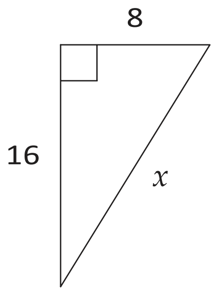
3.



4.



5.



15.4 Are the triangles right-angled?

You learnt in Sections 15.1 and 15.2 that in a right-angled triangle the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

How can we tell whether a triangle is right-angled if we are given the lengths of the sides? One way is to use the "converse" of the Pythagoras theorem.

The **converse** states that if the sum of the squares of the lengths of two sides equals the square of the length of the longest side, then the triangle is a right-angled triangle.

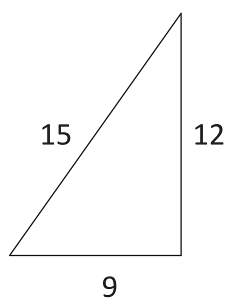
A converse is a statement that swaps around **what is given** in a theorem and **what must be determined**.

We can also state the converse as follows:

If a triangle has side lengths a, b and c such that $c^2 = a^2 + b^2$, then the triangle is a right-angled triangle.

In the questions that follow, you have to determine whether triangles are right-angled or not. Study the following example first:

Example: Determine whether the triangle is right-angled or not.

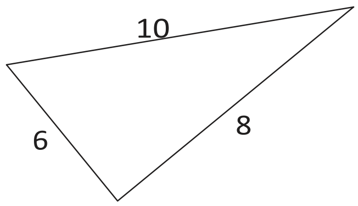


(Length of longest side)^2 = $(15)^2$ = 225
Sum of the squares of the lengths of the other two sides
= $9^2 + 12^2$
= 81 + 144
= 225
(Longest side length)^2 = Sum of squares of other two sides lengths
And this can be written as $15^2 = 9^2 + 12^2$
$\therefore$ The triangle is right-angled.

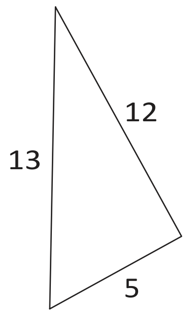
right-angled or not?

Determine whether the triangles are right-angled or not.

1.



2.



3. A triangle has sides measuring six, nine and 15 units.

4. Which of the following lengths of sides of a triangle will form a right-angled triangle? Answer without doing any calculations and explain your answer.

(a) 4, 2, 2
(b) 6, 8, 10
(c) 9, 12, 15
(d) 3, 4, 6
(e) $3x$, $4x$, $5x$
(f) 30, 40, 50

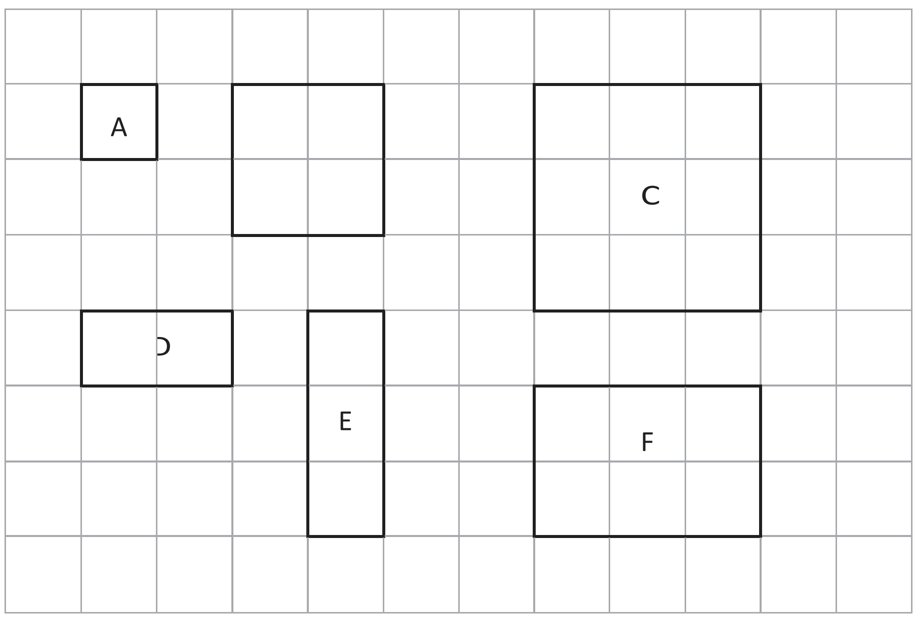
Chapter 16
Perimeter andarea of 2D shapes

16.1 Perimeter of squares and rectangles

The **perimeter** (P) of a flat shape is the distance around a shape. We measure it in units such as millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

explaining the formulae for perimeter

1. Each block in the grid below measures 1 cm × 1 cm. Calculate the perimeter of each shape by adding up thelengths and breadths.



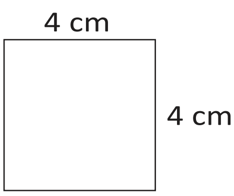
Copy and complete the table based on shapes A to F above.

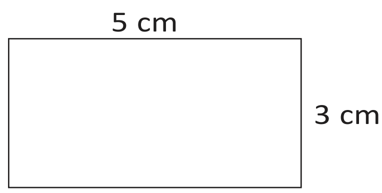
Shape	A	B	C	D	E	F
Length						
Breadth						
Perimeter						

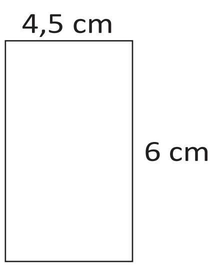
2. Explain to a partner why the following formulae for perimeter are correct:
   Perimeter of a square = $4s$ or (4 × length of a side)
   Perimeter of a rectangle = $2(l + b)$ or $2l + 2b$ (l is the length and b is the breadth)
3. Use the formulae in question 2 to calculate the perimeters of shapes A to F above.

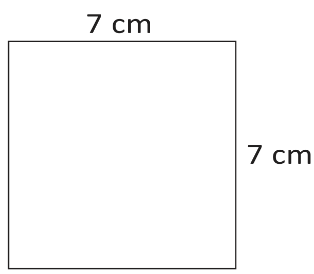
calculating perimeters using formulae

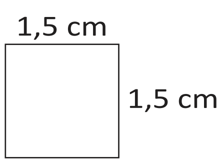
Use formulae to calculate the perimeters of the following shapes.

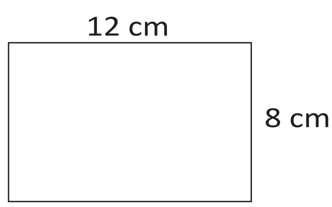
1.


2.


3.


4.


5.


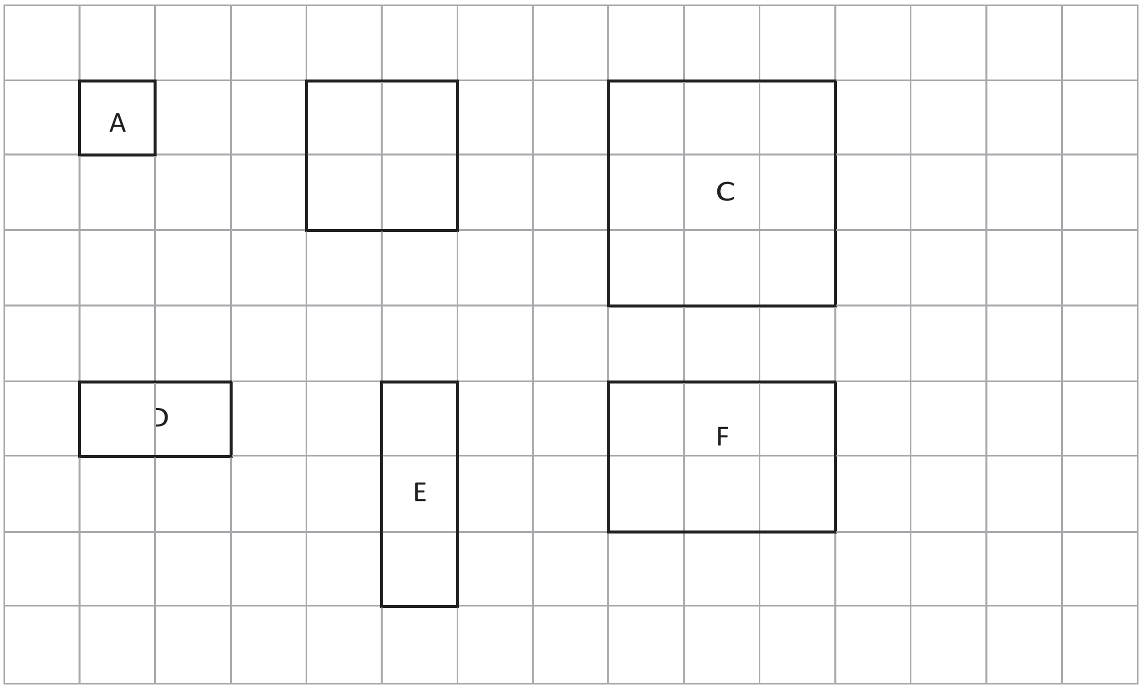
6.


16.2 Area of polygons

We use square units such as mm^2, cm^2, m^2 and km^2 to measure the **area** (A), or the size of a flat surface of ashape.

area of squares and rectangles

1. How many square units make up the area of the shapes on page 177?



2. Each square on the grid above measures 1 cm × 1 cm (or 1 cm^2). Write down the area of each shape above in square centimetres (cm^2).

Below are formulae for calculating area:

> Area of a square = s^2
> Area of a rectangle = $l \times b$

3. Calculate the areas of shapes C, E and F in question 1 using the formulae.

solving more perimeter and areaproblems

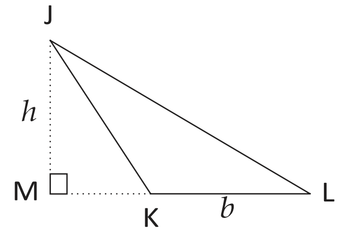
1. The perimeter of a square is 8 cm. What is the length of each side?
2. The area of a rectangle is 40 cm^2 and its length is 8 cm. What is its breadth?
3. The perimeter of a square is 32 cm. What is its length and area?
4. The area of a rectangle is 60 cm^2 and its length is 12 cm. What is its breadth and perimeter?
5. A rectangular yard has an area of 600 m^2. If the breadth is 20 m, find the length and the perimeter.
6. A square has an area of 10 000 m^2. What is the perimeter?

area of triangles

In Grade 7 you learnt how to calculate the area of a triangle with the following formula:

> Area of a triangle = $\frac{1}{2}$(base × perpendicular height) = $\frac{1}{2}(b \times h)$

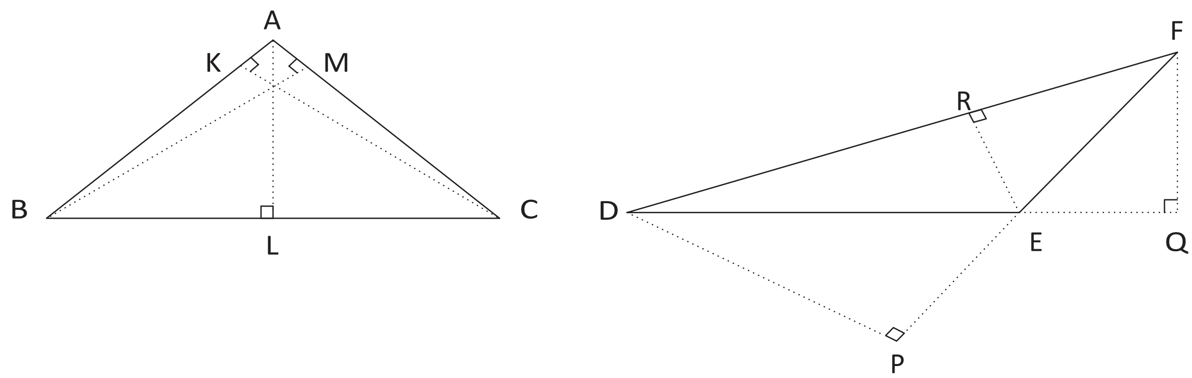
Any of the three sides of a triangle can be regarded as the **base**. The shortest distance from the vertex opposite the chosen base to the base is called the **height** of the triangle with respect to the chosen base. If the triangle is obtuse angled, the line showing the height is outside the triangle. For example, in ΔJKL, JM is the height with respect to the base KL.



To calculate the area of a triangle with the above formula, the height with respect to the chosen base must be used.

problems involving the area of triangles

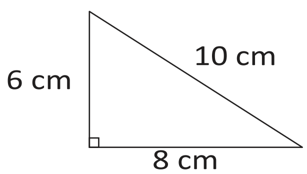
1. Copy and complete the table below by writing down the name of each base and its matching height in ΔABC and ΔDEF:



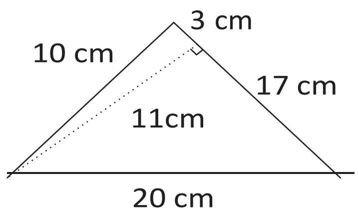
Base						
Height						

2. Calculate the area of the following triangles:

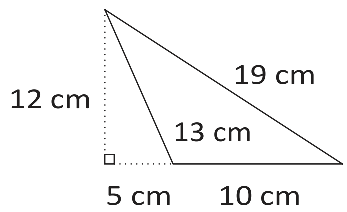
(a)



(b)



(c)

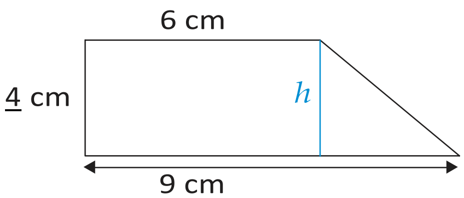


area of composite shapes

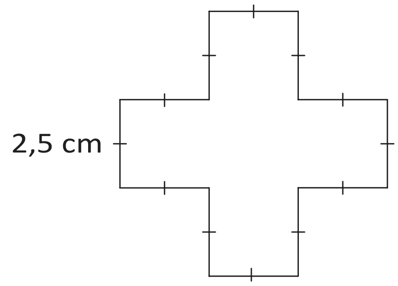
A **composite shape** is made up of a number of other shapes. Often, we can break up the shape into rectangles, squares or triangles to help us work out the area of the shape.

1. Trace the following shapes. Use a ruler and pencil to divide each of the shapes into rectangles, squares and/or triangles. The first one has been done for you.
2. Work out the length of the sides you need and then calculate the area of the shapes. Round off your answers to two decimal places where necessary.

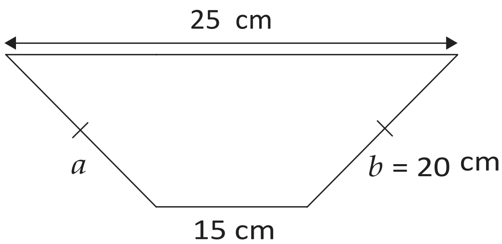
(a)



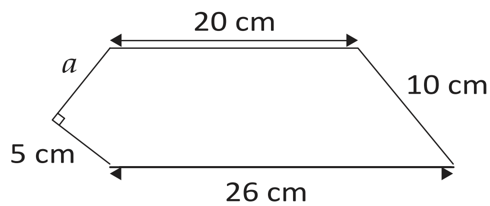
(b)



(c)



(d)



16.3 Perimeter of circles

parts of a circle

In Grade 7, you learnt about the different parts of a circle, including the following:

The **centre** of a circle is the point in the middle (centre) of the circle.

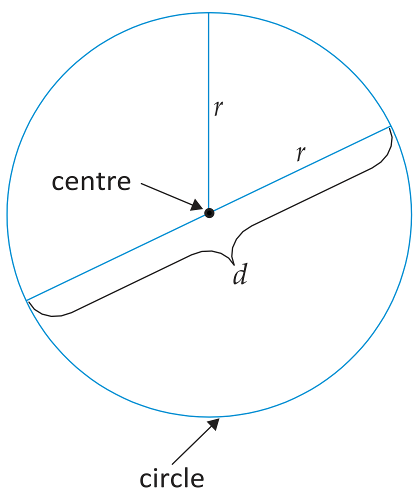
The **circumference** (C) is the distance around the circle. It is the length of the curved line that forms the circle.

The **radius** (r) is the line segment drawn from the centre of the circle to any point on the circle.

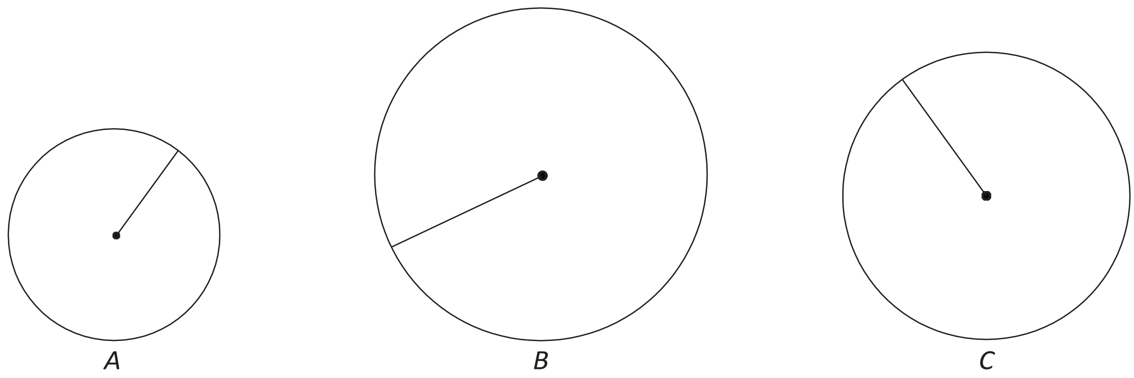
The **diameter** (d) is the line segment passing through the centre of the circle and joining any two points on the circle.

The length of the radius is always half the length of the diameter: $r = \frac{1}{2}d$

The length of the diameter is always twice the length of the radius: $d = 2r$



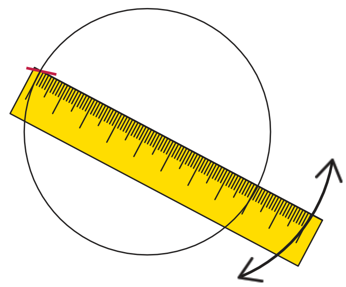
1. Use a ruler to measure the radii (plural of radius) given below. Then copy the table and write down the lengths of both the radii and diameters of the circles in the table.



Circle	A	B	C
Radius (mm)			
Diameter (mm)			

2. Write down the diameters of circles with the following radii:
   (a) $r = 8$ cm (b) $r = 1$ m (c) $r = 4,5$ cm (d) $r = 6,2$ m

relationship between a circle's circumference and diameter

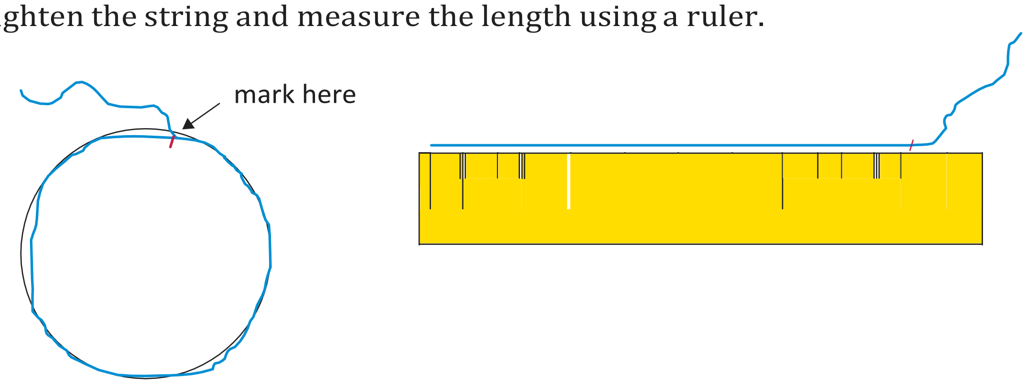


If you do not know where the **centre** of a circle is, you can determine it by measuring the diameter as follows:

- Mark a point on the circle from which to measure.
- Keeping the '0' of the ruler in place, move the other end of the ruler until you find the longest distance. This is the diameter.

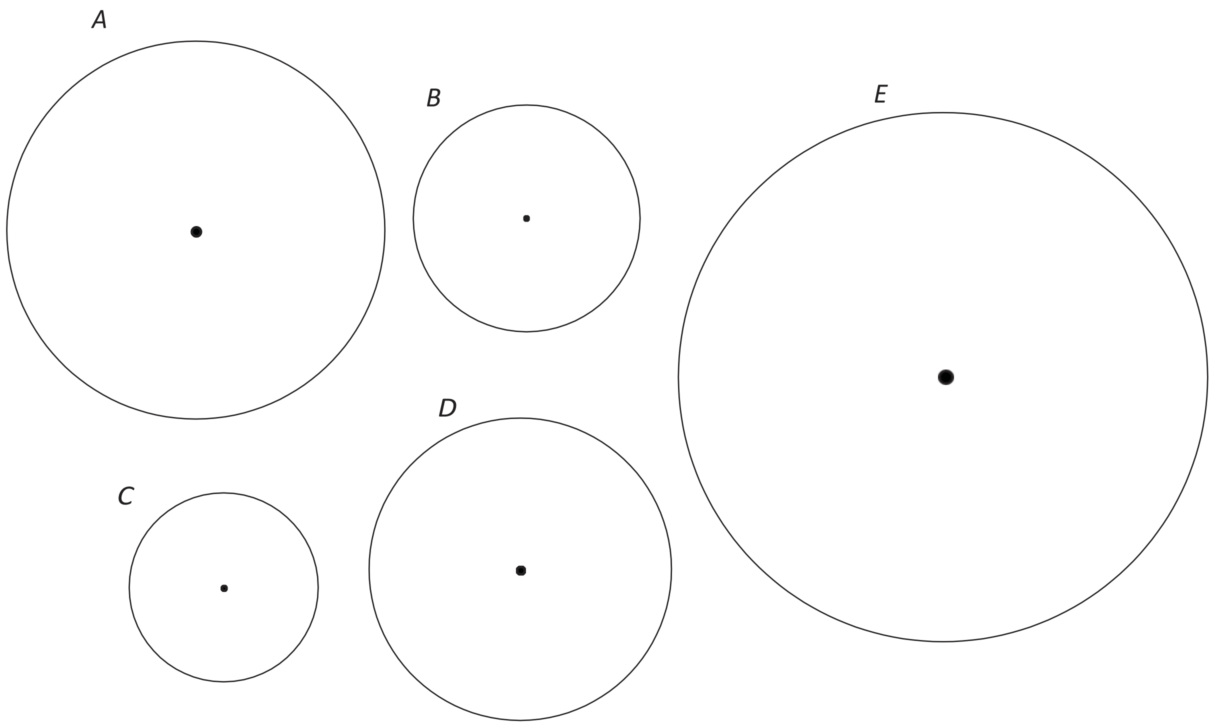
You can get a rough measurement of the **circumference** of a circle as follows:

- Use a string and lay it around the edge of the circle as accurately as possible.
- Mark the string when you reach the point where you first started measuring.
- Straighten the string and measure the length using a ruler.



Circles of different sizes are given below. The circumferences are shown in the table in question 2, rounded off to two decimal places. Copy the table.

1. Measure the diameter of each circle and write it in the table.



2. Use a calculator to work out the answers in the last column. (Round off to two decimal places.)

Circle	Diameter (cm)	Circumference (cm)	Circumference ÷ diameter
A		15,71	
B		9,42	
C		7,85	
D		12,57	
E		21,99	

3. What do you notice?

pi (π) and the formula for the circumference of a circle

In the previous activity, you should have found that the circumference of a circle divided by its diameter is always equal to the same number. This number is a constant value and is called **pi**. *Pi*is a Greek letter and its symbol is π.

You also worked with values rounded off to two decimal places (hundredths). However, π is an **irrational number**. This means that the numbers after the decimal comma go on and on without ending and without repeating. On a calculator, you will find that the value for π is given as 3,141592654 (correct to nine decimal places).

When we use π in our calculations, we usually round it off as $\pi \approx \frac{22}{7}$ or 3,14.

In the previous activity, you found that, for any circle, $\frac{C}{d} = \pi$ (the circumference divided by its diameter is equal to the constant, π). Therefore, if we multiply the diameter of a circle by π, we should get the circumference of the circle:

$$\begin{aligned}\text{Circumference of a circle } (C) &= \pi d \\ &= \pi(2r) \\ &= 2\pi r\end{aligned}$$

using the formula for the circumference of a circle

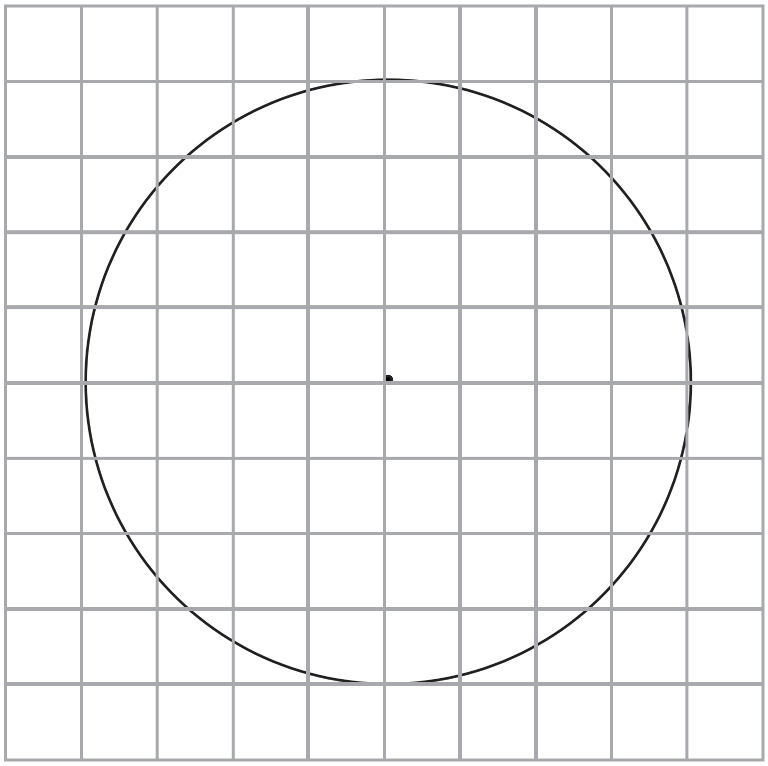
In the following calculations, use $\pi = 3{,}14$ and round off your answers to two decimal places where necessary.

1. Calculate the circumference of a circle with:
   (a) a radius of 2 cm
   (b) a radius of 10 mm
   (c) a diameter of 8 cm
   (d) a diameter of 25 mm
   (e) a radius of 40 m
   (f) a diameter of 100 m
2. Calculate the radius and circumference of a circle with a diameter of:
   (a) 125 mm
   (b) 70 cm
3. Calculate the radius of a circle with a circumference of:
   (a) 110 cm
   (b) 200 m

16.4 Area of circles

investigating the formula for the area of a circle

1. Each square in the grid on the following page measures 1 cm by 1 cm (1 cm^2).
   (a) Count the number of squares inside the circle. Estimate what the parts of squares add up to. What is the area inside the circle?
   (b) What is the radius (r) of the circle?
   (c) How accurate is the above method for finding the area of a circle?
   (d) How can we improve on this method of using squares to approximate the area of a circle?



(e) Suppose instead of using 1 cm by 1 cm squares we use 0,5 cm by 0,5 cm squares to measure the area of the circle above. Which of the two measurements of area will be more accurate? Explain.

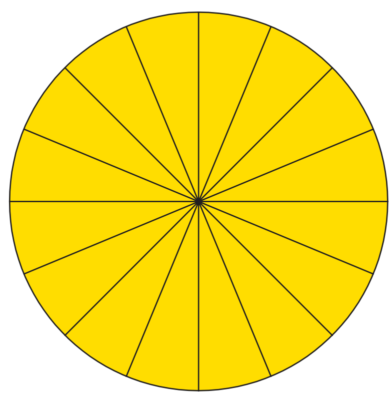
(f) Now suppose we use squares that are 0,25 cm by 0,25 cm. Which measurement will be the best estimate of the three?

We can estimate area by placing a square grid over the surface of which we want to estimate. We can then count approximately how many squares are needed to cover the surface we wish to measure.

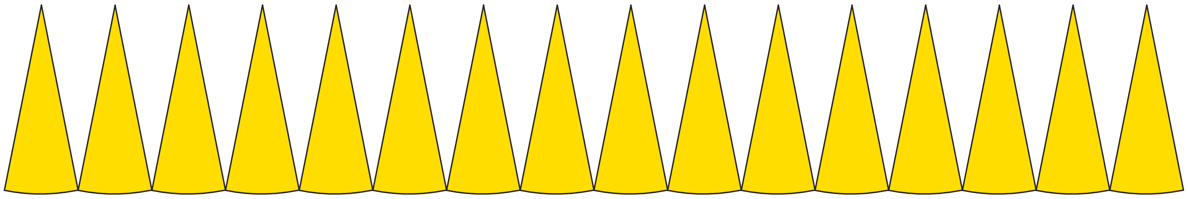
In the case of a curved surface like a circle, the area cannot be accurately determined in this way; it can only be estimated. The accuracy of the estimate depends on the size of the squares used.

In this activity we are going to develop a formula for calculating the area of a circle.

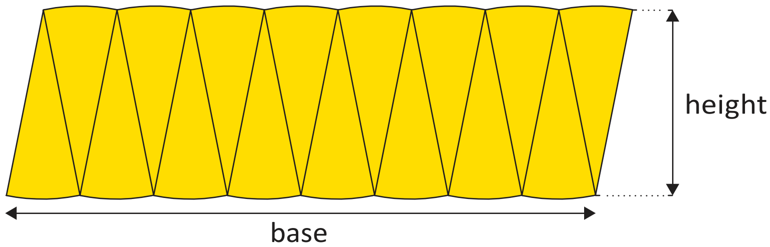
Consider the circle alongside. It has been divided into 16 identical sectors. We will use a technique that mathematicians sometimes use to transform a shape into one that they know something about in order to solve a problem.



The challenge here is that we want to find a way to calculate the area of a circle. We know how to find the area of a rectangle. Is there a way that we can redraw a circle so that it looks something like a rectangle? One way to go about this is to divide the circle into 16 identical sectors. We then cut the circle into 16 different pieces as shown below:



We then re-arrange the sectors like this:

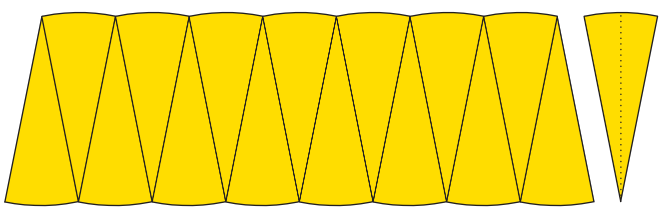


2. We have transformed the circle by cutting it into identical sectors and re-arranging them. What does this shape look like?

3. What does the:
   (a) height of the shape above match in the original circle?
   (b) base of the shape match in the original circle?

4. Is there a way in which we can make the challenge easier for ourselves?

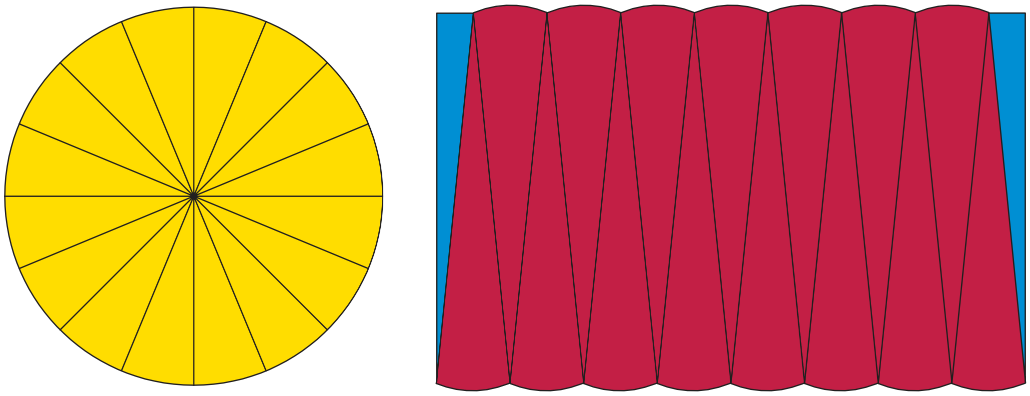
5. The last sector in the arrangement below is further divided in half.
   (a) What shapes are formed from dividing the sector?



   (b) What new shape will be formed by placing each half of the sector on either side of the shape above?

6. To what does the:
   (a) height of the new shape correspond in the original circle?
   (b) base of the new shape correspond in the original circle?

You have probably noticed that when we divide a circle into many sectors and then re-arrange the sectors, they form a rectangular shape. Try to make sense of the argument presented below.



$C = 2\pi r$

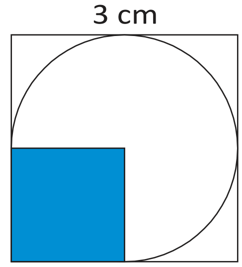
Area = $l \times b$

Area = $\frac{1}{2} \times 2 \times \pi \times r \times r$

Area = πr^2

7. (a) Use the formula $A = \pi r^2$ to calculate the area of a circle with a radius of 4 cm. Use π = 3,14.
   (b) How close is this answer to the number of squares you calculated inside the circle in question 1 on page 182?

From now onwards we will use the **formula** $A = \pi r^2$ to calculate the **area of a circle**, where r is the length of the radius. You will be given the value of π to use in the calculations. The value of π is usually given correct to two decimal places as 3,14.



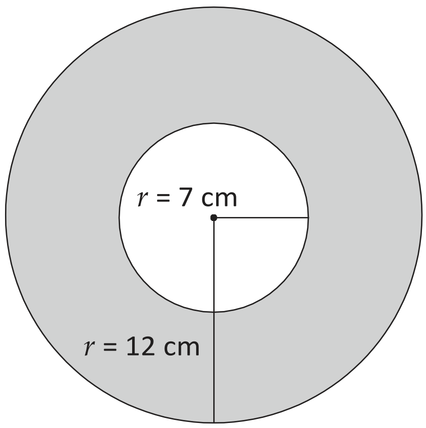
8. How can we interpret r^2 in the formula $A = \pi r^2$? Use the figure on the right to answer the following questions:
   (a) What is the radius of the circle?
   (b) The length of the blue square is 1,5 cm. What is its area?
   (c) What is the value of r^2?
   (d) If r is the radius of a circle, then r^2 is ...

using the formula for the area of a circle

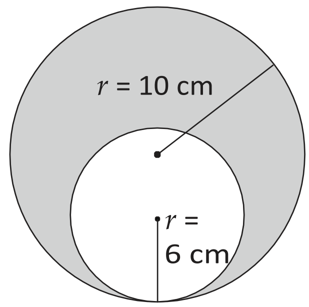
In the following calculations, use 3,14 as an approximation for π and round your answers off to two decimal places. Use a calculator where necessary.

1. Calculate the area of a circle with a radius of:
   (a) $r = 8$ cm (b) $r = 4,5$ cm
2. Calculate the radius of a circle with the following area:
   (a) 100 m^2 (b) 76m^2
3. Work out the area of the shaded parts of the following shapes:

(a)



(b)



16.5 Converting between square units

You already know how to convert between units we use to measure lengths or distances, for example mm, cm, m and km:

To convert	Do this	To convert	Do this
cm to mm	× 10	mm to cm	÷ 10
m to cm	× 100	cm to m	÷ 100
km to m	× 1 000	m to km	÷ 1 000

Use this knowledge to work out how to convert between square units (mm^2, cm^2, m^2 and km^2). Copy and complete the following conversion:

1. Convert cm^2 to mm^2
   1 cm^2 = 1 cm × 1 cm
   = 10 mm × 10 mm
   = ____

2. Convert m^2 to cm^2
   1 m^2 = 1 m × 1m
   = ____ cm × ____ cm
   = ____

3. Convert km^2 to m^2
   1 km^2 = ____ km × ____ km
   = ____
   = ____

4. Convert mm^2 to cm^2
   1 mm^2 = 1 mm × 1 mm
   = 0,1 cm × 0,1 cm
   = ____

5. Convert cm^2 to m^2

$1\ cm^2 =$ ____ cm × ____ cm

= ______________

= ______________

6. Convert m^2 to km^2

$1\ m^2 =$ ____ m × ____ m

= ______________

= ______________

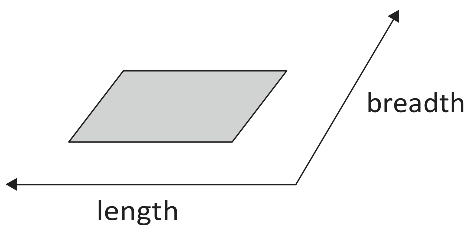
7. Copy and complete the following table:

To convert	Do this	To convert	Do this
cm^2 to mm^2		mm^2 to cm^2	
m^2 to cm^2		cm^2 to m^2	
km^2 to m^2		m^2 to km^2	

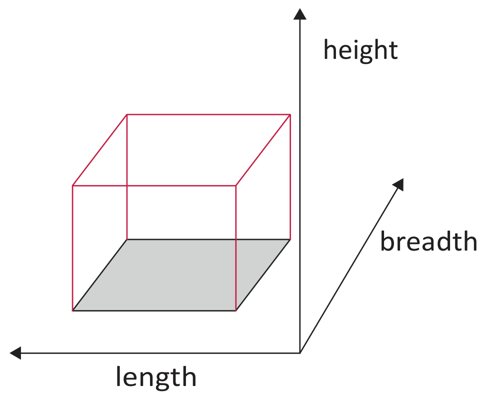
Chapter 17
Surface area and volume of 3D objects

17.1 From 2D to 3D measurements

Remember that 2D shapes have only length and breadth, while 3D objects have length, breadth and height.



2D shape



3D object

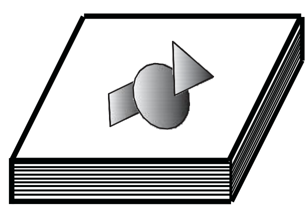
A 2D shape has only one surface. We call the size of this flat surface the **area** of the shape.

A 3D object has more than one surface. For example, a cube has six surfaces, or faces. The sizes of these surfaces on the outside of the 3D object are called its **surface area**.

A 2D shape is flat, so it takes up space in only two directions. But a 3D object has height as well, so it takes up space in a third direction also. The space that a 3D object takes up is called its **volume**.

investigating the surface area and volume of a book

Work with a partner. Choose a book each. The books must be different sizes.



1. Run your hand over all the outside surfaces of your book. How many surfaces (or faces) does your book have?
2. Estimate whether the surface area of your book is bigger or smaller than that of your partner's book.

3. If you were to cover the book with wrapping paper, explain how you would calculate the minimum size of paper you would need.
4. Estimate whose book takes up the most space. How could you calculate which book really takes up the most space?

17.2 Surface area of 3D objects

using nets to explore surface area

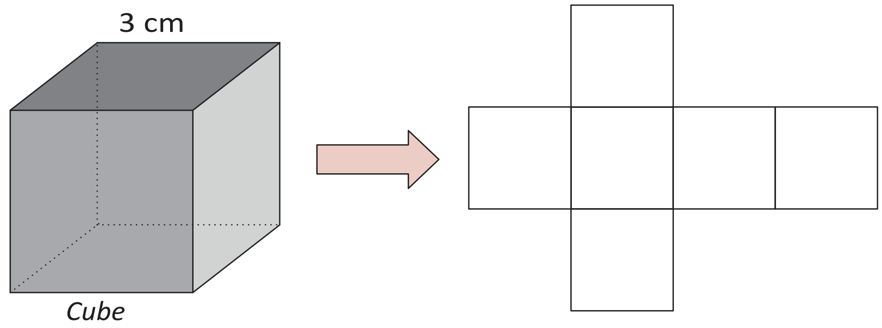
The **surface area** of an object is equal to the sum of the areas of all its faces. So we can use the net of an object to investigate its surface area.

A net is a flat shape that can be folded to make a 3d object.

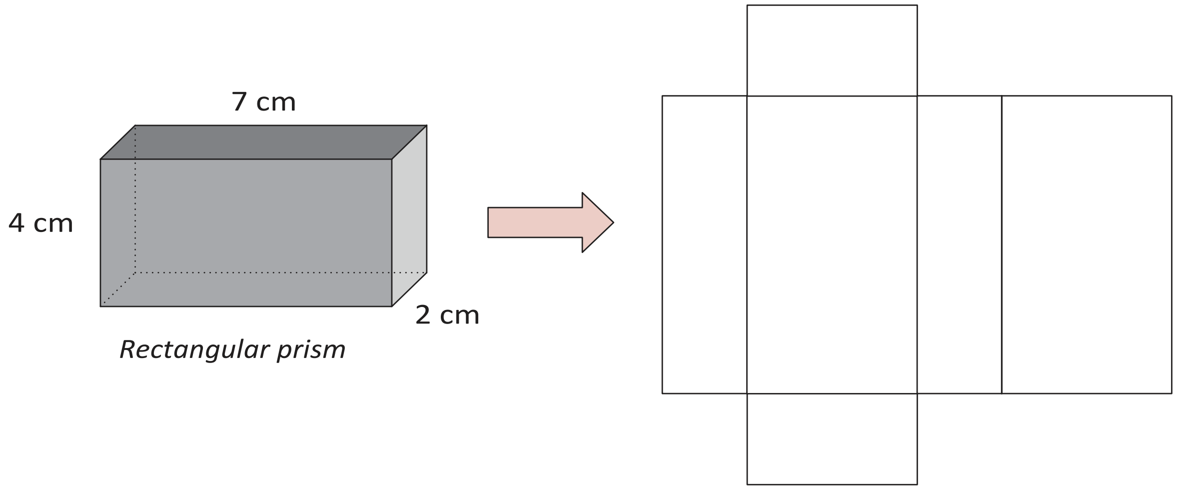
The following diagrams show 3D objects with their matching nets:

1. Use the measurements given to calculate the area of each face shown by the net. (Use your calculator if necessary and round off to two decimal places.)
2. Add up the areas to calculate the surface area of each object.

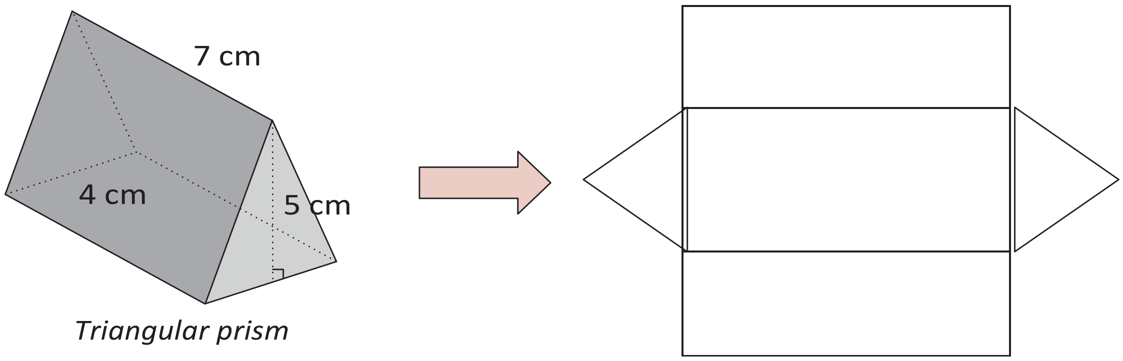
A



B



C

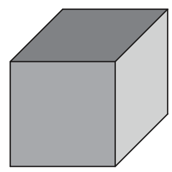


Triangular prism

deducing formulae for surface areas

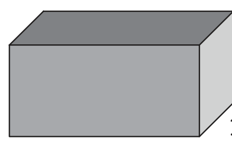
The surface area of a prism = the sum of the areas of all its faces

1. (a) Use the general formula above and the work you did on the cube on page 189 to determine which of the following formulae are correct. Write down the correct one(s).
   - Surface area of a cube = 4 × s
   - Surface area of a cube = $s \times s \times s \times s$
   - Surface area of a cube = 6 ×s^2
   - Surface area of a cube = s^6

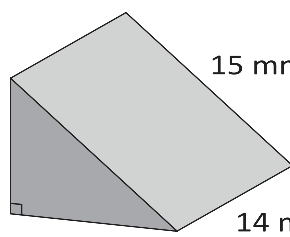
   

   (b) Explain your choice above.

2. (a) Write a formula for the surface area of any rectangular prism.
   (b) Explain your formula.

3. (a) Write a formula for the surface area of any triangular prism.
   (b) Explain your formula.

4. Use the formulae in questions 1 to 3 to calculate the surface areas of the cube, rectangular prism and triangular prism shown in questions 1 to 3.

   Surface area of cube: ...

   Surface area of rectangular prism: ...

   Surface area of triangular prism: ...

surface area calculations

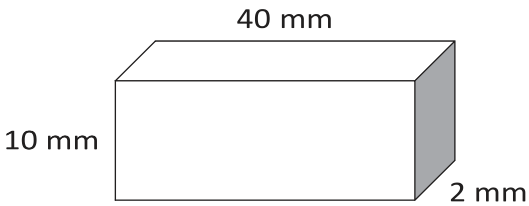
Work out the surface areas of the following four objects.
Give all answers in cm^2.

Remember:

$1\ cm^2 = 100\ mm^2$	$1\ mm^2 = 0{,}01\ cm^2$
$1\ m^2 = 10\ 000\ cm^2$	$1\ cm^2 = 0{,}0001\ m^2$
$1\ km^2 = 1\ 000\ 000\ m^2$	$1\ m^2 = 0{,}000001\ km^2$

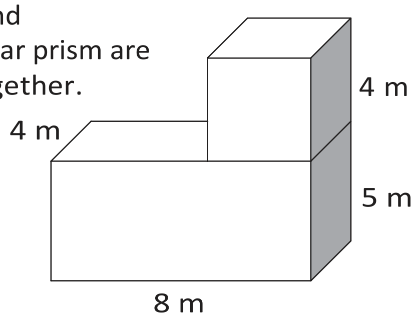
It may be a good idea to sketch the net for each object before doing the calculations.

1.

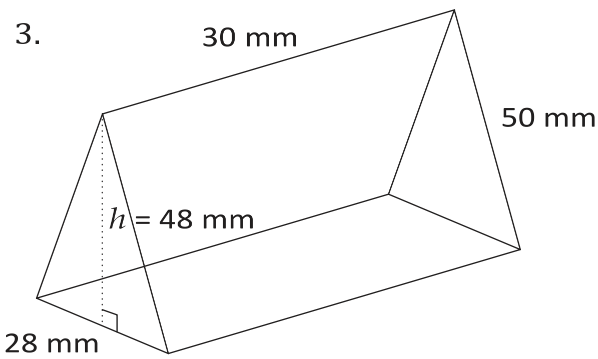


2.

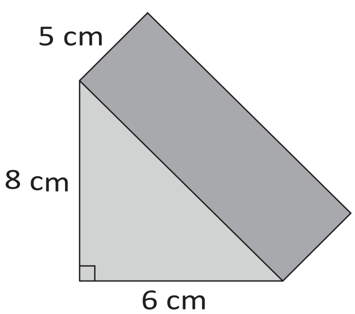
A cube and rectangular prism are glued together.



3.



4.

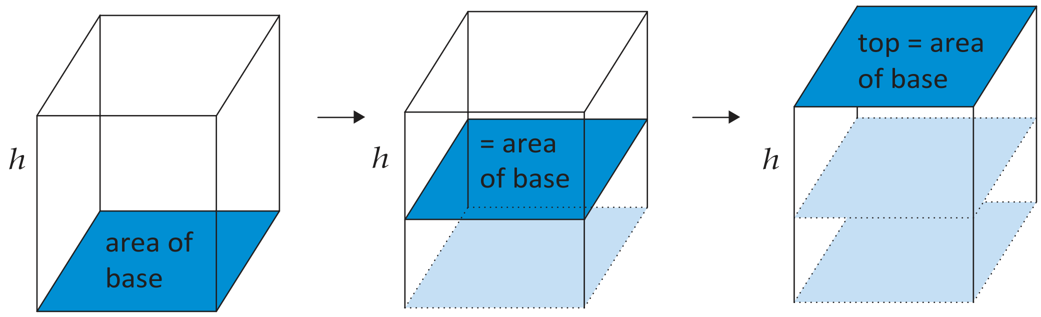


17.3 Volume of 3D objects

deriving formulae to calculate volume

Think of a prism and its base. If you were to move the base up to the top, between the lateral faces of the prism, the area of the base would remain exactly the same.

Lateral faces are faces that are not bases.

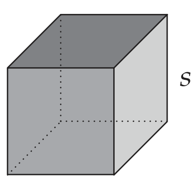


The volume of a prism = area of base × height

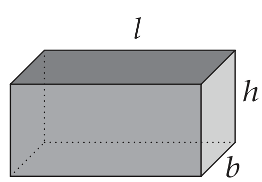
volume is the amount of space that an object takes up.

Use this general formula above to write the formula for the volume of a cube, a rectangular prism and a triangular prism.

A. Cube



B. Rectangularprism

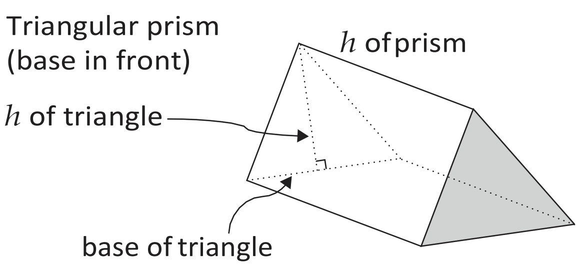


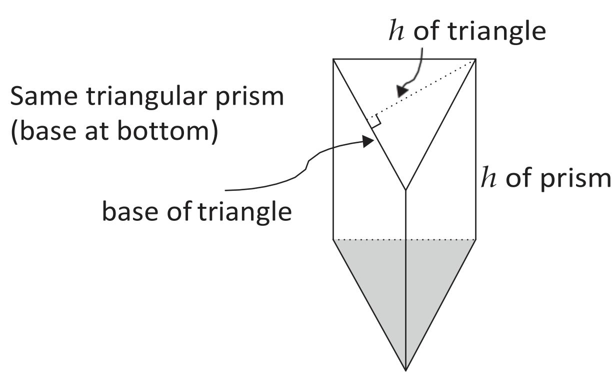
note about triangular prism

do not get confused between:

- the base of the prism and the base of the triangular face of the prism
- the height of the prism and the height of the triangular face of the prism.

C. Triangular prism





You should have found the following volume formulae:

Volume of a cube = s^3 or $s \times s \times s$

Volume of a rectangular prism = $l \times b \times h$

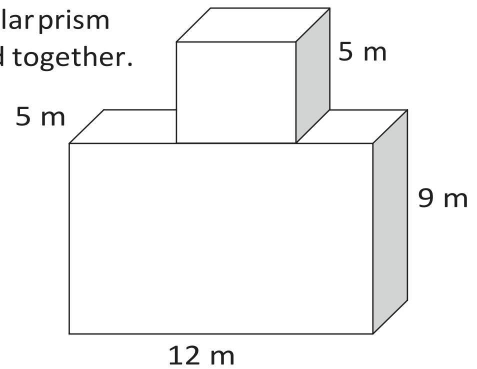
Volume of a triangular prism = $\frac{1}{2}(\text{base} \times h) \times \text{height of prism}$

Because we multiply three dimensions, the units used are cubic units, such as mm^3, cm^3 or m^3.

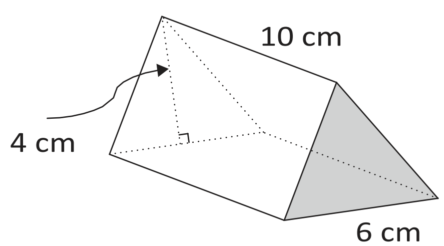
volume calculations

Calculate the volume of the following objects using the formulae given above:

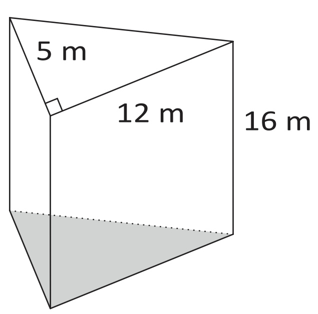
1. A cube and rectangular prism are glued together.



2.



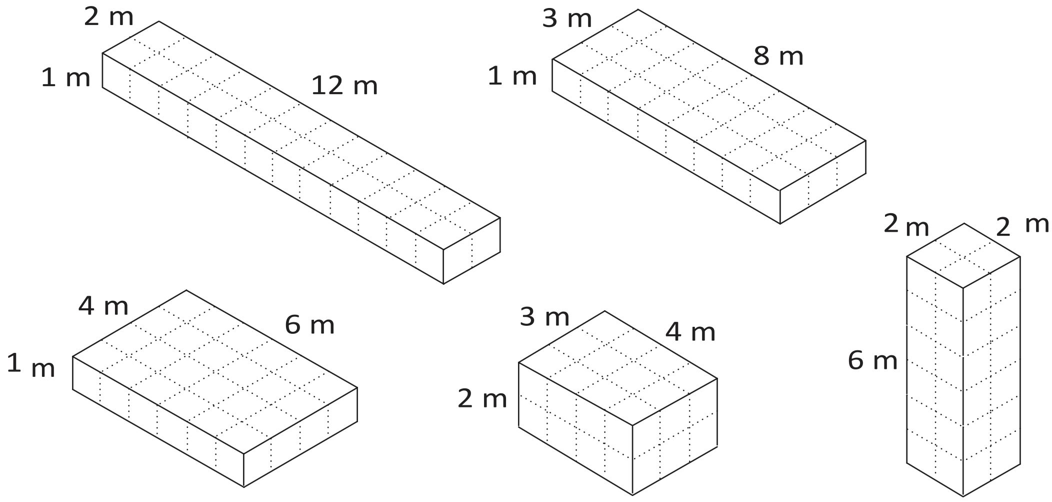
3.



17.4 Relationship between surface area and volume

Do objects with the same volume always have the same surface area? Do the investigation below in order to find out.

1. (a) Calculate the surface area and volume of the following five rectangular prisms by copying and completing the table below:



Length (m)	Breadth (m)	Height (m)	Surface area (m^2)	Volume (m^3)
12	2	1		
8	3	1		
6	4	1		
4	3	2		
2	2	6		

(b) In the last row of the table, write another set of dimensions (l, b and h) that will give the same volume but a different surface area as the ones already recorded.

2. Look at the completed table. What can you conclude about the surface area and volume of objects?

3. A rectangular prism has a volume of 8 m^3. Write down two possible sets of dimensions. Draw the prisms below with their dimensions written on the drawings.

4. The table on the following page shows surface area and volume calculations for cubes with different side lengths.

Side length of cube (m)	Surface area (m^2)	Volume (m^3)
1	6	1
2	24	8
3	54	27
5	150	125
8	384	512
10	600	1 000

(a) Look at the surface area column. Does the surface area increase or decrease as the side length of the cube increases?

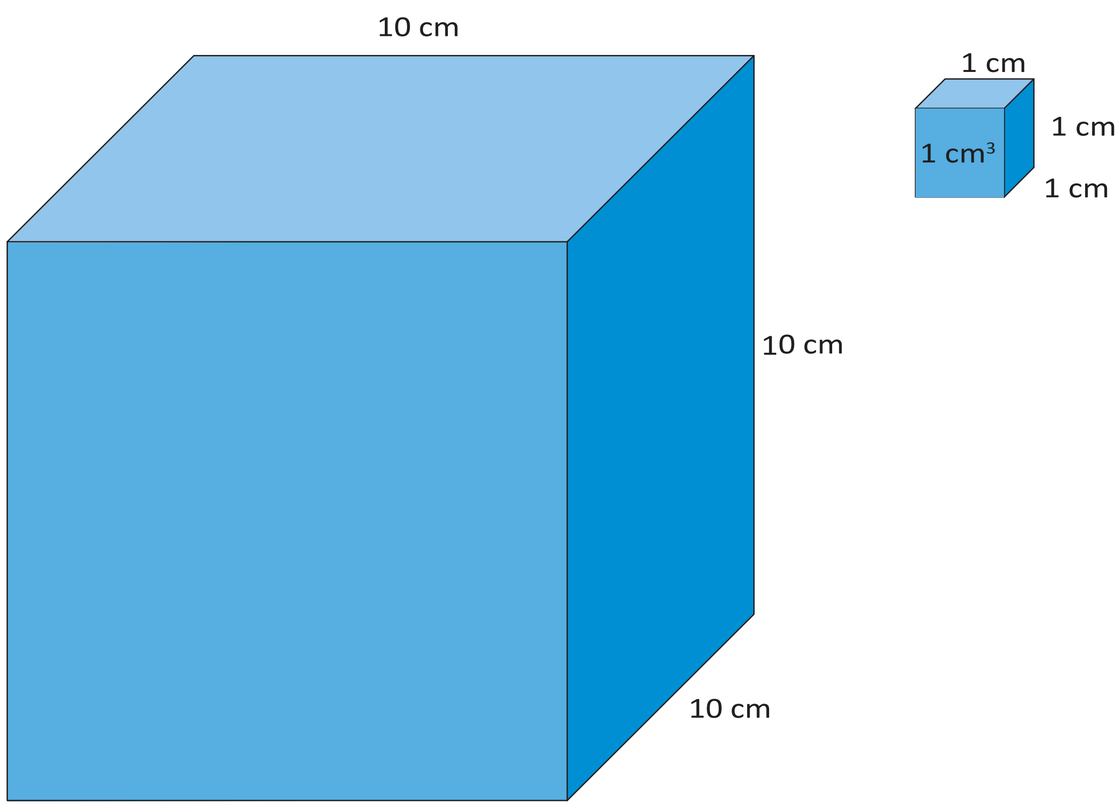
(b) Look at the volume column. Does the volume increase or decrease as the side length of the cube increases?

(c) Does **volume** or **surface area** increase more rapidly when the side length of the cube increases?

(d) Sketch a global graph of the volume of a cube versus its surface area.

17.5 Converting between cubic units

how many cubes?



1. The small cube has the dimensions of 1 cm × 1 cm × 1 cm and a volume of 1 cm^3. How many 1 cm^3 cubes will you need to form a large cube with dimensions 10 cm × 10 cm × 10 cm like the one shown on the previous page?
2. How many 10 cm × 10 cm × 10 cm cubes will form a 100 cm × 100 cm × 100 cm cube?
3. (a) To form a 1 000 cm^3 cube you need 1 000 cubes with a volume of 1 cm^3. If cubes of 1 000 cm^3 (10 cm × 10 cm × 10 cm) are then used to form a cube of 100 cm × 100 cm × 100 cm, how many 1 000 cm^3 cubes will there be?
   (b) What is the volume of this new cube?
   (c) How many cubes of 1 cm^3 will form a cube with a volume of 1 000 000 cm^3?
4. Which of the cubes given below has a bigger volume? Explain.
   A. A cube with a volume of 1 m^3
   B. A cube with a volume of 1 000 000 cm^3
5. (a) How many 1 mm × 1 mm × 1 mm cubes (1 mm^3) are needed to form a 1 cm × 1 cm × 1 cm cube?
   (b) What is the total volume of the 1 mm^3 cubes forming the 1 cm^3 cube?

practise converting between units

When working with volume, you often have to convert between different cubic units. Here are two examples of how you can work out equivalent units.

Converting cm^3 to mm^3:

$$\begin{aligned} 1\ cm^3 &= 1\ cm \times 1\ cm \times 1\ cm \\ &= 10\ mm \times 10\ mm \times 10\ mm \\ &= 1\ 000\ mm^3 \end{aligned}$$

$\therefore$ multiply by 1 000

Converting cm^3 to m^3:

$$\begin{aligned} 1\ cm^3 &= 1\ cm \times 1\ cm \times 1\ cm \\ &= 0{,}01\ m \times 0{,}01\ m \times 0{,}01\ m \\ &= 0{,}000001\ m^3 \end{aligned}$$

$\therefore$ multiply by 0,000001 or divide by 1 000 000

1. Write the following volumes in cm^3:
   (a) 3 mm^3 (b) 45 mm^3 (c) 0,6 m^3 (d) 1,22 m^3
2. Write the following volumes in mm^3:
   (a) 20 cm^3 (b) 151 cm^3 (c) 4,7 cm^3 (d) 89,5 cm^3
3. Write the following volumes in m^3:
   (a) 9 cm^3 (b) 50 cm^3 (c) 643 cm^3 (d) 1 967 cm^3
4. Write the following answers in cm^3:
   (a) 4 m^3 + 68 cm^3 (b) 12 m^3 + 143 cm^3

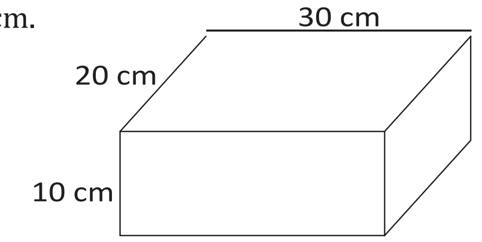
17.6 Capacity of 3D objects

difference between capacity and volume

Capacity is the amount of space available *inside* an object.

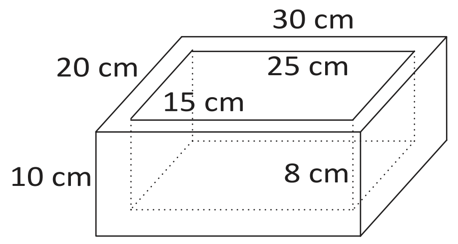
Volume is the amount of space that the object itself takes up.

1. A solid block of wood measures 30 cm × 20 cm × 10 cm.



   (a) What is its volume?

   The same solid block of wood is carved out to make a hollow container. The measurements inside the container are 25 cm × 15 cm × 8 cm.

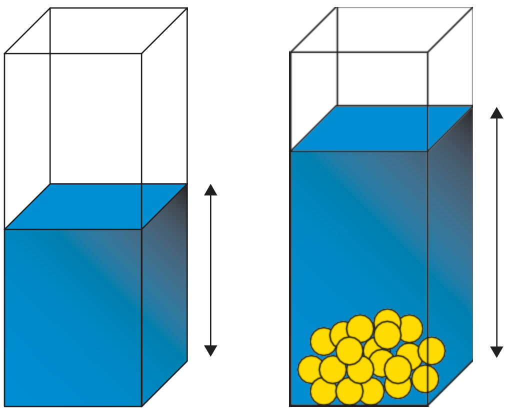


   (b) How thick are the walls of the container?
   (c) What is the capacity of the container?
   (d) If you filled the container with water, what volume of water would the container hold?

2. More of the wood is carved out of the container to make walls 1 cm thick at the sides and the bottom. Calculate the capacity of the container in litres.

displacement and more capacity calculations

Consider a glass vase half full of water. As soon as you place marbles into the water, the level of the water rises. This is not because the amount of water has changed, but rather because the marbles have taken the place of the water and have pushed the water higher up in the vase.



If one of the marbles has a volume of 1 cm^3, it would **displace** 1 ml of water.

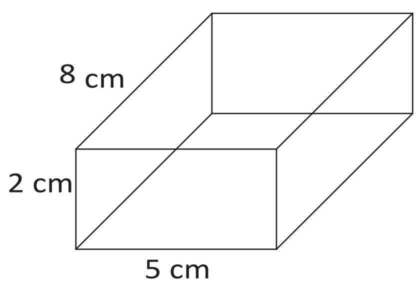
∴ We know that:

1 cm^3 = 1 ml
1 m^3 = 1 kl

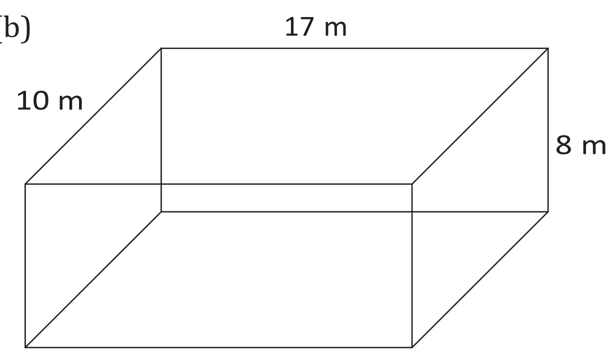
To displace means to move something out of its place.

1. Calculate the capacities of the following containers. The *inside* measurements are given. Write your answers in ml or kl.

   (a)



   (b)



2. Work out a possible set of inside measurements for a container with a capacity of 12 kl. Draw a sketch and write the measurements on it.

Chapter 18
Collect, organise and summarise data

The term **data handling** is used to describe certain ways of trying to make sense of large collections of observations (data) about things in the real world. Data can be about many different things, for example people's opinions on politics or the success rates of treating people with a certain kind of medicine. We use data to help us make decisions and solve problems about the world around us.

18.1 Collecting data

To find out more about any situation, we need to start by asking questions and collecting data. When you collect data, you need to consider:

- what you want to find out or the questions that you want to answer
- where you will find the data to answer the questions (for example, from people such as learners in your school, your family and community; or from published sources such as newspapers, books or magazines)
- who you will collect the data from (all of the people or a sample)
- how you will collect the data (such as using questionnaires or interviews).

sources of data collection

In some cases you can use data that has already been collected by another person or organisation.

Example 1

Your question is:

What is the most common form of transport that learners in South Africa use to travel to school?

For this question, you will find that this data already exists in a publication called *Census @ School 2009*, published by Statistics South Africa. You can then present and interpret the existing data.

Example 2

Your question is:

What is the most common form of transport that learners at my school use to travel to school?

Check with the principal whether such data has already been collected by the school. If the data does not exist, or is very old, you need to decide where to get the data from. You could then decide to collect the data yourself from your peers.

Copy the following table. For each of the following investigation questions, write down what or who would most likely be an appropriate source of information.

Question	Appropriate source of data collection
1. What is the favourite type of music amongst teenagers in my community?	
2. How much money do workers at Dress Factory earn per week?	
3. To what height do baobab trees usually grow?	
4. What are the ages of learners in Grades R to 7 in South Africa?	
5. How many people in South Africa have access to electricity?	
6. How many people in different African countries had malaria during the last five years?	
7. Has my school recycled more or fewer glass bottles this year than last year?	
8. What kind of household chores do seven- to ten-year-olds in my neighbourhood usually do?	
9. How many children in South Africa under the age of ten have been vaccinated to protect them from childhood diseases?	

populations and samples

The whole group of people (or things) that you want to find out about is usually referred to as the **population**.

A population is often quite large. The size of the population depends on what you need to find out. The larger your population, the more difficult it becomes to ask each member of that population the questions you want to ask.

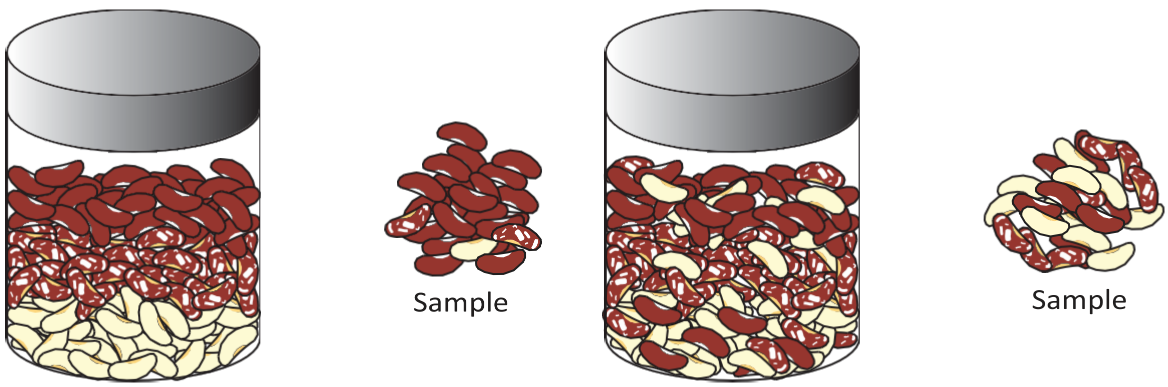
You could choose a smaller group of individuals from the population. Such a group, that is used to represent the whole population, is called a **sample**.

Examples

1. You may want to find out how much time the learners in a school spend on doing homework. If there are many learners in the school, you may be unable to ask them all about it. What you can do in this case is to talk to some learners in each class in the school. You may, for instance, speak to five learners from each class.
2. Health researchers may collect information about children by doing a survey of households selected randomly in each community.

random samples

A sample has to be chosen carefully to make sure that it represents the population. To understand what this means, think about what happens if you choose some beans from a jar in which the different kinds of beans are in separate layers. If you take a sample of the beans at the top, the sample is not representative. If the beans are all mixed up, then each bean has an equal chance of being chosen. The sample will be representative.



Example

You could select a random sample of learners from your school in the following two ways:

1. You may write the names of all the learners on separate paper strips. You then put all the strips in a plastic bag, mix them up, and draw 30 strips without looking at the names before you have finished.
2. You could select every fifth name from each of the class lists.

Look at the investigation questions. Which of the samples given do you think will reflect the whole population more appropriately? Copy the following table. Tick your choice and give a reason.

Sample 1	Sample 2	Reason
1. What is the type of music liked by most teenagers in my community?		
50 teenagers at a local school	25 teenagers each from two different local schools	

2. How much money do the 200 workers at Dress Factory earn per week?		
The workers at every fourth workstation in the factory	The 50 workers that gather outside during lunch break	
3. To what height do baobab trees usually grow?		
All baobab trees in a marked-off area	Every second baobab tree in a marked-off area	
4. What are the ages of learners in Grades R to 7 in South Africa?		
All the Grade R to 7 learners in my school	Ten learners in each grade from Grade R to 7 at three different schools	
5. Has my school recycled more or fewer glass bottles this year than last year?		
All the glass bottles recycled in one month this year and in the same month last year	The glass bottles recycled in one month this year and in any other month last year	

questionnaires

We can use different methods to collect data, for example questionnaires, face-to-face interviews or telephonic interviews. In this section, you will work with questionnaire questions that have multiple-choice responses.

Here are two questions with multiple-choice responses from which a **respondent** will choose.

A respondent is a person who responds to the questions.

How satisfied are you with our level of service? ☐ Not satisfied at all ☐ Fairly satisfied ☐ Very satisfied	What is the colour of your eyes? ☐ Brown ☐ Green ☐ Blue ☐ Other

1. Write a suitable question with multiple-choice responses to find out the following information:
   (a) What do teenagers spend their money on?
   (b) How much time do Grade 8s spend on homework every day?

2. Choose one of the questions from question 1 on page 201. Write down what you think would be the best sample to use if you had to conduct this investigation.

3. Use the multiple-choice question that you chose in question 2 to collect the data. Keep the results for the nextsection.

18.2 Organising data

The way we organise and summarise data depends on the kind of data that we have. It also depends on what we want to find out from the data. Work in groups to explore this. Do not worry about getting the answers right at this stage. You will learn about the different ways in which to organise and summarise data in this chapter.

Look at the following sets of data. For each one, discuss with your group and write down what you want to find out and what you think you would need to do to the data.

1. Data collected to find out which day would be best to have a soccer club practice:

   Twenty-five learners' preferred day for soccer practice

   Tuesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Friday Friday Friday Tuesday Thursday Wednesday Wednesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Tuesday Wednesday Monday Thursday Tuesday

2. Data collected to find out whether five-year-old children in a certain village have healthy body weights:

   Body weights of twenty-five children in kilograms, rounded off to the nearest 0,5 kg

   17 kg 16,5 kg 13,5 kg 14 kg 18 kg 18 kg 14 kg 21 kg 13,5 kg 15 kg 15 kg 14,5 kg 15,5 kg 19,5 kg 17 kg 17,5 kg 14 kg 14 kg 20 kg 14,5 kg 16 kg 18 kg 12 kg 16 kg 19 kg

3. Data collected to find out how many learners answer a certain type of question in less than 20 seconds:

   Time taken (in seconds) by a group of learners to answer a question

   20 25 24 33 13 26 10 19 39 31 11 16 21 17 11 34 14 15 21 18 17 38 16 21 25

4. Data collected to analyse the monthly salaries of employees of a small business:

   The monthly salaries of ten employees

   R8 000 R2 500 R75 000 R6 000 R7 500 R5 200 R4 800 R10 300 R15 000 R9500

tally marks, tables and stem-and-leaf displays

In Grade 7, you learnt about using tally tables and stem-and-leaf displays. We revise these two ways of organising data here.

We can use **tally tables** to record data in different categories. We draw a tally mark (|) for each item we count. We group tally marks in groups of five to count them quickly.

A **stem-and-leaf display** is a way of listing numerical data. If the numbers in a set of data consist of digits for tens and units (such as 23, 25, 34), the column on the right (the leaf column) shows the units digits of the numbers, and the column on the left (the stem column) shows the tens digits of the numbers.

Examples of tally marks:
A count of three = | | |
A count of five = ~~||||~~
A count of seven = ~~||||~~ | |

If the numbers in a data set consist of three digits (such as 324, 428, 526), both the hundreds and tens digits are written in the stem column and the units digits are written in the leaf column, for example: 32 | 4 would show 324.

Let's see how to display these numbers using the stem-and-leaf method:

12, 13, 20, 34, 35, 47, 49, 51, 53, 53, 53, 56, 59

The numbers range from 12 to 59, so the first digits represent the numbers 10 to 50.

Key: 1 | 2 means 12

stem	leaves
1	2, 3
2	0
3	4, 5
4	7, 9
5	1, 3, 3, 3, 6, 9

Values with the same stem are written in the same row. different leaves with the same stem are separated by a space or a comma.
In the first row, 1 | 2, 3 shows 12 and 13.

The stem column shows the tens digit of each value.

The leaf column shows the units digit of each value.

Notice that the stem-and-leaf display also shows us what the data set looks like. We can quickly see that most numbers are in the 50s and that there is only one number in the 20s.

1. Look back at the three sets of data on page 202. Copy and fill in the table on page 204 to show which form of data organisation you can use for each of these. Write a short explanation.

	Tally table	Stem-and-leaf display
A. Preferred day for soccer practice		
B. Body weight of children		
C. Time taken to answer a question		

2. Use the data set about preferred days for soccer practice:

Twenty-five learners' preferred day for soccer practice

Tuesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Friday Tuesday Friday Tuesday Thursday Wednesday Wednesday Tuesday Tuesday Wednesday Monday Thursday Tuesday Tuesday Wednesday Monday Thursday Tuesday

(a) Copy the following tally table. Organise the data using the tally table.

Preferred day	Tally	Frequency

(b) Which day should they choose to have soccer practice? Why?

(c) Which day would be the worst day for soccer practice? Why?

3. Zandile collected data about the number of garments that each of her workers produced per day. The answers were as follows:

61, 58, 48, 59, 49, 51, 54, 67, 55, 70, 59, 60, 62, 59, 62, 63, 64, 48, 64, 55

(a) Record the data in the form of a stem-and-leaf display.

Key:

(b) Complete: Most values occur in the ...
(c) How many garments do the fastest and slowest workers make?

4. Use the data you collected in the investigation in question 3 at the top of page 202.
   (a) Decide whether a tally table or a stem-and-leaf display will organise the data best, and record the data.
   (b) What does your tally table or stem-and-leaf display show about your data?

grouping data in intervals

When there are many values in a data set, it is often useful to group the data items into **class intervals**.

Example

Height in centimetres	Frequency
130–140	6
140–150	13
150–160	31
160–170	30
170–180	10

The class interval does not include the highest number in each case. So, the height of 150 cm falls into the interval 150–160 cm; not into the interval 140–150 cm.

This is a grouped frequency table. It represents 90 data values, but the values themselves are not shown. Instead, we show the frequency or the number of values falling into that interval.

1. The table shows the body weights (in kilograms) of athletes competing in a tournament.

55,2	56,1	58,4	59,3	60,6	61,2	61,7	63,4
63,2	64,2	65,9	66,5	66,7	67,3	67,8	68,0
70,5	72,9	73,4	74,1	74,8	75,9	76,7	78,7

   (a) Group the weights into 5 kg intervals. List the intervals.
   (b) Use a table to show the frequency of each class interval. It is useful to fill in the tallies first and then count up the frequencies, so that you do not leave any data items out.

Body weights of athletes	Tally	Frequency

   (c) In which intervals are the highest numbers of athletes?

2. The following data shows the time taken (in minutes and seconds) by runners to complete a race:

34:30	34:59	35:36	36:58	40:08	40:55	41:33	43:18
44:26	45:40	48:13	48:49	49:15	50:08	52:09	53:36

(a) Group the times into suitable intervals. List the intervals.

(b) Record the grouped data in the form of a table.

(c) How long did the highest number of runners take to finish the race?

3. Take another look at the data about the time (in seconds) that learners took to answer a certain question:

20 25 24 33 13 26 10 19 39 31 11 16 21 17 11 34 14 15 21 18 17 38 16 21 25

(a) Group the data into three intervals of ten seconds. Copy and fill in the following table to show the grouped data:

(b) Do you think that learners will need at least 40 seconds to answer this type of question? Explain.

(c) Were there more learners who took at least 20 seconds or more to answer the question than learners who took less than 20 seconds? Explain.

18.3 Summarising data: measures of central tendency and dispersion

one number speaks for many: the mode and the median

1. A farmer wants to know whether he used good quality seed when he planted pumpkins. So he counts the number of pumpkins on each of a sample of 20 pumpkin plants. The numbers of pumpkins are given below:

6 7 3 7 4 7 7 8 7 5 7 7 6 7 8 5 4 7 6 7

(a) Arrange the data values from smallest to biggest, to get a clearer picture.

(b) The farmer says to his wife: *Most of the plants have seven pumpkins, so it is not too bad.* Do you think this is a good summary of the data, or should the farmer give some more information?

In some data sets some values or items are repeated often. The value or item that occurs most often is called the **mode**. Some data sets have more than one mode, and many data sets have none.

Instead of "most often" we can also say "most frequently".

(c) Do you think that if the farmer had said the following, his wife would be somewhat better informed about the pumpkin plants?

The number of pumpkins varies from three to eight, but there are seven pumpkins on most of the plants.

2. Look at the Mathematics test results, out of 30, of a small class of 21 learners:

15	7	11	7	13	4	8	9	3	7	25
7	6	10	8	9	23	19	7	5	7	

Bongile scored 9 out of 30 in the test, which is poor. Can he claim that his mark is in the top half of the class? Explain your answer well.

A set of data can be separated into a top half and a bottom half by arranging the data items from smallest to largest and finding the number between the two halves.

For example, the following data set:

23 35 44 21 28 32 38 41 39 42 24 27

... can be rearranged like this:

21	23	24	27	28	32		35	38	39	41	42	44
bottom half							*top half*					

The number that sits halfway between the upper item in the bottom half and the lower item in the top half in this case is 33,5 (calculation: [32 + 35] ÷ 2 = 33,5).

The number that separates a set of data into an upper half and a lower half is called the **median**.

Half of the items are above the median and half of the items are below the median. To find the median, the data items need to be arranged from smallest to largest.

If a numerical data set has an odd number of items, the median is equal to the number in the middle of the set, when the items are arranged from smallest to largest:

3 4 5 6 7 7 7 7 7 7 **8** 8 9 9 10 11 13 15 19 23 25

3. Write down any 11 different numbers so that the median is 24.

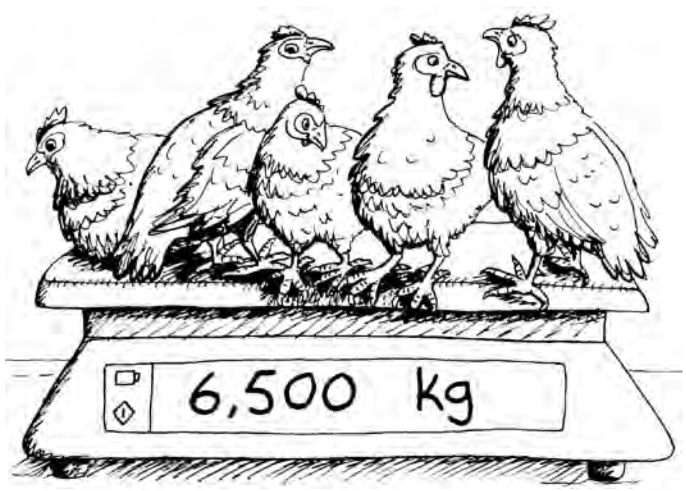
4. The body weights of 25 children are given below, all rounded off to the nearest 0,5 kg. This data was collected to find out whether five-year-old children in a certain village have healthy body weights or not.

17	16,5	13,5	14	18	18	14	21	13,5
15	15	14,5	15,5	19,5	17	17,5	14	
14	20	14,5	16	18	12	16	19	

Rearrange these data items into a bottom half and a top half, and state what the median body weight is.

5. (a) Does the data set shown above question 2 on page 207 have a mode, and if it has what is it?
   (b) What is the mode of the data set in question 2 on page 204?

if they were all equal ... but they are not



1. Five chickens are placed on a scale and the scale shows 6,500 kg, which is the same as 6 500 g. What can you say if someone asks you:
   What does each of the chickens weigh?

2. A roadside stall owner has ten watermelons to sell. They are not all of the same size and he did not pay the farmer the same price for each watermelon. So the stall owner weighs the watermelons and decides to sell them at the following prices:

   R16 R16 R18 R15 R14 R14 R16 R14 R13 R14

   (a) Check whether you agree that he will get R150 for the ten watermelons together.
   (b) The stall owner now decides to make the prices of all the watermelons the same, to simplify advertising and selling. What should he make the price of each watermelon, if he still wants to receive R150 for all of them together?

3. Susan bought six pumpkins at the market. Her husband, Abraham, asks her what she paid for each pumpkin. Susan says:
   They actually came at different prices, and I have forgotten the prices now. But I know I paid R72 in total so it would have been the same if I paid R12 each. So you can say that on average I paid R12 each.

   (a) Check if Susan's answer to her husband is correct. The actual prices she paid for the different pumpkins are given below:

   R7 R15 R10 R16 R9 R15

(b) How do you think Susan came to the R12 she used when she answered her husband's question?

(c) Check if Susan's answer would have been correct if the actual prices of the pumpkins were as follows:

R11 R12 R13 R11 R12 R13

When she gave an answer to her husband's question, Susan used the number 12 as a "summary" to represent the six different numbers 7; 15; 10; 16;9 and 15. The number 12 is a good representation of 7; 15; 10; 16; 9 and 15 together because:

$$7 + 15 + 10 + 16 + 9 + 15$$
$$= 12 + 12 + 12 + 12 + 12 + 12$$

If each value in a data set is replaced by the same number and the total remains the same, the "replacement number" is called the **mean** or **average**.

It can be calculated by dividing the total (sum) of the values by the number of values in the data set: Mean = sum of values ÷ number of values. (In the example above, the mean is 72 ÷ 6 = 12.)

Like the median, the mean (average) may not be equal to any of the actual values in the data set.

4. Look again at question 1 on page 208 about the five chickens. If you want to now give a different answer than before, write it down.

5. A journalist investigated the price of white bread at different stores in two large cities. The prices are given in cents at ten different shops in each city:

City A:	927	885	937	889	861	904	899	888	839	880
City B:	890	872	908	910	942	924	900	872	933	948

(a) If you just look at the above data, do you think one can say that white bread is cheaper in the one city than in the other? Look carefully, and give reasons for your answer.

(b) Calculate the mean price of white bread for the sample in each of the two cities.

(c) Find the median bread price in the sample for each of the two cities.

6. Geoffrey is a stock famer. He buys 21 goats at a mean price of R830 each.

(a) How much do the 21 goats cost, in total?

(b) One of the goats was a stud goat for which Geoffrey paid R4 800. What was the mean price of the other 20 goats?

7. (a) Find the mean and the median of the following data set:

1 1 1 1 1 1 1 1 2 2 2 2 2 2 130

(b) Write ten numbers so that the mean is much smaller than the median.

(c) Write ten numbers so that the mean is much bigger than the median.

(d) Write ten different numbers so that the mean is equal to the median.

8. Look at the times taken by the different learners in Grade 8A in a certain school, in seconds, to do question 7(b) on page 209:

20	30	36	14	20	14	29	39	15	37	35	24	
29	29	18	16	38	13	24	27	22	38	29	11	38

Look at the times taken by the different learners in Grade 8B in the same school, in seconds, to do question 7(b) on page 209:

20	22	39	22	16	37	36	15	14	13	16	10	14
26	11	14	31	17	11	28	39	20	35	26	20	

Which class works the fastest, Grade 8A or Grade 8B? Explain your answer very well.

how wide is the data spread?

1. Two samples were taken from the eggs produced on two different egg farms, to investigate the masses of the eggs coming from the two farms.

   The mean mass of the eggs from farm A is 50,6 g and the median mass is 52,0 g.

   The mean mass of the eggs from farm B is 50,3 g and the median mass is 52,0 g.

   (a) Do these figures indicate that the eggs from the two farms are similar, or that they differ?

   (b) The actual masses of the eggs in the two samples are given below. Check whether the mean and median masses given above are correct.
   Masses of the sample of eggs from farm A, in grams:

   51 54 45 53 49 54 55 46 54 45

   Masses of the sample of eggs from farm B, in grams:

   53 52 55 44 57 41 59 43 47 52

   (c) In what way do the masses of the eggs from the two farms differ?

The **range** of a set of data is the **difference** between the **maximum** (highest or top value) and the **minimum** (lowest or bottom value).

The values in the data set below vary from 36 to 60, hence the range is 60 − 36 = 24:

36 36 39 39 43 45 46 47 52 52 53 55 57 60

2. The following data shows the exam marks of two groups of learners:

Group 1:	30	31	35	50	55	58	60	70	78	80	88	88	90	90
Group 2:	55	55	56	57	59	59	59	67	69	75	80	80	80	81

   Compare the two groups by copying and completing the following statements:

   (a) In group 1 the marks vary from ______ to ______, a range of ______.

   (b) In group 2 the marks vary from ______ to ______, a range of ______.

3. These two sets of data show the prices of houses that have been sold in Towns A and B in one month:

Town A:	R321 000	R199 000	R181 000	R303 000
	R299 000	R248 000	R283 000	R315 000
	R405 000	R380 000	R322 000	

Town B:	R88 000	R122 000	R175 000	R166 000
	R107 000	R105 000	R1 114 000	R100 000
	R151 000	R1 199 000	R146 000	

(a) Read through the prices in each list and write down anything that comes to your mind when you look at the two sets of figures.

(b) You have been asked by the local newspaper to write a short paragraph on the prices of houses in the two towns. You want to make it quick and easy for the readers to get some sense of the prices in the two towns. Write your newspaper paragraph neatly.

The mean price for houses in the list for Town A is R296 000. This is very close to the median of R303 000. All the prices in Town A are within R115 000 of the mean.

The mean price for houses in the list for Town B is R315 727, which is more than double the median price of R146 000. Nine of the 11 houses in Town B cost far less than the mean, while the prices in Town A are more evenly spread on both sides of the mean.

4. Someone asks you about the house prices in Towns A and B, and you say: *The mean house price in Town A is R296 000, and the mean house price in Town B is R315 727.*

(a) Does this statement provide good information about the difference in pricing of houses in the two towns? In what way may it actually be misleading?

(b) What causes the mean to be a misleading way of describing the data for the house prices in Town B?

Data items like the house prices of R1 114000 and R1 199 000 in the list for Town B in question 3 are called **outliers** (or extreme values). Outliers are data values that are much lower or much higher than any other values in the data set. The mean is not a good way to summarise a set of data with outliers.

5. (a) Is there an outlier in this set of monthly salaries of the employees at a small business?

R8 000	R2 500	R75 000	R6 000	R7 500
R5 200	R4 800	R10 300	R15 000	R9 500

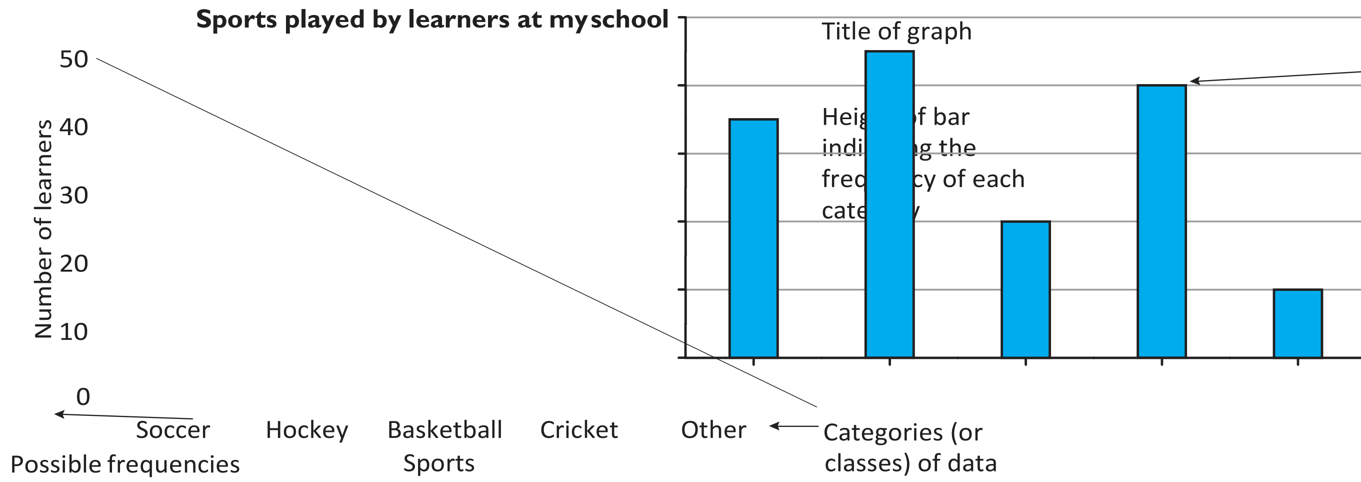
(b) Would the median prices of the two data sets be a good way to indicate the main difference between house prices in Towns A and B in question 3? Explain your answer.

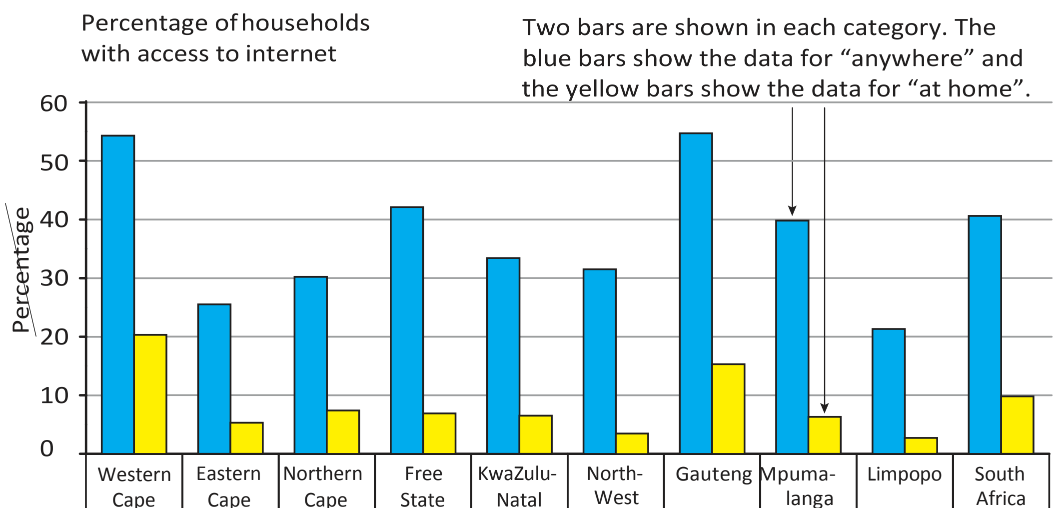
Chapter 19
Represent data

19.1 Bar graphs and double bar graphs

revising bar graphs and double bar graphs

A **bar graph** usually shows categories (or classes) of data along the horizontal axis, and the frequency of each category along the vertical axis, for example:





	Western Cape	Eastern Cape	Northern Cape	Free State	KwaZulu-Natal	North-West	Gauteng	Mpuma-langa	Limpopo	South Africa
Anywhere	54,3	25,5	30,2	42,1	33,4	31,5	54,7	39,8	21,3	40,6
At home	20,3	5,3	7,4	6,9	6,5	3,5	15,3	6,3	2,7	9,8

A key explains the colours used to distinguish the two sets of data.

Provinces

A **double bar graph** shows two sets of data in the same categories on the same set of axes. The graph at the bottom of the previous page shows the percentage of households *with* access to the internet at home, or for which at least one member has access elsewhere, by province in 2012.

representing data in bar graphs and double bar graphs

1. Road accidents are a big problem in South Africa, especially during the holiday season. Statistics about road accidents are published to make people aware of this problem.
   (a) Copy the table. Round off the numbers in the second column to the nearest hundred and write the result in the third column.

Year	Number of road accident deaths	Rounded-off number
2002	3 661	
2003	4 445	
2004	5 234	
2005	5 443	
2006	5 639	

   (b) Draw a bar graph of the rounded-offnumbers.
   (c) What trend do you notice in thisdata?
   (d) For this form of representation, do you think it makes a difference that you have rounded the data off? Explain.

2. Road accident data can be analysed in different ways. The table below shows the kinds of vehicles and the number of accidents that they were involved in, for 2011. The data is from the "Arrive Alive" campaign. Copy the table:

Vehicle type	Number of accidents	Rounded-off number
Cars	6 381	
Minibuses	1 737	
Buses	406	
Motorcycles	289	
LDVs and Bakkies	2 934	
Trucks	861	
Other and unknown	1 161	
Total	**13 769**	

(a) A large proportion of the data involves “other and unknown” vehicle types. What could the reason be for this?
(b) What information is missing from the table? What would we need to know to get a better picture of these accidents?
(c) Which vehicle was involved in the highest number of accidents? Does this mean that this vehicle is the least safe? Explain.
(d) Round off the data in the table to the nearest 100. Then draw a bar graph of the rounded-off data.

3. Statistics South Africa released the data below in their 2012 General Household Survey (GHS):

Percentage of people 20 years and older with no formal schooling

	WC	EC	NC	FS	KZN	NW	GAU	MPU	LIM
2002	4,4	12,5	16,5	10,0	11,8	14,6	4,5	17,1	20,1
2012	1,5	6,4	8,5	4,8	7,8	8,8	1,9	10,6	11,6

(a) Why do you think the data from 2012 is compared to 2002?
(b) Plot a double bar graph of this data.
(c) Explain the data for Limpopo, by copying the sentences below and filling in the missing percentages:
The percentage of people over 20 who had no formal schooling in Limpopo in 2002 was ___ ___ . In 2012, the survey showed that the percentage of people with no formal schooling was ___ ___ . The difference in the percentages is ___ ___ .
(d) From the graph, which provinces showed the least change in the percentage of people with no formal schooling? Explain how you know this and give a suggestion about why this could be so.

19.2 Histograms

what histograms represent

A histogram is a graph of the frequencies of data in different **class intervals**, as shown in the example on the following page. Each class interval is used for a range of values. The different class intervals are consecutive and cannot have values that overlap. The data may result from counting or from measurement.

A histogram looks somewhat like a bar graph, but histograms are normally drawn without gaps between the bars.

Example

Look at the following numbers of oranges harvested from 60 trees in an orchard:

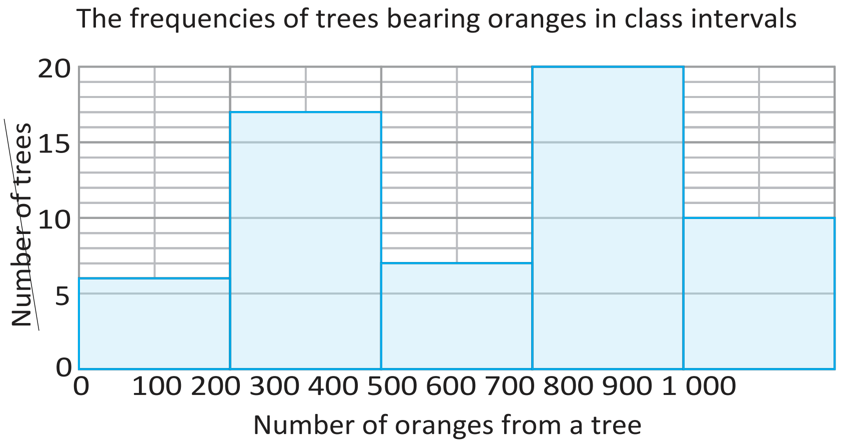
830	102	57	726	400	710	333	361	295	674	927	945
276	792	787	765	540	785	305	104	88	203	224	974
852	716	790	145	755	661	728	637	319	221	766	764
397	734	856	775	330	659	211	918	345	360	518	822
818	727	346	279	804	478	626	324	478	471	69	462

The frequencies of trees with numbers of oranges in specific class intervals are shown in the following table:

Number of oranges	Number of trees
0–200	6
200–400	17
400–600	7
600–800	20
800–1 000	10

We follow the convention that the top value (alsocalled the **upper boundary**) of each class interval is not included in the interval. The value of 400 is therefore included in the interval 400–600 and not in the interval 200–400.

Lets look at how this data is then represented in the form of a histogram:



representing data in histograms

1. In the 2009 Census@School, the learners from Grades 3 to 7 at a certain school were asked how long (in minutes) it takes them to travel to school. The following table shows the results from a sample of 120 learners:

Time in minutes	Frequency
0–10	15
10–20	48
20–30	34
30–40	14
40–50	6
50–60	1
60–70	2

(a) In which interval is a travel time of 30 minutes counted?
(b) Draw a histogram to illustrate this data.
(c) Describe in your own words what the histogram shows.
(d) What would you expect this data to look like for a school in a farming area?

2. Company A manufactures light bulbs. They want to see how many hours (h) their light bulbs last, as they would like to use that data to promote their light bulbs. They investigate a sample of 200 light bulbs straight from the factory. Look at the following data they collect:

Lifetime (h)	300–350	350–400	400–450	450–500	500–550
Frequency	15	25	70	50	40

(a) Draw a histogram to represent this data.
(b) Company B, which makes similar light bulbs, carries out a similar experiment and gets the following results. Draw a histogram to represent this data:

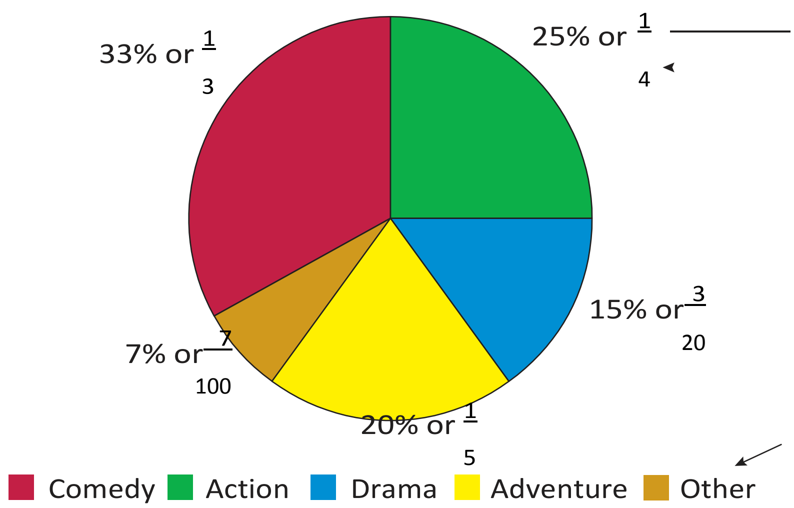
Lifetime (h)	300–350	350–400	400–450	450–500	500–550
Frequency	7	11	24	18	0

(c) Comment on the differences between the two histograms.

19.3 Pie charts

A **pie chart** consists of a circle divided into sectors (slices). Each sector shows one category of data. Bigger categories of data have bigger slices of the circle. The whole graph shows how much each category contributes to the whole.

Star High School learners' most popular types of movies



This pie chart shows five categories of data.

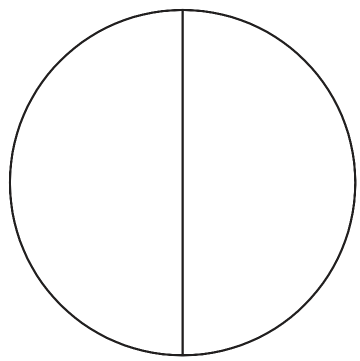
The size of each slice is equal to the fraction or percentage of the whole that the category forms.

The key (or legend) shows the category that each colour represents.

estimating the size of slices in a pie chart

In Grade 7, you learnt how to estimate the fractions or percentages of a circle in order to draw pie charts.

1. (a) Copy and complete the following pie chart to show that $\frac{1}{2}$ of the class walk to school, $\frac{1}{4}$ travel by train and $\frac{1}{4}$ travel by car.



   (b) Determine what percentage of learners:
   - walk
   - travel by train
   - travel by car.

   (c) There are 40 learners in the class. Determine how many:
   - walk
   - travel by train
   - travel by car.

2. The following data shows the highest level of schooling completed by a group of people:

Highest level of schooling completed	Number of people	Fraction of whole	Percentage of whole
Some primary school grades	36		
All primary school grades	54		
Some high school grades	72		
All high school grades	18		
Total	**180**		

(a) How many people make up the whole group?
(b) Copy the table. Complete the third column by working out the fraction of the whole group that each category makes up.
(c) Complete the fourth column by working out the percentage of the whole group that each category makes up.
(d) Draw a pie chart showing the data in the completed table. (Estimate the size of the slices.)

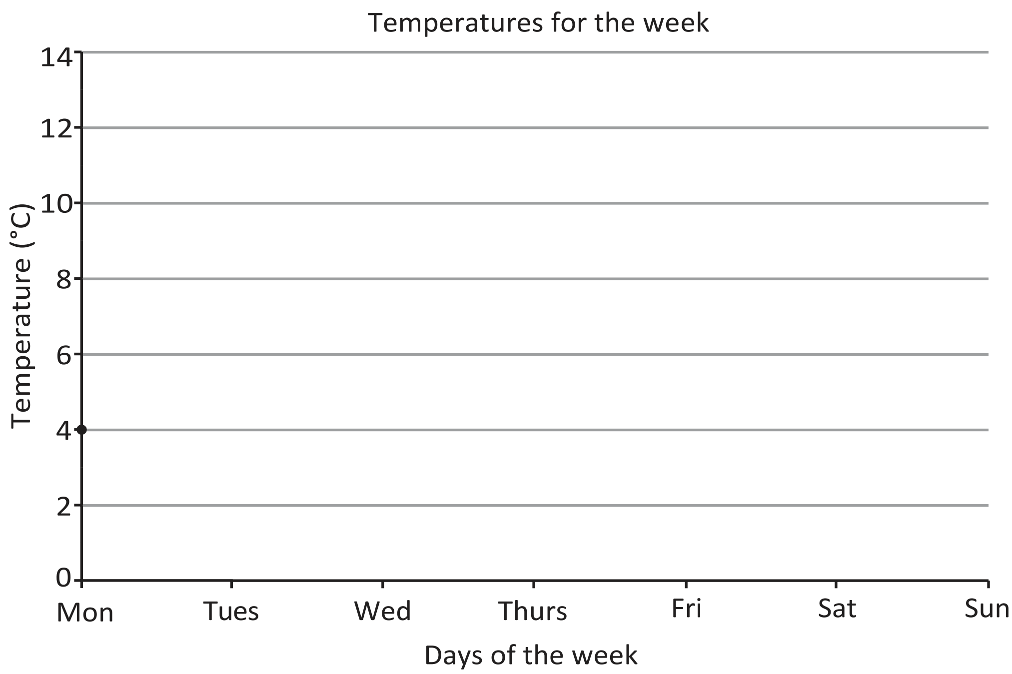
19.4 Broken-line graphs

plotting data points

The following table shows the average temperature in Bethal recorded every day for one week:

Day	Mon	Tues	Wed	Thurs	Fri	Sat	Sun
Temperature (°C)	4	10	12	9	13	13	11

1. Copy the axes below. Plot the data on the set of axes given. Make a dot for every point that you plot.



2. Use a ruler to join the dots in order.

You now have drawn a broken-line graph!

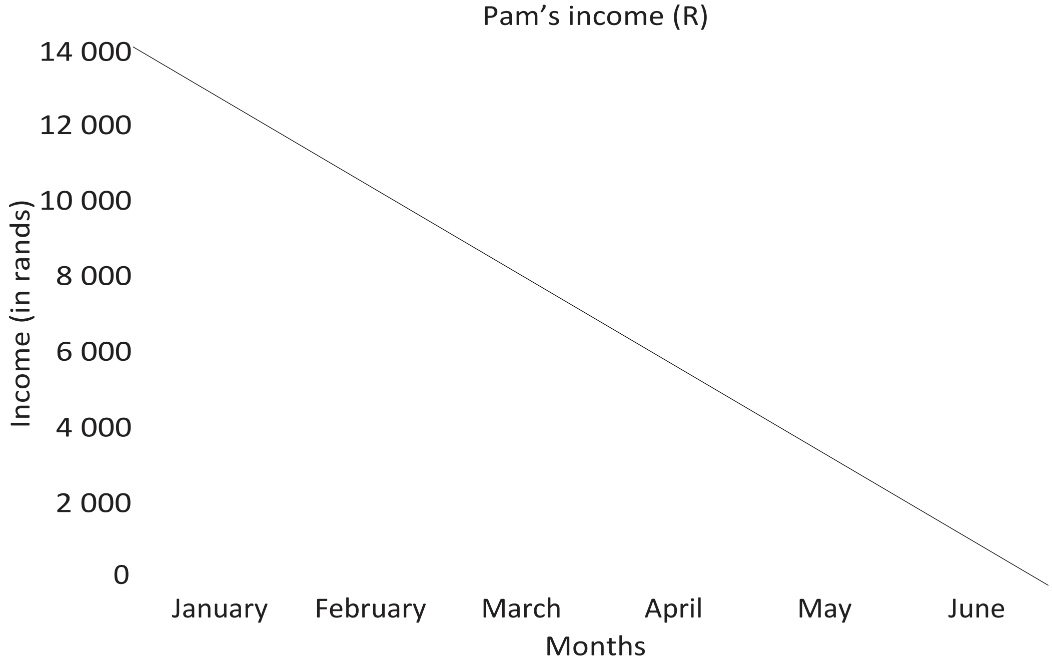
A **broken-line graph** is a line that joins consecutive data points plotted on a set of axes. Broken-line graphs are useful to show how something has changed or stayed the same over time.

drawing broken-line graphs

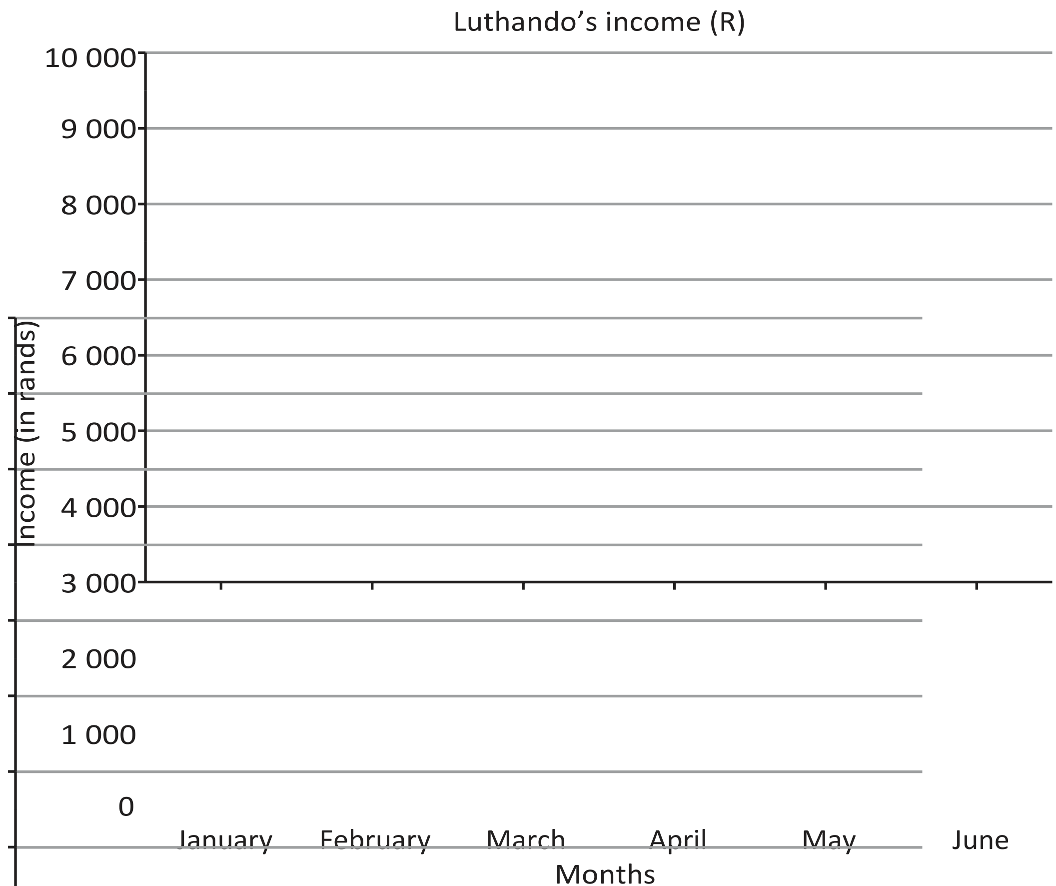
The following table shows the income of Pam's small business and Luthando's small business over six months:

Month	January	February	March	April	May	June
Pam's income (R)	12 000	12 000	9 000	6 000	7 000	9 000
Luthando's income (R)	6 000	7 000	8 000	8 000	9 000	9 000

1. Copy the axes below and draw a broken-line graph showing Pam's income:



2. Copy the axes below and draw a broken-line graph showing Luthando's income:



3. Whose income seems to be increasing steadily per month?

comparing different ways of representing data

The following table shows data from the 2012 GHS (Statistics South Africa).

1. Is it possible to find the mean, median and mode of this data? Explain.

Mode of school transport for learners in numbers and percentages

Mode of transport	Statistic (numbers in thousands)	Usual transport to school
Walking	Number	10 549
	Percentage	68,9
Bicycle/motorcycle	Number	90
	Percentage	0,6
Minibus taxi/sedan taxi/bakkie taxi	Number	1 129
	Percentage	7,4
Bus	Number	434
	Percentage	2,8
Train	Number	94
	Percentage	0,6
Minibus/bus provided by institution/government and not paid for	Number	209
	Percentage	1,4
Minibus/bus provided and paid for by the institution	Number	88
	Percentage	0,6
Vehicle hired by a group of parents	Number	1 344
	Percentage	8,8
Own car or other private vehicle	Number	1 371
	Percentage	8,9
Subtotal	Number	15 308
	Percentage	100

2. What are two good graphs you could use to represent this data? Explain your answer.
3. Describe the advantages of each of these ways (the two graphs and the table) for this particular set of data.
4. Draw the two graphs that you named in question 2.

Chapter 20
Interpret, analyseand report on data

20.1 Critically analysing how data is collected

Data collection methods can sometimes result in bias and misleading data. This is not always intended by the researcher – it often happens when the source of the data was not carefully checked, or the method of collecting data has not been planned carefully.

In Chapter 18 you learnt that a sample must be large enough to be representative and must be randomly selected from the population. If data is collected from only one part of a population, it could be biased towards that part. The researcher has to be aware of all the places where bias could occur, and should design the data-handling process so that it does not happen.

When you read reported statistics, always be aware that you need information about *how* the data was collected, *when* it was collected and *how* the sample was chosen.

Data can change over time, so you should also be aware of when it was collected. This information should be given in any report on data.

data sources and collection methods

1. Read the following paragraph and answer the questions that follow:

   A recent study revealed that 50% of high school learners smoke cigarettes, 45% drink alcohol and 60% abuse drugs. This is an indication of the general poor health and social problems of the teenage population in our country.

   (a) Do you agree that the figures are high enough to conclude that the habits of these teenagers are unhealthy?

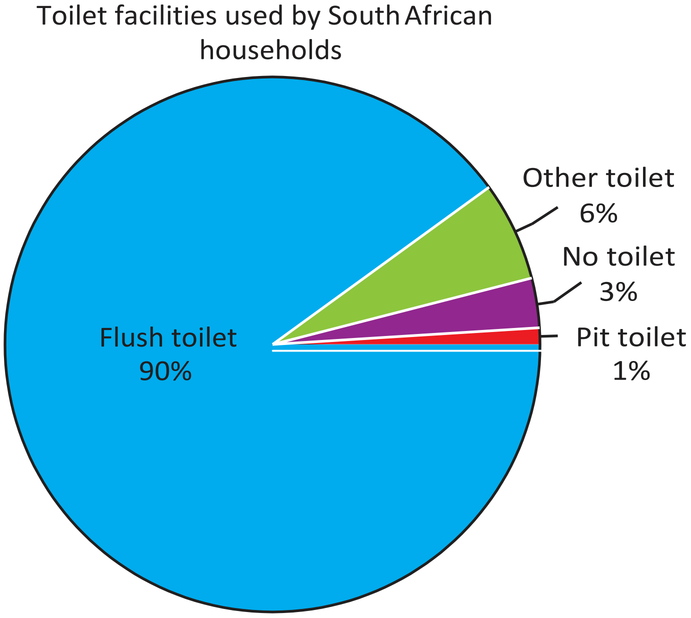
   (b) Can we tell the following from the data above?

   - What was the sample of this study?
   - Where was the data collected?
   - When was the data collected?

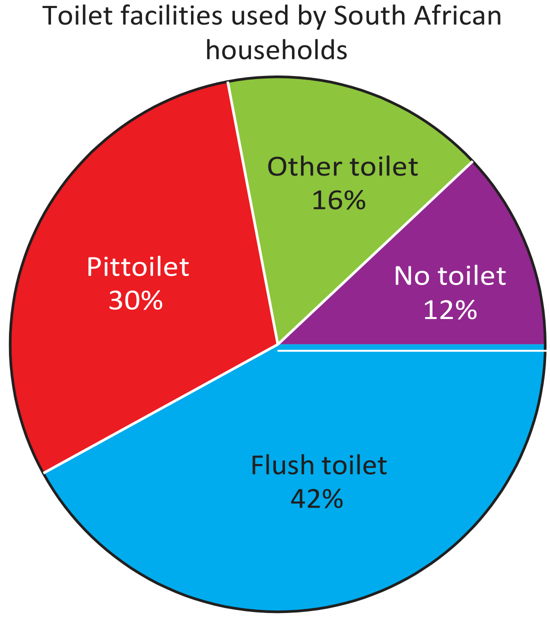
   (c) If the sample consisted of ten teenagers all located in an area known for drug and alcohol abuse, would the data be a good reflection of all teenagers in the country?

(d) Describe what you think would be a better sample.
(e) Why is it important to know the date of when this data set was collected?

2. The following pie charts show the toilet facilities in households in South Africa.

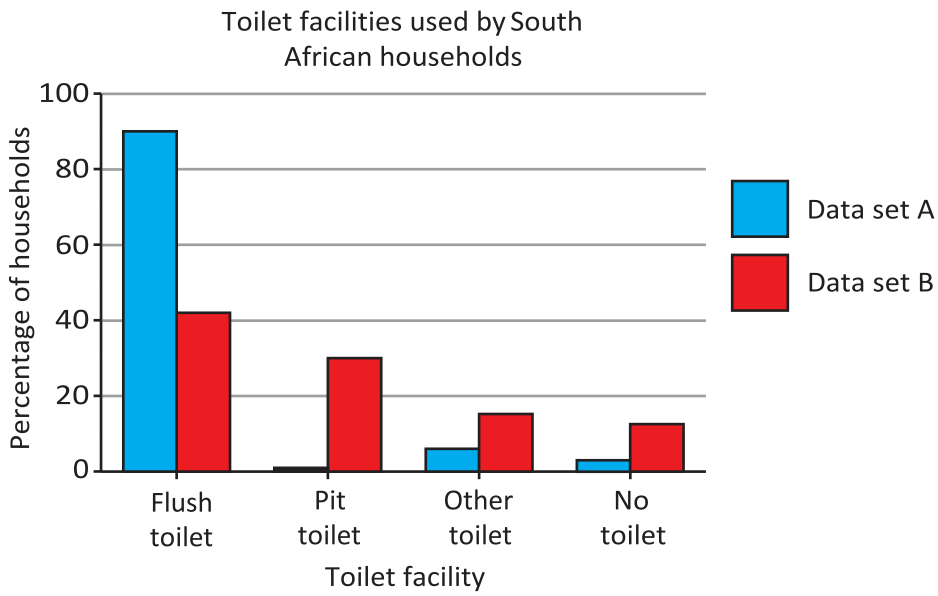


Pie chart A



Pie chart B

(a) According to Pie chart A, what type of toilet facility do most people have and what percentage of households is this?
(b) How will your answer for (a) be different if you use Pie chart B to answer the question?
(c) Write a short report in one paragraph about the data in the pie charts.



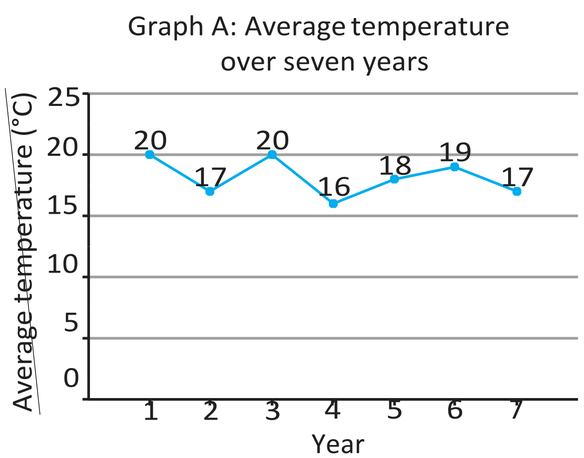
(d) What can you conclude from the data in Pie chart A?
(e) What can you conclude from the data in Pie chart B?
(f) How could the year of data collection account for the differences in the data?
(g) The graph at the bottom of the previous page shows the same data as the two pie charts. Does the diagram of the pie charts or the double bar graph allow us to compare the two sets of data more easily?
(h) Do the pie charts or the double bar graph show the percentage of types of toilet facilities used in South Africa better?

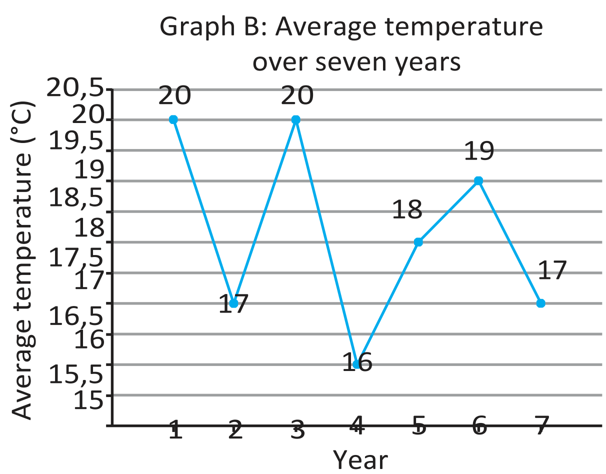
20.2 Critically analysing how data is represented

Graphs do not always show what they seem to show at first glance! If you look a bit closer, you might see that they are misleading you into drawing an incorrect conclusion. Work through the following activity to see how this can happen.

manipulation in data presentation

These graphs show the average temperatures recorded in the same place, at the same time.





1. Do both graphs show exactly the same data?
2. Why do the graphs look sodifferent?
3. Which of the graphs would people use to emphasise that there are big differences in the temperatures over the years? Explain your answer.
4. Suggest a way to change the vertical scale of Graph A to emphasise even more that there are no big differences between the temperatures over the years.
5. Write a short report on Graph A. Also include a prediction of the temperatures in Years 8 and 9.

20.3 Critically analysing summary statistics

It is sometimes necessary to inform another person about a set of data that you have worked on. In doing that, you may want to save the other person from having to look at all the values in the data set. You also want to emphasise some aspects of the data. It is for these purposes that we use summary statistics like the following:

- measures of central tendency (typical values): **mode**, **median** and **mean**
- measures of dispersion (values indicating the spread of data): the smallest and largest values and the difference between them (the **range**)

Summary statistics do not provide full information on data. Some information is always lost and so summary statistics can be misleading, especially if there are **outliers**; in other words, values that differ a lot from the majority of the values.

how summary statistics can be misleading

1. The manager of a small business is asked what monthly salaries his employees get. His answer: *The mean of the salaries is R13 731.*

(a) Do you think the manager's answer is a good description of the salaries?

(b) In order to have some sense of the salaries paid at the firm, which one of the following would you prefer to know: the *median*, the *mode*, the *range*or the *lowest and highest* salaries?

2. The actual monthly salaries of the 13 staff members in the small business from question 1 are given below:

R3 500	R3 500	R3 500	R3 500	R3 500
R4 200	R4 200	R4 200	R4 400	R12 000
R28 000	R44 000	R60 000		

In what ways may you be misinformed if you do not know the above figures, but only know that the mean salary is R13 731?

3. If only one summary statistic is used to provide information about the salaries at the firm, which of the following do you think would be the best to give? Give reasons for your choice.

A. the mode

B. therange

C. the median

D. the lowest and highest salaries

4. Look at the different monthly salaries of employees at another small business:

R34 000	R35 000	R3 400	R31 000	R32 000

(a) Why would the mean not be a good way to summarise this data?
(b) Calculate the mean salary.

5. The following data shows the number of boxes of chocolates sold by a store in ten consecutive months:

   42 38 179 40 43 40 48 39 41 42

   (a) Which would be the better summary description of the data, the mean or the median? Explain your answer.
   (b) Write a good summary description of the data without using the median.
   (c) Would it make sense to leave out the outlier, 179, when calculating the mean of the monthly sales? Explain your answer.

manipulation in summary reports on data

The mode, median and mean each highlight different bits of information about the same set of data. They can be very different from one another, depending on the kind of data set you have.

Sometimes people choose the statistic that does not show the typical values, but rather the value that works best forthem.

1. Thivha sells restored furniture. He reports that he usually sells seven items per week, and that he has the data to prove it. The receipts show that he made 52 sales over a period of eight weeks.
   (a) Can you tell from the data above whether Thivha is truthful or not about the sales?
   (b) You examined the receipts for the eight weeks closely, and find the following number of sales per week:

       3, 4, 4, 4, 4, 5, 6, 22

       Determine the mode and median of the data set.
   (c) Do you think the mode, median or mean is a better reflection of Thivha's typical sales figures per week? Explain your answer.

2. The following data shows the amount of pocket money that a group of learners receives per week:

   R0 R0 R5 R10 R10 R10 R10 R20 R20 R50

   (a) Determine the mode, median, mean and range of the data set.
   (b) The teenager who receives R5 a week wants to convince her parents to give her more money. Which of the summary statistics would she use? Explain your answer.
   (c) Which summary statistic do you think best represents the weekly pocket money in this group of learners? Explain your answer.

Chapter 21
Functions and relationships

21.1 Calculating output values

formulae and tables

The statement $y = 2x + 6$ can be true for any value of x, provided one chooses the appropriate value of y. The statement is true for certain combinations of values of x and y. Such a statement is called a **formula**.

A formula is a description of how the values of a dependent variable can be calculated for any given values of the other variable(s) on which it depends.

1. Which of the following are formulae for the function illustrated in the table?

   A. $y = 15x$ B. $y = -5x + 20$ C. $y = 5(20 - x)$ D. $y = 5x + 10$

x	1	2	3	4	5	6
y	15	10	5	0	−5	−10

2. For each of the tables that follow, determine which of the following formulae could have been used to complete the table. The letter symbol x is used to represent the input numbers and the symbol y represents the output numbers.

   A. $y = x^2$ B. $y = 10x$ C. $y = 10x - 1$
   D. $y = x^2 + 2$ E. $y = 5x + 2$ F. $y = -5x + 2$
   G. $y = 3^x$ H. $y = 3^{x+1}$

(a)

Input value	1	4	11	30	40	60
Output value	7	22	57	152	202	302

(b)

Input value	1	6	9	12	18	20
Output value	1	36	81	144	324	400

(c)

Input value	1	6	9	12	18	20
Output value	3	38	83	146	326	402

(d)

Input value	3	11	19	27	45	70
Output value	30	110	190	270	450	700

(e)

Input value	3	11	19	27	45	70
Output value	29	109	189	269	449	699

(f)

Input value	1	2	3	4	5	6
Output value	3	9	27	81	243	729

21.2 Different forms of representation

flow diagrams, tables, words and formulae

1. This question is about the relationship between two variables. Some information about the relationship is given in the following flow diagram:

   input number → × 3 → + 2 → output number

   (a) Copy the table. Use the instructions in the flow diagram to complete the table:

Input value	1	2	3	4	5	10	23	50	86
Output value									

   (b) Describe by means of a formula how to calculate the output number for any input number. (Let x represent the input numbers and y the output numbers.)

   (c) Describe verbally how to calculate theoutput number for any input number.

   (d) What input number will make the statement $3x + 2 = 71$ true?

   (e) What input number will make the statement $3x + 2 = 260$ true?

When there is only one output number for any input number, the relationship between the two variables is called a **function**.

2. Some information about the relation between output and input values in a certain function is given in the flow diagram:

   input number → + 2 → × 3 → output number

   (a) Copy the table. Use the flow diagram to complete the following table:

Input value	1	2	3	4	5			50	86
Output value						36	75		

(b) Describe by means of a formula how the input and output numbers are related. Use the letter y for the output numbers and x for the input numbers.

(c) Give a verbal description of how the input and output numbers are related.

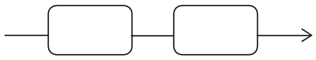
(d) Themba wrote the formula $y = (x + 2)3$ to describe how the input and output numbers are related. Is Themba correct? Explain.

3. A certain function g is represented by means of the formula $y = 2(x - 4)$.

(a) Copy and complete the table for g:

Input value	1	2	3	4	5	6	14	44	54
Output value				0	2	4	20	80	100

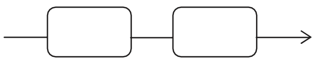
(b) Copy and complete the flow diagram for g (fill in the operators):

input value  output value

4. (a) Copy and complete the table for the relation given by the formula $y = 2x - 4$:

Input value	1	2	3	4	5	6	14	44	54
Output value									

(b) Copy and complete the flow diagram (fill in the operators):

input value  output value

(c) Give a verbal description of how to complete the table.

5. Copy and complete the following table:

<table>
<tr><th>Formula</th><th>Flow diagram</th><th>Table</th><th>Verbal description</th></tr>
<tr><td>$y = 4x$</td><td></td><td>

x	0	3,5	7	0,3
y	0	14		

</td><td></td></tr>
<tr><td></td><td></td><td>

x	2	3	4	5
y	1	2	3	4

</td><td></td></tr>
<tr><td></td><td></td><td>

x	2	3	4	5
y	7	11	15	19

</td><td>Multiply the input number by 4 then subtract 1.</td></tr>
</table>

$y = 2(x + 1)$	→ + 1 → × 2 →	x: −2, −1, 0, 1 y: −2, 0, 2, 4	
$y = 2x + 2$	→ × 2 → + 2 →	x: −2, −1, 0, 1 y:	Multiply the input number by 2 and then add 2.
$y = 2x + 1$	→ ☐ → ☐ →	x: −2, −1, 0, 1 y:	Multiply the input number by 2 and then add 1.

In Sections 21.1 and 21.2, you have used four different ways to represent functions, namely:

- a formula
- a table
- a flow diagram
- a verbal representation.

Later in this term you will represent functions by using **coordinate graphs**.

21.3 Completing more tables

looking at different formulae at the same time

1. Copy the table. Use the given formulae in each column to complete the table. Some rows have been completed for you. You may use a calculator.

x	$y = 10x$	$y = 10x^2$	$y = 10^x$
−7	−70	490	0,0000001
−6		360	0,000001
−5		250	0,00001
−4		160	0,0001
−3		90	0,001
−2		40	0,01
−1		10	0,1

x	$y = 10x$	$y = 10x^2$	$y = 10^x$
0	0	0	1
1	10	10	10
2	20	40	100
3	30		1 000
4	40		10 000
5	50		100 000
6	60	360	1 000 000
7	70	490	10 000 000

2. In each case, choose the correct answer from those given in brackets.

   As the input value increases by equal amounts (say from 1 to 2, 2 to 3, 3 to 4, and so on), the output value for:

   (a) $y = 10x$(increases by equal amounts/increases by greater and greater amounts)

   (b) $y = 10x^2$ (increases by equal amounts/increases by greater amounts)

   (c) $y = 10^x$ (increases by equal amounts/increases by greater and greater amounts)

3. (a) Copy and complete the table below. Some examples have been done for you.

x	$y = -2x - 1$	$y = -2x$	$y = -2x + 1$
−4	$-2 \times -4 - 1 = 7$	$-2 \times -4 = 8$	$-2 \times -4 + 1 = 9$
−3	$-2 \times -3 - 1 = 5$	$-2 \times -3 = 6$	$-2 \times -3 + 1 = 7$
−2			
−1			
0			
1	$-2 \times 1 - 1 = -3$	$-2 \times 1 = -2$	$-2 \times 1 + 1 = -1$
2			
3			
4			

   (b) Describe the relationships between the corresponding output numbers in the three columns.

4. (a) Copy and complete the followingtable:

x	$y = 2^{x-1}$	$y = 2^x$	$y = 2^{x+1}$
−1	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$
0	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$
1			
2			
3			
4			
5			
6			

(b) Describe the relationships between the corresponding output numbers in the three columns.

21.4 Solving some problems

looking at some situations

1. The formula $y = 38 - 2x$ describes the relationship between y and x in a certain situation.

   (a) Copy and complete the following table for this situation:

x	10	5	15		8	
y				36		2

   (b) What is the output value if the input value is 8?
   (c) For what input value is the output value equal to 28?
   (d) Which input value makes the statement $36 = 38 - 2x$ true?

2. Consider rectangles which each have an area of 24 square units. The breadth of the rectangles (y) varies in relation to the length (x) according to the formula $xy = 24$. Copy and complete the following table to represent this situation:

Length (x)					6	8	12	24
Breadth (y)	24	12	8	6				

3. Consider rectangles with a fixed perimeter of 24 units. The breadth of the rectangles (y) varies in relation to the length (x) according to the formula $2(x + y) = 24$.

Copy and complete the following table to represent this situation:

x	1	2	3	4	6					
y						5	4	3	2	1

4. The formula $b = 180° - \frac{360°}{n}$ gives the size b of each interior angle in degrees for a regular polygon with n sides (an n-gon).
   (a) Copy and complete the following table:

Number of sides (n)	3	4	5	6	10	12
Angle size (b)						

   (b) What is the size of each interior angle of a regular polygon with 20 sides, and a regular polygon with 120 sides?
   (c) If each interior angle of a polygon is 150°, how many sides does it have?

5. As you may know, metals contract when temperatures are low and expand when temperatures are high. So, when engineers build bridges they always leave small gaps in the road between sections to allow for heat expansion.
   For a certain bridge, engineers use the formula $y = 2{,}5 - 0{,}05x$ to determine the size of the gap for each 1 °C rise in temperature, where x is the temperature in °C.

   (a) Copy and complete the following table to show thesize of the gap at different temperatures:

Temperature (°C)	3	4	10	15	25	30	35
Gap size (cm)							

   (b) What is the size of the gap at each of the temperatures shown below?
   0 °C 18 °C −2 °C 50 °C
   (c) At what temperature will the gap close completely?

6. The formula $y = 0{,}0075x^2$, where x is the speed in kilometres per hour and y is the distance in metres, is used to calculate the braking distance of a car travelling at a particular speed. Use a calculator for this question.

The braking distance is the distance required for a vehicle travelling at a certain speed to come to a complete stop after the brakes have been applied.

Example

What is the braking distance if someone drives at 80 kilometres per hour?

On your scientific calculator you must punch in 0,0075 followed by × sign followed by (80) followed by x^2. The calculator will do the following:

$y = [0{,}0075 \times (80)^2] = (0{,}0075 \times 6\,400) = 48$

∴The braking distance is 48 m.

(a) What is the braking distance at a speed of 100 kilometres per hour?
(b) Calculate the braking distance at a speed of 60 kilometres per hour.
(c) Copy and complete the table below. Give answers to two decimal places where necessary.

Speed in km/h	10	20	30	40	50	60	100
Braking distance in m							

Refer to the table in question (c) to answer question (d) below.

(d) A car travels at a speed of 40 kilometres per hour. A sheep, 7 m away on the side of the road, suddenly runs onto the road. Will the car hit the sheep or will the driver be able to stop the car before it hits the sheep? Explain.
(e) A car travels at a speed of 90 km/h in an area that has school children crossing the road. What distance does the driver need to stop the car so that it does not hit the children?

7. (a) Copy the table below. Use the formula $y = 1{,}06x$ to complete the table:

x	100	200	300	400	500	1 000	5 000	10 000
y								

(b) What is the value of y if $x = 750$?
(c) What is the value of y if $x = 2\ 500$?
(d) Represent the formula $y = 1{,}06x$ by means of a flow diagram.

(e) If $y = 583$, what is the value of x?
(f) If $y = 954$, what is the value of x?
(g) The statement $1\ 060 = 1{,}06x$ is given. For what value of $x$is the statement true?
(h) The statement $530 = 1{,}06x$ is given. For what value of $x$is the statement true?

8. The formula $y = 0{,}1x + 5\ 000$ is given. What is the value of y in each case below?

(a) $x = 10$

(b) $x = 100$ (c)

$x = 1\ 000$ (d)

$x = 10\ 000$

9. The formula $y = 1,14x$ is used to calculate the price y of goods including VAT in rands, where x is the price in rands before VAT.
   (a) How much will you pay at the counter for goods that cost R38 without VAT, and for goods that cost R50 without VAT? You may use a calculator.
   (b) Copy and complete the following table for the prices of goods with VAT:

x	1	2	3	4	5	6	7	8	9
y									

10. Use your answers for question 9(b) to find the prices with VAT for goods with the prices before VAT was added (below). Do not use a calculator at all in this question.
   (a) R40
   (b) R400
   (c) R70
   (d) R470

11. (a) An article costs R11,40 with VAT included. What is the price before VAT was added?
   (b) An article costs R342 with VAT included. What is the price before VAT was added?

12. Consider the function represented by $y = 75 - 0,1x$. What is the value of y if:
   (a) $x = 0$?
   (b) $x = 750$?
   (c) Copy and complete the following table for the function represented by $y = 75 - 0,1x$:

x	0	10	20	50	100	200	500	700	800
y									

   (d) For what value of x is $75 - 0,1x = 0$?
   (e) For what value of x is $75 - 0,1x = 100$?

Chapter 22
Algebraic equations

22.1 Revision

setting up equations to describe problem situations

1. Farmer Moola has already planted 100 apple trees and 250 orange trees on his fruit farm. He decides to plant 20 more apple trees every day, as can be seen in the following table:

Number of days (x)	0	1	2	3	4	5	6	7	8
Number of trees (y)	100	120	140	160					

He also decides to plant ten orange trees a day, as shown below:

Number of days (x)	0	1	2	3	4	5	6	7	8
Number of trees (y)	250	260	270	280					

(a) Write a rule for calculating the number of apple trees after x days. Write the rule in the form of a formula. Represent the number of trees with the letter symbol y.
(b) Write a formula for finding the number of orange trees after x days.
(c) How many orange trees are there on the fourteenth day?
(d) After how many days will Farmer Moola have 260 apple trees?

After how many days will farmer Moola have 1 000 apple trees in his orchard?

It will take some time to work this out by counting in twenties. You can easily make a mistake and not even be aware of it. Another way to find the information is to figure out for which value of x it will be true that $100 + 20x = 1\,000$. To do this you may try different values for x until you find the value that makes $100 + 20x$ equal to 1 000. It is convenient to enter the results in a table as shown below. Anna first tried $x = 10$ and saw that 10 is far too small. She next tried $x = 100$ and it was far too big. She then tried 50.

Number of days (x)	10	100	50		
Number of apple trees (y)	300	2 100	1 100		

2. What number do you think Anna should try next, in her attempt to solve the equation $100 + 20x = 1\ 000$?

3. How many days after he had 250 orange trees, will farmer Moola have 900 orange trees on his farm?

4. In 2004, there were 40 children at Lekker Dag Crèche. From 2005 onwards, the number of children dropped by about five children per year. Explain what each number and letter symbol in the formula $y = 40 - 5x$ may stand for.

5. Cool Crèche started with 20 children when it was opened in 2008. The number of children in Cool Crèche increases by three children every year. Explain what each letter symbol and number in the formula $y = 20 + 3x$ stands for.

6. Farmer Thuni already has 67 naartjie trees and 128 lemon trees on his fruit farm. He decides to plant 23 new naartjie trees and 17 new lemon trees every day during the planting season.
   (a) Which quantities in this situation change as time goes by, and which quantities remain the same?
   (b) How many naartjie trees and how many lemon trees will he have in total, ten days after the planting season has started?
   (c) Write formulae that can be used to calculate the total numbers of naartjie and lemon trees after any number of days during the planting season. Use letter symbols of your own choice to represent the variables.
   (d) What information about the situation on farmer Thuni's farm can be obtained by solving the equation $67 + 23x = 500$?
   (e) Set up an equation that can be used to find out how many days into the planting season it will be when farmer Thuni has 500 lemon trees. Use a letter symbol of your own choice to represent the unknown number of days.

Quantities that change are called **variables**, and are represented with letter symbols in formulae and equations. Quantities that do not change are called **constants**, and are represented by numbers in formulae and equations.

solving equations by inspection

To **solve** an equation means to find the value(s) of the unknown for which an expression has a given value.

One method of solving an equation is to try different values of the variable until you find a value for which the expression is equal to the given value, or for which the two expressions have the same value. This is called **solving by inspection**.

The value of the variable for which an expression is equal to a given value, or for which two expressions have the same value, is called the **solution** or the **root** of the equation.

1. In each case, determine whether the value of x given in brackets is a root or solution of the equation or not. Justify your answer, in other words, say why you think the number is or is not a root of the equation. In cases where the given number is *not* a root or solution, find the solution by trying other values.

   (a) $3x + 1 = 16$ $(x = 5)$
   (b) $7x = 91$ $(x = 13)$
   (c) $10x + 9 = 7x + 30$ $(x = 6)$
   (d) $-10x - 1 = 29$ $(x = 3)$
   (e) $7 + 2x = 9$ $(x = 1)$

2. Find the solution of each equation by inspection:

   (a) $x - 1 = 0$
   (b) $x + 1 = 0$
   (c) $1 + x = 0$
   (d) $1 - x = 0$

3. In each case, check if the number in brackets makes the equation true. Explain your answer.

   (a) $8 + x = 3$ $(x = 5)$
   (b) $8 + x = 3$ $(x = -5)$
   (c) $8 - x = 3$ $(x = 5)$
   (d) $8 - x = 3$ $(x = -5)$
   (e) $8 - x = 13$ $(x = -5)$
   (f) $8 - x = 13$ $(x = 5)$

22.2 Solving equations

additive and multiplicative inverses

One way of thinking about the **additive inverse** of a number is to ask the question: *What do I need to add to the given number to get 0?*

1. What is the additive inverse of each of the following? Explain your answers.

   (a) 5
   (b) -5
   (c) 17
   (d) 0,1
   (e) $\frac{5}{6}$
   (f) $-2\frac{1}{4}$

We can think of the **multiplicative inverse** of a number by asking the question: *What do I need to multiply the number by to get 1?*

2. What is the multiplicative inverse of each of the following? Explain.

(a) 5 (b) −5

(c) $\frac{5}{6}$ (d) $\frac{1}{8}$

You can solve the equation $2x + 5 = 45$ in the following way:

$2x + 5 = 45$

$2x + 5 - 5 = 45 - 5$ — Subtract 5 from both sides to have only the term in x — This step can also be understood as $5 + (-5) = 0$

$2x + 0 = 40$

$\frac{2x}{2} = \frac{40}{2}$ — Divide both sides by 2 to have x only — This step can also be understood as $2 \times \frac{1}{2} = 1$

$x = 20$

3. Solve the equations below. Check that the value of x that you get is the solution.

(a) $5x + 2 = 32$ (b) $3x - 5 = -11$ (c) $5x = 40$

(d) $5x - 12 = 28$ (e) $\frac{3}{5}x = 15$

exponential equations

Example 1: Solve the equation $2^x = 8$.

Solution: $2^x = 2^3$ [Write 8 in terms of base 2, i.e. as a power of 2]

$x = 3$ [2 raised to the power of 3 is 8]

Solving exponential equations is the same as asking the question: *To which power must the base be raised for the equation to be true?*

1. Solve for x:

(a) $4^x = 64$ (b) $3^x = 27$ (c) $6^x = 216$

(d) $5^x = 125$ (e) $2^x = 32$ (f) $12^x = 144$

Example 2: Solve the equation $2^{x+1} = 8$.

Solution: $2^{x+1} = 2^3$

$x + 1 = 3$

$x = 2$

2. Solve for x:

(a) $4^{x+1} = 64$ (b) $3^{x-1} = 27$ (c) $2^{x+5} = 32$

22.3 Generating tables of ordered pairs

preliminary activity

1. Copy and complete the following table for the given values of x:

x	−3	−2	−1	0	1	2	3	6	10
$-3x + 2$									

2. What is the value of $15x + 3$ for:

   (a) $x = 2$? (b) x = multiplicative inverse of 15?

3. For what values of x are the following equations true?

   (a) $15x + 3 = 33$ (b) $15x + 3 = 4$

In questions 1 and 2 you evaluated an expression for given values of x.

In question 3 you determined a value of x that makes the equation true. In other words, you solved for x.

In completing a table of values you may be confronted with questions similar to questions 1, 2 and 3.

from a formula to a table of values

1. Copy and complete each of the tables below for the given formula.

   (a) $y = x$

x		−3	−2	−1	0	1	2		10	
y	−9							8		15

   (b) $y = x + 2$

x		−3	−2	−1	0	1	2		10	
y	−5							8		15

   (c) $y = x^3$

x		−3	−2	−1	0	1	2		6	
y	−216							125		1 000

2. Use the table in question 1(c) to solve for x in each case:

   (a) $x^3 = -1$ (b) $x^3 = 8$ (c) $x^3 = 0$

3. Copy and complete the table for $y = 2x$:

x		−3	−2	−1	0	1	2		10	
y	−14							8		26

4. Use the table in question 3 to answer the following questions:
   (a) What value of x makes the equation $2x = 20$ true?
   (b) For what value of x is $2x = 0$?

5. Copy and complete the table for $y = -x - 2$:

x		−3,5	−2	−1	0	1,2	2		6,9	
y	5							−8		−15

6. For what values of x are the following equations true?
   (a) $-x - 2 = 0$ (b) $-x - 2 = 5$ (c) $-x - 2 = -4$

7. Copy and complete the table for $y = x^2$:

x	−4	−3	−2	−1		1		3	4	13
y					0		4			169

8. Refer to the table in question 7 to answer the questions that follow:
   (a) Which different values of x make the equation $x^2 = 16$ true?
   (b) Solve for $x^2 = 9$.
   (c) Solve for $x^2 = 169$.
   (d) What is the solution of $x^2 - 1 = 3$?

9. Some tables of ordered pairs are given below. For each table, find out which of the following formulae was used to make the table. Write the number of the question and the correct formula.

   $y = -5x - 2$ $y = 5x + 2$ $y = 2x + 5$ $y = 2x - 5$
   $y = 2x - 5$ $y = -5x + 2$ $y = -3x + 2$ $y = 3x + 2$

(a)

x	−4	−3	−2	−1	0	1	2	3	4	5
y	−3	−1	1	3	5	7	9	11	13	15

(b)

x	−4	−3	−2	−1	0	1	2	3	4	5
y	−18	−13	−8	−3	2	7	12	17	22	27

(c)

x	−4	−3	−2	−1	0	1	2	3	4	5
y	−10	−7	−4	−1	2	5	8	11	14	17

(d)

x	−4	−3	−2	−1	0	1	2	3	4	5
y	18	13	8	3	−2	−7	−12	−17	−22	−27

(e)

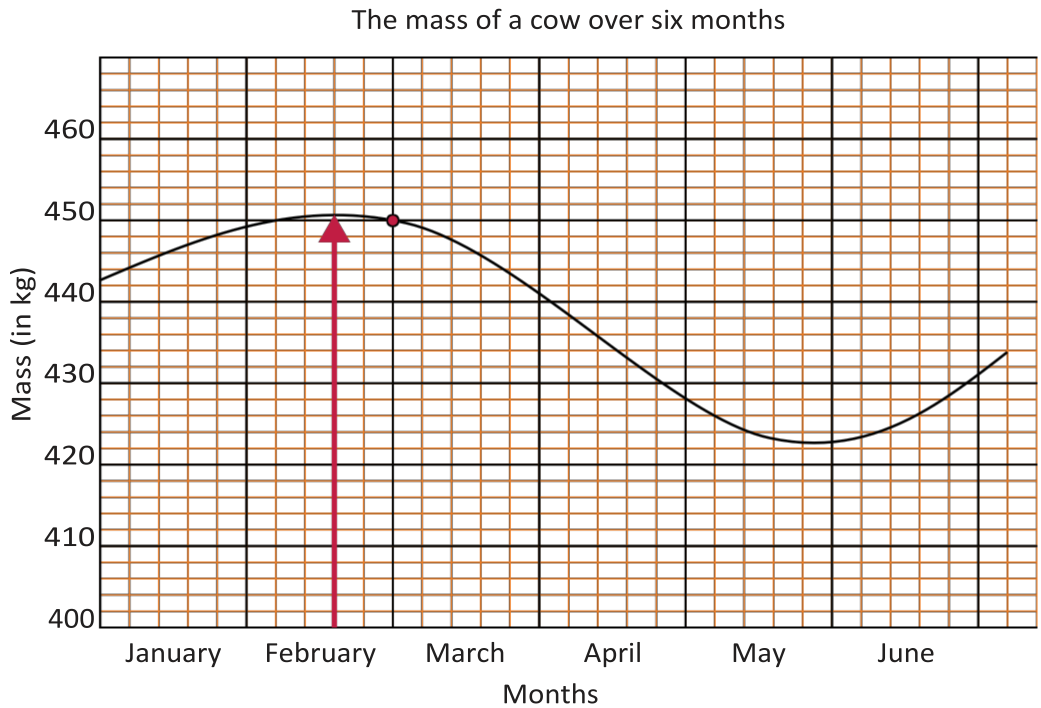
x	−4	−3	−2	−1	0	1	2	3	4	5
y	−13	−11	−9	−7	−5	−3	−1	1	3	5

Chapter 23
Graphs

23.1 What we can tell with graphs

interpreting graphs

1. Mrs Maleka is a dairy farmer. She cares for her cows and weighs all of them daily. Here is a graph of one cow's mass in kilograms over a period of six months. At the end of February, the mass of the cow was 450 kg, as shown by the red dot.



(a) The cow's mass reached a maximum a few days after the middle of February, as shown by the red arrow on the graph. When, in the period shown on the graph, did the cow's mass reach a minimum?

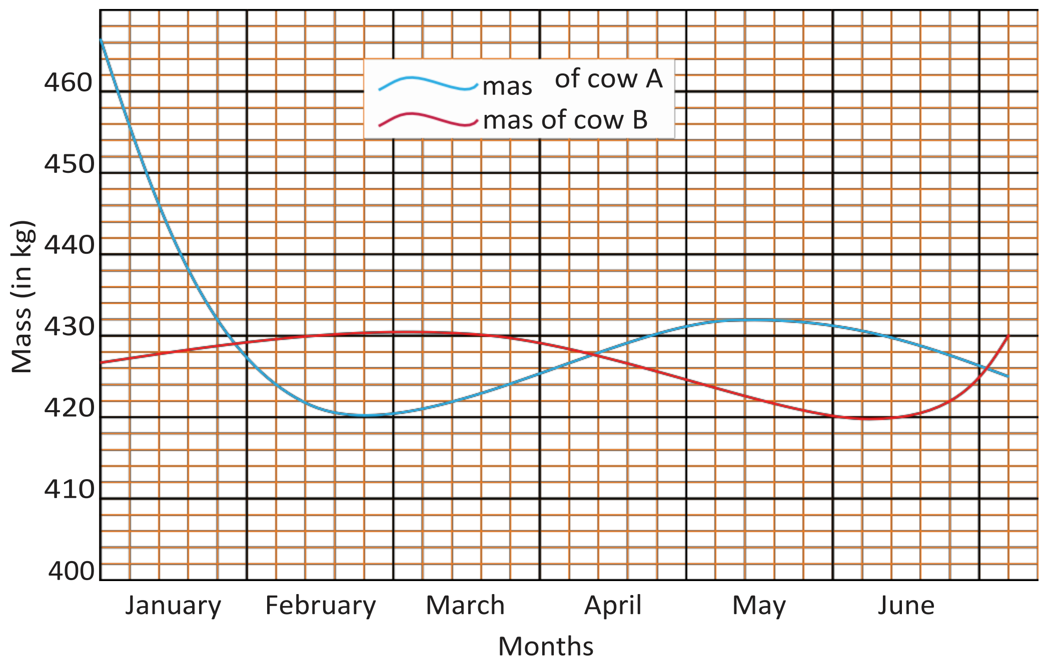
(b) During most of February the cow weighed slightly more than 450 kg. During which month did the cow weigh less than 430 kg, for the whole month?

(c) Throughout the month of June, the mass of the cow increased. During which other month did the mass of the cow also increase, right through the month?

(d) During which months did the mass of the cow decrease right through the month?

2. The blue and red curves below are graphs that show how the mass of two cows varied over the same period of time.

The mass of two cows over six months

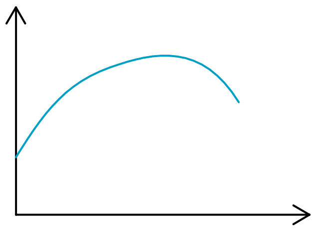
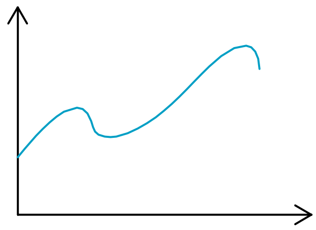
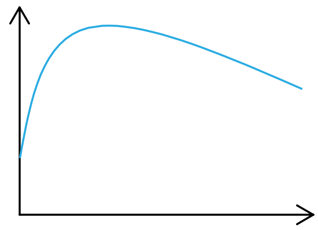
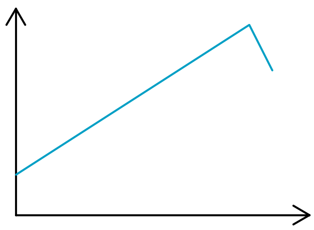
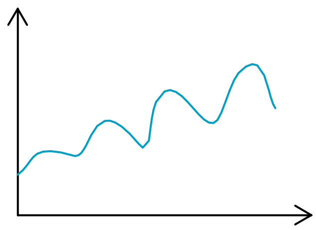
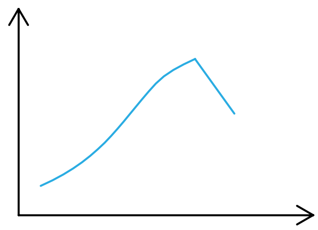
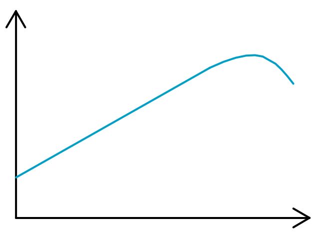
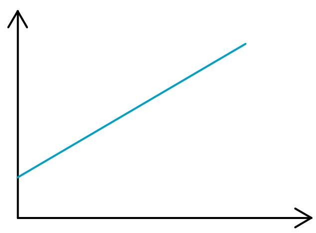


(a) Which cow was the heaviest at the end of February?
(b) When was cow B heavier than cow A?
(c) During which months did the mass of cow A decrease for the whole month?
(d) When did cow A's mass start to increase again?
(e) During which month did cow B's mass begin to decrease while cow A's mass increased for that whole month?
(f) When did cow A's mass catch up with cow B's mass again?
(g) When did cow A stop gaining weight and start losing weight again?
(h) When did cow B's mass catch up with cow A's mass again?

3. A traffic department keeps track of the traffic density on different roads. Two traffic officers are posted somewhere along each main road and they count and record the number of cars that pass in each direction during each 15-minute interval. They use tally marks to do this, as you can see in the example below:

	//// ~~////~~ ~~////~~	/// ~~////~~ ~~////~~ ~~////~~ ~~////~~	~~////~~ // ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~	~~////~~ //// ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~	/ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~	~~////~~ /// ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~
Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	14	23	37	59	71	48

Which of the graphs below do you think is the best representation of the data at the bottom of page 243, on traffic flow?

A

B

C

D

E

F

G

H

I


4. Which of the graphs above is the best representation of each of these traffic flow reports?

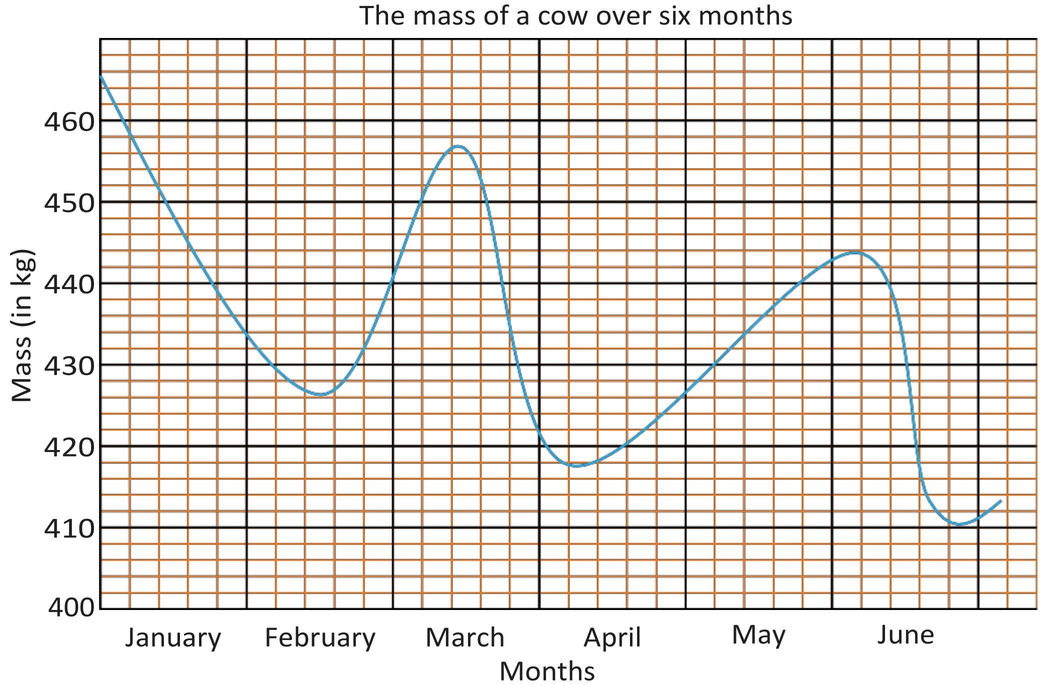
(a)

Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	42	53	64	75	86	75

(b)

Time	06:00 to 06:15	06:15 to 06:30	06:30 to 06:45	06:45 to 07:00	07:00 to 07:15	07:15 to 07:30
Cars	42	123	158	147	136	124

5. Study the graph for another cow on the following page:
   (a) During which periods did the cow lose weight?
   (b) During which of these periods did the cow lose weight more slowly?
   (c) During which of the periods did the cow lose weight most rapidly?
   (d) Compare the two periods when the cow gained weight.
   (e) Is there anything else about the graph that may indicate that this cow has health problems?



how graphs show increases and decreases

A graph on a system of coordinates shows the way in which one quantity (called the **dependent variable**) changes when another quantity (called the **independent variable**) increases. A quantity can change in the following ways:

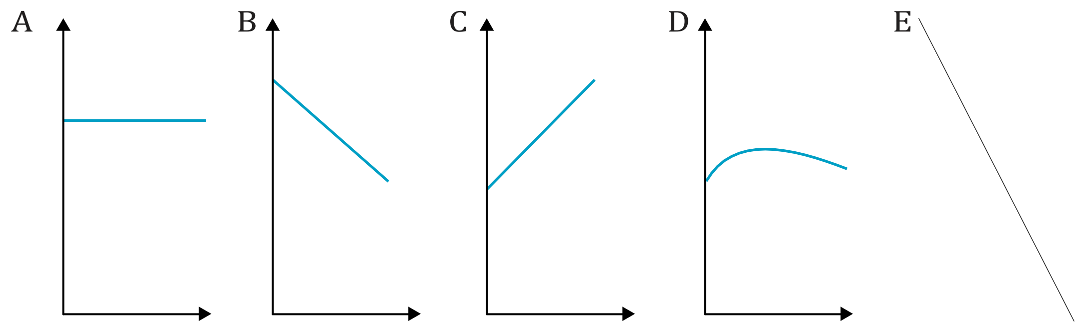
- It can increase or decrease.
- It can increase at a constant rate, for example the total amount saved if the same amount is saved every week ormonth.
- It can decrease at a constant rate, for example the length of a burning candle.
- It can increase (or decrease) at a varying rate, for example the increase in the area of a square as the side length increases.

When a quantity increases or decreases at a **constant rate**, it is called **linear** change or variation, and the graph is a **straight line**. When the rate of change is **not constant**, it is called **non-linear** change, and the graph is **curved**. If there is no change in the output variable, the graph is a horizontal straight line.

1. Draw a graph to match each of the following descriptions:
   (a) The quantity increases, and increases more rapidly as time progresses.
   (b) The quantity first increases slowly at a constant rate, and then increases at a faster, constant rate.
   (c) The quantity decreases faster and faster.
   (d) The quantity increases, and the rate of increase gradually diminishes.

2. Look at the statements and graphs about patterns of change in the petrol price per litre over a period. Match each statement with the appropriate graph given below. Time is represented on the horizontal axis in all these graphs, and petrol price on the vertical axis.

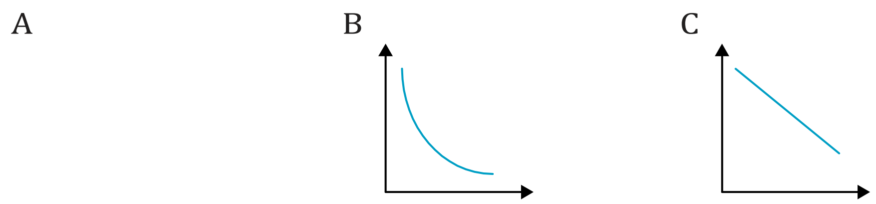
   (a) The price did not change.
   (b) The price rose at a constant rate.
   (c) The price decreased at a constant rate.
   (d) The price dropped very fast at first and then at a slower rate.
   (e) The price rose at a decreasing rate up to a point and then started to drop at an increasing rate.

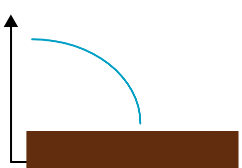


3. Copy and complete the following table in respect of the graphs in question 2:

Graph	Represents a linear or non-linear relation	Reason
A		
B		
C		
D		
E		

4. (a) Which graph represents a quantity that decreases at a constant rate?
   (b) Which graph represents a quantity that decreases at an increasing rate?
   (c) Which graph represents a quantity that decreases at a decreasing rate?





23.2 More features of graphs

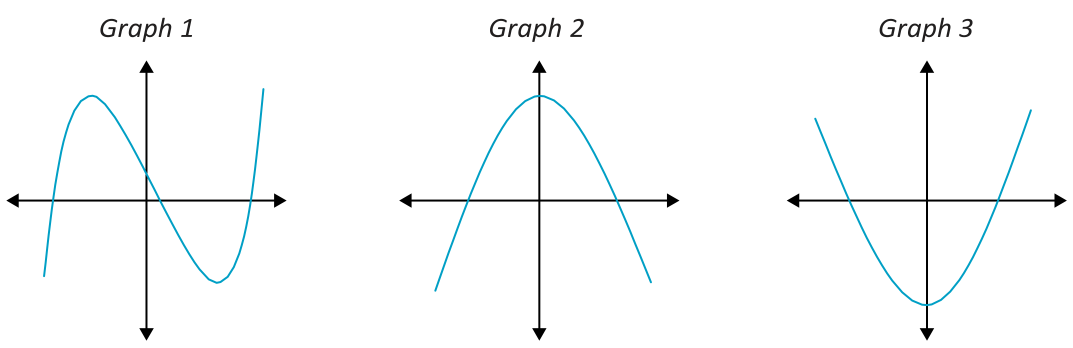
local maximum and minimum values

A graph has a **maximum value** when it changes from increasing to decreasing.

A graph has a **minimum value** when it changes from decreasing to increasing.

A graph can have more than one minimum or maximum value.

1. Consider the graphs below. Describe how the dependent variable behaves in each case by indicating which graph corresponds to which description.
   (a) The variable has a maximum value because it changes from increasing to decreasing.
   (b) The variable has a minimum value because it changes from decreasing to increasing.
   (c) The variable has a maximum value as well as a minimum value because it changes from increasing to decreasing and then from decreasing to increasing.



2. Draw graphs that match the following descriptions:
   (a) A quantity changes in a non-linear fashion, at one stage switching from decreasing to increasing and then to decreasingagain.
   (b) A quantity changes from increasing at a constant rate to decreasing at a constant rate and then becomes constant.

discrete or "continuous"

1. Which of the items in the list provided can you count, and which quantities need to be measured?
   (a) The number of cement bags sold
   (b) Heights of learners in Grade 8
   (c) Times taken for athletes to complete a 400 m hurdles race during the Olympic Games

Quantities can be counted, measured or calculated.

(d) The number of sweets in various 500 g bags
(e) The distance travelled by learners to school
(f) Cars passing at a scholar patrol crossing
(g) The cost of an exercise book in rands and cents
(h) Temperature

Copy the following table and write your answers in it:

Can only be counted	Can only be measured

2. Say if the following make sense or not. Explain.
   (a) 501,3 learners attended a rugby match played by the senior team.
   (b) The distance from school to the nearest shopping mall is 10,75 km.
   (c) 2 004,75 cans of cola were sold during a fundraising event.

Quantitative data is numerical data such as a person's marks in a Mathematics test. Quantities that can be counted are sometimes said to be **discrete**: they do not allow values in between any two consecutive values. You cannot have 2,6 people for example. Quantities that allow many values between any two values are sometimes said to be **continuous**.

The terms "discrete" and "continuous" are used in different meanings than these in formal mathematics.

23.3 Drawing graphs

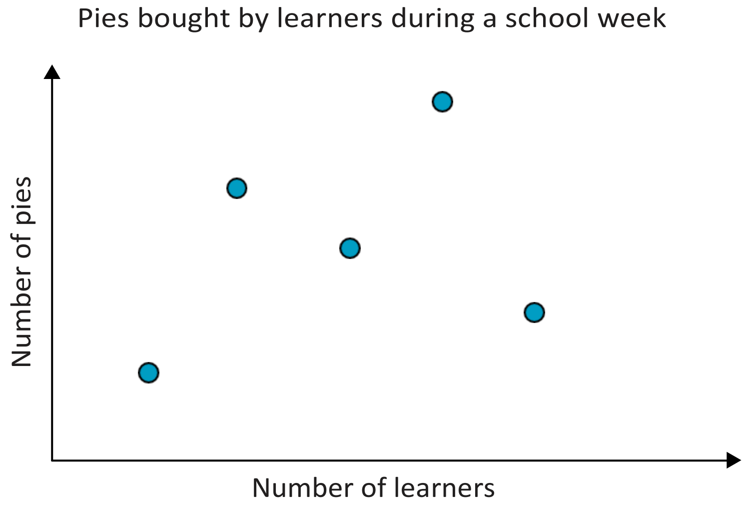
drawing global graphs

When we draw a graph of continuous data, it is a solid line or curve.

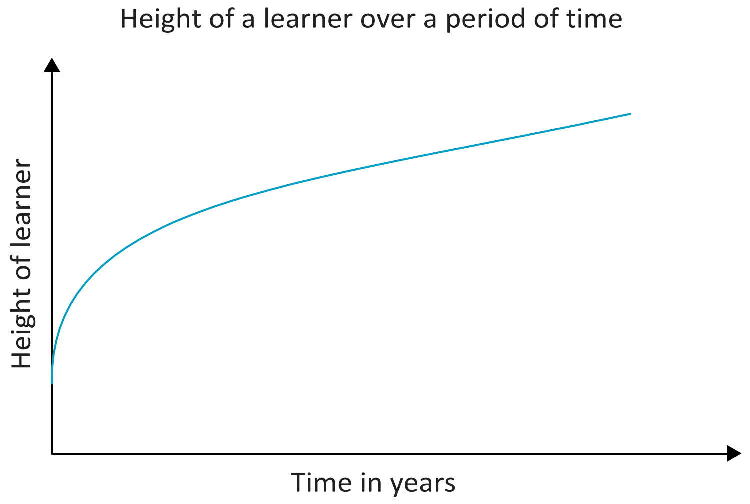
The graph of discrete data is a set of distinct points.

Consider the situations below.

Situation 1



Situation 2



1. (a) What type of data is graphed in situation 1?
   (b) What type of data is graphed in situation 2?
   (c) Why do you think the graph in situation 2 is a solid line?
   (d) Why are the points in situation 1 not joined?

2. Draw a rough graph for each of the following situations:
   (a) The height of a young tree and its age
   (b) The level of water in a dam, over a period without any rain
   (c) The temperature under a tree over a period of 24 hours

graphs of ordered pairs

Input and output values can be written as a pair. The first number in a pair represents the input number and the second number represents the output number. We therefore say that the pair of numbers is ordered.

Making a graph of **ordered pairs** is another way to show how the input and output values are related.

When drawing a graph of ordered pairs, work as follows:

- First identify the input values (x) and output values (y). In most cases the input values will be given and the output values are calculated using the formula given.
- The output values are written on the y-axis (the vertical axis) and the input values are written on the x-axis (the horizontalaxis).
- Plot the ordered pair. Suppose the ordered pair is (3; 6). To plot this pair, put your finger on the number 3 on the x-axis and another finger on the number 6 on the y-axis. Move your finger on the number 3 in a line straight up and move your finger on the number 6 straight across. Where your two fingers meet, make a point. You can describe this point with the ordered pair (3; 6).

1. On graph paper plot the ordered pairs given below:

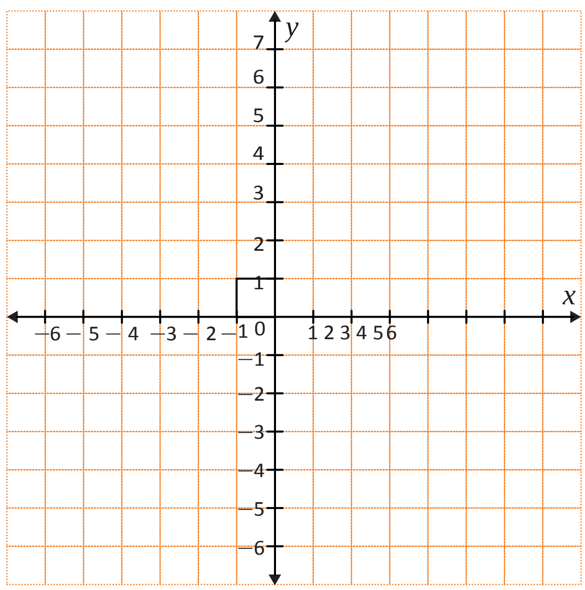
   (a) A(0; 3)

   (b) B(3; 0)

   (c) C(−2; 1)

   (d) D(4; −4)

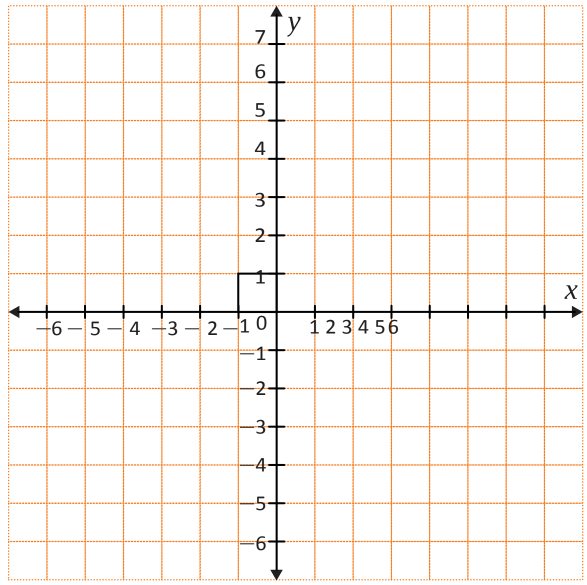
   (e) E(−3; −2)



2. (a) Copy and complete the table below for $y = x + 3$:

x	y	$(x; y)$
−4		
−3		

−2	1	(−2; 1)
−1		
0		
1	4	(1; 4)
2		
3		
4		



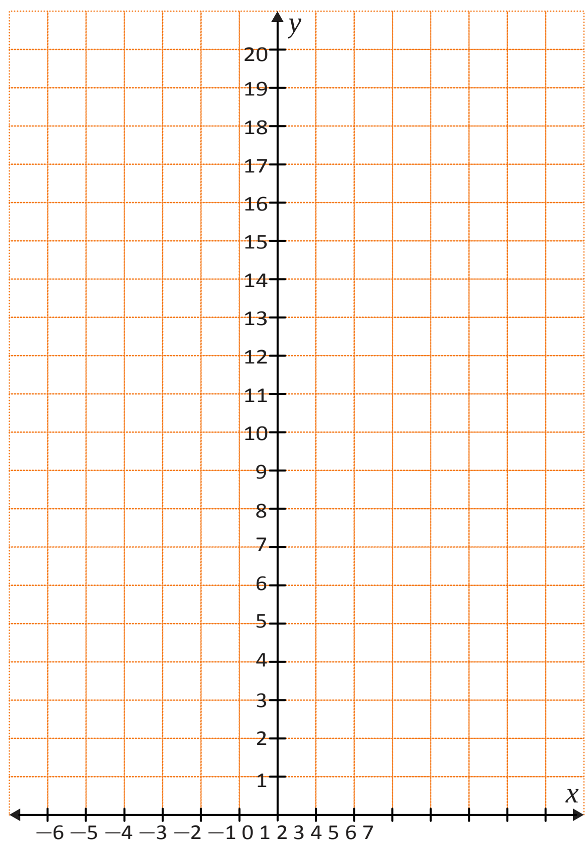
(b) Copy the coordinates onto graph paper. Plot the ordered pairs on the given coordinate system.

(c) Join the points to form a graph.

(d) The ordered pair (1; 6) is not on the graph because when we substitute the value of x ($x = 1$) in the formula $y = x + 3$, we get 4 instead of 6 [$y = 1 + 3 = 4$]. Is the ordered pair (100; 103) on the graph? Explain.

3. (a) Copy and complete the table below for the formula $y = x^2 + 3$:

x	y	$(x; y)$
−4		
−3		
−2	7	(−2; 7)
−1		
0		
1	4	(1; 4)
2		
3		
4		

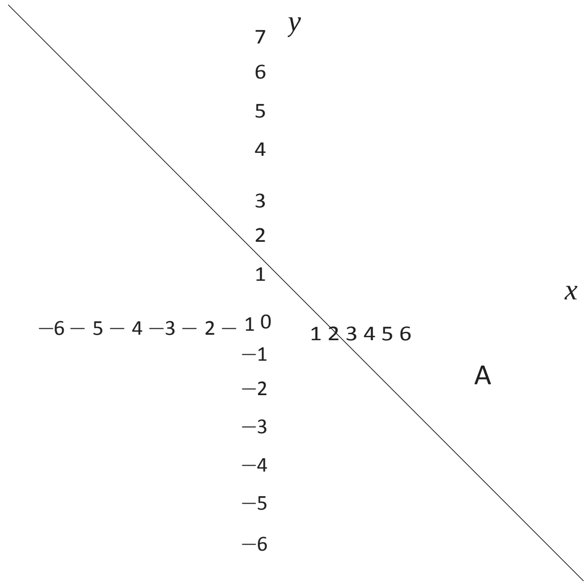


(b) On graph paper, copy the axis system. Then plot the coordinates on the axis system. Join the points to form a graph.

(c) Is the point (10; 103) on the graph? Explain.

4. (a) Copy and complete the table below for the formula $y = -x + 3$:

x	y	$(x; y)$
−4		
−3		
−2	5	(−2; 5)
−1		
0		
1	2	(1; 2)
2		
3		
4		



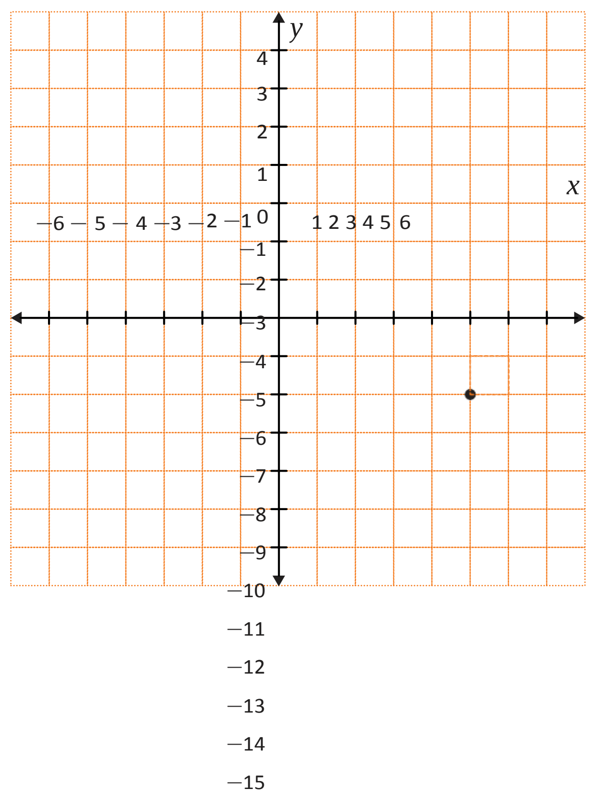
(b) On graph paper, copy the axis system. Then plot the ordered pairs on the axis system.

(c) Join the points to form a graph.

(d) What are the values of the ordered pair A on the graph?

5. (a) Copy and complete the table below for the formula $y = -x^2 + 3$:

x	y	$(x; y)$
−4		
−3		
−2	−1	(−2; −1)
−1		
0		
1	2	(1; 2)
2		
3		
4		

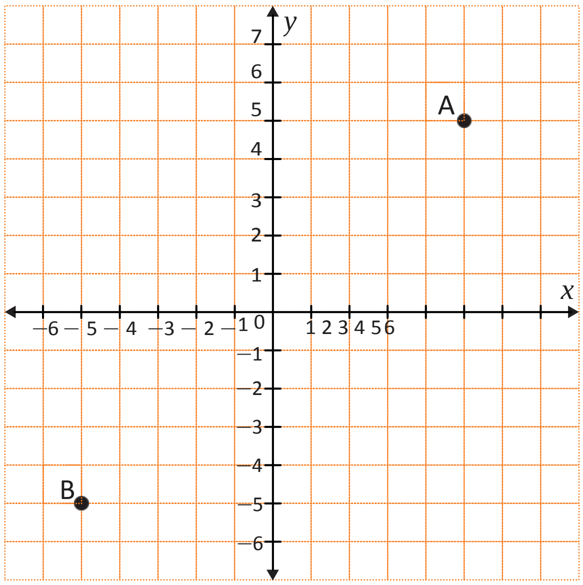


(b) On graph paper, copy the axis system and then plot the ordered pairs on the axis system.

(c) Join the points to form a graph.

6. (a) Copy and complete the table below for the formula $y = x$:

x	y	$(x; y)$
−4		
−3		
−2	−2	(−2; −2)
−1		
0		
1	1	(1; 1)
2		
3		
4		



(b) On graph paper, copy the axis system. Then plot the ordered pairs on the axissystem.

(c) Join the points to form a graph.

(d) Write down the values of the ordered pairs A and B on the graph.

Chapter 24
Transformation geometry

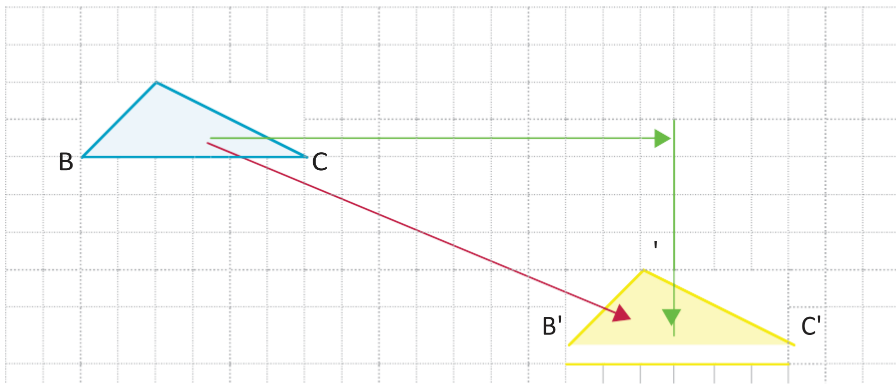
24.1 Transformations and coordinate systems

what are transformations?

A figure can be moved from one position to another on a flat surface by **sliding** (translating), **turning** (rotating) or **flipping** it over (reflecting), or by a combination of such movements. These and other kinds of movements are also called **transformations**.

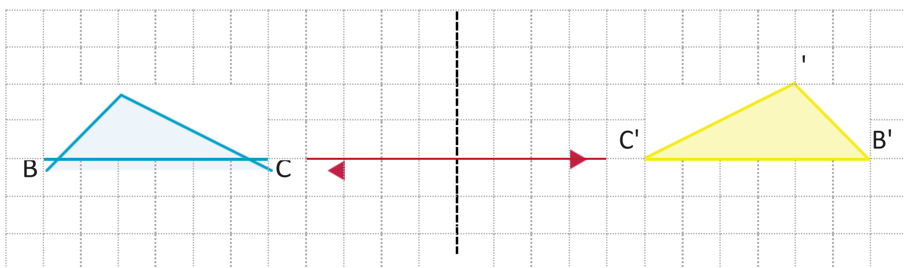
A slide is also called a **translation**.

A slide can also be performed in steps, as indicated by the green arrows.



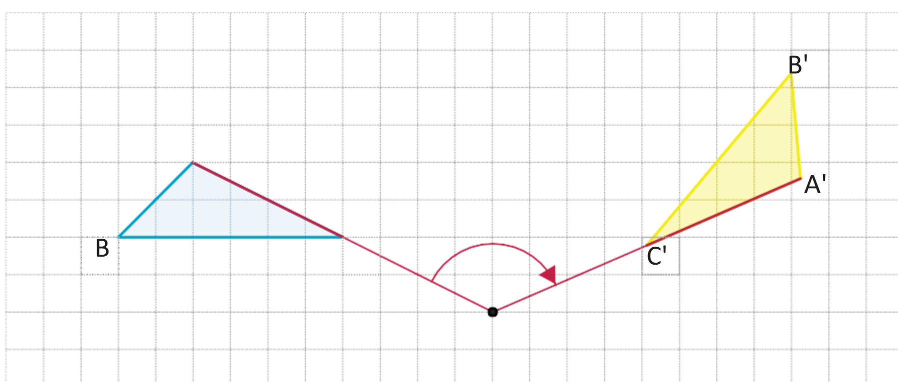
A flip-over is also called a **reflection**.

You may also think of folding the paper over on the dotted line.



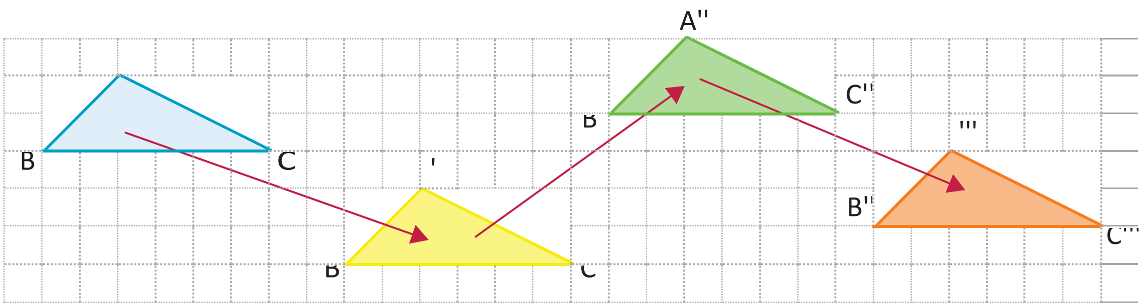
A swing or turn is also called a **rotation**.

The object is swung (rotated) clockwise or anticlockwise around a point called the **centre of rotation**. (It is as if you hold the object on a string.)



In its new position, the figure is called the **image** of the original figure. In the diagrams on page 254, the original figures are blue and the images are yellow. Slides, turns and flips do not change the shape or size of a figure. Hence, in these transformations, the original figure and its image are always congruent.

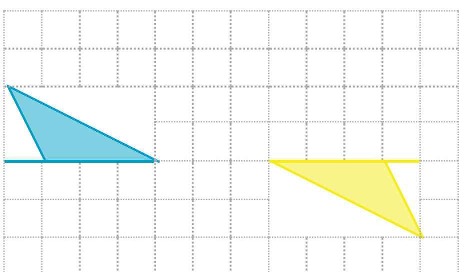
To name the image, we use the same letters as in the original figure, but we add the prime symbol (') after each letter. For example, the image of ΔABC is ΔA'B'C'. If there is a second image, we add two prime symbols, for example ΔA''B''C''. If there is a third image, we use three prime symbols, for example ΔA'''B'''C''', and so on.

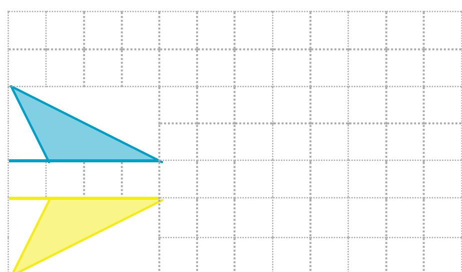


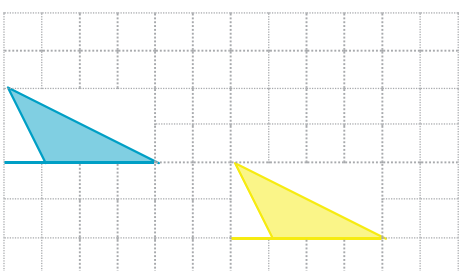
The grid in the background makes it possible to describe the different positions of the figure clearly. To do that, a **system of axes** can be drawn on the grid to form a **coordinate system**, as you will see on the next page. But first, answer the question below.

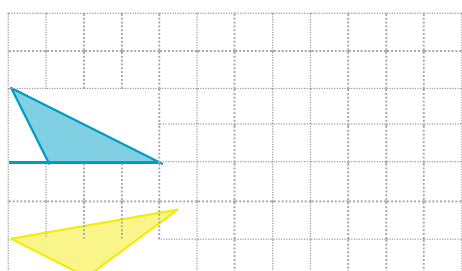
> A coordinate system consists of numbered horizontal and vertical lines that are used to describe position.

In each case, state whether the triangle was translated, reflected or rotated:

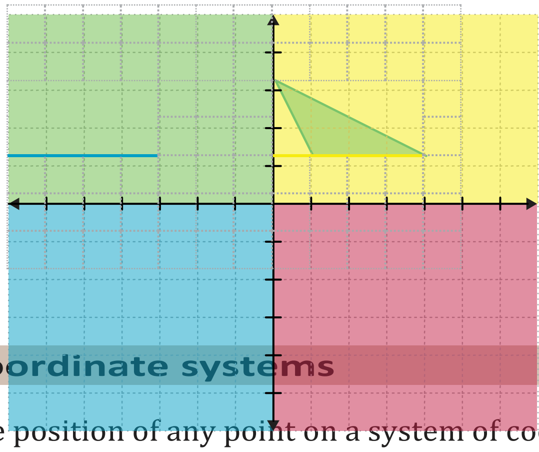
1. 

2. 

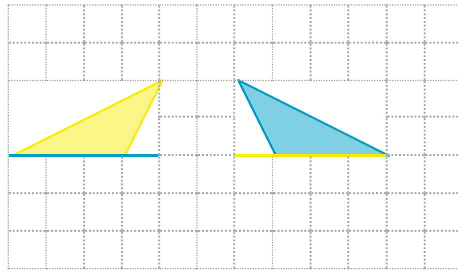
3. 

4. 

5.

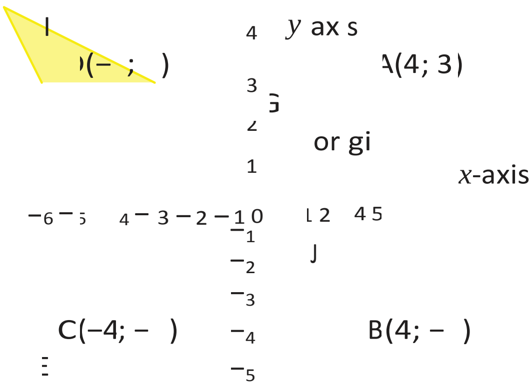


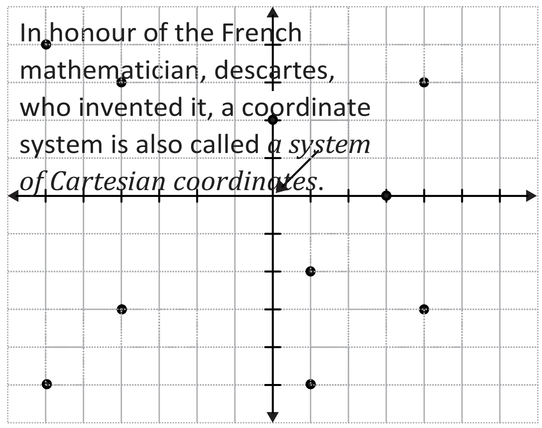
6.



coordinate systems

The position of any point on a system of coordinates can be described by two numbers, as demonstrated below for the points A, B, C and D.



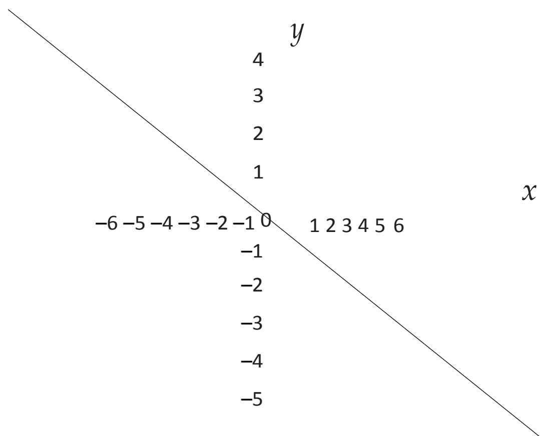


In honour of the French mathematician, descartes, who invented it, a coordinate system is also called *a system of Cartesian coordinates*.

The horizontal axis on the coordinate system is called the x-axis and the vertical axis is called the y-axis. The ordered pair (4; 3) indicates that the value of the x-coordinate is 4 and the value of the y-coordinate is 3. A coordinate system is divided into four sections called **quadrants**.

1. For the coordinate system above, give the actual coordinates of points E, F, G, H, I and J.

The first quadrant is coloured yellow on the system on the right, the second quadrant green, the third quadrant blue and the fourth quadrant red.



2. Copy the coordinate system onto grid paper. Mark the following points on the coordinate system:

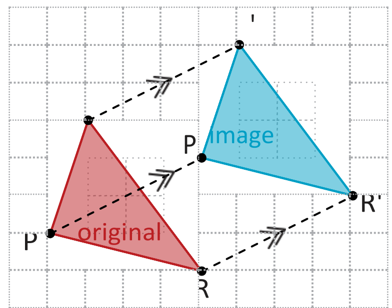
A(5; 2)	B(−4; 3)
C(−5; 1)	D(−3; −3)
E(−6; −2)	F(2; −3)
G(5; −2)	H(4; −6)

3. (a) In which quadrant are both coordinates positive?
   (b) In which quadrant are both coordinates negative?
   (c) In which quadrant is only the x-coordinate negative?
   (d) In which quadrant is only the y-coordinate negative?

24.2 Translation on the coordinate system

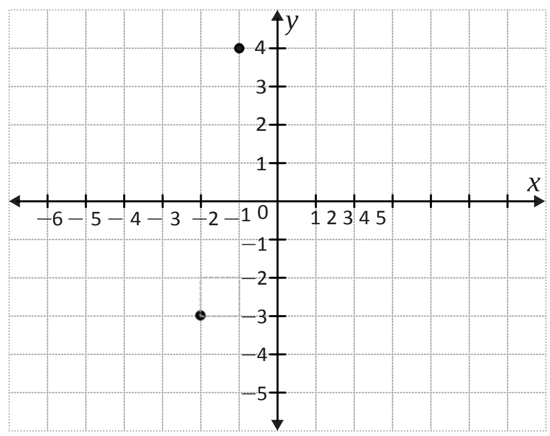
Revise the **properties of translation** from Grade 7:

- The line segments that connect any point in the original figure to its image are all equal in length. In the diagram: PP' = RR' = QQ'
- The line segments that connect any original point in the figure to its image are all parallel. In the diagram: PP'||RR'||QQ'
- When a figure is translated, its shape and size do not change. The original and its image are congruent.

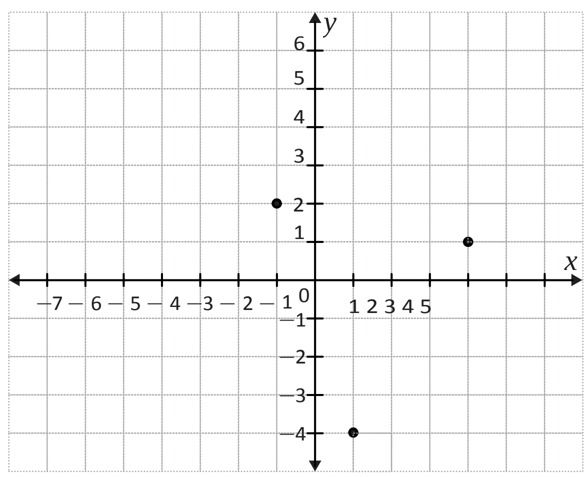


translating points on the coordinate system

1. Copy the coordinate system onto grid paper. Plot the image of each of the following translations:
   (a) R is translated three units down to R'.
   (b) R' is translated four units to the left, to R''.
   (c) W is translated five units to the right, to W'.
   (d) W' is translated six units up, to W''.



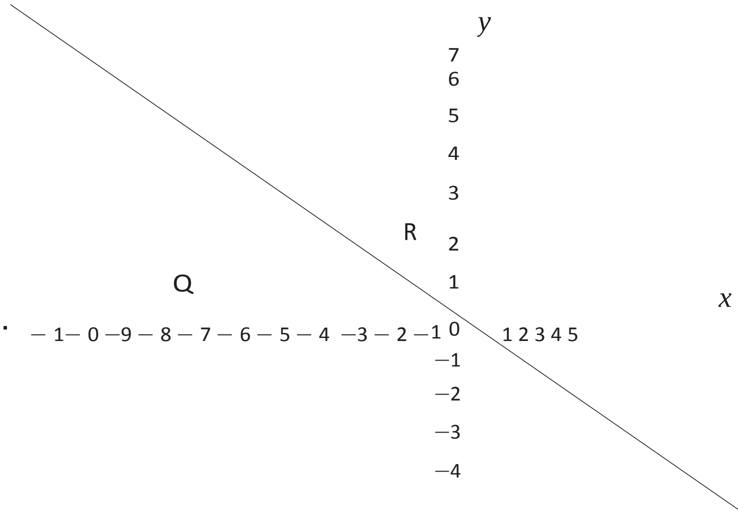
2. (a) Copy the coordinate system onto grid paper. Write down the coordinates of points A, B and C.
   (b) Translate A, B and C six units to the left and four units up.
   (c) Write down the coordinates of points A', B' and C'.
   (d) Join points A, B and C to form a triangle. Do the same with points A', B' and C'.
   (e) Are ΔABC and ΔA'B'C' congruent?



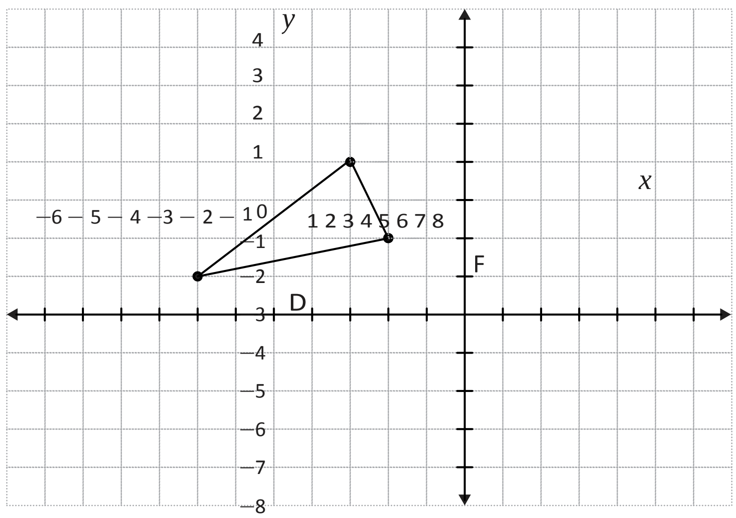
translating triangles on the coordinate system

When you plot the transformation of a shape, first plot the images of the vertices of the shape and then join the image points to create the shape.

1. (a) Copy the coordinate system onto grid paper. Translate ΔPQR six units to the right and two units down. What are the coordinates of the vertices of ΔP'Q'R'?
   (b) Translate ΔPQR four units to the left and three units up. What are the coordinates of the vertices of ΔP''Q''R''?



2. (a) Copy the coordinate system onto grid paper. Translate ΔDEF four units to the left and two units down. What are the coordinates of the vertices of ΔD'E'F'?
   (b) Translate ΔDEF three units to the right and four units up. What are the coordinates of the vertices of ΔD''E''F''?



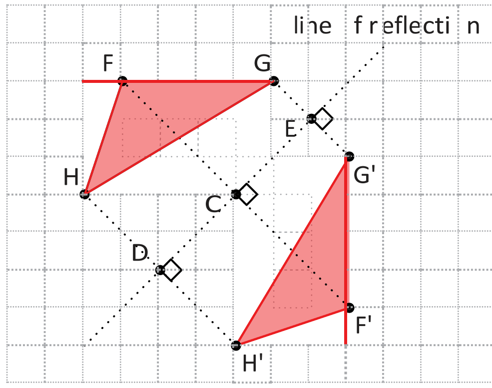
3. Copy the table and write down the coordinates of the vertices of ΔKLM after each translation described in the following table:

Vertices of triangle	Translated five units to the right and two units down	Translated four units to the left and three units down	Translated two units to the right and three units up
K(−1; 3)			
L(−2; −3)			
M(4; 0)			

24.3 Reflection on the coordinate system

Revise the **properties of reflection** from Grade 7:

- The image of ΔFGH lies on the opposite side of the **line of reflection** (mirror line).
- The distance from the original point to the line of reflection is the same as the distance from the image point to the line of reflection. In the diagram: GE = G'E; FC = F'C and HD = H'D.
- The line that connects the original point to its image point is always perpendicular ($\perp$) to the line of reflection. In the diagram: HH' $\perp$ line of reflection, FF' $\perp$ line of reflection and GG' $\perp$ line of reflection.
- When a figure is reflected, the figure and its image are congruent.



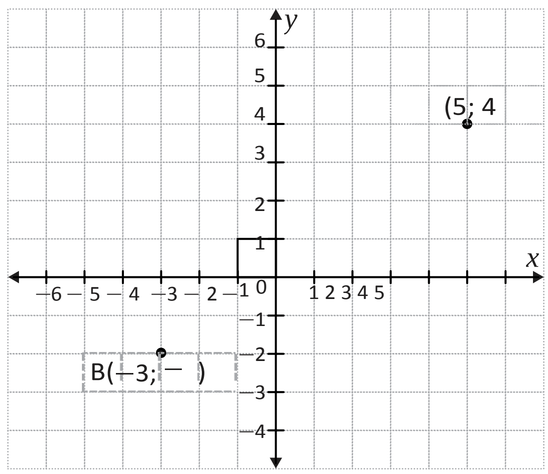
A line of reflection can run in any direction. This year, you will learn about reflections in the x-axis or in the y-axis only.

reflecting points in the x-axis or in the y-axis

Reflecting a point in the x-axis means that the x-axis is the line of reflection.

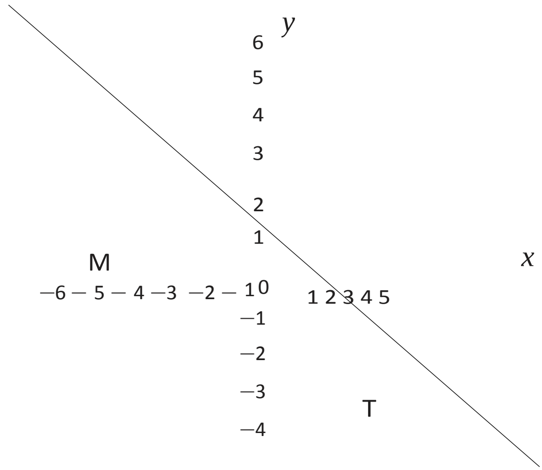
Reflecting a point in the y-axis means that the y-axis is the line of reflection.

1. The points A(5; 4) and B(−3; −2) are plotted on a coordinate system.



   (a) Copy the coordinate system onto grid paper. Reflect points A and B in the x-axis (horizontal mirror) and then in the y-axis (vertical mirror).
   (b) What are the coordinates of the images of point A and B when reflected in the x-axis?
   (c) What are the coordinates of the images of point A and B when reflected in the y-axis?
   (d) Compare the coordinates of points A and B with the coordinates of their images. What do you notice?

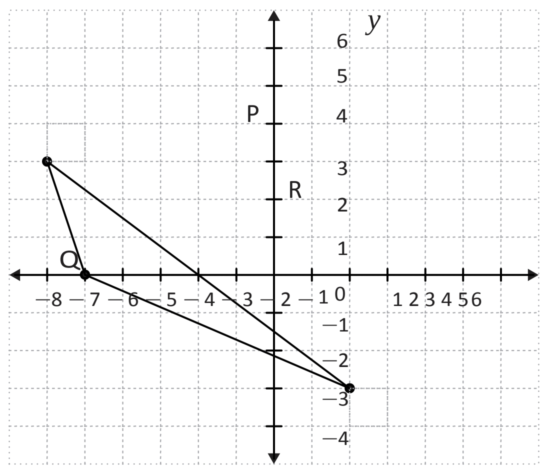
2. The points K, M and T are plotted on the coordinate system.
   (a) Write down the coordinates of points K, M and T.
   (b) Copy the coordinate system onto grid paper. Reflect each point in the x-axis and write down the coordinates of K', M' and T'.
   (c) Reflect points K, M and T in the y-axis and write down the coordinates of K'', M'' and T''.
   (d) Join points K, M and T to form a triangle. Do the same with points K', M' and T', and with points K'', M''and T''.
   (e) Are all three triangles congruent?



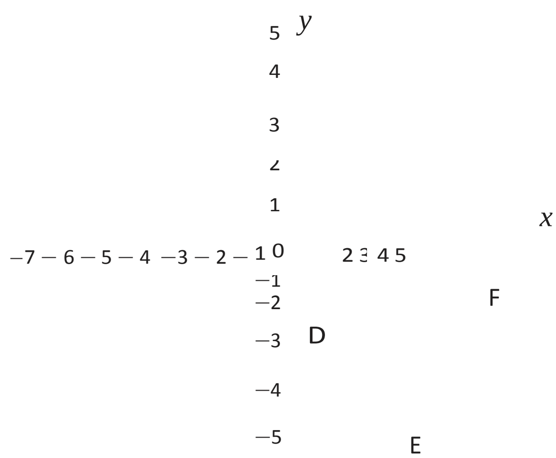
reflecting triangles in the x-axis or in the y-axis

When you reflect a triangle, first reflect the vertices of the triangle and then join the reflected points.

1. (a) Copy the coordinate system onto grid paper. Reflect ΔPQR in the x-axis.
   (b) Reflect ΔPQR in the y-axis.



2. (a) Copy the coordinate system onto grid paper. Reflect ΔDEF in the x-axis.
   (b) Reflect ΔDEF in the y-axis.



3. The coordinates of the vertices of three triangles are given in the tables below. Copy the tables. For each vertex, write down the coordinates of its reflection in the x-axis or in the y-axis asrequired.

(a)

Vertices of triangle	Reflection in the x-axis
K(−4; 5)	
L(2; −5)	
M(−5; −3)	

(b)

Vertices of triangle	Reflection in the y-axis
X(−1; 3)	
Y(−2; −3)	
Z(4; 1)	

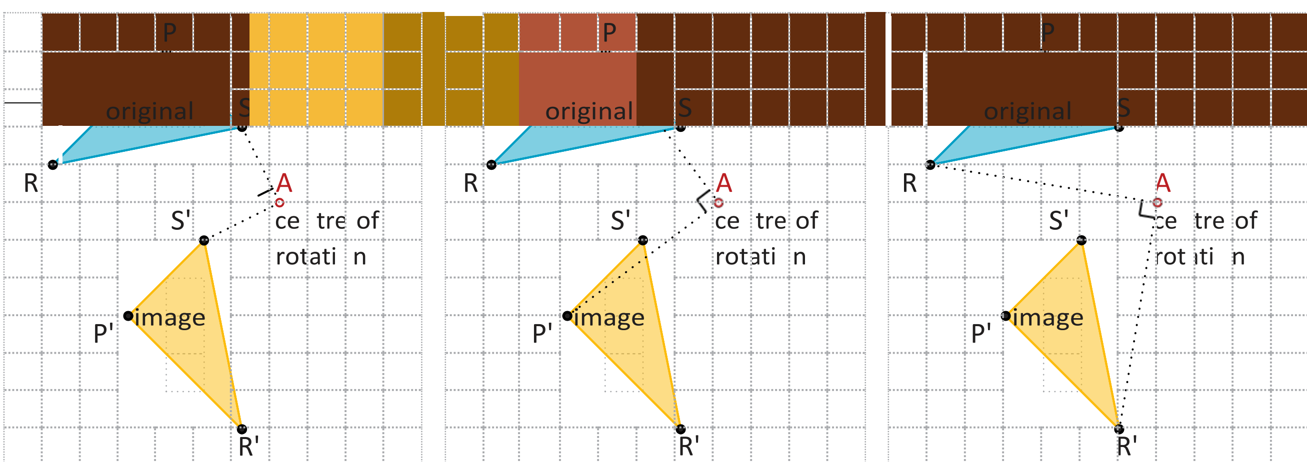
(c)

Vertices of triangle	Reflection in the y-axis	Reflection in the x-axis
D(−2; 5)		
E(0; −3)		
G(2; 0)		

24.4 Rotation on the coordinate system

The distance from the centre of rotation to any point on the original image is equal to the distance from the centre of rotation to its corresponding point on the image. In the diagrams below: SA = S'A, PA = P'A and RA = R'A.

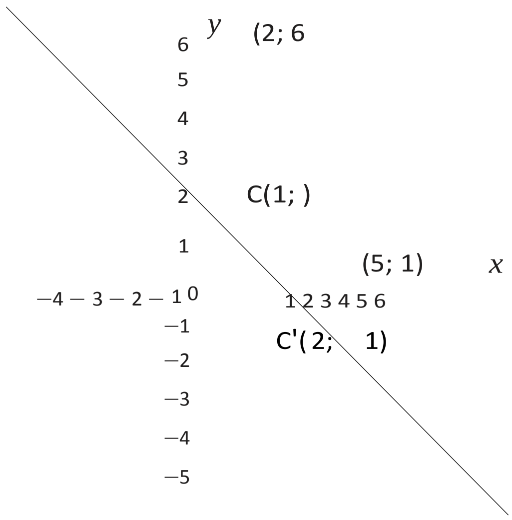
The angle that is formed between the line connecting an original point (S or P or R) to the centre of rotation A and the line connecting the image point (S', P', R') to the centre of rotation is equal to the angle of rotation. In the diagrams, the triangle was rotated through 90°, so SÂS' = 90°, PÂP' = 90° and RÂR' = 90°.



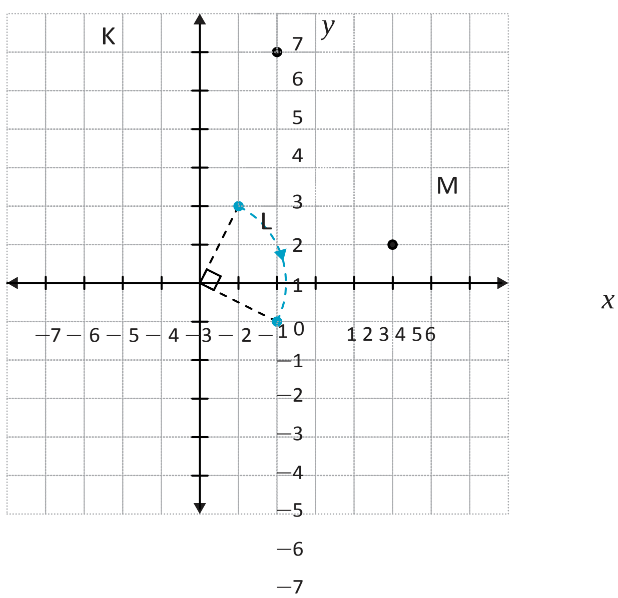
On the coordinate system, the centre of rotation can be any point. This year, you will focus on rotations about the point (0; 0), which is called the **origin**. A point, line segment or figure can be rotated clockwise or anticlockwise through any number of degrees about the centre of rotation.

rotating points and figures about the origin

1. In the diagram, point C has been rotated 90° clockwise about the origin. Copy the coordinate systems onto grid paper.
   (a) Rotate points A and B 90° clockwise about the origin.
   (b) Write down the coordinates of points A' and B'.
   (c) Join points A, B and C to form atriangle. Do the same with points A', B'and C'.
   (d) Are the triangle and its image congruent?
   (e) Compare the coordinates of points A, B and C with the coordinates of their images. What do you notice?

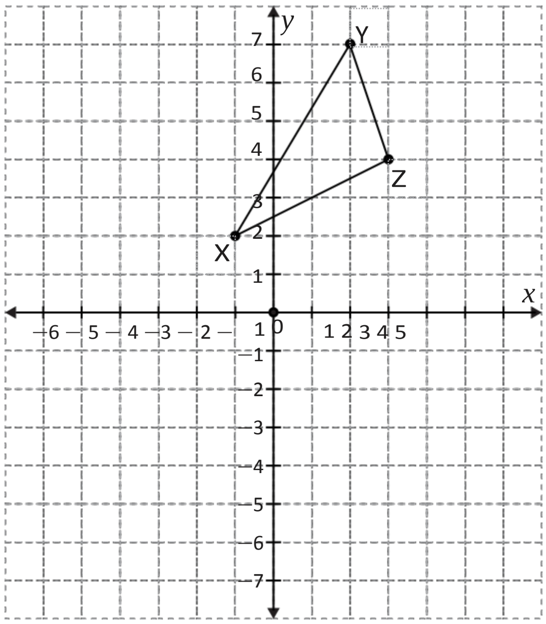


2. (a) Write down the coordinates of points K, L and M.
   (b) Rotate points K, L and M 90° anticlockwise about the origin.
   (c) Write down the coordinates of the image points.
   (d) Rotate points K, L and M 180° about the origin.
   (e) Write down the coordinates of K'', L'' and M''.
   (f) Can you explain why there was no need to say "clockwise" or "anticlockwise" in question (d)?

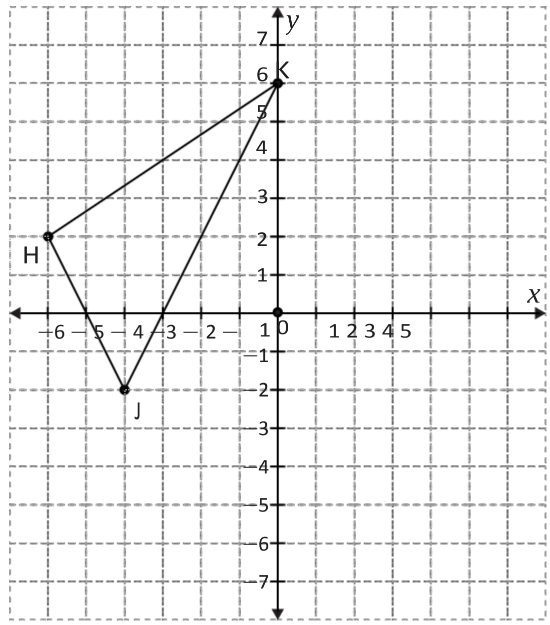


3. Copy the coordinate systems on the following page onto grid paper. Rotate the following triangles and write down the coordinates of the vertices of each triangle after the required rotation.

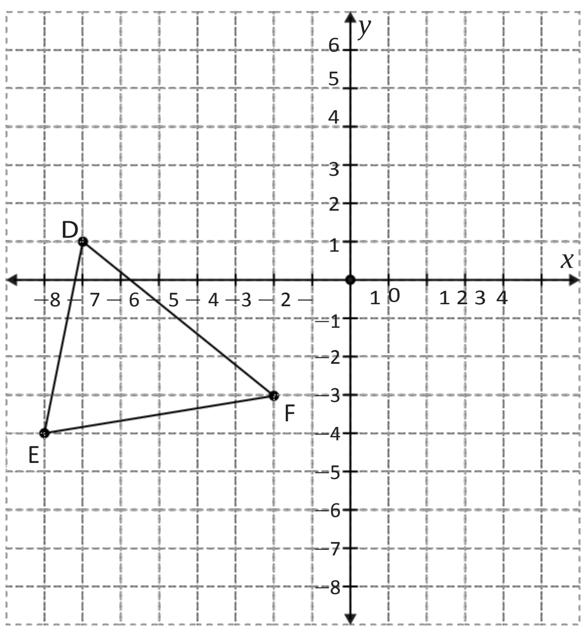
(a) 180° aboutthe origin



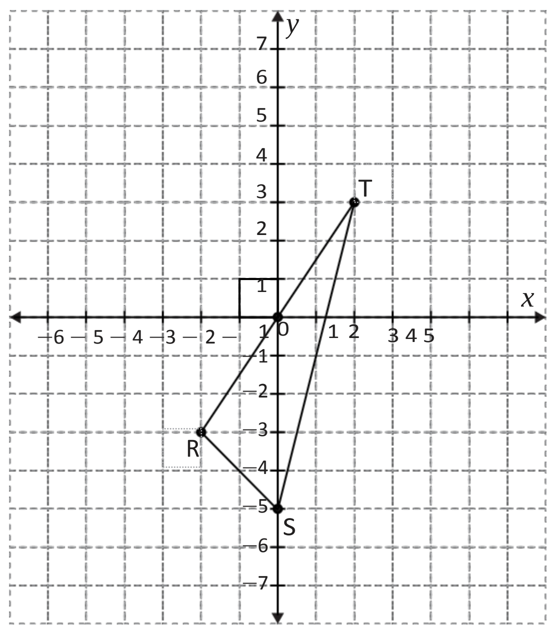
(b) 90° clockwise about the origin



(c) 90° anticlockwise about the origin



(d) 180° about the origin



4. Write down the coordinates of each image point after these transformations:
   (a) Rotation 180° about the origin: K(−1; 0); C(1; 1); N(3; −2)
   (b) Rotation 90° clockwise about the origin: L(1; 3); Z(5; 5); F(4; 2)
   (c) Rotation 90° anticlockwise about the origin: S(1; −4); W(1; 0); J(3; −4)
   (d) Rotation 180° about the origin: V(−5; −3); A(−3; 1); G(0; −3)

24.5 Enlargements and reductions

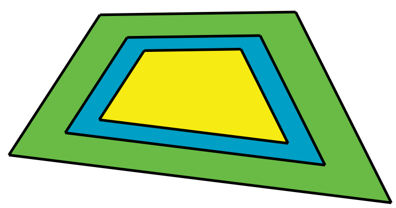
calculate and use scale factors

A figure may be made bigger or smaller without changing its shape.

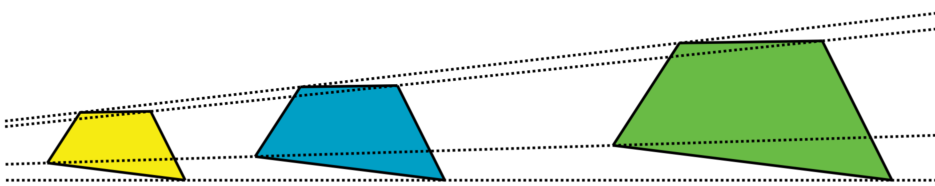
The yellow figure is a reduction *of the blue quadrilateral.*

The green figure is an enlargement *of the blue quadrilateral.*

A figure is only called an enlargement or reduction of another figure if the two figures have the **same shape**. The shapes can only be the same if all the corresponding angles are equal.



Even if the angles are equal, two figures may have different shapes. When the corresponding angles are equal, one figure is not necessarily an enlargement or reduction of the other.



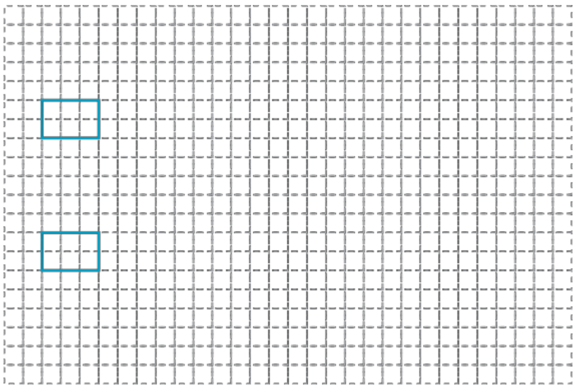
Although the angles are equal, the yellow and green figures above are *not* enlargements of the blue figure.

When a figure with straight sides is enlarged or reduced, the lengths of the sides are increased or decreased.

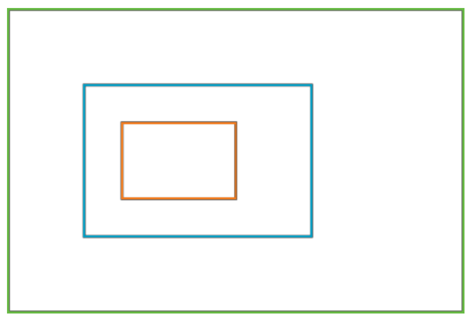
To find the lengths of the sides of the new figure, the lengths of the sides of the original figure are all multiplied by the same number. This number is called the **scale factor** of the enlargement or reduction.

The scale factor for an **enlargement** is bigger than 1. The scale factor fora **reduction** is smaller than 1.

1. On grid paper, draw a bigger rectangle ABCD, with each side five times as long as the blue rectangle. Also draw another bigger rectangle PQRS, with each side five units longer than the blue rectangle.

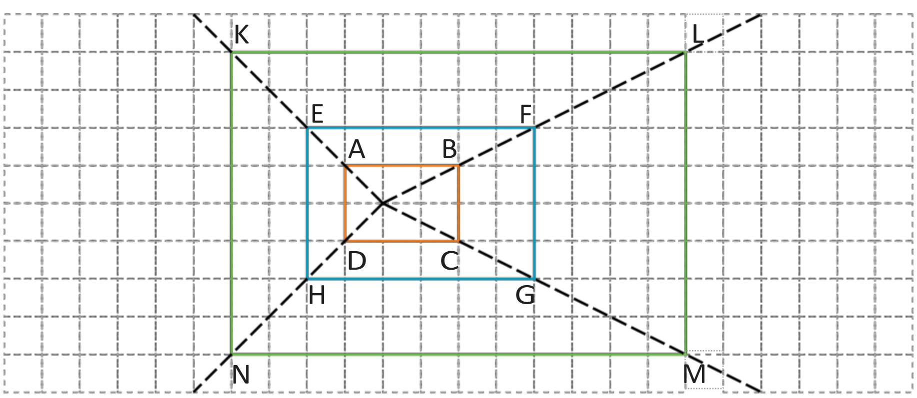


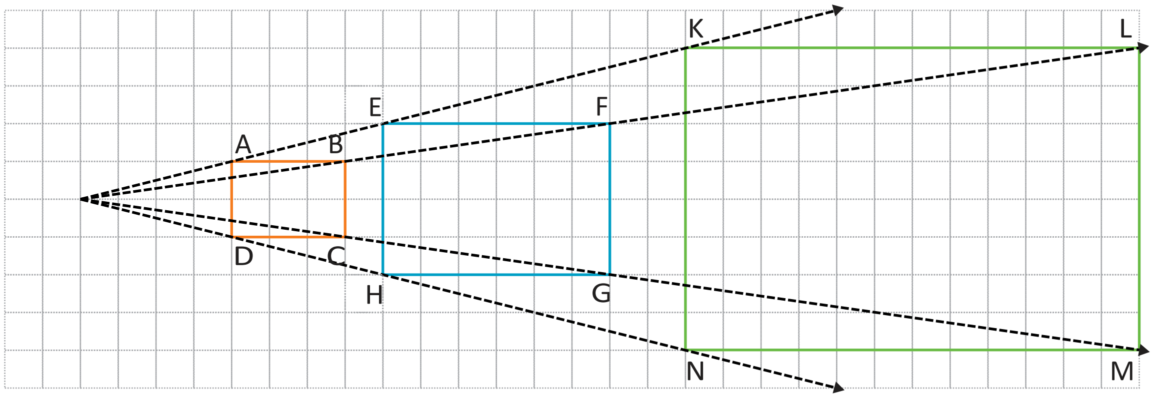
One figure is only called an enlargement or reduction of another figure if **the corresponding angles are equal** and **the ratio between the lengths of the corresponding sides is the same**, for all pairs of corresponding angles and sides in the two figures. This is demonstrated below.



The green rectangle on the right is an enlargement of the blue rectangle. The orange rectangle is a reduction of the blue rectangle.

In the following two diagrams, the same rectangles are shown on grids so that it is easy to compare the lengths of the corresponding sides and calculate the ratio between the lengths of thesides.





KLMN is an enlargement of EFGH.

Note that $\frac{LM}{FG} = 8 : 4 = 2$, $\frac{MN}{GH} = 12 : 6 = 2$, $\frac{NK}{HE} = 8 : 4 = 2$ and $\frac{KL}{EF} = 12 : 6 = 2$.

The ratio between the lengths of corresponding sides is 2, for all four pairs of corresponding sides.

We say: The **scale factor** of the enlargement from EFGH to KLMN is 2.

To avoid confusion, mathematicians normally state the dimensions of the image first when forming ratios.

ABCD is a reduction of EFGH.

Note that $\frac{BC}{FG} = 2 : 4 = \frac{1}{2}$, $\frac{CD}{GH} = 3: 6 = \frac{1}{2}$, $\frac{DA}{HE} = 2 : 4 = \frac{1}{2}$ and $\frac{AB}{EF} = 3 : 6 = \frac{1}{2}$.

The ratio between the lengths of corresponding sides is $\frac{1}{2}$, for all four pairs of corresponding sides. The scale factor of the reduction from EFGH to ABCD is $\frac{1}{2}$.

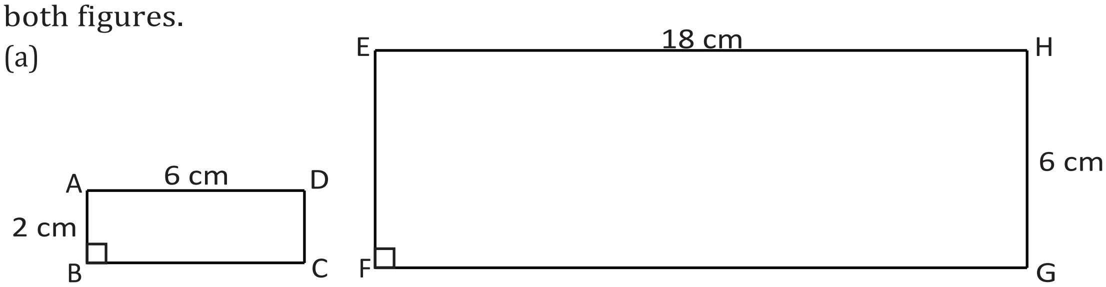
2. (a) What is the scale factor of the enlargement from ABCD to KLMN?
   (b) What is the scale factor of the reduction from KLMN to EFGH?

3. A rectangular shape on a photograph is 3 mm wide and 4 mm long. The photograph is enlarged with a scale factor of 5. What is the width and length of the rectangular shape on the enlarged photograph?

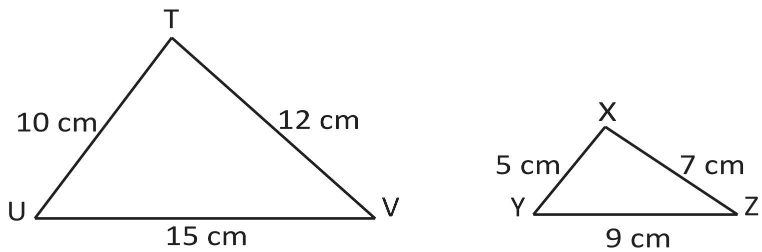
We work out the scale factor by calculating the ratios of the lengths of corresponding sides of the two figures. If the ratios are equal, we say that the corresponding sides are **in proportion**. This means that the second figure (the image) is a reduction or an enlargement of the first figure (the original).

4. Determine whether the second figure in each of the following pairs is an enlargement, a reduction, or neither of the two. Also work out the perimeters of both figures.

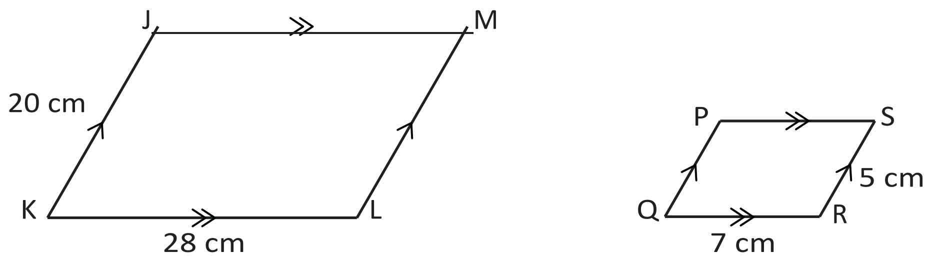
   (a)



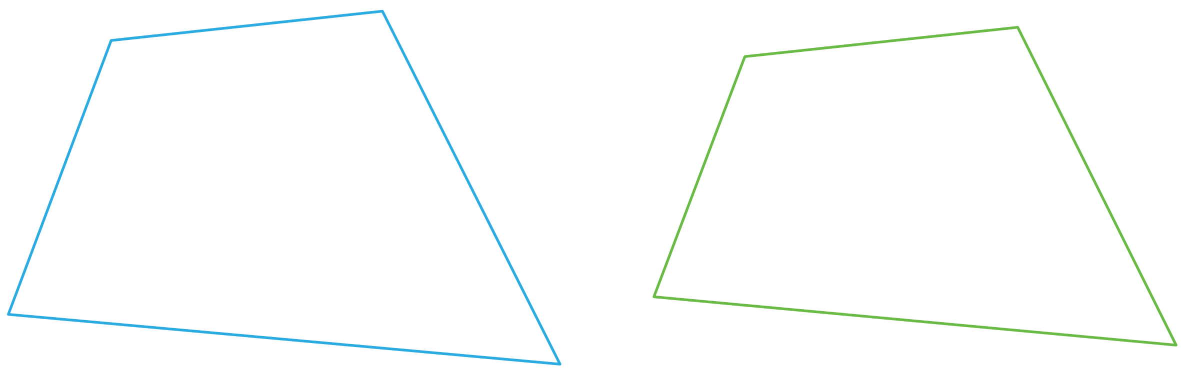
(b)



(c)

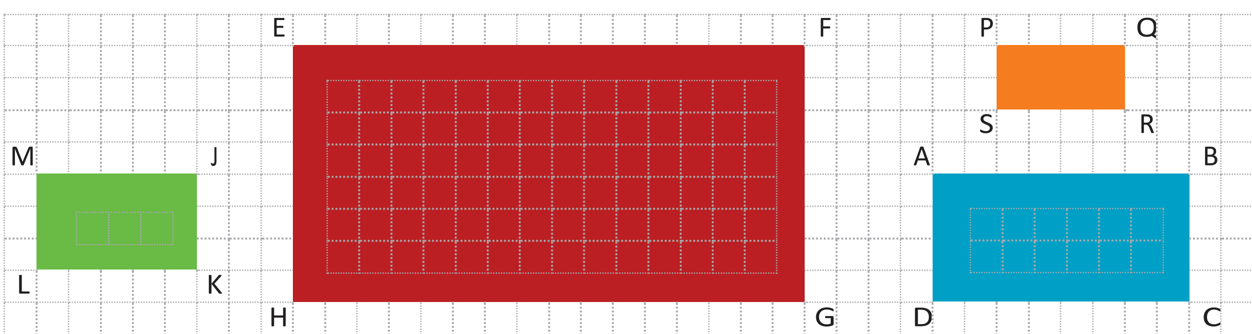


5. Takemeasurements and do calculations to establish whether the blue figure below is an enlargement of the green figure.



effect of enlargements or reductions on perimeter and area

Consider the following rectangles:



1. (a) Do you think EFGH is an enlargement of MJKL?
   (b) Do you think PQRS is a reduction ofEFGH?
   (c) Do you think EFGH is an enlargement of ABCD?

2. (a) Calculate $\frac{EF}{MJ}$, $\frac{FG}{JK}$, $\frac{GH}{KL}$ and $\frac{HE}{LM}$.
   (b) Is rectangle EFGH an enlargement of rectangle MJKL?
   (c) If EFGH is an enlargement of MJKL, what is the scale factor?
3. (a) Calculate $\frac{PQ}{EF}$, $\frac{QR}{FG}$, $\frac{RS}{GH}$ and $\frac{SP}{HE}$.
   (b) Is rectangle PQRS a reduction of rectangle EFGH?
   (c) If PQRS is a reduction of EFGH, what is the scale factor?
4. (a) Calculate $\frac{EF}{AB}$, $\frac{FG}{BC}$, $\frac{GH}{CD}$ and $\frac{HE}{DA}$.
   (b) Is rectangle EFGH an enlargement of rectangle ABCD?
   (c) If EFGH is an enlargement of ABCD, what is the scale factor?
5. Do you agree or disagree with the following statements?
   (a) Perimeter of enlargement/reduction = perimeter of original × scale factor
   (b) Area of enlargement/reduction = area of original × (scale factor)^2

calculating perimeters and areas of enlarged or reduced figures

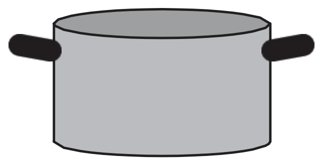
1. The perimeter of rectangle DEFG = 20 cm and its area = 16 cm^2. Find the perimeter and area of the enlarged rectangle D'E'F'G' if the scale factor is 3.
2. The perimeter of ΔJKL = 120 cm and its area = 600 cm^2. Determine the perimeter and area of the reduced ΔJ'K'L' if the scale factor is 0,5.
3. The perimeter of quadrilateral PQRS = 30 mm and its area is 50 mm^2. Find the perimeter and area of quadrilateral P'Q'R'S' if the scale factor is $\frac{1}{5}$.
4. The perimeter of ΔSTU = 51 cm and its area is 12 cm^2. Calculate the perimeter and area of ΔS'T'U' if the scale factor is $\frac{1}{3}$.
5. The perimeter of a square = 48 m.
   (a) Write down the perimeter of the square if the length of each side is doubled.
   (b) Will the area of the enlarged square be twice or four times that of the original square?
6. The perimeter of ΔDEF = 7 cm and ΔD'E'F' = 21 cm. What is the scale factor of enlargement? How many times larger is the area of ΔD'E'F' than the area of ΔDEF?
7. The perimeter of quadrilateral ADFS = 26 cm and the perimeter of quadrilateral A'D'F'S' = 13 cm. How many times larger is the area of quadrilateral A'D'F'S' than the area of quadrilateral ADFS?

Chapter 25

Geometry of 3D objects

25.1 Revision: 3D objects

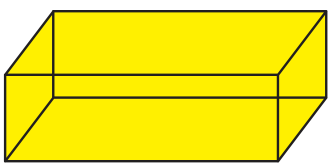
think of space while you look at pictures and drawings

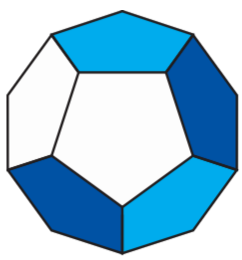


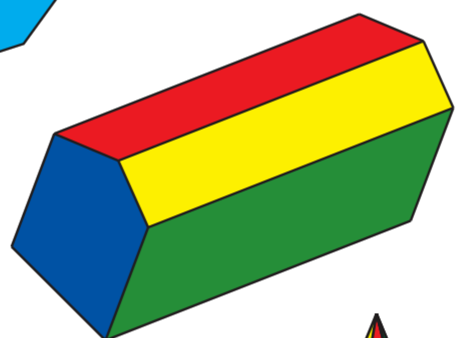
Most objects we see around us, like fruit, animals, trees, people and motor cars, have curved or round surfaces. Some objects, like a saucepan or other cooking vessel, have both round and flat surfaces. The circular bottom of a saucepan must be flat so that it makes good contact with the stove plate.

1. (a) Should the top of a table or desk be a flat or curved surface?
   (b) We eat with knives, forks and spoons. Which of these objects normally have curved surfaces?

This chapter is about objects that only have flat surfaces, like those shown below.





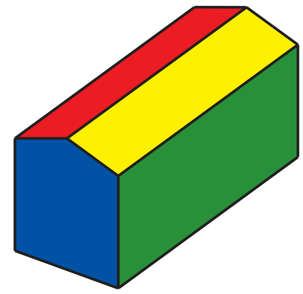


The front, right and top faces in the above drawing are made of clear plastic so that you can see the faces behind them.

Note this strange box with different colours on its different faces.

2. Do you think there is enough space for all the birds in the tent shown on the right-hand side?

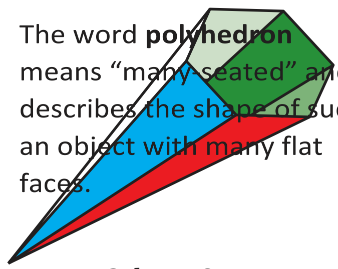
3. The unusual box, shown on the previous page, with flat faces (surfaces) only is shown again below. In the drawing of the same box on the right, dotted lines are used to indicate edges and surfaces that are hidden in the coloured drawing.



(a) How many faces (different flat surfaces) does this object have altogether?
(b) How many faces cannot be seen in the coloured drawing on the left?
(c) How many of the faces are rectangles?
(d) How many of the faces are pentagons?

A 3D object with **flat faces (surfaces) only** is called a **polyhedron** (plural: **polyhedra**).

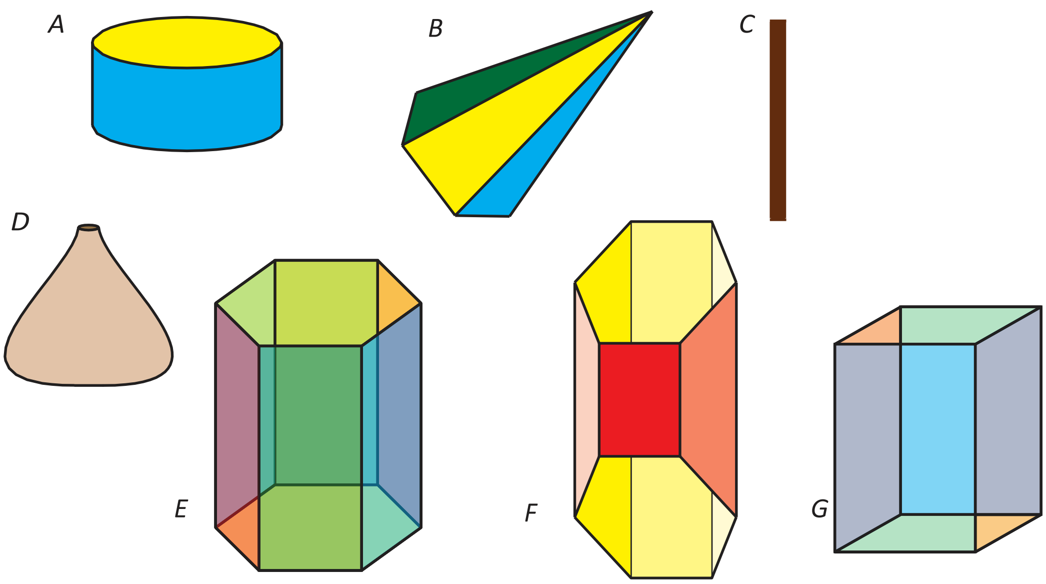
A straight **edge** is formed where two flat surfaces meet. The point where two or more edges meet is called a **vertex** (plural: **vertices**).

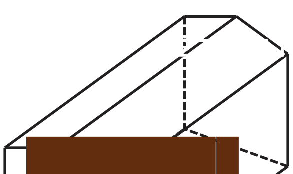


The word **polyhedron** means "many-seated" and describes the shape of such an object with many flat faces.

4. (a) How many edges does the coloured polyhedron in question 3 have?
(b) How many vertices does it have?

5. Which of the objects below are polyhedra?





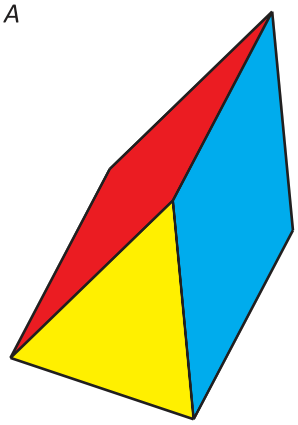
two special types of polyhedra

PolyhedralikeCandE atthebottom ofthepreviouspage arecalled **prisms**.
Polyhedra like B and G are called **pyramids**.

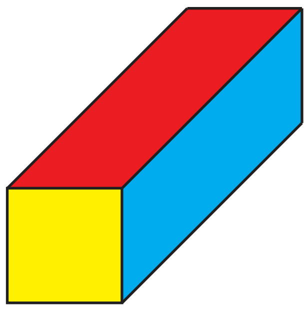
1. Describe the differences between prisms and pyramids.

Here are some more pictures of **prisms**:

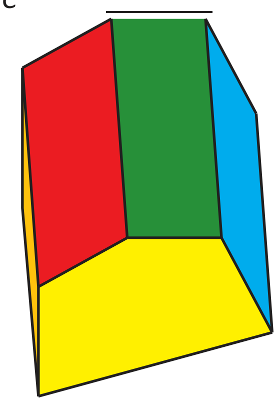
A



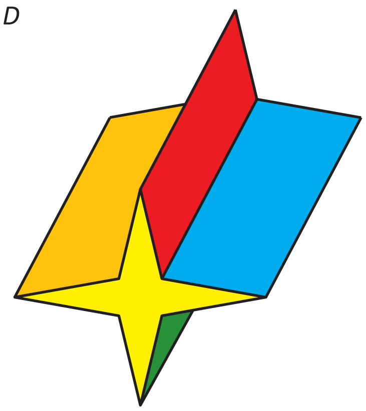
B



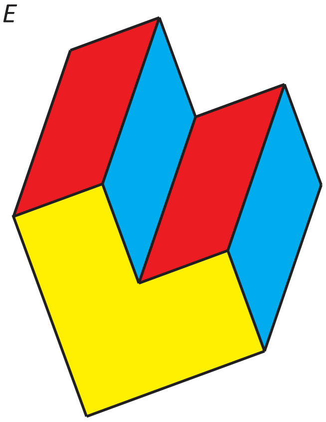
C



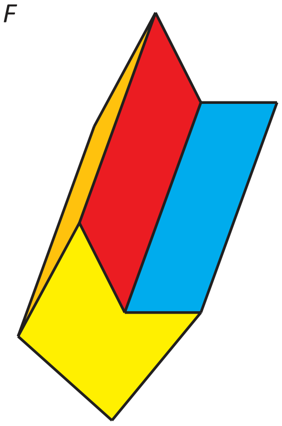
D



E



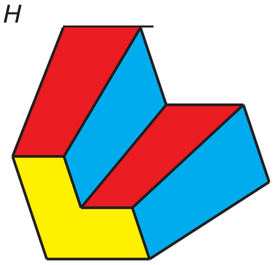
F



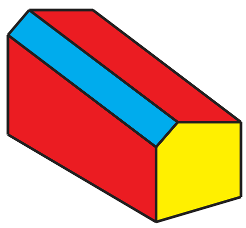
The objects shown by the four pictures below are polyhedra but they are *not* prisms or pyramids.

G

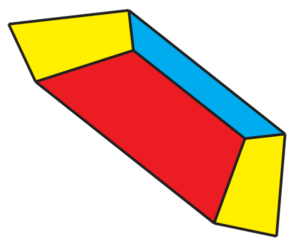
H



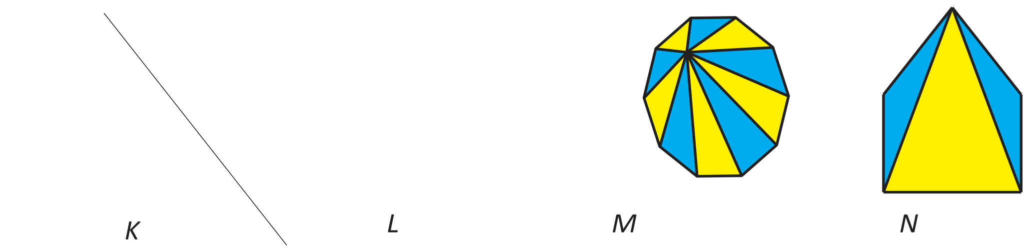
I

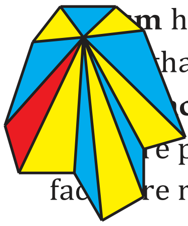


J



Here are some pictures of **pyramids**. More pictures of pyramids are shown at the bottom of this page and also on the next page.

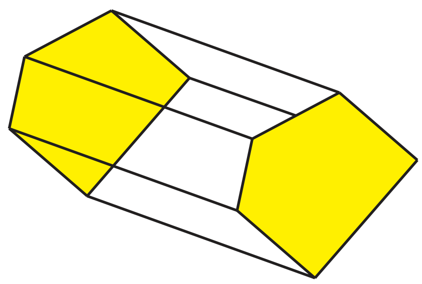




…**m** has two identical, parallel faces (called …hat are connected by parallelograms (called …**ces**). In the case of right prisms, the lateral …e perpendicular to the bases and the lateral fac…re rectangles.

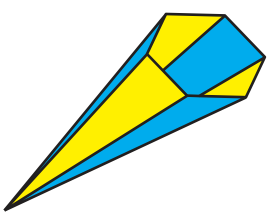
A prism with pentagonal bases like this one is called a **pentagonal prism** because the base is a pentagon.

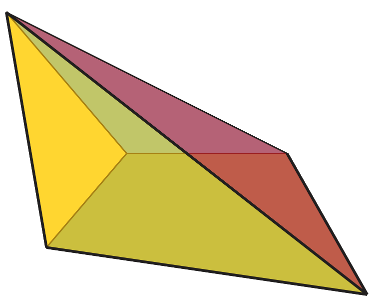
2. (a) Which pictures on the previous page also show pentagonal prisms?
   (b) Which picture on the previous page shows a hexagonal prism?
   (c) Which picture on the previous page shows an octagonal prism?

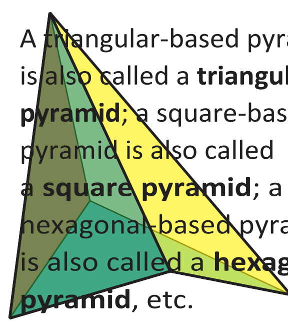


A **pyramid** has only one base. The lateral faces of a pyramid are triangles that meet at the **apex.**

The pyramid on the right is called a **hexagonal-based pyramid**.
Thetwopyramids belowhavequadrilaterals asbasesandare called **quadrilateral-based pyramids**.



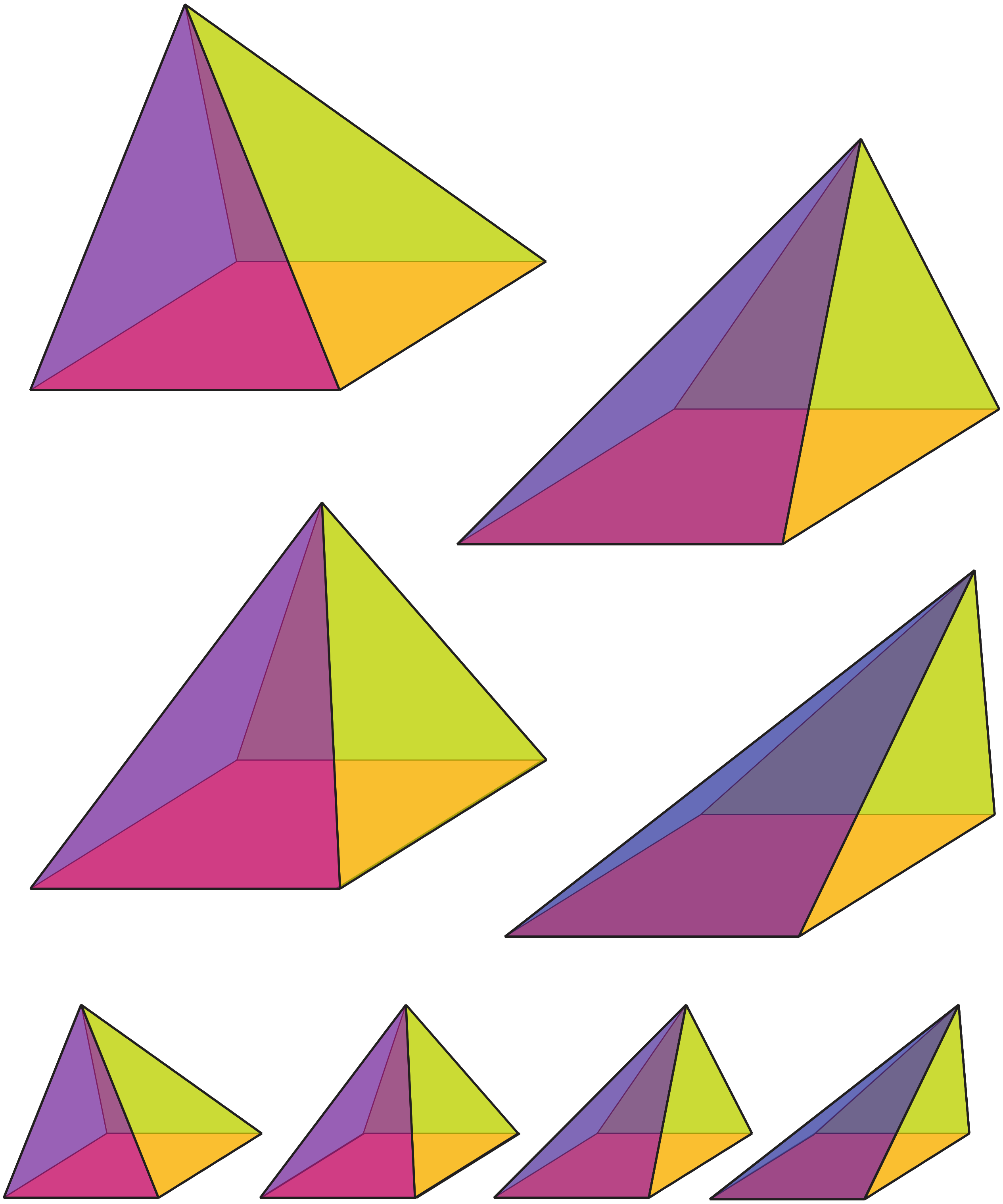




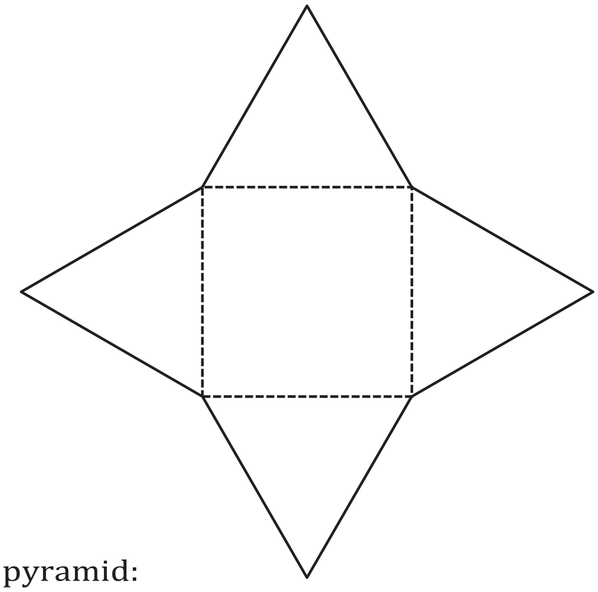
A triangular-based pyramid is also called a **triangular pyramid**; a square-based pyramid is also called a **square pyramid**; a hexagonal-based pyramid is also called a **hexagonal pyramid**, etc.

3. Which picture at the top of this page shows a hexagonal pyramid?

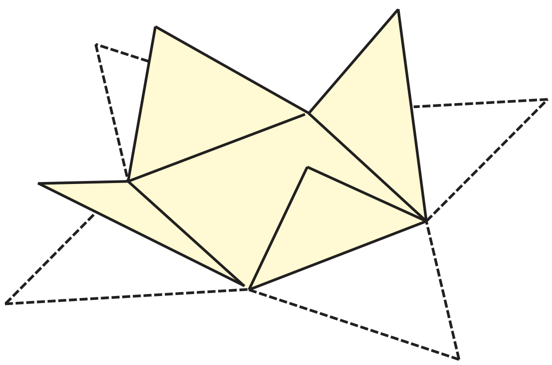
Pictures of different **square-based pyramids** are shown below.

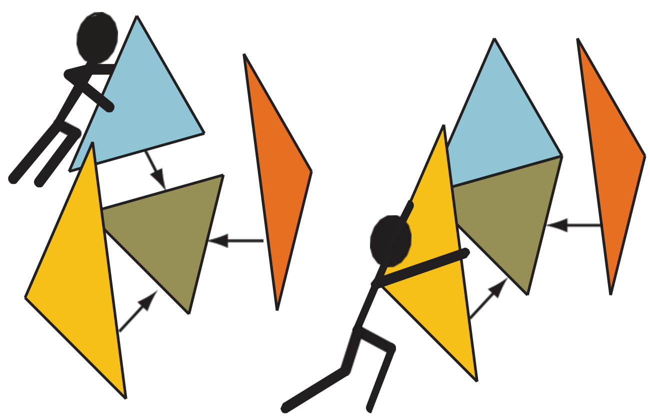


You can make a square-based pyramid by drawing and cutting out a diagram like the one on the left below, and folding the triangles up on the dotted lines, as shown on the right.



These men are building a **triangular-based** pyramid:

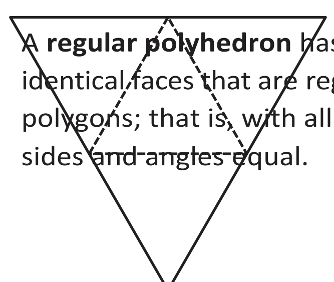




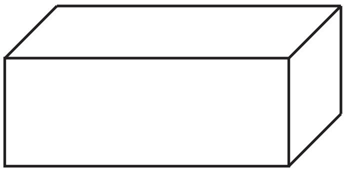
[redacted]-based pyramid is also called a
[redacted]**n**, which literally means “four-face”.
[redacted]ron with four identical faces that are equilateral triangles is called a **regular tetrahedron**.

If you copy and cut out a figure like the one on the right, and fold the triangles up on the dotted lines, you can make a regular tetrahedron. A diagram like this, that can be cut out and folded to make a model of a polyhedron, is called a **net**.

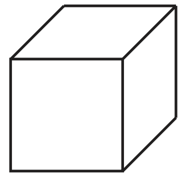
A **regular polyhedron** has identical faces that are regular polygons; that is, with all sides and angles equal.

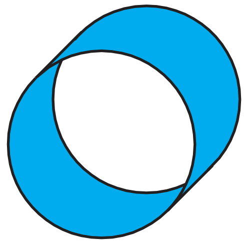


A rectangular prism is also called a **cuboid**.

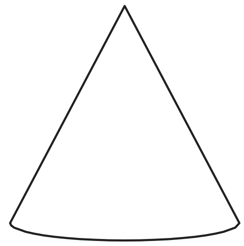


A cuboid with square faces is also called a **cube**.

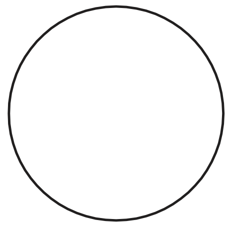




An object with two identical circular bases and one curved surface is called a **cylinder**.



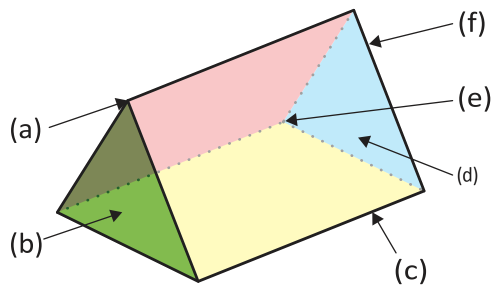
A "pyramid" with a round base is called a **cone**.



An object with the shape of a ball, in other words one curved surface with every point on its surface the same distance from its centre, is called a **sphere**.

Cylinders, cones and spheres are *not* polyhedra since they have curved surfaces. Remember, a polyhedron has faces, edges and vertices. The faces are the flat surfaces. An edge is a line along which two faces of a 3D object meet, and connects two vertices. A vertex is the point where the edges meet.

4. Write down and name the labels for parts (a) to (f) on the following figure:

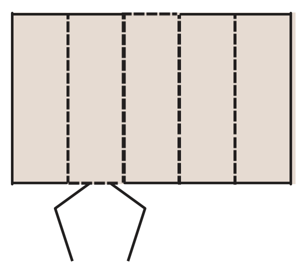


Triangular prism

5. Grade 8 learners made 3D objects from cardboard. Can you say which kind of figure the following three learners made?
   (a) Adam's object had eight vertices and 12 edges.
   (b) Lea's object had four vertices and four faces.
   (c) Mary's object had 12 edges and six congruent faces.

6. Copy and complete the table for prisms. Count the bases as faces too. If you find this difficult, it may help you to make quick rough sketches of nets for some prisms, like the sketches given below the table.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	5	6	9		
4	6		12		
5					
6					
8					
10					



7. Copy and complete the table for pyramids. Count the bases as faces too.

Number of sides in each base	Number of faces	Number of vertices	Number of edges	Faces + vertices	Edges + 2
3	4	4	6		
4					
5					
6					
7					
9					

8. Consider your answers for questions 6 and 7. Is the following statement true for both prisms and pyramids?

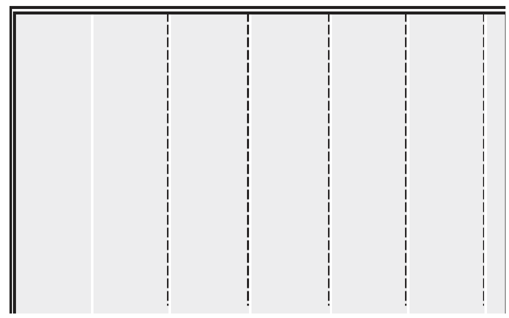
This statement is called **euler's formula** for polyhedra.

Statement: *The number of faces* + *the number of vertices* = 2 + *the number of edges*

9. Is Euler's formula true for the polyhedra G, H, I and J on page 271?

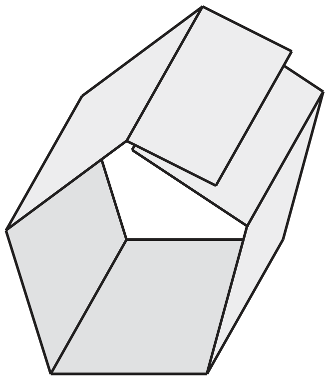
25.2 Nets and models of prisms and pyramids

a quick way to make prisms and pyramids



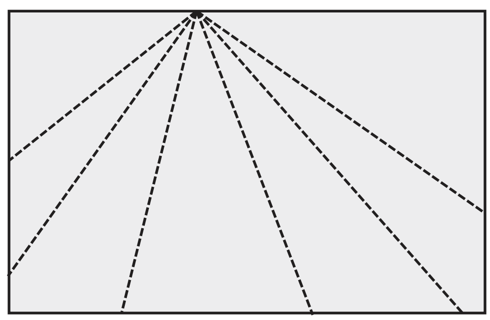
Fold sections about two fingers wide on a sheet of A4 paper, more or less as shown by the dotted lines in the sketch on the right.

Fold the sheet into a "tube" with five or six faces along its length, as shown below.

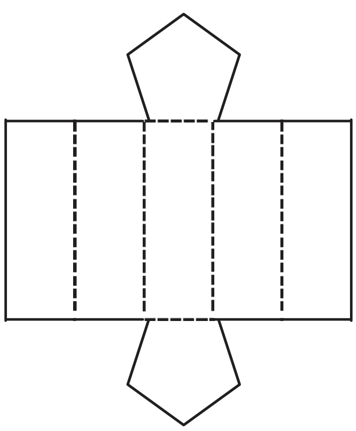


With a little extra work, you can now make a paper prism. You need to cut out two bases so that they fit well.

You can make prisms with triangular, square, rectangular, hexagonal and other shaped bases in this way.



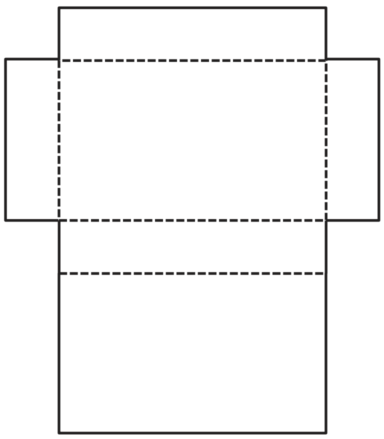
You can make a pyramid in the same way, but it is more difficult. Draw dotted lines on a sheet of A4 paper, as shown in the sketch on the right. Fold the paper along the dotted lines. It is quite difficult to know where and how to cut so that the base is a flatsurface.



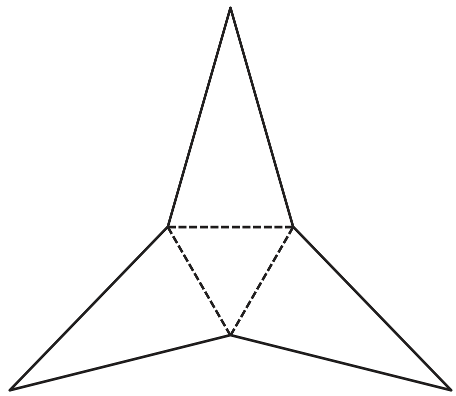
Apart from the difficulty of getting the base of the pyramid flat, the above method has the disadvantage that you have to use separate pieces of paper or other material to make one object. It would be better to make the whole object by folding one piece of paper. For example, a prism with a pentagonal base can be made by drawing, cutting out and folding a sheet of paper, as shown on the left. This diagram is called a **net** of a prism with a regular pentagonal base.

nets for different polyhedra

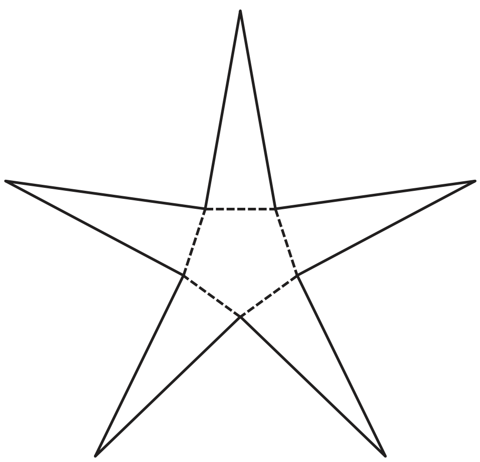
1. Name the polyhedron that can be made from each of the following nets:



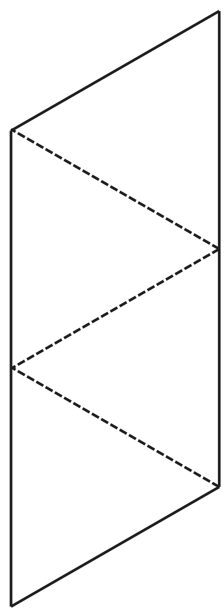
(a)



(b)

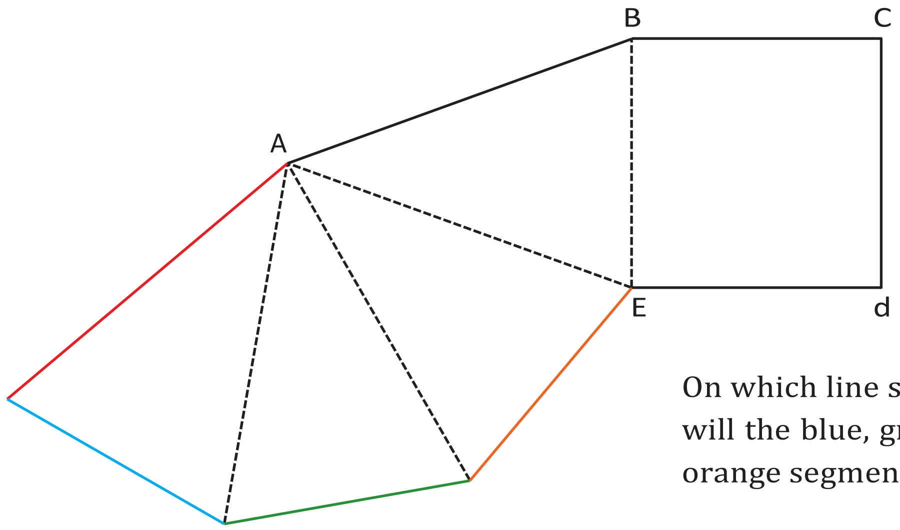


(c)



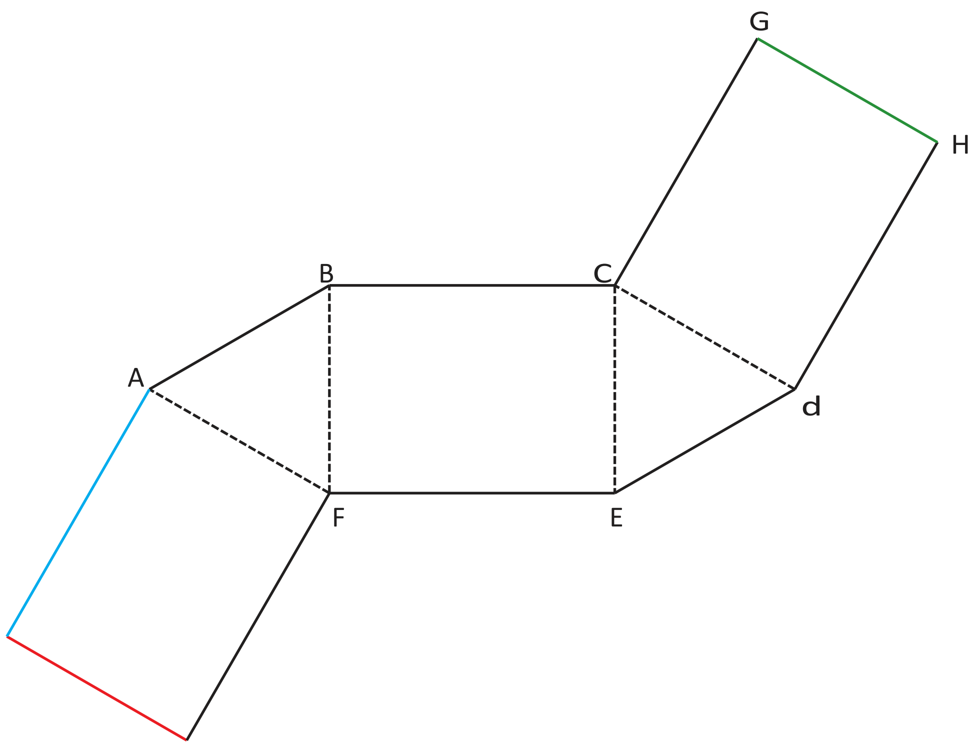
(d)

2. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.
   (b) When this net is folded to make a polyhedron, the red line segment will fall on AB to form an edge.



On which line segments will the blue, green and orange segments fall?

3. (a) Name the polyhedron that can be formed by cutting out the diagram below on the solid lines and folding it on the dotted lines.
   (b) On which segments will the red, blue and green segments fall to form edges of this polyhedron?



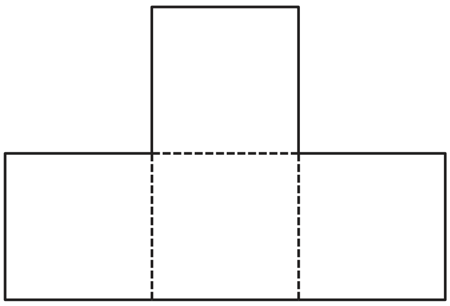
4. Some of the diagrams below and on the next page are the nets for the following objects:

- a square-based pyramid
- a hexagonal pyramid
- a cuboid
- a triangular prism
- a hexagonal prism
- a cube

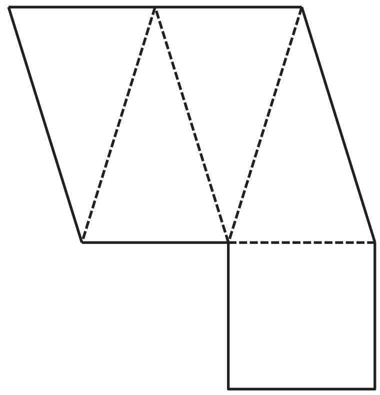
Under each diagram, write the name of the object for which the diagram is a net. There may be more than one net for some of the objects. Write "none" if the diagram is not a net for any prism or pyramid.

A diagram is only called a **net** of an object if the cut-out diagram can be folded to form **all** the faces of the object.

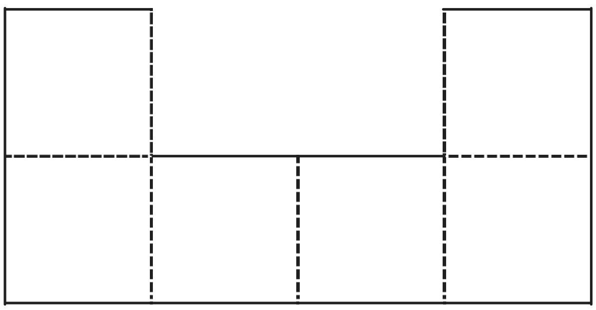
(a)



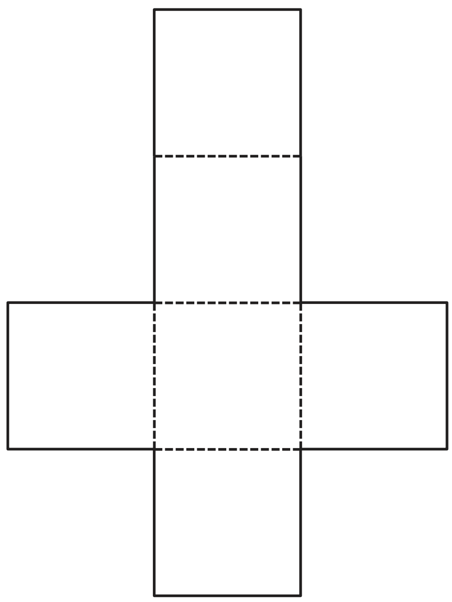
(b)



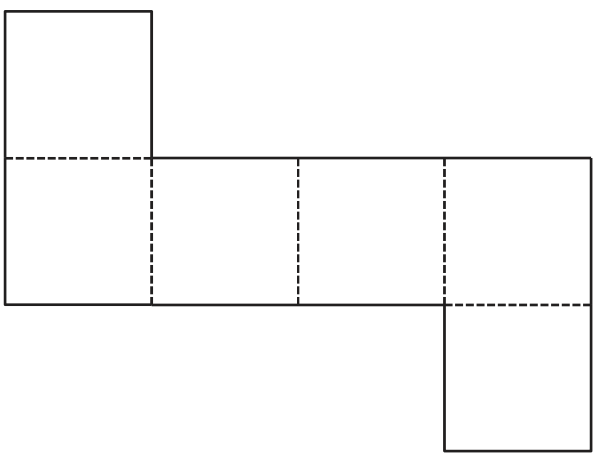
(c)



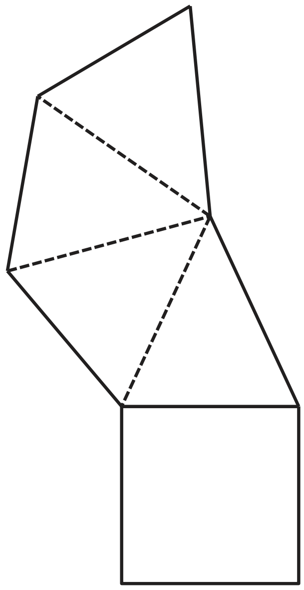
(d)



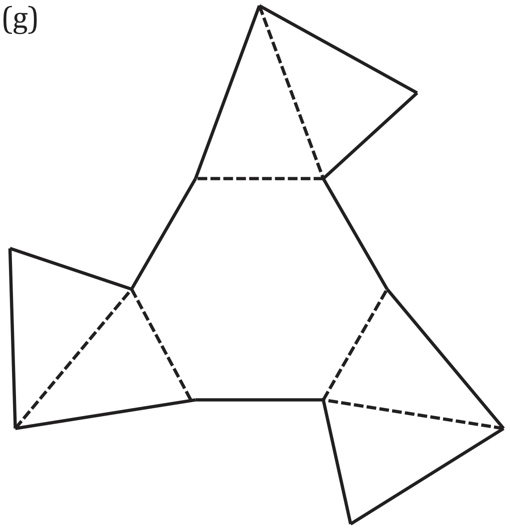
(e)



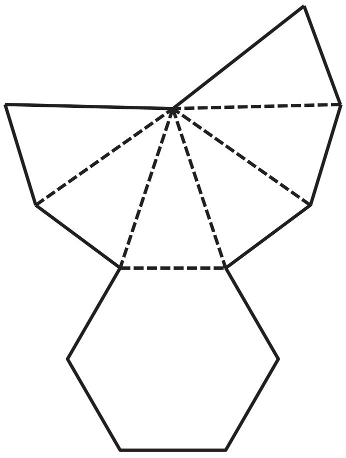
(f)



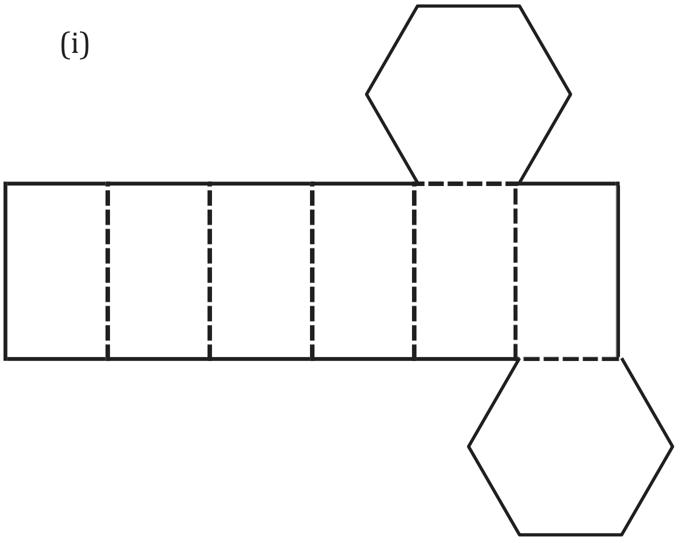
(g)



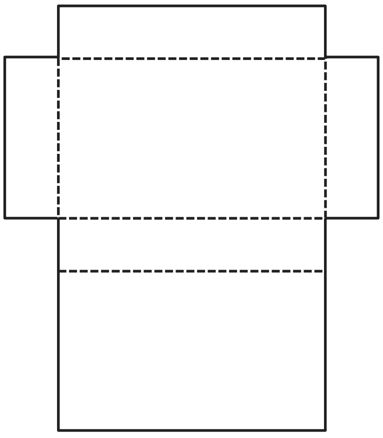
(h)



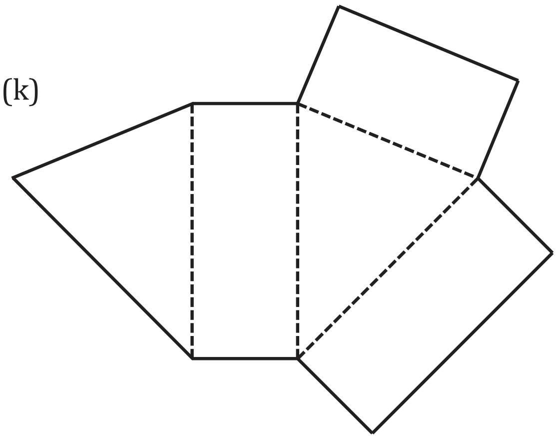
(i)



(j)

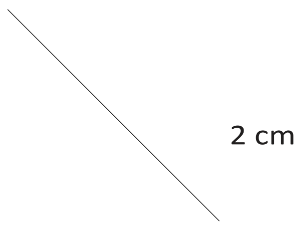


(k)

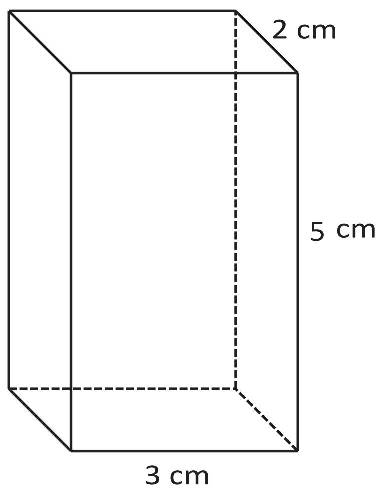


5. Draw a net for each of the following objects. Be accurate in your measurements.

(a) Cube



(b) Rectangular prism



(c) Triangular prism

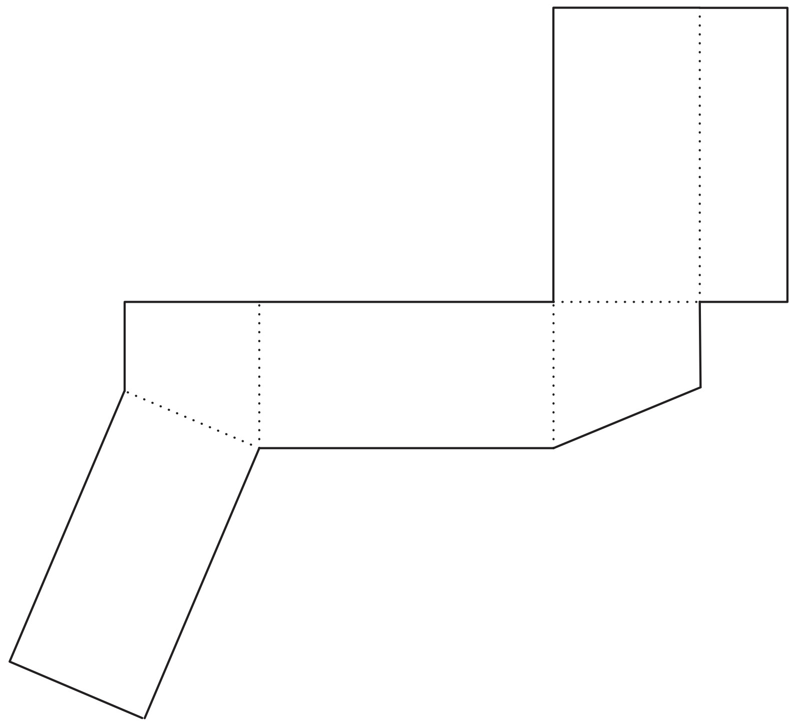
3 cm

3 cm 4 cm

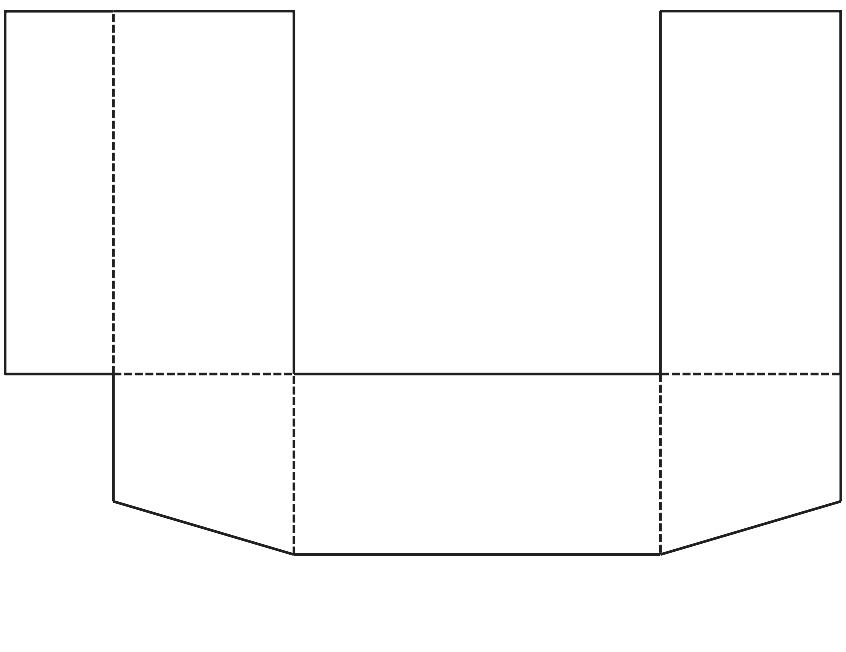
6. (a) Copy the nets in question 5 onto cardboard, but multiply the length of each side by 2. Be accurate in yourconstructions.

(a) Cut out, fold and use sticky tape to paste the nets to build the 3D models.

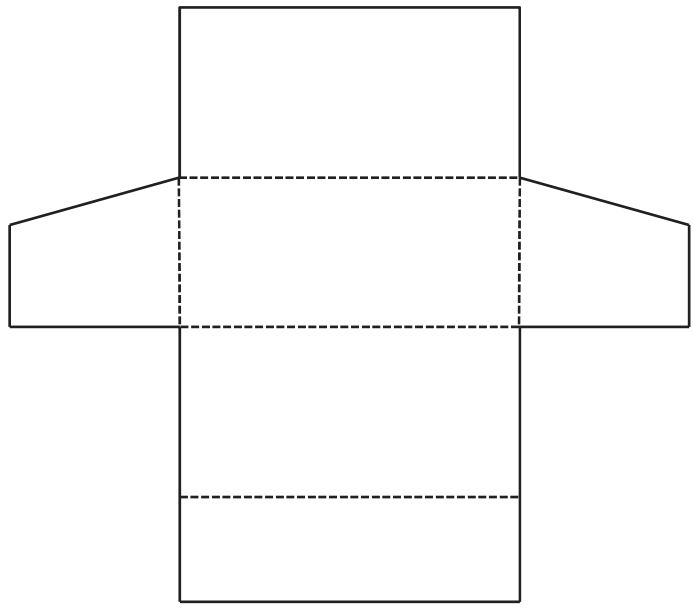
7. The first diagram below is a net for a prism with quadrilateral bases. Which of the diagrams (a), (b), (c) and (d) are nets for the same prism, and which diagrams are not nets for the prism?



(a)



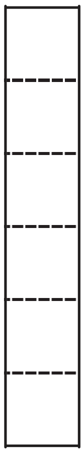
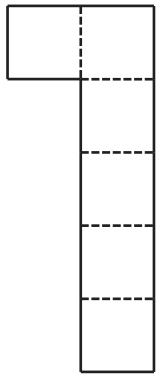
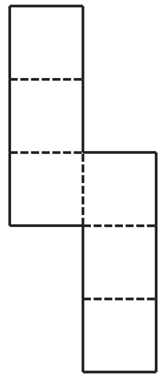
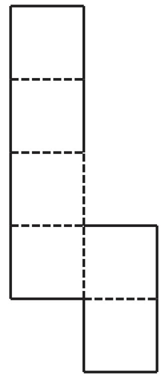
(b)

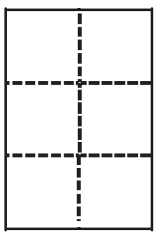
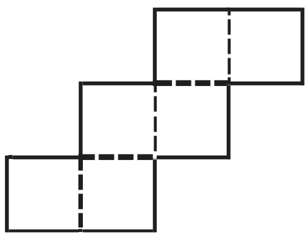
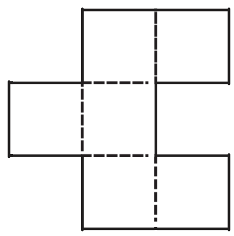
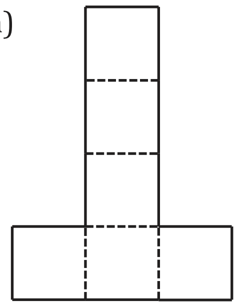


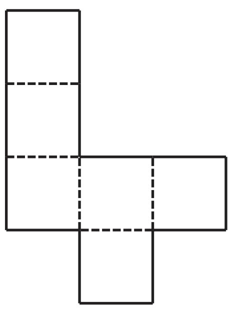
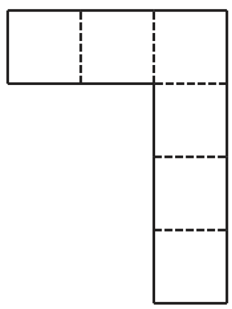
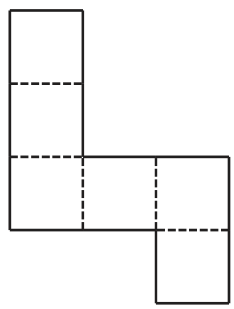
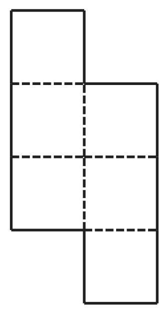
(c)

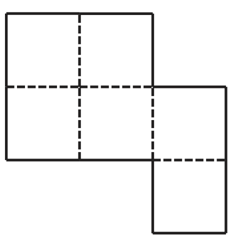
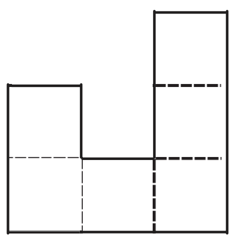
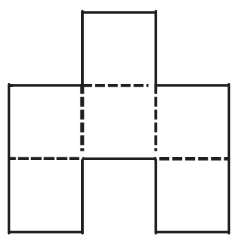
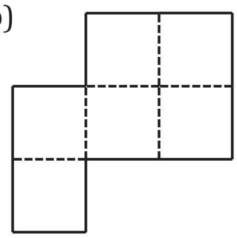
(d)

8. Which of these diagrams will work as nets for a cube?

(a)

(b)

(c)

(d)


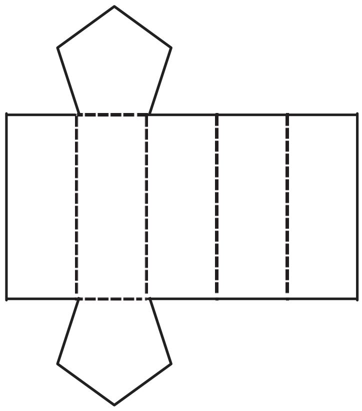
(e)

(f)

(g)

(h)


(i)

(j)

(k)

(l)


(m)

(n)

(o)

(p)


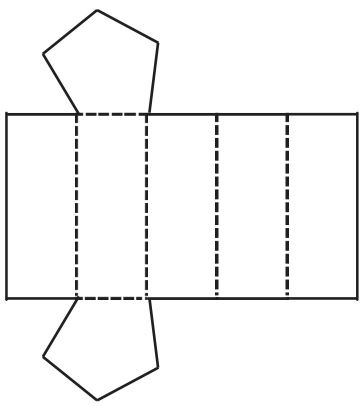
9. In each case below, state whether the diagram will work or not work as a net for making a pentagonal prism. The base need not be a regular pentagon. In the cases where the diagram will not work, explain why.

(a)

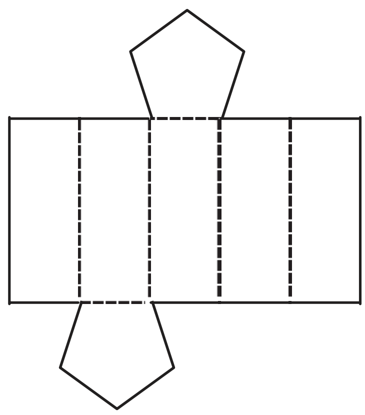


(b)

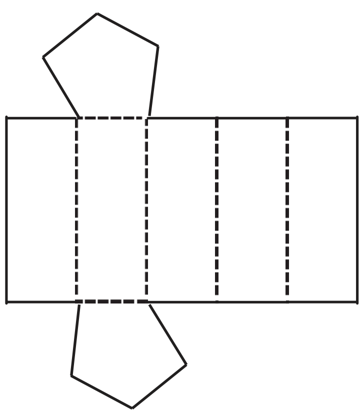
(c)



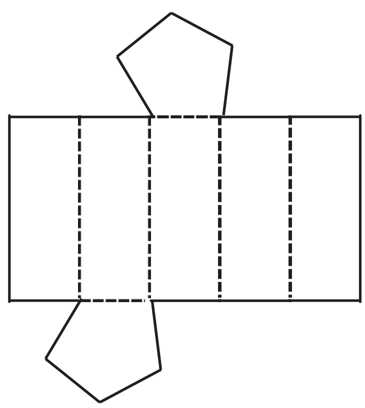
(d)



(e)

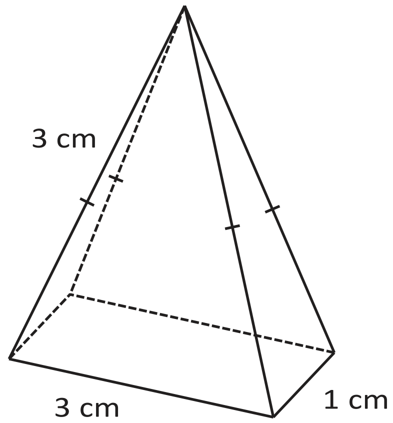


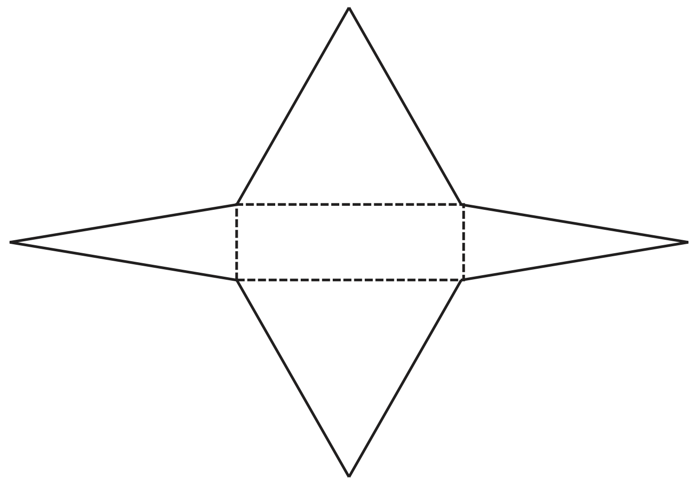
(f)



drawing nets and constructing 3d models of pyramids

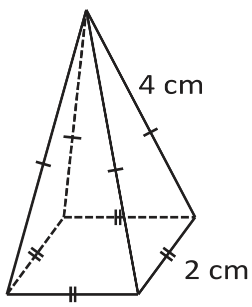
1. Trace the net and write the measurements on the sides of the net.



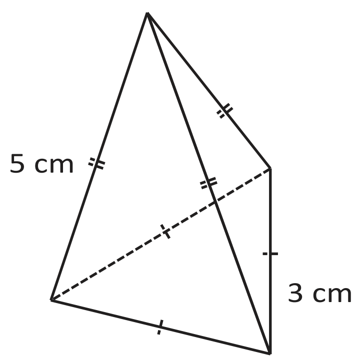


2. Draw accurate nets of the following pyramids:

   (a) Square-based pyramid



   (b) Triangular pyramid

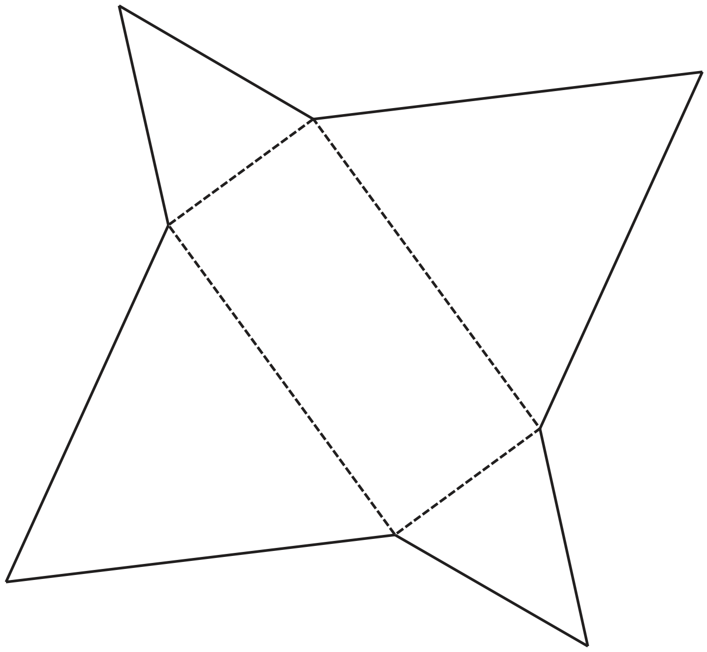


3. (a) Copy the nets you have drawn in question 2 onto cardboard or paper, but multiply the measurements by 2.
   (b) Then cut out, fold and paste the net to make a model of each 3D object.

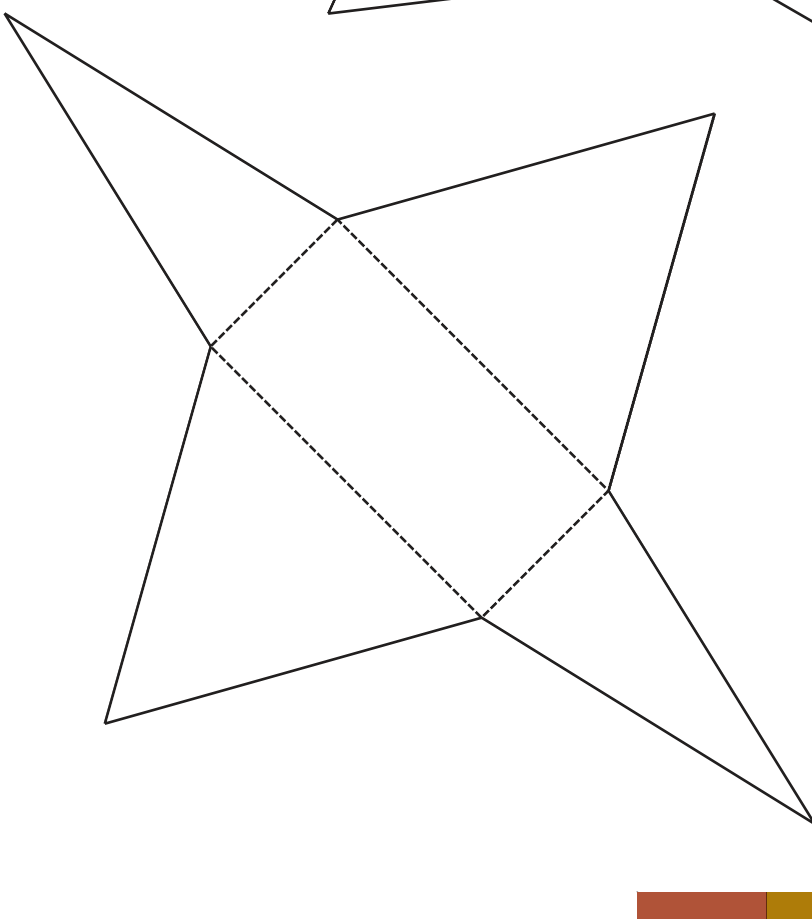
what makes a net work?

Which of the diagrams below will not work as nets to make a rectangular pyramid? You may have to take measurements to be sure in some cases, or make a copy and cut the diagram out and fold it. In each case where you say no, explain why you think it will not work.

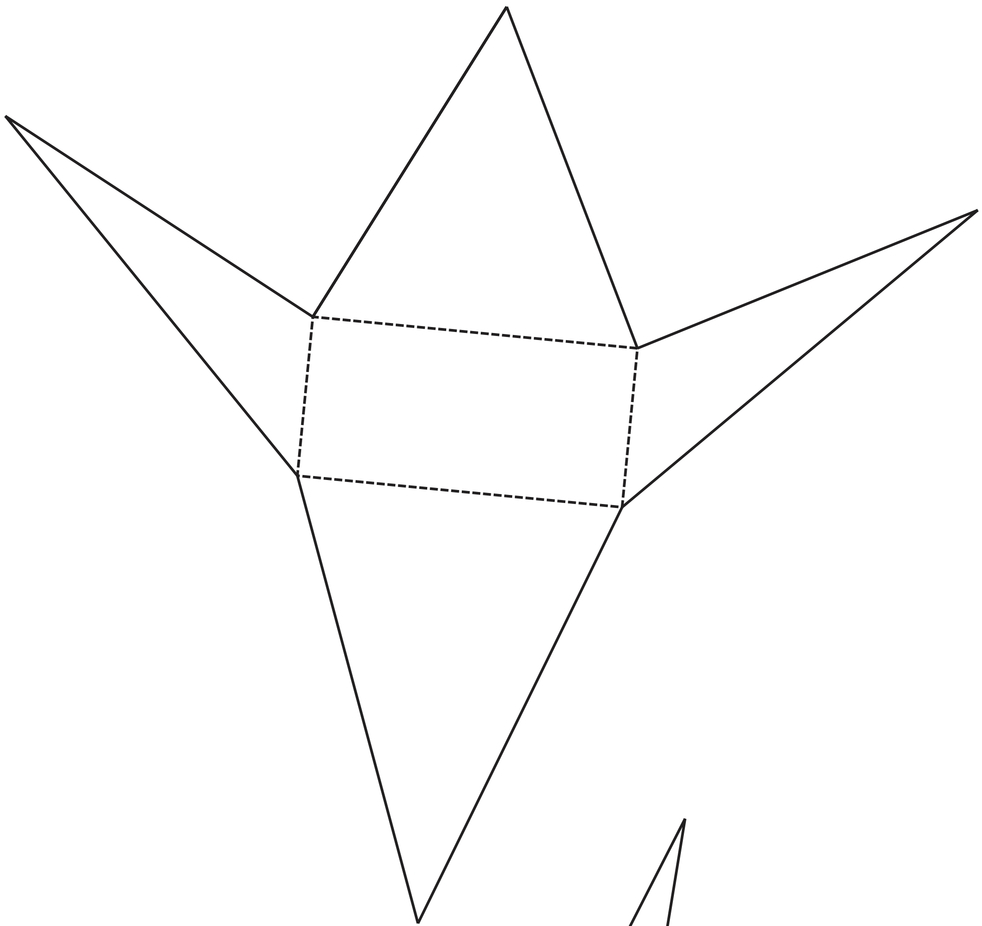
1.



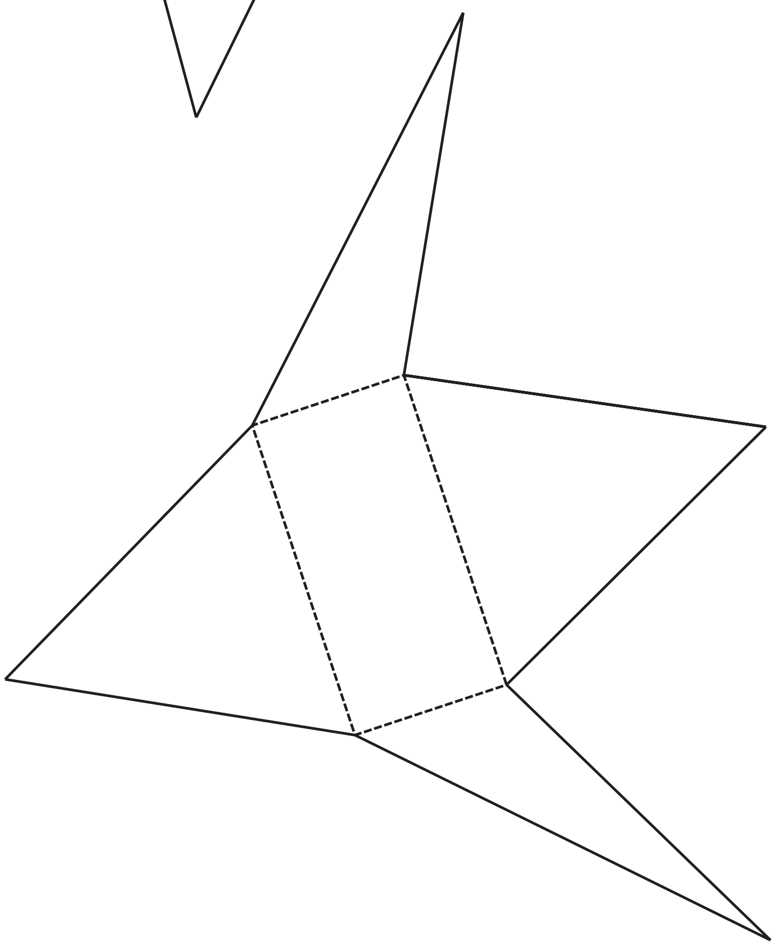
2.



3.



4.



circles and pyramids

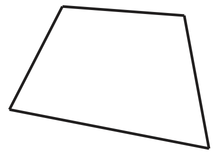
In order to meet at the apex, one side of each triangular face of a pyramid must be the same length as the closest side of the triangle next to it.

This means that certain line segments in the net of a pyramid must be equal.

1. Trace the diagram below. Mark the line segments that should be equal in the diagram, so that a pyramid can be made by folding a cut-out of the diagram on the dotted lines.

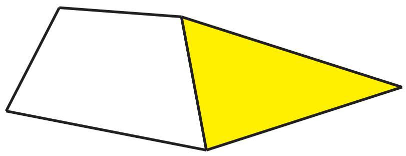
2. Make an accurate copy of the above diagram on stiff paper or cardboard. Cut it out and fold along the dotted lines. See if you can make a pyramid in this way.

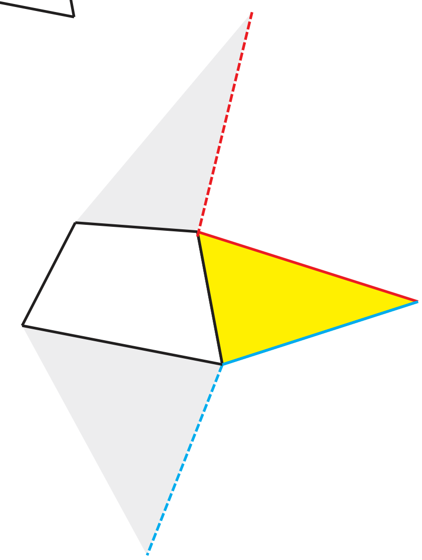
Any polygon can form the base of a pyramid.



If you want to draw a net for a pyramid, you can start by drawing a polygon.

Then draw any triangle on one side of the polygon.



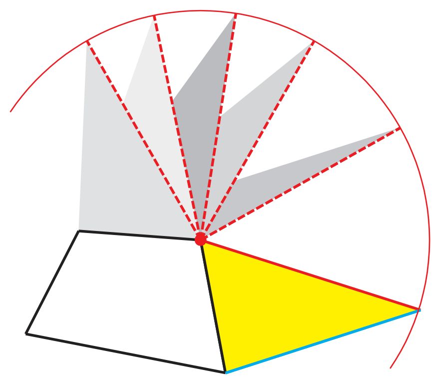


The triangles that will be adjacent to the first one must each have one side equal to the matching side of the first triangle, as indicated with the solid and dotted red and blue line segments in the sketch on the right.

The dotted line segments can be in other positions too, as long as they have the same lengths as the coloured sides of the first triangle.

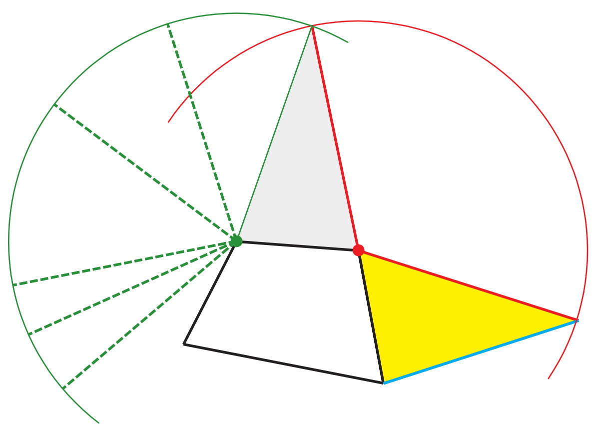
This means that once the first triangular face is drawn, there are many different possibilities for each of the two triangles that will be adjacent to it on the pyramid.

The circle with the red dot as midpoint, on the sketch on the right, shows the possibilities for one triangular face.



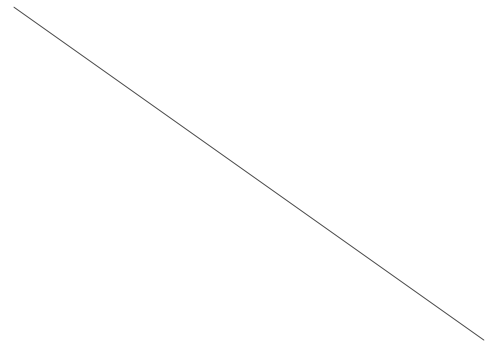
Once a triangle on the upper edge of the base is chosen, a circle can be drawn around the green vertex to indicate the possibilities for the third triangular face. The radius of this circle is the length of the second leg of the second triangle, shown in solid green on the sketch.

Any line segment drawn from the green circle to the green vertex can be a side of the third triangular face.



Only one triangular face remains to be drawn now. We will refer to it as the blue triangle.

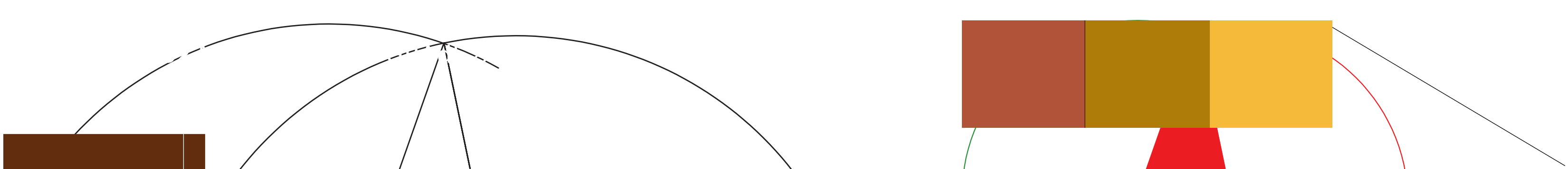
The black and blue dots on the sketch show where two vertices of the blue triangle should be.



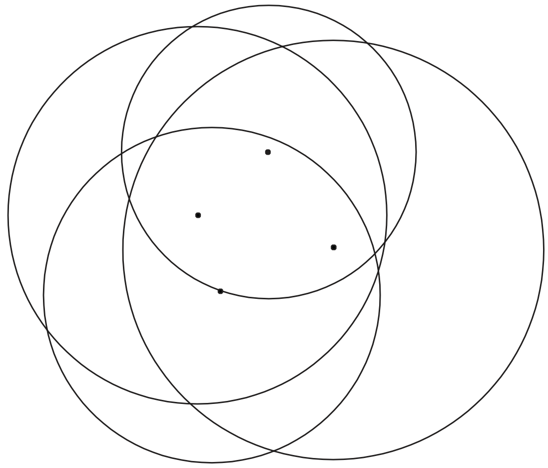
3. Copy the above sketch into your book and roughly draw the fourth triangular face for a pyramid. Think how long the sides should be so that the diagram will work precisely as a net to make a pyramid.

4. (a) How can the black dot and the green triangle help you to get some idea as to where the third vertex of the blue triangle should be?
   (b) How can the blue dot and the yellow triangle help you to get some idea as to where the third vertex of the blue triangle should be?

5. An enlargement of the sketch given at the top of this page is given below. Copy the enlargement into your book and use your pair of compasses to find the third vertex of the face that is not yet drawn, and complete the net for the pyramid. Then make a copy of the diagram on stiff paper or cardboard, cut it out and fold it to see if it forms a pyramid.



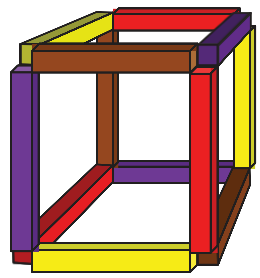
6. Join points on this sketch to draw a net for a pyramid.



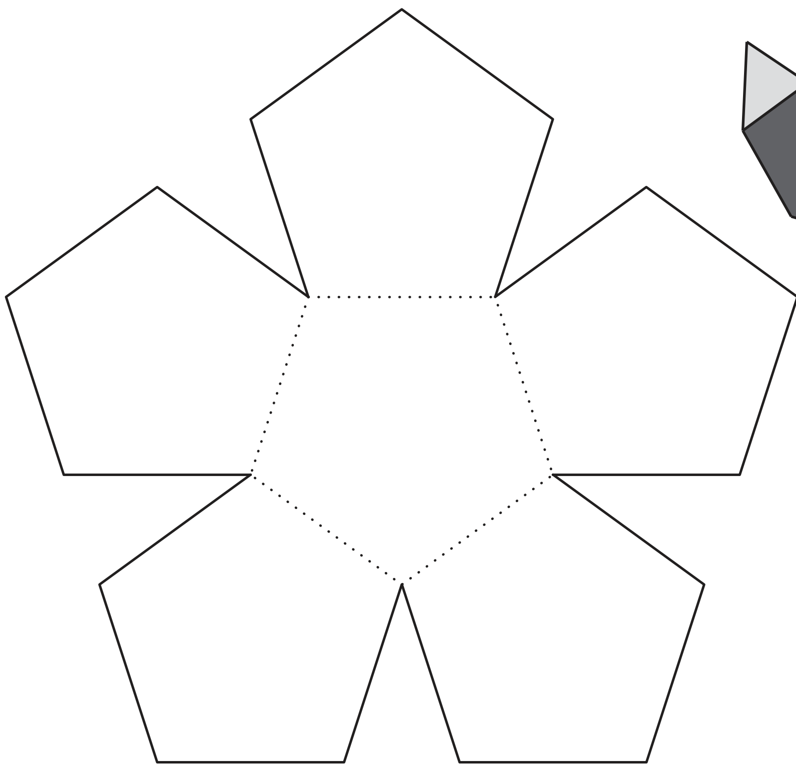
25.3 Platonic solids

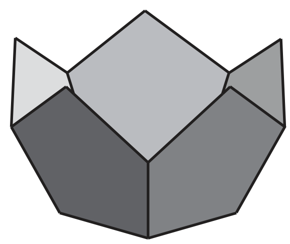
making polyhedra with identical faces and equal edges

A cube is a special type of polyhedron. It has six identical faces, and its 12 edges are all equal in length.

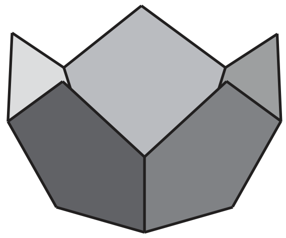


1. How many vertices does a cube have?
2. Can you think of an object which has faces that are identical triangles, and all its edges are equal? Try draw a rough sketch of the net for such an object.
3. (a) Make a copy of the diagram below. Cut it out and fold it along the dotted lines. Attach the faces with sticky tape to makean open container with pentagonal faces.

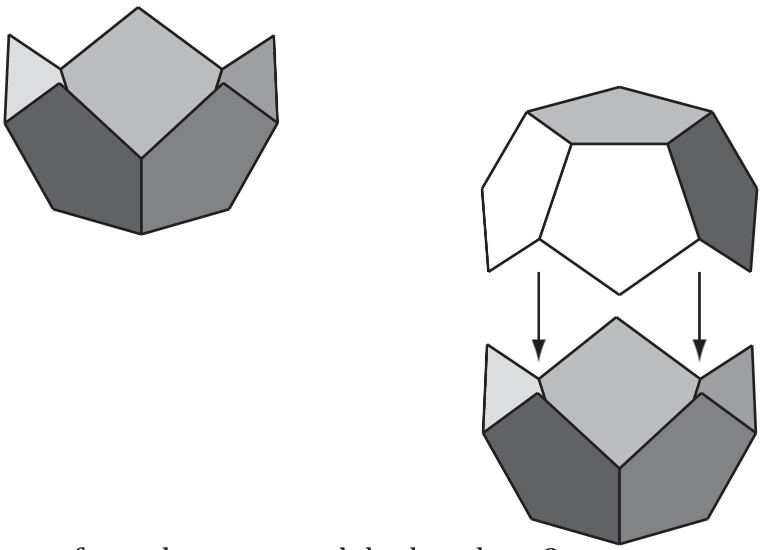




(b) Make another copy of the pentagon diagram, and make a second container.



(c) Turn one of your containers upside down and put the two together to form a polyhedron.

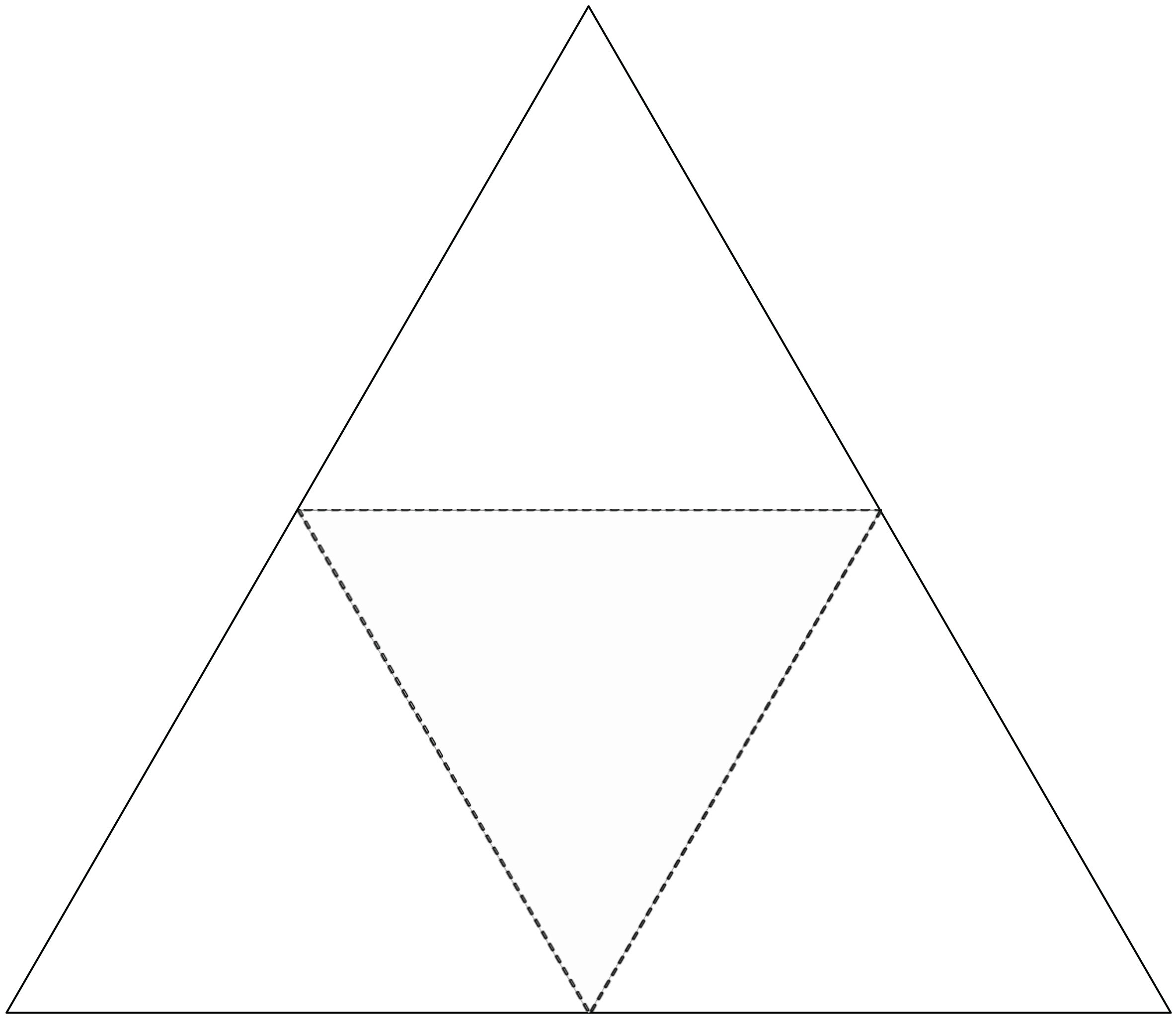


(d) How many faces does your polyhedron have?
(e) Are the faces identical, and what shape are they?
(f) Are the edges equally long, and how many edges are there?

A polyhedron of which all the faces are identical regular polygons is called a **Platonic solid**, because the Greek philosopher Plato was fascinated by such objects.

A **regular polygon** is a polygon with equal sides and equal angles.

4. Do you think the diagram below can be used as a net to make a Platonic solid? If it is possible, how many faces, how many edges and how many vertices will the polyhedron have?

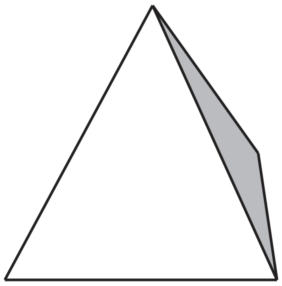


5. (a) Make two accurate copies of the above diagram. Cut out on the solid lines and fold on the dotted lines to form two identical polyhedra. Polyhedra like this are called **regular tetrahedra**.
   (b) Try to combine your two regular tetrahedra to make another solid, with six faces, nine edges and five vertices.

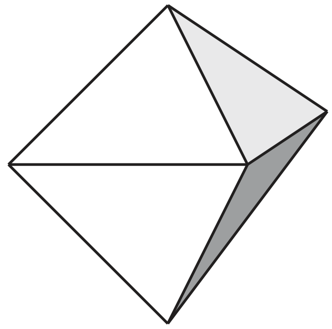
the platonic solids

The Platonic solids have special names, and these are given below.

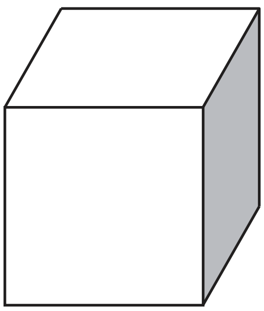
There are only five Platonic solids.



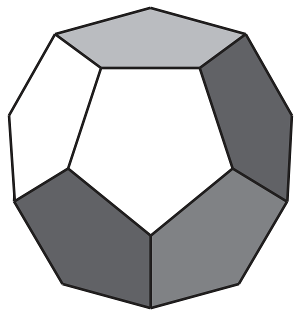
A **tetrahedron** consists of four equilateral triangles. It has six edges and four vertices.



An **octahedron** consists of eight equilateral triangles. It has 12 edges and six vertices.



A **hexahedron** (also known as a **cube**) consists of six squares. It has 12 edges and eight vertices.

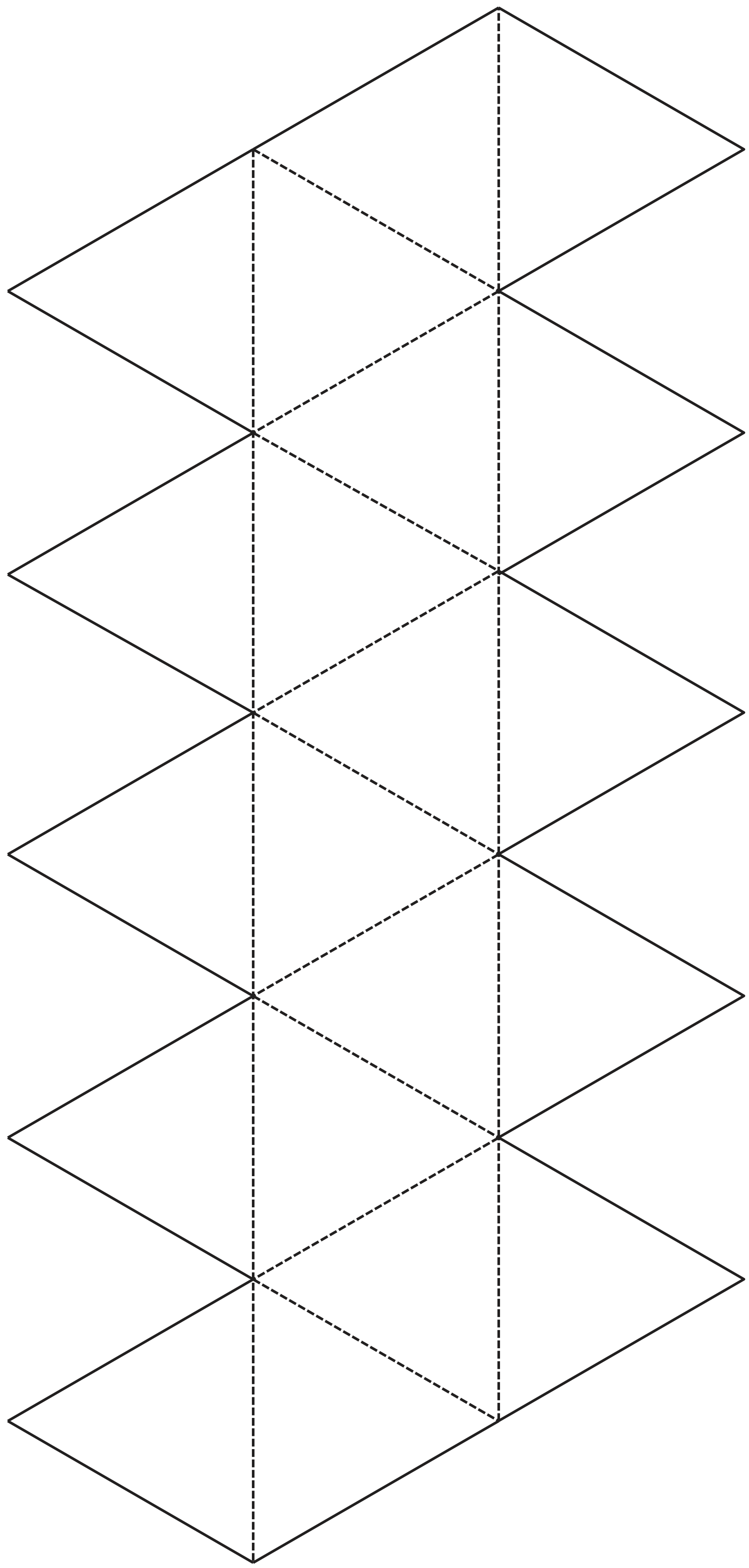


A **dodecahedron** consists of 12 regular pentagons. It has 30 edges and 20 vertices.

An **icosahedron** consists of 20 equilateral triangles. It has 30 edges and 12 vertices.

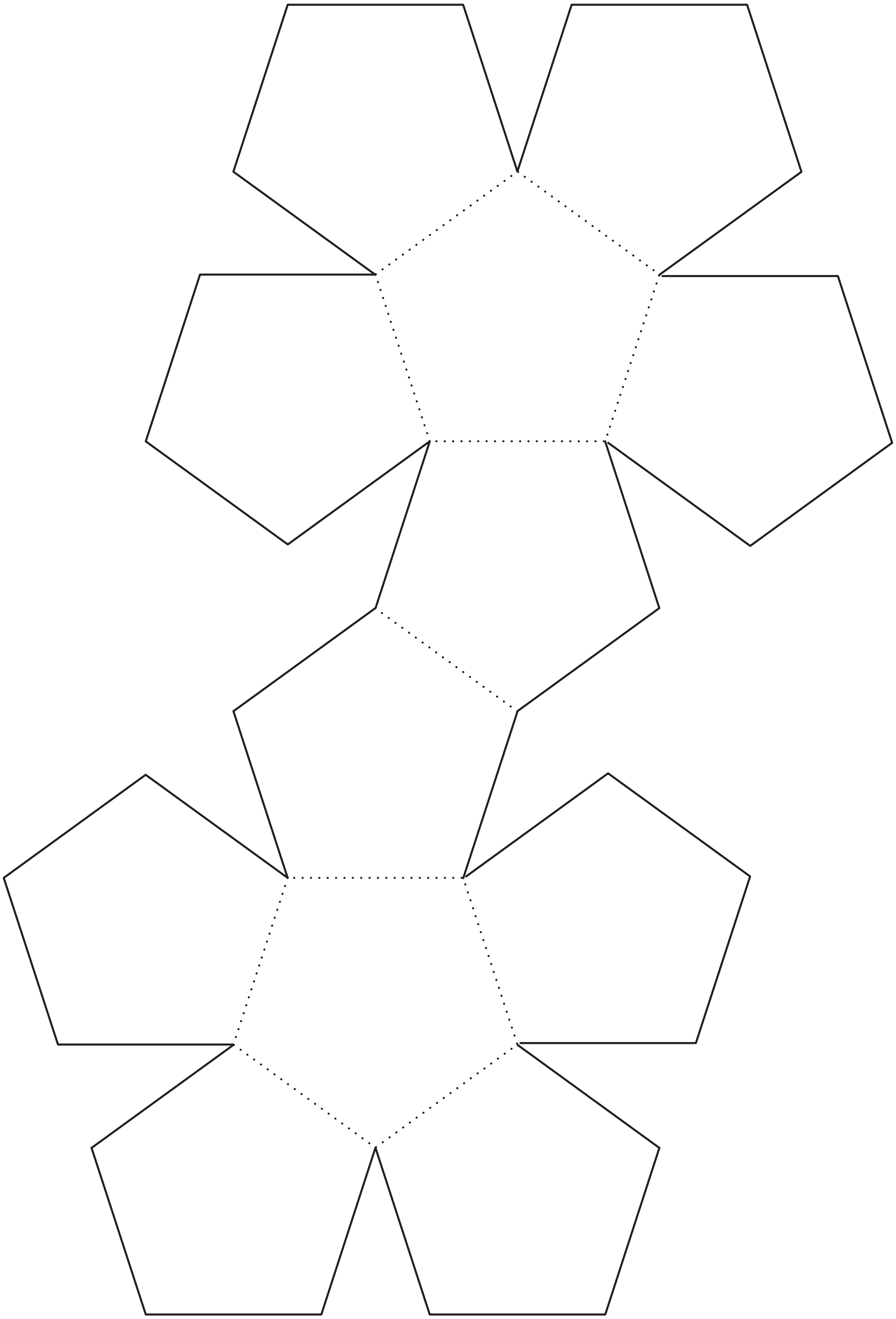
1. Nets for some of the Platonic solids are given on the following pages. Write the names of the objects next to the nets that can be used to make them.
2. Investigate whether Euler's formula is true for the Platonic solids.
   (Euler's formula: the number of faces + the number of vertices = 2 + the number of edges)

(a)



(b)

(c)



Chapter 26
Probability

26.1 How often different things can happen

different fractions of a whole number

Jayden lives close to the sea. He goes fishing every day. Some days he catches no fish, but on some days he catches several fish. He never catches more than five fish in a day.

He has decided that he will always stop fishing when he has caught five fish in one day.

1. What are the different possible outcomes of each of Jayden's daily fishing trips?
2. Jayden rolls a dice just once each day before he goes fishing.
   (a) What are the possible outcomes of rolling a dice once?
   (b) Are the six outcomes of rolling a dice equally likely?
   (c) Is there any reason for Jayden to believe that the outcome of his fishing trip on a day will be one less than the number that came up when he rolled the dice on that day?
3. Jayden keeps a record of the outcomes of his daily dice rolls. Here is a summary of his record for 60 consecutive days:

Outcome	1	2	3	4	5	6
Frequency	9	9	12	11	10	9

   (a) How many times was the outcome a 6?
   (b) What fraction is this of the total of 60 events?
   (c) On what fraction of the days was the outcome a 3?

The fraction of a number of events which have a specific outcome is called the **relative frequency** of that outcome.

4. What is the relative frequency ofa:
   (a) 5 in Jayden's series of 60 dice rolls?
   (b) 4 in Jayden's series of 60 dice rolls?

The **range** of a set of numbers is the difference between the smallest and largest numbers in the set.

5. What is the **range** of the relative frequencies of the different outcomes in Jayden's series of dice rolls? Express the range as a fraction in sixtieths, and as a percentage.

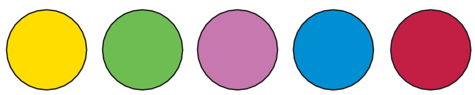
6. Do you think the six possible outcomes of Jayden's daily fishing trips are equally likely? Give reasons for your answer.

7. (a) Jayden has a book in which he keeps a record of the outcomes of his daily fishing trips. A summary of his record for a period of 200 consecutive days is given in the table below. Copy the table and write the relative frequencies of the different outcomes in the table, with each expressed as two hundredths.

Outcome	0	1	2	3	4	5
Frequency	30	32	68	54	12	4
Relative frequency						

(b) What is the range of the relative frequencies in this case? Express the range as a common fraction and as a percentage.

how often can we expect something to happen?



Imagine that you have five coloured buttons as shown above in a paper bag.

1. Imagine that you put your hand into the bag without looking inside, and grab one of the buttons.
   (a) Can you say which colour that button will be?
   (b) Discuss this with some classmates.

2. (a) What are the different possible colours of buttons that you could draw from the bag?
   (b) How many different possibilities are there?

3. Read the passage below, then answer the questions that follow.

   When you draw a button from the bag, we say you perform a ***trial****. The colour you draw is called the* ***outcome*** *of the trial.*

   (a) What are the different possible outcomes of the trial if you draw one button out of the bag?
   (b) Imagine that you put the first button back into the bag. If you now draw one button from the bag again, what are the possible outcomes of this new trial?
   (c) Imagine that you repeat the event a third time. What are the possible outcomes of this new trial?
   (d) Imagine that you perform many trials. What are the possible outcomes of each repetition?

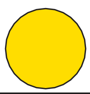
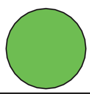
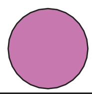
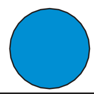
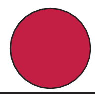
4. (a) When you draw one of the five buttons many times and put it back each time, do you think you will draw one colour more often than the others?
   (b) Discuss this with some classmates.

5. (a) Imagine that you draw a button out of the bag with five buttons and put it back, and repeat this 60 times. Approximately how many times do you think you will draw the red button?
   (b) Approximately how many times do you think you will draw the pink button?
   (c) Discuss this with some classmates.

When there is no reason to believe that any outcome will occur more often than any other outcome, the outcomes are said to be **equallylikely**.

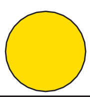
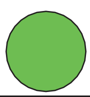
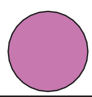
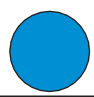
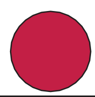
6. Susan decides to perform 160 trials on the bag with five buttons. In each trial she will draw one button from the bag, note its colour, and put it back. Lebogang decides she will perform 60 trials and Archie decides to perform 40 trials.

   Approximately how many times do you think each of them will draw each of the buttons? Copy the table below and enter your expectations in it:

					
Susan					
Lebogang					
Archie					

7. Here are the answers that eight different people gave for Archie, with his 40 trials.

"Close to 6" means it can be 6 or another number close to 6, for example 5 or 7 or 4 or 8.

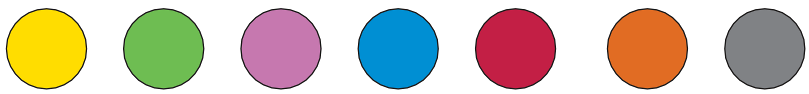
					
Answer A	close to 5	close to 5	close to 5	close to 5	close to 20
Answer B	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9	7, 8 or 9
Answer C	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8	6, 7 or 8
Answer D	close to 7	close to 9	close to 8	close to 10	close to 6
Answer E	close to 6	close to 6	close to 6	close to 6	close to 6
Answer F	6	9	7	8	10
Answer G	8	8	8	8	8
Answer H	close to 8	close to 8	close to 8	close to 8	close to 8

Which answers do you think are good answers, and which do you think are poor answers? For each answer, explain why you think it is good or poor.

8. (a) How much is one fifth of 160, one fifth of 60 and one fifth of 40?
   (b) Look at your own answers for question 6 again. Do you still agree with your answers? If you want to give different answers now, do so and explain what made you change your position.

9. Willem has decided to perform as many trials as he can in an afternoon, drawing one button each time out of a bag containing five coloured buttons.
   (a) In close to what fraction of the trials can he expect to get yellow as the outcome?
   (b) In close to what fraction of the trials can he expect to get red as the outcome?

10. Manare has decided to perform as many trials as he can in an afternoon, drawing one button each time out of a bag containing seven different coloured buttons.



   (a) In close to what fraction of the trials can he expect to get blue as the outcome?
   (b) In close to what fraction of the trials can he expect to get grey as the outcome?

11. Miriam has decided to perform as many trials as she can in an afternoon, drawing one button each time out of a bag containing 12 different coloured buttons.
   In close to what fraction of the trials can Miriam expect to get each specific colour as the outcome?

The number of times that a specific outcome is obtained during a series of trials is called the **frequency** of the outcome.

12. What is the frequency for each of the following colours in answer F, in question 7 on the previous page?
   (a) red
   (b) pink
   (c) yellow
   (d) blue

When the different possible outcomes of an event are equally likely, it is reasonable to expect that when the event is repeated many times, the frequencies for the different outcomes will be almost equal.

an investigation

1. Make eight small cards or pieces of paper. On each card write a different letter. Use the letters A, B, C, D, E, F, G and H. Put the cards in a paper bag. Imagine that you draw a card out of the bag, note the letter and put it back. Imagine that you perform 40 such trials, noting the outcome each time. Then you find the frequency for each letter. To what number do you think each of the frequencies will be close? The number you think of may be called the **expected frequency**.
2. What will be the expected frequencies for each letter if:
   (a) 200 trials are performed?
   (b) 1 000 trials are performed?
3. Now actually do the experiment described in question 1. Record your results with tally marks in a table like the one below. When you have finished, count the tally marks to find the **actual frequencies**.

	A	B	C	D	E	F	G	H
Tally marks								
Actual frequency								
Expected frequency	5	5	5	5	5	5	5	5

4. Write your actual frequencies on a slip of paper, in a table like as follows:

	A	B	C	D	E	F	G	H
Actual frequency								

5. The next step is to collect the slips of four different classmates. Copy the table on page 305, and write their frequencies in rows 1, 4, 7, 10 and 13 of the table, together with your own frequencies. **Do not do it yet**. When you put the five sets of results together, and add them up, you will have the actual frequencies out of 200 trials. You will write these in row 16.In the row for expected frequencies, write the numbers to which you think the frequencies will be close.
6. Now work with your four classmates, and complete rows 1, 4, 7, 10and 13.
7. In the first empty row after each actual frequency row, express the frequency as a fraction of the total number of outcomes in the experiment, which was 40 in each case. You need not simplify the fractions in rows 2, 5, 8, 11 and 14 of the table.
8. In rows 17 and 20, express the frequencies of rows 16 and 19 as fractions of 200.
9. In the remaining empty rows, express the fractions as percentages.

10. Calculate the ranges of the numbers in rows 3, 6, 9, 12, 15 and 18.

		A	B	C	D	E	F	G	H
1	Actual frequencies								
2									
3									
4	Actual frequencies								
5									
6									
7	Actual frequencies								
8									
9									
10	Actual frequencies								
11									
12									
13	Actual frequencies								
14									
15									
16	Total actual frequencies								
17									
18									
19	Expected frequencies								
20									
21									

11. In which row is the range the smallest? Try to explain why this is the case.

12. In which of the rows in question 10 are the numbers closest to the expected percentages in row 21?

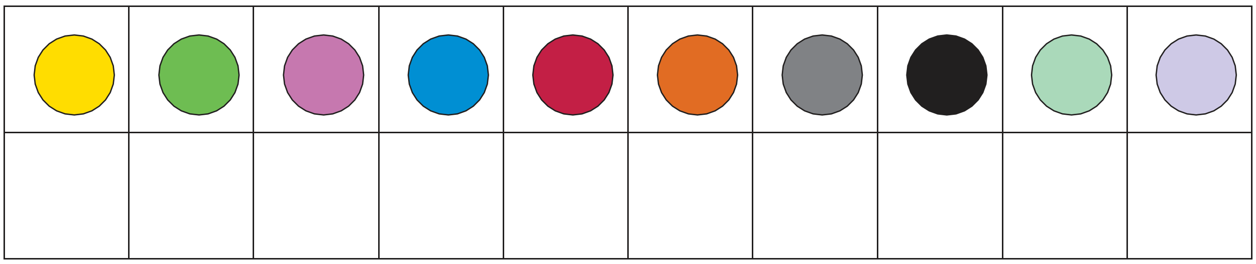
The expected relative frequency of an outcome is called the **probability** of the outcome.

13. Imagine that you have ten different coloured buttons in a bag, as shown below.

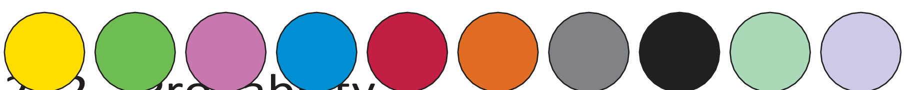
(a) Imagine that you draw one button out of the bag.
How many different equally likely outcomes are there for this trial?

(b) Imagine you draw one button out of the bag, look at the colour and make a tally mark in the column for that colour on a table like the one below. Imagine you do it many times. Approximately what fraction of the total number of tally marks do you expect to be in each column?

(c) The fraction you have specified in (b) is the probability of the outcome for each of the columns. Would you expect to get precisely that fraction in each column?



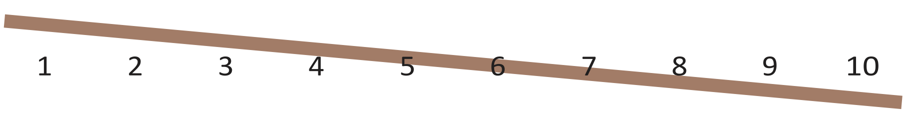
(d) Hashim says he expects to have **approximately** ten tally marks in each column, because the outcomes are equally likely. Do you agree with Hashim? Give reasons for your answer.



20.2 Probability

In this activity you have to think about the following situation:

There are ten coloured, numbered buttons in a bag: six yellow buttons, three blue buttons and one red button.



1. (a) What fraction of the total number of buttons is yellow?
   (b) What fraction of the total number of buttons is blue?
   (c) What fraction of the total number of buttons is red?

2. Suppose you put your hand into the bag without looking inside, take one button out and note its colour, and then put it back into the bag.

   If you repeat this trial many times, you will sometimes get a yellow, sometimes a blue button and sometimes a red button.

   (a) Do you think you will get blue more often than yellow? Explain your answer.
   (b) Do you think you will draw yellow about twice as often as blue?
   (c) Can you be certain which colour will be drawn? Explain your answer.
   (d) Share your ideas with two classmates.

Here is an experiment that you will do later. **Do not do it now**.

Put ten buttons like those on page 308, or pieces of paper or cardboard with the names of the colours written on them, in a bag. Put your hand into the bag without looking inside, and take one button out. Check what colour it is, copy the table below and make a tally mark in the column for that colour, and put the button back into the bag. Do this ten times.

Yellow	Blue	Red

Each time you perform a trial, a certain **event** takes place, and there are three possible events:

A. The event of the colour being yellow
B. The event of the colour being blue
C. The event of the colour being red

3. (a) In how many different ways can event A be achieved in one trial?
   (b) In how many different ways can event B be achieved in one trial?
   (c) In how many different ways can event C be achieved in one trial?

4. (a) Suppose you do the experiment and make ten trials. Do you think event A will happen three times or maybe four times, eventB will happen three times or maybe four times and event C will happen three times or maybe four times?
   (b) Share your ideas with two classmates.
   (c) Do you rather think event A will happen six times (or maybe five or seven times), event B will happen three times (or maybe two or four times), and event C will happen once (or maybe twice or not at all)?
   (d) Share your ideas with two classmates.

5. (a) Do the experiment that is described before question 3. Copy the table on the next page, and write the results in the second row of the table.
   (b) Repeat the experiment and write the results in the third row of the table.
   (c) Repeat the experiment three more times and enter the results in the table.
   (d) Complete the last two rows of the table.

Outcome	Yellow	Blue	Red
Frequency of each colour during the first ten trials			
Frequency of each colour during the second ten trials			
Frequency of each colour during the third ten trials			
Frequency of each colour during the fourth ten trials			
Frequency of each colour during the fifth ten trials			
Total frequencies out of 50 trials			
Total frequencies divided by 5			

When you did the experiment for the first time in question 5(a), you performed ten **trials**: you took a button out of the bag and inspected the colour.

Each time, there were three **possible outcomes** for the trial: the button could be **yellow**, it could be **blue** or it could be **red**.

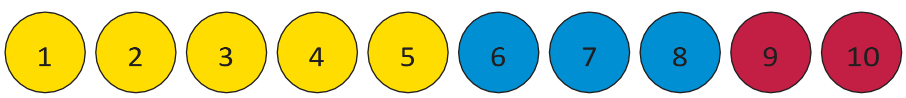
We can also say that three different events were possible: yellow, blue and red. But if we consider the numbers on the buttons, ten different outcomes are possible.

6. (a) How many different outcomes (numbered buttons) will produce the event yellow?
   (b) How many different outcomes will produce the event blue?
   (c) How many different outcomes will produce the event red?

7. (a) What fraction of the ten possible outcomes will produce the event yellow?
   (b) What fraction of the ten possible outcomes will produce the event blue?
   (c) What fraction of the ten possible outcomes will produce the event red?

The fractions you have given as answers for question 7 are the **probabilities** of the three different events.

8. (a) What is the probability of getting blue when one of the buttons below is drawn from a bag?



   (b) Describe in your own words what is meant by saying the probability of an event is $\frac{3}{20}$.