basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

TECHNICAL MATHEMATICS

GUIDELINES FOR PRACTICAL ASSESSMENT TASKS

GRADE 12

2022

These guidelines consist of 33 pages.

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1.   INTRODUCTION

The 18 Curriculum and Assessment Policy Statement subjects which contain a practical component all include a practical assessment task (PAT). These subjects are:

- AGRICULTURE: Agricultural Management Practices, Agricultural Technology
- ARTS: Dance Studies, Design, Dramatic Arts, Music, Visual Arts
- SCIENCES: Computer Applications Technology, Information Technology, Technical Sciences, Technical Mathematics
- SERVICES: Consumer Studies, Hospitality Studies, Tourism
- TECHNOLOGY: Civil Technology, Electrical Technology, Mechanical Technology, Engineering Graphics and Design

A practical assessment task (PAT) mark is a compulsory component of the final promotion mark for all candidates offering subjects that have a practical component and counts 25% (100 marks) of the examination mark at the end of the year. The practical assessment task for Technical Mathematics Grade 12 consists of three tasks (one task per term) which should be completed by end of Term 3. The tasks are COMPULSORY for ALL candidates offering Technical Mathematics in Grade 12.

The PAT is implemented during the first three terms of the school year. The PAT allows learners to be assessed regularly during the school year and it also allows for the assessment of skills acquired and it applies the science of Mathematics to the technical field where the emphasis is on application. It is therefore important that schools ensure that all learners complete the practical assessment tasks within the stipulated period to ensure that learners are promoted at the end of the school year. The planning and execution of the PAT differ from subject to subject.

The tasks should be administered under supervised conditions. Moderation may be done onsite.

2. TEACHER GUIDELINES

2.1 How to administer the PATs

- The following documents must be available for all formal tasks:

  o Task instructions explaining the procedures to be followed
  o The worksheets which include questions to be answered under examination conditions
  o The teacher guidelines with task instructions, worksheets and marking guidelines
    (The teacher guidelines MUST NOT be released to the learners.)
  o Teachers should compile marking guidelines (memoranda) for the real results of the task conducted (teachers should do the tasks themselves FIRST)
- The tasks must be done individually.
- Each learner must record his/her OWN INDIVIDUAL data and observations.
- Each learner must be provided with his/her OWN worksheet and answer the questions INDIVIDUALLY under examination conditions.
- Only once all the learners are ready to do the task and they are seated and ready to answer the questions may teachers hand out a worksheet to each learner. Examination conditions have to be applied.
- If it is not possible to do the task and complete the worksheet on the same day, the teacher must collect the learners' tasks. These tasks must be kept at school.

2.2 Moderation of the PATs

For moderation, the following documents are required in the teacher's file:

- Index indicating all tasks with raw and weighted marks
- All task instructions
- Marking guidelines for all tasks, with ticks and totals
- Composite working mark sheet for all learners showing raw and weighted marks
- Evidence of internal moderation

For moderation, the following documents are required in the learner's file:

- Index indicating all tasks with raw and weighted marks
- Answer sheets for all tasks

3. LEARNER GUIDELINES

3.1   This PAT for Grade 12 consists of THREE tasks.

3.2   The PAT contributes 25% towards the final promotion mark for Grade 12.

3.3   All the work in the PAT must be the learner's own. Group work will NOT be allowed.

3.4   Show ALL calculations clearly and include units. Round off answers to TWO decimal places. Use correct SI units where necessary.

4. EVIDENCE OF MODERATION

Learner's name:	
School:	

MARK ALLOCATION

TASK	MAX. MARK	WEIGHTING	LEARNER'S MARK (TEACHER)	MODERATED MARK (SCHOOL)	MODERATED MARK (DISTRICT)	MODERATED MARK (PROVINCE)
1	40	10				
2	30	7,5				
3	30	7,5				
TOTAL	100	25				
NAME						
SIGNATURES						
DATE						

DECLARATION OF AUTHENTICITY

I hereby declare that the tasks submitted for assessment is my own, original work and have not been submitted for assessment or moderation previously.

_____   _____
SIGNATURE OF LEARNER   DATE

As far as I know, the above declaration by the candidate is true and I accept that the work offered is his/her own.

_____   _____
SIGNATURE OF TEACHER   DATE

SCHOOL STAMP

5.   CONCLUSION

On completion of the practical assessment task learners should be able to demonstrate their understanding of the subject, enhance their knowledge, skills, values and reasoning abilities as well as establish connections to life outside the classroom and address real-world challenges. The PAT furthermore develops learners' life skills and provides opportunities for learners to engage in their own learning.

TECHNICAL MATHEMATICS

PRACTICAL ASSESSMENT TASK 1

GRADE 12

2022

TERM: 1

MARKS: 40

TIME: 1 hour

SURNAME AND NAME	
SCHOOL	

This task consists of 9 pages.

TECHNICAL MATHEMATICS TASK 1

TOPIC: ANALYTICAL GEOMETRY AND CIRCLES

AIMS:
- **To determine the equation of a circle with centre at the origin**
- **To determine the equation of a tangent to a circle at a point of contact**
- **To calculate the height of the minor segment of a circle.**

INSTRUCTIONS AND INFORMATION

1.   This PAT Task 1 worksheet consists of THREE activities.

2.   Do ALL the activities and answer ALL the questions.

3.   Resources required are ruler, compass, pencil, protractor and calculator.

4.   Clearly show ALL calculations, diagrams, etc. that you have used in determining your answers.

5.   Make sure your sketch is neat and constructed as per instruction.

ACTIVITY 1(A):
- **To determine the equation of a circle with centre at the origin**

Resources required: ruler, protractor, compass, eraser, pen and pencil

Step 1: Draw a circle with centre O at the origin and a radius of 5 cm on the Cartesian plane below.
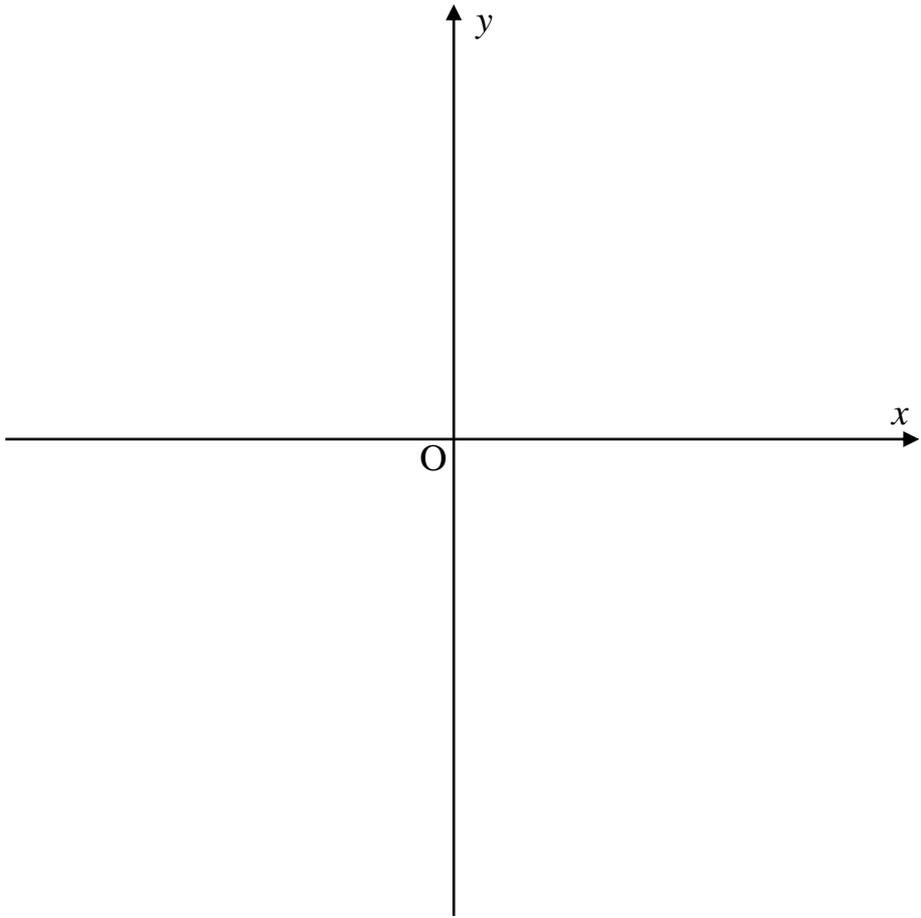
	Solution	Marks
		(2)

Step 2: Determine the value of r^2

	Solution	Marks
	$r^2 =$	(1)

Step 3: Plot and label the following sets of coordinates clearly on the circle in Step 1 as follows:

- Point A must be the x-intercept, $x > 0$
- Point B must be a point in the first quadrant, i.e. $x > 0$ and $y > 0$
- Point C must be a point in the 3rd quadrant (x and y are both integers).
- Point D must be a negative y-intercept. (4)

Step 4: Complete the table below for the points plotted in Step 3.

	Solution				**Marks**
	Coordinates	Value of x^2	Value of y^2	$x^2 + y^2$	
	A (;)				
	B (;)				
	C (;)				
	D (;)				(8)

Step 5: Compare the last column of Step 4 with the value of r^2 in Step 2. Hence, deduce the relationship between $x^2 + y^2$ and r^2, and give the equation of the circle drawn in Step 1.

	Solution	**Marks**
		(1)

Step 6: Use the equation of the circle in Step 5 to show, by calculation, whether point E (– 4 ; 2) lies on or inside or outside the circle.

	Solution	**Marks**
		(1)

ACTIVITY 1(B):

Given circle with $r = 13$ units and passes through point $(a ; -5)$, determine the value(s) of a using the relationship deduced in Activity 1(A) Step 5 above.

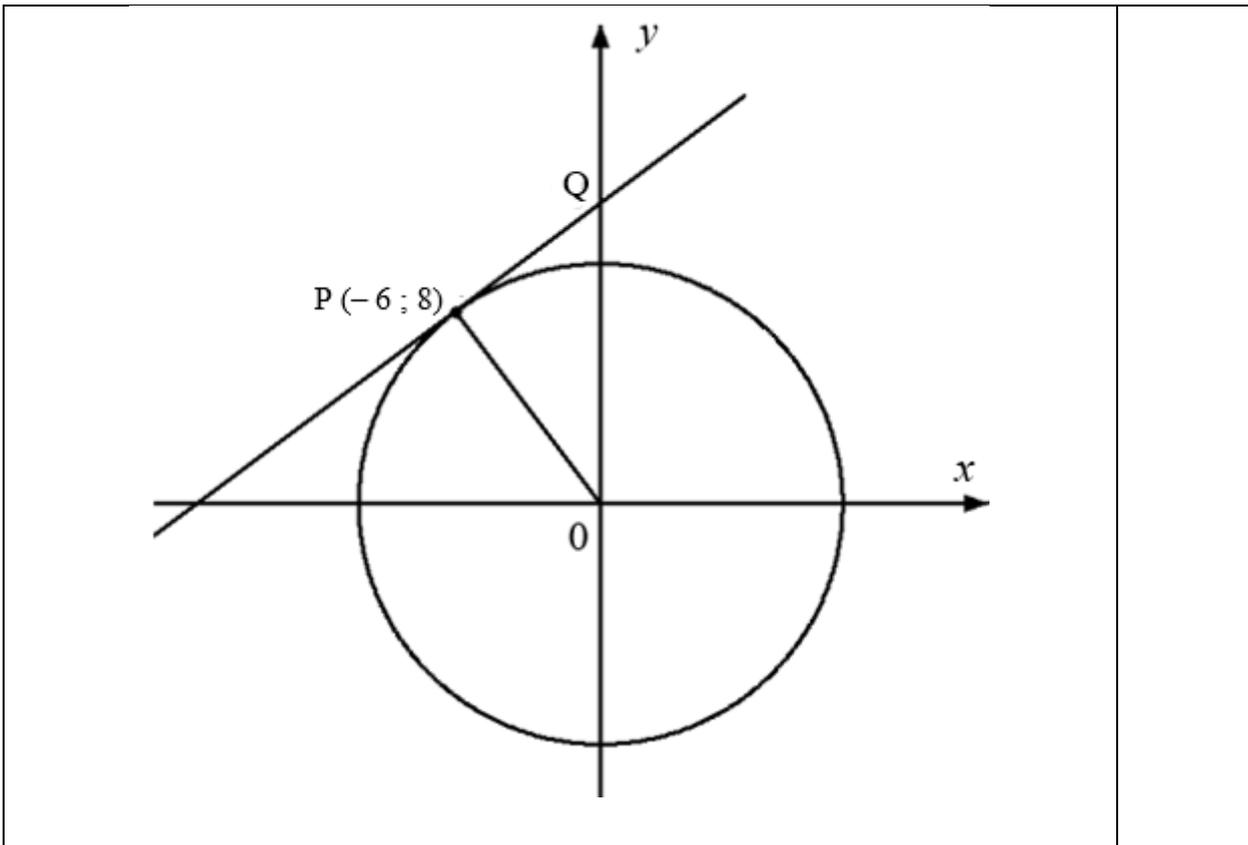|	**Solution**	**Marks**
		(4)

[21]

ACTIVITY 2:
- **To determine the equation of a tangent to a circle at a point of contact**

Resources required: ruler, protractor, compass, eraser, pen and pencil

The figure below represents a circle defined by $x^2 + y^2 = 100$ and a tangent to the circle at point P $(-6 \, ; \, 8)$.
OP is the radius of the circle. The tangent intersects the *y*-axis at point Q.

Step 1: Use your protractor to measure the magnitude of $O\hat{P}Q$, the angle between the radius and the tangent.

	Solution	Marks
	$O\hat{P}Q = $ _____	(1)

Step 2: Calculate the gradient of OP.

	Solution	Marks
		(2)

Step 3:   Hence, write down the gradient of the tangent at point P

Solution	Marks
	(1)

Step 4:   Use the information in Step 3 to determine the equation of the tangent to the circle at point P.

| Solution	Marks
	(3)

[7]

ACTIVITY 3:
- **To calculate the height of the segment**

Read the information below and answer the questions that follow.

The side wall of a tyre has a set of numbers, letters and symbols. These numbers and letters help you to decide which type of tyre you need to purchase for your car.

Below is a list and an explanation of each of the indicators from a tyre manufacturing company's website.
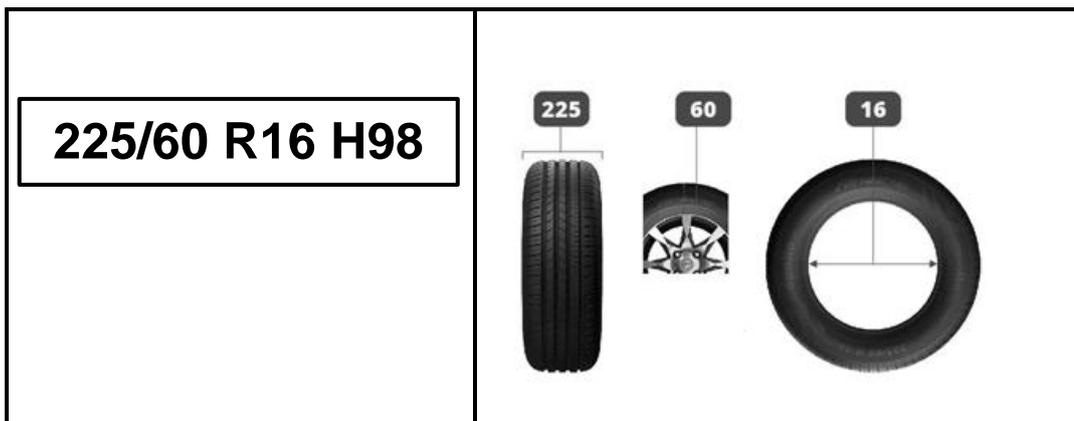
1. Width

The number 225 for this tyre represents the width, in millimetres. The width is the distance from the one side wall to the other, i.e. the part that is in contact with the road.

2. Aspect ratio

The aspect ratio, also known as the profile, refers to the height of the sidewall as a percentage (%) of the width. For the above tyre, the 60 is actually 60% of the width [60% x 225 = 135 mm].

3. Construction type

The letter R represents how the tyre is made. R means the tyre is a radial type of construction.
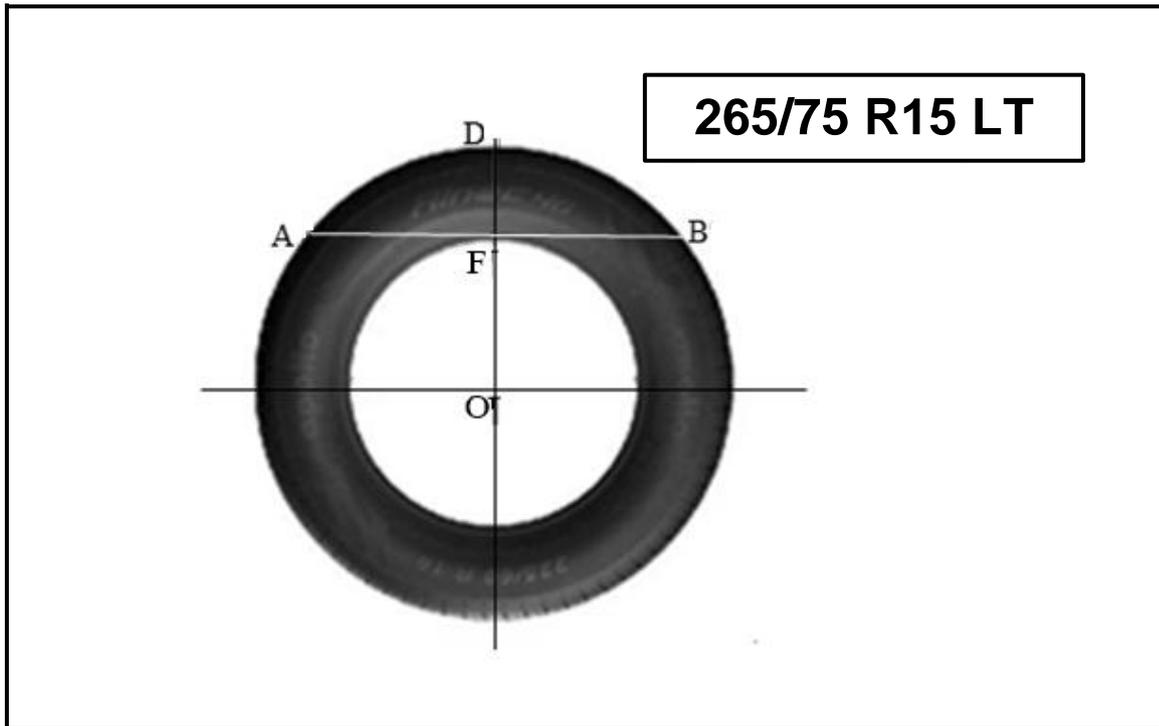
4. Rim

In this case, it is a 16-inch wheel. This is the diameter of the tyre's inner rim.

5. Speed rating

The letter H for this tyre represents the load index, that is the maximum speed at full load.

6. Load index

The number 98 for this tyre represents the load index, that is the maximum load-carrying capacity of the tyre.

265/75 R15 LT

3.1 Complete the following for the tyre in the picture using **265/75 R15 LT**.

	Solution	Marks
	Width of the tyre:……………..	
	Profile of the tyre:…………..	
	Diameter of the inner rim:……………	
	Symbol for speed rating:……………….	(4)

3.2 Convert the diameter of the inner rim to millimetres.
Use: 1 inch = 25,4 mm

	Solution	Marks
		(2)

3.3 Calculate the height FD of the side wall.

	Solution	Marks
		(2)

3.4   Determine the length of chord AB using $4h^2 - 4dh + x^2 = 0$

	Solution	**Marks**
		(4)

[12]

TOTAL: 40

TECHNICAL MATHEMATICS

PRACTICAL ASSESSMENT TASK 2

GRADE 12

2022

TERM: **2**

MARKS: **30**

TIME: **1 hour**

SURNAME AND NAME	

SCHOOL	

This task consists of 11 pages.

TECHNICAL MATHEMATICS TASK 2

TOPIC: INTEGRATION

AIMS:
- **To determine the definite integral $\int_a^b f(x)\,dx$ bounded by a function $y = f(x)$ and the x-axis between the ordinates $x = a$ and $x = b$**
- **To apply integration to solve real-life problems**

INSTRUCTIONS AND INFORMATION

1.   This PAT Task 2 worksheet consists of THREE activities.

2.   Do ALL the activities and answer ALL the questions.

3.   Resources required are calculators, grids (provided) and mathematical sets.

4.   Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

ACTIVITY 1

Introduction
- A review of **indefinite** integrals and **definite** integrals.

The term 'integration' refers to the converse operation of differentiation. Therefore, integrals may also be referred to as anti-derivatives.
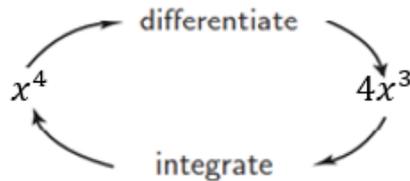
- An **indefinite** integral of a function $f(x)$ involves finding its anti-derivative in terms of x and is indicated as $\int f(x)\, dx$.

NOTE: When determining the integral of a function, the following basic rules of integration can be used for PAT Task 2:

Rule 1: $\int x^n\, dx = \dfrac{x^{n+1}}{n+1} + C \; ; \; n \neq -1$

Rule 2: $\int k\, dx = kx + C$

EXAMPLE:

Integral sign $\longrightarrow$ $\int -4x^3\, dx = -\dfrac{4}{4}x^4 + C$ $\longleftarrow$ Constant of integration

This term is called the integrand.

- A **definite** integral is an integral where the limits over which the integral has to be calculated is given. Therefore, the definite integral of a function $f(x)$ between the ordinates $x = a$ (lower limit) and $x = b$ (upper limit) is indicated as:

$$\int_a^b f(x)\, dx = \left[F(x) + C \right]_a^b$$
$$= \left[F(b) + C \right] - \left[F(a) + C \right]$$
$$= F(b) - F(a)$$

where $F(x)$ is the integral of $f(x)$ and C is the constant of integration

Given: $f(x) = 3x^2$

Determine **Step 1** and **Step 2**.

	Solution	Marks
Step 1	$\int f(x)\,dx = F(x) + C = \ldots\ldots\ldots\ldots$	(2)

	Solution	Marks
Step 2	• $F(3) = \ldots\ldots\ldots\ldots$	
	• $F(1) = \ldots\ldots\ldots\ldots$	
	• $\therefore \int_1^3 f(x)\,dx = \ldots\ldots\ldots\ldots\ldots\ldots$	
	$= \ldots\ldots\ldots\ldots\ldots$	
		(4)
		[6]

ACTIVITY 2

OBJECTIVE:
- To determine the area bounded by a function $f(x) = k$ (i.e. a horizontal line), the x-axis and the ordinates $x = a$ and $x = b$ by using a formula and a definite integral.

Resources required:
- Mathematical instruments (ruler and protractor are essential)
- Pen
- Pencil

Procedure:

Step 1: Sketch the graph of the function $f(x) = 2$ on the grid provided.

Step 2: Draw the vertical line (ordinate) $x = 3$ and thereafter shade the area bounded by the function $f(x) = 2$, the x-axis and the ordinates $x = 0$ and $x = 3$

	Solution	Marks
Step 1 and Step 2		(2)
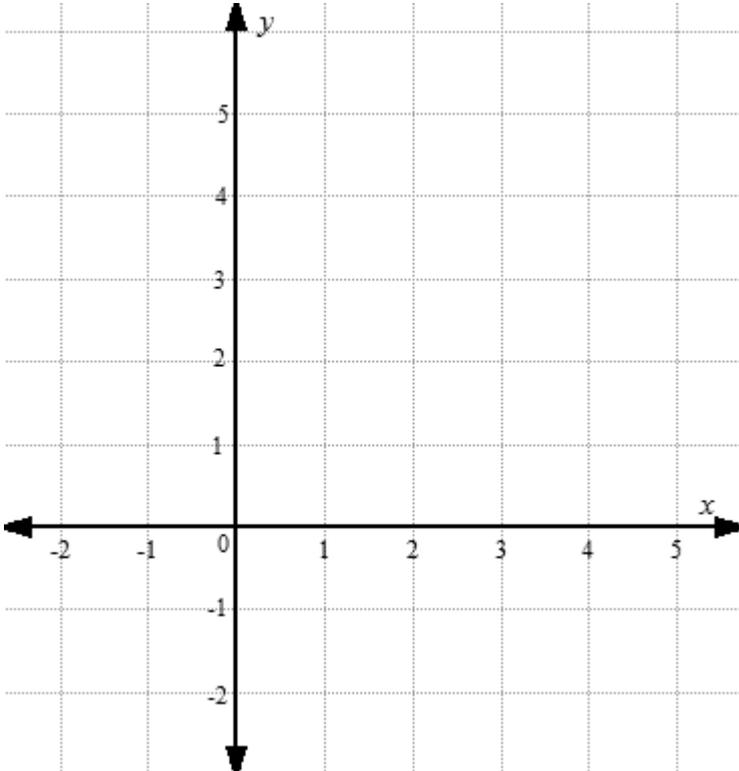
Step 3: What is the shape of the area bounded by the function $f(x) = 2$, the x-axis and the ordinates $x = 0$ and $x = 3$?

	Solution	Marks
		(1)

Step 4: Use an appropriate formula (for the shape mentioned above), to determine the area bounded by the function $f(x) = 2$, the x-axis and the ordinates $x = 0$ and $x = 3$.

	Solution	Marks
		(1)

Step 5: Determine $\int_0^3 2 \, dx$

	Solution	Marks
		(3)
		[7]

ACTIVITY 3

OBJECTIVE:
- To determine the area bounded by a function $f(x) = mx + c$ (i.e. a sloping line) and the x-axis and the ordinates $x = a$ and $x = b$ using a formula and definite integral.
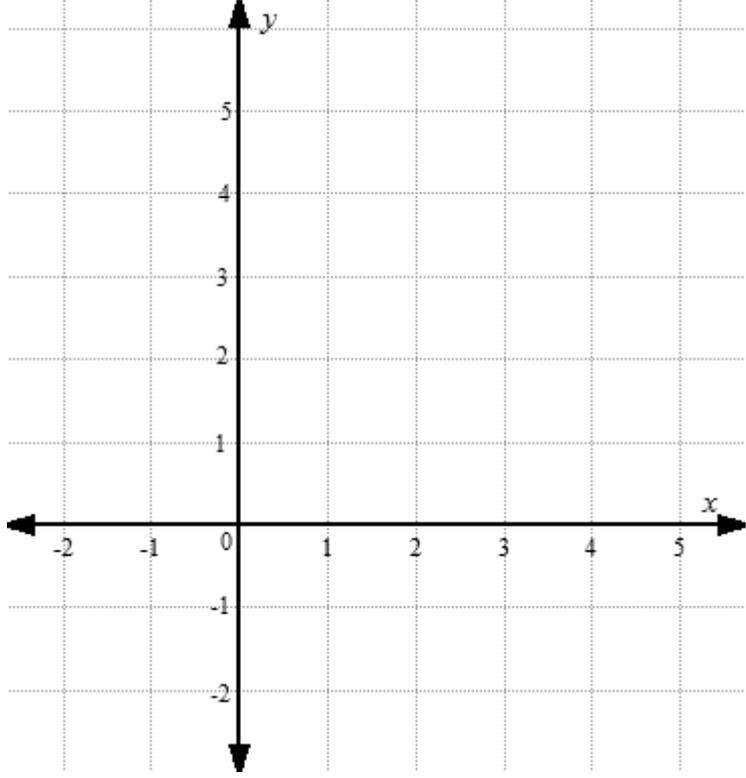
Materials required:
- Mathematical instruments
- Pen
- Pencil

Procedure:

Step 1: Sketch the graph of the function $f(x) = -x + 4$ on the grid provided.

Step 2: Shade the area bounded by the function $f(x) = -x + 4$, the x-axis and the ordinates $x = 0$ and $x = 4$

	Solution	Marks
Step 1 and Step 2		(2)

Step 3: What is the shape of the area bounded by the function $f(x) = -x + 4$, the x-axis and the ordinates $x = 0$ and $x = 4$?

Solution	Marks
	(1)

Step 4: Determine the area bounded by $f(x) = -x + 4$ and the x-axis and the ordinates $x = 0$ and $x = 4$ using an appropriate formula (for the shape mentioned in Step 3).

Solution	Marks
	(1)

Step 5: Determine the area bounded by the function $f(x) = -x + 4$, the x-axis and the ordinates $x = 0$ and $x = 4$ using definite integrals by calculating $\int_0^4 (-x+4)\ dx$

Solution	Marks
	(3)
	[7]

Observation and Conclusion

What can you deduce about the $\int_a^b f(x)\,dx$ and the area bounded by the function $f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ from Activity 2 and Activity 3?

Solution	Marks
	(1)
	[1]

ACTIVITY 4

OBJECTIVE:
- **To apply the knowledge of definite integrals in real life**

The world's tallest arch, the **Gateway Arch** (pictured below) is a 630-foot (192 m) monument found in Missouri, USA. It is clad in stainless steel and built in the form of a weighted catenary arch which is parabolic in shape.



A local company in South Africa is contracted to build a similar structure, but on a much smaller scale, as an iconic entrance to a mall.

The diagram below models the front view of the proposed structure on a Cartesian plane.

- The parabolic shaped outer arch is defined by the equation $f(x) = \left(-x^2 + 12x\right)$ m and cuts the x-axis at $x = 0$ and $x = 12$
- The parabolic shaped inner arch is defined by $g(x) = \left(-x^2 + 12x - 11\right)$ m and cuts the x-axis at $x = 1$ and $x = 11$

The shaded area represents the space between the two parabolas that needs to be covered by metal cladding.

Outer arch with equation:
$f(x) = \left(-x^2 + 12x\right)$

Inner arch with equation:
$g(x) = \left(-x^2 + 12x - 11\right)$
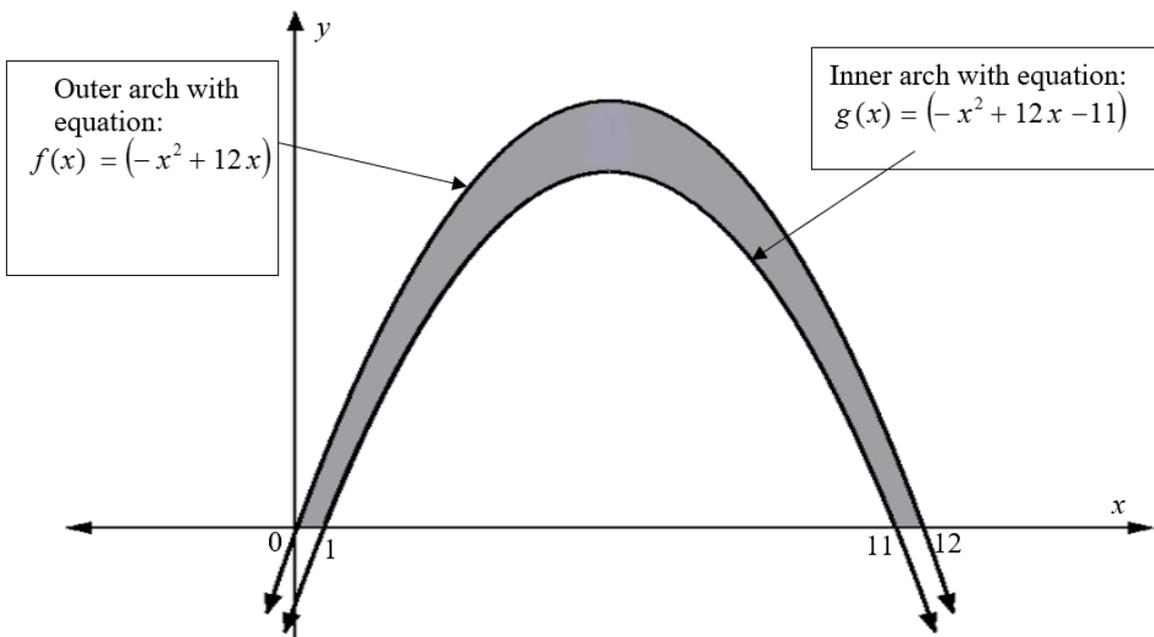
Step 1: Determine the area bounded by the curve of f and the x-axis between the points where $x = 0$ and $x = 12$

Solution	Marks
	(4)

Step 2:   Hence, determine the shaded area between the two parabolas that needs to be covered by metal cladding.

	Solution	Marks
		(5)
		[9]

TOTAL: 30

TECHNICAL MATHEMATICS

PRACTICAL ASSESSMENT TASK 3

GRADE 12

2022

MARKS: **30**

TIME: **1 hour**

SURNAME AND NAME	

SCHOOL	

This task consists of 7 pages.

TECHNICAL MATHEMATICS TASK 3

TOPIC: MENSURATION AND INTEGRATION

AIMS:
- **To apply and develop mathematical skills, reasoning and demonstrate an understanding of an irregular figure**
- **To calculate the area of an irregular figure by using the mid-ordinate rule and definite integral method**

INSTRUCTIONS AND INFORMATION

1.   This PAT Task 3 worksheet consists of TWO activities.

2.   Do ALL the activities and answer ALL the questions.

3.   Resources required are a ruler and calculator.

4.   Clearly show ALL calculations, diagrams, etc. that you have used in determining your answers.

5.   Make sure your sketch is neat and constructed according to the instructions.

ACTIVITY 1

Introduction

The mid-ordinate rule is a method of calculating approximate areas of non-standard geometric figures. In this task learners will use standard geometric figures to estimate the area of an irregular figure (non-standard geometric figure) and use the mid-ordinate rule to determine the area.

It can be calculated by using:

Area = width of segment x sum of heights of the mid-ordinates

AIM:
- **To determine the area of an irregular figure**

Resources needed: pencil, ruler and calculator

The diagram below models a piece of cardboard with a survey line (a line drawn from one end of the shape to another, approximately in the middle).

Step 1:   Measure and record the length of the survey line.

	Solution	Marks
		(1)

Step 2:   Divide the survey line into FIVE equal parts.

	Solution	Marks
	Answer on the diagram.	(1)

Step 3:   Measure and write down the length of the equal parts (**width of segment**).

	Solution	Marks
	Answer on the diagram.	(1)

Step 4:   Draw perpendicular lines (**ordinates**) to the survey line.

	Solution	Marks
	Answer on the diagram.	(2)

Step 5:   Measure and record the height of each segment.

	Solution	Marks
	Answer on the diagram.	(2)

Step 6:   Use dotted lines to draw and measure each mid-ordinate.

	Solution	Marks
	Answer on the diagram.	(3)

Step 7:   Use the mid-ordinate rule to determine the area of an irregular shape.

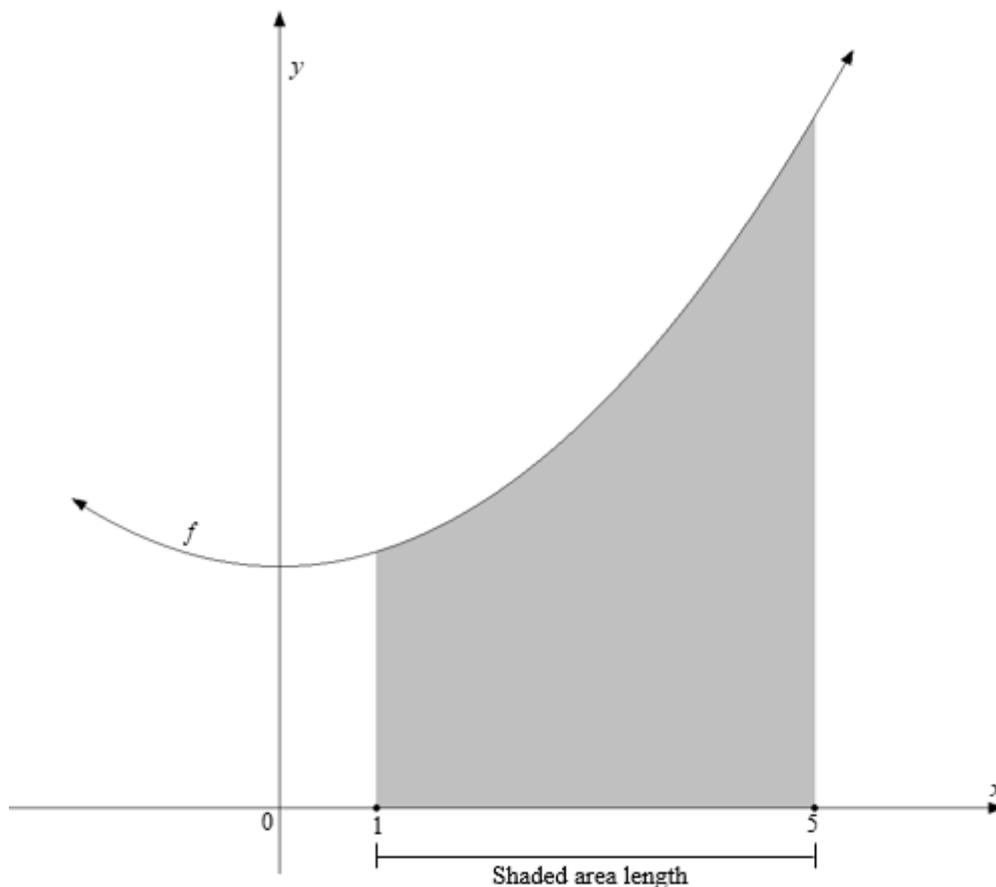|	Solution	Marks
		(4)
		[14]

ACTIVITY 2

AIM:
- **To calculate the area of an irregular figure by using the mid-ordinate rule and the definite integral method**

The mid-ordinate rule can be used to approximate the area bounded by the curve and the x-axis.

Given below is the shaded area bounded by the function defined by $f(x) = x^2 + 3$ and the x-axis and between $x = 1$ and $x = 5$

Shaded area length

2.1   Determine the midpoint of the limits $x = 1$ and $x = 5$. Use the letter **M** to label the midpoint on the diagram.

	Solution	Marks
	M, $x = $ ………….	(1)

2.2   Draw a vertical line from the curve to M, the midpoint calculated above.

	Solution	Marks
	Answer on the diagram.	(1)

2.3   Determine algebraically the heights of the shaded area at $x = 1$, $x = 5$
       and the midpoint calculated in QUESTION 2.1.

	Solution	Marks
		(3)

2.4   Draw the mid-ordinates, using dotted lines, and calculate their heights.

	Solution	Marks
	Answer on the diagram.	(2)

2.5   Use mid-ordinate rule to determine the area bounded by f and the x-axis
       between the ordinates $x = 1$ and $x = 5$

	Solution	Marks
		(3)

2.6   Use **definite integrals** to determine the area bounded by f and the x-axis
       between the ordinates $x = 1$ and $x = 5$

	Solution	Marks
		(4)

2.7   Compare the value of the shaded area bounded by the function
        $f(x) = x^2 + 3$ and the x-axis between $x = 1$ and $x = 5$ by using the
        mid-ordinate rule and **definite integrals**.

	Solution	Marks
		(2)
		[16]

TOTAL: 30