Primary Mathematics LTSM and Assessments Review

SUMMATIVE REPORT

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

Epoch trusts

Optima trust

ZENEX FOUNDATION

This intervention has been commissioned by the Department of Basic Education and funded by Zenex Foundation and the Epoch and Optima Trusts.

Date of publication: August 2023

Suggested citation:

Roberts, N., Tshuma, L., Moloi, Q., and Sommerdyk, S. (2023) *Primary Mathematics LTSM and Assessments Review: summative report,* Kelello: Johannesburg.

Enquires to:

Nicky Roberts
Director: Kelello

Email: Nicky@kelello.org
Twitter:@Kelello_

Website: www.kelello.org

Table of Contents

List of Figures

List of tables

Acronyms

AfL	Assessment for Learning
ANA	Annual National Assessment
EGMA	Early Grade Mathematics Assessment
FATs	Formal Assessment Tasks
C2005	Curriculum 2005
CAPS	Curriculum and Assessment Policy Statement
DBE	Department of Basic Education
DoE	Department of Education
LAB	Learner Activity Book
LOLT	Language of Leaning and Teaching
LTSM	Learning and Teaching Support Materials
MCC	Magic Classroom Collective
MSAP	Mental Starters Assessment Project
RNCS	Revised National Curriculum Statement
SACMEQ	Southern and East Africa Consortium for Monitoring Educational Quality
TIMSS	Trends in Mathematics and Science Studies
TMU	Teaching Mathematics with Understanding
UK	United Kingdom

Acknowledgements

An undertaking such as this requires a team effort. We are grateful to the colleagues in the Department of Basic Education and Zenex Foundation who offered their insights and funding support to this process. Without the Epoch and Optima Trusts and Zenex Foundation funding, this would not have been possible.

The Early Grade Mathematics Research Project (EGRMP) of the DBE constituted an LTSM and assessment working group which was particularly helpful, and comprised of: Lunga Swelindawo (DBE); Simphiwe Mlotshwq (DBE); Stephen Taylor (DBE); Bonita Goetham (DBE); Elspeth Khembo (DBE); Cebisa Mtumtum (DBE); Lesang Sebaeng (DBE); Elspeth Khembo (DBE); Thabo Magobo (presidency, DPME); Nosipho Mbonambi (DBE); Keitumatse Modiba (DBE); Lumka Tshiki (Tshikululu); Fatima Adams (Zenex); Asiya Hendriks (JET) and Andile Dube (Unicef).

The following DBE colleagues also made themselves available for further consultations one-on-one or in adhoc meetings: Bonita Goetham – DBE curriculum; Luvuyo Cebo, Doris Mahlobogoane, Tshitema Makondelela, Virginia Mputla, Ntebo Mkhondo and Tumi Montja (DBE workbooks), Mark Chetty – DBE assessment; Elspeth Khembo – DBE curriculum; and Asiya Hendricks, Lunga Swelindawo, and Stephen Taylor.

In addition, we formed an expert reference group who contributed regularly to the research process. The experts included: Deva Govender (NECT); Mzamzni Jully Mdaka (NECT); Asiya Hendricks (JET); Fatima Adams (Zenex); Ingrid Saphire (Wits); Anil Kanjee (TUT); Msebenzi Rabaza (UFS); Melanie Graven (RU); Roelien Herholdt (JET); Permie Isaac (Funde Wande); Nolukaholo Faith Hlungulu (WSU); Janet Ramollo (TUT); Cally Kuhne (Independent); Simphiwe Mlotshwq (DBE); Lunga Swelindawo (DBE); Stephen Taylor (DBE); Bonita Goetham (DBE); Elspeth Khembo (DBE); and Mark Chetty (DBE).

I am particularly grateful for the detailed and careful research work undertaken by the Kelello team: Stacey Sommerdyk as project manager; Qetelo Moloi lead the assessment review; Lindiwe Tshuma lead the LTSM review and was supported by Corin Mathew, Agatha Lebethe facilitated the expert workshops.

I trust that this document makes a contribution to better understanding the LTSM and assessment landscape for primary school mathematics in South Africa, and that its recommendations are implemented to further strengthen our collective endeavours to improve access to quality mathematics learning.

Nicky Roberts

Project leader, Kelello director

Introduction

This is a summative rather than an exhaustive report which offers a high level overview of the research conducted relating to Learning and teaching Support Materials (LTSM) and Assessment for primary school (Grades R to 7) mathematics in South Africa. It offers a bird's eye view of the research elements which have been undertaken for the Department of Basic Education, and points to the supporting documentation which have informed this summary. Policy makers, programme implementers, researchers, subject advisors and provincial departments of education may find this useful when choosing to invest in, or make selections from, existing LTSM and assessment offerings.

The LTSM and Assessment Reviews, commissioned by the Department and Basic Education, and funded by Zenex Foundation, sought to provide an overview of available LTSM and Assessments as these are the key ingredients for improving learner outcomes in primary school mathematics. The terms of reference for this review recognised the high variation of practices in terms of what and how LTSM is used, across South African schools. In addition, the knowledge that many primary schools only make use of DBE workbooks (which were designed as supplementary resources) was noted with concern. The review of current practice and analysis of selected offerings, as summarized here, is intended to assist educational stakeholders to make informed decisions about their selections of the LTSM.

Those seeking more detail, beyond this summative report, ought to consult the following working papers which complement this summary:

- A primary mathematics LTSM learning brief of 10 LTSM offerings.
- A primary mathematics Assessment learning brief of 9 assessment offerings.
- A synopsis of the findings in this report.

Figure 1: Reviews supported by engagements with South African primary mathematics experts (DBE and beyond)

This report summarises the high level findings of the assessment review and LTSM review which were informed by, and discussed with, early grade mathematics experts and DBE officials across various branches and forums. Each review developed an initial understanding of current practice which was then refined, drawing on literature and then an in-depth analysis of selected assessment instruments and LTSM offerings were given. For the LTSM review, the working group agreed to focus the in-depth analysis on workbooks, as they are the most dominant materials used in primary classrooms in South Africa. For the Assessments review, three assessments were identified by the working group to represent a cross section of assessment approaches and purposes. The LTSM research was extended further with a cross sectional analysis of one topic – whole number division – across the workbooks from Grade R to 6 to analyse a particular conceptual thread across the 3 selected LTSM offerings.

The assessment in-depth study gives an detailed account of what the purpose of the assessment is, the theoretical underpinnings that characterize the assessment, empirical evidence on the validity and credibility of the instrument, how the instrument has been used and includes excerpts of the structure and administration protocols of the assessment instrument. The key observations emerging from the assessment review were:

- **No evidence of perceived "over-assessment**" of the national system at primary school level since the discontinuation of the Annual National Assessments (ANA). However, there was some concern about volume of assessments administered at provincial and school level.
- The **DBE Systemic Evaluation** process and instrumentation is applauded. Most of the other assessment instruments that have been developed are for Assessment of Learning purposes and are linked to relevant LTSM.

- Concern relating to compliance requirements in relation to the **SA-SAMS capture of mathematics assessments**. This has resulted in assessment of all 5 content areas in every term in every grade. Using "-1" allows to exclude an SA-SAMS field.
- Evidence of **a range of investments,** which are largely uncoordinated initiatives in instrument design, to **develop validated assessment instruments** for various purposes (Marco-D, EGMA, standardized CAPS tests). Lack of mathematics benchmarks/norms.

The learning briefs – for primary mathematics LTSM and assessment – offer a high level synopsis of critical features of each offering. They are intended to offer a 'menu', for making decisions about available LTSM suitable to a particular context, and available assessments suitable for a particular purpose. Further, they provide an indication of the different types of materials and assessments which have been developed, and make clear the areas requiring further investment to further improve these products. The key observations emerging from the LTSM review were:

- The DBE learner workbook is welcomed and available in schools as a primary resource. It was developed as a supplementary resource but is being as a primary resource.
- Other mathematics LTSM has been developed (some of which have shown impact on learning outcomes) and impact evaluations are occurring (eg GPLMS, TMU pilot, NECT pilot, NumberSense, Bala Wande, NMI).
- Therefore, there is duplication of resources. Budget and printing is required for the DBE workbooks as well as the particular Learner Activity Book.

Therefore, the DBE should completely redesign the DBE workbook to be the primary LTSM resource (taking into account the lessons from GPLMS, NECT, TMU pilot and interventions which have evidence of impact). The redesigned DBE workbook set should comprise of a learner activity book, teacher guide and mathematics kit. The **learner activity book** must include:

- **A rhythm of engagement:** Term by term and week by week Learner Activity Book and teacher guidance;
- Carefully designed **conceptual threads** (learning trajectories for each topic or content area);
- A **limited number of key representations** which are systematically developed across each phase. Carefully selected "go-to" strategies that develop number sense and build towards flexibility;
- **Support for bilingual and multilingual learning** e.g. Let's talk maths sections, bilingual assessment elements, multilingual dictionaries for teachers; and,
- **Instructional prompts for teachers** in the LAB (through **worked examples** and instructional signaling) and detailed pedagogical support in the **teacher guide.**

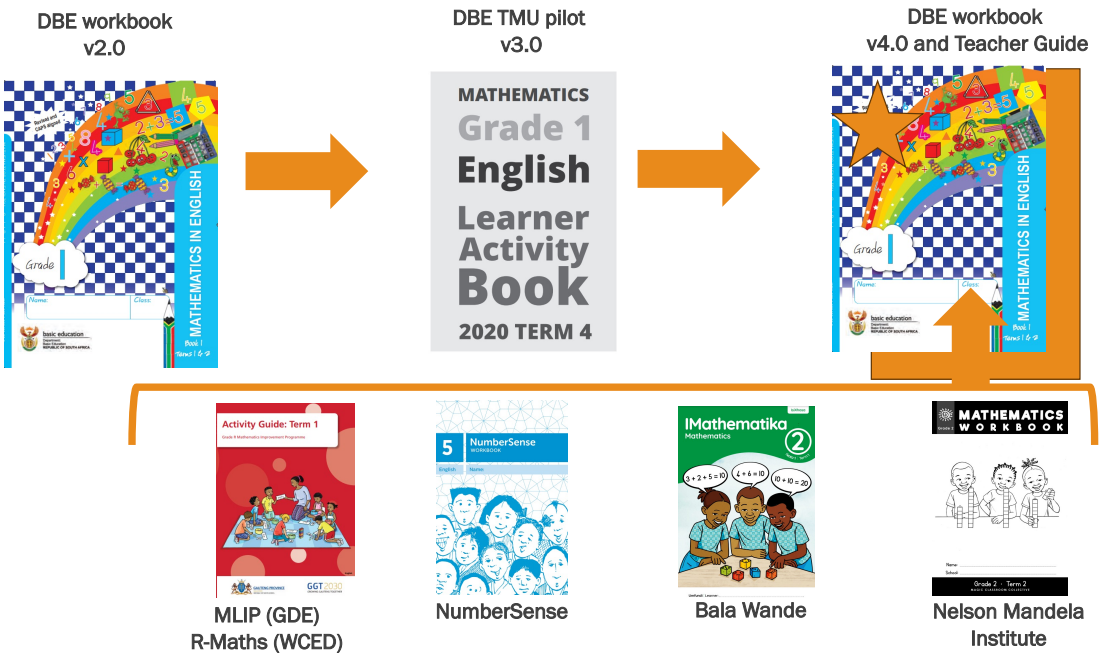


Figure 2: Workbook redesign

The **teacher guide** must provide additional pedagogical guidance, termly conceptual checkpoints, with Formal Assessment Tasks for teacher administration and marking. Recommendations for the minimum requirements for **mathematics kits** – specific to each grade level – are provided.

When looking across the assessment review and the LTSM review, several general observations stand out:

- The DBE workbook represents and major positive shift in South African schooling;
- Too many mathematics interventions can distract teachers and instructional leaders with completing messages;
- The last decade has seen significant advances in how to work to improve mathematics learning outcomes at school in primary schools;
- Following significant curriculum change in early democratic period, there has been decades of stability. Curriculum reviews tend to be seen as "disruptions" to the system, or emergency responses (e.g. to Covid) rather than inevitable curriculum strengthening processes. There is related a lack of alignment between mathematics Curriculum, LTSM and assessment instruments.

In response to these general findings, it is recommendation that the DBE publish and invest in predictable and well-planned cycles of revision for curriculum, assessment and LTSM in 5 and 10 year cycles. It is further recommended that these revision cycles be aligned with curriculum reviews of operational cycles.

These revision cycles must be given substantive time and involve detailed research work (and not be short term responses to immediate threats). The predictable cycles of curriculum-assessment-LTSM strengthening and review, will allow time to harvest the lessons emerging from evidence of successful implementation in schools. Alignment between curriculum, assessment and LTSM that is informed by

evidence of what works is critical. For this to take place in predictable cycle (which are not threatening as disruptive, as there is inevitable strengthening of the existing approaches). Decisions must be informed by:

- Assessment data generated through systemic evaluations and other validated instruments; and
- Impact evaluation findings (generated through research and innovation interventions on curriculum-assessment-LTSM design, and ongoing teacher development).

To ensure the stability of primary school system, carefully planned and coordinated changes to curriculum, LTSM, and assessment are recommended for incremental improvement to take place. A goal of **90% stability to 10% research and innovation** is envisaged. This would mean that each school engages with only one LTSM pack (Learner activity book, teacher guide and mathematics kit) for a 5-to-10-year period. 90% of schools use the National DBE offering (the revised DBE workbook described above), while up to 10% of schools participate in research and innovation studies. Such innovation studies are designed to test curriculum, assessment and LTSM innovations, which If improved learning outcomes are not evident after 3 years, the study is stopped. If improved learning outcomes are evident, the study continues, and its findings inform the next curriculum-assessment-LTSM review cycle.

In sum, the LTSM and assessment reviews found that there is:

- **Much positive work on LTSM and assessment from the DBE** (workbook, NECT LABS, Revised trackers, TMU framework, TMU pilot, systemic evaluation, AfL practices)
- **Much positive work and research in the mathematics eco-system** (GPLMS, NumberSense, MCC, Bala Wande, Numeracy chairs, NMI FATs, EGMA etc.)

Now the time is ripe to bring these together in coordinated and predictable **5- and 10-year cycles of curriculum, assessment and LTSM review**, that draw on local evidence of impact. As such it is encourage a **90-10 split of stability and innovation**, drawing on the research lessons and commissioned research on mathematics conceptual threads and language issues.

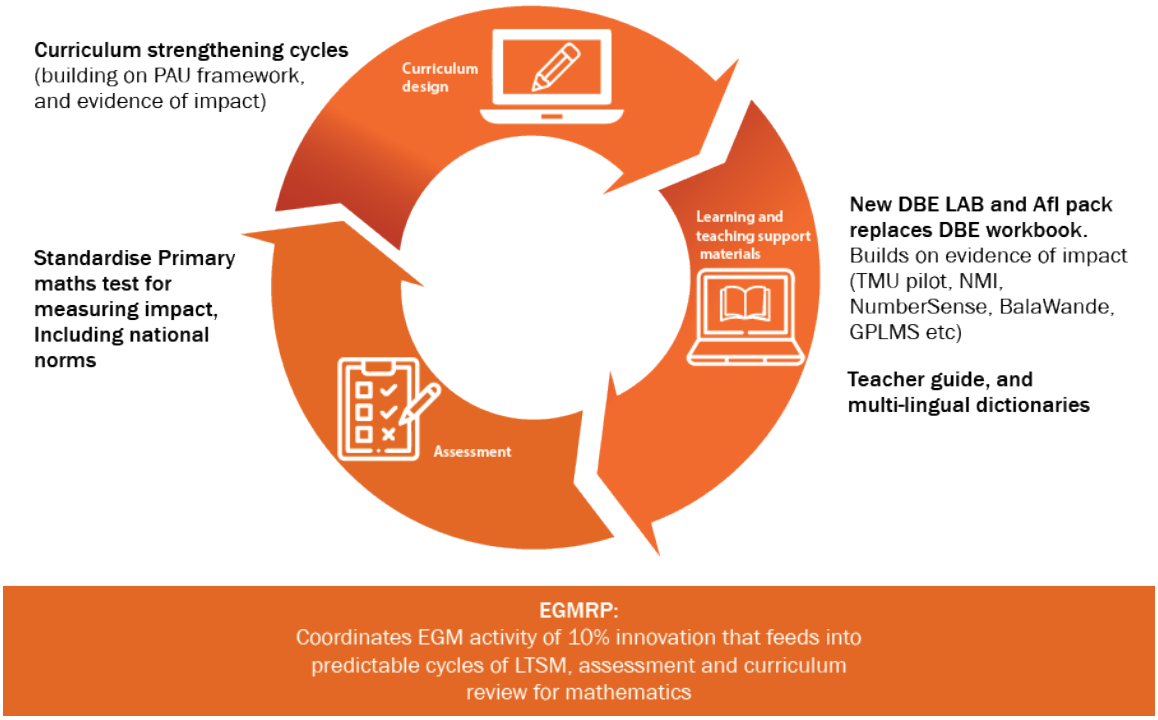


Figure 3: Curriculum-LTSM-Assessment Cycle

The proposed next steps for the EGMRP project are organised in terms of issues relating to assessment; issues relating to LTSM, and issues informing both.

Issues on Assessment:

1. Publish the **Systemic Evaluation findings** and ensure these findings feed into curriculum and LTSM processes, so there is a feedback loop into curriculum, LTSM and teacher development (90% stability).
2. Create and maintain a repository of quality **mathematics formal assessment tasks** (such as the NMI FATS) (90% stability). Use the learning briefs and assessment evaluation framework from this review.
3. Ensure that schools are not assessing every mathematics topic every term by publicising use of **SA-SAMS** relating to use of "-1" to exclude a topic in assessment data capture (90% stability).
4. Support/commission the design of **validated mathematics assessment instruments to measure impact** (building on the EGMA, but expanding this for Grades 3, 6 and 9) (10% innovation). **Establish national norms/benchmarks.**
5. Evaluate the impact of **M-SAP intervention at Grade 3,** and if evidence is positive, expand to include Grades 1&2.

Issues on LTSM:

6. **Extend and fully update the DBE workbook (v 4.0) to be a Learner Activity Book and Assessment for Learning pack.** Shift it from a supplementary resource to be THE primary printed resource (90% stability). Include multi-lingual dictionaries and teacher guides (90% stability), which could be provided online.
7. Complete the **impact evaluation (qualitative and quantitative) for the TMU pilot** (v3.0). Use these findings to inform the DBE workbook (v.4.0) (10% innovation). Collect findings and/or commission **impact evaluations** of mathematics LTSM and training interventions showing promise, at district-level scale[i] (10% innovation).
8. Collect findings and/or commission **impact evaluations** of maths LTSM and training interventions showing promise, at district-level scale[1] (10% innovation).
9. Encourage use of the **LTSM evaluation framework developed** for this review (for new and updated versions of maths LTSM) (10% innovation).
10. Extend this LTSM review to include a review of mathematics textbooks – particularly in the Intermediate Phase.

Issues informing curriculum, LTSM and assessment:

11. Commission and support research on mathematics **conceptual threads** (learning trajectories) relating to a limited number of representations, 'go-to' calculation strategies, geometry and measurement. This should extend and support the curriculum strengthening work. Trial and research innovations in schools (10% innovation)
12. Commission and support the development of communities of practice and research on the **expression of mathematics in African languages** (bi-lingual and/or multilingual) **LTSM and teacher training**. This should extend and support the work on **mother tongue instruction in mathematics**. Trial and research innovations in schools (10% innovation)
13. Continue the **quarterly EGMRP activities** (incl indabas) to ensure that DBE:
    a. leads the research agenda;
    b. maintains 90% stability; and
    c. responds to the 10% innovation which then informs the evidence informed cycles of curriculum, assessment and LTSM review.

[1]Bala Wande (RCT is underway), and GDE MLIP (cohort evaluation study is underway), NumberSense, JumpStart, Nelson Mandela Institute, and M-SAP are ready for impact evaluations).

Research design

The research team put forward a research design which was refined through engagement with the participating parties.

Data collection

The research drawing upon four bodies of evidence was collected through the process of the review:

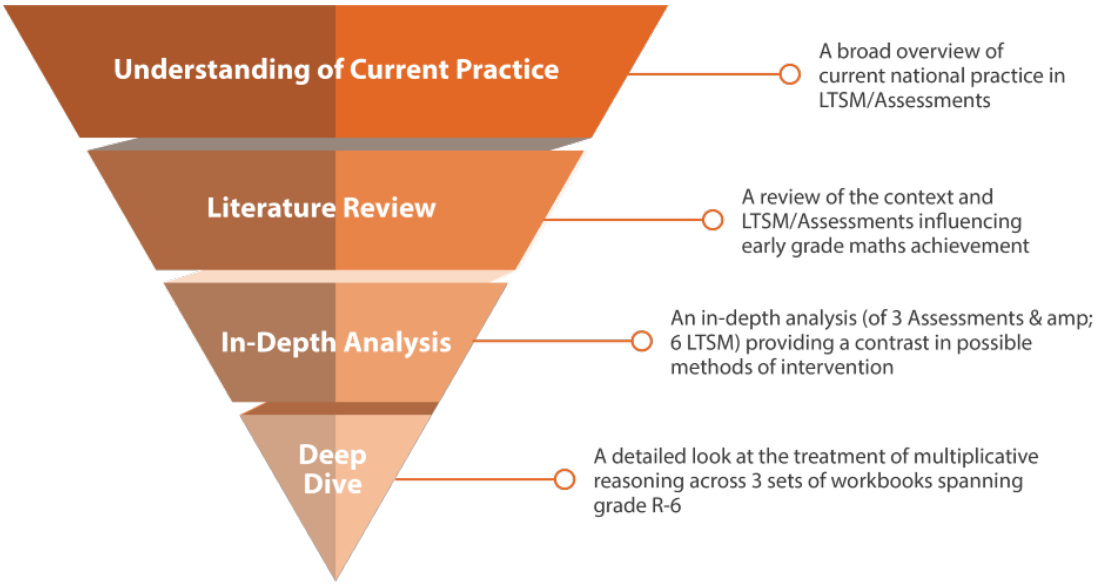


Figure 4: The four processes undertaken as part of the LTSM and Assessment reviews

While the first three elements relate to both LTSM and Assessments, the final element focused specifically on LTSM.

Consultation processes

The research team were supported in various ways to guide the enquiry and serve as validation relating to current practice. A project management team was convened by the DBE, which includes high level representatives, the DBE, Zenex Foundation, provincial departments of education and other key stakeholders involved in LTSM and Assessment.

A group of South African primary mathematics experts participated in three workshops as well as the final review of research outcomes. To ensure the inclusion of a voice from teachers, two surveys (one focusing on LTSM and the other focusing on assessment) were administered at the 2022 congress of the Association for Mathematics Educators of South Africa (AMESA) congress. The contributing parties assisted in ensuring that the current practice was well understood, and in defining selection criteria for the in-depth analyses.

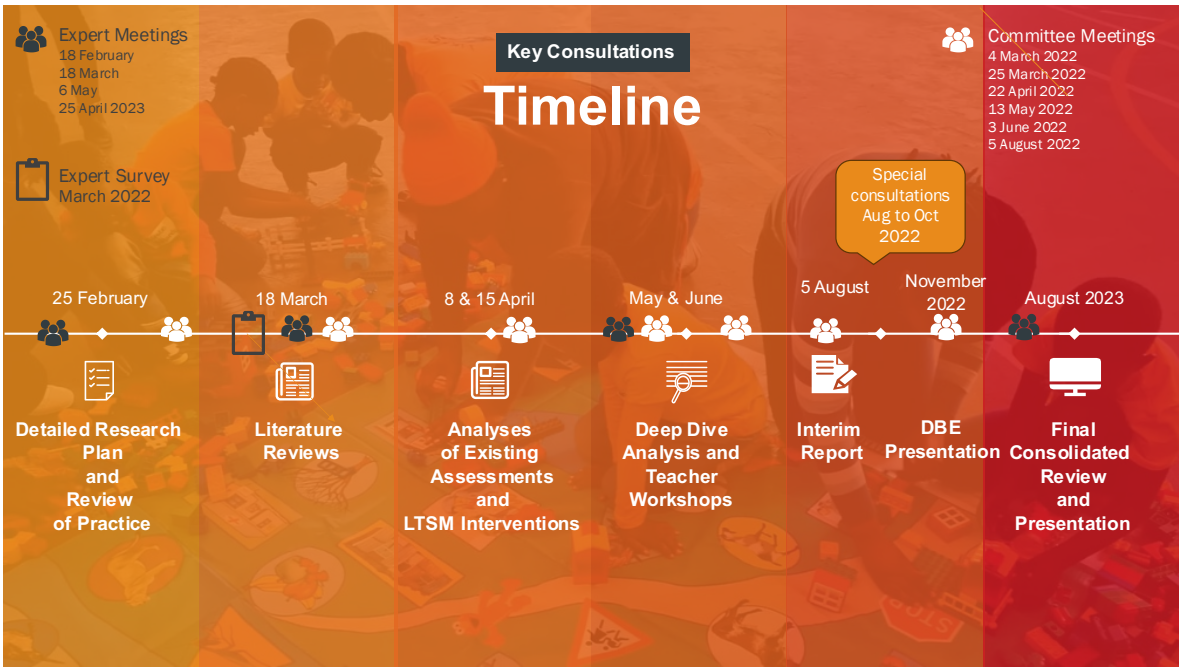


Figure 5: The consultations undertaken during the reviews

Selections for in-depth study

LTSM

A number of considerations were involved in the selection of six LTSM for the in-depth study. These considerations were agreed upon in consultation with the project management team. Primarily we were interested in primary level mathematics LTSM that have been designed for mainstream South African schools, specifically in no fee schools of Quintiles 1-3. We were also interested in early grade mathematics materials which have been developed in African Languages. We excluded LTSM which were designed for commercial gain or only available exclusively in English or Afrikaans. Instead, we focused on multilingual materials developed by the government, university, and NGO sectors.

Within these sectors, we sought to represent both materials already distributed at larger scale as well as smaller scale, research-lead experiments as well as materials developed by NGOs that might be scalable. Workbooks were selected as the unit of analysis, as they are the most used resources in early grade mathematics classrooms. Textbooks, particularly in grades R-3, are not widely used in Foundation Phase, where children are still learning to read and write.

A short list was presented to both the expert panel and the working group. A final selection was made by the working group. Resources and time constraints played a role in the number of items selected.

The final selection of LTSM for the In-Depth Study included:

1. DBE Workbooks
2. Teaching Mathematics for Understanding (TMU) Pilot

3. Magic Classrooms Collective (MCC)
4. Primary Mathematics Research Project (MPRP)
5. NumberSense
6. Bala Wande

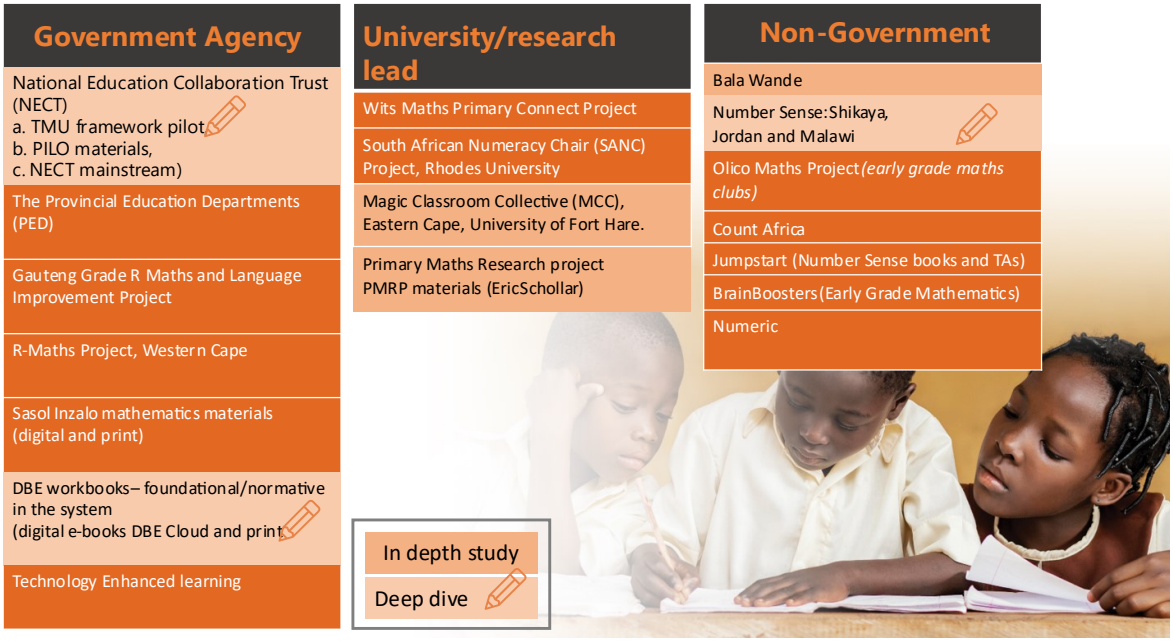


Figure 6: Primary mathematics LTSM shortlist and LTSM selected for detailed enquiries

DBE Workbooks and the TMU framework represent the Government led LTSMs distributed at national level; MCC and PMRP represent the small-scale researcher led experiments, and NumberSense and Bala Wande represent the non-government agency produced LTSMs that have the potential to be upscaled. The review was based on secondary documents, including journal articles and evaluation (internal and external) based on the LTSM. The review included and particularly focused on the design principles of the LTSM, distribution and uptake in schools, cost effectiveness, use in multilingual settings, teacher support regarding the use of the LTSM, mathematical manipulatives and possibility for future investment in online learning.

To complement the LTSM in-depth study, the working group decided to select three of the LTSM included in the in-depth study for a more detailed analysis of the progress of a single mathematical concept as it unfolded in workbooks from Grade R through to Grade 6. The three LTSM selected for further analysis included: DBE Workbooks, TMU Pilot from the NECT, and NumberSense. The selection process, again, was made in consultation with the working group and took into consideration:

- the **scope** of the materials (priority being given to materials that cover the whole Grade R to 6 range);
- the **distribution** of the materials (the DBE workbooks being used nationally, the TMU Pilot being tested for national distribution, and the NumberSense providing a counter point with materials widely distributed); and,

- the **relevance** of the materials (materials in use and with plans to continue use for the foreseeable future).

The mathematical concept selected for the deep dive into these three sets of workbooks was the treatment and progression of multiplicative reasoning, specifically whole number division calculations. This concept was selected for two reasons: first, it complemented the significant research work already conducted on additive relations; and second, success in multiplicative reasoning at a formative level is a strong indicator of success in mathematics at a higher level. Analysis of each workbook involved collating all the instances where whole number division calculations were present and noting:

- objectives/outcomes stated,
- concept development using sharing and/or grouping and strategies encouraged for calculation procedures,
- screenshots of related questions and,
- representations.

To augment the development of these documents, the researchers conducted two surveys with teachers who attended workshops organized through the Association for Mathematics Educators of South Africa (AMESA) in which teachers were introduced to specific LTSM and Assessments.

Assessments

In relation to assessments, the DBE recognised that various kinds of assessment are used in South Africa: school-based assessment, local systemic assessments, and international assessments. The quality of school-based assessment has been identified as an area of concern, and their utility for both formative and summative assessment purposes was noted. It is at this level that evidence of learning is collected about individual learners, which ought to guide further teaching. The broader assessments which are administered across a range of schools (district, provincial and national assessments) are intended to provide information about system functioning, an as an accountability mechanism for educators and school principals.

As with the selection of LTSM for in-depth study, assessments were excluded which were designed for economic gain and therefore out of the reach of the majority of schools. The working group agreed to exclude systemic assessments as they have already received substantial attention. Therefore, it was decided to focus the in-depth study on the classroom level assessments designed for summative and formative analysis.

1. **Early Grade Mathematics Assessment EGMA**
   a) EGMA Junior: Grade 1-3
   b) EGMA Senior: Grade 4-6
   c) EGMA One: Grade R-1
2. **Mental Starters Assessment Project (MSAP):** Grade 3
3. **The Magic Classroom Collective:** Formal Assessment Tasks (FATS)
4. **JET Grade Mathematics tests:** Grade R-6
5. **The Marko-D SA test:** Early childhood to Grade R
6. **ELOM:** WCED R-Math/GDE: Grade R
7. **Western Cape Systemic Evaluations:** Grade 3 & 6
8. **Number Sense:** Grade level Formal Assessment Tasks (FATS)
9. **DBE diagnostic assessment question banks** (Grade 3 and 6)
10. **Singaporean primary mathematics tests:** Tightly linked to LTSM
11. **Research-based Early Mathematics Assessment (REMA):** Grade R -1

Figure 7: Primary mathematics assessments shortlist and assessments selected for detailed enquiry

The working group identified three assessments for in-depth study to represent a range of approaches (see Figure 7 above). The choice of the assessments was informed by a judicious consideration of the purpose that each assessment serves as well as the space that they collectively occupy in the assessment landscape.

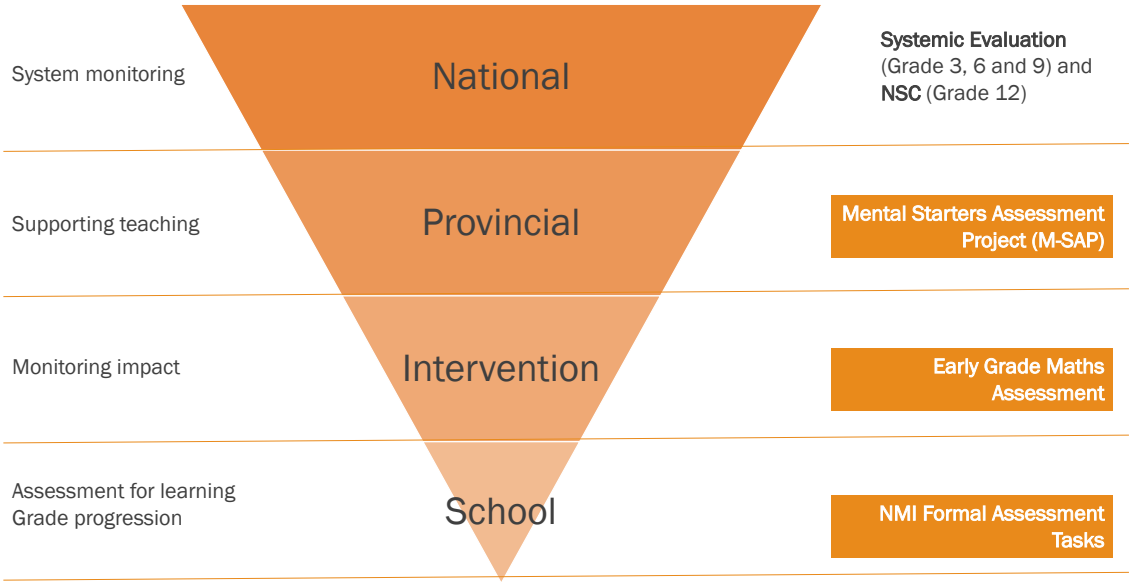


Figure 8: Assessment for different purposes at various levels of the school system

On the one hand, the EGMA has been designed to assess a narrower range of the mathematical content areas and is used mainly for the evaluation of the impact of educational interventions at system level, specifically at district as well as project levels in South Africa. On the other hand, the MCC FATs have been designed to assess all content topics in the curriculum and are thus used as curriculum-based measures (CBM). Unlike the EGMA and the MCC FATs, the MSAP is not a stand-alone tool but an assessment-driven approach to professional (pedagogical) development of teachers. So, while the EGMA and the MCC FATs provide important summative information for measuring changes at system and curriculum levels, respectively, the MSAP provides information of a formative nature for immediate use to leverage change at the classroom level.

LTSM and assessment quality standards

The United Nations Sustainable Development Goal (SDG) 4 stipulates quality education as a reality for all by 2030 (Education 2030, 2016). Quality Learning and Teaching Source Materials (LTSM) and assessment are essential for achieving SDG 4. Ensuring that every institution has appropriate learning materials and assessment processes is a key strategy for reaching the targets of SDG 4. Schools should be adequately resourced with books, relevant learning materials, open educational resources and technology that are non-discriminatory, learning conducive, learner friendly, context specific, cost effective and available to all learners – children, youth and adults' (Education 2030, 2016: 33). Through SDG 4, the United Nations (UN) highlights the importance of inclusivity and equitability in addition to improved literacy levels. Thus, equitable and inclusive education for all should promote lifelong learning and the urge to gather more knowledge. Similarly, the Department of Basic Education (DBE); through its strategic plan for 2030, related annual plans and Theory of Change, has identified its principles of quality education that place learners as the key focus of such an education.

Reflections of the quality of LTSM and assessment in South Africa are part of considerations for quality education in general. First, we provide a broad overview of the principles underpinning quality education, which the Department of Basic Education has developed for virtual schooling. These are considered relevant to these reviews, as virtual schooling necessarily requires excellence in LTSM and assessment practices with these being the primary ways in which curriculum is communicated to learners. In the face-to-face context the LTSM and assessment is significantly supported by teachers and instructional leaders. The quality of the LTSM and assessment in the ordinary schooling context remains central to learning opportunities offered. We then make clear how LTSM and assessment are seen as an integral part of the DBE's theory of change for improving learning outcomes in South Africa. Once again the human actors – learners and teachers – are central, but the curriculum, and related assessment and LTSM remain key ingredients. This is followed a brief explanation of what is envisaged to be quality assessment and quality LTSM. Once again we draw from the existing work of the DBE, in its definition of quality standards for assessment and LTSM for virtual schools. Together, these statements of quality narrow down from the broad vision of quality education, and the theory of change for education, to the specific expectations for quality LTSM and assessment. When investing in and selecting LTSM and assessment options, these decisions ought to be considered in relation to the broader context of quality education.

Principles underpinning quality education

There are important principles underpinning quality basic education in South Africa. The principles may be used to inform decision making, and as a basis for reviewing already existing LTSM and assessment. Figure 9 illustrates the DBE principles underpinning quality education (as originally designed for virtual schools and technology enhanced learning).

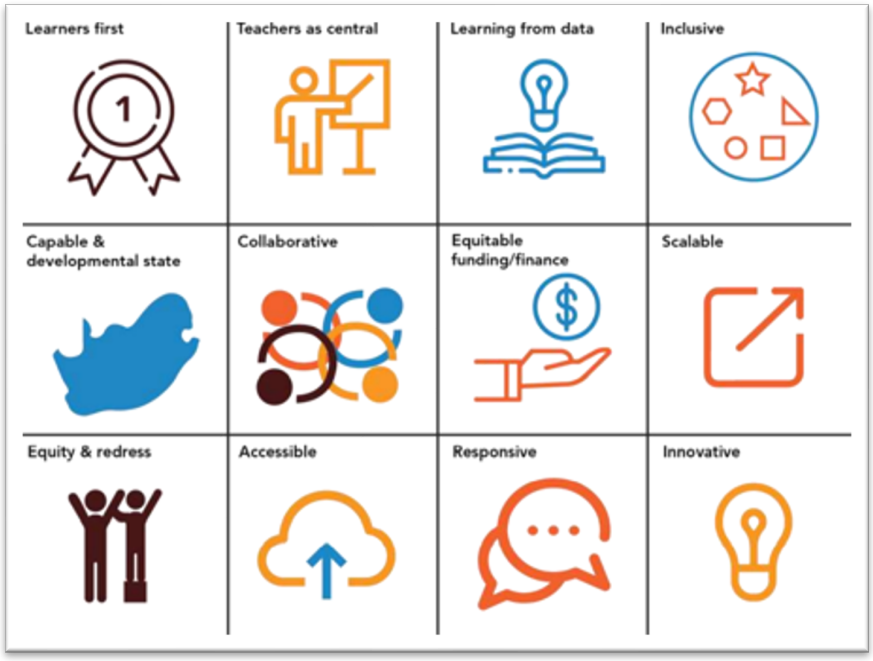


Figure 9: Principles underpinning quality education (which can be applied to LTSM and assessments)

According to these principles, education (including LTSM and assessment) should prioritize learners while teachers remain as central agents in mediating the content developed. Education should be informed by research and include different ability learners. Instead of working in silos, Government departments at national and provincial level need to form strategic collaborations with universities and development agencies to pull together the available expertise and already developed content. Such collaborations would enable developers to specialize in the specific content or assessment they are tasked to develop. Educational offerings (including LTSM and assessment) should be cost effective. LTSM and assessment should be responsive to end user recommendations; these should be incorporated in the periodical reviews. Education should leverage on the innovation opportunities so that the material fits the needs of the 21st century classrooms, which includes effective use of technology to enhance learning.

DBE Theory of change: the role of LTSM and assessment

LTSM and assessment in South African fit within a broader theory of change outlined by the Department of Basic Education. The DBE's Theory of Change serves to highlight the critical areas to focus on in the planning phases of LTSM development initiatives. The Theory of Change is a simplified tool used to explain important processes that facilitate educational change, and how better quality

education will be attained. As illustrated in Figure 10, the Theory of Change presents the importance of improving learning gains and ultimately improving learners' opportunities in life through the acquisition of literacy and mathematics competencies, and beyond that, specific subject knowledge and life skills.

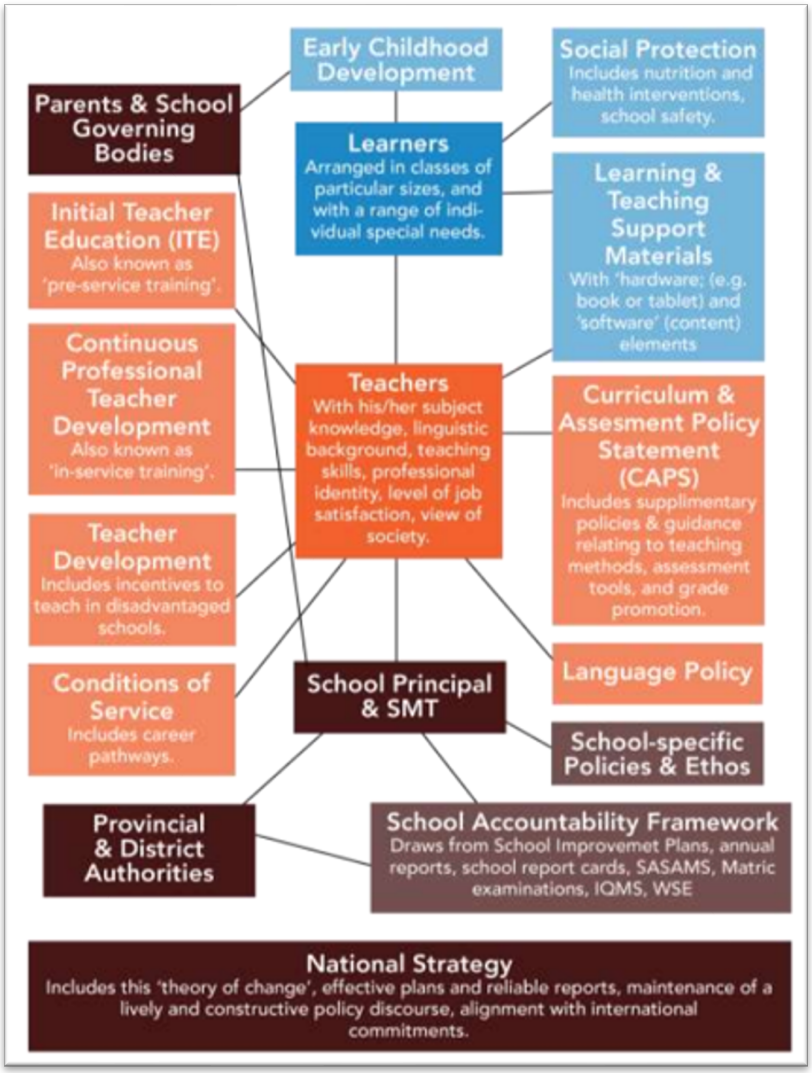


Figure 10: Theory of Change (Department of Basic Education)

The theory of change places great emphasis on school safety, health and nutrition. The learning abilities of learners from poor socio-economic backgrounds will be impacted positively by a nutritional meal provided by the school. Learners will benefit from quality LTSM if (primary mathematics) content is well taught and most importantly if they have participated in Early Childhood Development (ECD) initiatives. Furthermore, the Theory of Change prioritizes health and psychosocial interventions at primary school level because if psychosocial issues remain unresolved, learners may suffer from long-term learning impairments. Before learners access quality primary mathematics LTSM, their social needs should be addressed, right from the reception years.

The Theory of Change also implies that at primary school level, learning gains are influenced by the teacher's capabilities, the availability of LTSM, and class size. Teachers' capabilities are influenced by both the pre- and in-service training they receive. While pre-service training provides the knowledge needed for effective teaching, 66% of the teachers working in public schools received their Initial Teacher Education (ITE) before 2000 (DBE, 2020). From 2038, all teachers practicing in public schools will be expected to have received the updated teaching methods and technologies. It is therefore important to offer in-service training to enable teachers to continuously update their skills. LTSM development initiatives may leverage on these in-service training interventions to offer the required teacher support on implementing the strategies suggested in the LTSM.

The availability of effective LTSMs relies on investments in developing quality material, funding, and distribution systems to get the material to the learners. LTSM development initiatives should ensure that the content produced is effective in different linguistic, socio-economic and cultural contexts. The positive impacts of LTSMs are likely to be enhanced if learners can take materials home. Regarding language, LTSM development should facilitate a better transition from mother tongue education to the prescribed Language of Learning and Teaching (LoLT), by improving the teaching of English as a First Additional Language (EFAL) in the Foundation Phase, and most importantly by improving the teaching of the mother tongue at Foundation Phase (FP) level. A firm foundation in the mother tongue improves chances of mastering additional languages. Quality basic education will enable the country to measure its progress against SDG 4 indicators on the attainment of literacy and mathematics at primary school level.

DBE quality standards

To accompany the principles underpinning quality education, the DBE has developed quality standards as outlined in Figure 11. The quality standards put forward by the DBE (originally for virtual schooling and technology enhanced learning) make clear the key components which need to be reflected on for any assessment of quality schooling. The interconnection between curriculum design, LTSM, curriculum delivery and assessment is also evident. Curriculum informs both the LTSM and assessment. How curriculum is delivered (by each school, each teacher, and communicated in LTSM and assessment) impacts on effectiveness. In order to achieve quality education for all learners, all links of the LTSM chain: definition, design, creation, development, production, distribution, storage, and classroom usage must be carefully considered (Read, 2015).

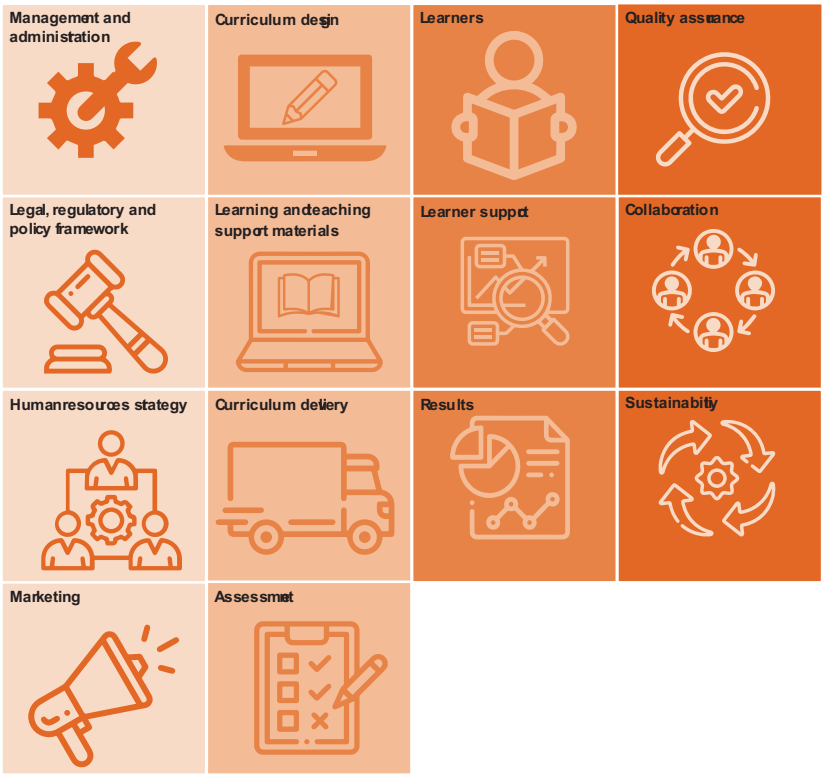


Figure 11: Quality standards for schools (Department of Basic Education)

While each component in Figure 11 has an articulated quality standard, we expand here only on the specific quality standards for LTSM and assessment in column two.

Quality standard: Learning and teaching support material

LTSM broadly refers to a variety of educational resources, which are necessary to enhance teaching and the understanding of the subject content while aiding teaching and learning (Modisaotsile, 2012; DBE, 2012). LTSM facilitate teaching and learning and are interchangeably known as: 'instructional materials', 'educational materials' or 'curriculum materials'. Textbooks, workbooks and teacher guides are considered as the key LTSM components required for curriculum implementation.

Primary mathematics LTSM are critical to learner success and must be designed in a way that maintains the learners' interest to keep them motivated and continue learning towards success. The

LTSM must therefore be presented in an accessible and coherent way, bearing in mind the role of the teacher in mediating the learning process. Principles of the universal design for learning should be adhered to when developing or when selecting previously developed materials.

Elements of the LTSM quality standard

1. Content is aligned to the learning outcomes of the curriculum and is fit for the delivery modality;
2. Learning materials specify the outcomes, how learners can navigate the materials and include self-assessment by learners;
3. Assessment is used to determine whether learners should progress with their learning or whether they need to revise the learning content;
4. The notional hours for each learning area provide guidance to learners on how long they can take with each chunk of learning, although there should be flexibility to accommodate differentiated learner pacing;
5. Supplemental learning resources are provided for learners who want to explore beyond the immediate course requirements;
6. The content of materials is up to date and accurate, does not promote prejudice, and is sensitive to diverse cultures;
7. The materials are engaging so that they motivate learners who are learning on their own;.
8. Online LTSM (at Intermediate Phase level) are tested for heuristics and usability and for their suitability for use on multiple devices.

We proposed the inclusion of one additional standard:

9. LTSM is available in all South African languages of teaching and learning, and bilingual and multilingual competencies are explicitly developed.

While textbooks, workbooks and teacher guides are considered as the key LTSM components, this analysis focuses on the workbooks. Learner workbooks contain guidelines for learners to carry out work or programmed assignments. The workbooks are also meant to assist teachers track learner progress and provide extra support where necessary. Workbooks are a hybrid between a textbook and an exercise book; providing learners with opportunities to complete activities in the books themselves. (Taylor et al, 2017). Thus, workbooks are a simple way to structure learning activities for learners. The section below focuses on the emerging recommendations for the development of quality mathematics learner workbooks

Quality standard: Assessment

To comply with the requirement for an integrated education system, curriculum and assessment must be in line with the provisions of Section 6A “Curriculum and Assessment” of SASA (Act 84 of 1996). Assessments, being an integral part of the teaching and learning process should be properly managed. The assessment must reflect the current national standards of assessment. The course content, assessment questions and aims of the course must all be aligned. To ensure the integrity of these

assessments are maintained, there must be security, rules and regulations set in place. There must be a credible system that regulates the internal and external moderation processes during assessments.

Elements of the Assessment standard

1. Assessment is recognized as a key motivator of learning and an integral part of the teaching and learning process. It is used to inform teaching practice and improve the curriculum.
2. Assessment information (including learning outcomes, assessment criteria as well as assessment procedures and dates) is provided in all courses, modules or topics.
3. The level of challenge of the assessment in a curriculum is appropriate for the grade level and phase of schooling.
4. There is a range of formative and summative assessment tasks and methods which ensure that all learning outcomes are validly assessed.
5. Assessment is designed in terms of predetermined outcomes and criteria.
6. A range of parties is involved in the assessment of learners: for example, there might be self-assessment, peer assessment, or teacher assessment.
7. For each curriculum, there is at least one integrated assessment procedure which is a valid test of the key purposes of the curriculum.
8. There are effective procedures for recognizing prior learning and for assessing current competence.
9. Care is taken that the assessment activities are designed and administered in ways that do not disadvantage learners in a range of contexts.

We proposed the inclusion of one additional standard:

10. Language of assessment is consistent with terminology and language of teaching and learning, with consideration given to bilingual or multilingual competencies.

South African context

Since 1994, the Department of Education (DBE) has implemented a number of policies, including pro-poor initiatives to promote equity and improve the quality of education in previously disadvantaged schools (Visser, 2018). The initiatives include the introduction and implementation of the following policy documents:

- National Curriculum and Assessment Policy Statement (CAPS) (2012);
- Teaching Mathematics with Understanding (TMU) (2018); and
- The policy on Learning and Teaching Support Material (LTSM) that supports the provision of core LTSM to schools.

Although the implementation of these policies was meant to address issues related to the provision of quality education, the gap between policy and practice remains wide in many parts of the country.

Focusing more specifically on the last decade, the field of primary mathematics in the arena of South African basic education has undergone radical changes in terms of curriculum consolidation, a decisive focus on clarifying subject content to be taught, crystallising how LTSM is produced and how

assessment is to be conducted. It has come to emphasise a learner-centred approach to teaching and learning in all subjects, and particularly in the teaching and learning of early grade mathematics.

The introduction of the Curriculum and Assessment Policy Statement (CAPS) is premised on the understanding that assessment is integral to effective teaching and meaningful learning. Assessment is critical to directing not only what is taught and learnt in the classrooms, but also to providing information that the education system requires for setting evidence-based targets and providing appropriate educational resources. For instance, almost half of the targets that the DBE has set in the *Schooling 2024 towards 2030* strategic plan are measured in terms of the results of standardised national and international assessments. The assumption and expectation is that the results are credible and are based on valid and reliable instruments.

Primary mathematics LTSM in South Africa, 2010 to 2022

Learner Teacher Support Materials (LTSM) is a broad term that includes a variety of educational resources, which are necessary to enhance teaching and the understanding of the subject content while aiding teaching and learning (Modisaotsile, 2012). LTSM facilitates teaching and learning in schools, including materials for learners with special educational needs (DBE, 2012). These materials are also interchangeably known as: “instructional materials,” “educational materials,” or “curriculum materials”.

Quality LTSM are crucial for achieving the Sustainable Development Goal (SDG) 4. Ensuring that every institution has appropriate learning materials and instructional technology is a key strategy for reaching the targets of SGD4. According to the Education 2030 Framework for Action, schools should be adequately resourced, with books, other learning materials, open educational resources and technology that are non-discriminatory, learning conducive, learner friendly, context specific, cost effective and available to all learners – children, youth, and adults’ (Education 2030, 2016: 33).

Effectiveness of resources for mathematical learning lies in their use in the classroom teaching and learning context; (Adler, 2000). Kruger (2003) states that commitment to the teaching and learning process and provision of relevant resources contribute positively towards the achievement of quality education. In schools where quality learning and teaching prevails, learners become independent, positive, and active in their own learning.

In order to achieve quality education for all learners, all links of the LTSM chain – including definition, design, creation, development, production, distribution, storage, and classroom usage – must be carefully considered (Read, 2015). The LTSM are used by teachers and learners in the teaching and learning process, to make the process more effective and productive. They comprise of textbooks or learner books, workbooks, and teacher guides – all considered important for the implementation of the curriculum. Textbooks, workbooks, and teacher guides are core LTSM required for curriculum coverage. The South African government has made ambitious promises about textbook provisioning (Zuze, 2018). Recent policies established by the DBE have meant to address issues related to the provision of quality education, however, the gap between policy and practice remains wide in many parts of the country.

Types of LTSM

For the purposes of this review, we have identified seven types of LTSM that are particularly relevant to primary mathematics in South Africa:

Textbooks

Textbooks are the most visible aspects of a curriculum and are often considered the main script that shapes the teaching and learning processes (UNESCO, 2017). Quality textbook development and provision involves four main steps: development (based on curricular frameworks); procurement systems (state or private sector, approved textbooks list); distribution and access (arrival in schools, issuance to learners); and storage and conservation. (UNESCO, 2015)

The DBE School Monitoring Survey (2018) defines a textbook as "a published book in which learners should not be writing." Since 2011, South African schools have been resourced to procure textbooks. The DBE is currently working towards providing one textbook per learner per subject in every school; however, the process of procurement and delivering these materials varies from province to province. Where textbooks of the appropriate quality are not provided, systemic inequalities and social exclusion are perpetuated (Glewwe, Kremer, & Moulin, 2007).

Workbooks (Learner Activity Books)

Workbooks contain exercises, often with spaces for learners to write answers, to help them practice what they have learnt. The assessment activities in the workbook assess aspects of all the elements, performance criteria, skills and knowledge and performance requirements of the content area. To demonstrate competence in the content, a learner must undertake all activities (formative and summative) in the workbook and have the learner's work deemed satisfactory by the teacher. In South Africa, learner workbooks for mathematics are distributed at no cost, and are meant to be used as a supplementary resource.

Teacher guides

Teacher guides support teachers in their teaching practices. Effective teachers' guides should: contain explicit communication of conceptual goals with direct links to proposed activities, provide knowledge and support to help understand and implement teaching plans, reinforce pedagogical content knowledge, give guidance on the practice and understanding of relevant pedagogical activities, present alternatives and freedom of choice, and engage teachers in ongoing reflection, (UNESCO, 2015). For inexperienced teachers, teacher guides are intended to compensate for the missing knowledge/practice.

Lesson plans

Lesson plans is the title given to a statement of the achievements to be realized and the specific means by which these are to be attained as a result of the activities engaged in day-by-day under the guidance of the teacher. A lesson plan focuses the teacher's attention on:

- Outcomes or results in terms of the learner;
- Definite processes and procedures with recognition of activity as the basis of learning;
- Appropriate learning activities and strategies to obtain feedback on learner achievement; and
- The learner in the foreground and the teacher in the background as guide and director only of the learning activity. (Ambrose, Bridges, Lovett, DiPietro & Norman, 2010)

Lesson plans include pedagogical aspects to help the teacher manage the classroom and to transmit knowledge as efficiently as possible.

Multimedia and digital resources

Multimedia and digital resources include electronic LTSM such as e-textbooks (GDE, 2012). According to UNESCO (2015), these are a growing source of knowledge for teachers and learners. Several studies show that greater access to information and communication technologies in schools can help reduce the digital divide between low- and high-income groups (UNESCO, 2014a; Jacob, 2016). The digital era has challenged conventional textbook practices. Textbooks need updating more frequently and need to support collaborative and interactive pedagogical methods (Smart and Jagannathan, 2018).

Mathematics manipulatives

Manipulatives are artifacts used in mathematics education: they are handled by learners in order to explore, acquire, or investigate mathematical concepts or processes and to perform problem-solving activities drawing on perceptual (visual, tactile, or, more generally, sensory) evidence. Mathematical manipulatives are classified into two main categories:

- Concrete manipulatives are physical artifacts that can be concretely handled by learners and offer a large and deep set of sensory experience; and
- Virtual manipulatives are digital artifacts that resemble physical objects and can be manipulated, usually with a computer mouse. (Bartolini & Martignone, 2020)

Supplementary materials

Supplementary materials include books, newspapers, informational pamphlets, and other materials printed in mother tongue and instructional languages reflecting local customs and concerns. They enrich teaching, engage learners in multi-dimensional learning, build learners' abilities to apply their knowledge (Elliott and Corrie, 2015), and are thus critical for literacy outcomes (Read, 2015). Studies show that investments in reading books and school libraries have an even greater correlation with increases in learner achievement in lower grades than investments in textbook provision (Read & Treffgarne, 2011; Read, 2015).

Current access to LTSM

In overcrowded classrooms, availability of a textbook may mean that learners at least have a structured approach to covering their curriculum; and they do not have to spend long periods in class copying notes from the chalk board. Availability of textbooks also implies that teachers have more time and flexibility to focus on lesson coverage (Zuze, 2018).

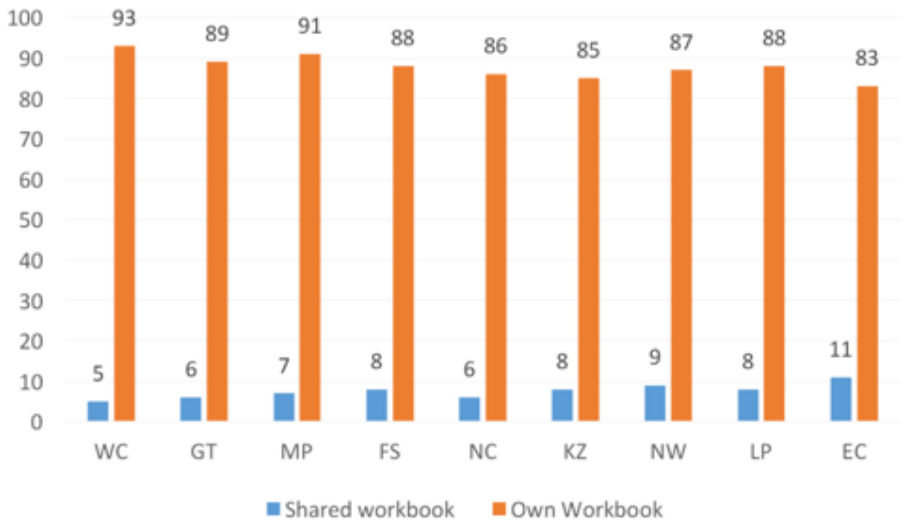


Figure 12: Percentage of learners with access to a workbook by province, TIMSS 2015[2]

In 2012 and 2014, textbooks were not delivered on time to schools in the Limpopo province; since then, textbook shortages have been reported in other provinces such as KwaZulu-Natal, and this is an indication that textbook availability is a national issue.

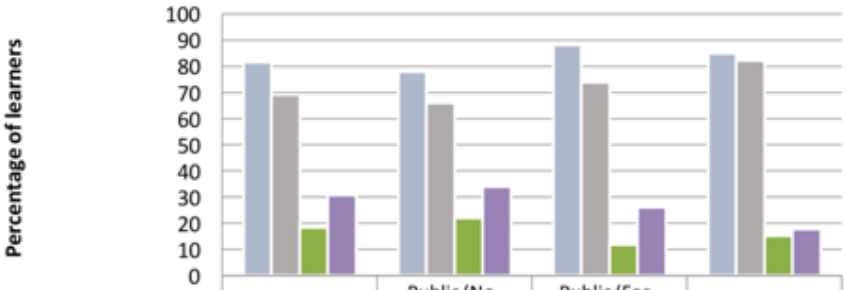
Table 1: Percentages, standard errors and confidence intervals of Grade 3 learners with Numeracy textbooks, DBE School Monitoring Survey (2018)

PROVINCE	PERCENTAGE WITH NUMERACY TEXTBOOKS (CALC GLOBALLY)	PERCENTAGE WITH NUMERACY TEXTBOOKS (CALC PER SCHOOL)	STANDARD ERROR	95% CONFIDENCE INTERVAL	
				Lower	Upper
EC	34.77	34.28	3.99	26.46	42.10
FS	28.21	27.62	3.23	21.27	33.96
GT	33.85	33.20	3.28	26.77	39.63
KZ	61.23	60.06	3.40	53.39	66.73
LP	19.28	21.04	3.15	14.86	27.22
MP	16.79	15.96	2.74	10.58	21.33
NC	46.35	45.52	3.88	37.91	53.14
NW	57.24	69.03	3.50	53.16	66.90

[2] Source: http://www.hsrc.ac.za/uploads/pageContent/9327/textbook%20figure1.jpg

WC	76.21	76.18	2.78	70.73	81.63
SA	42.07	42.69	1.24	40.25	45.13

In 2018, the DBE School Monitoring Survey reported that 43% schools have access to mathematics textbooks for Grade 3 learners nationally, with notable deviations from that norm with only 16% of schools in Mpumalanga having access to textbooks and 76% of schools in Western Cape having access.



	National	Public (No-fee)	Public (Fee-paying)	Independent
I have my own maths textbook	82	78	88	85
I have my own science textbook	69	66	74	82
Share or do not have a maths textbook	18	22	12	15
Share or do not have a science textbook	31	34	26	18

Figure 13: Percentage of learners who own a textbook by school type, TIMMS 2015[3]

Among Grade 5 learners who took part in the TIMSS study, the majority of learners had access to their own textbook. Ninety-three per cent (93%) of learners in the Western Cape and 91% in Mpumalanga had greater access to an individual textbook compared to 85% of learners in KwaZulu-Natal and 83% in the Eastern Cape. The fact that 17% of Grade 5 learners in the Eastern Cape Province did not have their own text is a cause for great concern given that this province had the poorest results in TIMSS 2015.

Primary mathematics assessments in South Africa, 2010 to 2022

The period from 2010 to 2022 has seen radical changes in terms of curriculum consolidation, a decisive focus on clarifying subject content to be taught, crystallising how assessment is to be conducted and emphasising a learner-centred approach to teaching and learning in all subjects, and particularly in the teaching and learning of early grade mathematics. The introduction of the CAPS is premised on the understanding that assessment is integral to effective teaching and meaningful learning. Assessment is critical to directing not only what is taught and learnt in the classrooms, but also to providing information that the education system requires for setting evidence-based targets

[3] Source: http://www.hsrc.ac.za/uploads/pageContent/9329/textbook%202.jpg

and providing appropriate educational resources. For instance, almost half of the targets that the DBE has set in the *Schooling 2024 towards 2030* strategic plan are measured in terms of the results of standardised national and international assessments. The assumption and expectation is that the results are credible and are based on valid and reliable instruments. Test validity is evidence that supports inferences that can be made from the results of the test. "Validity is an argument, not any statistic" (Bond & Fox, 2015:374)

Alderson (2005) provides a working definition of diagnostic assessment which summarizes its purposes and use in second/foreign language assessment:

> Diagnostic assessment consists of tests that are designed to identify both strengths and weaknesses in a learner's knowledge and (skills). Focusing on strengths will enable the identification of the level a learner has reached, and focusing on weaknesses or possible areas for improvement should lead to remediation or further instruction. Diagnostic tests are meant to enable a detailed analysis and report of responses to tasks, and must give detailed feedback which can be acted upon. The content of diagnostic tests may be based on material which has been covered in instruction or which will be covered shortly (pp. 256-257).

The purpose of assessment is to collect data that will be used to improve teaching and enhance learning as crisply covered in the following observations:

> Assessment is integral to the learning process, begins with clarity of purpose, [and] should gather information to inform practice (Ministry of Education, Singapore, 2013)
> "CAPS" captures the "embeddedness" of assessment into the curriculum. (Department of Basic Education, 2011)

There are two primary types of assessment: *assessment for learning* (AfL) and *assessment of learning* (AoL).

- **Assessment for Learning** takes on a formative approach to assessment and occurs during lessons where appropriate interventions can be effective to guide learning as it occurs, and
- **Assessment of Learning** is summative in nature focusing on standardisation, particularly with regard to monitoring achievement of set goals as is the case the Department's strategic plan *Action Plan 2019 Towards 2030*.

In the light of the assertion made above, it is important to foreground some contextual and historical developments related to curriculum content, assessment and performance in early grade mathematics in South Africa.

School-based assessments

School-based assessments are developed at school level by teachers and the results are used to inform further teaching and to communicate the progress of an individual learner to their parents. The development of school-based assessments is guided by the Curriculum and Assessment Policy Statement (CAPS) and the Framework for Teaching Mathematics with understanding, international assessments, most notably the EGMA, provide important tools for tracking progress. While open soured assessments offer some opportunities, the majority of systemic assessments currently used in South Africa are at the project or institution level.

CAPS

The *Curriculum and Assessment Policy Statement (CAPS), Grade 1-3 and Grade 4-6 Mathematics* documents define assessment, distinguishing between formal and informal assessments that teachers must administer and either clarify or exemplify ways of teaching specific subject content (DBE, 2011). However, little is said about the technical aspects of the assessment tasks such as the validity and reliability of the tasks that teachers are expected to compile. The assumption seems to be made that teachers will have the necessary expertise to carefully design, compile and validate the assessment tasks that they construct. The expectation is that, therefore, exposure to teacher-compiled tasks will help all learners learn. Yet there is tacit agreement among all who are involved in education that assessment varies widely from school to school and from teacher to teacher. Beyond the assumptions and expectations regarding the quality of assessment instruments (all subjects), there is a need to not only ensure some standardization in assessment but also to identify capacity for the development of standardized instruments to validate assessment results, especially if this information is required across different classrooms.

Largely CAPS provides guidance and techniques for teaching specific content areas but does not guide development of standardised assessments. No examples of recommended standardized assessment instruments are provided even though CAPS provides sound principles and examples that could be followed in developing quality assessment items (not necessarily instruments). For instance, CAPS provides detailed guidance on how important assessment tasks such as Projects and Investigations (*Grade 4-6 Mathematics*) should be used to teach learners skills of organizing and recording ideas, communicating ideas, showing clear calculations to demonstrate understanding of mathematical concepts and finally being able to generalise and draw conclusions. But there is a dearth of similarly detailed guidance on how to develop appropriate skills for developing, administering and validating assessment instruments for use both at school level and ultimately across schools. CAPS provides no replicable standardized assessment instruments.

Teaching Mathematics with Understanding (TMU)

The *Mathematics Teaching and Learning Framework for South Africa – Teaching Mathematics with Understanding* (TMU) special focus is on effective teaching, learning and assessment in *mathematics* as opposed to all subjects as CAPS does. TMU provides pedagogically sound techniques for formative assessment. These are techniques that enable the teacher to identify gaps or deficiencies in learners while the lesson is going on (*formative assessment*) and include:

1. Careful planning and considered selection of manipulatives to assess conceptual understanding. Learners with sound conceptual understanding can compare, relate and connect both objects and ideas.
2. Repetitive practices to assess *procedural fluency* in mathematics. This involves and leads to learners developing automaticity in working with numbers. Mental math will be an example of automaticity.
3. Questioning methods that promote independent thinking and problem-solving rather than the recall of formulas in mathematics.
4. Exploiting the power of language to help learners clarify their thinking as they communicate with and convince others about how they interpret mathematical concepts and relations.

The TMU is replete with excellent examples on how to use these techniques. However, when it comes to assessment of a summative nature in particular (apparently tests, examinations, etc.), the TMU makes a general statement that: "*All teachers must bear in mind that assessment should be unbiased, fair, transparent, valid and reliable*" (p.79) and no further clarification or interpretation of all the involved concepts is given. And yet these are concepts that play a critical role in setting and administering instruments that can lead to trustworthy, valid and reliable results. This is particularly so if the assessments are meant to be used across schools. So, there is strong guidance and techniques for *classroom-based assessment* as outlined above but the TMU does not directly guide development and use of standardised assessments. No specific instruments are available but TMU provides sound principles and examples to be followed in developing quality assessment items (not instruments).

Provincial, district and local assessments

Various provincial departments of education develop and administer their own "common assessments" for various purposes. These usually comprise of Formal Assessment Tasks (FATs) which are then administered at school level by teachers. The Western Cape Education Department administers systemic assessments for mathematics at Grade 3 and Grade 6 to monitor trends in mathematics attainment at Grade 3 and Grade 6. Most standardised assessments are instruments developed by either universities or other institutions that are implementing specific programmes (as seen in the shortlist of standardised assessment instruments.

International standardised assessments

South Africa participates in the Trends in International Mathematics and Science Study (TIMSS). This allows for the tracking of trends at the system level for mathematics achievements at Grade 5 and Grade 9 levels. In addition, the Southern and Eastern Africa Consortium for Monitoring Education Quality (SEACMEQ) assessments enables tracking of trends over time relative to other African countries

There are released items on the respective websites but current instruments are kept confidential. The studies and their instruments are designed to be comparable to national curricula. Manuals are available for developing assessment instruments framed in a comprehensive test blueprint.

Early grade mathematics assessment (EGMA)

EGMA occupies a unique space in that it can be used for both school-based and standardised assessment. The instrument has been piloted across different regions in Africa and has most of the desired features of a valid and reliable instrument. A variation of EGMA specifically re-designed to elicit information on emergent mathematics at Grade 1 level has been developed in South Africa; EGMA Junior has been developed for learners in Grades 1-3; and, EGMA Senior has been developed for learners in Grades 4-6. These assessments provide educationally sound coverage of both Foundation Phase and Intermediate Phase with variations on the same instrument and provide an opportunity for tracking performance over time and across grades using the same basic instrument, with a good spread of common items.

EGMA shares all the principles and features of good assessment in mathematics with both CAPS and specifically TMU which include, but are not limited to:

- Timely feedback to enable intervention while the lesson is going on;

- Focus on specific skills, e.g. specific number operations in the case of EGMA; and
- Ability to employ specific strategies such as grouping learners for mutual support where feedback shows common misconceptions, etc.

Global Proficiency standards for Mathematics

There are substantial amounts of assessment examples of items framed against global standards for mathematics at primary school level. The fact that this is an open source may limit the extent to which the items can be used as they are (as some candidates may have already accessed and practised them) but the important thing is that the standards and the exemplars provide an opportunity to develop new items against the same standards.

Findings and discussion

Discussion of the findings from the LTSM and Assessment reviews with the DBE officials through various structures and forums, coupled with engagements with the expert working group and teachers, demonstrate the complexity and contestations which exist in the South African primary school landscape. All education role players hold the interests of the majority of South African children in mind and show deep concern for the lack of progress evident in mathematics learning outcomes. What they consider is the best course of action with regard to LTSM and assessment investments and choices differs. In this discussion, we offer a synopsis of the findings and outcomes of the various discussions and engagements.

General findings

The stability of our fragile primary school system is paramount, so changes to LTSM and assessments must be carefully planned and coordinated with simple messaging for incremental improvement.

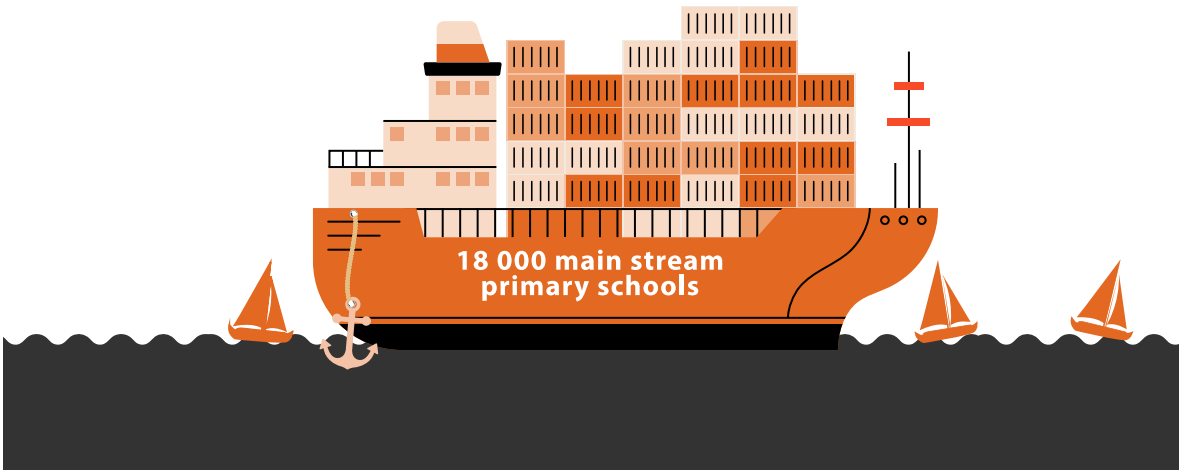


Figure 14: Metaphor of mainstream schools as the big ship "primary mathematics"

What is agreed is that the entire system – of 18 000 primary schools – need to be protected and very carefully guided in relation to mathematics teaching, the LTSM being used and advocated for and in relation to assessment practices and instruments. Rapid changes to curriculum or radical revisions to approaches are not considered helpful. Systemic change takes time. Careful consideration for how the system can absorb and respond to a limited number of key interventions and messages is required. What can be achieved at the small scale of 20-50 schools does not translate into the same gains at provincial and national system level. The fragility of the schooling system and paucity of mathematics knowledge for teaching in primary schools is well recognised and well documented. There is considerable agreement that school improvements requires careful design and implementation, much reach the mathematics teacher in her classroom, and support her with well structured, high quality LTSM and assessment tools. In addition, it is well recognised that the primary mathematics teachers require support from instructional leaders within their schools (the department heads, school management teachers and subject advisors). Ongoing teacher development, coupled with school-based coaching is highly valued.

The DBE workbook represents a major positive shift in South African schooling. The presence of the DBE workbooks in all primary schools in South Africa is highly valued and is recognised as a major and pivotal investment into schooling. This is appropriately heralded as a major DBE intervention which has improved access to learning opportunities and has been referred to as "changing the grammar" of primary schooling. (Pretorius and Spaull, 2022). This innovation and investment is applauded.

Too many mathematics interventions can distract teachers and instructional leaders with competing messaging. There are concerns about the plethora of interventions and approaches to improving primary mathematics and that these may create mixed messages and confusion within the system. These accompany deeply felt concerns about the lack of reliable evidence relating to primary school attainment in mathematics. The concerns about the unintended consequences of the Annual National Assessments are shared, and much hope is placed in the sample-based systemic evaluation processes which the DBE has piloted and will continue to lead.

The last decade has seen significant advances in how to work to improve mathematics learning outcomes at scale in primary schools. While the uncoordinated nature of the "too many interventions" is recognised, it is clear that the last decade has led to significant advancements as we have moved beyond diagnosing "the primary mathematics problem", with significant research, design work and effort put into finding scalable models for incremental improvement. This is evident from

- the DBE itself (with workbooks, a stable mathematics curriculum, annual teaching plans),
- the provincial departments of education (with several province wide improvement programmes targeting mathematics through LTSM, assessment and teacher development),
- public universities (through funded numeracy chairs at Wits and Rhodes, and focus on African language dominant classrooms at University of Fort Hare), and
- the NGO and donor or funding community (with numerous corporate social investment initiatives targeting mathematics.

Intentionally and proactively plan for curriculum reviews which are informed by evidence from assessment and use of LTSMs. The education stakeholders participating in these review processes have proposed collaborating to contribute to agreed cycles of engagement.

A process of Curriculum Strengthening currently being undertaken which, according to the Draft Curriculum Strengthen Blueprint, aims:

> To deliberately modernise knowledge and infuse competencies and cross-cutting priorities into the curriculum, pedagogy, and assessment, a framework is needed that will enable alignment across terminology, research, policy, evaluation, and practice.

Like the Curriculum Strengthen process, we also see the need for coordinated change.

Firstly, the need for primary mathematics Curriculum-LTSM-Assessment communities of practice – which cut across government, university and NGO sectors – is recognised. These review processes enabled a series of mathematics curriculum-LTSM- assessment workshops which could be extended to be DBE primary mathematics indabas (convened by the DBE), which facilitate information sharing about what is being done, emerging findings and ways in which collaborations can be forged and strengthened. These are envisaged to be virtual engagements which may take place quarterly or bi annually. When research and funding is being invested in particular assessment instruments, LTSM packs or professional development processes, these ought to benefit the entire system.

Secondly, the need for a predictable way in which changes are made at the curriculum, LTSM and assessment policy levels has been raised. The period of stability in curriculum which has been in place from 2012 with the release of CAPS is welcomed. However, it is simultaneously acknowledged (by external experts and role players within the DBE and provinces) that predicable cycles of improvement are now necessary. Covid-19 necessitated a reactive approach to an emergency situation." Annual teaching plans were revised, curriculum and assessment expectations were adjusted, and several interventions sought to support a shift to more remote and independent learning. As we slowly recover from the emergency situation of the pandemic, the stakeholders engaged in this process have muted predictable cycles of curriculum, LTSM and assessment review, which align with the five year plans of government structures. The goodwill and investment from the private sector can be coupled with the research in public universities to allows for incremental improvement at scale in support of the DBE and provincial department mandates. This requires a rhythm of engagement so that the collaborating partners have a structured way in which to engage.

The ideal model harnesses the strengths from DBE's large scale LTSM development and nationwide distribution; the commercial publishers' LTSM development targeting school procurement and the research interventions' small scale and trialled LTSM development. The ideal model supports and promotes design and innovation from the research initiatives. One of the features of the funder based LTSMs that the ideal model draws upon is the opportunity for developing online workbooks. In the ideal model, the implementation and distribution of LTSMs would be coordinated by provincial education departments while the national department of education provides the financial and administrative support. The research findings would guide the approval of LTSMs for inclusion in the national catalogue and ultimately into the nationwide distributed LTSMs.

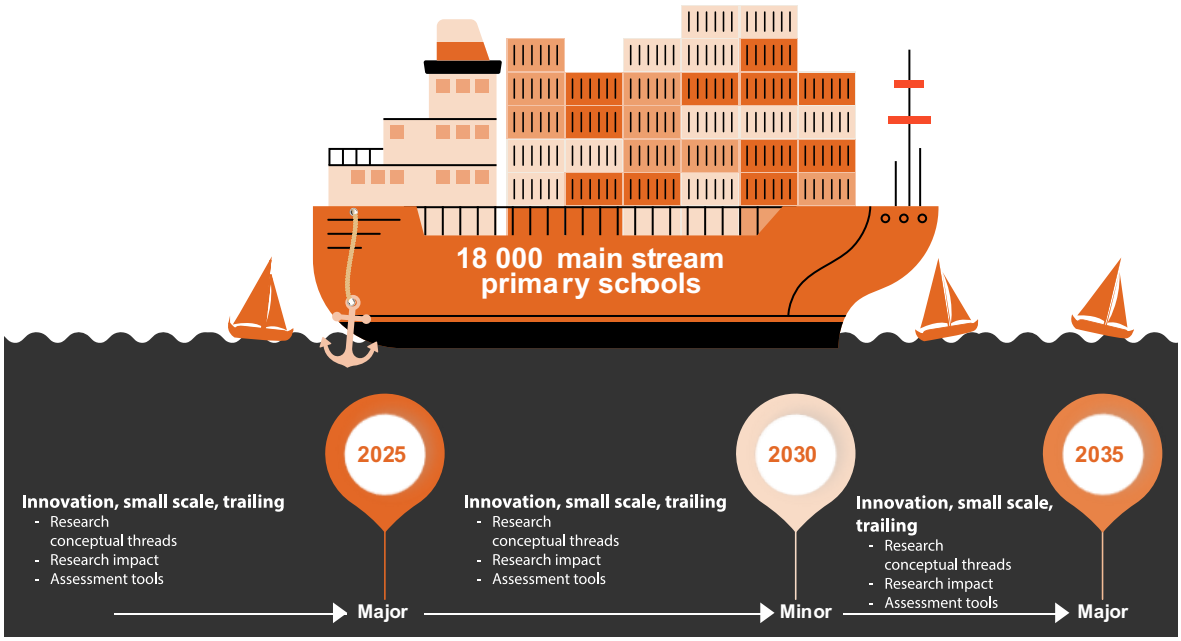


Figure 15: A proposed cycle of incremental improvement for Primary mathematics LTSM and assessments

The strengths of the ideal model lies in the collaboration of different stakeholders in LTSM development, promoting and scaling excellence from school level, through districts and provincial levels. The proposed model also provides a wider selection of approved LTSMs available for procurement by schools. An integrated model is what the country needs as the next step in LTSM research agenda The recommendations emerging from this analysis focus on design principles, assessment, teacher support regarding the use of LTSM, LTSM delivery in schools, collaborations in LTSM development and concept thread research.

Carefully plan and coordinate changes to curriculum, LTSM and assessment with simple messaging for incremental improvement. The goal is for a 90:10 split for stability:innovation, to protect the stability of our fragile primary school system, while also innovate and research alternative curriculum and LTSM offerings. 90% of teacher would use the updated DBE Workbook (which would be improved by a 5-year cycle) as their primary resource with the support of secondary sources which would be included in differentiated practice books/software (for independent practice) and textbooks. Figure 17, below, outlines simple messaging to learners and teachers, with aligned support through CPD training and subject advisors. This approach ought to be trialed with Foundation Phase. When the Intermediate Phase mathematics textbook review is complete, whether to expand this to Intermediate Phase can then be considered.

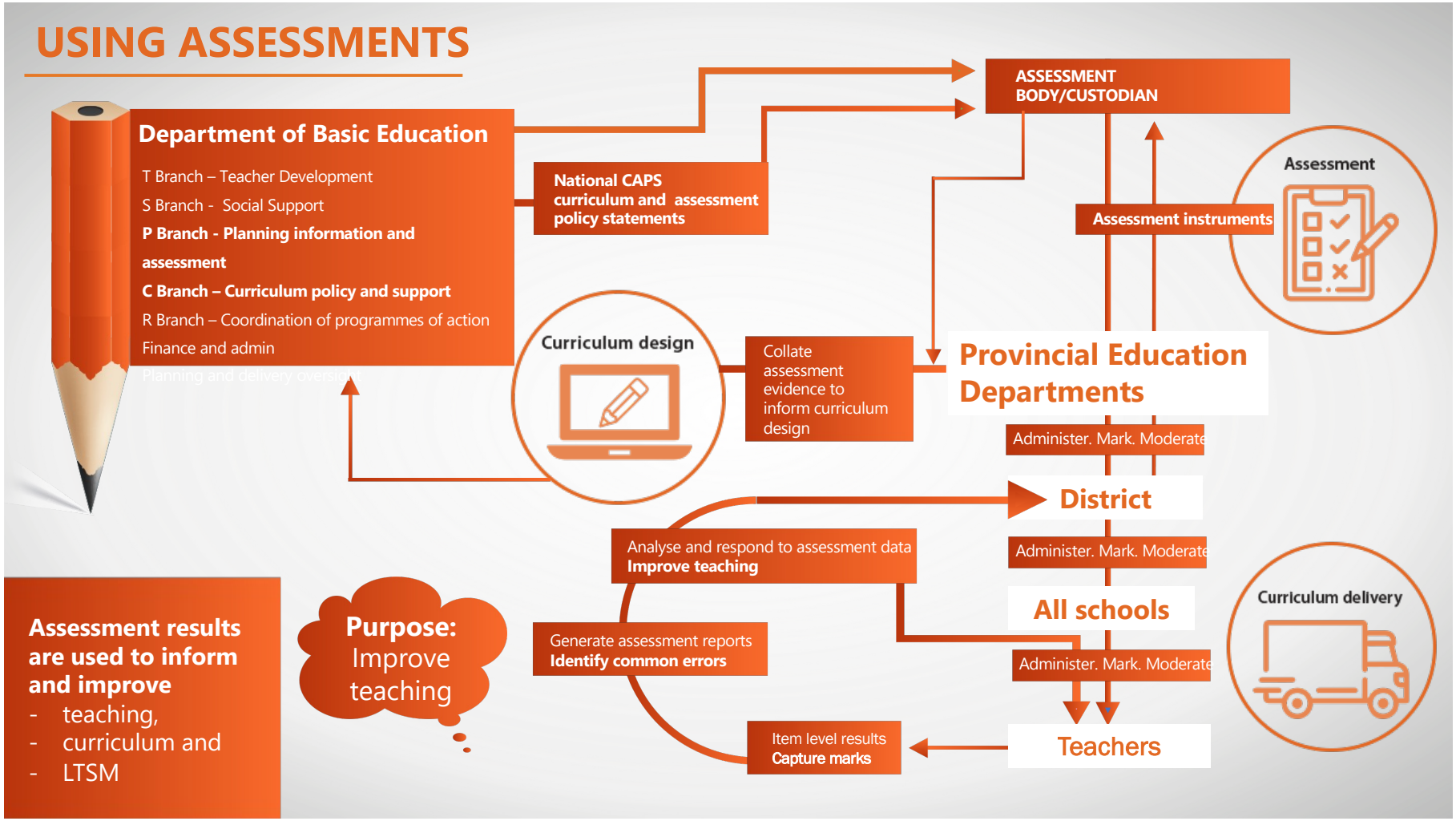


Figure 16: Envisaged LTSM and assessment landscape in South Africa

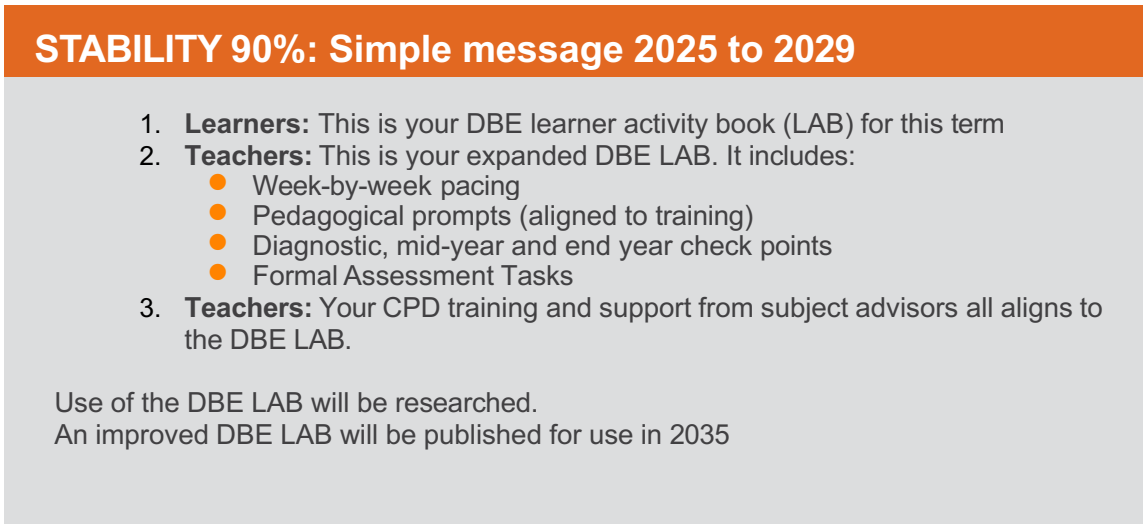


Figure 17: Envisaged LTSM and assessment landscape in South

The next section will specifically address how assessment is used in primary mathematics contexts, and how these principles of 5- and 10- year review, with intentional and proactive planning with the view supporting the primary mathematics through clear communication of which resources to use how.

Assessment findings

Assessment can be a powerful tool, especially when it speaks to curriculum and LTSM. An in-depth analysis of the three identified assessments has covered important aspects relevant for assessment that helps to monitor learner performance in mathematics. In the process outlined above, three instruments were identified for in-depth analysis: M-SAP, EGMA and MCC- FATS. These three instruments were designed for different purposes but together cover the spectrum of assessment that the education system requires. All three assessments have been trialled for at least ten years each and have been found to be valid measures for the different purpose for which they were designed (curriculum-base measure, impact evaluation, and assessment-driven approach to teacher development respectively). As part of the research process, the various purposes of assessment, as well as the models for assessment delivery were described and discussed. These have been included in Annexure 2.In the literature on these instruments, the experts and designers concur on important observations.

The central role played by the assessment branch in the DBE, in driving the assessment processes for South Africa is recognised. The Annual National Assessments (ANA's) were developed to allow for annual standardised assessment data at each grade level to guide teachers and prioritise schools for additional support. However, using an assessment at individual learner and school level for accountability purposes lead to unintended consequences. Stopping these assessment was therefore deemed appropriate. The DBE systemic evaluation process which is currently underway will fill the identified gap of the lack of reliable primary level mathematics attainment levels, which can be tracked over time. It is expected to use appropriate randomised sampling to allow for provincial level data to be reported.

A set of diagnostic assessments (including Grade 3 and Grade 6 mathematics) also have been published by the DBE.[4] This offers a useful definition of assessment for eLearning as

> the process of gathering information about a learner's learning from a variety of sources, using a variety of approaches, or 'assessment tools', and interpreting that evidence to enable both the teacher and the learner to determine where the learner is in his or her learning; where the learner needs to go; and how best to get there.

This DBE initiative recognizes that teachers can adjust instructional strategies, resources, and environments effectively to help all learners achieve grade specific outcomes, but only if they have accurate and reliable information about what their learners know and are able to do at a given time. The diagnostic assessments are therefore intended to:

- reveal the misconceptions learners bring as prior knowledge to a class,
- measure the conceptual gains of a class as a whole, and
- identify concepts that are weak areas of understanding for the individual learner or as a class/grade.

South African also participate in the Trends in International Mathematics and Science Study (TIMMS), including a Grade 5 mathematics assessment. To date this is the only standardised assessment with a national sample. It shows worrying stagnation in mathematics attainment at the Grade 5 level in South African primary schools. The difference in mean results for 2015 of 376 (3.5) and TIMSS 2019 of 374 (4.7) are not statistically significant.

There is general agreement that the approach to promoting mathematics learning and assessment at school level, in primary school mathematics in South Africa, has been uncoordinated and largely led by specific entities and groups of research bodies. Evidence of the uncoordinated approach can be seen from the distinct methodologies, the selection of language(s), grades and curriculum domains that are the foci of interventions for improvement. There is a need for greater coherence in how curriculum and LTSM innovations are researched and learners are assessed to judge efficacy across interventions.

Knowledge about available assessment instruments, which would allow for wider uptake and use, is scant. There has not been consistent system-wide communication on and awareness of existing assessment resources of high quality that schools and teachers can tap into in order to improve the assessment experiences of learners and improve teaching and learning.

The SA-SAMS assessment data capture system is considered to lead to a problematic 'compliance culture' at school level. The national IT platform (SA-SAMS) on which assessment data is captured and made accessible to users is not responsive to the realities of what gets taught in schools during the different school terms in a year. The likely unintended effect is that schools will be forced to populate the system with data that reflects compliance rather than genuine learning realities and thus ensuing interventions may be misdirected. By way of example, teachers report having to upload one mark for *every* content area in the mathematics curriculum per term. Therefore necessitating that every area is taught in every term. While this may be a user problem, it is reported widely enough to raise a concern about the way in which assessment capture is dictating curriculum coverage and sequencing. This problem was surfaces through various of our engagements with different stakeholders. IT subsequently was pointed out to us, that teachers able to leave a topic area without a mark by simply entering "-1". Doing so will ensure that no mark is captured for that topic in that particular term. Our concern therefore shifts to the need to ensure

[4] DBE 2021, https://www.education.gov.za/DiagnosticTestItems.aspx

that this technical work around is widely communicated, understood and adopted by teachers in schooling.

South Africa has examples of mathematics assessment which incorporate the main uses of assessment. The suite of three assessments that have been analysed in depth covers a spectrum that incorporates the main uses of assessment:

- For monitoring and comparing impact of various research-based curriculum and LTSM interventions (in the 10% innovation space);
- For monitoring learner performance in relation to the curriculum expectations; and
- For using assessment as a professional development process to promote formative assessment.

Many of the assessment instruments deliberately focus on Number, Operations and Relationships. The prudence of selecting domains of early grade mathematics that have predictive validity versus assessing the curriculum in its totality has been the trend over the last decade, and it has led to remarkable improvements in teaching and assessment. The Early Grade Mathematics Assessment (EGMA), which prioritises Number, Operations and Relationships, has been used to assess impact of several research interventions which have shown significant impact in terms of improvement in learner performance. Initiatives to extend these assessments to Spatial reasoning (encompassing measurement, shape, space and geometry) and attending to Data Handling, should be pursued as this area has also been reported to have significant predictive validity. MCC models a unique approach to Formal Assessment Task assessment that covers the whole curriculum. It can be used at school level, on individual and groups of learners and in ways that are learner-paced and context-sensitive.

The M-SAP is used for formative purposes to promote assessment- driven pedagogy and develop teacher AfL strategies. The Assessment has been piloted widely at Grade 3 level across all provinces in South Africa, with emerging evidence of positive results in improving learner outcomes. It leverages knowledge of Number Structure to develop:

- Rapid recall fluencies;
- Strategic calculations; and
- Strategic thinking.

A collaborative effort is needed between the DBE and the M-SAP Research and Innovation Intervention (referred to here as a Community of Practice) **to facilitate greater access to the M-SAP assessments.** To support the process the DBE should:

- Collaborate with the Maths Research and Innovation Interventions to extend the M-SAP to include Grades 1-4;
- Link M-SAP research findings to Continuing Professional Teacher Development (CPTD); and,
- Initiate the integration of M-SAP as an assessment approach into cycles of improving the mathematics curriculum, LTSM (Learner Workbook) and Assessment instruments (FATS).

The M-SAP CoP should support this process through:

- Management of a repository of M-SAP data;
- Publication of indicative mental fluency norms for each grade level, based on the national data set; and,
- Publishing and sharing the data on mental fluencies to inform cycles of improvements.

There is a lack of alignment between assessment findings and curriculum review process. Curriculum reviews have not been made to benefit optimally from the assessment findings and learnings made from the initiatives of the research bodies and the experiences of teachers and learners in the public school system. Consequently, curriculum reviews have been seen as "disruptions" rather than necessary cycles that need to be planned for in the evolution of the national curriculum responding to the needs of the population. Any curriculum is necessarily a "best guess" at grade level expectations. This can be significant refined to better support learning if reliable assessment data is used to inform each curriculum revision cycle.

Evidence generated from impact evaluations is required to identify and validate effective LTSM interventions, allowing the learning from small scale experimental interventions to feed into the development of the best DBE Workbooks possible. To date, the EGMA has been used for summative purposes in measuring the impact of interventions that seek to improve mathematics outcomes. The instrument has been piloted in South Africa and in other African countries showing acceptable levels of reliability and validity in all those contexts. It leverages competency in Number, Operations and Relationships (NOR) because of the researched "predictive power" of this sub-domain which also forms 60% of mathematics in CAPS. There is a clear need for the various interventions in curriculum, LTSM and teacher development (part of the 10% innovation space) to be monitored and compared for impact. In the absence of a standardised assessment instrument, it is not possible to fairly compare impact. This is an issue faced by the TMU pilot, research-based intervention supported by the private sector and for province wide interventions (such as GPLMS, MLIP or R-Maths, and WCED's Back on Track). It is therefore important that a series of impact evaluation instruments, which measure impact on mathematics learning outcomes are developed and managed. Such assessment instruments can be used for monitoring impact in interventions that are designed to change practice and realise improved performance. The EGMA provides a possible starting point for instrument development, but requires refinement and expansion.

Table 2: EGMA Versions and Target Learner Groups

EGMA Version	Target Learner Groups
EGMA 1	Grade 1
EGMA Junior	Grades 1-3
EGMA Senior	Grades 4-6

An assessment instrument should be developed for use in impact evaluations to generate evidence of successful intervention techniques, which should be fed into curriculum, LTSM and assessment review cycles. As such, an instrument is intended to monitor and compare the efficacy of curriculum interventions this should be managed by the curriculum branch (in consultation with teacher development). To facilitate this process, the DBE should:

- Initiate/support the establishment of a "primary mathematics test" Community of Practice (CoP)[5] that will facilitate the rollout and creating of refined and parallel versions of primary mathematics test instruments in all the SA official languages;
- Encourage the standardised use of the "primary mathematics test" instruments for impact evaluations; and,
- Initiate the integration of revised "primary mathematics test" benchmarks into cycles of improving the mathematics curriculum, LTSM (Learner Workbook) and Assessment instruments (FATS).

Furthermore, the "primary mathematics test" CoP should:

- Establish grade level norms/benchmarks (using a nationally representative sample) which specify how much learning is expected at specific stages as learners progress through primary schooling;
- Guard both the integrity and the validity of the "primary mathematics test" instruments by keeping the instrument confidential, define standardised administration requirements; and,
- Design the necessary instruments for the "primary mathematics test" CoP. This will involve extending the EGMA test design to include topics like Shape, Space, Measurement and Data Handling.

The ultimate purpose of assessment is to collect information that must be used to improve mathematics teaching and enhance mathematics learning. It is important to note that the primary mathematics instrument and related community of practice is not an intervention for its own ends. Rather it is a necessary means to allow more rigorous research and comparison of impact across the existing mathematics improvement interventions – which trial innovations in curriculum, LTSM and assessment, and teacher development. The interventions fall within the 10% innovation space and share a common goal of improving mathematics learning outcomes and improving teacher knowledge and practice. Such interventions are done on a small scale, but their impact must be measured so that their lessons and approaches can then be scaled for systemic impact.

[5] The name could be "Primary mathematics validated test" CoP or similar. The main point it to use the work done on the EGMA to revise the instrument to accompany a wider topic focus (including special reasoning, geometry and measurement) and ensure that there are validated instruments which can be used in all impact evaluations of primary mathematics interventions.

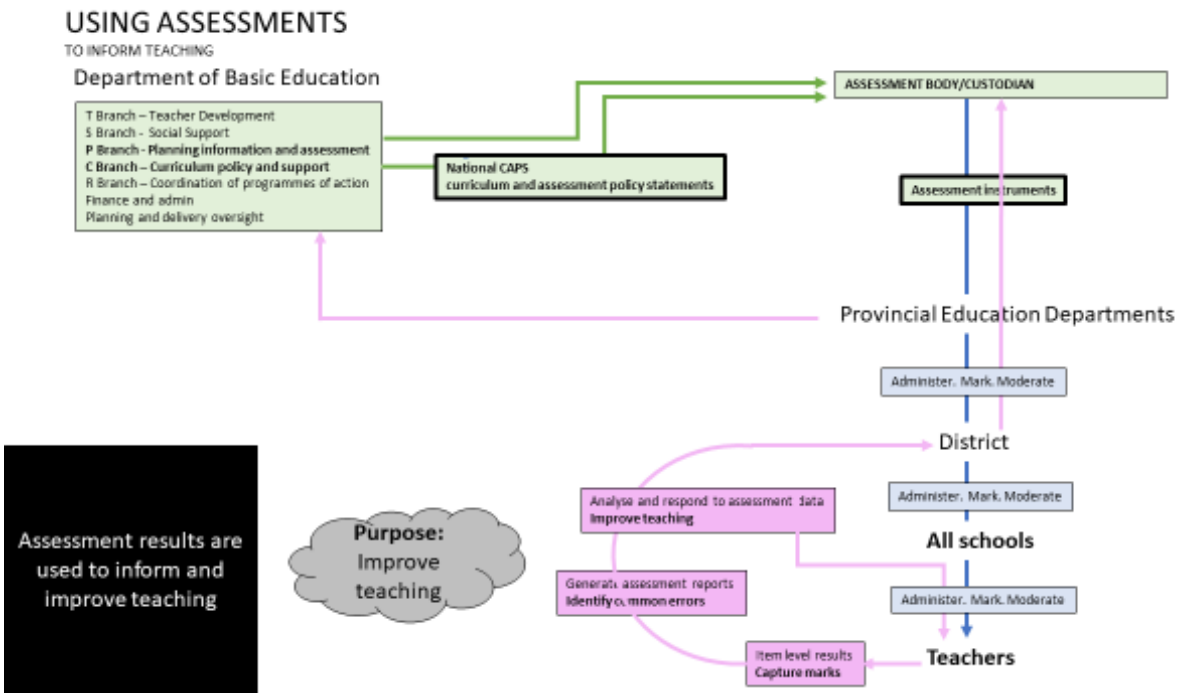


Figure 18: Using assessment results to improve teaching

There is researched evidence that the greatest weakness in education systems that prioritise assessment is the non-use or under-utilisation of assessment results to improve teaching. In addition to schools being equipped with sufficient capacity and skills on how to analyse and use assessment to provide relevant feedback and intervention to learners, there is need for a feedback loop to districts (subject advisors) to know where teachers need support and respond promptly.

There is a need for assessments that monitoring learner performance in relation to the curriculum expectations. The MCC- FATs are CAPS-based assessments that have been piloted at the Foundation Phase in English and isiXhosa in a sample of 15 deep rural schools in the EC. The assessment has been used in a "learning laboratory" design that leverages researched benefits of multilingualism in mainstream no-fee schools in the EC. In terms of efficacy, systemic evaluation scores showed more than 20% increase from baseline in the MCC Project schools compared to the rest of EC mainstream schools between 2009 and 2014.

The DBE should facilitate the use of FATs in the classroom by:

- Compiling and managing a repository of quality mathematics FATs (from PDEs and interventions like MCC and TMU);
- Considering to extend the development, trialling and eventual use of the quality FATs across all SA languages;
- Promoting the use of quality FATs results for term-to-term monitoring of learner progress and curriculum coverage; and,
- Adapting the SA-SAMS platform to host FATs results so that they are accessible to Subject Advisors and Curriculum support officials who must support schools to teach effectively.

This would enable schools to use the FATs results from the end of a term for diagnostic purposes at the beginning of a new term and thus ensure seamless progress across terms.

LTSM findings

When considering what makes good LTSM in mathematics, it is important to reflect on the underlying design principles. These necessarily are a response to the curriculum and assessment frameworks (and alignment between these is critical). Much evaluation work of mathematics LTSM has been an assessment of the extent to which the LTSM is CAPS compliant. However it is important to extend the design principles to reflect on the extent to which mathematics learning and better mathematics teaching is facilitated through the use of the LTSM. Such enquiry goes well beyond considerations for CAPS compliance, and is fundamental to contributing to evidence-informed improvements in curriculum, LTSM and assessment.

As an example of LTSM design work extending beyond CAPS compliance, Brombacher – the designer of the NumberSense workbooks – has made his design principles explicit. Brombacher and Roberts (2022) write that mathematics learning materials must:

- Provide "on the job" training opportunities for teachers;
- Allow for flexible implementation based on an objective assessment of the child's developmental level;
- Allow of a lot of regular practice;
- Be designed according to developmental trajectories; and
- Be responsive to the regulatory context in which teachers work – materials must be seen to address the curriculum.

Bala Wande has designed LTSM, and reviewed the LTSM by experts, and conducted follow-up evaluation work on the efficacy of the LTSM. The purpose of the design is to equip teachers to teach reading-for-meaning and calculating-with-confidence in Grades R-3 in South Africa. The LTSM include workbooks, a teachers guide, and a manipulatives set aligned to the material per grade

Porteus (forth-coming) – the designer of the Magic Classroom Collective mathematics workbooks - has also made her design principles explicit in relation to three lenses: pedagogical, mathematical and language related. First, she summarises the South African instructional realities, referring to these as constraints, and then she expounds on her resulting design principles, which are framed as design opportunities.

Table 3: Instructional realities and resulting design principles (Porteus, forth coming)

	Instructional Realities	Design Principles
1. Pedagogical Constraints and Opportunities		
1.1	Mainstream classrooms have severe resource constraints; productive instructional tools are often limited to a chalkboard.	The fundamental teaching strategy proposed within the instructional toolkit must be achievable within the resource constraints of mainstream classrooms.
1.2	Mainstream teachers struggle with establishing an "instructional base step," building momentum through	The instructional toolkit must assist teachers to build an instructional "base-step," embedding an achievable lesson

	consistent and sensible instructional days, weeks, and terms.	structure within reasonable teaching rhythms and expectations for days, weeks and terms.
1.3	The pedagogical culture of most mainstream teachers is confined to whole-class oriented, teacher-directed pedagogical practice.	The fundamental instructional method embedded within a toolkit must be achievable within a whole-class oriented and teacher-directed pedagogical culture. At the same time, it must have opportunities for pedagogical stretch for teachers who are ready to grow.
1.4	Teachers struggle with both formative and summative assessment strategies.	The toolkit must assist teachers to improve their assessment practice, scaffolding ways for teachers to learn from assessment activities.
2. Mathematical Constraints and Opportunities		
2.1	Most teachers have little claim on mathematical meaning making; they have learned mathematics mainly through memorisation.	The toolkit must serve as a "teacher development programme," designed to re-imbue meaning in mathematics. The workbooks must present a limited set of strategies and representations, where representation, strategy and procedure are tightly aligned.
2.2.	Most teachers do not understand how mathematical concepts develop for children. When a learner is not able to do a task at grade level, teachers are largely unable to assist.	An instructional toolkit must make explicit mathematical learning trajectories, how learning progresses for any conceptual thread. In so doing, materials must share theory (not only isolated teaching examples) with teachers.
2.3	Most learners continue to learn mathematics through memorisation, reproducing mathematical alienation at national scale.	An instructional toolkit must maximise the potential for mathematical meaning making among children, even when teachers mathematical meaning making remains fragile.
2.4	The daily pace of mathematical teaching is slow, with little chance to practice and build fluency.	An instructional toolkit must assist teachers to adopt activities provide opportunities for learners to practice and build fluency, especially with speed.
3. Leveraging Language and Dialogic Engagement: Constraints and Opportunities		
3.1.	Teachers have an ambivalent and under-developed relationship with African home language LOLT for teaching mathematics in the primary phase.	The instructional toolkit must assist teachers to leverage African languages for primary mathematics learning by both normalising and developing an accurate and friendly instructional register for teaching mathematics in African language LOLT classrooms.

3.2	Most teachers' way of talking about mathematical concepts is not accurate, nor coherent to children.	An instructional toolkit must signal to teachers the high-value instructional narratives (sentences or phrases) that bring a given conceptual step to life in the classroom.
3.3	The culture of dialogic engagement between teachers and children is currently highly constrained.	The toolkit must both provide ways for children and teachers to externalise their thinking, as well as signal an instructional discourse that can expand productive classroom talk over time.

The context for which Porteus designs is for African language dominant (isiXhosa) mainstream classrooms in rural Eastern Cape (Ramadiro & Porteus, 2007).

We take Porteus's three lenses, and included some of the design features from Brombacher and Bala Wande, and use these as a set of criteria for reflecting on and or making choices about LTSM offerings we reviewed for the deep dive. The set of criteria may also be helpful to designers planning new learner workbooks or revision of existing offerings. In so doing, we aim to illustrate how workbooks are designed for particular purposes and that these purposes then inform the related design choices. We have coded each of the workbooks reviewed using: red (absent), yellow (partially present) and green (present). It is important to recognise that the red (absent) does not imply weakness: the absence of a particular feature may be entirely appropriate for the intended purpose.

General

There are several general issues worth putting upfront when reflecting on any LTSM offering, all of which reflect the particular intended purpose of the workbooks.

Looking across the workbooks reviewed, four learner workbook purposes are evident:
(1) supplementary practice book;

(2) learner activity book and daily lessons to drive teacher instruction;

(3) differentiated practice book series, for independent learner work; and

(4) Learner workbook series, embedding daily & weekly teaching strategy.

Table 3 provides an overview of these general design criteria in relation to each workbook offering. Reflecting across the six LTSM offerings there are several observations worth making explicit.

Table 4: Reflecting on general LTSM attributes informed by workbook purpose

Grade range reviewed	**DBE workbooks** Grade R to 6	**TMU pilot** Grade 1 to 3	**NumberSense** Books 00-24 (Grade R to 6)	**Magic Classroom Collective** Grade R to 3	**Bala Wande** Grade R to 3	**PMRP** Grade 4 to 6
Purpose	Supplementary learner activity book	Daily lessons to drive teacher instruction	Differentiated book series, offering a page a day for independent learner practice	Learner workbook series, embedding daily /weekly teaching strategy	Learner workbook series, embedding daily /weekly teaching strategy	Number work lessons to drive teacher instruction
Does the LTSM…	DBE workbooks	TMU pilot	NumberSense	Magic Classroom Collective	Bala Wande	PMRP
…have credible research/ evidence of impact?	No impact assessment	Pilot currently underway, no learner immpact data published yet	Impact at small scale (Shikaya and JumpStart) and large/national scale (Jordan) established. Malawi national implementation currently underway. See papers by Brombacher, Roberts and Moloi	Large impact on learner outcomes at small scale in rural context established. See paper by Porteus	Pilot currently underway, no learner outcomes data published yet. See baseline papers by Ardington and Sapire	Impact on learner outcome 1 at district scale established. See thesis by Schollar
…have proposed next steps for research?	n/a. Complete redesign drawing on lesson from researched workbooks	Complete pilot and report on impact	RCTs in South Africa: JumpTrak and Shikaya models at, at least, district level	RCT in South Africa at, at least, district level	Complete pilot and report on impact	No longer active
…meet CAPS grade level expectations?	Yes - all in one book per term	Yes - all in one book per term	Yes: Mapping of CAPS requirement to every page. Separate workbooks for geometry.	Yes - all in one book per term	Yes - all in one book per term	Not intended -based on outcomes-based education
…follow CAPS Annual Teaching Plan (pedagogical guidelines)?	No	No - experimenting with revised sequencing	No	No	No	No
...offer an alternative (re-sequenced and newly structured) ATP by term, week and day?	Not intented - supplementary activity book	Clusters concepts aiming for completion or "signoff" on a topic in each grade.	Spiral curriculum for learner practice: Teach/practice at the right level	Clusters concepts and resequences annual plan	Clusters concepts and resequences annual plan	Focused on the Number Operations and Relationships content area

The DBE workbooks were designed to be supplementary. Subsequently workbooks have been developed and used to support curriculum coverage and pacing. The importance of a workbook as more than a supplementary resource, and as a key driver of teacher instruction in mathematics

is now clear. This strengthens the call to revise the DBE workbook to move beyond supplementary and be recognised as a core resource which draws on the attributes of the workbooks which have been developed subsequently and researched to have impact.

The workbooks reviewed that are intended to drive teacher instruction follow the CAPS, and offer alternative annual teaching plans (to the exemplar ATP offered in CAPS). Another key consideration is alignment of the LTSM content to the intended curriculum. In South Africa, much curriculum alignment review work has considered the extent to which there is tight adherence to the CAPS exemplar teaching plan (pedagogical guidelines) which offers but one example an annual teaching plan with week-by-week curriculum coverage and pacing. In more recent developments, there has been far greater appetite for considering alternative annual teaching plans. This has in part been necessitated by learning losses as a result of Covid-19 (where revised annual teaching plans were developed by the DBE), but also as a result of use of the CAPS framework and realizing that its curriculum coverage and pacing is interpreted a-historically without teachers connecting major sections within the curriculum.

By way of example, the R-maths and Grade R mathematics improvement projects both created alternative annual teaching plans, which met the articulated Grade R curriculums standards specified by CAPS, but which did not follow its suggested learning programme. The same approach has been adopted by the TMU pilot, MCC, Bala Wande and NumberSense. All demonstrate alignment with the CAPS statement of learning outcomes, but none follow the suggested pedagogical guidelines which offer one possible learning programme. This reordering and connecting of related concepts across a grade level is welcomed. The suggested learning programme (pedagogical guidelines) of CAPS was not tested or trialed. It was offered simply as a support to teachers as they used CAPS initially. Now that CAPS is embedded into the South African system, it is appropriate that the learning programmes (and related annual teaching plans) are being improved and refined. The flexible use of the instructional rhythms – while still meeting grade level expectations is appropriate. It is a sign of the system maturing, and improvements are being made on annual teaching plans.

With this in mind, the workbooks which are being researched to have impact on learner outcomes are strongly supporting teachers in their pacing and coverage. They tend to develop particular topic areas for longer periods. For example the TMU workbooks seek to work for extended periods on one topic, and "sign this off" before shifting to new topics. Other workbooks offer a more integrated and spiralling curriculum pacing. The NumberSense workbooks offer sets of exercises to practice which are carefully scaffolded, but offer a daily routine of counting, calculating and problem solving. Each page will integrate attention to measurement contexts, and does not treat measurement as an isolated topic. The MCC workbooks take into account the realities of what is possible in a rural context, with 8 imithamo (weeks) per term. There is a deliberate focus on fluencies and calculations involving 2s, 5s and 10s to deeply embed the base 10 thinking, required for working in higher number ranges.

There are three workbook offerings (NumberSense, Magic Classroom Collective and PMRP) which have documented impact on mathematics learner outcomes. Some workbook offerings (TMU pilot and Bala Wande) draw on lessons from previous research projects, and are currently being piloted for learner impact. The primary mathematics landscape in south Africa is now mature enough to expect documented impact on learner outcomes. The TMU pilot follows on from work conducted in Gauteng on (GPLMS) and Bala Wanda has sought to draw on lessons across previous interventions. Several of the interventions which have established small scale impacts

are now ripe for RCT research at district or provincial level. It is clear that developing excellent LTSM takes time, and several design trials. The DBE ought to be drawing on the development work which has been undertaken in the contexts which have evidence-based results on learner outcomes.

Workbooks which have been developed more recently try and minimise the number of "moving parts" which teachers need to use. The NumberSense workbooks were originally designed to focus on number work. These were then extended to include geometry and measurement workbooks, and augmented with mental mathematics starters. The complexity for teachers of moving between an ATP, a teacher guide, lessons plans and the learner activities has been noted. MCC has found that 85% of the teachers they work with did not read or use the teacher guide. As such, there is a move to try to communicate more of what is intended for the teacher – through the learner workbook, and to integrate elements such as mental mathematics into a single resource.

Pedagogy

When reflecting on the pedagogy aspects of the LTSM we reflect on the extent to which the learner workbook supports teacher development, instructional rhythms and offers instructional guidance. Once again is appropriate to notice that learner workbooks had this aspect as their explicit intent, and which were designed simply as supplementary learner activity books, or as differentiated practice workbooks. Table 4 provides an overview of these pedagogy design criteria in relation to each workbook offering. We discuss each of these pedagogy attributes in turn in the section below.

Pedagogy: Does the LTSM work within the classroom constraints?

When choosing or designing LTSM for mathematics it is important to consider the resources available in the mathematics classroom. Look at the LTSM to see if they use pen and paper only or require specialized equipment.

Table 5: Reflecting on pedagogy LTSM attributes informed by workbook purpose

How does the LTSM support pedagogy?	DBE workbooks	TMU pilot	NumberSense	Magic Classroom Collective	Bala Wande	PMRP
	Grade R to 6	Grade 1 to 3	Books 00-24 (Grade R to 6)	Grade R to 3	Grade R to 3	Grade 4 to 6
	Supplementary learner activity book	Daily lessons to drive teacher instruction	Differentiated book series, offering a page a day for independent learner practice	Learner workbook series, embedding daily /weekly teaching strategy	Learner workbook series, embedding daily /weekly teaching strategy	Number work lessons to drive teacher instruction
Does the LTSM workbook…						
…work within the classroom constraints?	Expects use of pictures / images in workbooks as paper-based manipulatives	Depends on low-cost manipulatives (eg paper-based ten strips, ten frames, arrays)	uses word problems in imaginable problem contexts. Drawing is encouraged in early grades. No manipulatives are expected.	Encourages drawing, some reference to unifix, posters, charts (paper based) and unifix (non-paper based) manipulatives	Depends on access to Bala Wande toolkit which includes Diennes blocks, beadstrings, tenframes etc[6]	Focus is on formal written algorithms, there is no reference to manipulatives
…build instructional rhythms?	termly	termly, weekly, daily lesson	1 page per day	termly, weekly, daily lesson	termly, weekly, daily lesson	termly, weekly, daily lesson
…support the teaching style dominant/ desired in your context?	Not specified.	Lesson plans provide instructions for groupwork / scaffolding work	Allows for differentiated working, allows different groups of learners to to work at their own pace.	Teacher guides provide instructions for groupwork / scaffolding work	Teacher guides provide instructions for groupwork / scaffolding work	Assumes that learners are assigned to ability groups for differentiated working
…include suitable assessment strategies which can be easily integrated by teachers?	Not intented - supplementary activity book	revision, formative, summative	FATS available, separately	Mini quiz, practice page, in books, FATS available separately	sAssessment and consolidation worksheets	weekly assessment activities provided

Most workbooks are designed for successful use just with pen and paper, but some workbooks expect access to particular manipulatives or tool kits.

Access to some basic manipulatives (such as unifix) coloured pencils, pairs of scissors, glue and 2D shapes for tracing are valued in the early grades. Measurement instruments (jugs, cylinders, scales, metre sticks, thermometres, clocks) are necessary in intermediate phase. However in the absence of a minimum mathematics took kit available in each classroom, it is appropriate that most workbooks do not assume that these are available. We discuss a recommended minimum kit in Annexure 3.

[6] It is possible to use the Bala Wande LAB and teacher guide without having Diennes blocks. A teacher could improvise using paper cut outs, or simply refer to the Diennes block manipulatives as shown in the diagrams

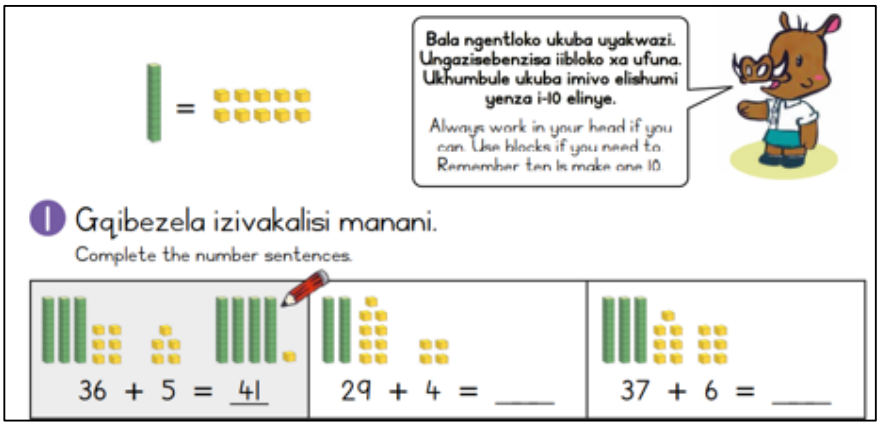


Figure 19: Bala Wande example using Diennes blocks

In this example from Bala Wande, it is expected that Foundation Phase learners all have access to sets of Diennes blocks (base ten kits). The Bala Wande LTSM makes extensive use of these specialized resources and expects children to "use blocks if you need to". The costs of the toolkit, how these are managed in a classroom ought to be considered when selecting an LTSM offering which requires specialised materials. Of course, the Bala Wande workbooks can be used in the absence of Diennes blocks. Teachers could improvise by using paper cut outs or just referring to the Diennes blocks diagrams (which appear in the LAB). What types of manipulatives and which representations to develop, is a key design decision.

Manipulatives enrich the learning environment. Young learners are more engaged when using manipulatives, and their perception of their learning environment improves in the areas of enjoyment, understanding and efficiency (Cockett & Kilgour, 2015). The TMU framework greatly supports the use of mathematical manipulatives in South African schools for the purposes of promoting play-based learning. However, it is well documented that how the manipulatives are used is fundamental to developing children's mathematics (the mathematics is not automatically imbued in the artifacts). The publication and distribution of the minimum manipulatives packs is recommended for each grade in primary school mathematics. Ensure there is careful specification of the numbers of each required (for learner, class, grade and phase levels). A draft is presented in Annexure 3. Budget for the manipulative pack to be distributed with the learner workbooks every 5 years. Align the distribution of mathematics toolkits with the envisaged curriculum. LTSM and assessment reviews (in 5 and 10 year cycles).

Pedagogy: Does the LTSM build instructional rhythms?

To reflect on this criterion, it is worth asking: Does the LTSM make clear a clear rhythm of teaching – over the year, over the term, over a week and in a lesson?

Most researched workbooks deliberately build instructional rhythms. Instead of using daily lesson plans as lesson organisers, LTSM development can strategically use the term and week as lesson organisers. Weekly and termly rhythms of engagement are less pressurised; and they allow the teacher some space to provide remedial teaching as part of the teacher's ongoing reflection and evaluation of the teaching and learning process.

Rhythm of engagement by term

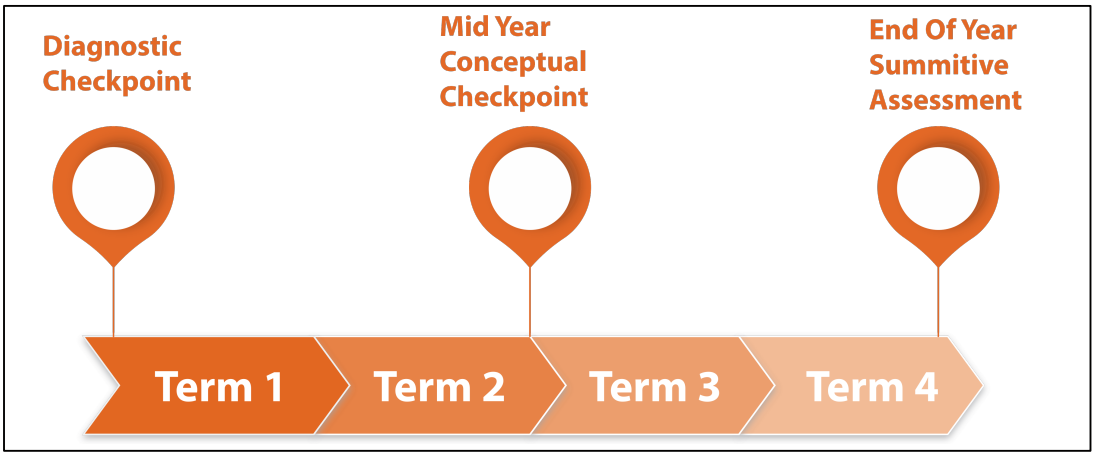


Figure 20: Rhythm of engagement by term

The rhythm of engagement by term implies that primary mathematics teachers; under the guidance of Provincial Education Department (PEDs) could assess learners as diagnostic assessment at the beginning of the year, a conceptual checkpoint midyear, and as a summative assessment against the conceptual checkpoints at the end of the year. Having such conceptual checkpoints, may assist teachers to address identified misconceptions and conceptual gaps (with appropriate guidance on how to take a step back to remediate).

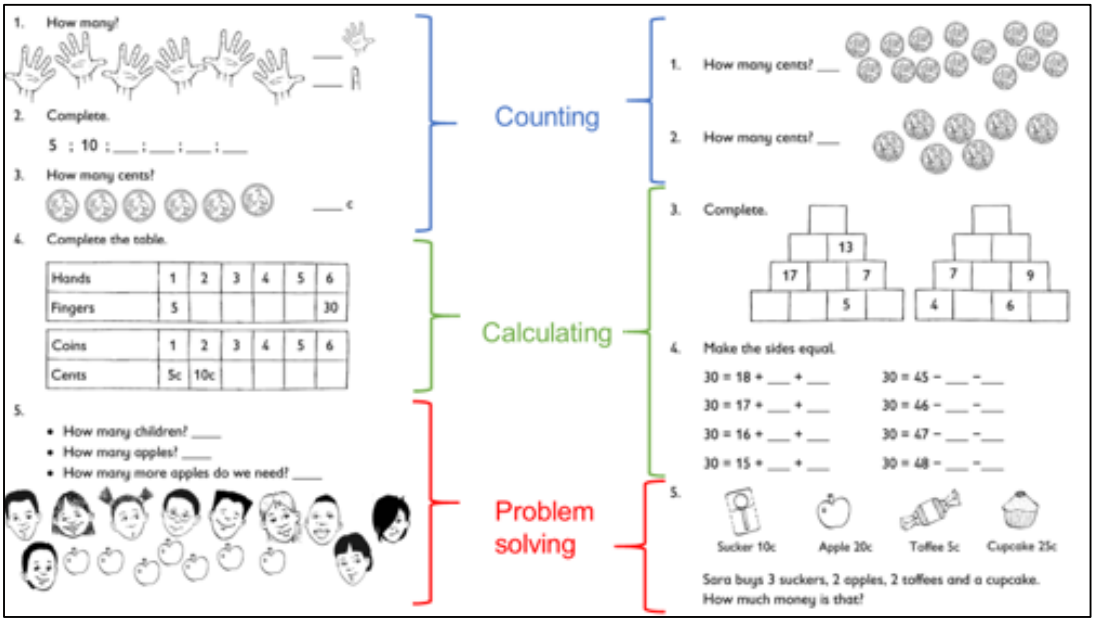


Figure 21: Sample pages from the NumberSense

Almost all South African mathematics LTSM use the four term format. Where there are difference in this structure, these relate to the number of productive weeks which are assumed to be possible in each term. The South African school calendar provides for 10 weeks per term. However when considering disruptions to schooling, was well as the need for diagnostic and summative assessment designing for a full 10 weeks of teaching is not viable. Bala Wande designs for 10 weeks each term, with the tenth week being dedicated to revision. MCC designs for 8 weeks per

term and provides formal assessment tasks which can be administered over a few days each term. The TMU pilot provides for 40 lessons each term (with 5 lessons per week, this assumes 8 teaching weeks per term).

In addition to termly and weekly content organisers, rhythm of engagement by page can also be used to organize workbook content. NumberSense workbooks provide a good example of rhythm of engagement by page. Brombacher and Roberts (2022) explain that "The NumberSense materials are expressly designed to enable learners to independently complete a page a day for each school day of the year." Brombacher and Roberts (2022) argue that "repeating the same page structure allows teachers to develop a ritual or working on a page a day, while ensuring that all three tasks – counting calculating and problem solving have been given some attention". Presenting workbook content in this sequence, allows the learner to systematically practice counting and calculation tasks before attempting problem solving activities. The skills assessed range from lower-order skills that require less cognitive processing (counting) to higher-order skills that require deeper learning and a greater degree of cognitive processing (calculating and problem solving); (Adams, 2015).

Bala Wande is explicit about the structure of each lesson. Here is an example from Day 1 of doubling where the "mental maths" and "show me a number" are followed by a game, then concept development and then worksheets.

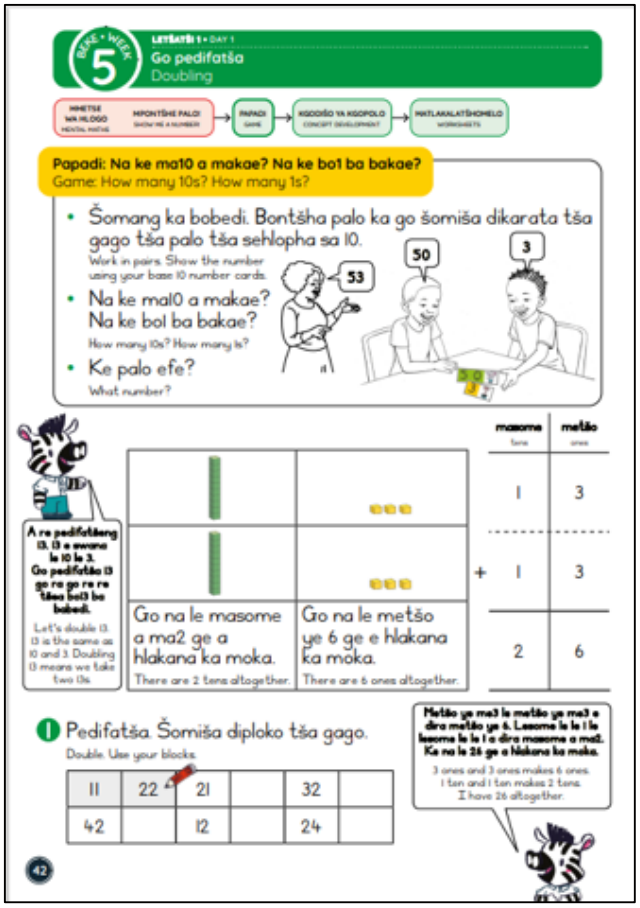


Figure 22: Sample pages from the Bala Wande

Notice the use of a zebra character offers the teacher's instructional narrative. The strategy for doubling a number is approached as taking two of that number and adding them together. The addition strategy is breaking up both numbers into tens and ones. The representation is column addition. The procedure is to first identify the tens and the ones and write these in a columns labelled tens and ones. The number is repeated and then the ones are added before the tens are added. There is no concrete or imaginable context in which the doubling function is structured.

Pedagogy: Does the LTSM include regular Assessment activities?

Workbooks should include practice assessment activities that are properly linked to the worked examples. The assessment activities should include easy medium and difficult items, intended to assess the least-able, average and most-able learners. Recommended formative assessments should be included in the workbooks, providing support to teacher in how to integrate these into the rhythm of engagement.

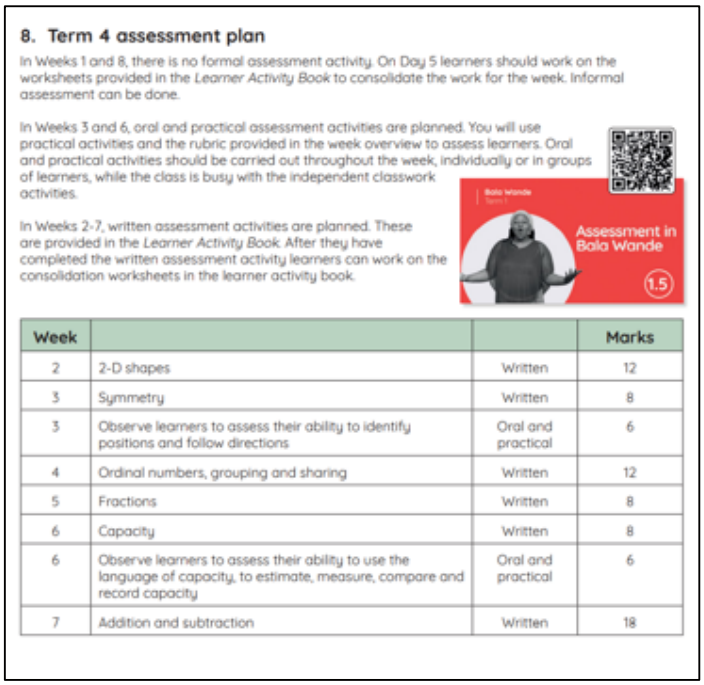
All the researchers learner workbooks include various kinds of assessments. At times these are offered separately from the workbook (in a set of FATS or in the teacher guide). The TMU pilot learner activity books include regular assessment tasks. The assessment page is blank, and the assessment task is provided to the teacher.

Figure 23: MCC example of weekly "assess and practice" page Grade 2 LAB

Each umthamo in the MCC material includes both a mini assessment (focused on the work of the umthamo). In addition, in each week, there is a practice page which is focused more broadly, reviewing content across previous imithamo. Notice that although the focus of the week was "we

can multiply and divide 2s, 5s, and 10s" the practice page includes additive relations, as well as number patterns.

Most of the reviewed LTSM includes formal assessment activities for each term. Workbooks need to have assessment activities for each school term. There is a need for diagnostic activities administered at the beginning of each term and summative assessments administered at the end of the term. The end of term summative assessments may be used formatively at the beginning of the following term.



8. Term 4 assessment plan

In Weeks 1 and 8, there is no formal assessment activity. On Day 5 learners should work on the worksheets provided in the *Learner Activity Book* to consolidate the work for the week. Informal assessment can be done.

In Weeks 3 and 6, oral and practical assessment activities are planned. You will use practical activities and the rubric provided in the week overview to assess learners. Oral and practical activities should be carried out throughout the week, individually or in groups of learners, while the class is busy with the independent classwork activities.

In Weeks 2-7, written assessment activities are planned. These are provided in the *Learner Activity Book*. After they have completed the written assessment activity learners can work on the consolidation worksheets in the learner activity book.

Week			Marks
2	2-D shapes	Written	12
3	Symmetry	Written	8
3	Observe learners to assess their ability to identify positions and follow directions	Oral and practical	6
4	Ordinal numbers, grouping and sharing	Written	12
5	Fractions	Written	8
6	Capacity	Written	8
6	Observe learners to assess their ability to use the language of capacity, to estimate, measure, compare and record capacity	Oral and practical	6
7	Addition and subtraction	Written	18

Figure 24: Bala Wande term 4 assessment plan

Bala Wande teacher guides have an explicit assessment plan each term. This is explained in the Bala Wande teacher guide. An example mark capture sheet and how to use these assessments for a mark towards each content are is provided.

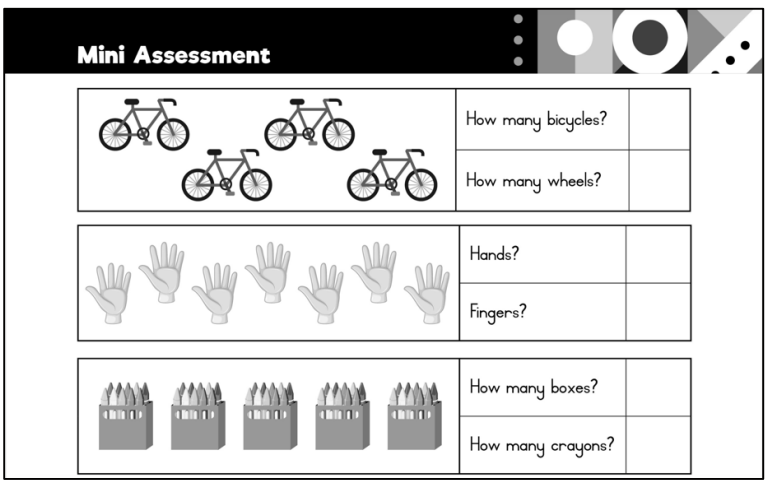


Figure 25: MCC Mini assessment Grade 2 workbook

Both NumberSense and MCC LTSM materials include term by term Formal Assessment Tasks for each grade. These are separate from the LABS so they can be administered under invigilated conditions. The TMU workbooks provide both termly assessment activities as well as lessons preparing learners for the upcoming terms as illustrated in Figure 26:

Contents

Figure 26: TMU pilot learner activity book Grade 2

LTSM developers express difficulties with align the assessment tasks with the requirements for data in SA-SAMS. The South African School Administration and Management System (SA-SAMS) is a freely available electronic platform that aims to assist schools with administration and reporting, including the systematic recording of learner performance scores. Its most recent update in 2021 produced SA-SAMS 21.1.1, a version which incorporated revisions of the Annual Teaching Plans (ATPs), Programs of Assessment (POAs) and amendments to Section 4 of the Curriculum and Assessment Policy Statements (CAPS). The treatment of assessment activities in LTSM development should inform the periodic revisions of SA-SAMS to promote system wide alignment. SA-SAMS should be aligned to a more flexible curriculum sequencing.

Grade 2 TERM 1 FAT (and terms 2, 3, and 4) all assess all 5 content areas

MATHEMATICS MARKING SHEET

Formal Assessment Task: Term 2				
DAY/SECTION		CAPS Goals	SCORE	LEVEL
		Number, Operations & Relationships	/60	/7
Day 1	A	Number Sense	/20	
	B	Addition and Subtraction	/20	
Day 2	C	Multiplication	/20	
Day 3	D	Patterns, Functions and Algebra	/15	/7
	E	Shape and Space	/10	/7
Day 4	F	Measurement	/10	/7
	G	Data Handling	/5	/7
		TOTAL:	/100	/7
		Bonus Questions	/2	

Figure 27: MCC formal Assessment task marking sheet

Mathematics

When reflecting on the mathematical aspects of the LTSM, the underlying philosophy relating to what is mathematics and how it ought to be taught becomes pertinent. The CAPS specifies the mathematical content to be taught, as well as broad approach to mathematics.

There remain significant differences in how the mathematics is treated, and how developers envisage that children will come to learn mathematics. For example, there is a fundamental difference in the approached between

- LTSM designers who consider the primary purpose of mathematics to be accurate and efficient calculation using formal written algorithms; and
- LTSM designers who consider the primary purpose of mathematics to be develop mathematical thinking (doing what a calculator cannot do) which involves problem solving and justifying ones strategies to make reasoning clear.

This is what makes evidence of impact critical to selecting appropriate LTSM. One must ask: Has the LTSM material been shown to have impact on learner outcomes?

LTSM development should always be accompanied by teacher support focusing on reinforcing the strategies suggested in the LTSM and the implementation of the LTSM across different contexts. Workbook development needs to be accompanied by comprehensive teacher support opportunities reinforcing the suggested strategies, as is the case with the MCC Project. Teacher support should focus on capacitating primary mathematics teachers on:

- reflective use of print, manipulative and online formats of LTSM;
- how to adapt and design learner activities;
- how to manage diagnostic assessments;
- how to diagnose problems and address them during the teaching and learning process; and
- scaffolding the learning content.

Table 5 provides an overview of these mathematics design criteria in relation to each workbook considering how the LTSM supported mathematical sense making for both teachers and learners.

Table 6: Reflecting on mathematics sense making for teachers in LTSM attributes informed by workbook purpose

How does the LTSM support the mathematical sense making of teachers and learners? Does the LTSM workbook…	**DBE workbooks** Grade R to 6 Supplementary learner activity book	**TMU pilot** Grade 1 to 3 Daily lessons to drive teacher instruction	**NumberSense** Books 00-24 (Grade R to 6) Differentiated book series, offering a page a day for independent learner work	**Magic Classroom Collective** Grade R to 3 Daily lessons to drive teacher instruction	**Bala Wande** Grade R to 3 Daily lessons to drive teacher instruction	**PMRP** Grade 4 to 6 Daily lessons to drive teacher instruction
…include support and signaling to the teacher?	Not intend ed - supplementary activity book	In teacher guide (concept booklet)	In teacher training materials and mental warmups	Detailed explanation of procedure, representation and strategy though using characters	Detailed explanation of procedure, representation and strategy though using characters	In teacher guide
…explicitly develop well researched learning trajectories (conceptual threads)?	None evident	Guided by Japanese approaches, C-P-A (Concrete-Pictoral-Abstract).	Create meaning from the real or imaginable word problem context, work with functions, relationships and patterns, spiralling curriculum (highest number range for counting, lower number range for calculating, lowest number range for word problems).	Develop mental imagery for discrete and continous contexts from concrete or imaginable contexts. Focus on 2s, 5s and 10s, (using number pictures and number lines).	Develop mental fluencies and written calculation procedures making use of concrete manipulatives as to infuse written calculation methods with meaning.	Context-free calculations followed by word problems focusing on the 4 basic operations. A stong emphasis on formal written algorythms for all 4 operations
…provide plenty of opportunities for children to make meaning from the tasks and instructional narrative?	Practice activities available, but not structured	Emphasis on bare calculations and break up both number (place value) strategies.	Encourages a range of strategies. Compares different approaches through contrasting learner methods in higher levels. Uses clueboards, addition chains and pyramids, function machines and tabulations	Systematically develops a "go-to" strategy. Break up the second number for addition and subtraction. Skip counting (using clueboards and arrays) for multiplication and division.	Emphasis on bare calculations and break up both number (place value) strategies.	Practice activities available but not structured

…allow for some differentiated teaching within a class?	Designed as supplementary - so could be busy book for stronger learners	Extension tasks are included in the teacher guide	Yes. Strong "teach at the right level" design, with differentiated series of books 00-24	Minimal and whole calss pacing assumed. 2-page spread is designed for first page to be core and 2nd page consolidation.	Teacher guide includes sets of extension activities for each day	Activities organised into different levels of difficulty, remediation and enrichment exercises provided

We now discuss each of the mathematics related questions in turn.

Mathematics: Does the include mathematical support and signalling to teachers?

Given that we know that the mathematical knowledge of teachers in South African primary schools is very weak, it is important that the LTSM materials offer support and signalling to teachers about the mathematics. Once of the ways to do this is to offer detailed explanations through worked examples.

Workbooks should contain (relevant, suitable and adequate) worked examples that guide learners in their independent, individual work unaided by the teacher. Workbooks should providing bare calculations alongside word problems to assist learners in understanding concept development. Adequacy and relevance of suitable worked examples will reinforce learners' grasp of content. Worked examples also serve as serve as source of feedback for the learners to look back after making efforts to solve a problem. In particular, a worked example should make explicit three components: the representation; the strategy; and the procedure (Roberts 2019)

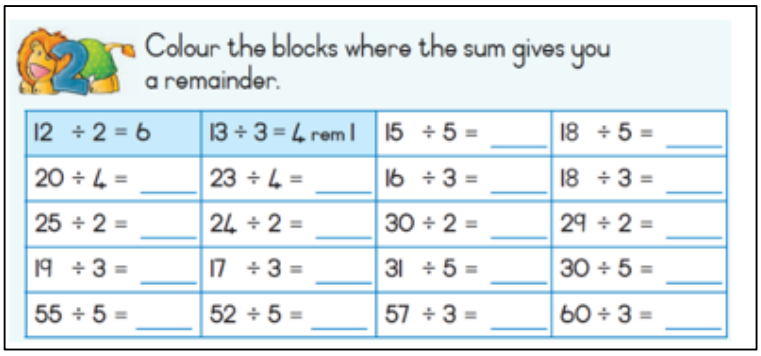


Figure 28 DBE workbook Grade 3

This example from the DBE workbook, provides two examples with the correct answers, but it offers no support on how these answers are arrived at. So while a worked example is present, this does not offer support or signalling to the teacher.

In contrast, this MCC workbook offers a specific strategy; representation and procedure for division

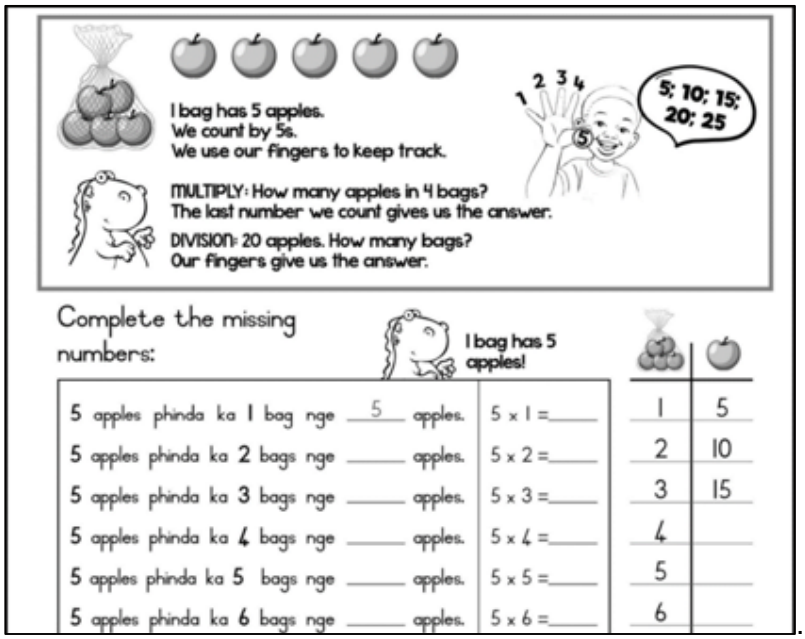


Figure 29: MMC workbook Grade 2

In this MCC example, all three elements are made explicit:

- the **strategy** for division is skip counting upwards to reach the target;
- the **representation** is tracking the counts on fingers; and
- the **procedure: t**o divided 20 by 5: We count by 5s to reach a target of 20. So we say 5, 10, 15, 20 while we keeping track of the 1 , 2, 3, 4 counts on our fingers. When we reach the target we stop. The answer is on our fingers".

The dinosaur character – Zibalo's - words are the expected instructional narrative for the teacher.

Also noteworthy in this MCC is example are several features important for early grade mathematics in Figure 29. A concrete or imaginable context of packing apples with 5 apples in each bag is provided. With this context in place, a function of ×5, is then explored systematically. The instructional sentence: "5 apples repeated in 2 bags is…" offered in isiXhosa. The notion of multiplication as repeating a unit, leverages the isiXhosa linguistic expression "phinda ka" (repeated) and this avoids the linguistic confusion creates by 5×3 being expressed as "5 groups of 3", as is common in English. A number sentence (symbolic syntactical representation) follows the linguistic expression, and finally a "t-table" making explicit the relationship between bags and apples is offered (t-tables and clue boards are encouraged in CAPS, for later use in long division and as a necessary requirement for functional thinking in Senior Phase).

Finding 1: Across the LTSMs reviewed various ways in which LTSM supports teacher development are evident, which are linked to their varying design purposes.

- The workbooks are used simply to provide practice tasks for learners (the is the case for the DBE workbooks which were designed as "busy books");
- The LTSM is accompanied by a teacher guide (this is the case for Bala Wande and the TMU pilot materials);

- The LTSM offers some differentiated practice activities for independent learner work (NumberSense and PMRP) and,
- The LTSM include instructional prompts and support for the teaching of concepts in the learner activity books themselves (this is the case for MCC, NumberSense and Bala Wande).

We discuss and exemplify each in turn. Again when choosing an LTSM offering, the desired teacher support and intent of the learner workbooks is a fundamental consideration.

(1) LTSM is used to provide practice tasks to learners

The DBE workbooks do not offer any strategies, representations or procedures. They offer opportunities for practicing division calculations.

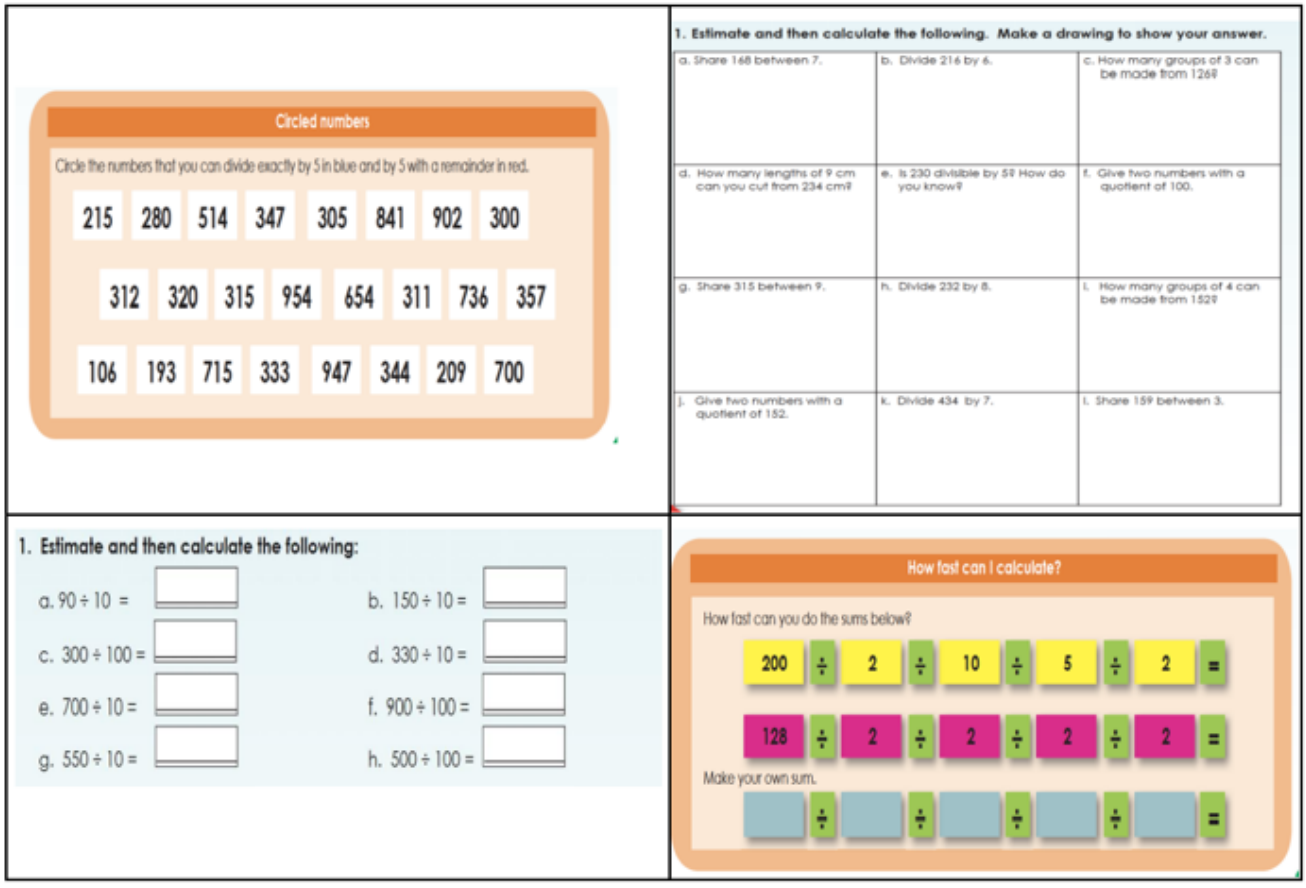


Figure 30: DBE workbook Grade 6

In Grade 6, at the point when children are expected to use the standard written algorithm for long division, then some instructional scaffolding is presented through the use of speech bubbles. Notice the introduction of a T-table or "clue-board", for long division. This was not presented in earlier grades (although entrenching its use for division, would have supported short division calculations).

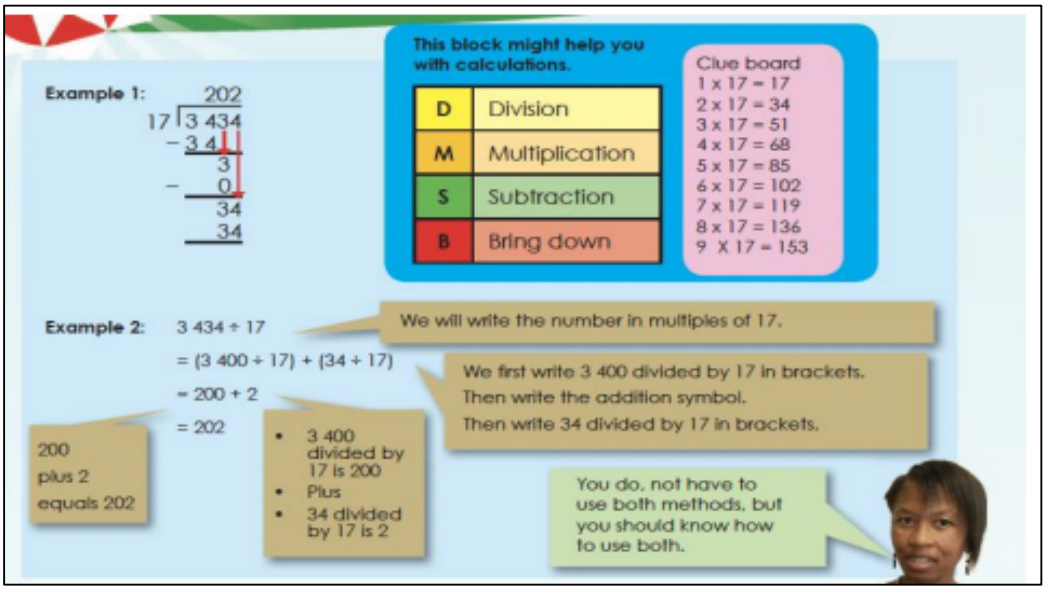


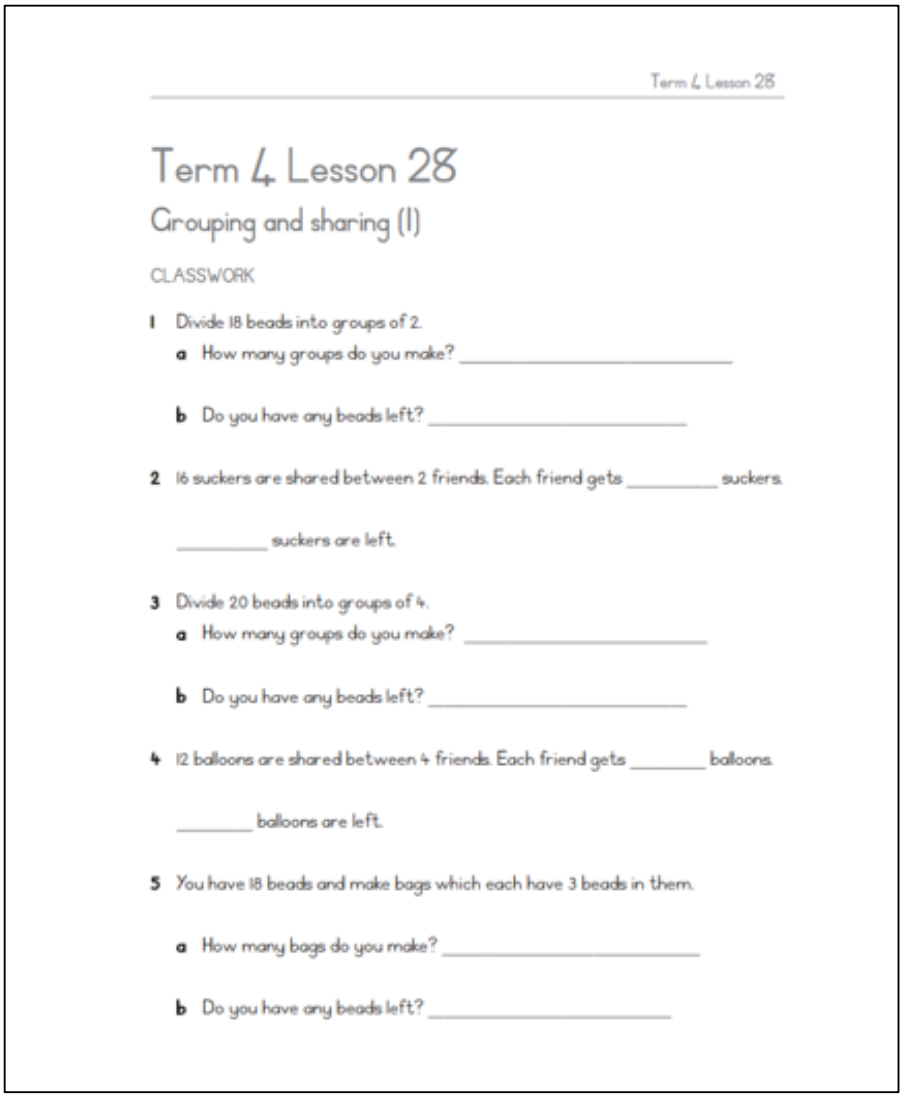
Figure 31: DBE workbook Grade 6

(2) **The LTSM is accompanied by a teacher guide or concept guide**

In this example from Bala Wande, the teacher guide offers a short video on the concept development activity, as well as a photographic sequence of what is expected.

For the TMU pilot materials, division is approached in relation to sharing and grouping problem contexts. There is no preferred strategy, procedure of representation offered in the LAB.

In the Bala Wande LABs halving in Grade 2 (see Figure 32 below) is approached using base ten blocks and place value tables. Whether teachers will use the teacher guide, and be able to move between the learner book and the guide – and how much support they will need to attain this, are important considerations.



Term 4 Lesson 28

Term 4 Lesson 28

Grouping and sharing (I)

CLASSWORK

1 Divide 18 beads into groups of 2.

a How many groups do you make? _____

b Do you have any beads left? _____

2 16 suckers are shared between 2 friends. Each friend gets _____ suckers.

_____ suckers are left.

3 Divide 20 beads into groups of 4.

a How many groups do you make? _____

b Do you have any beads left? _____

4 12 balloons are shared between 4 friends. Each friend gets _____ balloons.

_____ balloons are left.

5 You have 18 beads and make bags which each have 3 beads in them.

a How many bags do you make? _____

b Do you have any beads left? _____

Figure 32: TMU pilot Grade 4 LAB

Compare the instructional scaffolding evident in this TMU workbook, to that which is offered to the teacher for doubling in the MCC and Bala Wande examples presented earlier. The TMU pilot encourages different strategies for division, but only makes these options explicit in the teacher guide (content booklet).

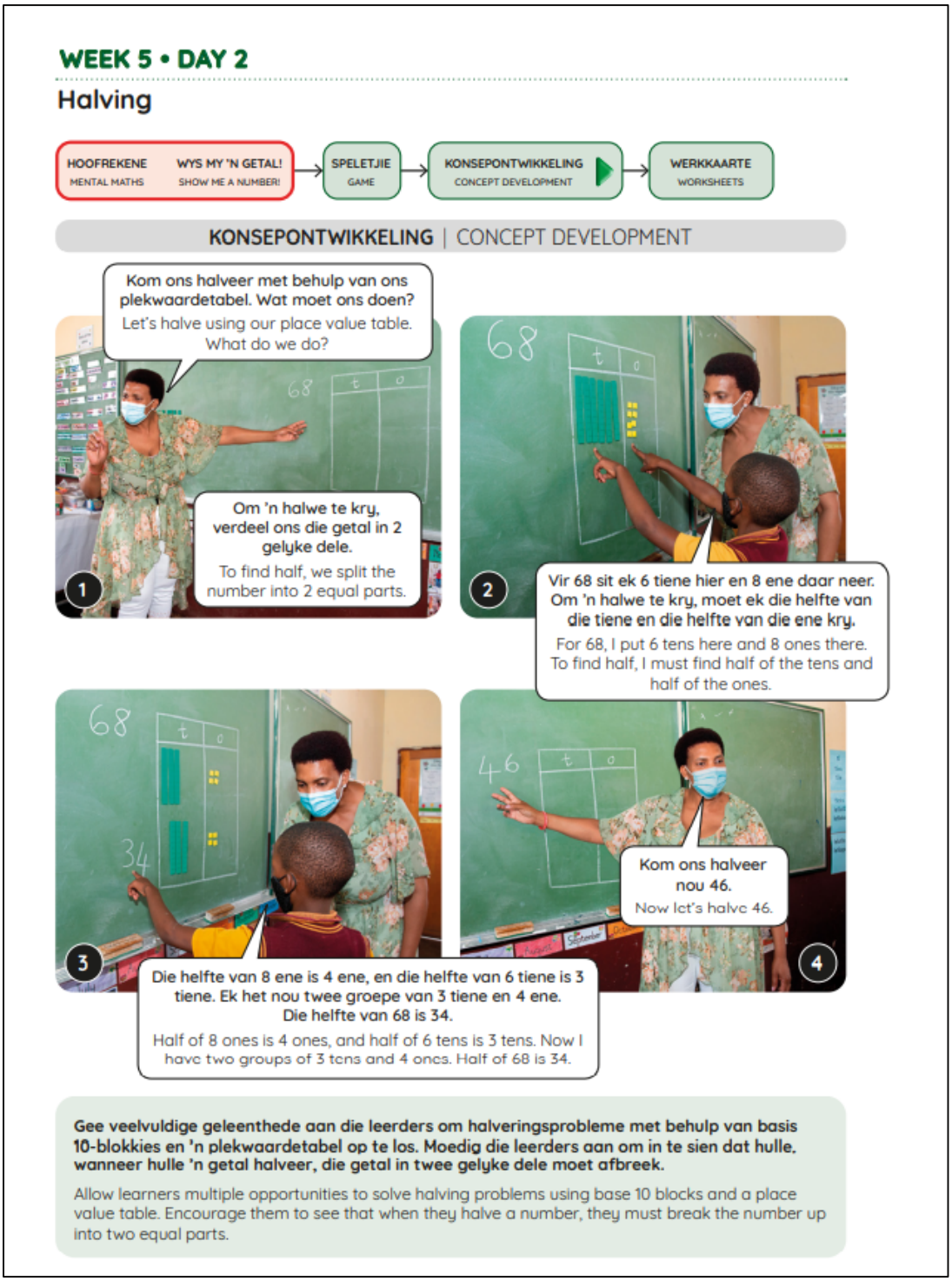
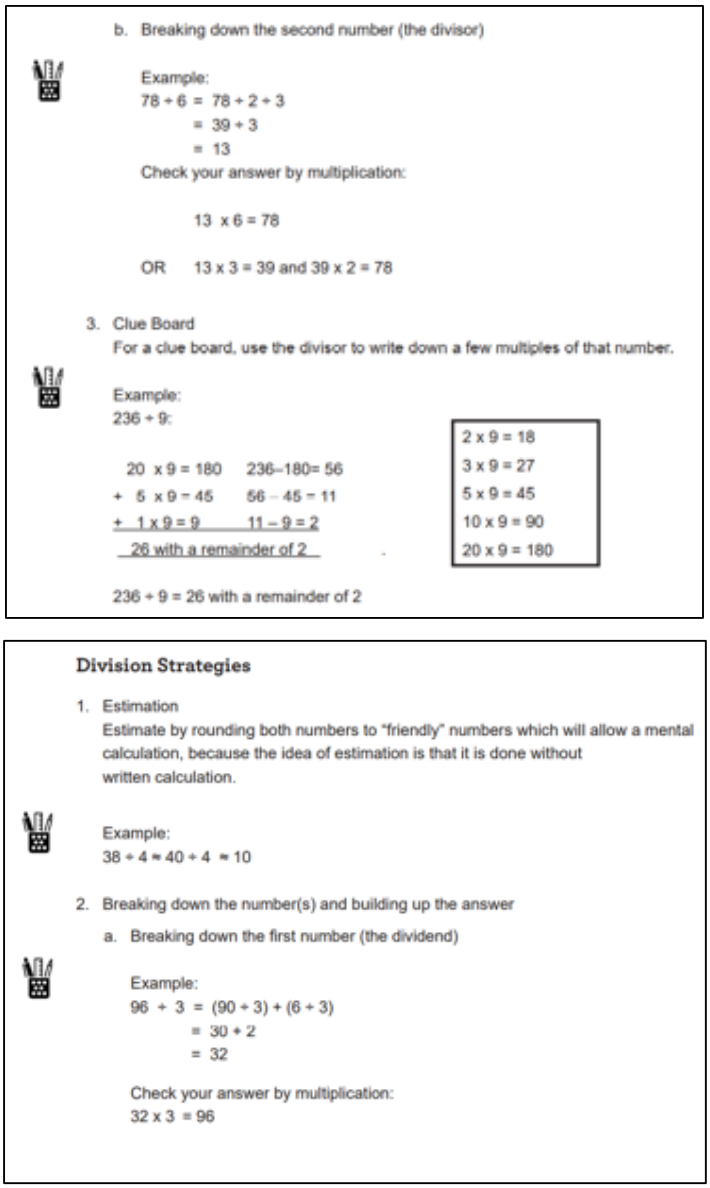


Figure 33: Example of photo-narrative in Bala Wande teacher guide



b. Breaking down the second number (the divisor)

Example:
78 ÷ 6 = 78 ÷ 2 ÷ 3
= 39 ÷ 3
= 13
Check your answer by multiplication:

13 x 6 = 78

OR 13 x 3 = 39 and 39 x 2 = 78

3. Clue Board
For a clue board, use the divisor to write down a few multiples of that number.

Example:
236 ÷ 9:

20 x 9 = 180 236–180= 56
+ 5 x 9 = 45 56 – 45 = 11
+ 1 x 9 = 9 11 – 9 = 2
26 with a remainder of 2

2 x 9 = 18
3 x 9 = 27
5 x 9 = 45
10 x 9 = 90
20 x 9 = 180

236 ÷ 9 = 26 with a remainder of 2

Division Strategies

1. Estimation
Estimate by rounding both numbers to "friendly" numbers which will allow a mental calculation, because the idea of estimation is that it is done without written calculation.

Example:
38 ÷ 4 ≈ 40 ÷ 4 ≈ 10

2. Breaking down the number(s) and building up the answer

a. Breaking down the first number (the dividend)

Example:
96 ÷ 3 = (90 ÷ 3) + (6 ÷ 3)
= 30 + 2
= 32

Check your answer by multiplication:
32 x 3 = 96

Figure 34: TMU pilot Mathematics content booklet – targeted support Grade 4

Three different strategies are offered: Estimation, breaking down and building up, and using a clue board. The representations are number symbols. The procedure is demonstrated symbolically (with each step of the process shown using number symbols and operations on a new line), but is not expressed in words. There is no guidance as to when these strategies may or ought to be applied (in which grade level) or for which kinds of calculations. Teachers are expected to be familiar with these strategies and invoke any one of them, as needed when engaging with the children as they work through the learner activity book.

(3) The LTSM offers some differentiated practice activities for independent learner work (NumberSense and PMRP)

Brombacher and Roberts (2022) claim that the NumberSense workbooks are also intended to support the teacher.

First, the pages of the learner workbooks are, in effect, the teacher's lesson plan. The lesson begins with a teacher-led activity that sets the learner up to independently complete the workbook page as a consolidation of the teacher-led activity. Second, having teachers work through the materials to prepare for their lessons supports the development of a richer more robust understanding of the mathematics that they are teaching.

The NumberSense books provide scaffolding for teachers by presenting different learners strategies to the same calculation. This is more common in the higher grades, as the calculation strategies become more complex and require some written work.

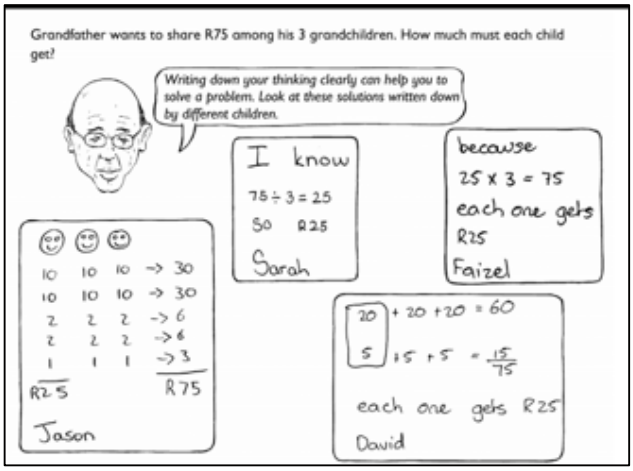


Figure 35: Book 24 NumberSense (approximately Grade 6)

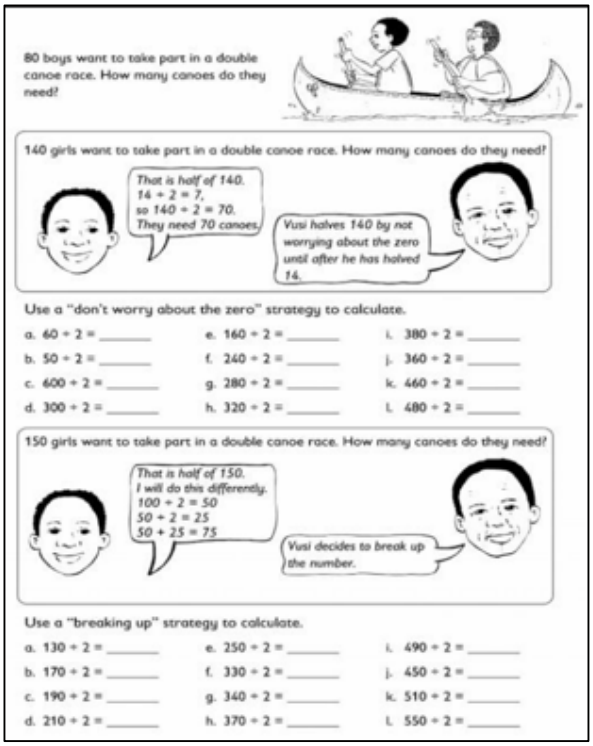


Figure 36: Book 17 NumberSense (approximately Grade 3 or 4)

In this example of division tasks from NumberSense, 24 calculations are offered on one page. To scaffold how these are approached, two different division strategies are provided: "don't worry about the zero" and "break up the number". The representations used are symbolic syntactical – showing, and procedure is implicit in that each step of the process is expressed using number symbols and operations. Later in the NumberSense series (at Grade 6 level), different solutions are offered for the same division calculation (75 ÷ 3 = ...). In this way, the NumberSense workbooks offer some teacher scaffolding as to the approach to a division calculation. Which approach is more efficient for the particular number involved in the discussion, may be discussed by the skilled teacher (but is not prompted in the workbook).

(4) In Bala Wande and MCC the LTSM include instructional prompts and support for the teaching of concepts in the learner activity books themselves

In these two reviewed LTSM learner workbooks, teacher development is intentionally a key component of the learner workbook design.

The Bala Wande LTSM toolkit includes teacher videos, a teacher guide, and has characters in the learner workbooks which offer instructional narrative (key sentences) to the teacher during the lesson.

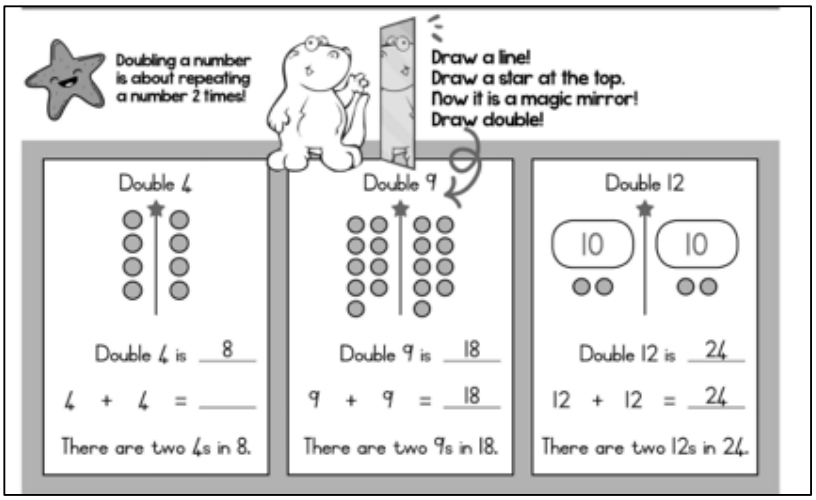


Figure 37: MCC Grade 1 Learner activity book

The MCC workbooks provide teacher support directly in the learner workbook. This example of how doubling and halving is approached in the MCC materials, makes clear the support on strategy, procedure and representation which teachers can glean from the pages of the leaner workbook.

Doubling is explained as "repeating a number 2 times". The representation used is a number picture (structured by 5s and 10s) and a metaphor of a magic mirror line is used. Doubling when numbers are bigger than 10 (using the repeated 10 structure) is shown to make clear the developmental trajectory from doubling single digit numbers to doubling double digit numbers. With the doubling concept in place, further opportunities to double are offered.

In the same umthamo, the concept of halving is introduced. The dinosaur character Zibalo provides the instructional narrative to the teacher "When I share equally between 2 children, each child receives half". This phrasing allows the sentence to be repeated for sharing equally between 3,4 or 5 children.

In addition, once the sharing metaphor for halving is secure, the grouping approach for halving is introduced. While this is not necessary for correctly halving, it is a conceptually important part of understanding division. The sharing metaphor becomes inefficient for larger numbers. All division algorithms use a grouping metaphor. For this reason, the instructional narrative for a grouping metaphor for halving is included. Once again, Zibalo- offers the instructional narrative. The teacher is able to explain division by repeating Zibalo's explanation: When we divide, we separate into equal groups. How many 2s in 8?

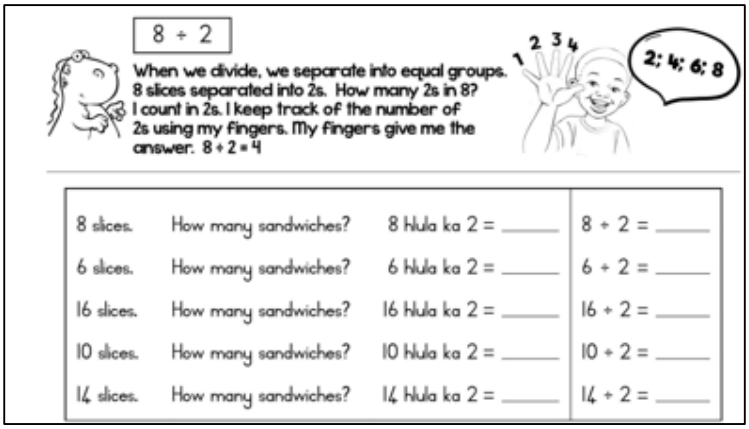


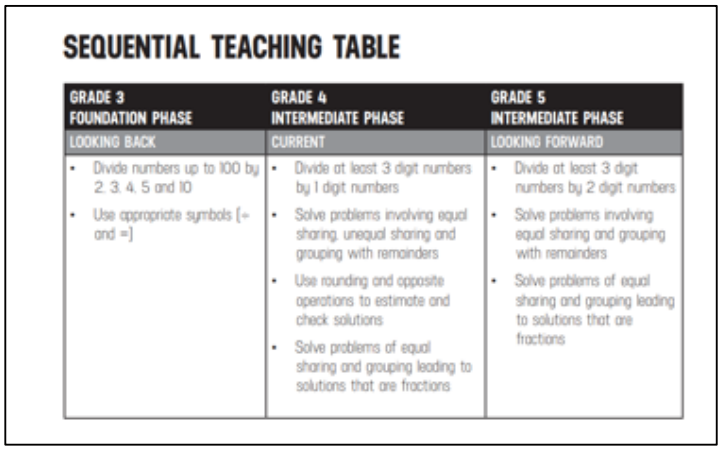
Figure 38: MCC Grade 2 Learner activity book

The strategy for calculating now makes use of the grouping metaphor for division and requires skip counting in twos to reach a target. The procedure is articulated by Zibalo: "I count in 2s. I keep track of the number of 2s using my fingers. My fingers give me the answer." The representation used is oral counting and using fingers to keep track of the counts.

The MCC offers the most detailed and explicit scaffolding and support to teachers, offering very limited "go-to" calculation strategies. In other LTSM, a wider range of strategies are presented (particularly in the teacher guides), and there is far less explicit guidance in the learner workbook.

Mathematics: Does the LTSM explicitly develop well research learning trajectories (conceptual threads) within and across grades?

In LTSM development, mathematical coherence relates to the strategies, representations and procedures used to explain the content. The mathematics can only be coherent if it is accurate. The representations, strategies and procedures should be conceptually coherent and mathematically connected in a way that allows mathematical ideas to build conceptually and promote learner understanding (Henderson Pinter, Merritt, Berry III, & Rimm-Kaufman, 2018). For a coherent presentation of concepts in primary mathematics workbooks, the representations, procedures and strategies used to explain the problem and the action followed to solve the problem should be aligned.



SEQUENTIAL TEACHING TABLE

GRADE 3 FOUNDATION PHASE	GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Divide numbers up to 100 by 2, 3, 4, 5 and 10 • Use appropriate symbols (÷ and =)	• Divide at least 3 digit numbers by 1 digit numbers • Solve problems involving equal sharing, unequal sharing and grouping with remainders • Use rounding and opposite operations to estimate and check solutions • Solve problems of equal sharing and grouping leading to solutions that are fractions	• Divide at least 3 digit numbers by 2 digit numbers • Solve problems involving equal sharing and grouping with remainders • Solve problems of equal sharing and grouping leading to solutions that are fractions

Figure 39: TMU pilot sequential teaching table for Grade 4 whole number division

Some designers may consider a statement of content (learning objectives) over time to constitute a learning trajectory. They may simply indicate the expected progression from one grade to the next within a particular topic. For example, the TMU pilot content books offer a sequential teaching table for each topic in a grade, which provides the curriculum expectation for the focal grade, the previous grade and the grade which follows.

However, the above is not a learning trajectory but rather a specification of grade expectations. It describes what content needs to be covered, but it does not scaffold by being explicit about strategy, representation and procedures – how these concepts are developed over time.

The deep dive analysis – into the particular mathematics topic of whole number division – provides far better insight into what is meant by a learning trajectory. We consider the three LTSM offerings – The DBE workbook, the TMU framework, and the NumberSense learner workbooks – to illustrate what is meant by "quality research informed learning trajectories" (and how to recognise their absence).[7]

Learning trajectory for division in DBE workbooks

First, we consider the DBE workbooks. Recall that these were designed as supplementary learning materials and not intended to support teacher instruction.

[7] The deep dive analysis considered only learner tasks related to whole number division. This excluded attention to divisibility rules, division used to introduce the fraction of a unit or the fraction of a group, common fractions, decimal fractions or percentages.

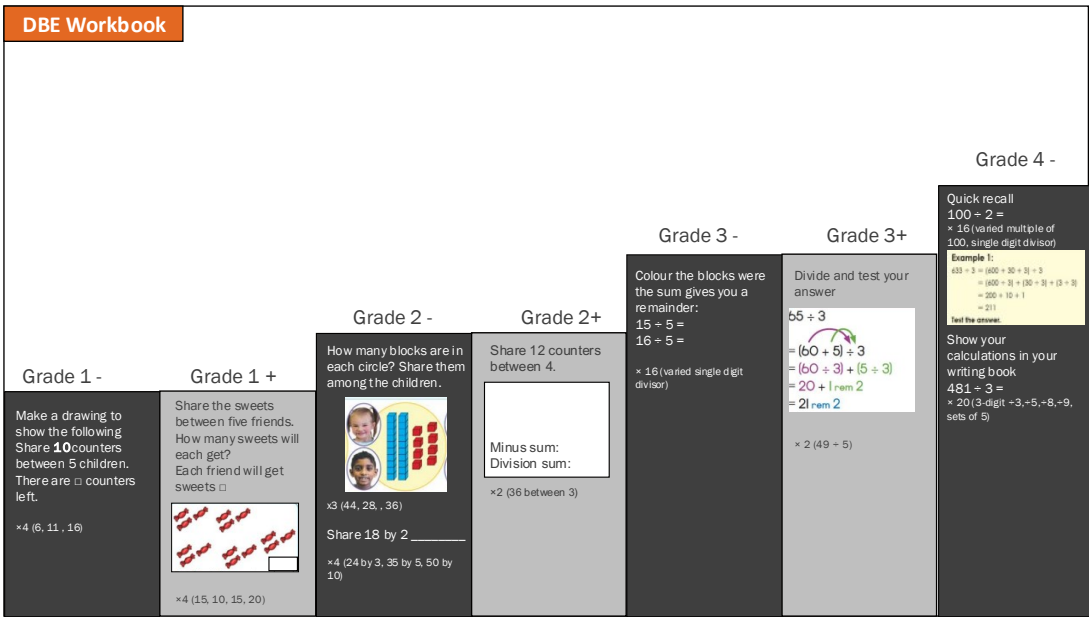


Figure 40: DBE workbooks – Progression of tasks on division with whole numbers Grade 1 to Grade 3

What is striking about this mapping of the division tasks over time is the paucity of opportunities to divide from Grade 1 to 3. In addition, there is little consistency in the strategies, procedure and representations used.

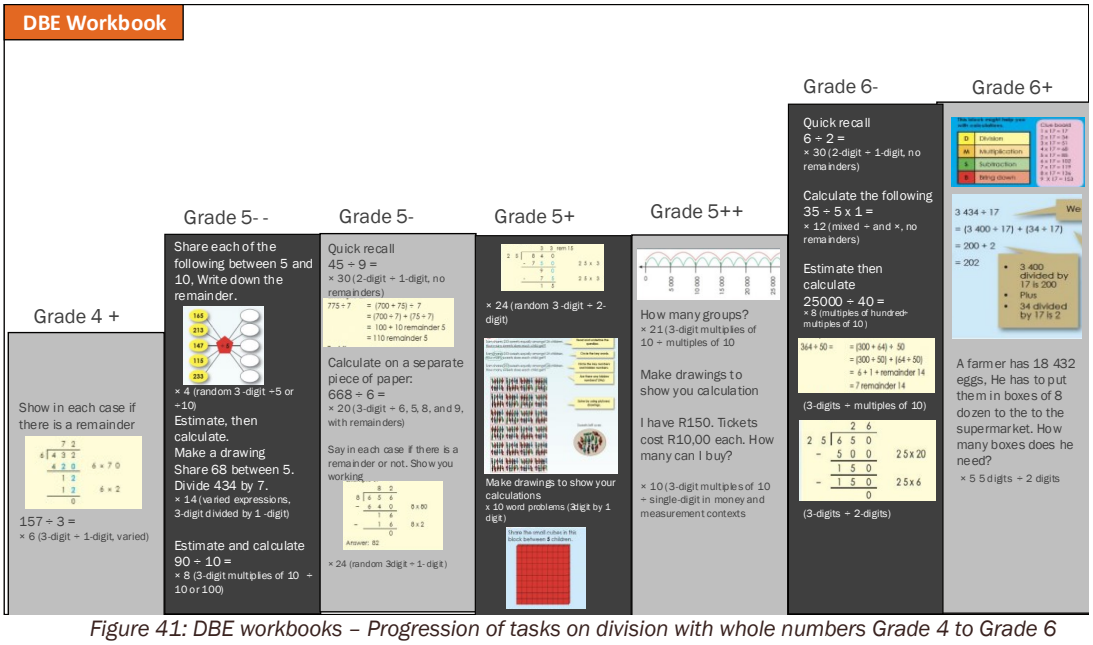


Figure 41: DBE workbooks – Progression of tasks on division with whole numbers Grade 4 to Grade 6

The DBE workbook does not offer a coherent learning trajectory of whole number division: there are few division tasks, grouping and sharing are not made explicit, division is treated separately from multiplication, division is not offered in meaningful contexts beyond Grade 1 and the focus on Grades 3 onwards is on procedural fluency with two division calculation strategies (the long division algorithm and flexibly breaking up the divisor).

We know that learners and teachers tend to work with large numbers as discrete objects (a long string of ones) without structuring the using the place value of 1s, 10s, and 100s. Therefore, we expect that when working with discrete objects, the LTSM instructional prompts support structuring in 2s, 5s, 10s and 100s or just using place values of 1s, 10s and 100s.

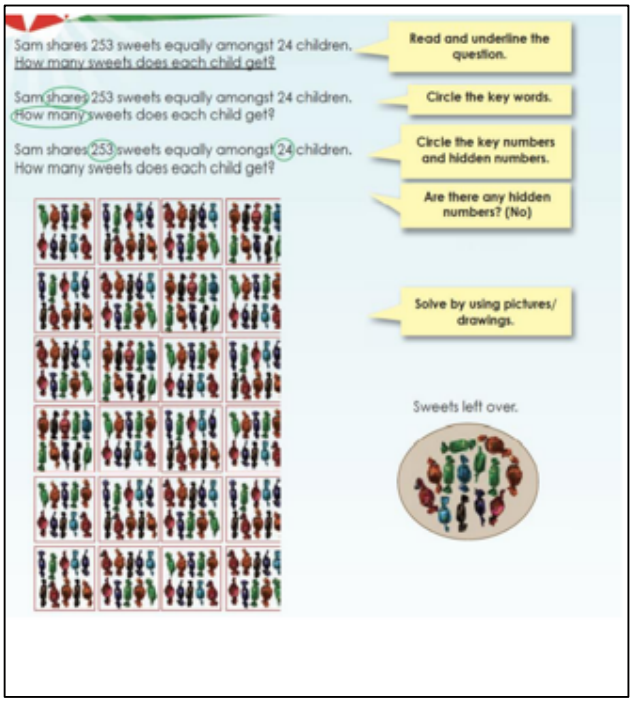


Figure 42: DBE workbook Grade 5

The DBE Grade 5 workbook offers an example which clearly shows that the learning trajectory for division is not well considered. This example shows an example of long division involving a 3-digit number where an unstructured drawing in ones is offered as the strategy.

This example is problematic not just because of its use of discrete objects – a 4 by 4 array of groups of 10 ones – but because it uses a sharing metaphor for division. None of the written algorithms use a sharing metaphor. They all use grouping, considering how many of the divisors can be subtracted from the dividend. While sharing is how division is usually introduced, in the Foundation Phase, it is the grouping metaphor which supports the development of structured and efficient calculation strategies, either involving skip counting up to reach a target or repeated subtraction from the dividend.

Learning trajectory for whole number division in TMU pilot

As presented earlier, the TMU pilot provides three strategies for division of whole numbers in its concept book for teachers: Estimation, breaking down the numbers and building up the answer and clue boards. The TMU pilot offers more opportunities to divide, but there are still very few division tasks in the early grades.

TMU pilot

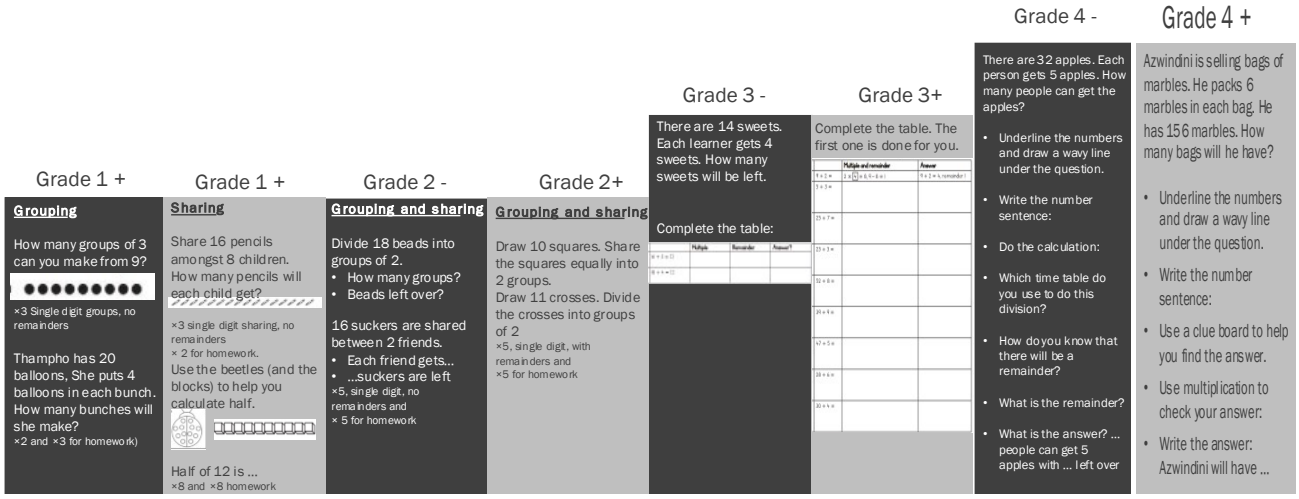


Figure 43: TMU pilot – Progression of tasks on division with whole numbers Grade 1 to Grade 4

The TMU workbook offers a more coherent learning trajectory of whole number division: there are more division tasks, grouping and sharing are made explicit, and division tasks are presented in meaningful contexts in Grade 1 and then as isolated word problems thereafter. There is more evidence of a learning trajectory as children are first exposed to division by sharing (their dominant model for division) and encouraged to share iconic representations between small groups of people. Later in Grade 1 the division concept of grouping is offered, and again, iconic representations are arranged into equal groups. By Grade 2, both sharing and grouping models are used. In Grade 3 and Grade 4 division, contexts are presented as real or imaginable problem situations, and the strategy, procedure and representations to be used are left to the children.

The TMU workbook's learning trajectory for whole number division focuses on symbolic syntactical representations, and on procedural fluency for three division calculation strategies: Estimation breaking down and building up, and using a clue board[8]. The TMU does not exploit the inverse relationship between multiplication and division beyond using multiplication to check division. There is no evidence of supporting functioning thinking (which is described in some detail in relation to NumberSense LTSM).

Learning trajectory for whole number division in NumberSense

Several aspects of the NumberSense approach to mathematics are made clear when reflecting on the learning trajectory for whole number division across the set of workbooks.

[8] A clue board is a representation used to support division calculations involving large numbers. It is referred to as strategy in the TMU pilot.

The NumberSense learning trajectory for whole number division is the most developed and clearly draws on mathematics education research (particularly in the domains of mathematical thinking and early algebra). This is evident in the following:

- The use of real or imaginable problem contexts as a starting point is clear, with familiar contexts being revisited at a higher number range over time;
- Repeating the same problem contexts while slowly increasing the number range and introducing new representations (from iconic draws to tabulations and function machines, to number sentences using number symbols and operations);
- Division is presented as the inverse of multiplication and not on its own;
- Use of a tabulated clue board, function machine and array as powerful representations, which are required in later grades and into secondary school;
- Structuring iconic representations into groups of 2s, 5s or 10s;
- Offering a range of strategies, representations, procedures and representations (mostly using symbolic syntactical number symbols and operators), though using learner examples of different ways to solve the same calculation; and
- Supporting functional thinking by constraining the problem context to a particular multiplicative function (such as x2, x5 and x10 initially and extended to other relations later) which can be explored.[9]

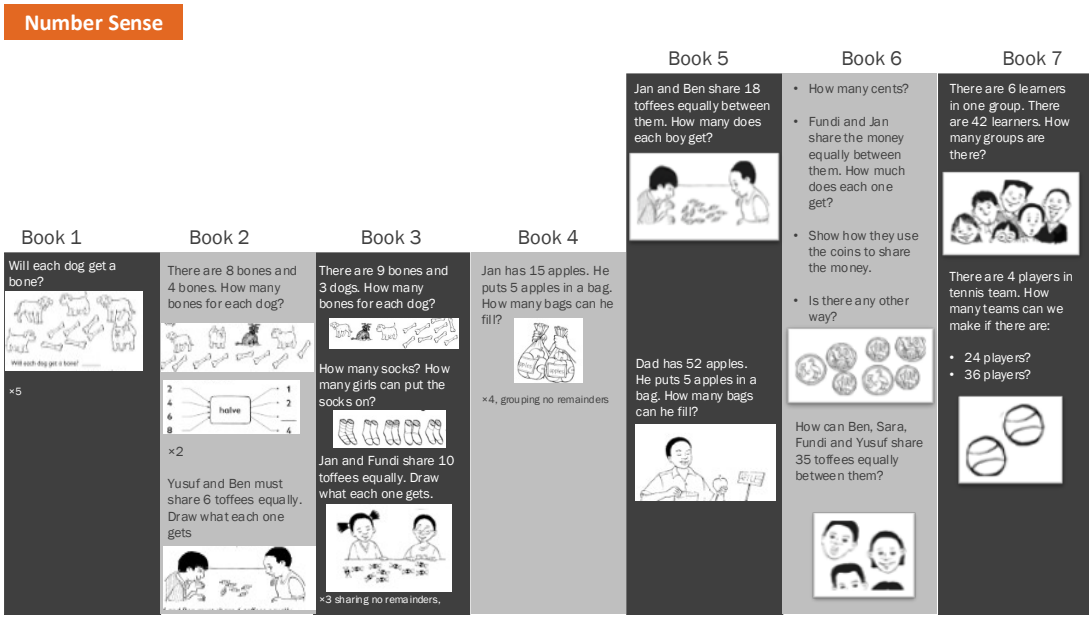


Figure 44: NumberSense – Progression of tasks on division with whole numbers from Book 1 to Book 7 (Approximately Grade 1 to Grade 4)

[9] This approach is highlight valued in the transition from arithmetic to algebra. This task's design draws on the Variation Theory in mathematics (where a particular dimension of a problem context is constrained in order to be able to reflect on the effect of varying each dimension in turn).

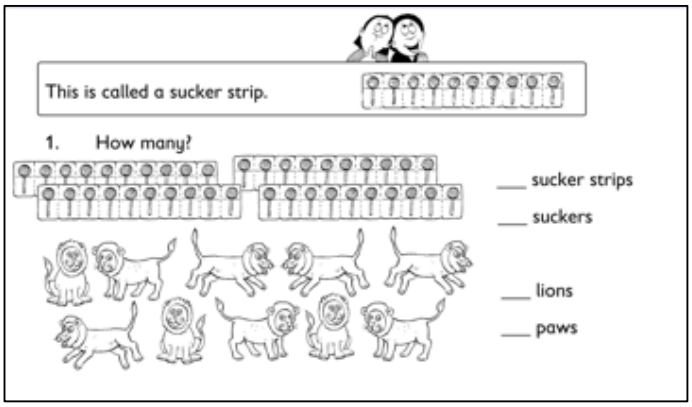


Figure 45: NumberSense Book 3

It is clear that the NumberSense workbooks made extensive use of real or imaginable problem contexts to introduce the mathematical concept of division. There is careful scaffolding of the same "dogs and bones" situation, which shifts from one to one matching to equal sharing. No specific strategy is offered to learners, but the iconic drawings are included to encourage matching, and learners are encouraged to "Draw what each one gets". While sharing and grouping are not explicitly named, the problems offered include both situations, so both processes are modelled by children in the early grades. The function machine representation is introduced early in Grade 1 and revisited repeatedly.

The iconic representation of pairs of socks in Book 3 is typical of the structural arrangement in the NumberSense materials. These are examples of pairs (structured by 2s). When working on multiplication and division contexts, structuring by 5s and 10s is a frequent feature of the problem context (eyes and people for ×2, hands and fingers for ×5, suckers and sucker strips for ×10, mammals and paws for ×4).

The same problem situations are repeated with increasing number ranges and representations. For example, consider the learning trajectory evident for the relationship between hands and fingers (×5 function). In Book 5, there are 7 pages which include the hands: fingers structure.

The NumberSense design is for the average learner to complete one book each term. So over the course of a year, children encounter the hands: finger problems more than 20 times (with increasing sophistication).

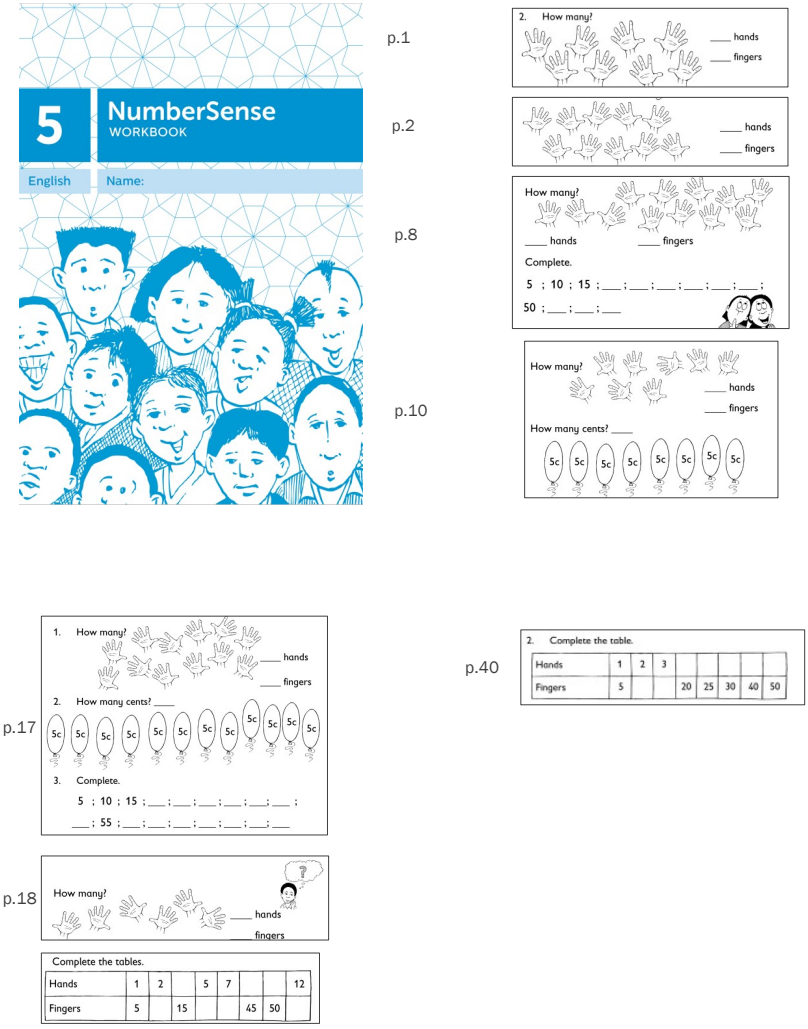
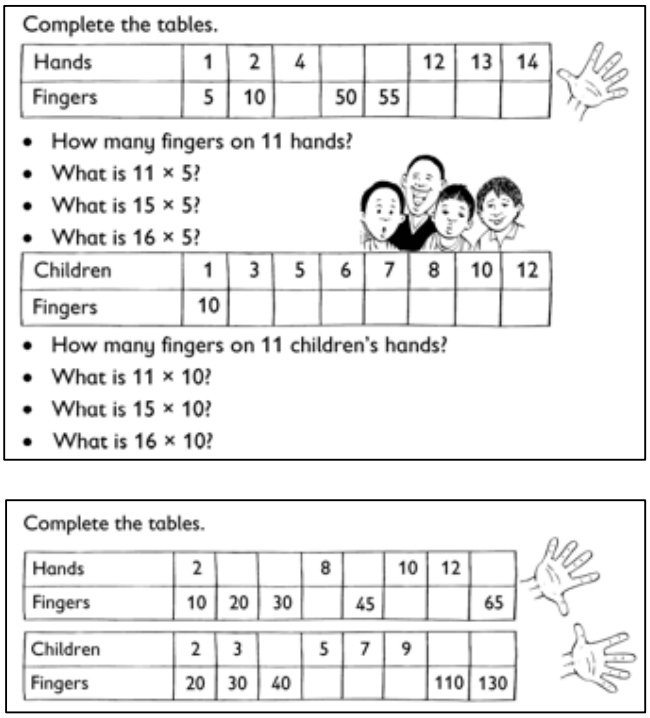


Figure 46: NumberSense: Book 5 treatment of "hands: fingers" problem

Notice:

- the slight shifts from having an iconic drawing depicting the actual numbers of hands and fingers, to the skip counting pattern (p. 8);
- the shift to an equivalent structure of 5c balloons (p. 10 and p. 17); and
- the introduction of the tabulation representation (where both multiplication and division calculations are expected) and where the hands are not presented in ones (p. 18 and p. 40)

Book 5 also includes many other contexts involving the ×5 function (lollies in boxes, 5c balloons, etc.). This hands: finger problem is repeated in Book 6 (twice), Book 7 (twice) and Book 8 (thrice). By Book 9, the finger: hand problem appears in tabulated form, alongside a comparison to the hand: children problem.



Complete the tables.

Hands	1	2	4			12	13	14
Fingers	5	10		50	55			

- How many fingers on 11 hands?
- What is 11 × 5?
- What is 15 × 5?
- What is 16 × 5?

Children	1	3	5	6	7	8	10	12
Fingers	10							

- How many fingers on 11 children's hands?
- What is 11 × 10?
- What is 15 × 10?
- What is 16 × 10?

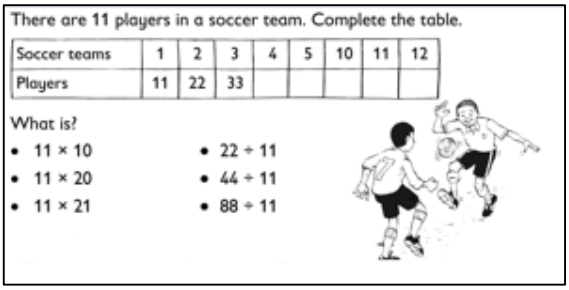
Complete the tables.

Hands	2			8		10	12	
Fingers	10	20	30		45			65

Children	2	3		5	7	9		
Fingers	20	30	40				110	130

Figure 47: NumberSense Book 9

Later in Book 9, the familiar hand: fingers problems are repeated twice. In addition, this familiar problem context is used to introduce the multiplication operator symbol. Notice that children are expected to complete division calculations alongside the multiplication calculations.



There are 11 players in a soccer team. Complete the table.

Soccer teams	1	2	3	4	5	10	11	12
Players	11	22	33					

What is?

- 11 × 10
- 11 × 20
- 11 × 21
- 22 ÷ 11
- 44 ÷ 11
- 88 ÷ 11

Figure 48: NumberSense Book 20 (Grade 5/6)

This example from Book 20 is typical of the way in which division is treated as the inverse of multiplication. A structural relationship is established: "There are 11 players on a soccer team". The representation offered is a multiplication table, which allows for the function ×11 to be explored at multiple points.

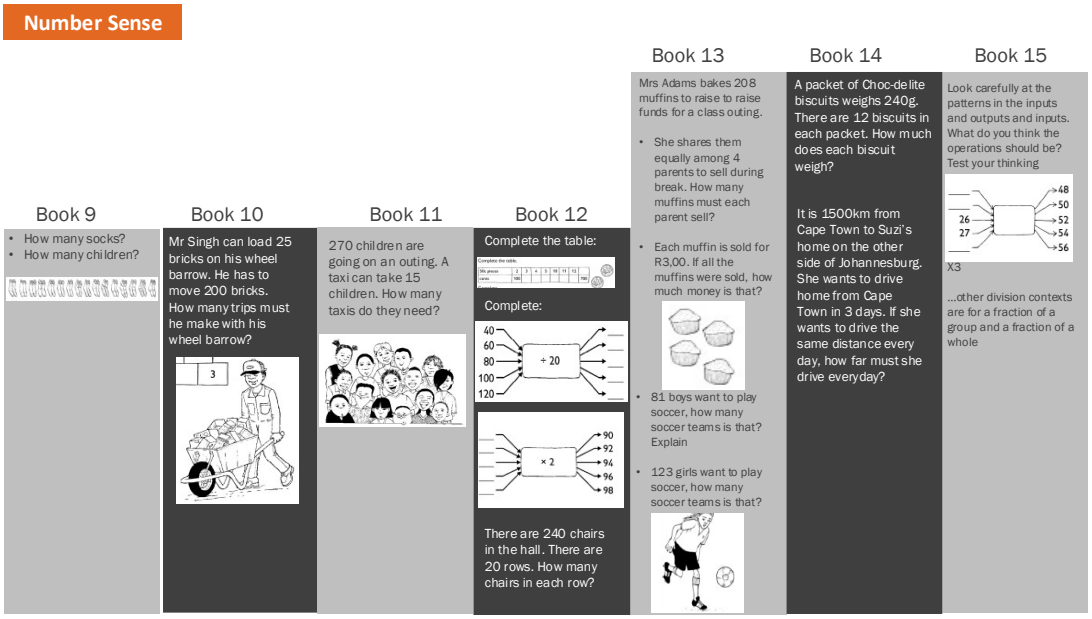


Figure 49: NumberSense – Progression of tasks on division with whole numbers from Book 9 to Book 15 (Approximately Grade 4 to Grade 6)

Learners are encouraged to reflect on division as the inverse of multiplication by being expected to answer 22 ÷ 11 =.... 44÷ 11 (by drawing from their tabulation). They are extended when asked to calculate 88 ÷11 which can be done using the pattern established for the ×11 function.

The tabulation in Book 17 is a further example of a problem context which encourages functional thinking. This is an extension of the "what if" questioning evident in Book 7 (the structural relationship is constrained by 6 players in a team, and the number of people available to participate in explored through "what if" questioning). Notice the repetition of the tabulations and function machine representations. With these representations, the inverse relationship between multiplication and division is made explicit. As book numbers increase, there are fewer examples of iconic drawings involving the actual number of discrete objects, and iconic drawings are simply used as context prompts (drawings and calculation strategies are left to the children). Also, there is a clear shift towards use of number symbols and operations in Book 20 (Grade 5), and now possible strategies to conduct division calculations in various ways are offered through comparison of different example responses to the same calculation.

Mathematics: Does the LTSM provide plenty of opportunities for children to make meaning from the tasks and instructional narrative?

Workbooks containing many practice questions that are related to the materials being learned make learning more concise than in textbooks (Susantini, Isnawati & Lisa; 2016). Assessments included in LTSM development should therefore consider the adequacy of worked examples, practice assessment activities and their alignment to the curriculum.

The NumberSense LTSM offers far more tasks for learners relating to whole number division across all grades than the TMU pilot. The DBE workbooks have the fewest whole number division tasks. From the deep dive analysis presented above relating to the learning trajectory for

whole number division, it is clear that different LTSMs offer very different opportunities for learning mathematics, both in quality and quantity.

Mathematics: Does the LTSM all for differentiated teaching within a grade?

NumberSense and PMRP both offer differentiated sets of practice exercises for children. Given the well-documented underperformance in mathematics, it is useful to note the differentiated offerings provided by NumberSense and PMRP. Learners are offered workbooks with exercises appropriate for their developmental level. The teacher instructs on a particular topic and pays attention to a key mathematics concept, but the exercises to practice and apply mathematical learning are differentiated. This may be an appropriate choice in well-functioning classrooms.

However, while "teaching at the right level" is important, significant differentiation is difficult for teachers to manage well. Those who are cautious of such approaches raise concerns about learners being streamed or placed on an underperforming track with low expectations. While it is recognised that there are likely to be three attainment groups – with the core, those achieving below the core and those above the core, the practicalities of managing such are challenging to most teachers. The Bala Wande offering includes extension activities in the teacher guide. These are a set of calculations which can be put on the board for students who complete their work faster than others. The MCC workbooks are designed for whole class teaching and pacing. However, each 2-page spread is designed so that the first page is critical content to be completed by all learners, and the second page is consolidation. The second page may not be completed by the weaker students, and they still worked on the core concept.

Language: African languages and English

Does the LTSM…use language(s) suitable to your context (in either a multilingual or bilingual setting)?

South Africa is a multilingual nation and the majority of its public schools are in multilingual settings. Multilingual settings consist of diverse languages in addition to the LoLT. Teachers in multilingual classrooms need to use questioning techniques that probe and encourage learners to talk using the multiple languages available. Primary mathematics classrooms should use heteroglossic approaches (Sapire & Essien, 2021), including translanguaging. All the available languages in a primary mathematics classroom should be used as resources for meaning making and mathematical expression.

How does the LTSM support bilingual/multilingual dialogical and mathematical engagement?	DBE workbooks	TMU pilot	NumberSense	Magic Classroom Collective	Bala Wande	PMRP
	Grade R to 6	Grade 1 to 3	Books 00-24 (Grade R to 6) Differentiated book series, offering a page a day for independent learner work	Grade R to 3	Grade R to 3	Grade 4 to 6
	Supplementary learner activity book	Daily lessons to drive teacher instruction		Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction
Does the LTSM workbook…						
…use language suitable to your context?	Multiple monolingual - all African languages (x9)	Multiple monolingual - all African languages (x9)	Multiple monolingual - all African languages (x9)	Bi-lingual English isiXhosa	Bi-lingual English African languages (x3)	Monolingual - English
…deliberately develop African language and English registers for mathematics?	Translation from English, without development of African Language or English registers	Translation from English. Teacher guide includes glossary of terms used for each section.	Translation from English, without development of African Language or English registers	Let's talk maths in English and isiXhosa	Lets talk maths in English and an African language	Learner talk activities only available in English
…signal high value instructional sentences?	None	In teacher guide	In teacher guide	In learner workbook	In learner workbook and teacher guide	Provides summaries of the rules
…provide ways for children to externalise their thinking and signal mathematical talk?	Not intended - supplementary activity book	Practice, unstructured discrete object drawings, shifting to bare calculations	Practice using selection of representations, for example: multiplication tabulation, clueboard, and functions	Practice using structured number pictures and number lines (emphasis on 2,5,10s), whole-part-part diagrams, arrays, multiplication tabulations, t-tables, functions	Practice using bare calculation with emphasis on multifix, whole part-part diagrams, place value tables for written calculations	Provides activities that promote "listen and learn" and "talk and do". Focuses on procedures for standard written algorithms for each operation.

Figure 50: Summary of primary mathematics LTSM attributes: Language

The availability of LTSM mathematics materials at Grade R to 3 levels in all South African languages is welcomed. However, it is noted that these may be termed "multi-monolingual" resources as the books are produced in only one language, and 11 different books are available for each of the South African languages. Additional attention to the need to shift to bilingualism (with English in mathematics) is required. The development of learner workbooks at the FP level should accommodate all 11 official languages of the country. IP level workbooks should incorporate the transition from mother tongue education to the use of English or Afrikaans as LoLT. Mathematics workbook development should take into consideration under resourced multilingual classrooms where the learner's and teacher's home languages may not be the language of instruction. A workbook needs to highlight and explain newly introduced vocabulary

and mathematics register. A workbook should use simple and clear design features that support bi/multilingual engagements without cluttering the workbook.

South African primary school classrooms are not linguistically homogenous. Urban environments tend to be multilingual with greater access to English, while more rural environments are African language dominant, where the transition to English is a shift to bilingualism. Workbook development should cater to the specific needs of schools in urban and rural environments.

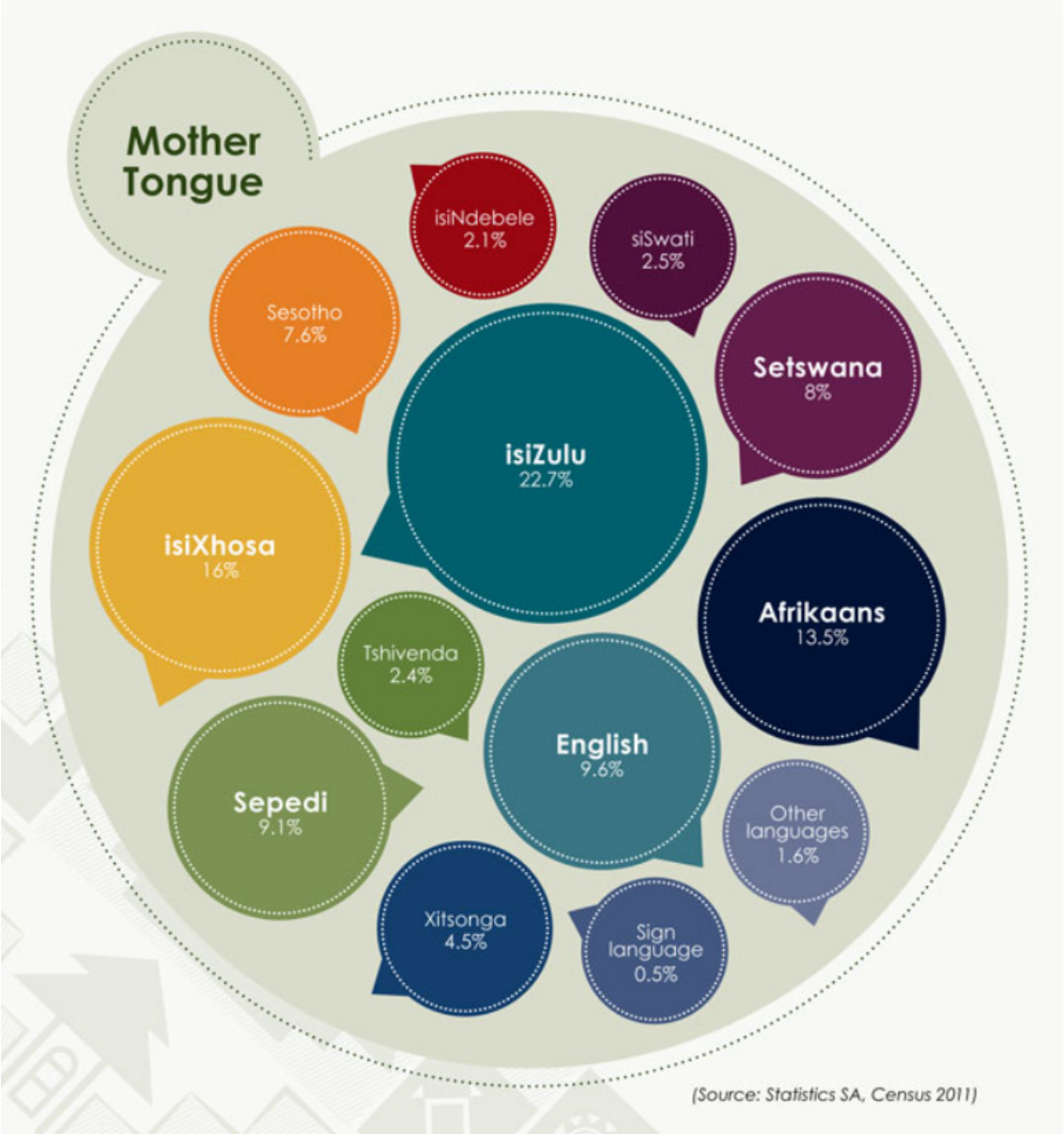


Figure 51: Stats SA Language Diagram[10]

[10] Source: https://www.awesomesouthafrica.co.za/2014/06/south-african-languages/

Language context of most urban environments

Classrooms in urban settings have heterogeneous language contexts, characterized by multiple languages in one school or classroom. In such classrooms, there should be greater reliance on English with translation into the dominant African language. Bala Wande workbooks provide a good example of versioning. The diversity that is characteristic of urban environments calls for teacher support in using heteroglossic approaches. Schools within the inner city and in communities with several African languages spoken require targeted support in using the different languages available as resources for making meaning in mathematics.

Regarding primary mathematics workbook development for urban settings, further investigation is required to establish:

- how bilingual LTSM works in urban settings;
- how teacher questions and prompts for multilingual engagement should be phrased; and
- the learning gains for learners immersed in English at FP level.

Language context of most rural environments

Classrooms in rural settings have homogenous language contexts characterized by a specific dominant African language, such as isiXhosa in the Eastern Cape Province. Learners in such classrooms have limited access to English (as a foreign language). The gradual transition from using mother tongue instruction to using English or Afrikaans should focus on carefully building a particular African language register and discourse as a starting point. This critical language development process should be led by a team of experienced mathematics education researchers and linguists who investigate and trial the different ways of expressing primary mathematics ideas. FP Mathematics teachers should create many opportunities for learners to express mathematical ideas in the dominant African language with a gradual introduction to expressing mathematics ideas in English. Primary mathematics content should be scaffolder towards an increased use of English.

Regarding primary mathematics workbook development for rural settings, further investigation is required to establish:

- how bi/multilingual engagements can be incorporated into workbooks without making the workbook text heavy; and
- how the gradual transition from an African Language into bilingual African language texts works in homogenous language settings.

Does the LTSM…deliberately develop African language and English registers for mathematics?

Some mathematics in the LTSM for the Foundation Phase is available only in one language. The African language versions are often translations from English.

The LTSM materials developed for primary mathematics offer a trove of translated mathematical courses into African languages. There are concerns about the quality and consistency of these translations and the extent to which the expression of African language mathematics discourse is being systematically developed and leveraged to support mathematical meaning making.

There are LTSM offerings which adopt a more bilingual approach to the materials.

- MCC (working in isiXhosa dominant classrooms) offers isiXhosa workbooks with deliberate scaffolding for bilingual isiXhosa-English discourse; and
- Bala Wande has developed every workbook in bilingual African Language-English. Every sentence is offered in an African language, and the English translation is included in a smaller font.

Other mathematics LTSM for the Foundation Phase is available in a bilingual format. For example, this Bala Wande mathematics workbook repeats every African language sentence in English (using a smaller font).

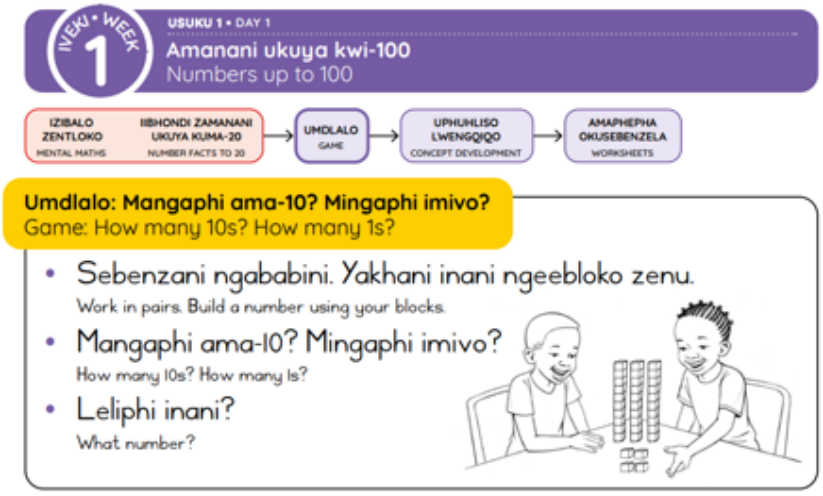


Figure 52: Bala Wande Grade 3 Learner activity book

This example from the isiXhosa-English Grade 3 activity book illustrates this approach to bilingual mathematics LTSM.

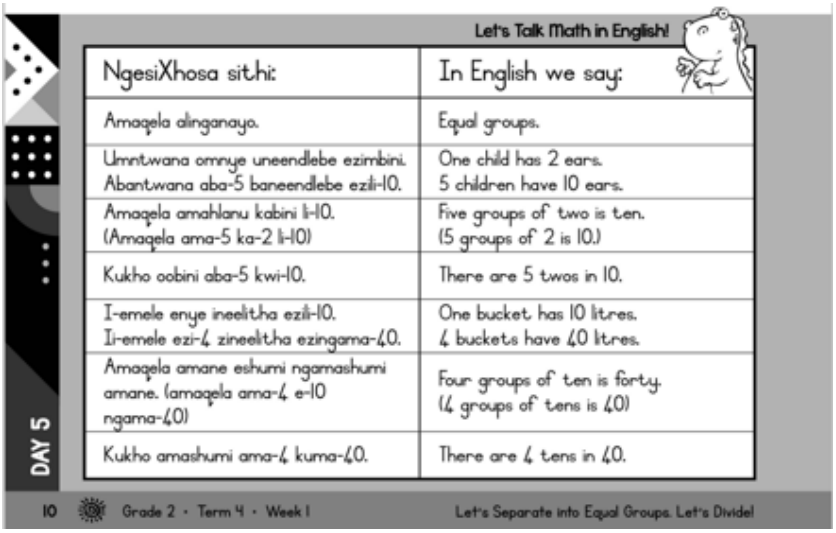


Figure 53: Magic Classroom Collective: Grade 2 learner activity book

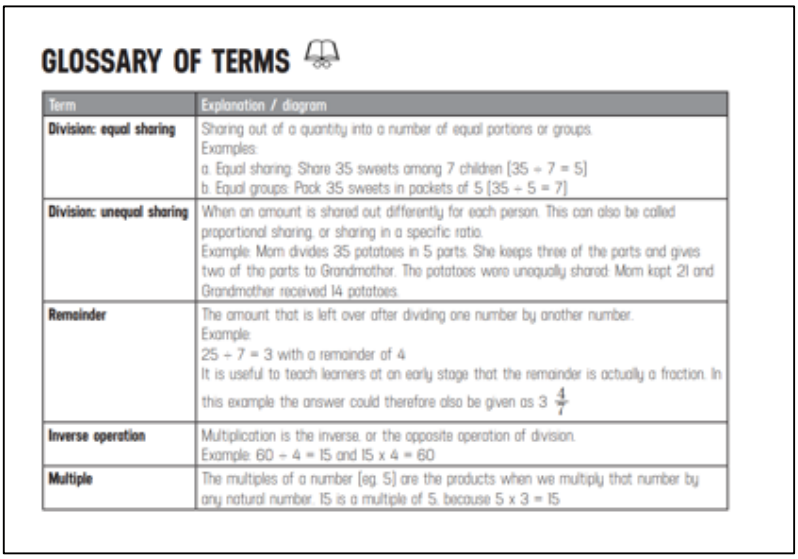
Some mathematics LTSM for Foundation Phase seeks to build on African language home languages fluency, but support the later transition to English. The MCC workbooks are in isiXhosa. However, there are dedicated "Let's talk maths" sections which draw attention to the language being used and offer translations of key concepts in English. Therefore, key concepts are provided in both languages in this MCC workbook.

The introduction of English translations of isiXhosa mathematical expressions is scaffolder over Grades 2 and particularly in Grade 3. This is in anticipation of the later transition to learning mathematics in English in higher grades.

Does the LTSM…signal high value instructional sentences?

To support language development in both English and an African language, it is helpful when LTSM resources explicitly offer a sentence which the teacher is expected to say. The learners can bring this to mind when reflecting on similar tasks.

There have been several investments into mathematics dictionaries and glossaries of mathematical terms, some of which are then presented in multiple languages.



GLOSSARY OF TERMS

Term	Explanation / diagram
Division: equal sharing	Sharing out of a quantity into a number of equal portions or groups. Examples: a. Equal sharing: Share 35 sweets among 7 children [35 ÷ 7 = 5] b. Equal groups: Pack 35 sweets in packets of 5 [35 ÷ 5 = 7]
Division: unequal sharing	When an amount is shared out differently for each person. This can also be called proportional sharing, or sharing in a specific ratio. Example: Mom divides 35 potatoes in 5 parts. She keeps three of the parts and gives two of the parts to Grandmother. The potatoes were unequally shared: Mom kept 21 and Grandmother received 14 potatoes.
Remainder	The amount that is left over after dividing one number by another number. Example: 25 ÷ 7 = 3 with a remainder of 4 It is useful to teach learners at an early stage that the remainder is actually a fraction. In this example the answer could therefore also be given as $3\frac{4}{7}$
Inverse operation	Multiplication is the inverse, or the opposite operation of division. Example: 60 ÷ 4 = 15 and 15 x 4 = 60
Multiple	The multiples of a number (eg. 5) are the products when we multiply that number by any natural number. 15 is a multiple of 5, because 5 x 3 = 15

Figure 54: TMU pilot: Grade 4 concept booklet: targeted support

South African mathematics educators are only just starting to seriously grapple with offering mathematics in multiple languages. There have been a few decades of investment in African language translations from mathematics expressed in English. There are now also numerous multilingual mathematics dictionaries. For example, NumberSense and Bala Wande have multilingual mathematics dictionaries. The TMU pilot materials include a multilingual dictionary and a glossary of terms in their lesson plans. The teacher guide (referred to as the "concept booklet: targeted support") for the TMU pilot materials offers a glossary of terms for each topic covered. The following is an example from "Topic 5: Whole Number Division" in Grade 4.

Drawing attention to the meaning of particular mathematical terms and phrases is important and helpful. However, this is not sufficient. Investments are required to systematically develop the African language registers for mathematics. The translations of the primary mathematics curriculum, and the LTSM materials (recognized as flawed) provide an excellent starting point for

such work. We need to understand the differences between translations and how these can be improved to harness the affordances of the mathematical expressions of particular languages.

There are emerging examples of the detailed research work which is required to develop African language registers for mathematics (MCC has worked with isiXhosa teachers, mathematics experts and isiXhosa linguists).

There is evidence of emerging detailed research work which is required to develop the mathematics for particular African Languages. This has been the work of MCC over several decades, where teaching mathematics and literacy in isiXhosa dominant classrooms has been the focus. The learning materials emerging from this intervention, are not translations from English but carefully developed through long term engagement between teachers, mathematics and linguistic experts. The evidence of impact on learner outcomes is now published. Extending, learning from this approach and investing similarly in other African languages is required.

Drawing on the MCC approach, the Bala Wande materials also include a "let's talk math's" in English and an African language. This is expected to support bilingual development of mathematics phrases and sentences in Foundation Phase, in anticipation of the shift to Mathematics in English. MCC offers similar linguistic prompting.

Figure 55: Bala Wande: Grade 3 learner activity book

In response to the rich evidence of the need for and existence of success with new methods of teaching primary mathematics in South Africa, **it is now appropriate to draw on the lessons learnt from across the smaller interventions (researched to be having impact) to fundamentally revise and improve the DBE mathematics workbooks.**

Summary of findings

This review into available LTSM and assessment for primary mathematics is timely. It is hoped that it will allow for further improvement in what is offered, and greater coordination between different parts of the system. The importance of keeping the big ship "primary school mathematics" stable is well noted: Incremental change can only be possible with simple messaging, adequate and well considered LTSM assessment, teacher training and school-based coaching. Competing messages and competing interventions are not in the national interest. At the same time, there are several ways in which the richness of the interventions in the system can be contained and harnessed to

inform the incremental improvement necessary at systems level. There are smaller "yachts", call them design interventions, which are able to be more agile in trial and research of materials, design principles, assessment approaches and instrument experimentation at small scale without distracting the course of the big ship. When these design interventions design for scale, understanding the significant constraints faced at provincial and national level, they have much to offer the future direction of the "big ship". For researched and evidence based lessons – established at small scale – to have large scale relevance, systems and rhythms of engagement to harvest and incorporate promising approaches are required.

General findings

1. The DBE workbook represents a major positive shift in South African schooling.
2. Too many mathematics interventions can distract teachers and instructional leaders with competing messaging.
3. The last decade has seemed significant advances in how to work to improve mathematics learning outcomes at scale in primary schools.

Assessment findings

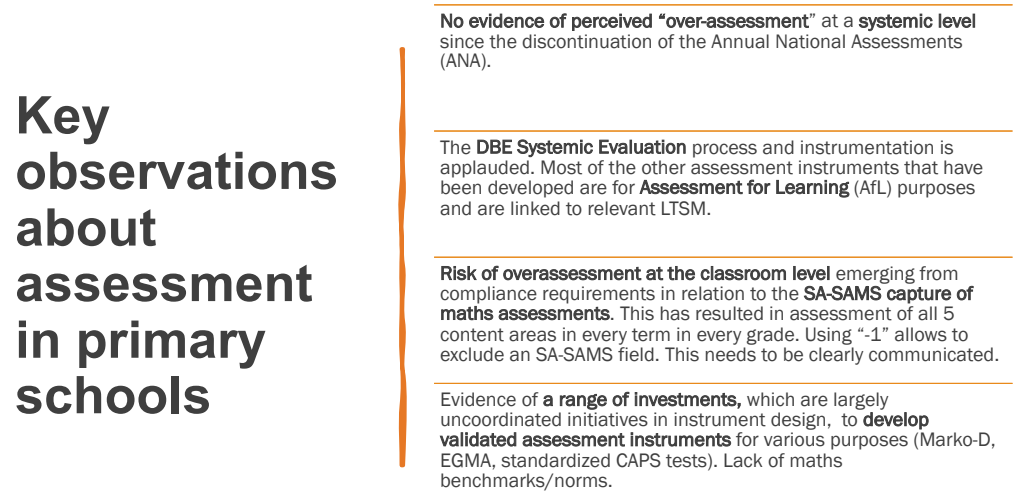


Figure 56: Key observations about assessment in primary schools

4. Following significant curriculum change in the early democratic period, there has now been decades of stability. Curriculum reviews are seen as "disruptions" to the system, or emergency responses (e.g. to Covid). There is a related lack of alignment between Curriculum, LTSM and assessment instruments.
5. **Mechanisms for assessment development, delivery and utilization tend to be uncoordinated. The review found that in relation to Math's assessment:**
    a. The standardized instruments (for impact measures) tend to be isolated and uncoordinated initiatives,
    b. SA-SAMS was not aligned with realistic delivery of math's curriculum, and

c. Need to further support Assessment for Learning practices.

6. **There are different assessment instruments design for different purposes:**

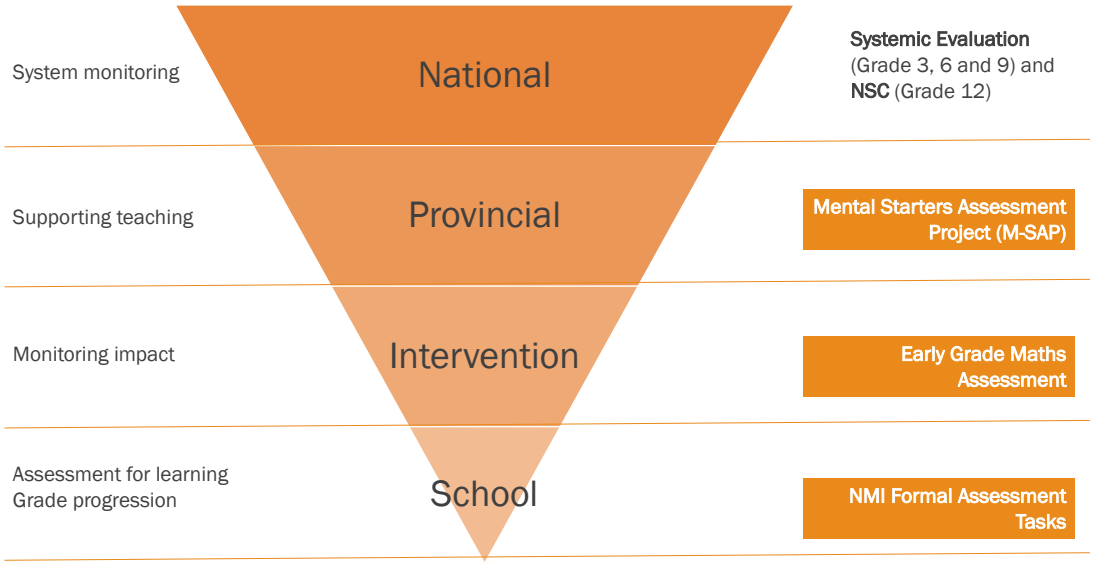


Figure 57: Illustration of layers of assessment required at various levels of the primary school environment

6.1 **M-SAP is used for formative purposes to promote assessment- driven pedagogy and develop teacher AfL strategies.** The Assessment has been piloted widely at Grade 3 level across all provinces in SA with great success and positive results in terms of reliability and validity. It leverages knowledge of Number Structure to develop:

- Rapid recall fluencies,
- Strategic calculations, and
- Strategic thinking.

6.2 **The development of an assessment is required for summative purposes in measuring the impact of interventions** that seek to improve mathematics outcomes in a timely manner. The instrument should be piloted in SA to show acceptable levels of reliability and validity in relation to interventions designed to test alternative curriculum, LTSM and teacher development options in all those contexts.

6.3 **The MCC- Formal Assessment Task**s are CAPS-based assessments that have been piloted for over ten years at the FP in English and isiXhosa in a sample of 15 deep rural schools in the EC. The assessment has been used in a "learning laboratory" design that leverages researched benefits of multilingualism in mainstream no-fee schools in the EC.

LTSM findings

7. There have been significant **advances in the availability and quality of mathematics LTSM materials:**

7.1. The DBE learner workbook is welcomed and available in schools as a primary resource. It was developed as a supplementary resource, but is being used as a primary resource

7.2. Other mathematics LTSM has been developed (some of which have shown impact on learning outcomes) is occurring (e.g. TMU pilot, NECT lesson plans, NumberSense, Bala Wande, NMI).

7.3. As a result, there are some duplication of resources. Budget and printing is required for the DBE workbooks as well as the particular Learner Activity Book,

8. There are **improvements in LTSM design**. The review found that
   8.1. Recently developed workbooks minimize the number of "moving parts" which teachers need to use;
   8.2. Most researched workbooks deliberately build instructional rhythms;
   8.3. Many include checkpoints and or formal assessment tasks; and,
   8.4. Across the LTSMs reviewed various ways in which LTSM supports teacher development are evident, and these are linked to their varying design purposes.
      8.4.1. The LTSM is accompanied by a teacher guide (Bala Wande, NumberSense and TMU pilot materials);
      8.4.2. The LTSM include instructional prompts and support for the teaching of concepts in the learner activity books themselves (such as MCC).
9. Most workbooks are designed for **successful use just with pen and paper,** but some workbooks expect access to particular manipulatives or tool kits (See Annexure 3 for minimum standards).
   9.1. **Manipulatives** enrich the Foundation Phase learning environment, but there are **specific needs** in each grade.
   9.2. There is a need for **measuring equipment**, **2D shapes and 3D objects** – especially for Intermediate Phase.
10. The availability of LTSM mathematics materials at Grade R to 3 levels, in all **South African languages** is welcomed. However, it is noted that these may be termed **"multi-monolingual"** resources as the books are produced in only one language, and 11 different books are available for each of the South African languages.
11. In relation to **language issues, the review found that**:
   11.1. South African primary school classrooms are not **linguistically homogenous**: Urban environment tend to be multilingual with greater access to English, while more rural environments are African language dominant, where the transition from English is a shift to bilingualism.
   11.2. The LTSM materials developed for primary mathematics offer a **trove of translated mathematics** into African languages. There are concerns about the **quality and consistency of these translations** and the extent to which the expression of African language mathematics discourse is being systematically developed and leveraged to support mathematical meaning making.
   11.3. There are LTSM offerings which adopt a **more bilingual approach** to the materials (Bala Wande and MCC).
   11.4. There are emerging examples of the **detailed research** work which is required to develop African language registers for mathematics (MCC has worked with isiXhosa teachers, mathematics experts and isiXhosa linguists; Bala Wande has worked on multilingual dictionaries)
   11.5. There have been several investments into **mathematics dictionaries and glossaries** of mathematical terms, some of which are then presented in multiple languages.
12. From the **deep dive analysis** of three workbooks on the learning trajectory for whole number division, it is evident that different LTSMs offer very **different opportunities for learning mathematics both in quality and quantity.**

Summary of recommendations

General recommendations

1. **Publish and invest in predictable and well-planned cycles of revision for curriculum, assessment and LTSM in 5 and 10 year cycles**
   - **1.1.** The DBE should implement medium- to long-term curriculum reviews of operational cycles of five (5) and ten (10) years.
2. **Carefully plan and coordinate changes to curriculum, LTSM and assessment with simple messaging for incremental improvement**
   - **2.1.** Aim for a 90:10 split for stability: innovation, as the stability of our fragile primary school system is paramount.

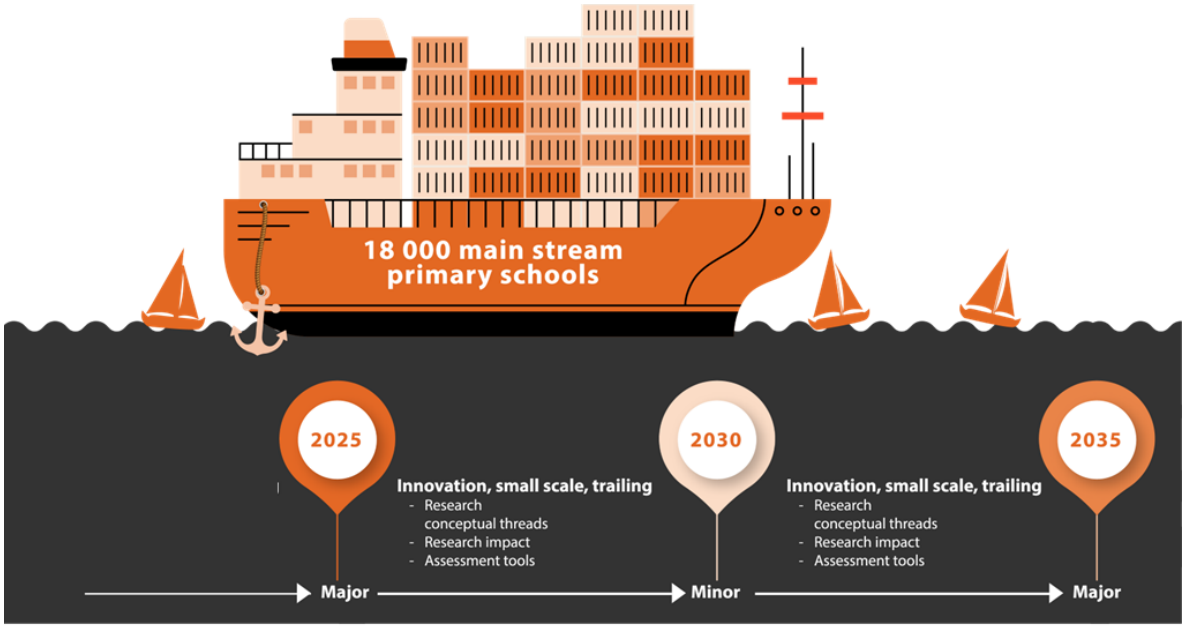


Figure 58: A proposed cycle of incremental improvement for Primary mathematics LTSM and assessments

3. **Allow innovation, through research interventions, to research impact of alternative LTSM and assessment practices:**
   - **3.1.** Draw on the research and lessons learnt from across the innovations to fundamentally revise and improve the DBE mathematics workbooks at Foundation Phase to be the primary a ubiquitous resource. Once a review of mathematics textbook use in the Intermediate phase has been concluded, consider whether a similar process is adopted for the Intermediate Phase.

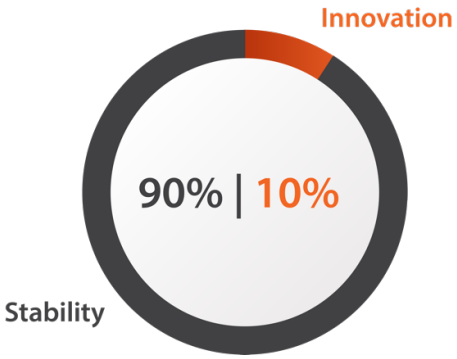


Figure 59: 90:10 Illustration of Stability to Innovation

5. **Intentionally and proactively plan for curriculum reviews which are informed by evidence from assessment and use of LTSM:**
   - **5.1.** Use assessment data generated through the systemic evaluation process, and standardized instruments to inform decisions on curriculum reviews.
   - **5.2.** Use impact evaluation findings (generated through design interventions) on LTSM design, use and teacher support, to inform decisions on curriculum review.

Assessments Recommendations:

5. **The DBE should share quality assessment for learning instruments**
   - **5.1. A collaborative effort is needed between the DBE and the M-SAP Research and Innovation Intervention (referred to here as a Community of Practice) to facilitate greater access to the M-SAP assessments.**

     The DBE should:

     - **5.1.1.** Serve as a "clearinghouse" for all assessment instruments that schools can access;
     - **5.1.2.** Conduct due quality assurance on assessment instruments that are intended for use by mainstream schools;
     - **5.1.3.** Publish a periodic catalogue of all assessment instruments of acceptable quality and allow schools to select instruments that they can utilize for specific purposes;
     - **5.1.4.** Co-design the SA-SAMS platform with experts in primary mathematics education such that the expected quarterly uploading of marks is aligned with and responsive to the mathematics topics that are taught per term; and,
     - **5.1.5.** Encourage the uptake and use of the 'mathematics assessment review framework (see Annexure 1).

     The PEDs should:

     - **5.1.6.** Create awareness about large-scale assessments that are conducted in the province and the purpose of the assessments.
     - **5.1.7.** Establish processes and capacity of analysing the assessment data at district level such that the results are timeously fed back to schools to inform teaching and enhance learning.

5.2. **Assessment instruments need to be developed to allow impact evaluations to generate evidence of successful intervention techniques, which should be fed into curriculum, LTSM and assessment review cycles.**

The DBE should:

5.2.1. Initiate/support the establishment of a Primary Mathematics Test Community of Practice (CoP) that will develop a set of test instruments in all the SA official languages used as LOLTs;

5.2.2. Encourage the standardised use of mathematics tests for impact evaluations; and,

5.2.3. Initiate the integration of the mathematics.

The Primary Mathematics assessment COP should:

5.2.4. Establish grade level norms/benchmarks (using a nationally representative sample) which specify how much learning is expected at specific stages as learners progress through primary schooling. This is intended to allow for comparison in experimental designs which are testing alternative curriculum, LTSM and teacher training interventions (in the 10% innovation space);

5.2.5. Guard both the integrity and the validity of the tests by keeping the instrument confidential and define standardised administration requirements;

5.2.6. Ensure test design to include topics like Shape, Space, Measurement; and, Data Handling…So shift to a Primary Mathematics test instruments (likely at Grade 1, Grade 3, Grade 6 and 9).

5.3. The DBE should facilitate the use of **Formal Assessment Tasks** (such as the MCC FATS):

5.3.1. Compile and manage a repository of quality mathematics FATs (from PDEs and interventions like MCC and TMU)

5.3.2. Consider extending the development, trialling and eventual use of the quality FATs across all SA languages

5.3.3. Promote the use of quality FATs results for term-to-term monitoring of learner progress and curriculum coverage

5.3.4. Adapt the SA-SAMS platform to host FATs results so that they are accessible to Subject Advisors and Curriculum support officials who must support schools to teach effectively.

5.3.5. Ensure that the use of "-1" in SA-SAMS to allow teachers to not submit marks for every topic in every grade, is communicated to all schools.

5.3.6. Ensure that provinces and districts support schools **to use the FATs results** from the end of a term for diagnostic purposes at the beginning of a new term and thus ensure seamless progress across terms.

LTSM Recommendations

6. Redesign the DBE workbooks for the Foundation Phase to be the primary resource (taking into account the lessons from NECT, TMU pilot and interventions that have evidence of impact). These should include:

6.1. A Learner Activity book (like those used for TMU pilot, Bala Wande, MCC);

6.2. A teacher guide; and,

6.3. Grade-specific mathematics kit.

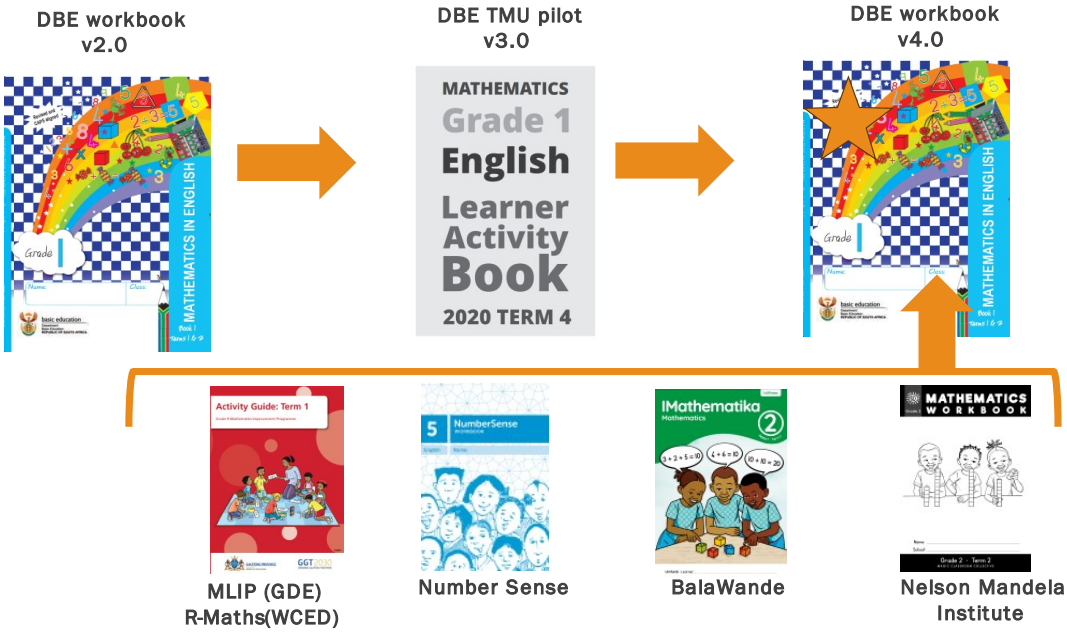


Figure 60: Proposed creation of DBE workbook version 4.0

The revised DBE workbook should take into account the lessons from various innovation initiatives.

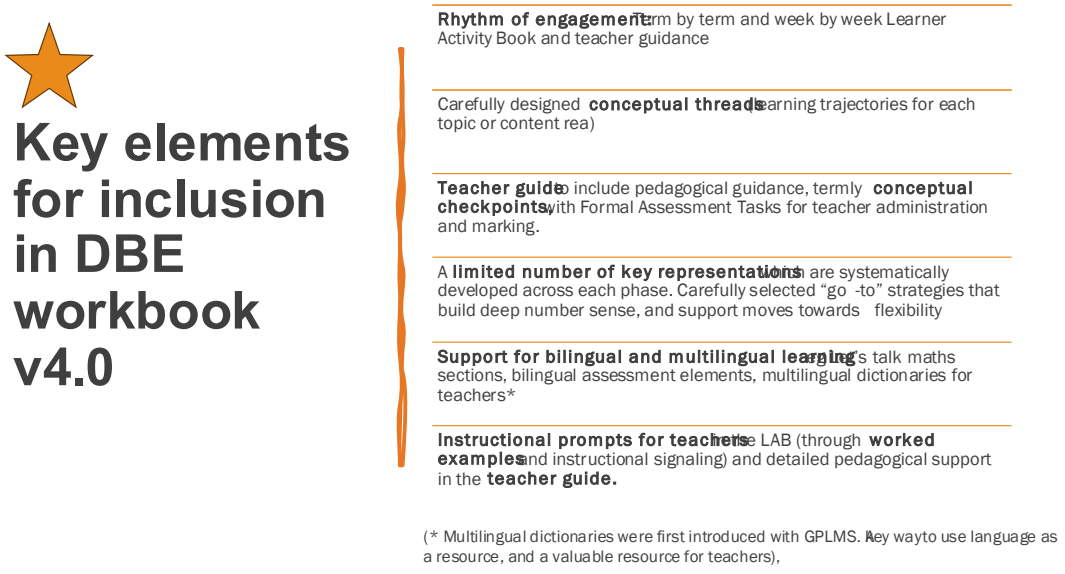


Figure 61: Key elements for inclusion in DBE workbook version 4.0

7. Guide the research agenda for mathematics LTSM – with particular focus on mathematics expressions in African languages and conceptual threads (learning trajectories) in mathematics:

7.1. Research the uptake and use of mathematics textbooks in the Intermediate Phase to establish whether learner activity books should be considered the primary resource at this phase.
7.2. Publish calls for research into mathematical expression in African languages as a priority area for research and innovation to systematically trial and develop learner workbooks in different African languages. This requires systematic research into the development of mathematics registers for particular African Languages.
7.3. Encourage investment into African languages communities of practice for mathematics LTSM and assessment design and innovation (with a bare minimum of 4 focal areas). Multiple African languages and English multilingualism (learners' and teacher's home languages may not be shared or the language of instruction as is common in urban environments).
7.4. Promote research in and development of bilingual and multilingual assessments for semi-urban and township contexts alongside use of dominant local languages in rural context.
7.5. Publish calls for research (analysis of existing CAPS and LTSM materials as well as proposed evidence-based improvements) into mathematics learning trajectories for key ideas, go-to calculation strategies (that build number sense and conceptual understanding of place value and inverse operations) and a related limited number of representations (from Grade R to Grade 6).

Conclusion

This LTSM and assessment review represents an important milestone in the South African primary mathematics landscape. It was commissioned by the Department of Basic Education, with funding support from Zenex Foundation. It has enabled DBE officials across all branches – teacher development, LTSM, curriculum and assessment – to reflect deeply on the available LTSM offerings, assessment and potential future direction for these key investments into quality mathematics education. At the same time, the review process has allowed participants involved in the design of mathematics LTSM and assessments to share ideas with each other, and spread knowledge of what is currently available for use in primary schools. There is now recognition of pivotal importance of the mathematical foundations laid in the primary school years. This requires coordinated national efforts to improve the quality mathematics learning opportunities and efforts to improve teacher knowledge for primary mathematics teaching. Excellent primary mathematics LTSM and assessment are key ingredients for such efforts.

It is clear that there is significant state investment into mathematics LTSM workbooks in South Africa. There are also pockets of innovation and research into mathematics LTSM and related teacher development which have been found to have impact on learner outcomes at small scale. The time is now ripe to leverage these innovations at a greater scale – through Randomized Controlled Trials and significant reworking of the DBE workbook. A series of mathematics LTSM design principles have been put forward. These provide a starting point for continuous review and improvement of mathematics LTSM. Minimum recommendations for what a primary mathematics kit should include has been proposed. There is also evidence of ways of working with primary mathematics assessments in diverse ways – to measurement impact of interventions, to provide system wide tracking over time, to use assessment as a means of teacher development, and common Formal Assessment tasks to support school-based assessment processes. Of concern, is

the lack of coordination between these various assessment initiatives. An ongoing community of practice for primary mathematics LTSM and assessment is proposed, to ensure that the primary mathematics research is leveraged for implementation at scale. For this to be possible, a regular series of primary mathematics indabas together with an agreed framework for major and minor revisions to the curriculum framework and related LTSM and assessment instruments it envisaged.

There is willingness and appetite for greater coordination and engagement between government, academic and civic role players. We trust that this review facilitates sharing of knowledge relating to what is available, how to reflect on quality and provokes debate and further research amongst the primary mathematics role players in South Africa.

In sum, the LTSM and assessment reviews found that:

- **Much positive work on LTSM and assessment from the DBE** (workbook, NECT LABS, Revised trackers, TMU framework, TMU pilot, systemic evaluation, AfL practices)
- **Much positive work and research in the mathematics eco-system** (GPLMS, NumberSense, MCC, Bala Wande, Numeracy chairs, NMI FATs, EGMA etc)

Now the time is ripe to bring these together in coordinated and predictable **5- and 10-year cycles of curriculum, assessment and LTSM review**, that draw on local evidence of impact. As such it is encourage a **90-10 split of stability and innovation**, drawing on the research lessons and commissioned research on mathematics conceptual threads and language issues.

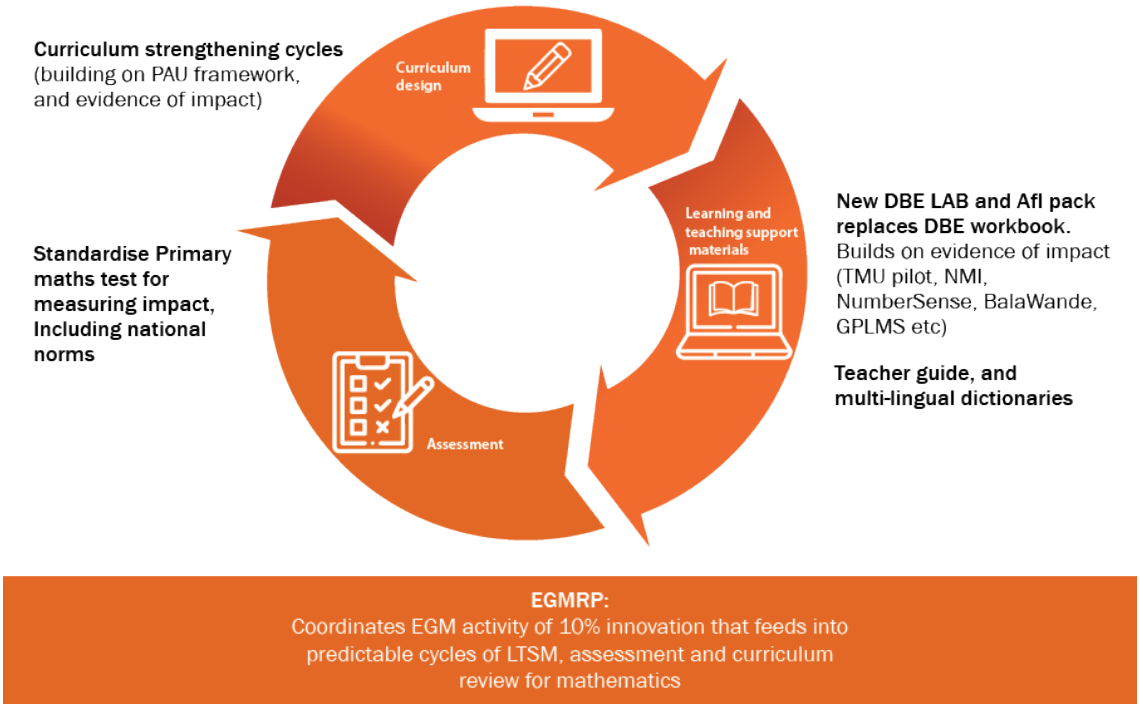


Figure 62: Curriculum, LTSM and Assessment relationship

The proposed next steps for the EGMRP project are organised in terms of issues relating to assessment; issues relating to LTSM, and issues informing both.

Issues on **Assessment:**

- Publish the **Systemic Evaluation findings** and ensure these findings feed into curriculum and LTSM processes, so there is a feedback loop into curriculum, LTSM and teacher development (90% stability)
- Create and maintain a repository of quality **mathematics formal assessment tasks** (such as the NMI FATS) (90% stability). Use the learning briefs and assessment evaluation framework from this review.
- Publicise use of **SA-SAMS** relating to use of "-1" to exclude a topic in assessment data capture (90% stability)
- Support/commission the design of **validated mathematics assessment instruments to measure impact** (building on the EGMA, but expanding this for Grades 3, 6 and 9) (10% innovation). **Establish national norms/benchmarks.**
- Evaluate the impact of **M-SAP intervention at Grade 3,** and if evidence is positive, expand to include Grades 1&2

Issues on **LTSM**

- Extend and fully update the **DBE workbook** (v 4.0) to be a **DBE LAB, teacher guide* and mathematics kit.** Shift it from a supplementary resource to be THE primary resource (90% stability).
- Complete the **impact evaluation (qualitative and quantitative) for the TMU pilot** (v3.0). Use these findings to inform the DBE workbook (v.4.0).
- Collect findings and/or commission **impact evaluations** of mathematics LTSM and training interventions showing promise, at district-level scale.
- Encourage use of the **LTSM evaluation framework developed** for this review (for new and updated versions of mathematics LTSM).

Issues **informing curriculum, LTSM and assessment**

- **Review the primary mathematics curriculum** (extending the current curriculum strengthening process), taking into account the evidence emerging from this review. Alignment between LTSM- Assessment-Curriculum is required.
- Commission and support research on mathematics **conceptual threads (learning trajectories)** relating to a limited number of representations, 'go-to' calculation strategies that build number sense (number for algebra) and moves toward flexibility. Trial and research innovations in schools (10% innovation).
- Commission and support the development of communities of practice and research on the **expression of mathematics in African languages (bi-lingual and/or multilingual) LTSM and teacher training**. This should extend and support the work on **mother tongue instruction in mathematics**. Trial and research innovations in schools (10% innovation).
- Continue the **quarterly EGMRP activities (including indabas)** to ensure that DBE:
    - leads the research agenda,
    - maintains 90% stability, and
    - responds to the 10% innovation which then informs the evidence informed cycles of curriculum, assessment and LTSM review.

References

Adams, N. E. (2015). Bloom's taxonomy of cognitive learning objectives. *Journal of the Medical Library Association*: JMLA, 103(3), 152.

Alderson, J. C. (2005). *Diagnosing foreign language proficiency: the interface between learning and assessment*. London: Continuum.

Bartolini M.G., Martignone F. (2020)Manipulatives in Mathematics Education.In: Lerman S. (eds)Encyclopaedia of Mathematics Education. Springer, Cham.https://doi.org/10.1007/978-3-030-15789-0_93

Bond, T. G., & Fox, C. M., 2015. *Applying the Rasch Model: Fundamental measurement in the human sciences* (3rd Ed). Routledge: New York.

Bowie, L., & Reed, Y. (2016). How much of what? An analysis of the espoused and enacted mathematics and English curricula for intermediate phase student teachers at five South African universities. *Perspectives in Education*, *34(1), 102-119*.

Brombacher, A. (2015). National intervention research activity for early grade mathematics in Jordan. *ors: Xuhua Sun, Berinderjeet Kaur, Jarmila Novo*, 415.

Brombacher, A. and Roberts, N. (2022). NumberSense: A focus on Shikaya in Spaull, N., & Taylor, S. (Eds) *Large scale interventions to improve early reading and mathematics,* Oxford University Press.

Cockett, A., & Kilgour, P. W. (2015). Mathematical manipulatives: Creating an environment for understanding, efficiency, engagement, and enjoyment. *Teach collection of Christian Education,* 1(1), 47-54. Retrieved from https://research.avondale.edu.au/teachcollection/vol1/iss1/5

Department of Basic Education (DBE) (2011). *National Curriculum Statement Grades R - 12:* Curriculum and Assessment Policy (January 2011).

Department of Basic Education (DBE) (2019). Mathematics Teaching and Learning Framework for South Africa. *Teaching Mathematics for Understanding.*

Department of Basic Education (DBE) (2018). School Monitoring Survey 2017/2018: Technical Report. Pretoria: Department of Basic Education.

Department of Basic Education (DBE), (2012). *The Learning and Teaching Support Materials Policy.* Johannesburg.

DNA Economics (2019). Qualitative Evaluation of the Mathematics Teaching and Learning Framework for South Africa: *Draft evaluation report.* Prepared for Tshikululu Social Investments.

Duncan, G. J., & Magnuson, K. (2011). The nature and impact of early achievement skills, attention skills, and behavior problems. *Whither opportunity*, 47-70.

Education 2030. (2016). *Incheon Declaration and Framework for Action for the implementation of Sustainable Development Goal 4:* Ensure inclusive and equitable quality education and promote lifelong learning.

Fleisch, B., Taylor, N., Sapire, I. and Herholdt, R., (2011). Evaluation of back to basics mathematics workbooks: A randomised control trial of the primary mathematics research project. *South African Journal of Education*, 31(4), pp.488-504.

Graven, M., & Venkat, H. (2021). Piloting national diagnostic assessment for strategic calculation. *Mathematics Education Research Journal*, *33*(1), 23-42.

Henderson Pinter, H., Merritt, E. G., Berry III, R. Q., & Rimm-Kaufman, S. E. (2018). The importance of structure, clarity, representation, and language in elementary mathematics instruction. *Investigations in Mathematics Learning*, 10(2), 106-127.

Jacob, O. N., & Dahir, N. (2021). Challenges Preventing Public Primary School Students from Using Information Communication Technology for Learning in Nigeria and the Way Forward. *Electronic Research Journal of Social Sciences and Humanities*, 3, 45-52.

Khoza, S. B. and Mpungose, C. B. (2020). Digitalised curriculum to the rescue of a higher education institution, *African Identities*, DOI:10.1080/14725843.2020.1815517

Lee, J. S., & Ginsburg, H. P. (2009). Early childhood teachers' misconceptions about mathematics education for young children in the United States. *Australasian Journal of Early Childhood*, *34*(4), 37-45.

Miller, S. M. (2008). Teacher learning for new times: Repurposing new multimodal literacies and digital-video composing for schools. In J. Flood, S.B. Heath, D. Lapp (Eds.) *Handbook of research on teaching literacy through the communicative and visual.*

Modisaotsile, B. (2012). *The Failing Standard of Basic Education in South Africa.* Africa Institute Of South Africa Briefing NO 72.

Moloi, Q., & Roberts, N. (2021). A validation process towards the modification of the Grade 1 Early Grade Mathematics Assessment. In *Proceedings of the 26th Annual National Congress of the Association for Mathematics Education of South Africa, 14th–16th July* (pp. 502-512).

NEEDU. *National Report 2013.* Teaching and Learning in Rural Primary Schools.

Platas, L. M., Ketterlin-Gellar, L., Brombacher, A., &Sitabkhan, Y. (2014). Early grade mathematics assessment (EGMA) toolkit. *RTI International, Research Triangle Park, NC*.

Platas, L. M., Ketterlin-Geller, L. R., &Sitabkhan, Y. (2016). Using an assessment of early mathematical knowledge and skills to inform policy and practice: Examples from the early grade mathematics assessment. *International Journal of Education in Mathematics, Science and Technology*, *4*(3), 163-173.

Popov, N. (2000). A Review of the System of Higher Education in Bulgaria. *Civic Education Project*, Volume 1, Issue 4 of Discussion series.

Ramadiro, B., & Porteus, K. (2017). *Foundation phase matters: Language and learning in South African rural classrooms.* Magic Classroom Collective Press.

Read, T. (2015). Where have all the textbooks gone? Toward sustainable provision of teaching and learning materials in sub-Saharan Africa. Washington, DC: World Bank.

Roberts, N. (2019, October). Assessing Early Grade Mathematics Learner Outcomes Using m-Learning. In *Interactive Mobile Communication, Technologies and Learning* (pp. 136-146). Springer, Cham.

Roberts, N. & Moloi, Q. (2021) 'JumpStart Programme Impact on the Mathematics Learning Outcomes: A cross sectional quasi-experiment using the Early Grade Mathematics Assessment' in Qhobela, M., Ntsohi, M.M.E., &Mohafa, L.G. (2021). *Book of Proceedings of the 29th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education*. Lesotho: SAARMSTE. pp. 122-138

Sapire, I. & Essien, A. (2021). Multiple monolingualism versus multilingualism? Early grade Mathematics teachers' and students' language use in multilingual classes in South Africa. In Essien A. & Msimanga (eds), *Multilingual Education Yearbook 2021: Policy and Practice in STEM multilingual contexts*. Springer Nature, Switzerland, pp. 75-95. https://doi.org/10.1007/978-3-030-72009-4_5.

Schollar, E. 2008. *Final report: The primary mathematics research project 2004–2007 – towards evidence-based educational development in South Africa*. Johannesburg: Eric Schollar and Associates.

Smart, A., & Jagannathan, S. (2018). *Textbook policies in Asia: Development, publishing, printing, distribution, and future implications*. Asian Development Bank.

Spaull, N., van der Berg, S., Wills, G., Gustafsson, M., Kotze, J. (2016). Laying Firm Foundations: Getting reading right. *The final report to the ZENEX Foundation on poor student performance in Foundation Phase literacy and numeracy.*

Susantini, E., Isnawati, and Lisa, L. (2016). Effectiveness of Genetics Student Workbook to Improve Creative Thinking Skills of Teacher Candidate Students. Journal of Science Education, 17(2), 74-78.

Taylor et al, (2017). *Improving early grade reading in South Africa* [P2.10.SA.IE] UNESCO, (2015). Learning and Teaching Materials. https://learningportal.iiep.unesco.org/en/print/pdf/node/8624

Visser, M. (2018).High-performing learners also need support.Human Sciences Research Council(HSRC).http://www.hsrc.ac.za/en/review/hsrc-review-april-june-2018/high-performing-learners

Wilson, P. H., Sztajn, P., Edgington, C., & Myers, M. (2015). Teachers' uses of a learning trajectory in student-centered instructional practices. Journal of Teacher Education, 66(3), 227–244.

Zuze, L. (2018). *Workbook and textbook availability - Adding learners' voices,* Human Sciences Research Council (HSRC). http://www.hsrc.ac.za/en/review/hsrc-review-april-june-2018/workbook-textbook-availability

Annexure 1: Mathematics LTSM and assessment evaluation frameworks

This LTSM review and assessment review of primary mathematics in South Africa, has led to the development of two evaluation frameworks:

1. The Mathematics LTSMS evaluation framework; and
2. The Mathematics assessment framework.

Both frameworks ought to be used (and where appropriate updated and extended) when mathematics materials are being evaluating. How the frameworks have been applied to particular LTSM packagaes and Assessment instruments is presented in Annexure 2.

Mathematics LTSM evaluation framework

Grade range reviewed	
Purpose	
Does the LTSM…	
…have credible research/ evidence of impact?	
…meet CAPS grade level expectations?	
…follow CAPS Annual Teaching Plan (pedagogical guidelines)?	
…offer an alternative (re-sequenced and newly structured) ATP by term, week and day?	
…include mental maths?	
How does the LTSM support pedagogy?	
Does the LTSM workbook…	
…work within the classroom contraints?	
…build instructional rythms?	
…support the teaching style dominant/ desired in your context?	
…include suitable assessment strategies which can be easily integrated by teachers?	
…include support and signalling to the teacher?	
…explicitly develop well researched learning trajectories (conceptual threads)?	
…provide plenty of opportunities for children to make meaning from the tasks and instructional narrative?	
How does the LTSM support bilingual/multilingual dialogical and mathematical engagement?	
Does the LTSM workbook…	
…include support and signalling to the teacher?	
…explicitly develop well researched learning trajectories (conceptual threads)?	
…provide plenty of opportunities for children to make meaning from the tasks and instructional narrative?	
…allow for some differentiated teaching within a class?	

Mathematics Assessment evaluation framework

Grade range reviewed	
Purpose	
Is this a trustworthy assessment instrument? (Validity)	
Is there research evidence to support the trustworthiness of the instrument?	
... evidence of validity	
... evidence of ability to predict future performance	
... evidence of ability to measure impact (of interventions)	
How does the assessment promote mathematics knowledge and skills?	
Does the Assessment...	
...meet curriculum grade level expectations?	
...offer a reliable measure of grade-to-grade hierarchy in mathematics knowledge and skills?	
...include mental maths?	
... measure essential mathematics knowledge and skills?	
How easy is the processing and analysis of the assessment data?	
Does the administration, capture and analysis of assessment data...	
...require teacher marking?	
...require specialist invigilation	
... require specialised tools?	
... involve specialised skills?	

Annexure 2: Deep dive synopses

The tabulations presented in this annexure offer a summary of the findings from the deep dive analysis conducted for the selected assessment and LTSM offerings. A colour coding of red, yellow and green is used to indicate presence, partial presence or absence of a particular attribute. It is important to note that the offerings are designed for specific purposes. Depending on their purpose particular attributes will necessarily be absent. As such the presence of red coding is not an indication of a problem, but an indication of the offering fitting its purpose. Selections should be made based on the intended use.

Assessment

	EGMA	M-SAP	MCC FATs
Grade range reviewed	Grade 1, EGMA Junior (Grade 2-3) and EGMA senior (Grade 4-6)	Grade 3	Grade 1-3
Purpose	Standardised mathematics instrument for impact evaluations	Professional development intervention with "pre-test/post-test" scripted lesson design	Standardised assessments of curriculum expectations by school term
Is this a trustworthy assessment instrument? (Validity) **Is there research evidence to support the trustworthiness of the instrument?**	**EGMA**	**M-SAP**	**MCC FATs**
… evidence of validity	Yes, widely researched	Yes, widely researched	Developed in response to teacher requests with teacher-researcher collective

… evidence of ability to predict future performance	Yes, widely researched	No - Not intended. Focused on mental mathematics	No - Not intended
… evidence of ability to measure impact (of interventions)	Widely used in a range of impact studies	Researched impact on learner outcomes established at small scale	No - Not intended. Focused on CAPS for each grade and as feedback for teacher and parent

How does the assessment promote mathematics knowledge and skills? **Does the Assessment…**	**EGMA**	**M-SAP**	**MCC FATs**
…meet CAPS grade level expectations?	No - Not intended. Focused on number sense (predictive of future mathematics attainment)	No - Not intended. Focused on mental mathematics	Yes, as intended. CAPS aligned by term and grade
…offer a reliable measure of grade-to-grade hierarchy in mathematics knowledge and skills?	Yes. Grade 1 test is subset of Junior (grade level comparison provided)	No - Not intended	No - Not intended
...include mental mathematics?	Yes - includes a rapid recall section	Yes - includes rapid recall, strategic calculating and strategic thinking sections	Yes - includes a rapid 2-minute tango recall section
… measure essential mathematics knowledge and skills?	Yes, measures more than 60% of expected mathematics knowledge	Yes, measures mental fluencies essential number sense and for later written calculation strategies.	Yes, measures grade level of expected mathematics knowledge in CAPS
How easy is the processing and analysis of the assessment data? **Does the administration, capture and analysis of assessment data…**	**EGMA**	**M-SAP**	**MCC FATs**
…require teacher marking?	No. Computer marking for table-based. Marking by test administrators (not teachers) for pen and paper	Yes - intended for teacher to gain knowledge of learner needs and errors. Includes teacher capture tool in Excel	Yes - teacher capture intended for teacher to gain knowledge of learner needs and errors. Marks are recorded for capture in SA-SAMS format.
…require specialist invigilation	Yes - trained test administrator in 1:10 ratio with children. No teacher presence.	No. Class teacher invigilates.	No, Class teacher invigilates

… require specialised tools?	Optional, format allows both paper-and-pencil and tablet use	Available in paper-and-pencil format	Available in paper-and-pencil format
… involve specialised skills?	Some specialised skills required (using Excel to define levels) and comparing raw scores to available benchmarks	No specialised skills required	No specialised skills required
How does the assessment feedback support planning and pegagogy? **Does the assessment feedback…**	**EGMA**	**M-SAP**	**MCC FATs**
… support teaching (pedagogy)	No - Not intended	Feedback supports teaching during lesson	Yes. Helps teacher know expected attainment levels per grade and term
… provide immediate identification of learners who need help during the lesson?	No - Not intended	Yes, helps identify learning needs immediately	No - Not intended
... help the teacher identify topics/ domains of mathematics that require re-teaching?	Yes. Facility analysis can show areas of strength and weakness	Provides feedback on uptake of particular mental mathematics strategies	Yes, helps identify domains for re-teaching
… help track learner progression over a term(s) or semester(s)?	No - Not intended	No - Not intended	Yes, used as school-based assessment for reporting to parents

How is the assessment responsive to the context? **Does the assessment …**	**EGMA**	**M-SAP**	**MCC FATs**
…work within the classroom constraints?	Either pen and paper or tablet based	Yes - worksheets need to be printed	Yes - tests need to be printed
…support the teaching style dominant/ desired in your context?	Small groups (Grade 1 and Junior) or whole class (Senior)	Whole class teaching design	Whole class teaching design
…include suitable reports which can be easily interpreted and responded to by teachers?	Learners are placed at a particular level. Specialist analysis required	Teacher marks, captures data and creates report	Teacher marks, and enters results on SA-SAMS
How does the assessment support bilingual/multilingual mathematical engagement? **Does the assessment …**	**EGMA**	**M-SAP**	**MCC FATs**
…use language suitable to the context?	Multi - monolingual - all African languages (x9)	Multi - monolingual - all African languages (x9)	Multi - monolingual - all African languages (x9)
…deliberately develop African language and English registers for mathematics	Translation from English	Translation from English	Designed from isiXhosa with researcher-teacher collective
…provide ways for children to externalise their thinking and signal mathematical talk?	No "show your working" is expectied shown. Grade 1 test adminsitrator may record the strategy used (if required). Includes timed fluency items.	By being timed and focusing on particular mental mathematics strategies, ways of externalising thinking and preferred methods are apparent	Yes. Some items include space to "show your working" (for expectation to number picture, number line or formal written calculation.

LTSM

Grade range reviewed	**DBE workbooks** Grade R to 6	**TMU pilot** Grade 1 to 3	**NumberSense** Books 00-24 (Grade R to 6)	**Magic Classroom Collective** Grade R to 3	**Bala Wande** Grade R to 3	**PMRP** Grade 4 to 6
Purpose	Supplementary learner activity book	Daily lessons to drive teacher instruction	Differentiated book series, offering a page a day for independent learner work	Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction (with differentiated workbooks)
Does the LTSM…	DBE workbooks	TMU pilot	NumberSense	Magic Classroom Collective	Bala Wande	PMRP
…have credible research/ evidence of impact?	No impact assessment	Pilot currently underway, no learner impact data published yet	Impact at small scale (Shikaya and JumpStart) and large/national scale (Jordan) established. Malawi national implementation currently underway. See papers by Brombacher, Roberts and Moloi	Impact on learner outcomes at small scale established. See papers by Porteus	Pilot currently underway, no learner outcomes data published yet. See baseline papers by Ardington and Sapire	Impact on learner outcome 1 at district scale established. See thesis by Schollar
…meet CAPS grade level expectations?	Yes - all in one book per term	Yes - all in one book per term	Yes: Mapping of CAPS requirement to every page. Separate workbooks for geometry.	Yes - all in one book per term	Yes - all in one book per term	Not intended - based on outcomes-based education

…follow CAPS Annual Teaching Plan (pedagogical guidelines)?	No	No - experimenting with revised sequencing	No	No	No	No
...offer an alternative (re-sequenced and newly structured) ATP by term, week and day?	Not intended - supplementary activity book	Clusters concepts aiming for completion or "signoff" on a topic in each grade.	Spiral curriculum for learner practice: Teach at the right level (differentiated pacing in class)	Clusters concepts (eg multiplication with division, addition with subtraction, measurement as a context for number work)	Clusters concepts and resequences annual plan	Focused on the Number Operations and Relationships content area
...include mental mathematics?	Not intended - supplementary activity book	No	Separate mental starters available as a teacher resource	Game of the week in the learner workbook	Games in the learner workbook	Mental mathematics included
How does the LTSM support pedagogy?	**DBE workbooks**	**TMU pilot**	**NumberSense**	**Magic Classroom Collective**	**Bala Wande**	**PMRP**
	Grade R to 6	Grade 1 to 3	Books 00-24 (Grade R to 6)	Grade R to 3	Grade R to 3	Grade 4 to 6
	Supplementary learner activity book	Daily lessons to drive teacher instruction	Differentiated book series, offering a page a day for independent learner work	Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction
Does the LTSM workbook…						
…work within the classroom contraints?	Expects use of pictures / images in workbooks as paper-based manipulatives	Expects use of array diagrams and ten phrase as paper-based manipulatives	There is no reference to manipulatives	Expects use of posters, charts (paper based) and counters (non-paper based) manipulatives	Expects use of a Bala Wande toolkit	Focus is on formal written algorithms, there is no reference to manipulatives

…build instructional rythms?	Termly	termly, weekly, daily lesson	1 page per day	termly, weekly, daily lesson	termly, weekly, daily lesson	termly, weekly, daily lesson
…support the teaching style dominant/ desired in your context?	Not specified,	Lesson plans provide instructions for groupwork / scaffolding work	Allows for differentiated working, allows different groups of learners to work at their own pace.	Teacher guides provide instructions for groupwork / scaffolding work	Teacher guides provide instructions for groupwork / scaffolding work	Assumes that learners are assigned to ability groups for differentiated working
…include suitable assessment strategies which can be easily integrated by teachers?	Not intented - supplementary activity book	revision, formative, summative	FATS available	Mini quiz, practice page, FATS		weekly assessment activities provided
…include support and signalling to the teacher?	Not intented - supplementary activity book	In teacher guide (concept booklet)	In teacher training materials and mental warmups	Detailed explanation of procedure, representation and strategy though using characters	Detailed explanation of procedure, representation and strategy though using characters	In teacher guide
…explicitly develop well researched learning trajectories (conceptual threads)?	None evident	Guided by Japanese approaches, C-P-A (Concrete-Pictorial-Abstract).	Create meaning from the real or imaginable word problem context, work with functions, relationships and patterns, spiralling curriculum (highest number range for counting, lower number range for calculating, lowest number range for word problems). Encourages a range of strategies. Compares different approaches through contrasting learner methods in higher levels. Uses	Develop mental imagery for discrete and continuous contexts from concrete or imaginable contexts. Focus on 2s, 5s and 10s, (using number pictures and number lines).	Develop mental fluencies and written calculation procedures making use of concrete manipulatives as to infuse written calculation methods with meaning.	Context-free calculations followed by word problems focusing on the 4 basic operations. A strong emphasis on formal written algorithms for all 4 operations
…provide plenty of opportunities for children to make meaning from the tasks and instructional narrative?	Practice activities available, but not structured	Emphasis on bare calculations and break up both number (place value) strategies.		Systematically develops a "go-to" strategy. Break up the second number for addition and	Emphasis on bare calculations and break up both number (place value) strategies.	Practice activities available but not structured

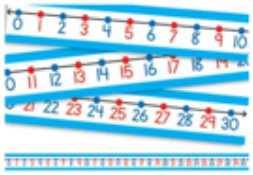
			clueboards, addition chains and pyramids, function machines and tabulations	subtraction. Skip counting (using clueboards and arrays) for multiplication and division.		
How does the LTSM support bilingual/multilingual dialogical and mathematical engagement?	**DBE workbooks** Grade R to 6 Supplementary learner activity book	**TMU pilot** Grade 1 to 3 Daily lessons to drive teacher instruction	**NumberSense** Books 00-24 (Grade R to 6) Differentiated book series, offering a page a day for independent learner work	**Magic Classroom Collective** Grade R to 3 Daily lessons to drive teacher instruction	**Bala Wande** Grade R to 3 Daily lessons to drive teacher instruction	**PMRP** Grade 4 to 6 Daily lessons to drive teacher instruction
Does the LTSM workbook…						
…include support and signalling to the teacher?	Not intended - supplementary activity book	In teacher guide (concept booklet)	In teacher training materials and mental warmups	Detailed explanation of procedure, representation and strategy though using characters	Detailed explanation of procedure, representation and strategy though using characters	In teacher guide
…explicitly develop well researched learning trajectories (conceptual threads)?	None evident	Guided by Japanese approaches, C-P-A (Concrete-Pictorial-Abstract).	Create meaning from the real or imaginable word problem context, work with functions, relationships and patterns, spiralling curriculum (highest number range for counting, lower number range for calculating, lowest number range for word problems).	Develop mental imagery for discrete and continuous contexts from concrete or imaginable contexts. Focus on 2s, 5s and 10s, (using number pictures and number lines).	Develop mental fluencies and written calculation procedures making use of concrete manipulatives as to infuse written calculation methods with meaning.	Context-free calculations followed by word problems focusing on the 4 basic operations. A strong emphasis on formal written algorithms for all 4 operations

…provide plenty of opportunities for children to make meaning from the tasks and instructional narrative?	Practice activities available, but not structured	Emphasis on bare calculations and break up both number (place value) strategies.	Encourages a range of strategies. Compares different approaches through contrasting learner methods in higher levels. Uses clueboards, addition chains and pyramids, function machines and tabulations	Systematically develops a "go-to" strategy. Break up the second number for addition and subtraction. Skip counting (using clueboards and arrays) for multiplication and division.	Emphasis on bare calculations and break up both number (place value) strategies.	Practice activities available but not structured
…allow for some differentiated teaching within a class?	Designed as supplementary - so could be busy book for stronger learners	Extension tasks are included in the teacher guide	Yes. Strong "teach at the right level" design, with differentiated series of books 00-24	None evident	Teacher guide includes sets of extension activities for each day	Activities organised into different levels of difficulty, remediation and enrichment exercises provided

How does the LTSM support bilingual/multilingual dialogical and mathematical engagement?	**DBE workbooks**	**TMU pilot**	**NumberSense**	**Magic Classroom Collective**	**Bala Wande**	**PMRP**
	Grade R to 6	Grade 1 to 3	Books 00-24 (Grade R to 6)	Grade R to 3	Grade R to 3	Grade 4 to 6
	Supplementary learner activity book	Daily lessons to drive teacher instruction	Differentiated book series, offering a page a day for independent learner work	Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction	Daily lessons to drive teacher instruction
Does the LTSM workbook…						
…use language suitable to your context?	Multiple monolingual - all	Multiple monolingual - all	Multiple monolingual - all	Bi-lingual English isiXhosa	Bi-lingual English African languages (x3)	Monolingual - English

…deliberately develop African language and English registers for mathematics?	African languages (x9) Translation from English, without development of African Language or English registers	African languages (x9) Translation from English. Teacher guide includes glossary of terms used for each section.	African languages (x9) Translation from English, without development of African Language or English registers	Let's talk maths in English and isiXhosa	Lets talk maths in English and an African language	Learner talk activities only available in English
…signal high value instructional sentences?	None	In teacher guide	In teacher guide	In learner workbook	In learner workbook and teacher guide	Provides summaries of the rules
…provide ways for children to externalise their thinking and signal mathematical talk?	Not intented - supplementary activity book	Practice, unstructured discrete object drawings, shifting to bare calculations	Practice using selection of representations: for example: multiplication tabulation, clueboard, functions	Practice using structured number pictures and number lines (emphasis on 2,5,10s), whole-part-part diagrams, arrays, multiplication tabulations, t-tables, functions	Practice using bare calculation with emphasis on multifix, whole part-part diagrams, place value tables for written calculations	Provides activities that promote listen and learn, and talk and do

Annexure 3: Minimum Manipulatives Pack

Phase	Grade	Paper Based	Non-Paper Based	Cross cutting for all learning areas
Foundation Phase	R	 Ten frames Dot cards Jumbo playing cards Posters (number freeze and maths discussion posters)	Counters Bead strings Multi-fix cubes Shape blocks Die 	Each learner: - Colouring pencils x12 - Pencils x4 - Erasor x4 - Sharpener - Glue sticks x4 - Pair of scissors
	1	 Ten frames Dot cards Number cards (0-20) Structured number lines (0-20) Playing cards	Counters Bead strings Multi-fix cubes Die 2D shape set 	Each learner: - Colouring pencils x12 - Pencils x4 - Erasor x4 - Sharpener - Glue sticks x4 - Pair of scissors - Ruler
	2	Number chart (100)  Number cards (0-20) Structured number lines (0-100) Playing cards	Multi-fix cubes (for structuring in 5s and 10s at learner level) Measuring instruments, 2D shape set 3D object set Die	Each learner: - Colouring pencils x12 - Pencils x4 - Pens x4 - Erasor x4 - Sharpener - Glue sticks x4 - Pair of scissors - 30 cm Ruler
	3	Number cards (1000) Number cards (0-20) Structured number lines (0-100)	Multi-fix cubes (to allow for structuring in 10s and 100s at demonstration/group level) Play money	Each learner: - Colouring pencils x12 - Pencils x4

		Ten frames 100 chart Playing cards Fraction Wall	Place value blocks Large teaching clocks and mini clocks Measuring instruments: measuring jugs and cylinders, balance scale, bathroom scale, metre sticks, ruler	- Pens x4 - Erasor x4 - Sharpener - Glue sticks x4 - Pair of scissors - 30 cm Ruler
Intermediate Phase	4	Fraction Wall Structured number lines (0-100) 100 chart Playing cards Dice	At Grade level (to be shared across classes): 2D shape sets x10 3D object sets x10 Large teaching clock x4 Geoboards x 40 Measuring instruments: measuring jugs and cylinders, balance scale, bathroom scale, metre sticks, ruler, trundle wheel, themometre Dice    	As above
	5			As above
	6			As above

NOTES:

1. There is much which is available to support early Grade mathematics. However we caution against substantive expenditure into brightly coloured plastic packaged by particular providers with particular and limited functions (for example Dienes blocks or Numeric).
2. We have tried to be conservative, in the hope of achieving universal access to the minimum, in ways that are cost effective for national scale.
3. There has been a notable decline in the availability of basic stationary (listed above as cross cutting). These are essential learning tools for mathematics and should be part of the mathematics minimum requirements.
4. We think that the **absolute minimum requirement** is for:
   4.1. **The cross cutting equipment for all learning areas** (pencils, colouring pencils, erasers, glue, sharpeners, rulers, pairs of scissors) is the biggest priority as without these, little is possible in mathematics or other learning areas
   4.2. **Multilink cubes** to be available in all Foundation Phase classes (with storage/packaging options for groups of 5 learners). Multilink cubes are more versatile for structuring in arrays, and should be sued for pace-ten structuring. Diennes blocks are not a requirement, if multi-fix cubes are available.

4.3. **2D shape sets.** We think that the shape sets which are being used by NumberSense in Malawi, show the minimum requirements. These are used for arrangement and tracing into workbooks for shape, space and geometry work.

SI Measuring instruments (time, mass, capacity, volume, length, temperature) to be available for use in intermediate Phase classes. This is particularly relevant forGrade 4, and can be in storage packs which enable sharing between Grades and classrooms.

5. There are several available mathematics kits on which to draw (TMU polit for paper-based manipulatives, Bala Wande, MCC, NumberSense, R-Maths WCED, Maths Improvement Project for GDE). These include many excellent posters and wall displays which would benefit classrooms, but are not essential.
6. LTSM designers would benefit from knowing that all learners have access to multilink and to 2D shape sets. The 24 2D shapes, in Figure 63 below, have been provided by NumberSense as part of the national intervention in that country. Shape and Space is a neglected area of the South African curriculum. Much can be done within workbooks, if it is known that children have access to the 2D shapes which they can arrange on a page and use to trace around.

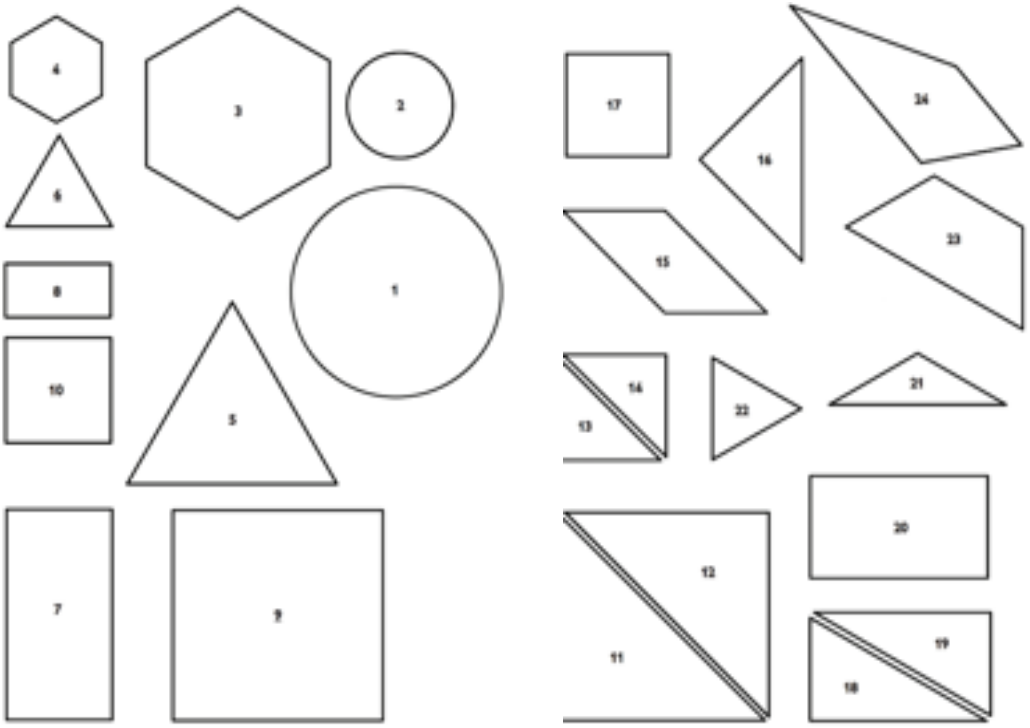


Figure 63: NumberSense numbered shape set (used in Malawi)

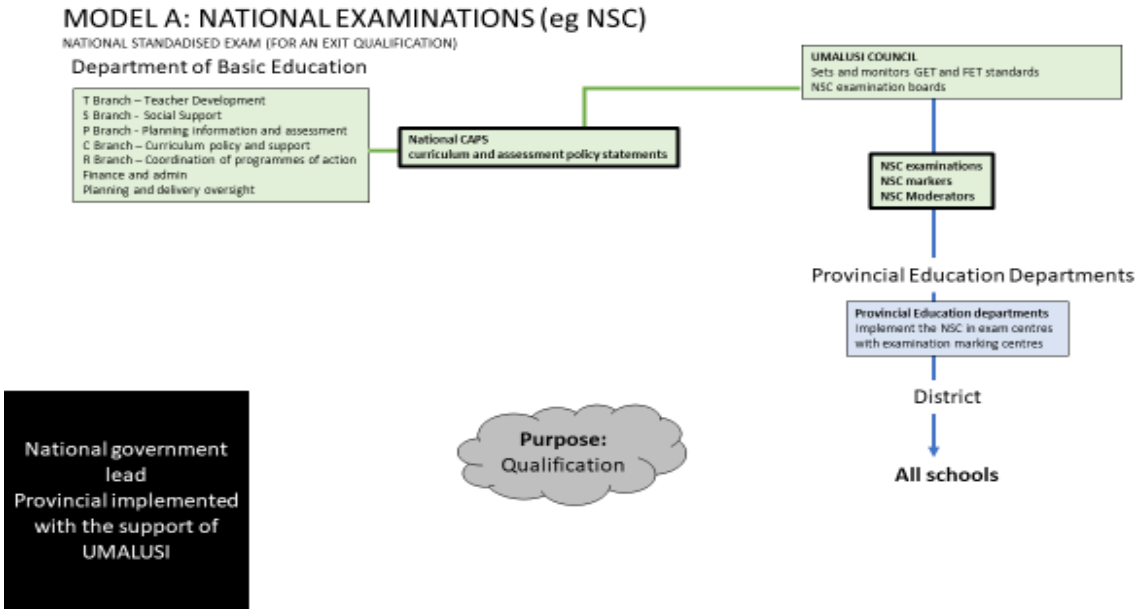
Annexure 4: Assessment delivery models

Proposed mechanisms for assessment delivery

We propose a systematic mechanism of identifying required assessments, developing them, assuring their quality, distributing to the ultimate users and then ensuring that feedback from assessment informs necessary action to improve teaching and enhance learning. As we have shown in the in-depth analysis of the three standardized assessments, the key determinant of either conducting or not conducting an assessment if the PURPOSE for which the results of the assessment will serve. On the basis of what the purpose of the assessment is, we propose five models of developing and administering assessments, including public examinations.

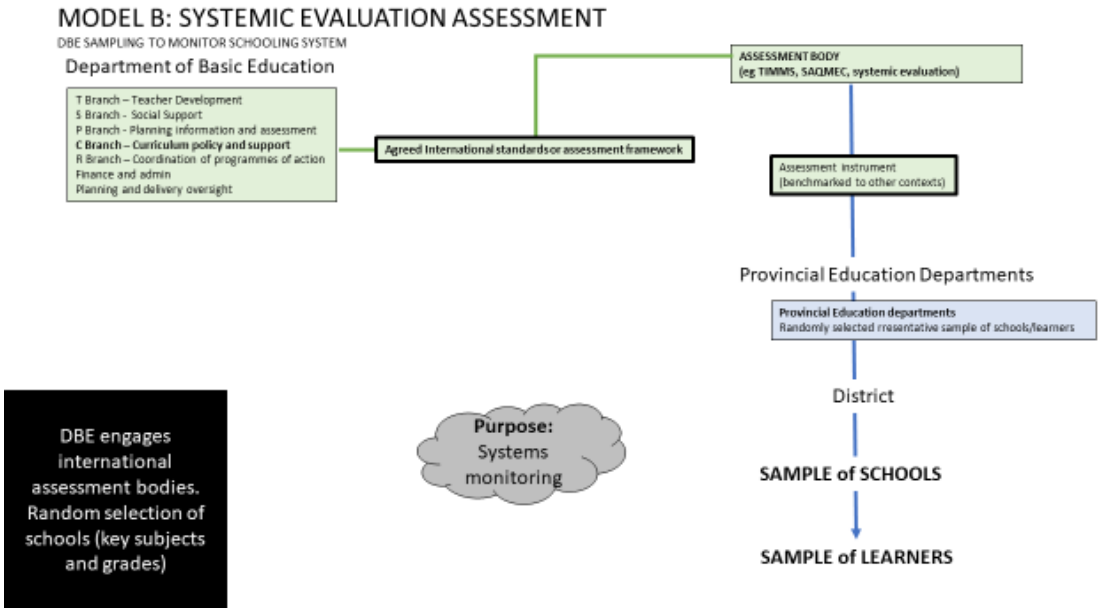
MODEL A: Assessment for a qualification (NSC & GETC)

This model is designed to deliver examinations that are intended to lead to acquisition of qualifications. The National Senior Certificate (NSC) is a classic example here and has been running with remarkable success for years. Extension of the processes to the envisaged GETC will should be relatively easy.



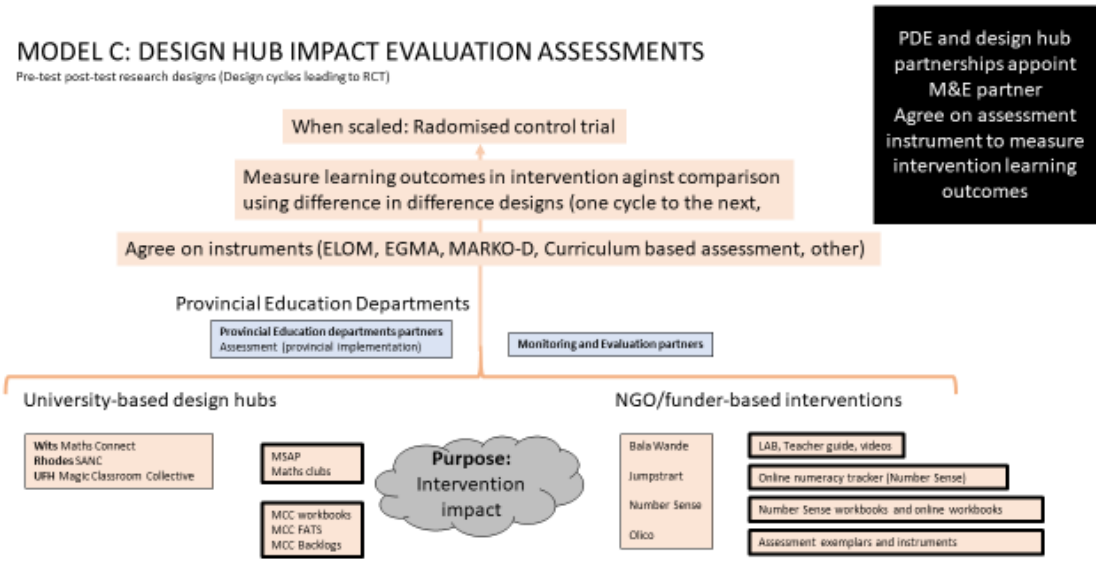
MODEL B: Systemic Evaluation Assessment

Model B is proposed for the development, management and administration of large-scale assessments whose purpose is to monitor the performance of the education system. The DBE Branch responsible for this function should draw on both international and national expertise to conceptualise the assessments, develop assessment frameworks and assessment suitable instruments for the selected grades.



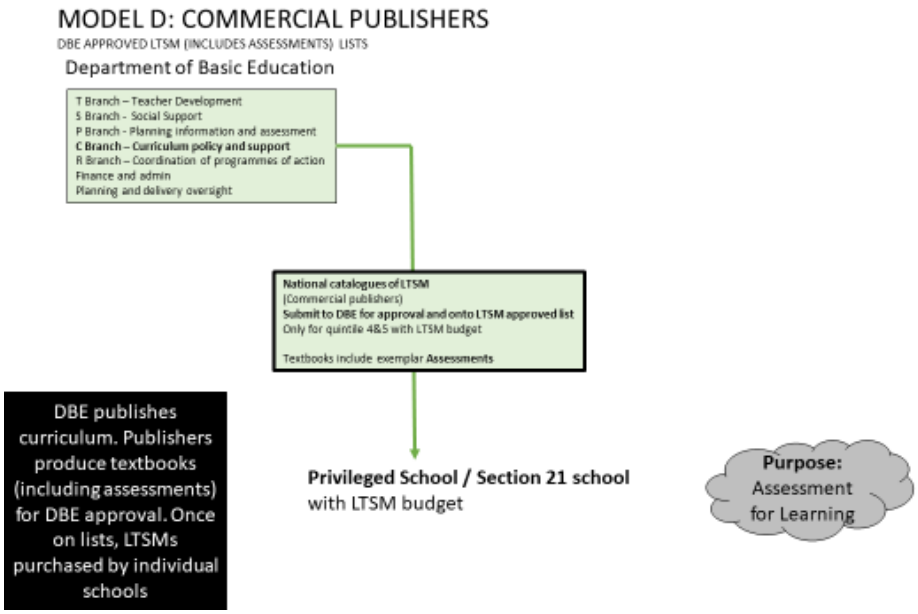
MODEL C: Design Intervention or Impact Evaluation Assessments

Model C is specifically proposed to harness local expertise and encourage innovation and collaboration among the different players in the terrain of assessment in early grade mathematics and also in other selected subjects. The mission in this model is to design research-based evaluations using some of the assessment instruments discussed in this analysis and others to evaluate the impact of different interventions. Provincial Departments of Education (PDEs) should feature prominently in these interventions to ensure a sustainable synergy with schools.



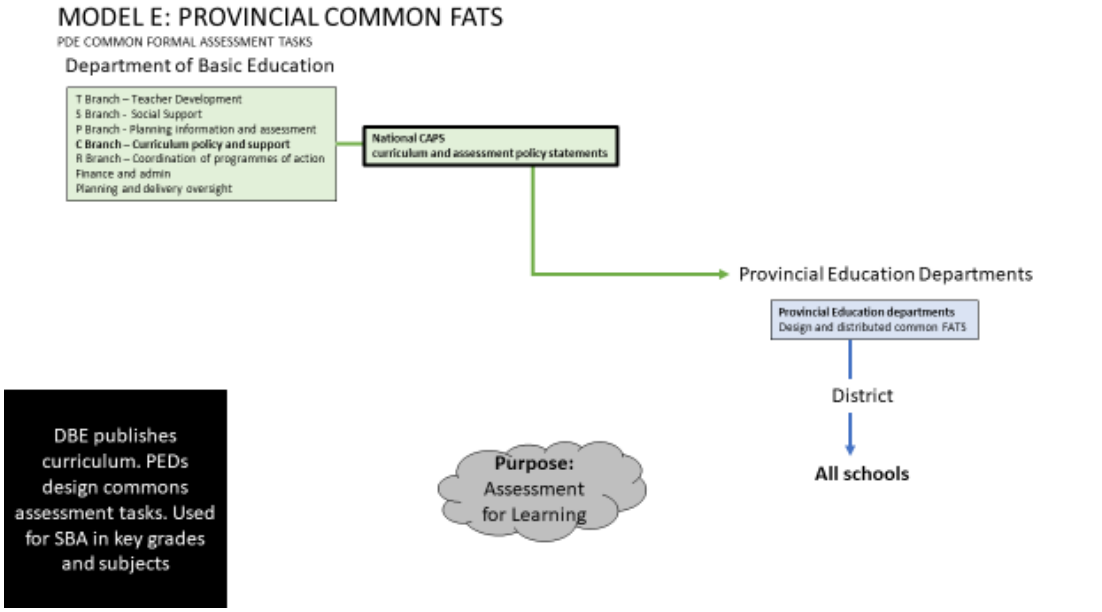
MODEL D: Commercial Publishers

Commercial publishers are a valuable role-player in the development and distribution of assessment instruments. While it's the responsibility of the DBE to design and develop curriculum materials (including assessments), The DBE should then encourage publishers to print materials which will be placed on catalogues for schools that can procure on their own (Section 21) to select and procure materials that they require. This applies to LTSM but will also apply to assessment-related resources and materials.



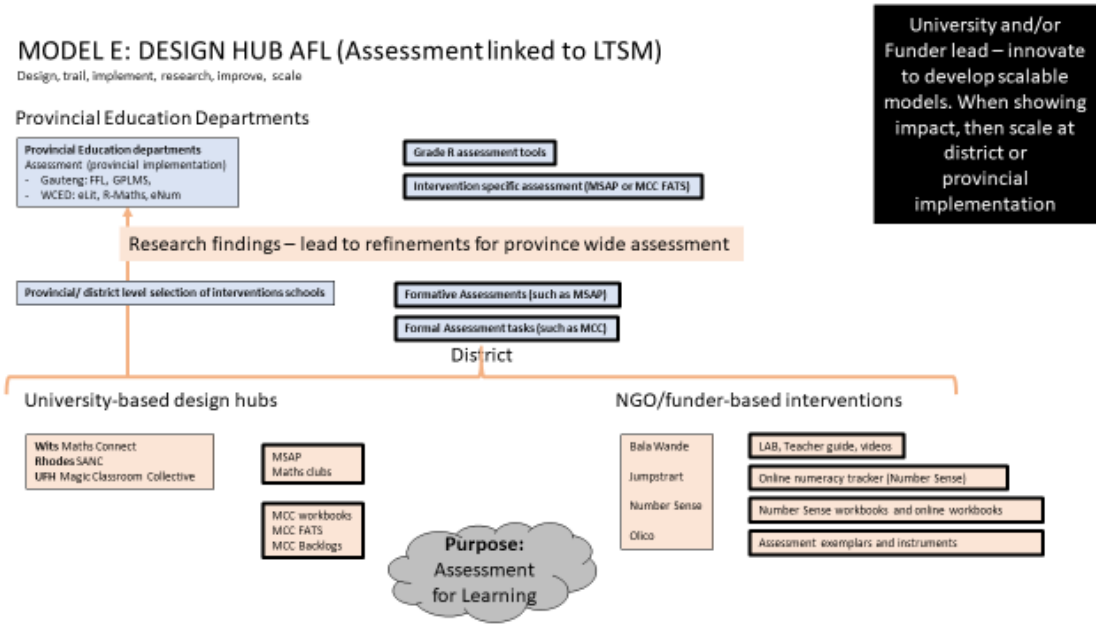
MODEL E: Provincial Common Formal Assessment Tasks

Using curriculum documents published by DBE (e.g. CAPS), PEDs should develop common Formal Assessment Tasks (FATs) to model quality assessments in schools. The purpose here is to equip schools with assessment materials that enable them to promote AfL strategies to improve performance in mathematics.



MODEL F: Design Intervention Assessment for learning (linked to LTSM)

The special role played by the use of English as the First Additional Language (AfL) in the majority of the schools in South Africa calls for a focused design intervention around the development and use of appropriate assessment instruments for the promotion of AfL.



Annexure 5: Experts and Steering Committee Members

Many thanks to the experts and steering committee members who contributed to this process: