TECHNICAL MATHEMATICS

TRIGONOMETRY

GRADE 12

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Read to Lead
A Reading Nation is a Leading Nation

2030 NDP

Foreword

In order to improve learning outcomes the Department of Basic Education conducted research to determine the specific areas that learners struggle with in Grade 12 examinations. The research included a trend analysis by subject experts of learner performance over a period of five years as well as learner examination scripts in order to diagnose deficiencies or misconceptions in particular content areas. In addition, expert teachers were interviewed to determine the best practicesto ensure mastery of thetopic by learners and improve outcomes in terms of quality and quantity.

The results of the research formed the foundation and guiding principles for the development of the booklets. In each identified subject, key content areas were identified for the development of material that will significantly improve learner's conceptual understanding whilst leading to improved performance in the subject.

The booklets are developed as part of a series of booklets, with each bookletfocussing onlyon one specific challenging topic. The selected content is explained in detail and include relevant concepts from Grades 10 - 12 to ensure conceptual understanding.

The main purpose of these booklets is to assist learners to master the content starting from a basic conceptual level of understanding to the more advanced level. The content in each booklets is presented in an easy to understand manner including the use of mind maps, summaries and exercises to support understanding and conceptual progression. These booklets should ideally be used as part of a focussed revision or enrichment program by learners after the topics have been taught in class. The booklets encourage learners to take ownership of their own learning and focus on developing and mastery critical content and skills such as reading and higher order thinking skills.

Teachers are also encouraged to infuse the content into existing lesson preparation to ensure in-depth curriculum coverage of a particular topic. Due to the nature of the booklets covering only one topic, teachers are encouraged to ensure learners access to the booklets in either print or digital form if a particular topic is taught.

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2. How to use this booklet

This booklet is designed to clarify the content prescribed for Technical Mathematics. In addition, it offers some tips on how to tackle real life problems on a daily basis. Candidates will be expected to have mastered the content prescribed for Grades 8-11.

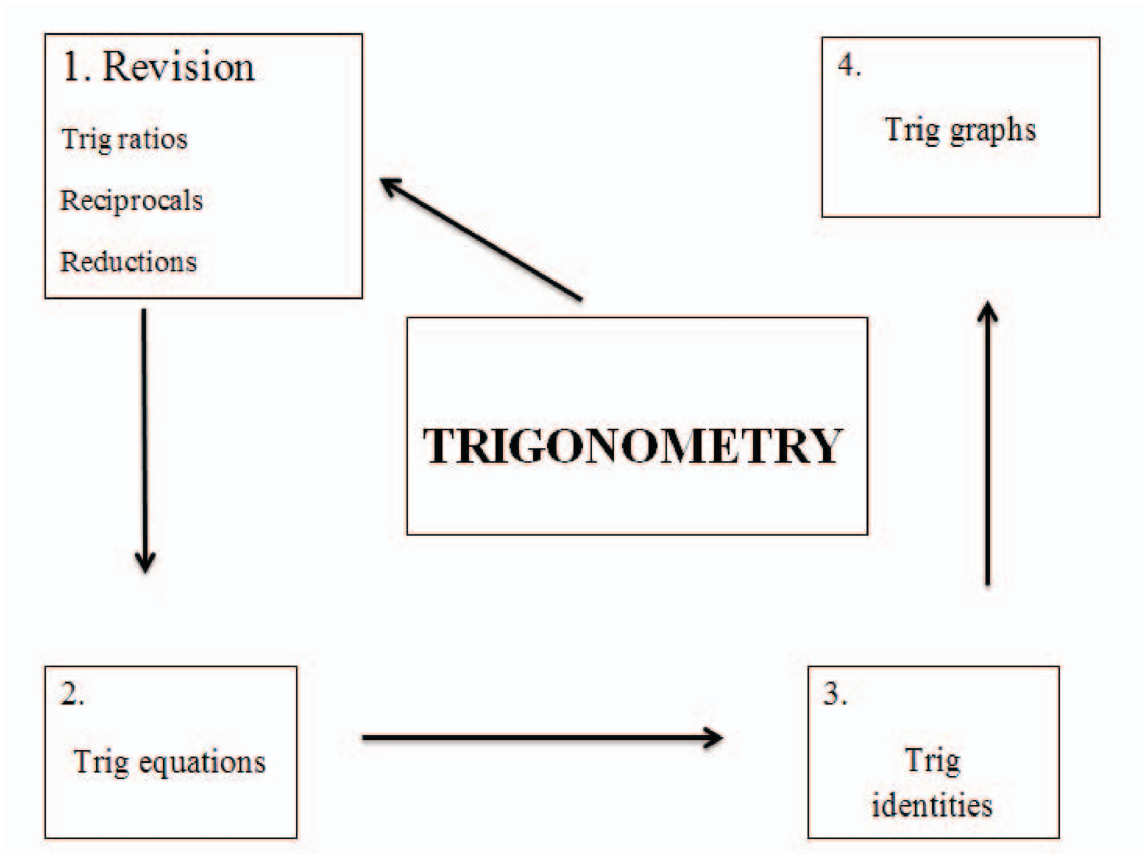
This booklet must be used to master some mathematical rules that you may not have been aware of. The prescribed textbook must also be used.

3. Study and examination tips

All learners should be able to acquire sufficient understanding and knowledge to:

- develop fluency in computation skills without relying on the use of a calculator;
- generalise, make conjectures, and try to justify or prove them;
- develop problem-solving and cognitive skills;
- make use of the language of Technical Mathematics;
- identify, investigate and solve problems creatively and critically;
- use the properties of shapes and objects to identify, investigate and solve problems creatively and critically;
- encourage appropriate communication by using descriptions in words, graphs, symbols, tables and diagrams;
- practise Technical Mathematics every day.

4. Mind map of Trigonometry



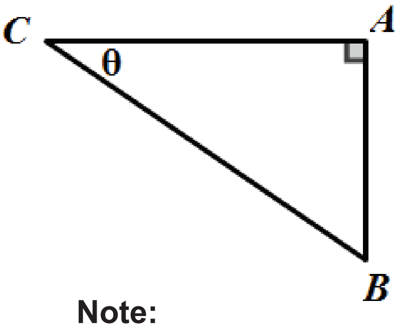
5.1. Trigonometric ratios

Important aspects of trigonometric ratios:

- Ratio means fraction. It can be expressed as $a:b$ or $\frac{a}{b}$.
- The term '**SOHCAHTOA**' will you help to remember the following 3 ratios:

$\sin\theta = \frac{O}{H} = \frac{y}{r}$ $\cos\theta = \frac{A}{H} = \frac{x}{r}$ $\tan\theta = \frac{O}{A} = \frac{y}{x}$

- $Opposite = O = y$ $Adjacent = A = x$ $Hypotenuse = H = r$



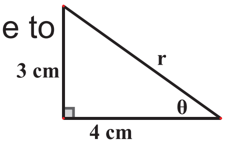
Note:
The following **trigonometric ratios** exist in right-angled triangles **only**:

$\sin\ \theta = \frac{AB}{BC}$ $\operatorname{cosec}\theta = \frac{BC}{AB}$

$\cos\ \theta = \frac{AC}{BC}$ $\sec\ \theta = \frac{BC}{AC}$

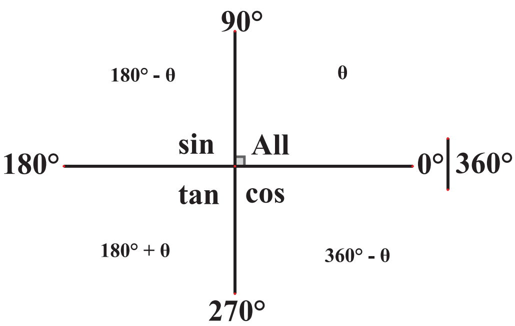
$\tan\theta = \frac{AB}{AC}$ $\cot\ \theta = \frac{AC}{AB}$

Examples
Use the diagram alongside to find the following:



No	Unknown	Solution	Marks
1.	r	$x^2+y^2=r^2$ $(4)^2+(3)^2=r^2$ $25=r^2$ $5=r$	1
2.	$\sin\theta$	$\frac{y}{r}=\frac{3}{5}$	1
3.	$\operatorname{cosec}\theta$	?	1
4.	$1-\sin^2\theta$	?	2
5.	$\frac{\sin\theta}{\cos\theta}$	?	2

Reduction of angles greater than $90°$:



Examples
Reduce the following angles to acute angles:

No	Unknown	Solution	Marks
1	$\sin 330°$	$=\sin(360°-30°)$ $=-\sin 30°$ $=\frac{1}{2}$	2
2	$\sec 170°$	$=\sec(180°-80°)$ $=-\sec 80°$	1
3	$\cot(360° - \theta)$	$=-\cot 30°$	2
4	$\operatorname{cosec} 240°$	?	2
5	$\cos(360°-\theta)$	?	2

Reciprocal identities:

$\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$ $\quad$ $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\quad$ $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\quad$ $\cot\theta = \dfrac{1}{\tan\theta}$

Complete the following:

$\cot\theta = \dfrac{1}{\ldots\ldots\ldots\ldots}$ $\quad$ $\operatorname{cosec}\theta = \dfrac{1}{\ldots\ldots\ldots\ldots}$

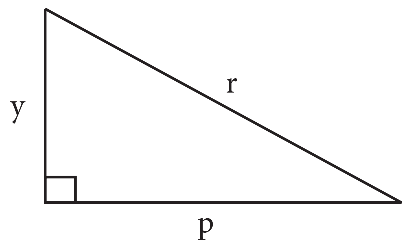
$\ldots\ldots\ldots = \dfrac{1}{\sin\theta}$ $\quad$ $\ldots\ldots\ldots = \dfrac{1}{\cot\theta}$

$\cos\theta = \dfrac{1}{\ldots\ldots\ldots}$ $\quad$ $\tan\theta = \dfrac{1}{\ldots\ldots\ldots\ldots}$

No. Activities

1. If $\sin 54° = \text{p}$, express the following in terms of p, **without using a calculator:**

a. $\sin 36°$ **Solution:** **Marks**



$\sin 36°$
$= \dfrac{y}{r}$
$= \dfrac{\sqrt{1-p^2}}{1}$ ✓✓ 2
$= \sqrt{1-p^2}$

b. $\tan 54°$ Answer: $= \dfrac{p}{\sqrt{1-p^2}}$ ✓✓ 2

c. $\cos 36°$ Answer: p 2

d. $\sin 594°$ Answer: $-p$

2. Solve the equation, if $A \in [-90°;0°]$:

a. $\sec^2 A - 3\tan A - 5 = 0$ Answer: $-45°$ 6

3. Simplify the following:

a. $\dfrac{\sin(180°-\theta)}{\sin\theta} + \dfrac{\cos^2\theta + \sin^2\theta}{2} - \dfrac{3\cos(360°-\theta)}{\cos(180°+\theta)}$ Answer: $= -\dfrac{1}{2}$ 6

b. $= \dfrac{\tan(180°+A).\cos(180-A).\sin(360-A)}{\sin(180-A)}$ Answer: $= \sin A$ 6

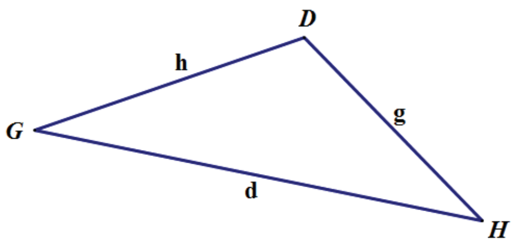
4. If $\cot\theta = \dfrac{-3}{2}$ and $\sin\theta > 0°$ calculate, by using the sketch, the value of:

a. $\cos\theta.\sin\theta$ Answer: $-\dfrac{6}{13}$ 6

5.2 Sine, Cosine and Area Rules

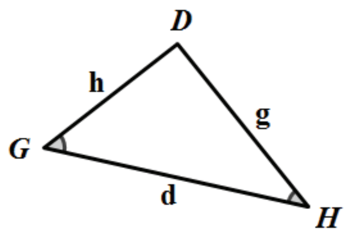
Note: **Use this method on non-right angled-triangles.**

Area rule



- Area of ΔDGH $= \frac{1}{2} dh \sin G$
- Area of ΔDGH $= \frac{1}{2} dg \sin H$
- Area of ΔDGH $= \frac{1}{2} gh \sin D$

Sine rule



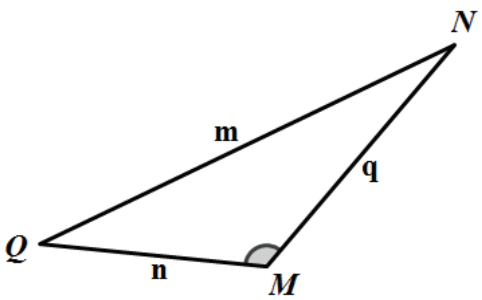
When to use the Sine rule?

1. When given two angles and one side, or
2. Two sides and a non-included angle.

Sine rule:

- $\frac{\sin G}{g} = \frac{\sin H}{h}$ OR
- $\frac{g}{\sin G} = \frac{h}{\sin H}$

Cosine rule



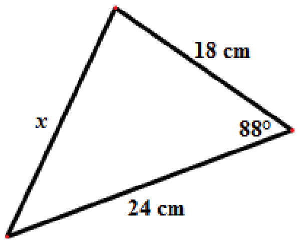
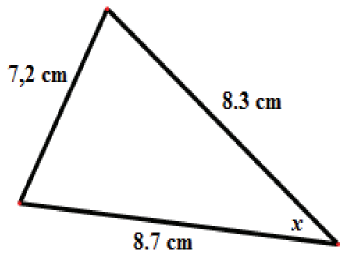
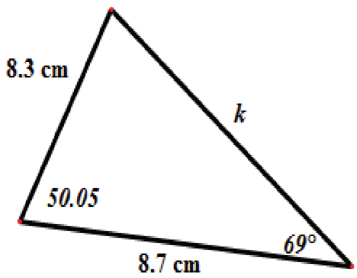
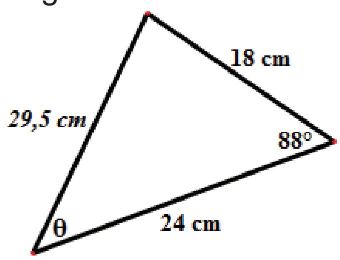
When to use Cosine rule?

1. When given three sides.
2. Two sides and the included angle.

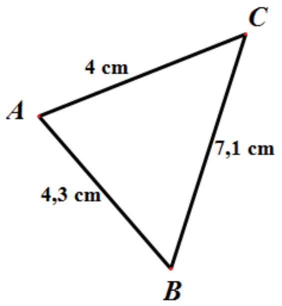
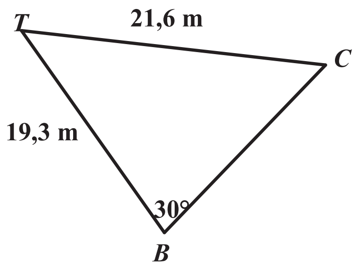
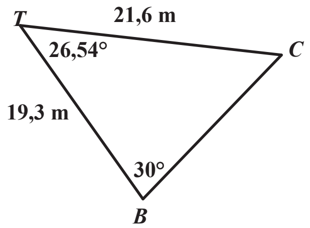
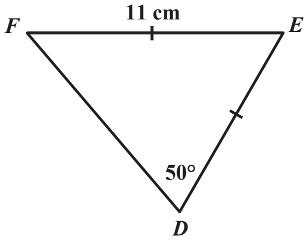
Cosine rule:

- $m^2 = n^2 + q^2 - 2nq \cos M$
- $n^2 = m^2 + q^2 - 2mq \cos N$
- $q^2 = m^2 + n^2 - 2mn \cos Q$

Examples of Sine and Cosine rules

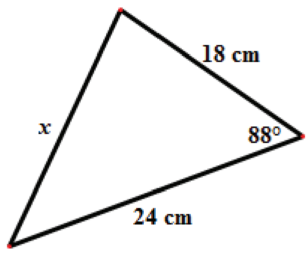
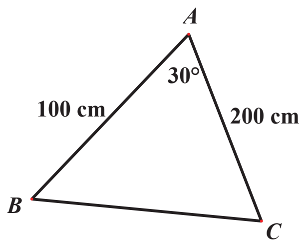
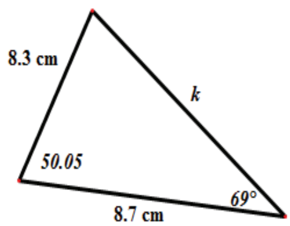
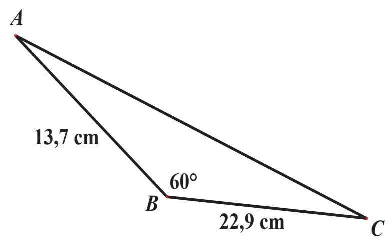
No.	Activity	Sine rule or Cosine rule	Solution	Marks
1.	Calculate the length of *x* 	Cosine rule	$a^2 = b^2 + c^2 - 2bc\cos A$ $x^2 = (18)^2 + (24)^2 - 2(18)(24)\cos 88°$ $x = \sqrt{(18)^2 + (24)^2 - 2(18)(24)\cos 88°}$ $x = 29{,}49\ cm$	✓Cosine rule formula ✓Substitution into correct formula ✓Answer
2.	Calculate the size of angle *x*. 	Cosine rule	$a^2 = b^2 + c^2 - 2bc\cos A$ $(7{,}2)^2 = (8{,}7)^2 + (8{,}3)^2 - 2(8{,}7)(8{,}3)\cos x$ $\cos x = \dfrac{(8{,}3)^2 + (8.7)^2 - (7{,}2)^2}{2(8{,}3)(8{,}7)}$ $x = 50{,}05°$	✓Cosine rule formula ✓Substitution ✓Answer
3.	Calculate the length of *k*. 	Sine rule	$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ $\dfrac{\sin 50{,}5°}{k} = \dfrac{\sin 69°}{8{,}3}$ $k = \dfrac{8{,}3\sin 50.05°}{\sin 69°}$ $k = 6{,}82\ cm$	✓Sine rule formula ✓Substitution ✓Answer
4.	Calculate the size of angle θ. 	Sine rule	$\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ $\dfrac{\sin \theta}{18} = \dfrac{\sin 88°}{29{,}5}$ $\sin \theta = \dfrac{18\sin\ 88°}{29{,}5}$ $\theta = 37{,}57°$	✓Sine rule formula ✓Substitution ✓Answer

Exercise 2.1

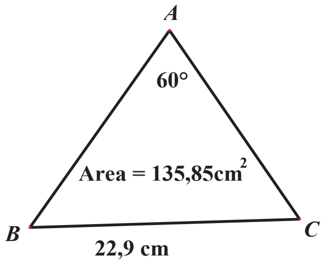
		Rule:		Marks
2.1	Calculate the size of $\hat{A}$. 		Answer: $\hat{A} = 117{,}57°$	3
2.2	Calculate $\hat{C}$. 	Rule:	Answer: $\hat{C} = 26{,}54°$	3
2.3	Calculate the length of BC. 	Rule:	Answer: $BC = 19{,}31$ m or $BC = 9{,}65\,m$	3
2.4	Calculate the size of $\hat{F}$. 	Rule:	Answer: $\hat{F} = 50°$	3

Examples of area rule

Calculate the area for each of the following correct to 2 decimal places, where necessary:

No.	Activity	Sine rule or Cosine rule	Solution	Marks
1.		Area rule	$A=\frac{1}{2}ab\sin C$ $A=\frac{1}{2}(24)(18)\sin 88°$ $A=215{,}87\,\text{cm}^2$	✓Area rule formula ✓Substitution into correct formula ✓Answer
2.		Cosine rule	$A\ of\ \Delta\, ABC=\frac{1}{2}bc\sin A$ $A=\frac{1}{2}(100)(200)\sin 30°$ $A=5000\ \text{cm}^2$	✓Area rule formula ✓Substitution into correct formula Answer
3.		Area rule	$A=\frac{1}{2}ab\sin C$ $A=\frac{1}{2}(8{,}3)(8{,}7)\sin 50.05°$ $A=27{,}68\ \text{cm}^2$	✓Area rule formula ✓Substitution into correct formula ✓Answer
4.		Area rule	$A=\frac{1}{2}ac\sin B$ $A=\frac{1}{2}(22{,}9)(13{,}7)\sin 60$ $A=135{,}85\,\text{cm}^2$	✓Area rule formula ✓Substitution into correct formula ✓Answer

5. Find the length of AB.



Area rule

$$A = \frac{1}{2} ac \sin B$$ ✓Area rule formula

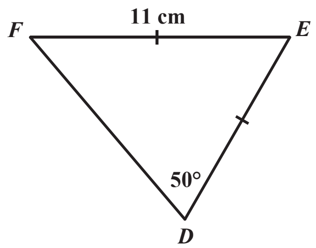
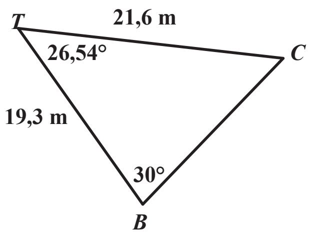
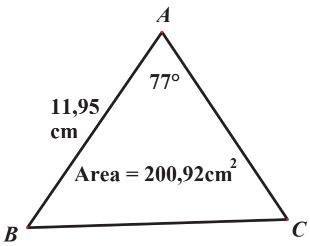
$$135,85 = \frac{1}{2}(22,9)(AB)\sin 60°$$ ✓Substitution into correct formula

$$\frac{135,85 \times 2}{22,9 \sin 60°} = AB$$

$$13,70\, cm = AB$$ ✓Answer

Exercise on Area rule

Calculate the area for each of the following correct to 2 decimal places, where necessary:

No.	Activity	Rule	Answer	Marks
2.1		Rule:	$\hat{A} = 59,58\,\text{cm}^2$	3
2.2		Rule:	$\hat{C} = 93,14\,\text{cm}^2$	3
2.3	Calculate the length of AC, if the area of triangle ABC is $200,92\,\text{cm}^2$. 	Rule:	$AC = 34,51\, cm$	3

5.3 Trigonometric identities

Trig identities are trig equations that are always true for any angle.

Study the trigonometric identities below. They are divided into two categories: quotient identity and square identities.

Quotient identity

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Square identities

- $\sin^2\theta + \cos^2\theta = 1$

From the above, the following can be done:

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \text{cosec}^2\theta$

Tips for solving trigonometric identities:

- Simplify both sides to look exactly the same as each other.
- If both sides look challenging, try to simplify both sides until they are the same.
- It is usually helpful to express tan in terms of sin and cos.
- Find the common denominator when fractions are added or subtracted.

Example	Explanation	Marks
Use trig identity: Simplify : $\cos^2\theta.\tan^2\theta$		(2)
Solution		
$=\cos^2\theta.\tan^2\theta$ $=\cos^2\theta.\dfrac{\sin^2\theta}{\cos^2\theta}$ $=\sin^2\theta$	$\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$	✓ $\dfrac{\sin^2\theta}{\cos^2\theta}$ ✓ $\sin^2\theta$
		(2)

Prove

Choose a side that looks complicated.

$$\frac{1-\cos^2\theta}{1-\sin^2\theta} = \tan^2\theta$$

LHS = $\dfrac{1-\cos^2\theta}{1-\sin^2\theta}$

Use square identities:

$\sin^2\theta = 1-\cos^2\theta$

$\cos^2\theta = 1-\sin^2\theta$

= $\dfrac{\sin^2\theta}{\cos^2\theta}$ ✓ $\sin^2\theta$ ✓ $\cos^2\theta$

= $\tan^2\theta$ = RHS

(4)

$$\frac{1}{1+\cos x} + \frac{1}{1-\cos x} = 2\cos ec^2 x$$

Choose LHS, as it looks complicated.

LHS = $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x}$

= $\dfrac{(1-\cos x)+(1+\cos x)}{(1-\cos x)(1+\cos x)}$ Common denominator ✓LCD

= $\dfrac{1-\cos x+1+\cos x}{1-\cos^2 x}$ Manipulation ✓$1-\cos^2 x$

= $\dfrac{2}{\sin^2 x}$ Simplification ✓2 ✓$\sin^2 x$

= $2\cos ec^2 x$

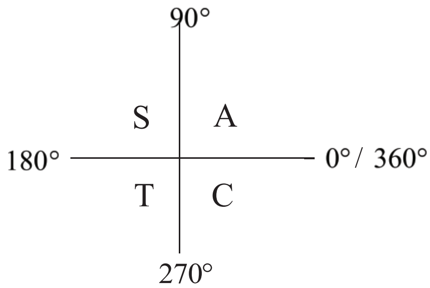
Activities	**Marks**
Prove the following identities:	
1. $\sin\theta(\sin^2\theta+\cos^2\theta) = \sin\theta$	(2)
2. $\cos x(\cot x+\tan x) = \cos ec\, x$	(4)
3. $\cos^2\theta+\tan^2\theta+\sin^2\theta = \sec^2\theta$	(4)
4. $(1+\cos\theta)(1-\cos\theta)\cos ec = \sin\theta$	(5)
5. $1-\sin\theta\cos\theta\tan\theta = \cos^2\theta$	(4)

6. $\dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta} = 1$ (3)

5.4 Trigonometric equations

Prior knowledge:

Revision of definitions, reciprocals and quadrants.



Reciprocals

Ratio	**Reciprocal**
sin	cosec
cos	sec
tan	cot

Quadrants

1. The first quadrant is between 0° and 90° degrees.

   All trig ratios are positive in that quadrant.
2. The second quadrant is between 90° and 180° degrees.

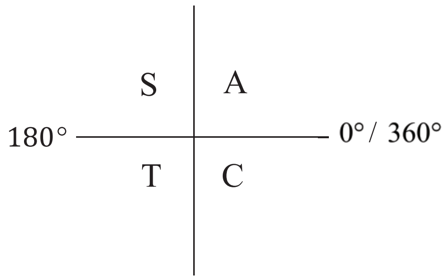
   Sin and cosec are positive in that quadrant.
3. The third quadrant is between 180° and 270°

   Tan and cot are positive in that quadrant.
4. The fourth quadrant is between 270° and 360°.

   Cos and sec are positive in this quadrant.

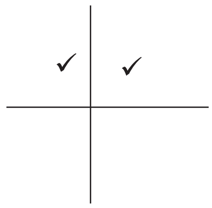
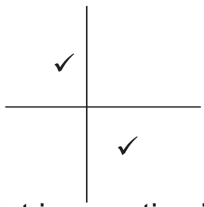
Procedure to solve:

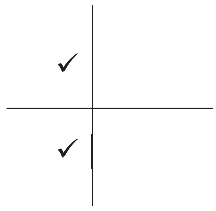
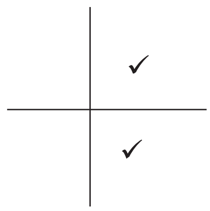
$sin\,\theta = 0{,}5\theta \in [0°: 360°]$



- ❖ The sine ratio is positive, so you should look at where it is positive between the given intervals.

  The sine ratio is positive in quadrant 1 and 2.
- ❖ Then, use a calculator to get the angle, by making θ the subject of the formula and activating $\sin^{-1} 0{,}5$
- ❖ The angle will be in first quadrant and is referred to as the reference angle. Use the same angle to find the size of the angle in the second quadrant.
- ❖ If cot, sec and cosec equations are given, they need to be written in terms of their reciprocal ratios as they appear on the calculator.

Example	**Explanation**	**Marks**
		(2)
Solve for θ : if		
$sin\,\theta = 0{,}5 \theta \in [0°: 360°]$	The value of the ratio is positive. Sin is positive in the first and second quadrant.	
$\theta = \sin^{-1} 0{,}5$ $\theta = 30°$ first quadrant and		✓ $30°$ ✓ $150°$
$\theta = 180° - 30°$ $\theta = 150°$	Second quadrant	
Solve for θ : if	(Round off the angle to two decimals.)	(5)
$cot\,\theta = -0{,}689 \theta \in [0°: 360°]$		
$\frac{1}{\tan\theta} = -0{,}689$		✓Writing reciprocal
$\tan\theta = -\frac{1}{0{,}689}$ $\theta = 55{,}43°$ ref angle	Cot is negative in the second and fourth quadrants. Then change to its reciprocal, as cot does not appear on the calculator.	✓Reference angle

$\therefore \theta = 180° + 55{,}43° or 360° - 55{,}43°$	Make tan the subject. Find the angle.	
$\therefore \theta = 235{,}43° \quad or \quad 304{,}57°$		✓Second quadrant ✓Third quadrant
	Two quadrants where tan is negative	✓Correct sizes
Solve for θ : if		(5)
$3\cos\theta = -1{,}5\theta \in [0°:360°]$	The coefficient of the ratio must be 1, so divide by 3	
$\cos\theta = -0{,}5$		✓Divide by 3
$\theta = \cos^{-1} 0{,}5$		
$\theta = 60°$ ref. angle		✓Reference angle
$\therefore \theta = 180° - 60° \quad or \quad 180° + 60°$	Use the calculator to find the reference angle	✓Second quadrant
$\therefore \theta = 120° \quad or \quad 240°$		
		✓Third quadrant ✓Correct sizes
Solve for θ : if	(Round off the angle to two decimals.)	(5)
$sec(\theta - 45°) = 1{,}302\theta \in [0°:360°]$		
	First identify angles where sec is positive. 	
$\frac{1}{\cos(\theta - 45°)} = 1{,}302$	Change sec to its reciprocal.	✓Reciprocal
$\cos(\theta - 45°) = \frac{1}{1{,}302}$	Make the angle the subject of the formula.	
$(\theta - 45°) = \cos^{-1} \frac{1}{1{,}302}$	Use the calculator to find the angle.	✓Reference angle
$\theta - 45° = 39{,}82°$ ref. angle		
$\therefore \theta - 45° = 39{,}82° or 360° - 39{,}82°$	Write the angle in its quadrant.	✓Fourth quadrant ✓ $84{,}82°$
$\therefore \theta = 84{,}82° \quad or \quad 320{,}18°$		✓ $320{,}18°$

Activities	Marks	Answers
Determine the size in each and round off the angles to two decimal place $\theta \in [0°: 360°]$		
1. $\sin\theta = 0{,}707$	(2)	$\theta = 44{,}99° or 135{,}01°$
2. $\cos\theta = 0{,}156$	(2)	$\theta = 81{,}03° \quad or \quad 278{,}97°$
3. $\tan\theta = 0{,}847$	(2)	$\theta = 40{,}25° \quad or \quad 220{,}26°$
4. $\frac{1}{2}\cos\theta = -0{,}138$	(5)	$\theta = 106.02° \quad or \quad 253{,}98°$
5. $\cot\theta - 1 = 2{,}158$	(5)	$\theta = 17{,}57° \quad or \quad 197{,}57°$
6. $\cos ec(\theta + 20°) = 1{,}145$	(5)	$\theta = 60{,}85° \quad or \quad 119{,}15°$

6 Message to Grade 12 learners from the writers

Technical Mathematics can be fun, as it requires you to pull together all your learning from the lower grades, in order to answer the Grade 12 examination questions. If you skipped one grade before Grade 12, I had left a void to ground the floor.

Please ensure that you know all the axioms and corollaries (rules), in order that you can answer all the questions. Answer the Technical Mathematics exemplar papers before you sit the final examinations. Write the exemplar in 3 hours and mark the script on your own using the memorandum, in order to gauge whether you are ready to sit the final paper. The memorandum is also available on the DBE website.

We assure you that this year's final paper will be similar to those of previous years in both format and style.

7 Thank you and acknowledgements

We hope the guidance provided in this booklet helps you in your final examinations.

Mr Leonard Gumani Mudau, Mr Mongameli Mbusi, Mr Muthige Ntshengedzeni Steven, Mrs Nontobeko Tom and Mr Zulu Bhekani all wish you well.

TECHNICAL MATHEMATICS

TRIGONOMETRY

GRADE 12

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